Nonperturbative Jet Radiation and the OPE

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B decays

The advance of the Heavy Quark Expansion of QCD allowed to achieve a high precision in the dynamic treatment of strong interaction in B decays, not based on global symmetries

b quarks are heavy, but not enough for safe asymptotics. Not only α_s-corrections may be sizable; nonperturbative corrections are significant

$$\left(\frac{M_B}{m_b}\right)^5 \approx 2 = 1 + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right) \ldots$$

A first-principles control over nonperturbative dynamics in B decays is required

The Heavy Quark Expansion is based on the smart application of the Wilsonian OPE

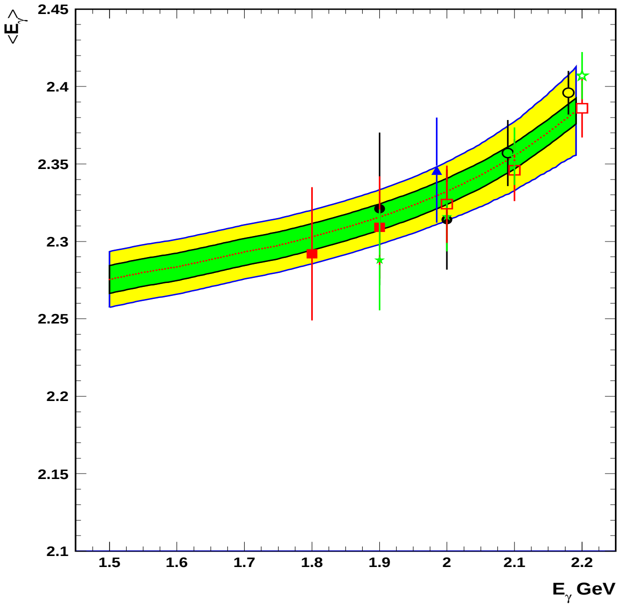
It has nothing to do with integrating α_s over the Landau singularity or with summing non-summable perturbative series

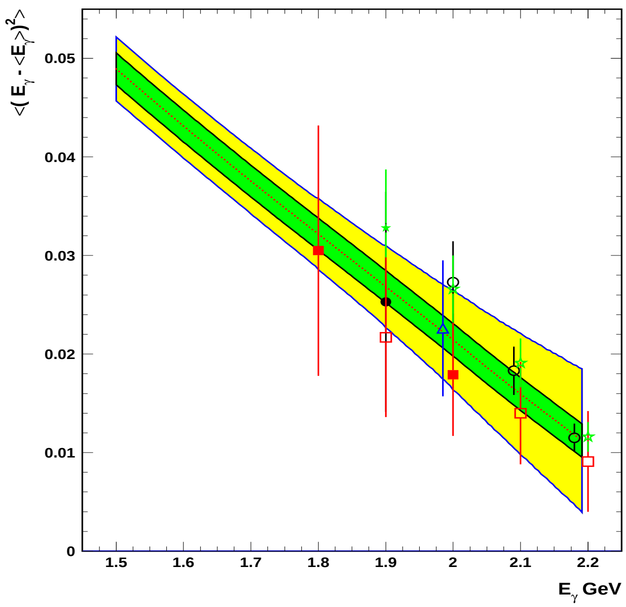
IR domain *is excluded* from the perturbative calculations

The OPE-based theory may seem to work too well

'Theoretical correlations'

Theory predictions for photon energy moments in $b \to s+\gamma$ based on HQP from $b \to c\,\ell\nu$ data vs. experiment





Some other good news:

A "$\frac{1}{2} > \frac{3}{2}$" puzzle — seems to be resolved (Belle, 2004)

see Bigi et al., hep-ph/0512270

ϱ^2, the slope in $B \to D^* \ell\nu$ – resolved in favor of theory*:

$$\varrho^2 = 0.925 \pm 0.075 \qquad \text{BaBar, hep-ex/0602023}$$

$b \to \text{light } q$ decays have some subtleties which represent the problems for applying the OPE

These motivated the theoretical study underlying this talk

* N.U. 2001

$b \to q$ decays, OPE and perturbative corrections

or

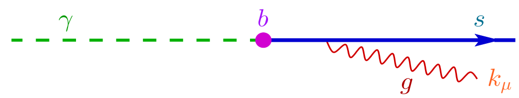
Strong coupling regime in gluon bremsstrahlung

$B \to X_s + \gamma$ decay distributions   similar questions in $b \to u\,\ell\nu$

Tree level spectrum: $\frac{d\Gamma}{dE_\gamma} = \delta(E_\gamma - \frac{m_b}{2})$ consequence of $M_X^2 = m_s^2 = 0$

modified by

- Perturbative effects: $b \to \gamma + s_{\rm virt} \to \gamma + s + g$



$M^2_{\{sg\}} > 0$

- Intrinsic motion of b: Doppler shift

Ali, Pietarinen; ACM 1979-1982

with $\vec{p}_b \neq 0$ $E_\gamma = \frac{m_b}{2} + \frac{\vec{p}_b \vec{n}_\gamma}{2}$

A counterpart of the DIS parton distribution leading to the nontrivial structure function of the b quark in B

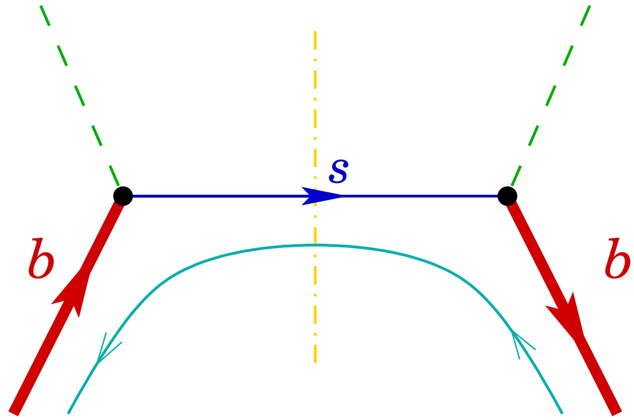
In fact, not $(\vec{\pi}\vec{n})$ enters, but $\pi_0 - \vec{\pi}\vec{n} = \pi_\mu n^\mu$, $n^2 = 1$

n is the unit vector along the light quark momentum

$\pi_\mu = iD_\mu - m_b v_\mu$ are the nonrelativistic energy and momentum operators for heavy quark

Comes from the s-quark propagator in the forward scattering amplitude

$$\bar{b}\ \ldots\ \frac{1}{m_b^2-(p_b+\pi_b-q)^2-\frac{i}{2}\sigma G}\ \ldots\ b\ ,$$



$$m_b^2-(p_b+\pi_b-q)^2-\tfrac{i}{2}\sigma G \simeq m_b^2-(p_b-q)^2-2\pi_\mu(p_b-q)^\mu+\ldots$$

p_b-q is the s-quark parton momentum in the decay

Looks like an interaction of the b quark rather than a property of the s-quark jet!

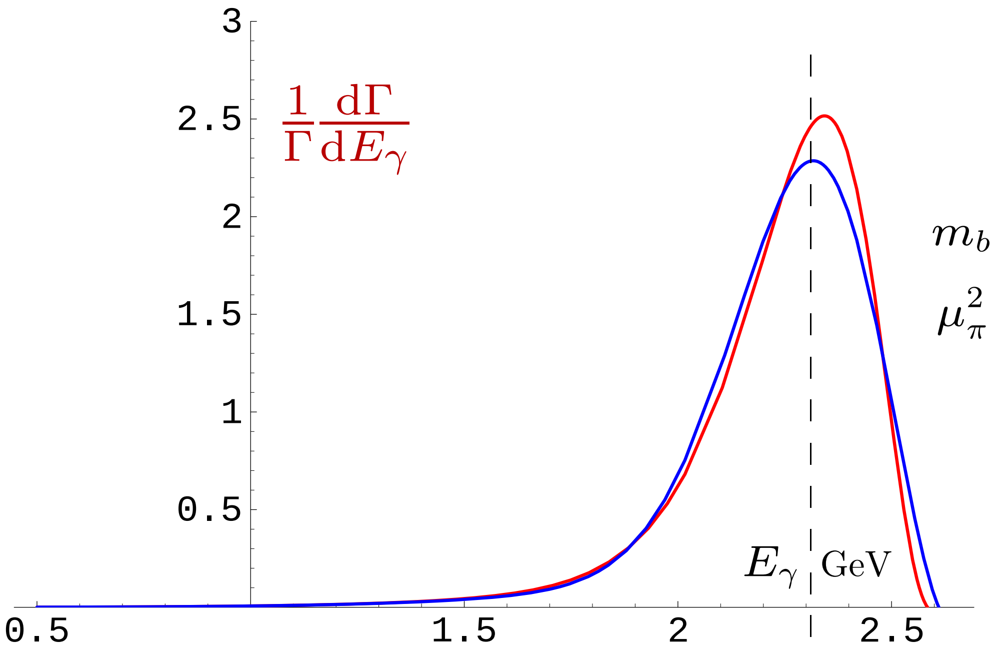
NB : This applies to *soft gluons* only

A peculiarity of the heavy-to-light decays: double-log Sudakov radiation

Emission resummation technique:

multiple gluon emissions

I will dwell on its different aspect



OPE: soft modes $\Longrightarrow$ nonperturbative effects
hard gluons $\Longrightarrow$ Wilson coefficients
(perturbative spectrum)

Works well in $b \to c\,\ell\nu$, but some problems for $Q \to q$ transitions: what are 'soft' and what are 'hard'?

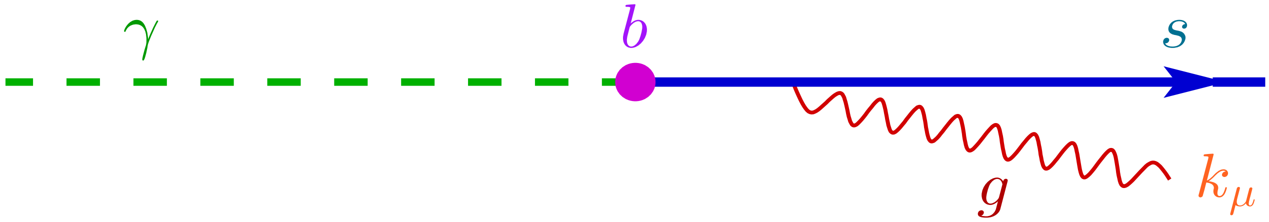
Original OPE is formulated in the Euclidean theory;
$k_\mu \gg \Lambda_{\rm QCD}$ implies small coupling

the same idea works for $b \to c$ decays, away from the light front: $|\vec{k}| > \mu$ – short distances; α_s enters at $Q^2 > \mu^2$

The problem appears for $b \to q$

assume m_b is very large

OPE expands in the fields' momenta $\Longrightarrow$ have to assume literally that all the components are small



$$\mathrm{d}W^{\rm pert} = \int \frac{\mathrm{d}\omega}{\omega} \int \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\, C_F \frac{\alpha_s(k_\perp^2)}{\pi}\; \mathrm{d}W_{\rm born}\;, \qquad k_\mu = (\omega, k_\perp, k_\parallel)$$

Even if $\omega \gg \mu$, but radiation angle θ is very small, $k_\perp$ can fall below $\mu_{\rm hadr}$

highly collinear gluons are strongly coupled

$\rm KLN$: cancellation between real and virtual corrections

at least the total decay width is not affected

What about moments of E_γ we calculate using Wilsonian $\rm OPE$? No virtual contributions to cancel...

$$\delta E_\gamma \sim \frac{k_\perp^2}{\omega} \sim \frac{\mu_{\rm hadr}^2}{\mu_{\rm Wils}} \lesssim \Lambda_{\rm QCD} \quad \stackrel{?}{\Longrightarrow} \quad \delta M_{E_\gamma}^{(n)} \sim \left(\frac{\mu_{\rm hadr}^2}{\mu_{\rm Wils}}\right)^n$$

additional nonperturbative effect on top of the $\rm OPE$?

A priori this cannot be excluded. OPE controls what it can, the effect of all-soft physics. If there are nonperturbative effects from large-frequency modes, they would be additional contributions.

A number of theoretical speculations (SCET): heavy quark parameters from $b \to c\,\ell\nu$ cannot be used in $b \to q$ decays. Only those between $b \to u\,\ell\nu$ and $b \to s+\gamma$ are allowed

similar concepts lie behind the 'shape-function' scheme

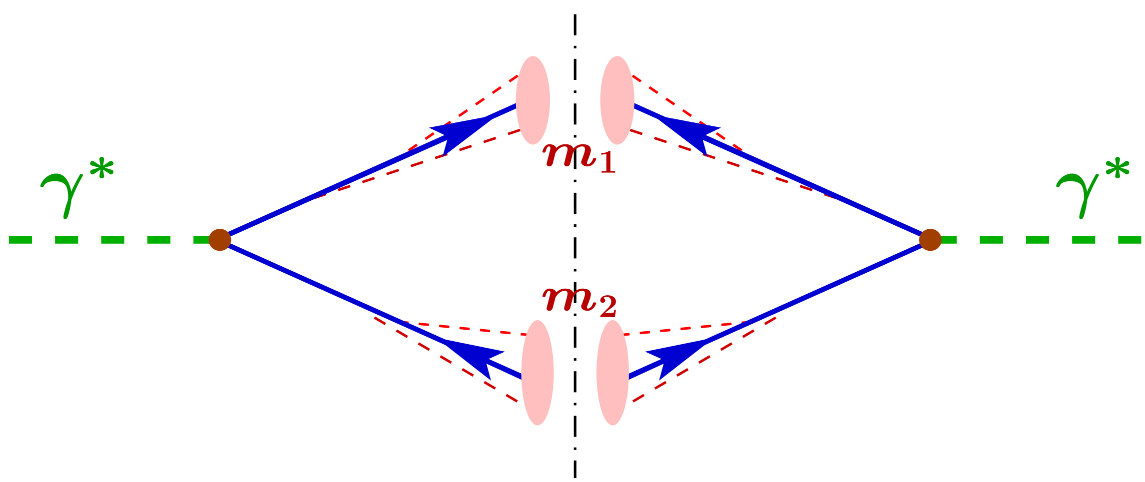
For a year, 2004 the V_{ub} (inclusive) programs at B factories were diverted to exclude $b \to c\,\ell\nu$ information and only use $b \to s+\gamma$ spectrum for $b \to u\,\ell\nu$ description!

Extra effects here would lead to certain paradoxes

Consider $\sigma(e^+e^- \to \mathrm{hadrons})$ from this perspective

no OPE for jets

Look at it now as a decay of the photon with the mass $\sqrt{s}$ into two jets:



$$\Gamma \sim 1 - \frac{m_1^2+m_2^2}{s} + \frac{(m_1^2+m_2^2)^2}{s^2} + \;...$$

If there are $\delta m^2_{\rm jet}$, we would normally have the phase-space–related $1/s$, $1/s^2$, ... corrections in the total rate, not only in jet distributions where no Euclidean OPE applies

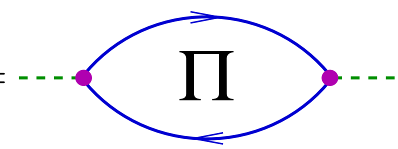
However, for the *total cross section* we know from the OPE that

$$\delta_{\mathrm{IR}} R(s) \sim \frac{1}{s^3} + \alpha_s \frac{1}{s^2}$$

since $D(Q^2) \sim 1 + (1{+}\alpha_s)/Q^4 + \ldots$

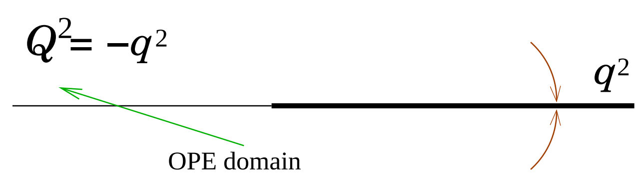
This is a consequence of analyticity and unitarity for the polarization operator of the currents

$$\Pi(q^2)(-q^2\delta_{\mu\nu}+q_\mu q_\nu) = \int \mathrm{d}^4x\, e^{iqx}\langle 0|iT\{J_\mu(x)J_\nu(0)|0\rangle =$$



$$R(s) \propto \mathrm{Im}\,\Pi(s), \qquad D(Q^2) = -Q^2\frac{\mathrm{d}}{\mathrm{d}Q^2}\,\Pi(Q^2)$$

If we know the asymptotics of $D(Q^2)$ in deep Euclid, we know power corrections in $R(s)$ at large s :



Cancellation between jet hadronization and corrections to factorization for jet evolution? Voodoo, would contradict the physical picture

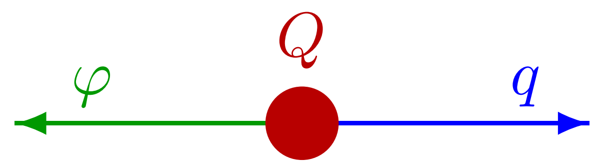
we consider energetic gluons!

OPE equates power corrections with all-soft vacuum modes...

No, integer moments of jet mass^2 are not affected!

Kinematics and the OPE

$Q \longrightarrow q + \varphi$



$m_\varphi = 0\,,\qquad m_Q \ggg \Lambda_{\mathrm{QCD}}$

no $1/m_Q$ corrections

$$\frac{\mathrm{d}\Gamma_{\mathrm{tot}}(E_\gamma)}{\mathrm{d}E_\gamma} = \int_{-\infty}^{\infty} \mathrm{d}k_+\, F(k_+)\, \frac{\mathrm{d}\Gamma_{\mathrm{pert}}}{\mathrm{d}E}\left(E_\gamma - \tfrac{k_+}{2}\right)$$

support of $F(k_+)$ is $(-\infty, \overline{\Lambda}\,]$

$\overline{\Lambda} \equiv M_B - m_b$

Kinematically

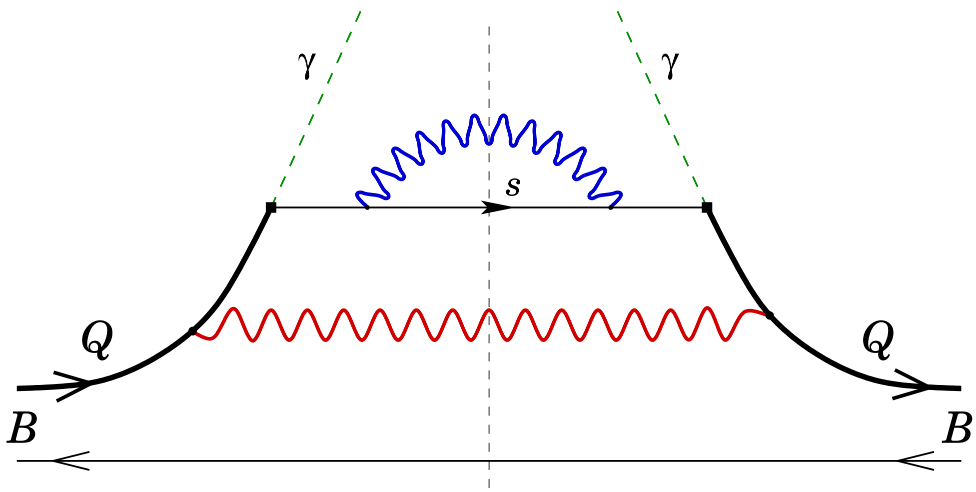
$$E_\gamma = \frac{M_B^2 - M_X^2}{2M_B} \qquad \text{hence} \qquad \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\gamma} \Longleftrightarrow \frac{\mathrm{d}\Gamma}{\mathrm{d}M_X^2}$$

Not distinguishing soft and hard effects we have the translation rule

$$\boxed{M_X^2 = m_b\, k_+}$$

Usual nonperturbative (OPE) window is

$M_X^2 \lesssim \mu_{\mathrm{hadr}} m_b\,;$ larger $M_X^2 \gg \mu_{\mathrm{hadr}} m_b$ come from hard bremsstrahlung



Soft gluons $|k_\mu| \lesssim \mu_{\rm hadr}$ are included into HQ distribution function $F(x)$ (Fermi motion). Other, hard gluons are in the Wilson coefficients (kernel)

$$F_0 = \int \mathrm{d}k_+ \; F(k_+) = 1 \; , \qquad F_1 = \int \mathrm{d}k_+ \; k_+ \, F(k_+) = 0$$

$$F_2 = \int \mathrm{d}k_+ \; k_+^2 \, F(k_+) = \frac{\mu_\pi^2}{3} \; , \quad F_3 = \int \mathrm{d}k_+ \; k_+^3 \, F(k_+) = -\frac{\rho_D^3}{3}$$

$$F_n = \int \mathrm{d}k_+ \; k_+^n \, F(k_+) = \frac{1}{2M_B} \langle B|\bar{Q} iD_z (iD_0 - iD_z)^{n-2} iD_z Q|B\rangle$$

$\dfrac{\mathrm{d}\Gamma_{\rm pert}}{\mathrm{d}E}$ accounts for all other modes

using the same separation scheme, $\omega = |\vec{k}| > \mu$

$$\frac{\mathrm{d}\Gamma^{\rm pert}}{\mathrm{d}M_X^2} = \int \frac{\mathrm{d}\omega}{\omega} \, \vartheta(\omega - \mu) \int \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \, C_F \frac{\alpha_s(k_\perp^2)}{\pi} \, \delta(M_X^2 - k_\perp^2 \tfrac{m_b}{2\omega})$$

$$\frac{m_b}{2\omega} = \frac{E_{\rm jet}}{\omega} \gg 1$$

Cannot go down to $M_X^2 < m_b \, \mu_{\rm hadr} \frac{\mu_{\rm hadr}}{\mu}$, or $k_+ < \mu_{\rm hadr} \frac{\mu_{\rm hadr}}{\mu}$

since there $k_\perp < \mu_{\rm hadr}$ contribute...

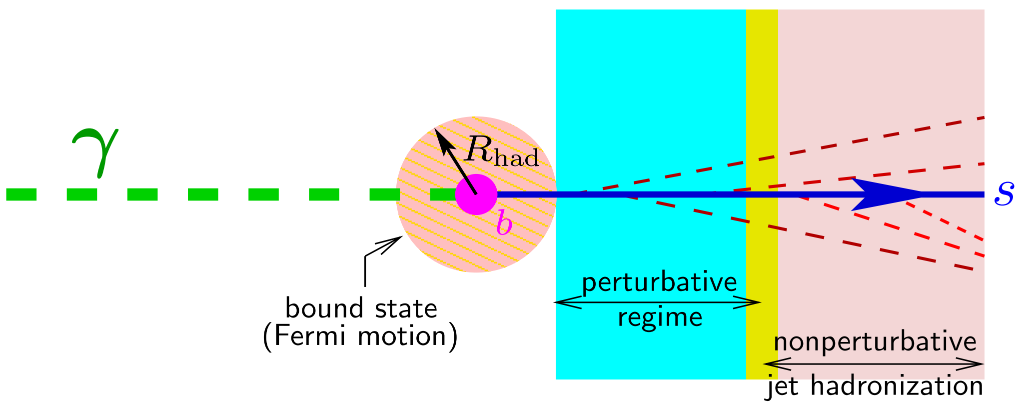
Physics: this is not the bound-state dynamics!

Travel distance

$$\underbrace{\frac{1}{M_X}}_{\tau_0} \cdot \underbrace{\frac{m_b}{2M_X}}_{\gamma\text{-factor}} = \frac{m_b\,\omega}{k_\perp^2\, m_b} = \frac{\omega}{k_\perp^2} \;\sim\; R_{\rm had}\cdot\frac{\omega}{\mu_{\rm hadr}}$$

much larger than $\mu_{\rm hadr}^{-1}$

The jet hadronization process is space- and time-separated from the bound state once $\omega \gg \mu_{\rm hadr}$



In the total width virtual corrections cancel these radiation effects no matter what α_s coupling is

$\langle M_X^2\rangle$ – ?   No virtual correction contributes...

actually not true for the first moment

In the 'canonic' description this would yield

$\delta\langle M_X^2\rangle \sim m_b\,\mu_{\rm hadr}\,\frac{\mu_{\rm hadr}}{\mu}$ of the 'known' sign

In fact this effect cancels in the integer moments of $\Gamma_{\rm pert}$;   these do not involve α_s at $Q^2 < \mu^2$

How to show this?

Need a description of the running-coupling gluon emission

Problem – these jet-radiation processes are not in Euclid, far far away

In Euclidean QCD we assume the gauge fields effects can be arbitrary at small k, while turn into usual (perturbative) ones at large k

At first sight the Minkowskian effects can be arbitrary if we admit that gluons may be nonperturbative at large ω

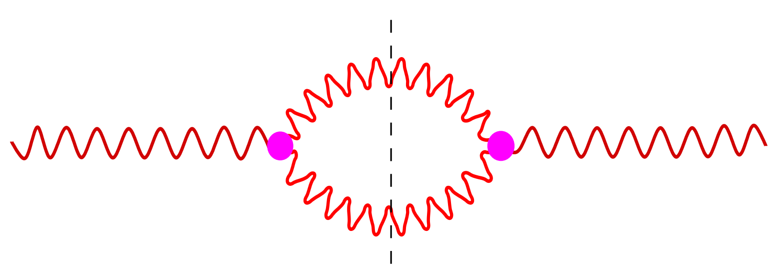
Nevertheless theory at least must be unitary and respect analytic properties
It is obtained by analytic continuation from the Euclidean QCD

A strong constraint!

DMW: dispersive approach, to respect this automatically
Dokshitzer, Marchesini, Webber (1996 and up)

Dressed gluon propagator:

$$\frac{\alpha_s^{\epsilon}(Q^2)}{Q^2} = \pi \int \frac{\mathrm{d}\lambda^2}{\lambda^2}\, \frac{\rho(\lambda^2)}{\lambda^2+Q^2}\,, \qquad \rho(s) = -\frac{1}{\pi^2}\mathrm{Im}\,\alpha_s^{\epsilon}(-s)$$



Instead of $\frac{\alpha_s}{k^2}$ we then put inside the diagrams

$$\int \mathrm{d}^4k \, \frac{\alpha_s(k^2)}{k^2} \ldots :$$

$$\pi \int \frac{\mathrm{d}\lambda^2}{\lambda^2} \rho(\lambda^2) \underbrace{\int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{1}{k^2+\lambda^2} \ldots}_{\downarrow}$$

the would be loop correction with the massive gluon

- Relation to the 'OPE'

Dikeman, Shifman, N.U. (1995)

Must require that the 'strong coupling regime' effects are absent at $Q^2 \gg \mu^2_{\mathrm{hadr}}$ (in Euclid)

$$\rho(\lambda^2) = \rho_{\mathrm{pert}}(\lambda^2) + \delta\rho(\lambda^2)$$

$$G(Q^2) = \frac{\alpha_s^{\epsilon}(Q^2)}{Q^2} + \frac{\delta\alpha_s^{\epsilon}(Q^2)}{Q^2}$$

assume $\delta\alpha_s^{\epsilon}(Q^2) \to 0$ fast at large Q^2;

the dispersion relation then says that

$$\boxed{\int \frac{\mathrm{d}\lambda^2}{\lambda^2} \lambda^{2n} \, \delta\rho(\lambda^2) = 0 \quad \text{for any integer } n}$$

in jet physics the nonperturbative effects are given by the $\log$-moments of $\delta\rho$

Consider bremsstrahlung with a massive gluon:

$$\mathrm{d}W_{\mathrm{brem}} = C_F \int \frac{\mathrm{d}\omega}{\omega} \underbrace{\int \frac{\mathrm{d}\lambda^2}{\lambda^2}\, \rho(\lambda^2) \int \frac{\mathrm{d}k_\perp^2}{k_\perp^2 + \lambda^2}}_{\int \mathrm{d}k_\perp^2 \;\; \frac{1}{\pi} \frac{\alpha_s^\epsilon(k_\perp^2)}{k_\perp^2}} \mathrm{d}W_{\mathrm{born}}$$

Soft / collinear bremsstrahlung is driven by $\alpha_s^\epsilon(k_\perp^2)$

Ok, for Γ_{tot} virtual corrections cancel this effect

Moments of M_X^2 :

$$\int \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\, k_\perp^{2n}\, \alpha_s(k_\perp^2)$$

saturated at large $k_\perp \sim m_b$, but we need α_s with a power accuracy here

Do explicitly:

$$\langle M_X^2 \rangle^{\mathrm{pert}} = C_F \int \mathrm{d}M_X^2 \int \frac{\mathrm{d}\omega}{\omega}\, \vartheta(\omega - \mu) \int \frac{\mathrm{d}\lambda^2}{\lambda^2}\, \rho(\lambda^2) \;\times$$
$$\int \frac{\mathrm{d}k_\perp^2}{k_\perp^2 + \lambda^2}\, M_X^2\, \delta\!\left(M_X^2 - (k_\perp^2 + \lambda^2)\tfrac{m_b}{2\omega}\right)$$

Integrating over M_X^2

$$\langle M_X^2 \rangle^{\mathrm{pert}} = C_F \int \frac{\mathrm{d}\omega}{\omega}\, \vartheta(\omega - \mu)\, \frac{m_b}{2\omega} \int \frac{\mathrm{d}\lambda^2}{\lambda^2}\, \rho(\lambda^2) \int \mathrm{d}k_\perp^2$$

Product of independent integrals! No $\alpha_s(k_\perp^2)$, minimal scale Q^2 is determined by the kinematics, it is μ at worst

$$\int \frac{\mathrm{d}\lambda^2}{\lambda^2}\, \rho(\lambda^2) = \lim_{Q^2 \to \infty} \delta\alpha_s^\epsilon(Q^2) = 0$$

Where the miracle hides?

Calculate the M_X^2-spectrum itself :

$$\frac{\mathrm{d}\Gamma^{\mathrm{pert}}}{\mathrm{d}M_X^2} = C_F \int \frac{\mathrm{d}\omega}{\omega}\, \vartheta(\omega-\mu) \int \frac{\mathrm{d}\lambda^2}{\lambda^2}\, \rho(\lambda^2) \int \frac{\mathrm{d}k_\perp^2}{k_\perp^2+\lambda^2}\, \delta(M_X^2 - (k_\perp^2+\lambda^2)\tfrac{m_b}{2\omega})$$

$$= \frac{C_F}{M_X^2} \int \frac{\mathrm{d}\omega}{\omega}\, \vartheta(\omega-\mu) \int \frac{\mathrm{d}\lambda^2}{\lambda^2}\, \rho(\lambda^2)\, \vartheta(M_X^2 - \tfrac{m_b}{2\omega}\lambda^2)$$

instead of

$$\frac{C_F}{M_X^2} \int \frac{\mathrm{d}\omega}{\omega}\, \vartheta(\omega-\mu)\, \underbrace{\int \frac{\mathrm{d}\lambda^2}{\lambda^2 + \frac{2\omega}{m_b} M_X^2}\, \rho(\lambda^2)}_{\frac{1}{\pi}\alpha_s^{\epsilon}\left(\frac{2\omega}{m_b} M_X^2\right)}$$

The radiation is driven by a different effective coupling $\tilde{\alpha}_s(k_\perp^2)$:

$$\delta\tilde{\alpha}_s(Q^2) = \pi \int_0^{Q^2} \frac{\mathrm{d}\lambda^2}{\lambda^2}\, \rho(\lambda^2) \qquad \text{vs.} \qquad \delta\alpha_s^{\epsilon}(Q^2) = \pi \int_0^{\infty} \frac{\mathrm{d}\lambda^2}{\lambda^2 + Q^2}\, \rho(\lambda^2)$$

the kinematic constraint to have definite M_X^2 (rather than definite $k_\perp$) changes the dispersion integral over λ

$\tilde{\alpha}_s$ and α_s^{ϵ} coincide 'with the log accuracy'
(up to 3 loops – Schwinger) *yet not in powers*

Integer moments of $\delta\tilde{\alpha}_s(Q^2)$ all vanish, while those of $\delta\alpha_s^{\epsilon}(Q^2)$ are 'positive'

Interpretation: in a sense, $\alpha_s^{\epsilon}(k_\perp^2)$ remains valid, but cannot be used other than for the total probability

one-gluon (massless) description is incompatible with running of α_s, a field-theory effect associated with the presence of a-few–particle states

These have different kinematics and reshuffle the distribution. Effective α_s becomes observable-dependent since using it implies we forcibly interpret the process in the massless one-gluon language

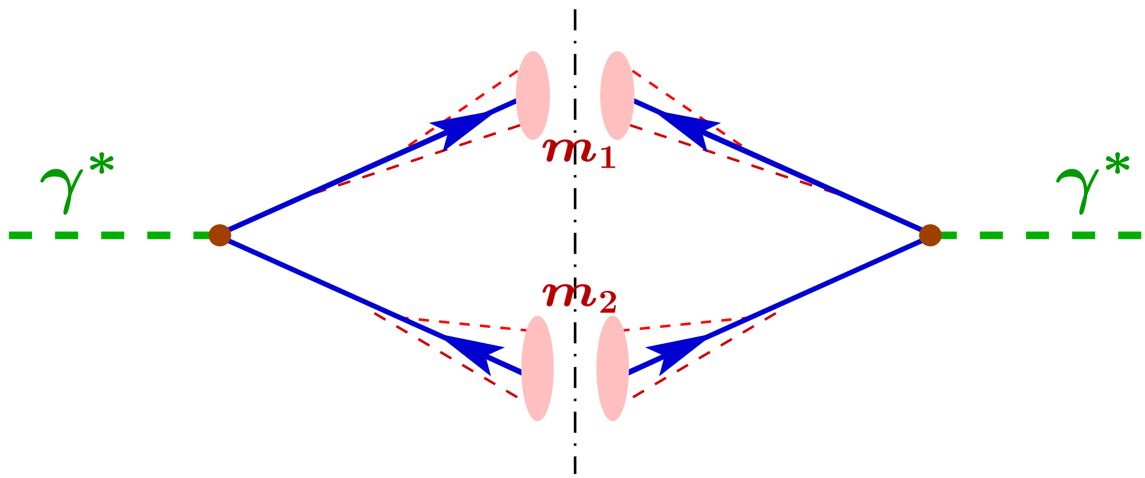
Physics behind: The coupling grows from $\alpha_s(E_{\rm jet})$ or from $\alpha_s(M_X)$ due to the final-state interaction, viz. gluon (jet) splitting. FSI do not change truly inclusive characteristics

Kinematics is driven by M_X^2, why we get the lower scale in α_s? From the factor $\frac{1}{k_\perp^2+\lambda^2}$, which (at $\lambda^2=0$) is much larger than $1/M_X^2$. This is due to degeneracy of states, $|q\rangle$ and $|q\,g\rangle$ where the quark and gluon are collinear. This is just called FSI

KLN implies that 'rescattering' does not affect the `total` probabilities and other truly local observables. That is what we get

Go back to $\sigma(e^+e^- \to \text{hadrons})$

$$\delta_{\rm IR} R(s) \sim \frac{1}{s^3} + \alpha_s \frac{1}{s^2}$$



$$\Gamma \sim 1 - \frac{m_1^2+m_2^2}{s} + \frac{(m_1^2+m_2^2)^2}{s^2} + \;\ldots$$

If there were $\delta m^2_{\rm jet}$, we would have had the related $1/s$, $1/s^2$, ... corrections in the total rate

We know they are absent to any order: power corrections come only from the soft ('Euclidean') gluons – that is what one calculates in the Euclidean OPE

We got this for inclusive jet moments as well

My conclusions

There may be some nonperturbative jet hadronization effects on top of the (Euclidean) OPE; point-to-point spectrum may depend on them

these have to be oscillating, though

Natural for nonperturbative hadronization, but

larger mass scale $\Delta M^2 \propto \mu_{\mathrm{hadr}} m_b$

The Wilsonian OPE is not affected yet. The usual moments are given by the local heavy quark expectation values, plus the perturbative corrections originating only from the short-distance domain

μ-dependence of moments is given by $\alpha_s^{\epsilon}(\mu)$

Additional nonperturbative effects may appear – just to the extent not to run in contradiction with the OPE

This appears to be a rather general feature of sufficiently inclusive jet physics. Inclusive B decays are only more transparent since admit OPE which allows to isolate and to take care of the effects of (truly) soft physics

Nonperturbatively the moments are affected only by the soft modes, but not by the *collinear* modes, even in the Minkowski processes

SCET and QCD are nonperturbatively different

Matching would include perturbative corrections in the nonperturbative regime

The standard expression used for the resummed distribution in the Mellin space

$$\Delta_N = \int_0^1 \mathrm{d}z\, z^{N-1} \Delta(z)$$

has the form

$$\ln \Delta_N(Q^2) = \frac{C_F}{\pi} \int_0^1 \mathrm{d}x\, \frac{x^N - 1}{1-x} \left[\int_{Q^2(1-x)^2}^{Q^2(1-x)} \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\, \alpha_s^\epsilon(k_\perp^2) - B\, \alpha_s^\epsilon(Q^2(1-x)) - D\, \alpha_s^\epsilon(Q^2(1-x)^2) \right]$$

While correctly describing the contribution of multiple emissions with different momenta, $\alpha_s(k_\perp^2)$ does not count precisely the contributions of a particular gluon "virtuality" μ^2 in the emission: it yields $\ln\left(\frac{Q^2}{\mu^2} + \frac{1}{1-x}\right)$ instead of $\ln \frac{Q^2}{\mu^2} + \ln \frac{1}{1-x}$; a different effective coupling $\tilde{\alpha}_s$ must be there. This is reshuffled into B and D through explicit integration using the logarithmic running of α_s with the scale, to any particular N^{k}LO, but potentially introduces large higher-order corrections

This is related to running of the coupling, and therefore is complementary to the standard analysis of resummation

I think it is advantageous to pass here to the proper effective coupling. This usually significantly reduces higher-order corrections and yields accurate numerical estimates already with much simpler quasi-LO calculations, at least for sufficiently inclusive distributions