# Towards Reggeon Field Theory in QCD

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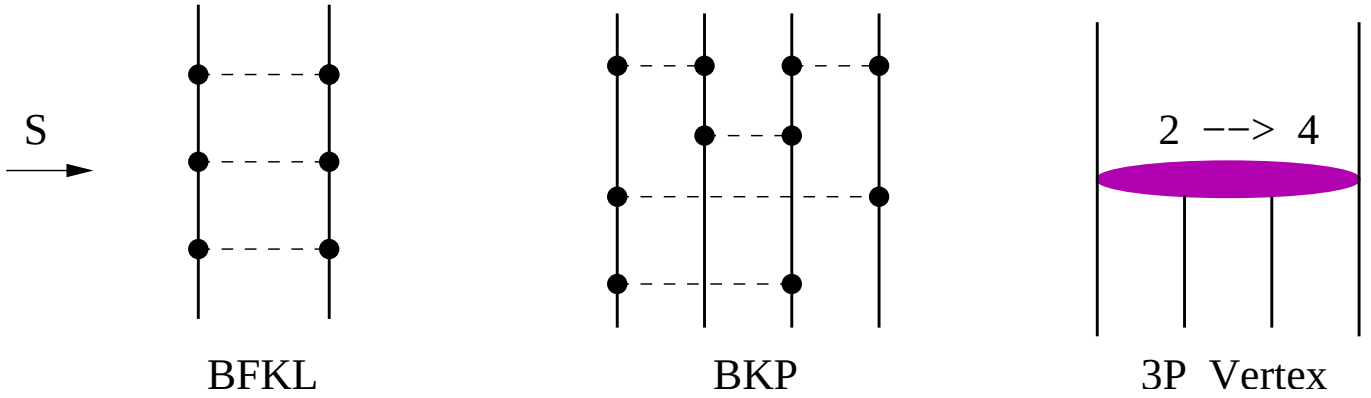
# RFT - History

60‘ V. Gribov: RFT with supercritical bare Pomeron $s^{\Delta}$, $\Delta > 0$.

QCD($s \to \infty$) $\longrightarrow$ RFT $\longrightarrow$ Confinement

$\longrightarrow$ String Theory

70‘ BFKL (Balitsky, Fadin, Kuraev, Lipatov) ladder - Hard Pomeron, $s^{c\,\alpha_s}$




BFKL BKP 3P Vertex

80‘ BKP (Bartels, Kwiecinski, Praszalowicz), GLR (L. Gribov, Levin, Ryskin)

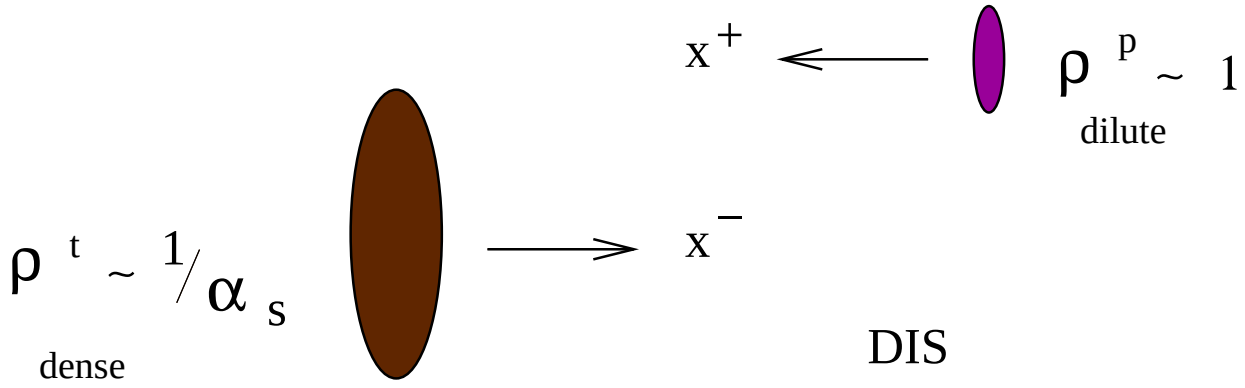
90‘ 3P Vertex (Bartels, Wusthoff, M. Braun), Lipatov‘s action, Mueller‘s dipole model,

B-JIMWLK (Balitsky, Jalilian Marian, Iancu, McLerran, Leonidov, Kovner).

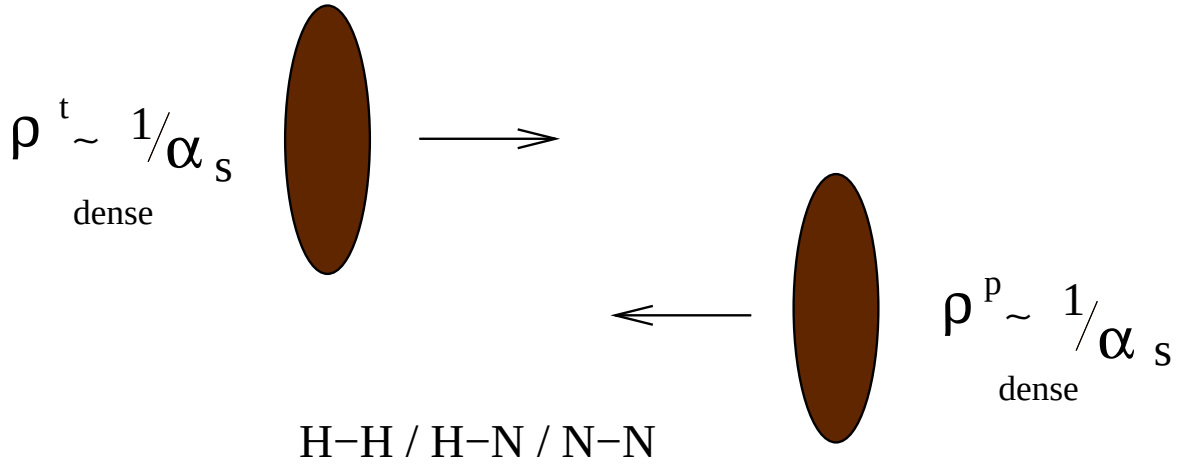
2005 JIMWLK+, KLWMIJ+, Dense-Dilute Duality (DDD), Self-Duality of RFT.

# High Energy Scattering

HERA - eRHIC




LHC - RHIC -TeVatron



Target ($\rho^t = \rho^+;\ k^+ > \Lambda$) Projectile ($\rho^p = \rho^-;\ k^- > \Lambda$)

$$\langle T| \quad \rightarrow \qquad\qquad \leftarrow \quad |P\rangle$$

$S$-matrix:

$$S(Y) \;=\; \langle T\, \langle\, P|\ \hat{s}(\rho^t,\, \rho^p)\ |P\rangle\, T\rangle$$

Projectile averaged $s$-matrix:

$$\Sigma^p_{Y\,-\,Y_0}(\rho^t) \;=\; \langle\, P|\ \hat{s}(\rho^t,\, \rho^p)\ |P\rangle \;=\; \int D\rho^p\ \ \hat{s}(\rho^t,\, \rho^p)\ \ W^p_{Y\,-\,Y_0}[\rho^p]$$

$$S(Y) \;=\; \int D\rho^t\ \ \Sigma^p_{Y\,-\,Y_0}[\rho^t]\ \ W^t_{Y_0}[\rho^t]$$

Boosting projectile $|P\rangle_Y \rightarrow |P\rangle_{Y+\delta Y}$ (QCD)

$$\frac{d\Sigma^p}{dY} = H_\Sigma^{HE} \; \Sigma^p$$

Boosting target $|T\rangle_Y \rightarrow |T\rangle_{Y+\delta Y}$ (QCD)

$$\frac{dW^t}{dY} = H_W^{HE} \; W^t$$

Lorentz invariance: $dS/dY_0 = 0 \rightarrow H_\Sigma^{HE} = H_W^{HE}$

$S$-matrix evolution: $H^{RFT} \rightarrow$ the RFT Hamiltonian

$$\frac{dS}{dY} = \int D\rho^t \; \Sigma^p_{Y-Y_0}[\rho^t] \; H^{RFT}[\rho^t, \delta/\delta\rho^t] \; W^t_{Y_0}[\rho^t]$$

# Dense/Dilute limit

$$H^{KLWMIJ} = H^{RFT}(\rho \to 0)\,; \qquad H^{JIMWLK} = H^{RFT}(\rho \to \infty)$$

$JIMWLK$ - Jalilian Marian, Iancu, McLerran, Leonidov, Kovner (1997-2002)

$KLWMIJ$ - A. Kovner and M.L., Phys.Rev.D71:085004, 2005

Evolution with Pomeron Loops (model):

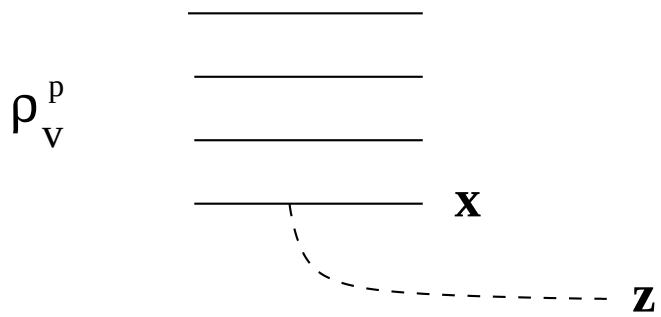
$$H^{RFT} \simeq H^{JIMWLK}(\rho \to \infty) \;'' + '' \; H^{KLWMIJ}(\rho \to 0)$$

# KLWMIJ Hamiltonian (dilute limit)

$$\langle \mathcal{O} \rangle_Y \equiv \langle P | \hat{\mathcal{O}}[\rho^p] | P \rangle_Y = \int D\rho^p \; \mathcal{O}[\rho^p] \; W_Y^p[\rho^p]$$

$$\frac{d \langle \mathcal{O} \rangle_Y}{d Y} = \int D\rho^p \; \mathcal{O}[\rho^p] \; H^{KLWMIJ} \; W_Y^p[\rho^p]$$

Boosting projectile $|P\rangle_Y \rightarrow |P\rangle_{Y+\delta Y}$




Linear evolution means $\delta\rho \propto \rho_v^p$

Emission amplitude is given by the Weizsaker-Williams field

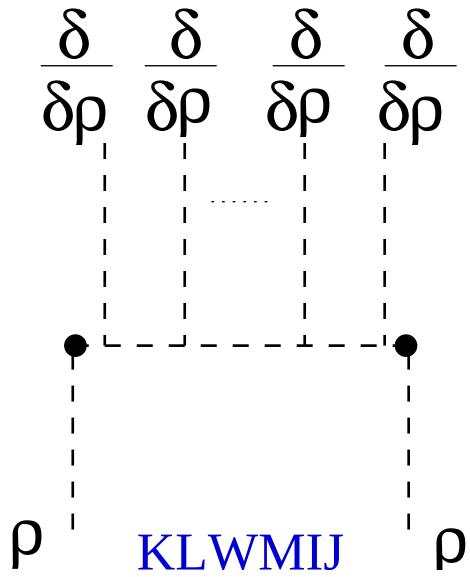
$$b_i^a(z) = \frac{g}{2\,\pi} \int d^2x \, \frac{(x - z)_i}{(x - z)^2} \, \rho^{p\,a}(x)$$

$$\rho^p \;\longrightarrow\; \rho^p \;+\; T^a\,; \qquad\qquad \hat{\mathcal{O}}[\rho^p] \;\longrightarrow\; \hat{\mathcal{O}}[\rho^p \;+\; T^a]$$

Dual Wilson line

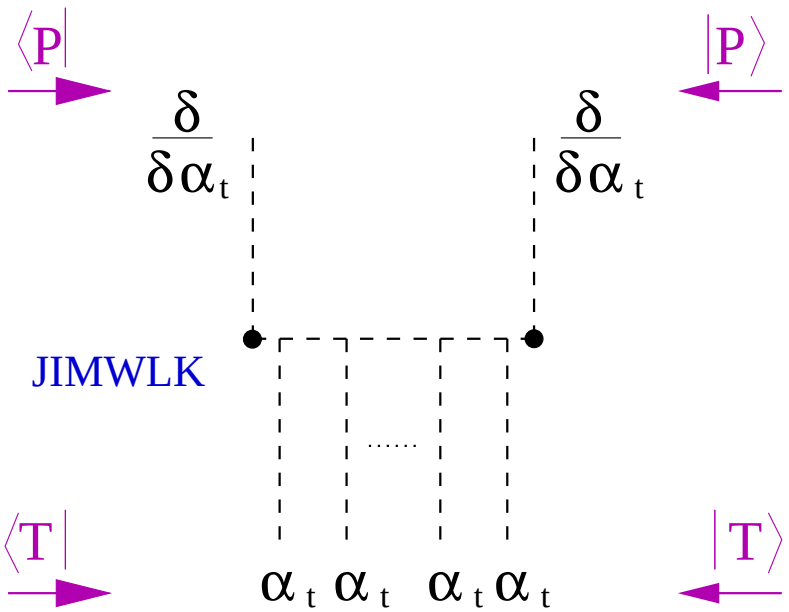
$$R(z) \;=\; P\,exp\left\{\int dx^-\,T^a\,\frac{\delta}{\delta\rho^a(z,\,x^-)}\right\} \qquad\qquad R(z)\,\rho^a(z) \;=\; \rho^a(z) + T^a$$

$$H^{KLWMIJ}\,[\,\rho,\,\delta/\delta\rho\,] \;=\; \int_z\; b_i^a(z)\,[1 \;-\; R(z)]^{ab}\;\; b_i^b(z)$$



# JIMWLK Hamiltonian (dense limit)

$$H^{JIMWLK}\left[\alpha^t,\ \frac{\delta}{\delta\alpha^t}\right] \;=\; \int_z b_i\left[\frac{\delta}{\delta\alpha^t}\right]\ [1\ -\ S(z)]\ b_i\left[\frac{\delta}{\delta\alpha^t}\right]$$




Eikonal scattering matrix for projectile's gluon

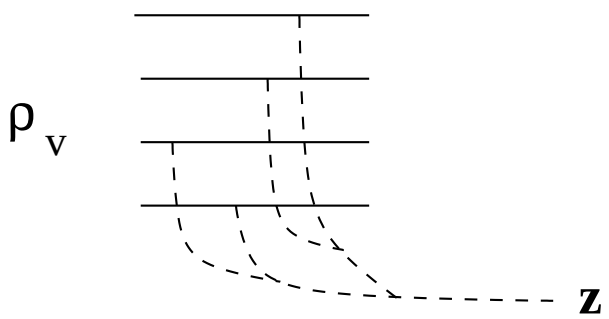
$$S^{ab}(z) \;=\; P\,exp\left\{i\int dx^-\,\alpha^t(z,\,x^-)\right\}^{ab}$$

The field $\alpha^t \equiv A^+$ is a (strong) classical external field created by the target

$$\Delta\,\alpha^t(z)\ ''=''\ \ \rho^t(z) \qquad\qquad (YMe\ \ LC)$$

# Beyond JIMWLK: $JIMWLK+$

A. Kovner and M.L., JHEP 0503:001,2005

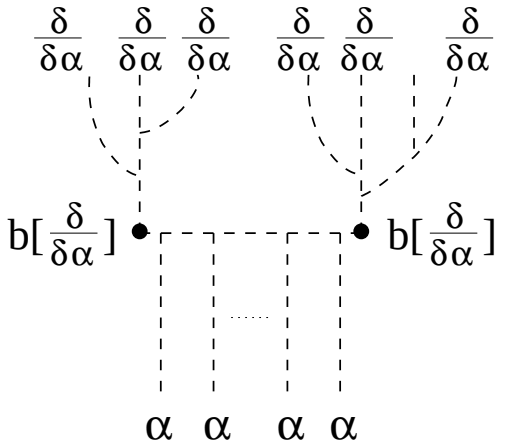



Coherent emission of a single gluon

$D_i[b]\, b_i^a \;=\; \rho_p^a$

$b$ is non-linear in $\rho^p \;=\; \rho_v \;\geq\; 1$

$$H^{JIMWLK+} \;=\; \int_z b_i \;\left[1 \;-\; S(z)\right]\; b_i$$




Y. Hatta, E. Iancu, L. McLerran, A. Stasto and D.N. Triantafyllopoulos, Nucl.Phys. A764: 423,2006;
I. Balitsky, Phys.Rev. D72:074027,2005

# DDD - Dense Dilute Duality

$$H^{JIMWLK}\,(\rho \to \infty) \;=\; \alpha_s \int_{x,y,z} \frac{(z-x)_i (z-y)_i}{(z-x)^2 (z-y)^2} \;\; \frac{\delta}{\delta \alpha^a(x)} \, [1 \,-\, S(z)]^{ab} \, \frac{\delta}{\delta \alpha^b(y)}$$

$$H^{KLWMIJ}\,(\rho \to 0) \;=\; \alpha_s \int_{x,y,z} \frac{(z-x)_i (z-y)_i}{(z-x)^2 (z-y)^2} \;\; \rho^a(x) \, [1 \,-\, R(z)]^{ab} \, \rho^b(y)$$

DDD transformation:

$$i\,\alpha \;\to\; \frac{\delta}{\delta\,\rho}\,; \qquad\qquad \frac{\delta}{\delta \alpha} \;\to\; i\,\rho \qquad\qquad S \;\to\; R$$

$$\boxed{H^{JIMWLK} \;\leftrightarrow\; H^{KLWMIJ}}$$

# Self-Duality of High Energy Evolution

A. Kovner and M.L. (PRL 94:181603,2005)

- Lorentz Invariance (LI)
- Eikonal Approximation (EA)
- Projectile - Target Democracy (PTD)

$$H^{RFT}(i\,\alpha\ ,\delta/\delta\,\alpha) \;=\; H^{RFT}(\delta/\delta\,\rho,\ i\,\rho)$$

Self-Duality $=$ $t$-channel unitarity?

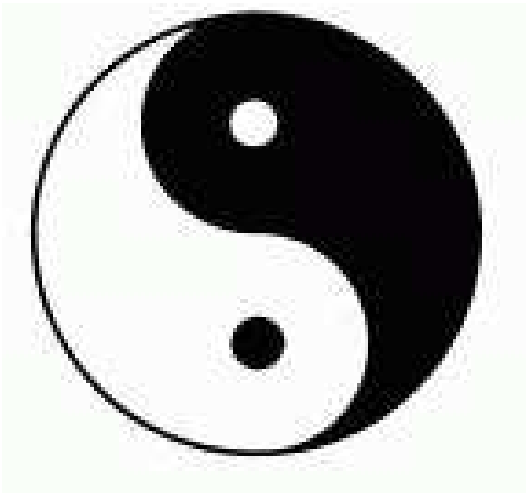
Harmonic oscillator

$E \leftrightarrow B$

# Reggeon Field Theory in QCD

A. Kovner and M.L. (hep-ph/0512316, hep-ph/0604085)

- Hamiltonian (2+1) dimensional interacting non-local field theory.
- The basic "quantum Reggeon field" is the unitary matrix $R$ ($S$).
  - Symmetry: DDD
  - Two zero energy degenerate vacua ("Yang" and "Yin"), DDD is spontaneously broken.
  - Spectrum of excitations is twice degenerate (gluons and "holes")



- More symmetries: $SU_V(N)$, $Z_2$; 2-d Conformal invariance?
- BFKL Pomeron is a tachyon

# Summary and Outlook

- Some progress has been achieved in understanding high energy limit of QCD:
  JIMWLK+, KLWMIJ+
  DDD, Self-Duality

- We are in a quest for a complete QCD - RFT.
  - Self-Duality
  - $s$- and $t$- channel unitarity
  - Symmetries? Conformal Symmetry? Integrability?

- Phenomenology (LHC, HERA, RHIC, TeVatron)