

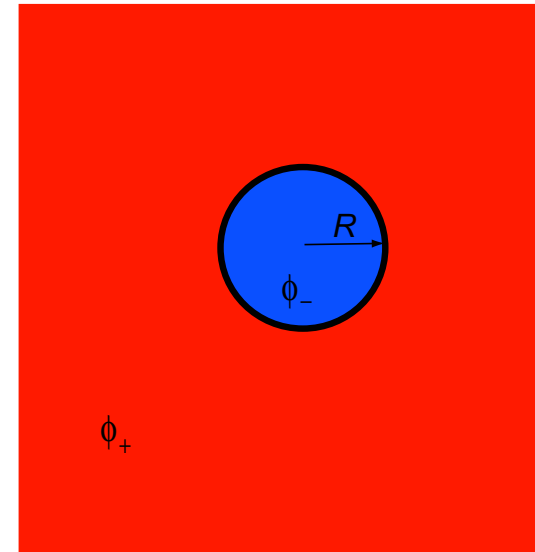
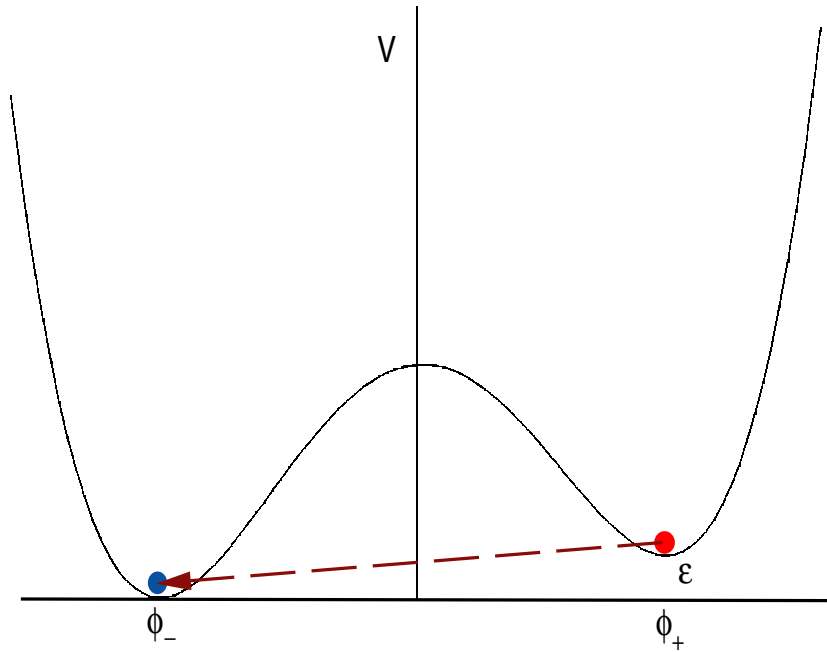
Particle decay in false vacuum

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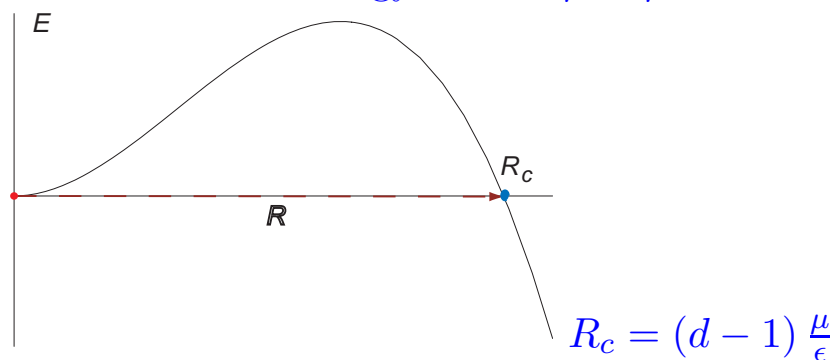
- Introduction. False vacuum.



d space-time dimensions:

Gain in volume energy: $-\text{Volume} \times \epsilon \propto \epsilon R^{d-1}$

Loss in surface energy: $\text{Area} \times \mu \propto \mu R^{d-2}$



The rate of critical bubble formation (per unit time \times volume) $w_0 \sim \exp(-\text{Action})$.

- Euclidean-space calculation

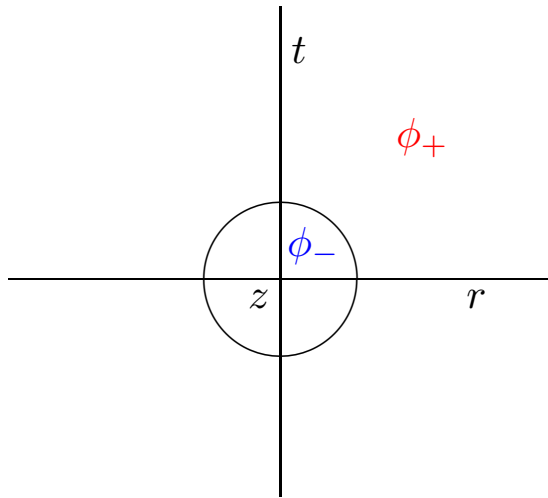
Decay rate = imaginary part ($\times(-2)$) of the false vacuum energy. The path integral

$$Z = \mathcal{N} \int e^{-S[\phi, \dots]} \mathcal{D}\phi \dots = \exp(-E_{\text{vac}}T)$$

$$\Rightarrow w_0 = 2 \text{Im}(\ln Z)/VT.$$

- Bounce

Stationary configuration: $O(d)$ -symmetrical solution of field equations with $\phi \rightarrow \phi_+$ at $r \rightarrow \infty$ and $\phi \approx \phi_-$ at $r = 0$.



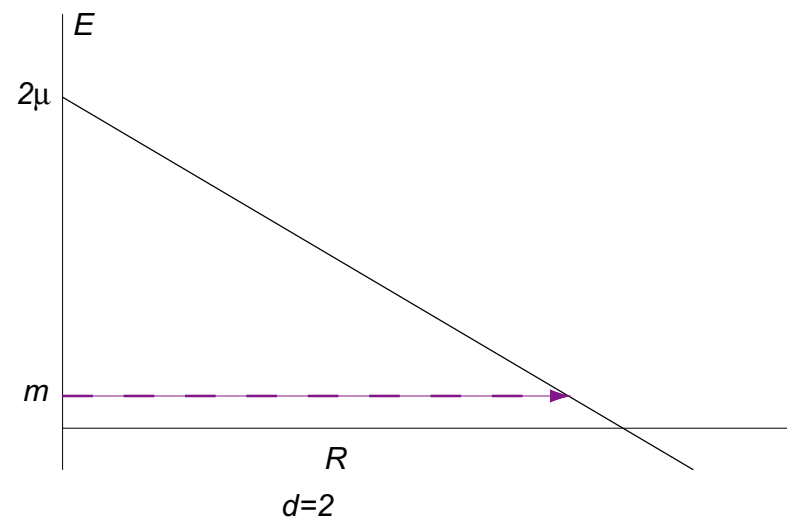
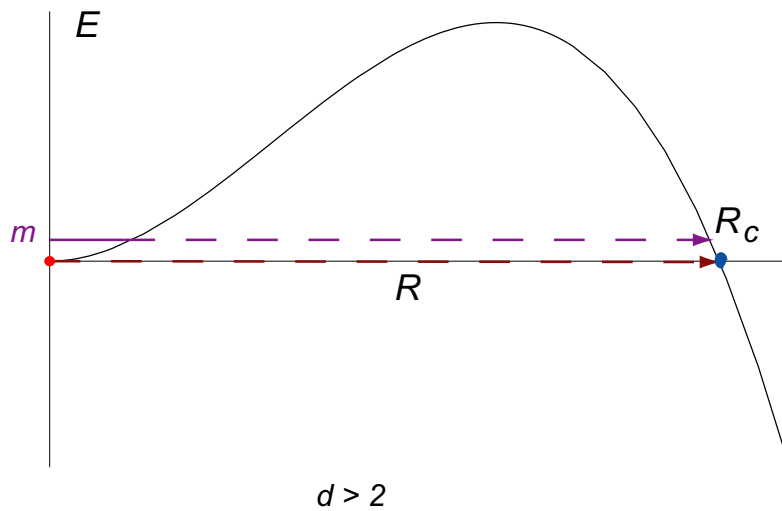
$$w_0 d^d z = \left| \frac{\det'(S^{(2)})}{\det(S_0^{(2)})} \right|^{-1/2} \exp(-S_B) \left(\frac{S_B}{2\pi} \right)^{d/2} d^d z$$

$d = 2$: $w_0 = \frac{\epsilon}{2\pi} \exp(-\frac{\pi\mu^2}{\epsilon})$. $d = 3, 4$: partial results.

- Catalysis by presence of a particle. (Particle decay \rightarrow true vacuum.)

Energy transfer when the particle field has zero mode on the wall. (Boson of the master field, or fermion.)

Initial state $\delta E = m$ (the particle mass). Final state: $\delta E = 0$ (particle 'rides' as a bound state on the bubble wall).



Decay rate: $\Gamma = K w_0$. K - catalysis factor.

For $mR \ll S_B$, but still $mR \gg 1$: $K = C \cdot \exp(2mR)$.

Calculation of C is the subject.

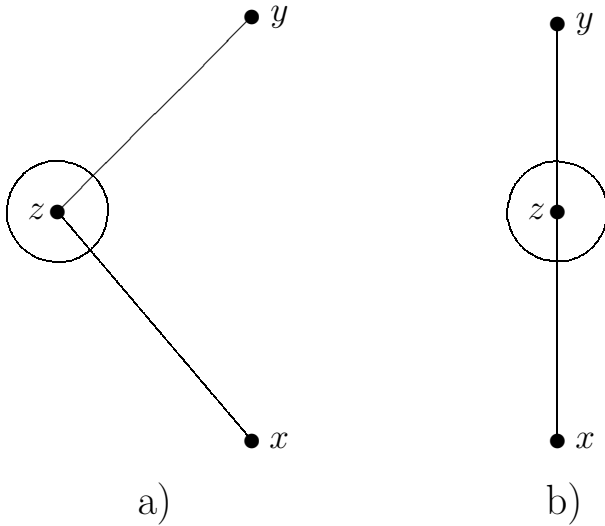
- $mR \ll S_B$ - arbitrary d . Distorsion of the bounce can be neglected.
- $m \approx 2\mu$ in $d = 2$.

- Boson

Consider the particle propagator in the ϕ_+ vacuum ($\sigma(x) = \phi(x) - \phi_+$):

$$D(x, y) = \frac{1}{Z} \int \sigma(x)\sigma(y) e^{-S[\phi, \dots]} \mathcal{D}\phi \dots$$

Bounce contribution at $L = |x - y| \gg R$



$$\delta D(x, y) = \frac{i}{2} w_0 \int d^d z F(x - z, y - z) D_0(x - z) D_0(y - z) ,$$

$D_0(x)$ - free propagator. Use saddle point in the dz_{\perp} integration. Then at $L \gg R$

$$\delta D(x, y) = \frac{i}{2} w_0 F_0 \int d^d z D_0(x - z) D_0(y - z) ,$$

with F_0 given by the alignment in Figure b). Compare with effect of $m^2 \rightarrow m^2 + \delta m^2$:

$$\delta_m D(x, y) = -\delta m^2 \int d^d z D_0(x - z) D_0(y - z)$$

- $\delta m^2 = -(i/2) w_0 F_0$, which corresponds to the particle decay rate $\Gamma = F_0 w_0 / (2m) \Rightarrow$ the catalysis factor K is found as

$$K = \frac{F_0}{2m}$$

For the bosons of the master field F_0 is found from asymptotic form of (classical) bounce field profile $\phi(r) - \phi_+ \rightarrow -2v \exp[-m(r - R)] \rightarrow C D_0(r)$.

$$D_0(r) = \frac{m^{d/2-1}}{(2\pi)^{d/2} r^{d/2-1}} K_{\frac{d}{2}-1}(mr)$$

$$\Rightarrow C = -4 (2\pi)^{d/2-1} m^{(3-d)/2} R^{(d-1)/2} v e^{mR}$$

$$F_0 = C^2 \Rightarrow$$

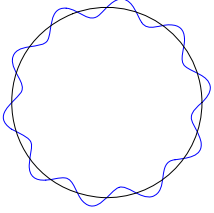
$$K = 2^{d+1} \pi^{(d-3)/2} \Gamma\left(\frac{d+1}{2}\right) m^{2-d} v^2 V_{d-1} e^{2mR}$$

$V_{d-1} = \pi^{(d-1)/2} R^{d-1} / \Gamma[(d+1)/2]$ is the spatial ($d-1$ dimensional) volume of the critical bubble.

Enhancement of K : classical $\exp(2mR)$ and an extra factor $m^{2-d} v^2$ - inverse of the (small) coupling constant for ϕ .

- Infrared complication in $d = 2$

Soft modes: distortion of the shape of the bounce. Eigenvalues $\lambda_n = c_n/R^2$.



$d = 2$ normalized eigenmode ($\mu \sim mv^2$):

$$\sigma_n(r) \sim \frac{m v}{\sqrt{\mu R}} e^{-m(r-R)} \sim \sqrt{\frac{m}{R}} e^{-m(r-R)}$$

\Rightarrow mode contribution to $\delta D(r_1, r_2)$:

$$\frac{R^2}{c_n} \sigma_n(r_1) \sigma_n(r_2) \sim c_n^{-1} (m R) e^{2mR} e^{-m(r_1+r_2)}$$

Classical part: $\sim v^2 e^{2mR} e^{-m(r_1+r_2)} \Rightarrow$

$$\frac{\text{Mode contrib.}}{\text{Classical}} \sim \frac{m R}{v^2}$$

Although, as expected, suppressed by the small coupling $1/v^2$, is infrared unstable at large R .

- Solution

Effective action for the soft modes (polar-coordinate parametrization of the bounce shape, $(r(\theta), \theta)$)

$$S = \int_0^{2\pi} \left(\mu \sqrt{r^2 + \dot{r}^2} - \frac{1}{2} \epsilon r^2 \right) d\theta = \frac{\pi \mu^2}{\epsilon} + \int_0^{2\pi} \frac{\epsilon}{2} (\dot{\rho}^2 - \rho^2) d\theta + O(\rho^4)$$

$R = \mu/\epsilon$, $\rho(\theta) = r(\theta) - R$, $\dot{\rho} = d\rho/d\theta$.

Modes: $\lambda_n \propto (n^2 - 1)$.

One negative mode: $\rho_0 = 1/\sqrt{2\pi}$.

Two types of $n > 0$ modes:

$$\rho_n^{(1)} = \frac{1}{\sqrt{\pi}} \cos n\theta, \quad \text{and} \quad \rho_n^{(2)} = \frac{1}{\sqrt{\pi}} \sin n\theta; \quad (n = 1, 2, \dots)$$

Only the fluctuations of the vertical size of the bounce contribute to $\delta D(x, y)$:

$$\langle [\rho(0) + \rho(\pi)]^2 \rangle \propto \sum_n \frac{[\rho_n(0) + \rho_n(\pi)]^2}{n^2 - 1}$$

The sum $\rho(0) + \rho(\pi)$ is not vanishing only for the negative mode and for the positive modes of the first type, $\rho_n^{(1)}$, with even n , i.e. $n = 2k$. Thus

$$\langle [\rho(0) + \rho(\pi)]^2 \rangle \propto -\frac{1}{2} + \sum_{k=1}^{\infty} \frac{1}{4k^2 - 1} = 0$$

The theory cures itself from the infrared problem by cancellation between one negative and the sum over the positive modes.

- Fermionic case. $d = 2$.

If a fermion field is present in the theory (no actual fermions in the false vacuum), such that the fermion mass $m(\phi)$ changes sign between ϕ_+ and ϕ_- , the fermion has a zero mode on the bubble wall and affects the spontaneous decay rate of the false vacuum: in $d = 2$ it makes $w_0 \rightarrow 2 w_0$. Two (degenerate) final states: $F(\text{kink}, \text{antikink}) = (+1/2, -1/2)$ and $F(\text{kink}, \text{antikink}) = (-1/2, +1/2)$.

- Fermion present in the false vacuum.

Consider the bounce contribution to the fermion Green function $G(x, y) = \langle \psi(x) \bar{\psi}(y) \rangle$.

Fermion zero mode, $[\sigma_i \partial_i + m(\phi)] \psi_0 = 0$

$$\psi_0(r, \theta) = C_f \sqrt{\frac{R}{r}} \exp \left\{ - \int_R^r m[\phi(r')] dr' \right\} \chi(\ell) \begin{pmatrix} e^{-i\theta/2} \\ e^{i\theta/2} \end{pmatrix}$$

$\chi(\ell)$ is a one-dimensional fermion field living on the bounce boundary and (nominally) depending on the length parameter $\ell = R\theta$ along the boundary. Classical equation for χ : $\dot{\chi} = 0$. C_f is the normalization factor:

$$2 C_f^2 \int \exp \left[-2 \int_R^r m(\phi) dr' \right] dr = 1 .$$

Switch to \tilde{C}_f :

$$C_f \exp \left\{ - \int_R^r m[\phi(r')] dr' \right\} \rightarrow \tilde{C}_f \exp [m_f(R - r)]$$

Generally

$$\tilde{C}_f^2 = \frac{m_f}{2} f \left(\frac{m_f}{m} \right)$$

f - dimensionless function of m_f/m with m standing for other mass parameters in the false vacuum.

For $m_f/m \rightarrow 0$, $f(m_f/m) \rightarrow 1$. In a ϕ^4 theory with $m =$ the boson mass

$$f(u) = \frac{2^{2u}}{\sqrt{\pi}} \frac{\Gamma(u + 1/2)}{\Gamma(u + 1)}$$

Contribution of the zero mode to $G(x, y)$:

$$\delta G(x, y) = -\frac{i}{2} \frac{w_0}{2} d^2 z \tilde{C}_f^2 e^{2m_f R} R \frac{e^{-|x-y|}}{\sqrt{|x-z||y-z|}} (1 + \sigma_1) g(0, \pi R)$$

with $g(\ell_1, \ell_2) = \langle \chi(\ell_1) \chi^\dagger(\ell_2) \rangle$ the propagator of χ .

$$g(\ell_1, \ell_2) = (1/2) \text{sign}(\ell_1 - \ell_2) \Rightarrow g(0, \pi R) = -1/2$$

Compare with $m_f \rightarrow m_f + \delta m_f$:

$$\delta_m G(x, y) = -\delta m_f d^2 z G_0(x-z) G_0(z-y) \rightarrow -\delta m_f d^2 z \frac{m}{4\pi} (1 + \sigma_1) \frac{e^{-|x-y|}}{\sqrt{|x-z||y-z|}}$$

$$G_0(x, y) = \frac{1}{2\pi} (-\sigma_i \partial_i + m) K_0(m_f |x-y|) \Rightarrow$$

$$\Gamma_f = \frac{\pi}{2} f\left(\frac{m_f}{m}\right) R w_0 \exp(2m_f R) = \frac{\mu}{2} f\left(\frac{m_f}{m}\right) \exp\left(-\frac{\pi \mu^2}{\epsilon} + 2m_f R\right)$$

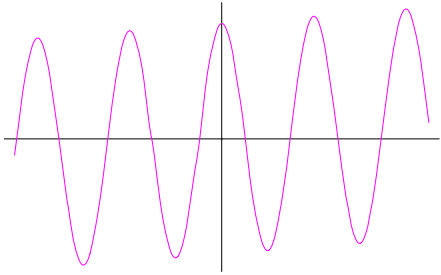
$$w_0 = (\epsilon/\pi) \exp(-\pi \mu^2/\epsilon) \text{ and } R = \mu/\epsilon$$

For the fermion indeed $K_f \sim R \exp(2m_f R)$.

Meson decay in sine-Gordon model

- Weak coupling

$$L_{SG} = \frac{1}{2} (\partial\phi)^2 + \frac{\alpha}{\beta^2} \cos(\beta\phi) + (\epsilon\beta/2\pi) \phi$$



The catalysis factor for a boson is

$$K = \frac{32}{\beta^2} \frac{\mu}{\epsilon} e^{2m_b\mu/\epsilon}$$

m_b = boson mass.

- Strong coupling

Equivalent: Thirring model in external electric field

$$L_{Th} = i\bar{\psi}\partial_\nu\gamma^\nu\psi - \frac{1}{2} g j^\nu j_\nu + \mu\bar{\psi}\psi + A_0 j_0$$

$\frac{\beta^2}{4\pi} = (1 + \frac{g}{\pi})^{-1}$, $j_\nu = \bar{\psi}\gamma_\nu\psi$, μ = soliton mass in the sine-Gordon model, $\partial_x A_0 = \epsilon$.

Small g : the SG boson is a shallow bound state of fermion-antifermion, $m_b = 2\mu - \mu g^2$.

The near-threshold dynamics of the fermion-antifermion (soliton-antisoliton) pair can be described by the nonrelativistic Hamiltonian

$$H = \frac{p^2}{\mu} - \epsilon x - 2g\delta(x)$$

Boson-induced vacuum decay = ionization of the bound state in $U(x) = -2g\delta(x)$ by ext. electric field ϵ .

Equation for the Green function ($E = -\kappa^2/\mu$):

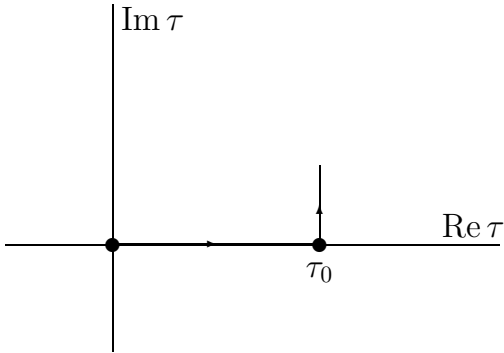
$$G(0, 0; -\kappa^2/\mu) = \frac{G_\epsilon(0, 0; -\kappa^2/\mu)}{1 - 2g G_\epsilon(0, 0; -\kappa^2/\mu)}$$

with $G_\epsilon(x, y; E)$ the Green function in linear potential ($-\epsilon x$)

$$G_\epsilon\left(0, 0; -\frac{\kappa^2}{\mu}\right) = \int_0^\infty \sqrt{\frac{\mu}{4\pi\tau}} \exp\left(\frac{\epsilon^2}{12\mu}\tau^3 - \frac{\kappa^2}{\mu}\tau\right) d\tau$$

The pole (in κ) is determined by

$$2g G_\epsilon\left(0, 0; -\frac{\kappa^2}{\mu}\right) = 1$$



$\tau_0 = 2\kappa/\epsilon$. The decay rate (due to ionization):

$$\Gamma = 2\mu g^2 \exp\left(-\frac{4}{3}g^3 \frac{\mu^2}{\epsilon}\right)$$

Summary

- Boson of the master field, arbitrary d :

$$K = 2^{d+1} \pi^{(d-3)/2} \Gamma\left(\frac{d+1}{2}\right) m^{2-d} v^2 V_{d-1}(R) e^{2mR}$$

- No infrared complication in $d = 2$: the IR cancels between one negative and the sum over the positive modes.
- Fermion in $d = 2$

$$\Gamma_f = \frac{\pi}{2} f\left(\frac{m_f}{m}\right) R w_0 \exp(2m_f R) = \frac{\mu}{2} f\left(\frac{m_f}{m}\right) \exp\left(-\frac{\pi \mu^2}{\epsilon} + 2m_f R\right)$$

$w_0 = (\epsilon/\pi) \exp(-\pi \mu^2/\epsilon)$ and $R = \mu/\epsilon$

- $d = 2$ Sine-Gordon staircase

Weak coupling ($m_b \ll \mu$)

$$K = \frac{32}{\beta^2} \frac{\mu}{\epsilon} e^{2m_b \mu/\epsilon}$$

Strong coupling in SG (weak coupling g in Thirring)

$$\Gamma = 2\mu g^2 \exp\left(-\frac{4}{3} g^3 \frac{\mu^2}{\epsilon}\right)$$