$B \to K^*\gamma$ **vs** $B \to \rho\gamma$

Roman Zwicky

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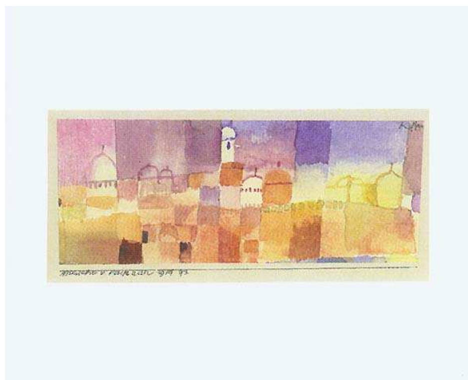

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   $b \to (s,d)\gamma$ famous FCNC
2. Exclusive $B \to V\gamma$ from QCD-factorization
   2.1 Focus "cleaner" observable

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3. ξ from LCSR - SU(3)-effect, focus ...
   3.1 Kaon DA, Gegenbauer moment a_1 - turbulent recent history
   3.2 Point large uncertainties due to f_V^T

$$T_1^V \sim f_V^\perp (1 + c_1 a_1^\perp) + \ldots \qquad \text{with} \quad c_1 \sim O(1)$$

   3.3 Brief remarks $1/m_b$-corrections
   3.4 Results

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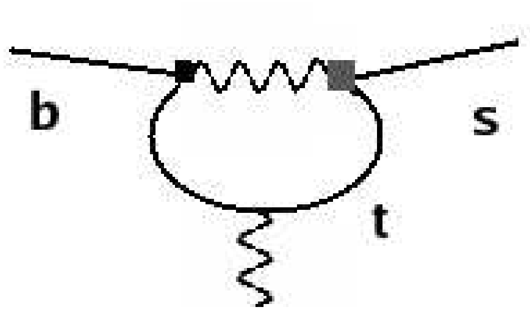
1. Overview $b \to (d,s)\gamma$

- Famous FCNC processes ... new physics ?



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$$\lambda_u^d + \lambda_c^d + \lambda_t^d = 0 \quad \mathrm{UT}$$

$$\mathcal{H}_{\rm eff}^{(d,s)} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \underbrace{\lambda_p^{(d,s)}}_{\rm CKM} \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,\dots,8} C_i Q_i \right] + \mathrm{BSM}$$

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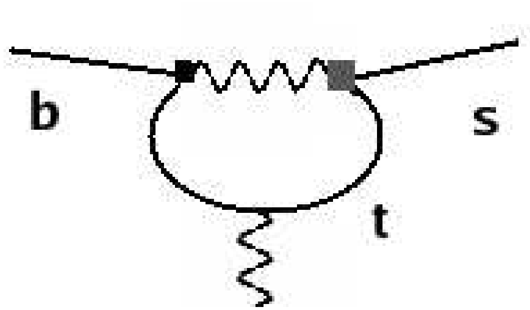

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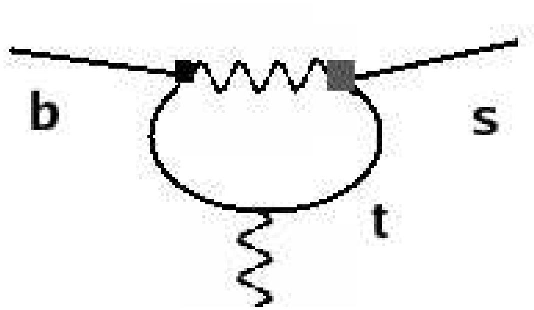

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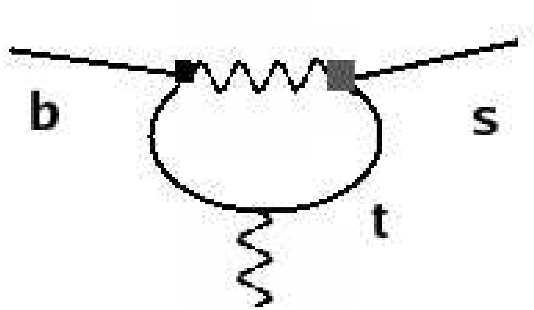

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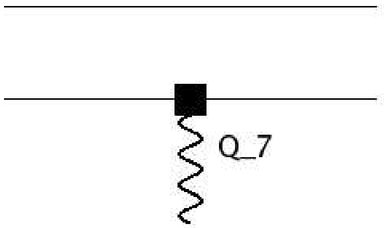
2. Exclusive case $B \to (K^*, \rho)\gamma$

- At α_s^0 LO m_b^{-1} only electric penguin contributes



$$Q_7 \equiv \frac{e^2}{8\pi^2} m_b (\bar{b}\,\sigma \cdot F\, q)_{V+A} \qquad q = (d, s)$$

$$\langle V\gamma | Q_7 | B \rangle = T_1^V(0)\,(\mathrm{kin}) \qquad \text{the semileptonic FF}$$

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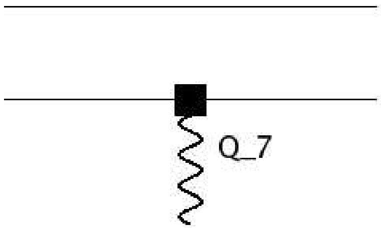

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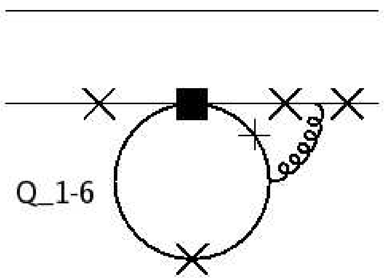
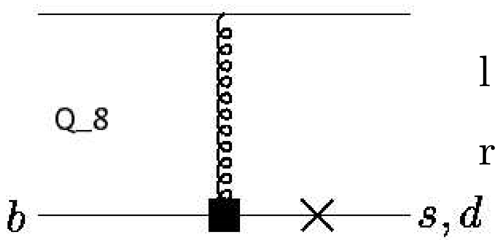
QCD-corrections, NLO α_s ?

- QCD factorization
  (Bosch et al, Beneke et al, Ali et al 01 ... Becher et al 05 (all order α_s)):

$$\begin{aligned}\langle V\gamma|Q_i|B\rangle &= T_i^I F(B\to V_\perp) + \int_0^1 d\xi du\,\phi_B(\omega)\phi_{V_\perp}(u) T_i^{II}(\xi,u) + O\left(\frac{\Lambda}{m_b}\right) \\ &= \delta C_i \langle V\gamma|Q_7|B\rangle\end{aligned}$$





l : example hard vertex int.

r : example hard spectator int.

2.1 The branching ratio

- With $a_7 \equiv C_7 + \delta C_i \dots \; K^*$ for definiteness

$$Br(B \to K^*\gamma) = [G_F^2 \dots] | \underbrace{(\lambda_c^s + \lambda_u^s)}_{\sim V_{\rm ts}} a_7(K^*) + \lambda_c^s b^c + \lambda_u^s b^u |^2 |T_1^{K^*}(0)|^2$$

  - $|V_{\rm ts}| = |V_{\rm cb}|(1 + O(\lambda^2))$ ok $|V_{\rm td}|$ unknown
  - b are $1/m_b$ corrections, control K^*, large (e.g. annihilation ρ)
  - uncertainty $T_1 \sim 10{-}15\%$ wilson coeff. $a_7 \sim 7 - 8\%$ ...experiment

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1993-	$\sim 5\%$	2005 -	$\sim 50\%$
$B(B \to K^{*\,0}\gamma)$	40.1 ± 2.0	$B(B \to \rho^0\gamma)$	0.38 ± 0.18
$B(B \to K^{*\,+}\gamma)$	40.3 ± 2.6	$B(B \to \rho^+\gamma)$	0.68 ± 0.34
units 10^{-6}		$B(B \to \omega\gamma)$	0.54 ± 0.22

- Belle 05 measurement big news
  (5σ in ρ^0-channel where Babar 04 found no events yet) $\Rightarrow$ wait sum-conf 06
- K^*-channel: may extract $T_1^{K^*} = 0.28 \pm 0.02(?)$
- ρ-channel: not conclusive by itself $\Rightarrow$

Ratio, constraint on $\left|V_{\rm td}/V_{\rm ts}\right|^2$ (other possible observables)

- $$R \equiv \frac{B(B \to (\rho,\omega)\gamma)}{B(B \to K^*\gamma)} = \left|\frac{V_{\rm td}}{V_{\rm ts}}\right|^2 \underbrace{\left|\frac{T_1^{B\to\rho}}{T_1^{B\to K^*}}\right|^2}_{(\xi^{SU(3)}_{B\to V\gamma})^{-2}} \left|\frac{a_7^c(\rho)}{a_7^c(K^*)}\right|^2 (1+\Delta R)\,{\rm Kin}$$
  - uncertainty in Wilson-coeff.& leading-$1/m_b$ cancels (maybe also NP !?)
  - isospin/CP-average taken to compensate poor statistics
  - $\Delta R\; 1/m_b$-corrections (CKM suppressed)
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- The penguin formfactor is

$$\langle V(p)|Q_7|B(p_B)\rangle = \mathrm{kin}\, T_1(0) \quad q = p_B - p \quad q^2 = 0\,\mathrm{photon}$$

  - calculated Light-Cone Sum Rules
  - unfort. L-QCD possible $q^2 > 16\mathrm{GeV}^2$ & problem unstable ρ small quark masses

main concern . . .

3. SU(3) breaking effects in $\xi_{B\to V}$ in LCSR (focus hadronic input)

- Main source from leading $V_\perp$ ($V_{||}$) distribution amplitude(s)

$$\langle 0|\bar{q}(z)\sigma_{\mu\nu}s(-z)|K^*\rangle = i(e_\mu q_\nu - \{\mu \leftrightarrow \nu\}) f_K^\perp(\mu) \int_0^1 du e^{i(2u-1)(qz)} \phi_K^\perp(u)$$

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$$\phi(u,\mu) = 6u\bar{u}(1 + \sum_{n\geq 0} a_n(\mu) C_n^{3/2}(2u-1)) \stackrel{\mu\to\infty}{\to} 6u\bar{u}$$

  - a_n G.-moments (det. diff.), $a_{\rm odd}(\pi) = 0$ not $a_{\rm odd}(K) \neq 0$ SU(3)-break
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   A. determinations of a_n indicate $a_0 \equiv 1 > |a_{1,2}| > |a_{3,4}|$.. (cons. conformal hierarchy)
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2. Model satisfying theoretical and experimental constraints e.g. Ball,Talbot 05

Case under consideration difference irrelevant (not sensitive endpts)

3.1 Determination of Gegenbauer moments a_1, a_2, ...

- Fit to an observable, be careful other hadr. uncert. do not contaminate
  Examples for a_2^π: $F_{\gamma\gamma^*\pi}$,$F_\pi^{\rm em}$,$F^{B\to\pi}$-shape
  More spectral data (bins) would be useful e.g. $B \to \pi e\nu$ others

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- Direct calculation from the matrix elements

$$\langle 0|\bar s z_\mu \gamma^\mu \gamma_5 (iz \overleftrightarrow{D})^n q|K(p)\rangle = (zp)^{n+1} f_K 2 \int_0^1 du (2u-1)^n \phi_K(u) \equiv N \cdot M_n$$

$$M_0 = 1 \qquad M_2 = \frac{1}{5} + \frac{12}{35} a_2$$
$$M_1 = a_1 \qquad M_4 = \frac{3}{35} + \frac{8}{35} a_2 + \frac{8}{77} a_4$$

  - In QCD sum rules (pioneered by Chernyak & Zhitnitsky $\sim$ 1980)
    Noticed that only first few moments give stable sum rules, $n > 4$ not useful
  - Lattice worked on it ~ 90 got contradicting results
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    Also here higher moments difficult (derivatives)
- New methods from exact operator relations for first moment (a_1) ...

a_1 from QCD sum rules

QCD-sum allows to estimate hadronic parameters (of low lying states)
Sketch: a_1 obtained from correlation function of the type

$$i \int_x \langle 0 | T \bar{q}(i \overleftrightarrow{D}_\mu) \Gamma_1 s(x) \, \bar{s} \Gamma_2 q(0) | 0 \rangle$$

inserting a complete set of states

$$\sim a_1 \frac{f_K^2}{q^2 - m_K^2} + \text{higher states}$$

Performing the operator product expansion for high virtualities $-q^2 = Q^2 << \Lambda_{QCD}^2$

$$\sim c^1(q^2) + \frac{c^{\bar{q}q}}{q^4} \langle \bar{m}_s q q \rangle + \ldots$$

and then estimating the “higher states” via a dispersion relation and analytic continuation of the the imaginary part of the OPE (Quark Hadron Duality) allows us to extract a_1

History of the results

When $\Gamma_{1,2}$ have same chirality the SR is diagonal and they have opposite chirality the SR is non-diagonal ...

Type	$a_1(K)(\mu_0)$	$a_1^{\parallel}(K^*)(\mu_0)$	$a_1^{\perp}(K^*)(\mu_0)$	Authors	Remarks
ND	0.17	0.19	0.2	Chernyak & Zhit. 84	sign mistake
ND	-0.18	-0.4	-0.34	Ball Boglione 03	NLO,unstable
D	0.05 ± 0.02	-	-	Khodjamiran et al 04	-
OPR	0.1 ± 0.12	0.1 ± 0.07	-	Braun Lenz 04	neglect $O(m_s^2)$
D	0.06 ± 0.03	0.03 ± 0.02	0.04 ± 0.03	Ball RZ 05	confirm 04, exte
OPR	0.07 ± 0.18	0.01 ± 0.05	0.09 ± 0.07	Ball RZ 06	incl $O(m_s^2)$

- ND: spectral-fct non-positive def. (cancellations, contamination higher states) ! which turns out to be the case $\Rightarrow$ **not consider** anymore
- D: pos. def. work fine are the best
- OPR: New method can't compete yet ...

New operator relations for a_1

$$M_1 \equiv \frac{3}{5}\, a_1^{\|}(K^*) = -\frac{f_K^\perp}{f_K^\|}\,\frac{m_s - m_q}{m_{K^*}} + 2\,\frac{m_s^2 - m_q^2}{m_{K^*}^2} - 4\kappa_4^{\|}(K^*)$$

$$\langle 0|\bar q (g G_{\alpha\mu}) i\gamma^\mu s|K^*(q)\rangle = e^\alpha f_K^\| m_{K^*}^3 \kappa_4^\|(K^*)$$

1. Take $O_{\mu\nu} = \frac{1}{2}\bar q \gamma_\mu \gamma_5 i \overleftrightarrow{D}_\nu\, s + \ldots$ with $O_\mu^{\ \mu} = 0$ playing role of energy momentum tensor

$$\langle 0|\partial_\mu O^\mu_\nu|K\rangle \stackrel{e.o.m}{=} \ldots$$

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.
- The $(\kappa_4)'s$ are estimated via several QCD Sum Rules, not very stable sensitive to $m_s, \alpha_s, \langle\bar{s}s\rangle/\langle\bar{q}q\rangle$
- $(\kappa_4)'s$ could also be estimated from Lattice ! Why not ? Overall Precision ?

3.2 Decay constants for the ρ and the K^*

MeV	$f_\rho^{\parallel}$	$f_{K^*}^{\parallel}$	$\mu = 1\mathrm{GeV})$	$f_\rho^{\perp}(\mu)$	$f_{K^*}^{\perp}(\mu)$
$\tau^- \to V^- \bar\nu$	$209 \pm 2^*$	217 ± 5	exp	–	–
QCD-SR	206 ± 7	222 ± 8	QCD-SR	165 ± 9	185 ± 10

* also obtained from $\rho^0 \to e^+e^-$ consistent within uncert. and isospin

L-QCD provides ratios (cancellation of chiral logs) (Becirevic et al 03, quenched)

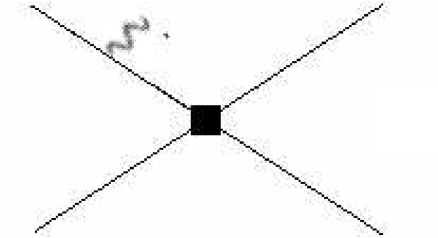
$$\left(\frac{f_\rho^T}{f_\rho^{\parallel}}\right)_{\mathrm{L-QCD}}(2\mathrm{GeV}) = 0.72 \pm 0.02 \quad \overset{\mathrm{compare}}{\leftrightarrow} \quad \left(\frac{f_\rho^T}{f_\rho^{\parallel}}\right)_{\mathrm{SR}}(2\mathrm{GeV}) = 0.69 \pm 0.04$$

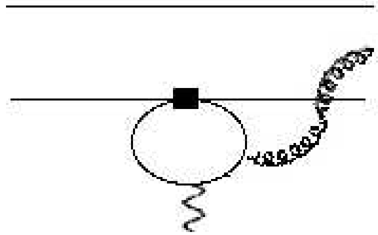
- Sum Rules: stable for $f_\rho^{\perp,\parallel}$, $f_{K^*}^{\parallel}$
- For $f_{K^*}^{\perp}$ nearby resonance needs to be added for stability
- Comparison with experiment ρ-channel ok (encouraging?)
- Comparison with L-QCD idem

$\Rightarrow$ Question-mark $f_{K^*}^T$ remains, effort from L-QCD highly desirable

3.3 $1/m_b$ corrections

- Annihilation and c,u-loop graphs of Q_2 num. rel. $B \to V\gamma$,





since $C_2 \sim 3|C_7|$ enhanced, (LO α_s^0 but NLO m_b^{-1})

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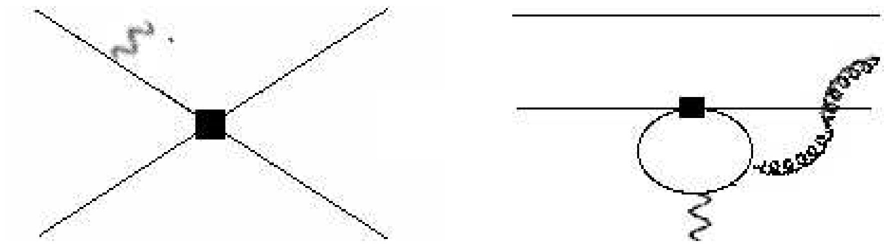

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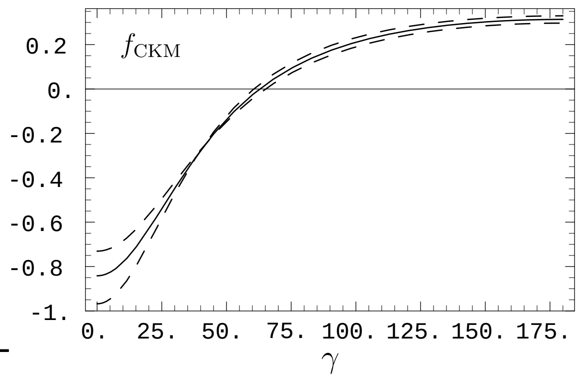
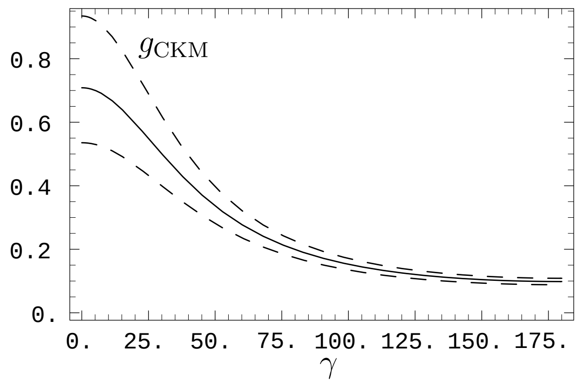
- The generically parametrized $1/m_b$-corr. ($\delta b = (b^u - b^c)/b^c(\rho)$)

$$(1 + \Delta R) = |1 + \delta b \frac{V_{\rm ud} V^*_{\rm ub}}{V_{\rm td} V^*_{\rm tb}}|^2 = (1 + f_{\rm CKM} {\rm Re}[\delta b] + g_{\rm CKM} \frac{1}{2} |\delta b|^2)$$





effective suppression

$\gamma_{\rm UT-fit} = 71 \pm 16^\circ$

Few short remarks (in preparation)

CKM-hierarchy in $B(B \to V\gamma)$

p	u	c
K^*	λ^4	λ^2
ρ	λ^3	λ^3

$\lambda \sim 0.2$

$Q_2 = (\bar{s}p)_{V-A}(\bar{p}b)_{V-A}$

A. c-loop: (Voloshin-effect inclusive case)
$1/m_c^2$-OPE charm-loop estimate $\langle V|(\bar{s}\tilde{G}_{\alpha\beta}DF^{\alpha\beta}b)_{V+A}|B\rangle$ QCD sum rules $\sim 2-5\%$
(KRSW 97) recalculate matrix element in LCSR (somewhat lower value)

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model dependence
problem also appears in the inclusive case ! (Negligible for K^* !)

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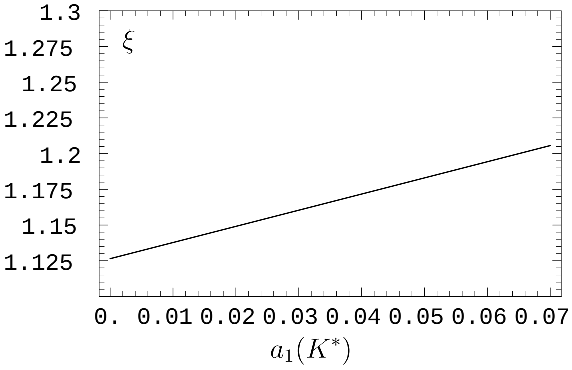
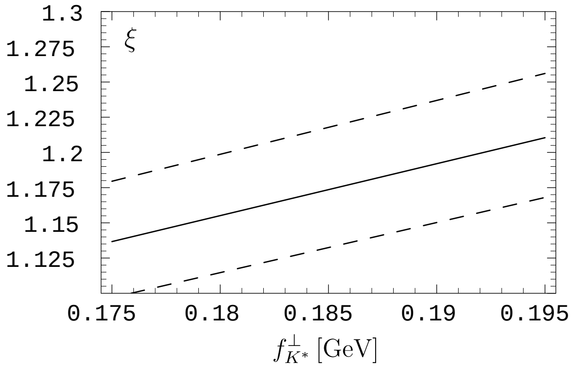
C. Annihilation (only p=u relevant)

- isospin larger $\rho^\pm$ than ρ^0 (charge & Wilson Coeff.)

3.4 The results (Ball RZ JHEP 06)

$$\xi = \frac{T_1^{K^*}(0)}{T_1^{\rho}(0)} = 1.17 \pm 0.09$$





bulk uncert. f_V^T

m_b largely cancels

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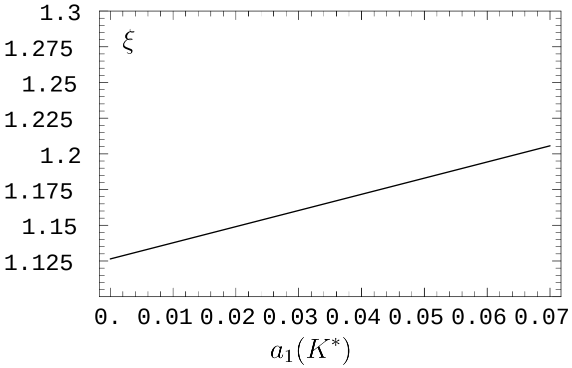

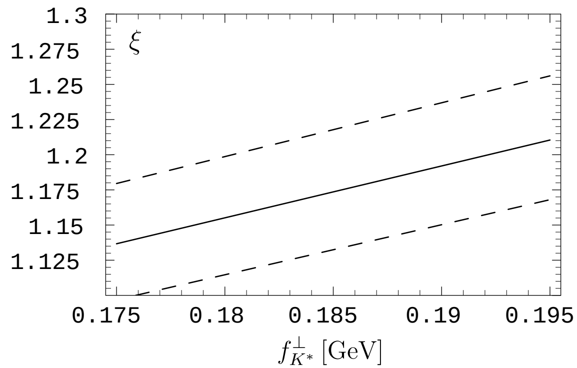

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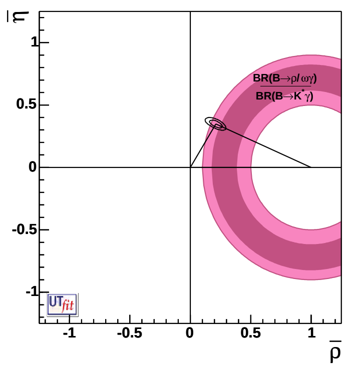
- $$R_{\rm th} = \left|\frac{V_{\rm td}}{V_{\rm ts}}\right|^2 (0.75 \pm 0.11_{(\xi)} \pm 0.03_{(a_7,\gamma,|V_{\rm ub}/V_{\rm cb}|)})$$

from where we deduce



$$|V_{\rm td}/V_{\rm ts}|^{\rm HFAG}_{B\to V\gamma} = 0.179 \pm 0.022_{\rm ex} \pm 0.014_{\rm th}$$

4. Comparison

	R^{HFAG}	R_{05}^{Belle}	R_{04}^{Babar}	UT-Fit	$f(\gamma, \lvert V_{\mathrm{ub}}/V_{\mathrm{cb}}\rvert)$	$\Delta m_s^{\mathrm{CDF}}$
$\lvert V_{\mathrm{td}}/V_{\mathrm{ts}}\rvert$	0.179	0.207	< 0.197	0.198	0.216	0.208
Δ_{th}	0.014	0.016		0.010	0.029	0.008
Δ_{ex}	0.022	0.027			$0.007\,(\Delta\gamma \sim 4^\circ)$	

B_s-mixing vs B_d-mixing (SM box graphs), basic equation

$$\underbrace{\frac{\Delta m_s}{\Delta m_d}}_{\sim 2/1\%} = \frac{m_{B_s}}{m_{B_d}} |V_{\mathrm{td}}/V_{\mathrm{ts}}| \times \underbrace{\frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}}}_{(\xi_{\Delta m_s}^{SU(3)})^2 = (1.2 \pm 3.5\%)^2} |V_{\mathrm{ts}}/V_{\mathrm{td}}|^2$$

- Again .. new physics could cancel in ratio (blind under d,s)
- Potential $R^{\mathrm{th}}_{B\to V\gamma}$:
  - theory: $\sim 0.010\,(6\%)$ with better $f_V^\perp$
  - experiment: $5\%_{K^*} \leq (5 + ..)\%_\rho$
  - $\Rightarrow$ HFAG, CKM-fitter, UT-fit can make their averages

Conclusions

A $B \to K^*\gamma$ vs $B \to \rho\gamma$ constraint on $|V_{\rm td}/V_{\rm ts}|$
B_s-Mixing vs B_d-Mixing stronger constraint, gap will get closer
Most importantly the two constraints are independent

B Considerable progress on leading Kaon DA – Gegenbauer moment a_1
Useful learning ground for $B \to K^* l^+ l^-$, $B \to K\nu\nu$ (Super-B)

C Effort from Lattice-QCD etc on $f_V^{(\perp,\|)}$ desired

D In order to asses other observables like Isospin(ρ),
$B \to \rho\gamma / B \to \rho l \nu \sim |V_{\rm td}/V_{\rm ub}|^2 + \ldots$ needs to control annihilation and u-quark loops

E The latter are also of independent interest as a guidance for the analogous problem in the inclusive $b \to d\gamma$

Bon appetit !

Backup slide

The puzzle about $\Gamma[B \to K^*\gamma]$

- It is possible to estimate $\Gamma[B \to K^*\gamma]$ in theory
  1. Take NLO calculation from Bosch et al
  2. Weak Annihilation insignificant in this channel (CKM-suppressed)
  3. $|V_{\rm ts}| = |V_{\rm cb}|(1 + O(\lambda^2))$ under reasonable control (3-4% error)

  with

  A. twist-3 radiative corrections to FF T_1

  B. new Gegenbauer moments a_1

$$
\begin{aligned}
\Gamma[B \to K^*\gamma]_{\rm theory} &= (4.9 \pm 30\% +) \cdot 10^{-5} \\
\Gamma[B \to K^*\gamma]_{\rm experi.} &= (4.0 \pm 5\%) \cdot 10^{-5}
\end{aligned}
$$

Looks consistent. Without A. and B. the deviation was significant!