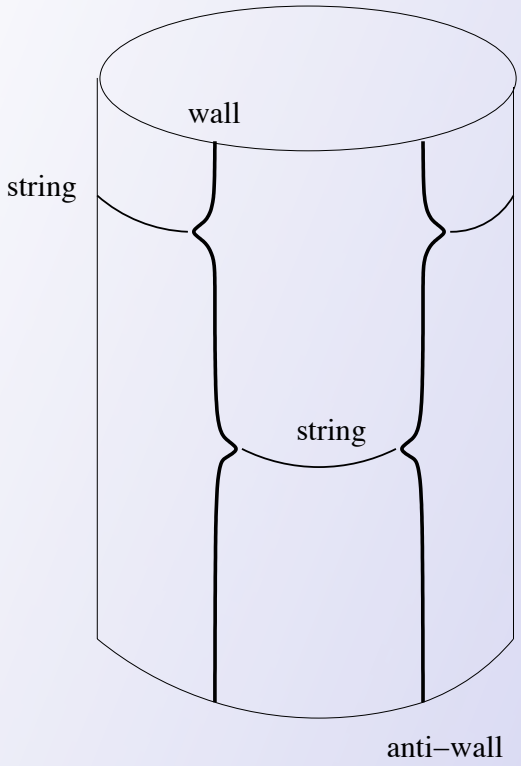
Bulk-Brane Duality in Field Theory

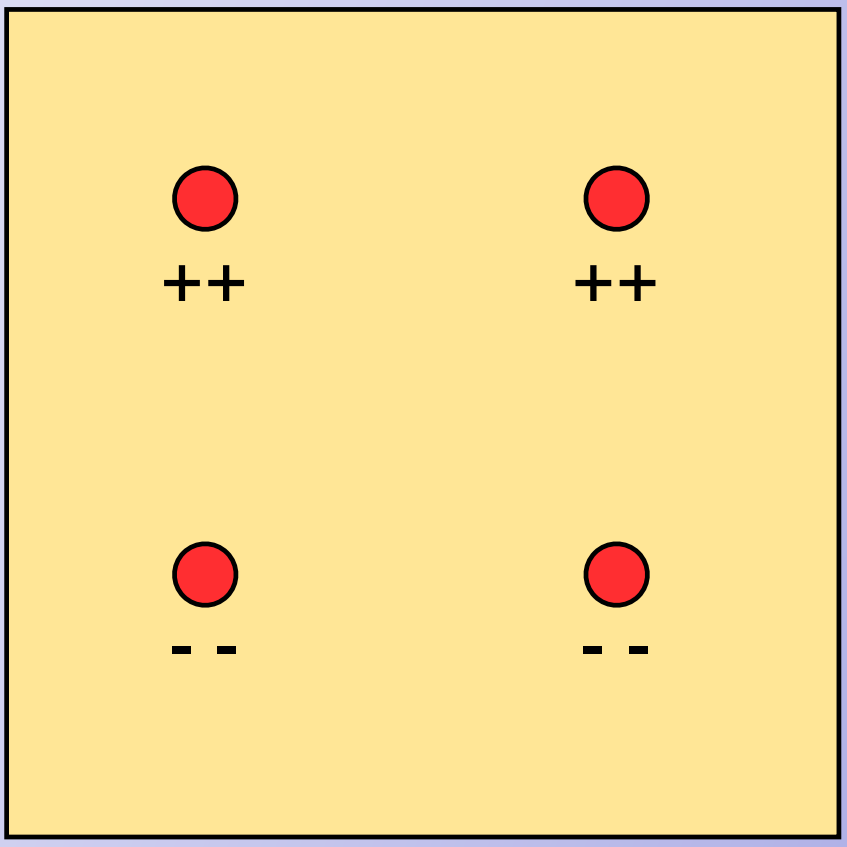
M. Shifman
Theoretical Physics Institute, University of Minnesota

with A. Yung

4D

3D





8 supercharges

walls & flux tubes

4 supercharges

SQED; CS

(Blurred) holography/Weak-strong coupling duality:

$4D$ $\mathcal{N}=2$ SQED with 2 flavors and FI term

$$S=\int d^4x\left\{\frac{1}{4g^2}F_{\mu\nu}^2+\frac{1}{g^2}|\partial_\mu a|^2+\bar{\nabla}_\mu\bar{q}_A\nabla_\mu q^A+\bar{\nabla}_\mu\tilde{q}_A\nabla_\mu\bar{\tilde{q}}^A\right.$$

$$\left.+\frac{g^2}{8}\left(|q^A|^2-|\tilde{q}_A|^2-\xi\right)+\frac{g^2}{2}\left|\tilde{q}_A q^A\right|^2+\frac{1}{2}\left(|q^A|^2+|\tilde{q}^A|^2\right)\left|a+\sqrt{2}m_A\right|^2\right\},$$

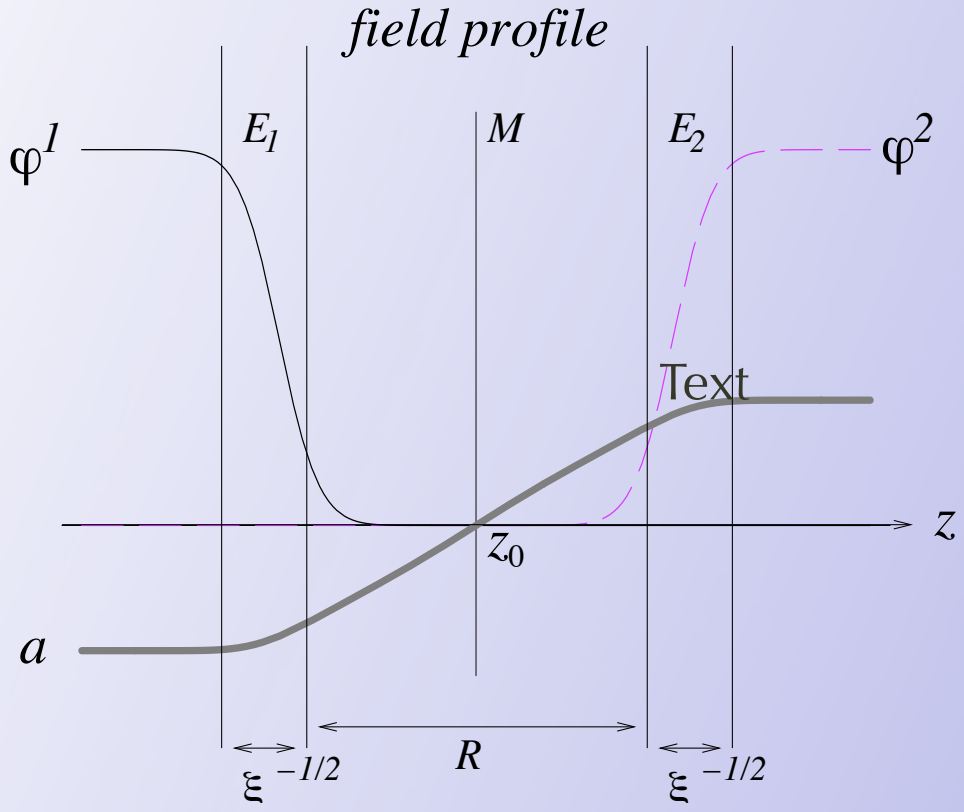
FI

$3D$ $\mathcal{N}=2$ dual SQED (with "hidden" CS)

$$-\frac{1}{4e^2}F_{mn}^-F^{-\,mn}+\frac{1}{2e^2}\left(\partial_n a_-\right)^2+\left|D_n s\right|^2+\left|\tilde{D}_n\tilde{s}\right|^2-2a_-^2\,\bar{s}s-2(m-a_-)^2\,\bar{\tilde{s}}\tilde{s}-e^2\left(|s|^2-|\tilde{s}|^2\right)^2$$

Basic Elements of the Construction:

(a) Domain walls ⋆ (SY)



Two edges (domains *E*) of the width $\sim \sqrt{1/\xi}$ are separated by a broad middle band *M* of the width $R \sim \Delta m/(g^2 \xi)$.

The tension $T = \Delta m\, \xi$

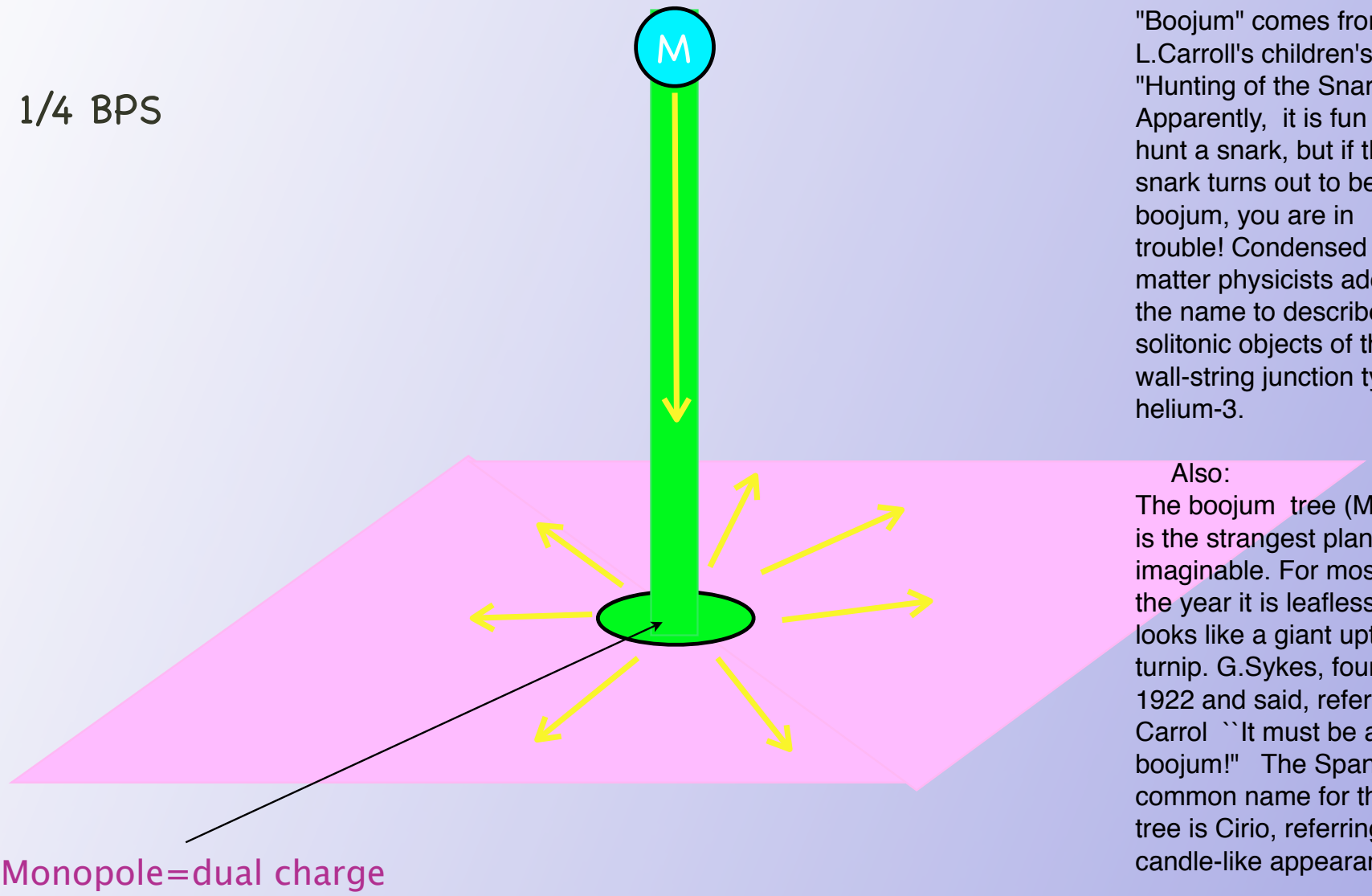
Moduli:

z_0 and σ ⇐ relative phase between ϕ^1 and ϕ^2

σ dualizes 3D photon a la Polyakov

(b) Wall-string junctions ** (SY,ST,ASY)

1/4 BPS

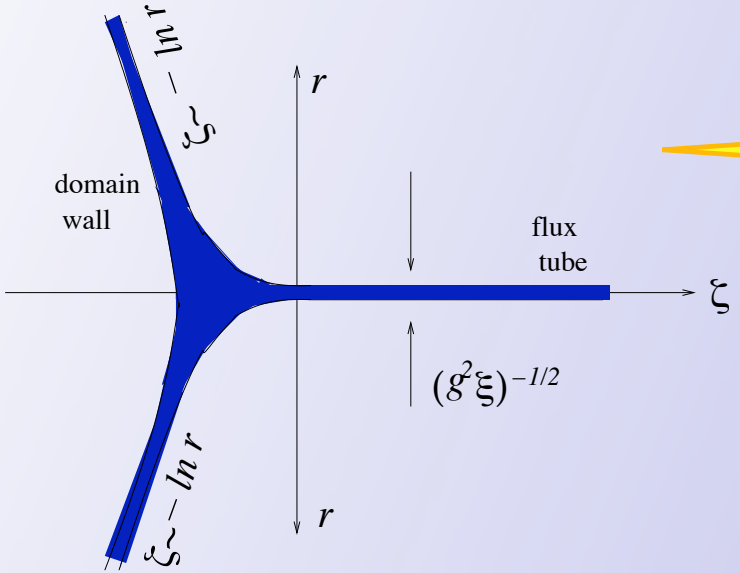


"Boojum" comes from L.Carroll's children's book "Hunting of the Snark." Apparently, it is fun to hunt a snark, but if the snark turns out to be a boojum, you are in trouble! Condensed matter physicists adopted the name to describe solitonic objects of the wall-string junction type in helium-3.

Also:
The boojum tree (Mexico) is the strangest plant imaginable. For most of the year it is leafless and looks like a giant upturned turnip. G.Sykes, found it in 1922 and said, referring to Carrol ``It must be a boojum!" The Spanish common name for this tree is Cirio, referring to its candle-like appearance.

World-volume theory on the wall:

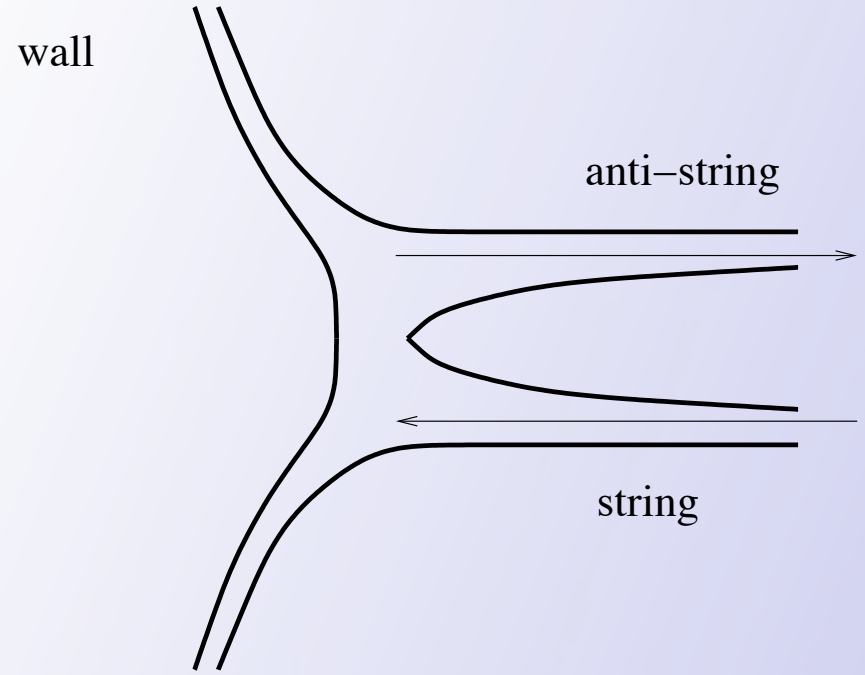
$$\frac{T_w}{2}\left(\partial_n z_0\right)^2-\frac{1}{4e^2}\left(F_{mn}^{(2+1)}\right)^2=\frac{1}{2e^2}\left(\partial_n a_{2+1}\right)^2-\frac{1}{4e^2}\left(F_{mn}^{(2+1)}\right)^2$$



In addition, the same logarithm

☺ ☺ $$F_{0i}^{2+1}=\frac{e_{2+1}^2}{2\pi}\frac{x_i}{r^2}$$

$$E_{(2+1)}^{G}=\int_{r_0}^{r_f}\frac{1}{2e_{2+1}^2}\left(F_{0i}\right)^2 2\pi r\,dr=\frac{\pi\xi}{\Delta m}\int_{r_0}^{r_f}\frac{dr}{r}=\frac{\pi\xi}{\Delta m}\ln\frac{r_f}{r_0}.$$

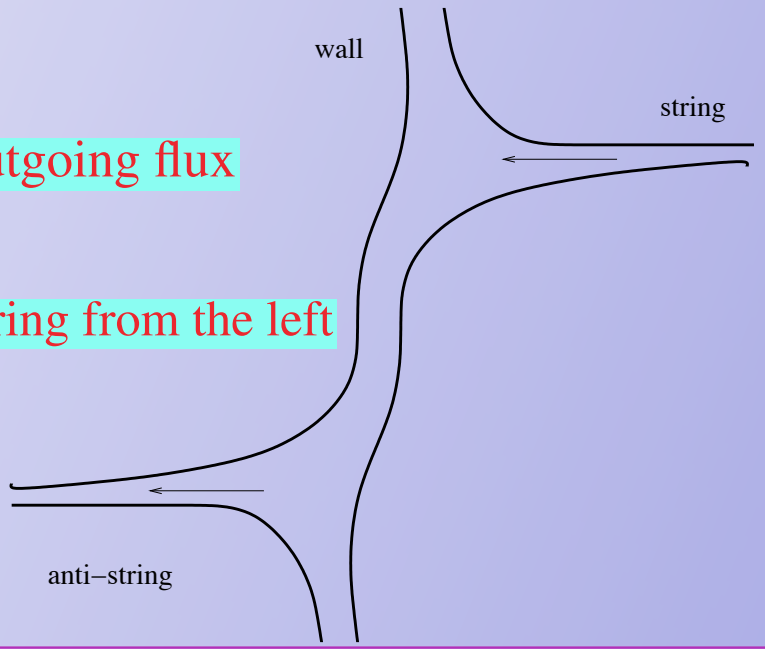


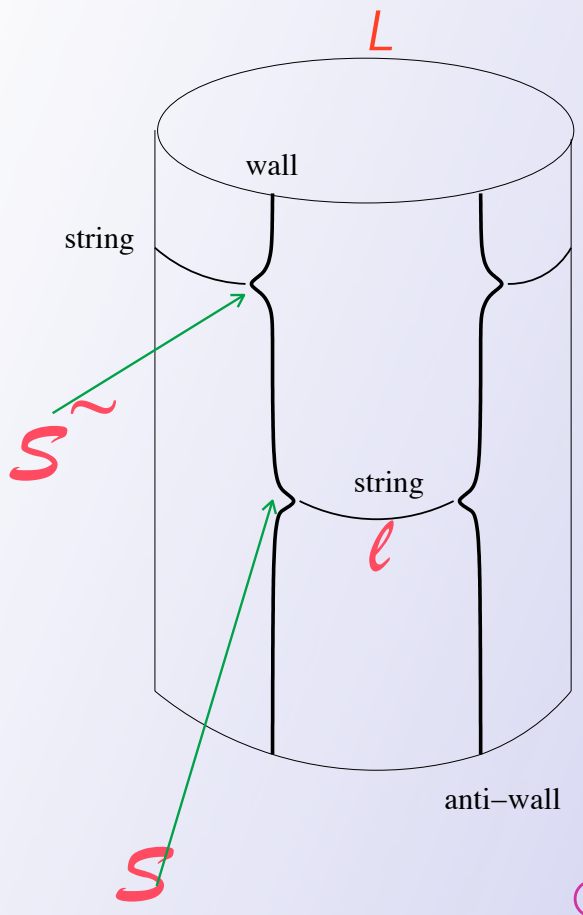
☺ String: $(n_e, n_s)=(+1, +1)$

☺ Antistring: $(n_e, n_s)=(-1,+1)$

$n_e = +1$, incoming flux, $n_e = -1$, outgoing flux

$n_s = +1$, string from the right, $n_s = -1$, string from the left





$L \gg \ell$

$$a_- \equiv \frac{1}{\sqrt{2}}\left(a^{(2)}_{2+1} - a^{(1)}_{2+1}\right) = \frac{2\pi\xi}{\sqrt{2}}\,\ell$$

$$A^-_n \equiv \frac{1}{\sqrt{2}}\left(A^{(1)}_n - A^{(2)}_n\right)$$

Crucial tests:

☺ $m_s = \sqrt{2}\,\langle a_- \rangle = 2\pi\xi\,\ell$

☺ If $m = 2\pi\xi L/\sqrt{2}$ then $m_{\tilde{s}} = 2\pi\xi\,(L-\ell)$

"real mass"

$$-\frac{1}{4e^2}F^-_{mn}F^{-\,mn} + \frac{1}{2e^2}\,(\partial_n a_-)^2 + |D_n s|^2 + \left|\tilde{D}_n\tilde{s}\right|^2 - 2a_-^2\,\bar{s}\,s - 2(m-a_-)^2\,\bar{\tilde{s}}\,\tilde{s} - e^2\left(|s|^2 - |\tilde{s}|^2\right)^2$$

3D 4D

$$e^2 = 4\pi^2 \frac{\xi}{\Delta m}$$

$$R^{-1} \sim g^2 \frac{\xi}{\Delta m}$$

We need (for string masses << other excit.)

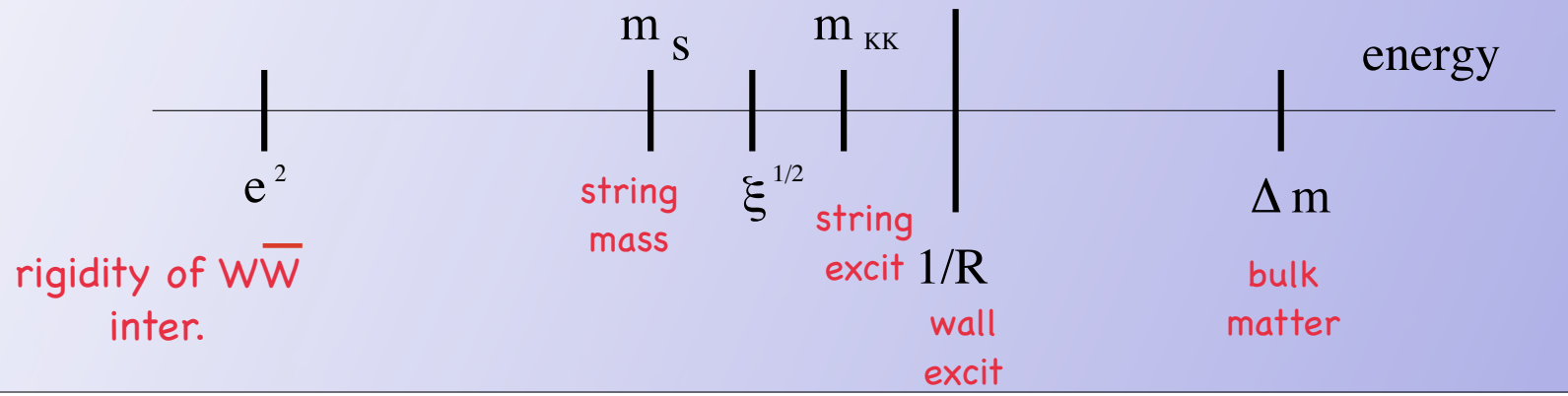
$$R^{-1} \gg \xi^{1/2}$$

not protected; assumed

If $g^2 \gg \Delta m/\sqrt{\xi} \gg 1$

Weak-strong coupling duality

$$e^2 R = \frac{16\pi^2}{g^2}$$



Physics of the world-volume theory

3D ferm. part

$$\frac{1}{e^2}\,\bar{\lambda}_-\, i\partial\!\!\!/\,\lambda_- + \bar{\psi}\, iD\!\!\!/\,\psi + \bar{\tilde{\psi}}\, iD\!\!\!/\,\tilde{\psi} - \sqrt{2}\left(a_-\bar{\psi}\psi + (m-a_-)\bar{\tilde{\psi}}\tilde{\psi}\right)$$

4 supercharges

$$\frac{1}{4\pi}\Big[\mathrm{sign}(a) + \mathrm{sign}(m-a)\Big]\,\varepsilon_{nmk}A_n^-\partial_m A_k^-$$

Induced CS

SUSY

$$\frac{D}{2\pi}\Big[|m-a_-| - |a_-|\Big] = \frac{D}{2\pi}(m-2a_-)$$

After integrating out S and S~

$$\frac{1}{2e^2}(\partial_n a_-)^2 - \frac{1}{4e^2}(F_{mn}^-)^2 + \frac{1}{2\pi}\varepsilon_{nmk}A_n^-\partial_m A_k^- + \frac{e^2}{8\pi^2}(2a_- - m)^2$$

Approximation not applic. if ℓ is close to 0 or L

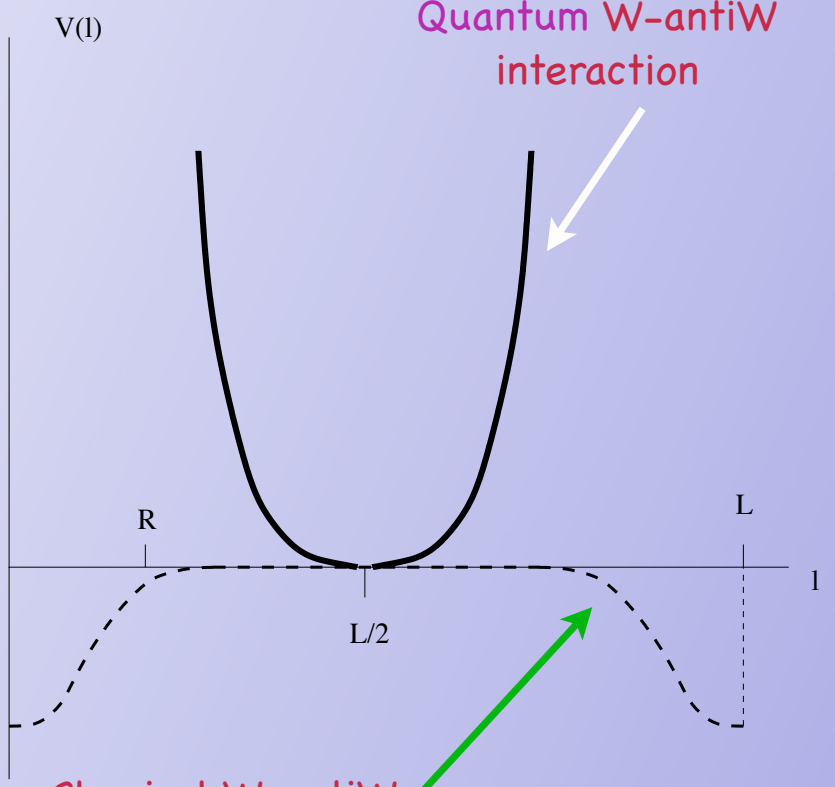
Approximation applic. on plateau

$$\langle a_- \rangle = \frac{m}{2}, \qquad \ell = \frac{L}{2}$$

Stabilization!

$$m_a = \frac{e^2}{\pi} \ll m_s$$

Infinite rigidity of strings; induces CS



Conclusions:

☺ Found strong-weak coupling holographic duality ("blurred")
in field theory; 4D SQED ↔ 3D SQED

☺ Wall-antiwall stabilization at quantum level;

☺ Wall-antiwall interaction due to string loop **nonexponential**;

☺ ☺ ☺ Generalization to non-Abelian?