Large n multi-vortices and multi-monopoles

Stefano Bolognesi

Niels Bohr Institute

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Introduction

Topological Solitons in a Gauge-Higgs theory:

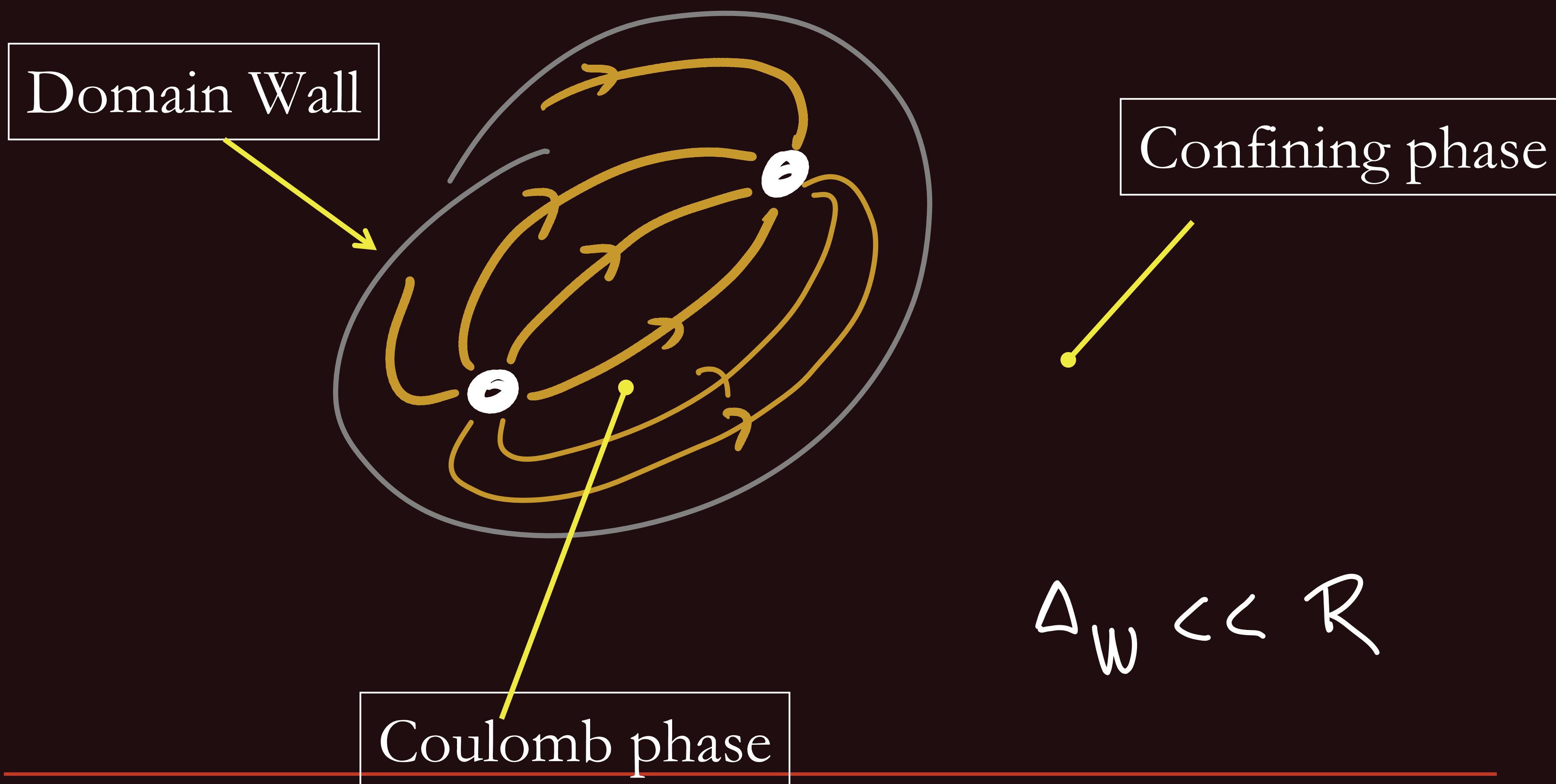
$$\mathcal{L} = -FF - |D\phi|^2 - V(|\phi|)$$

Vortex $\quad \pi_1(U(1)) = \mathbb{Z}$

Monopole $\quad \pi_2\left(\frac{SU(2)}{U(1)}\right) = \mathbb{Z}$

Large n

Bag Models ot Hadrons



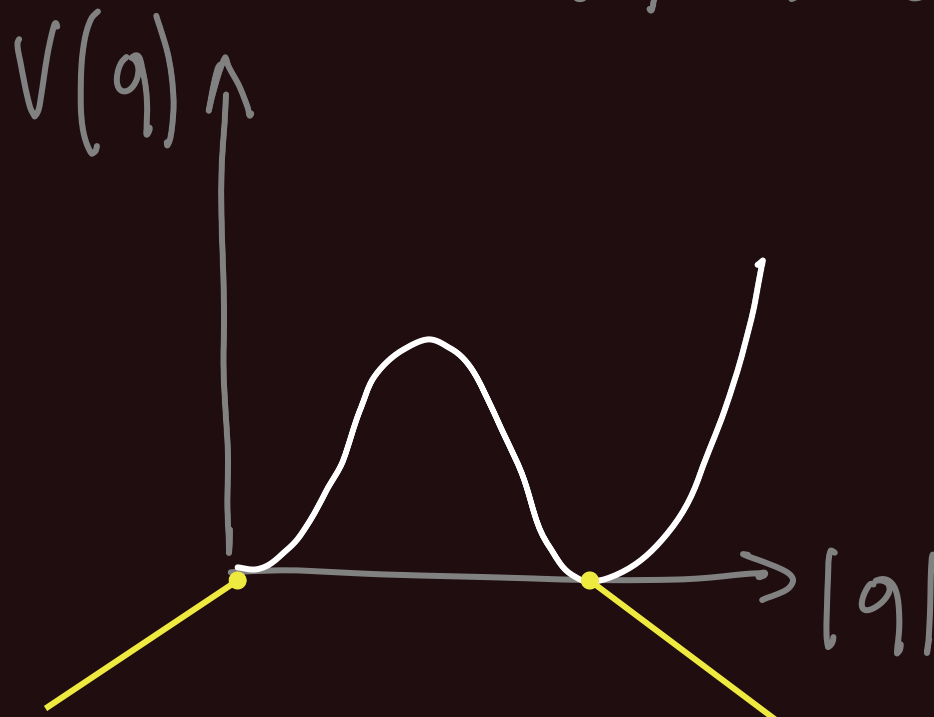
Genesis of the idea

$$\mathcal{N}=2 \xrightarrow{W(\Phi)} \mathcal{N}=1$$

Many discrete vacua survive

$$U(N) \longrightarrow U(1)^n$$

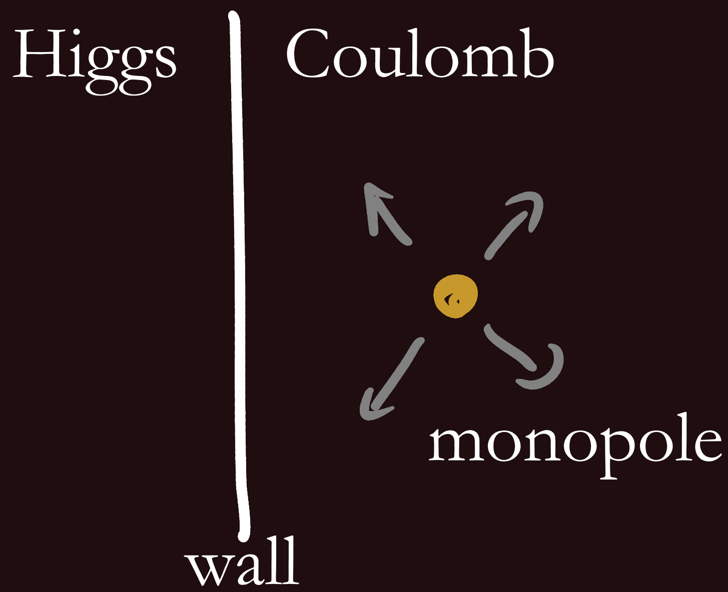
$\boxed{\text{Toy model}}$

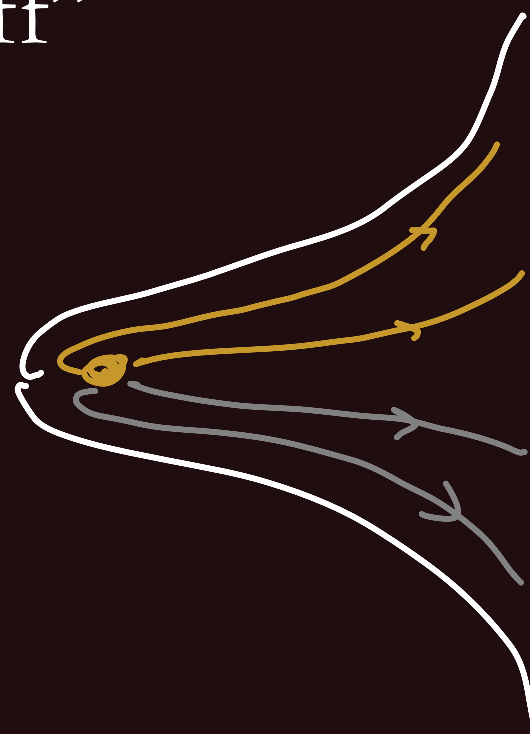


Coulomb phase

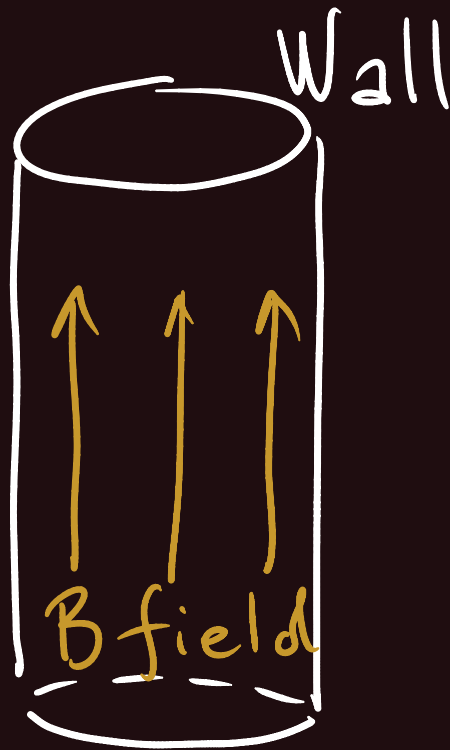
Higgs phase

Vortex and Wall are "made of the same stuff"





<u>Question:</u> Is there a limit in which this equivalence is exact?



$$T(R) = \underbrace{\frac{\Phi_B^2}{2\pi R^2}}_{\text{Repulsion}} + \underbrace{T_W\, 2\pi R}_{\text{Attraction}}$$

$$R_V \sim \frac{n^{2/3}}{T_W^{1/3}} \qquad T_V \sim T_W^{2/3}\, n^{2/3}$$

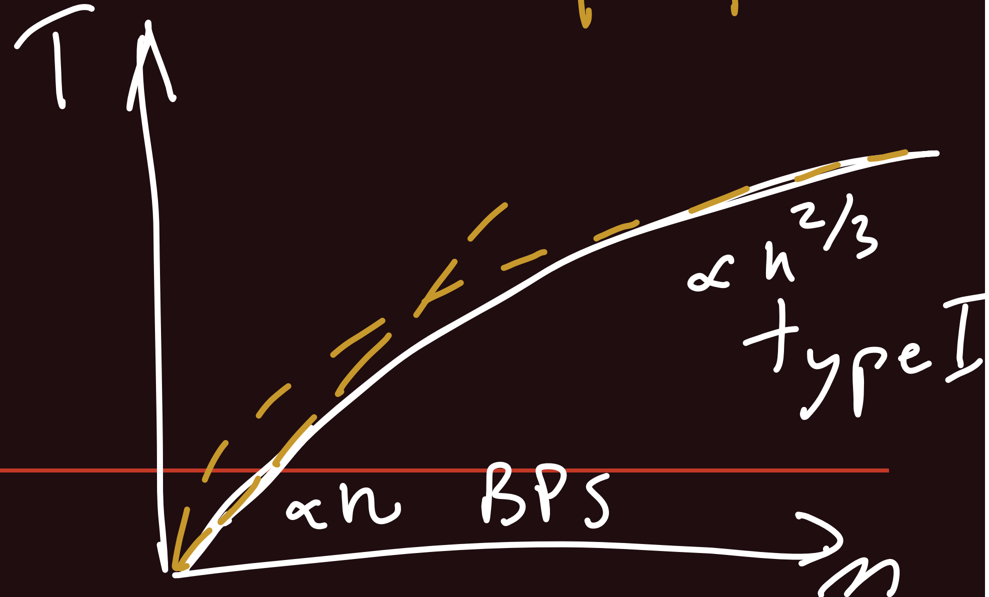
Softly Broken N=2

$$W = \tilde{Q}\,\Phi\, Q - m\,\tilde{Q}Q + \mu\,\Phi^2$$

Color-Flavor locked phase $\phi = m$ $\tilde{q}q = \mu m$

Coulomb phase $\phi = 0$ $q = \tilde{q} = 0$

$$\frac{e^2\mu}{m} \ll 1$$

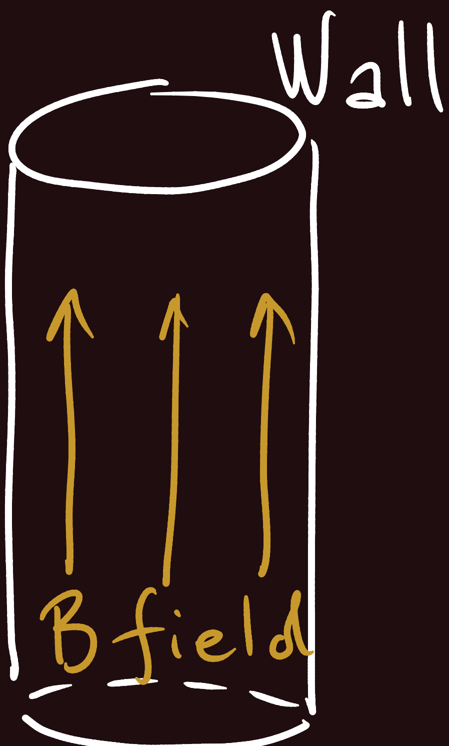


Almost BPS

And now the conjecture…

BPS potential

$$V(q) = \frac{e^2}{2}\left(|q|^2 - v\right)$$



$$T(R) = \frac{2\pi n^2}{e^2 R^2} + \frac{e^2 v^2}{2}\pi R^2$$

$$T_V = 2\pi n v$$

It saturates exactly the BPS bound!

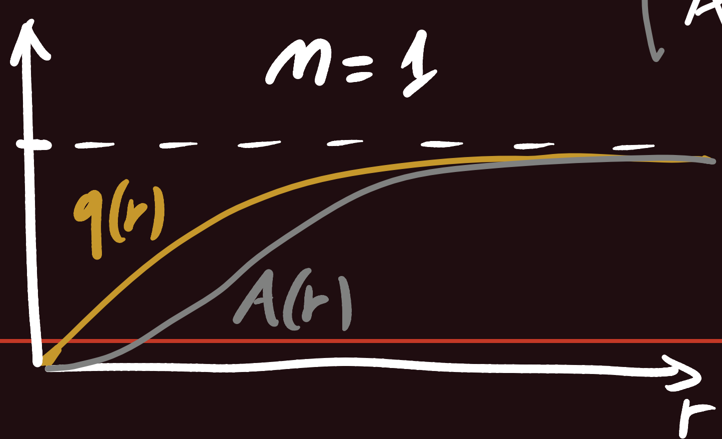
Multi-vortex ansatz

$$q = e^{in\theta} q(r) \qquad A_\theta = \frac{n}{r} A(r)$$

Profile functions

Differential Equations:

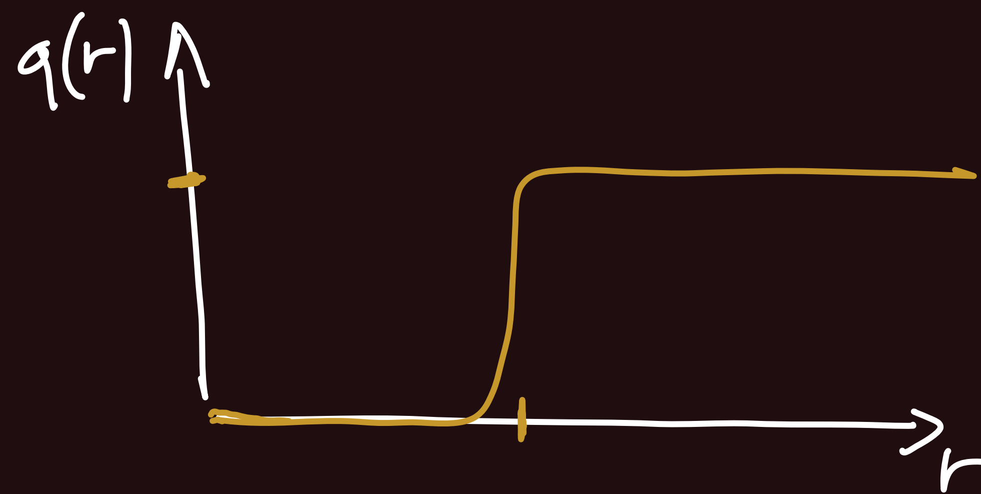
$$\begin{cases} q'' + \frac{1}{r} q' - n^2 \frac{(1-A)^2}{r^2} q - \frac{\delta V}{\delta q} = 0 \\ A'' - \frac{1}{r} A' + e^2 (1-A) q^2 = 0 \end{cases}$$



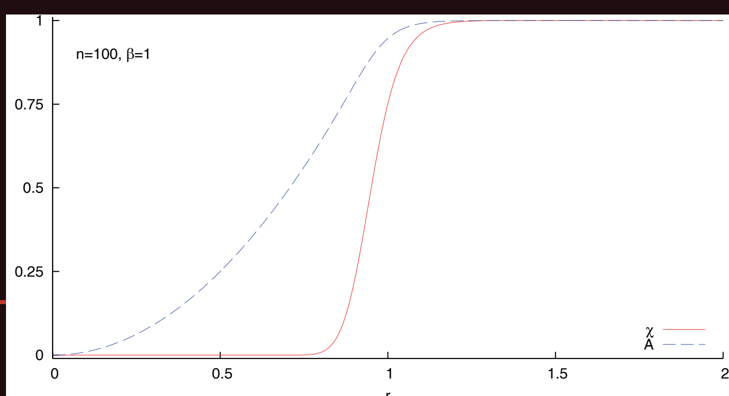
Numerical Proof

Length Rescaling

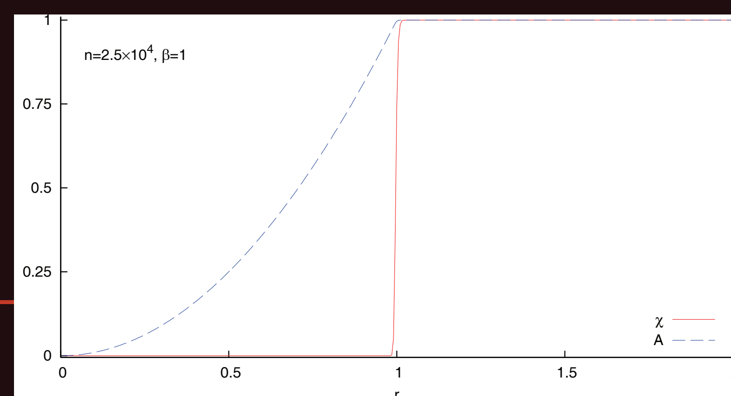
$$r \to \frac{r}{n}$$



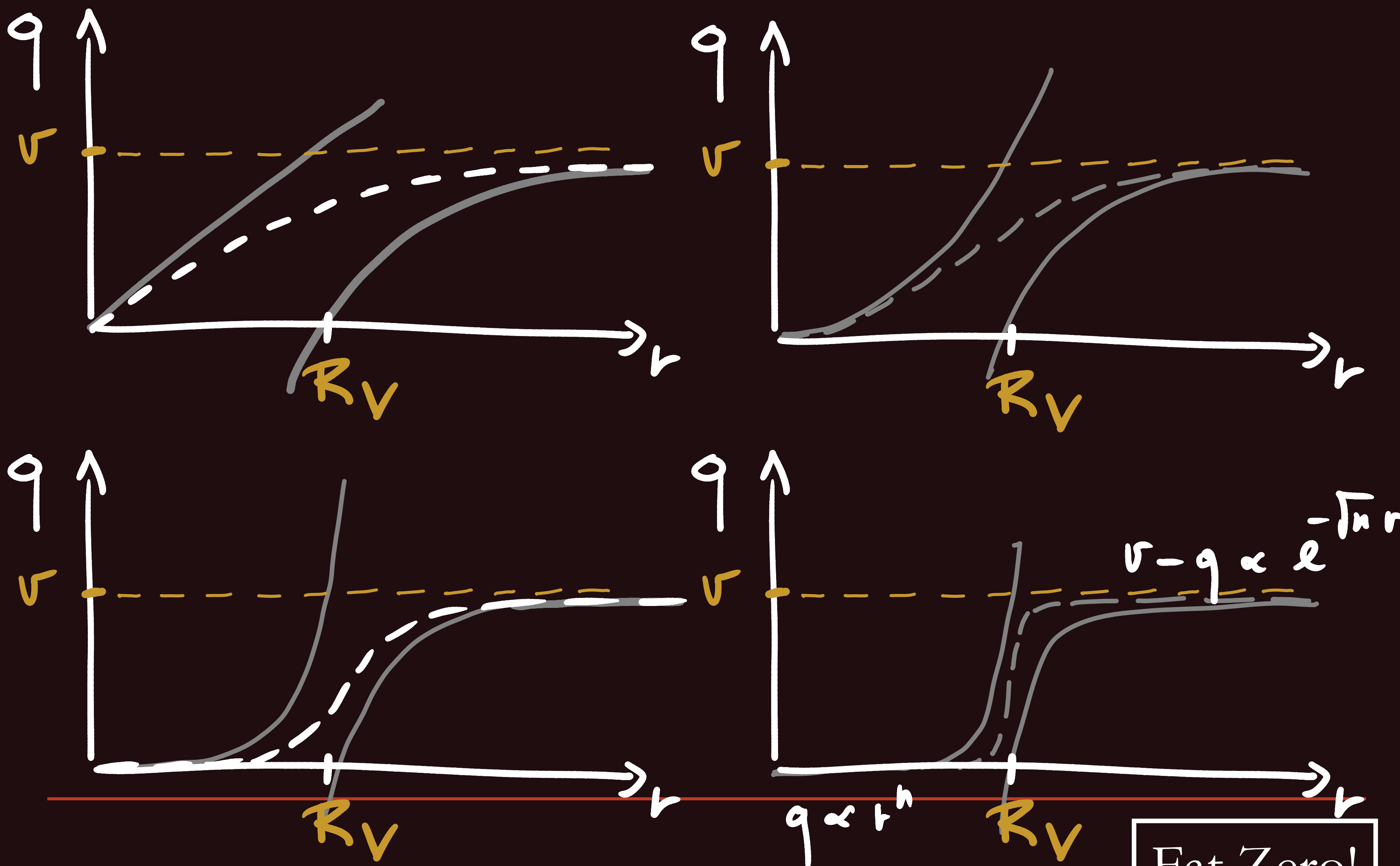
BPS n=100



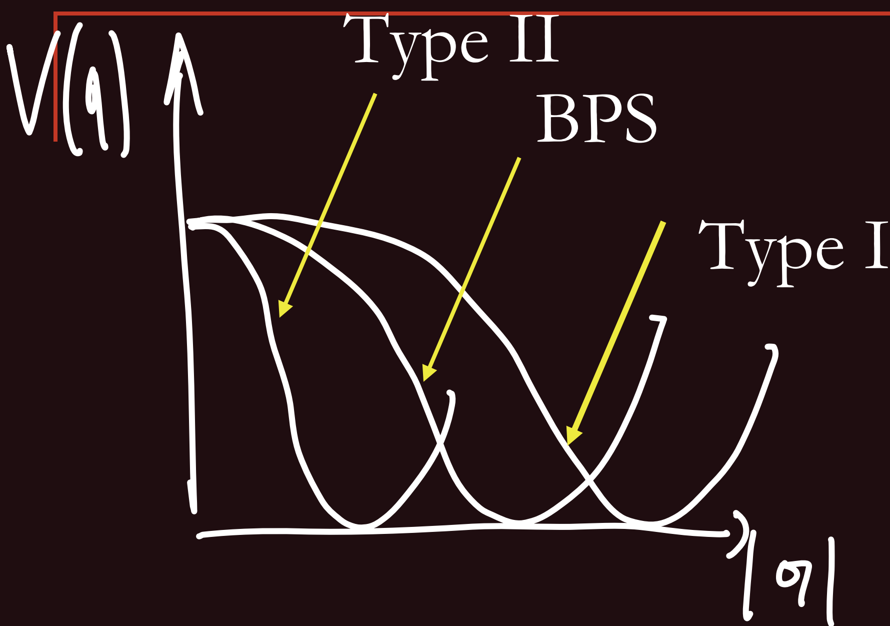
BPS n=25,000

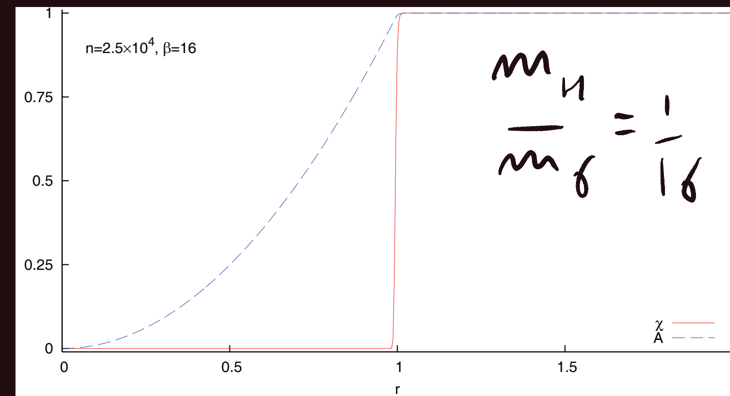


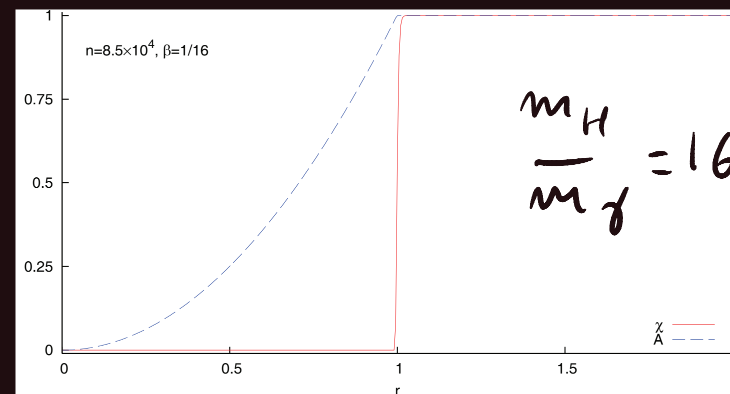
Why a step function?

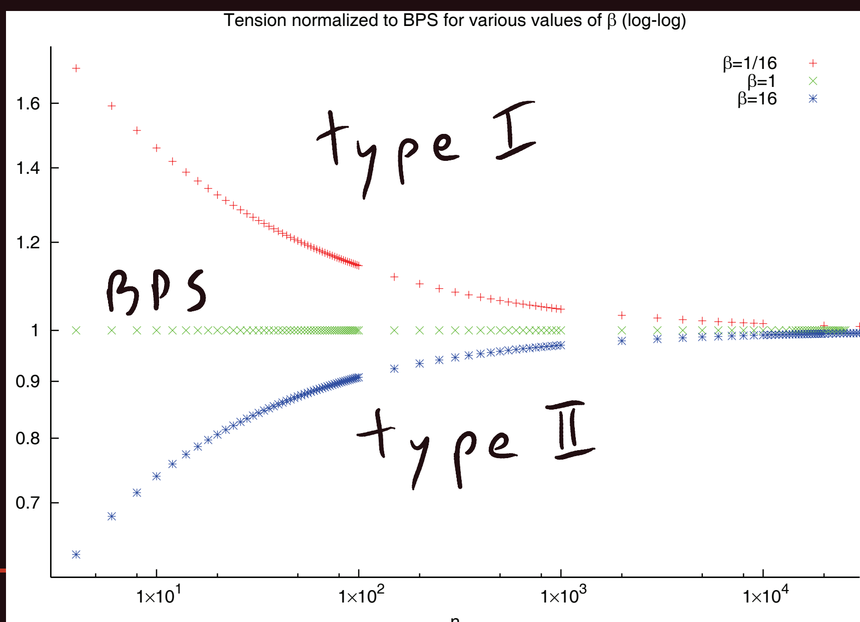


Fat Zero!









It works for every potential!

with S.B.Gudnason

Another example…

Confining Strings

$SU(N)$

k_string

$T(k,N)$

Tension of the k_string

Casimir formula

$$R(k,N)=\frac{T(k,N)}{T(1,N)}$$

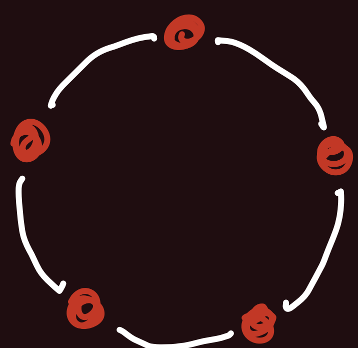
Sine formula

Assumtion: n_strings are vortices of a "dual" U(1)

Center of SU(N) $\mathbb{Z}_N$

$$\begin{pmatrix} e^{i\frac{2\pi k}{N}} & & \\ & \ddots & \\ & & e^{i\frac{2\pi k}{N}} \end{pmatrix}$$

Interpolating U(1)



$$\begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \\ & & & -(N-1) \end{pmatrix}$$

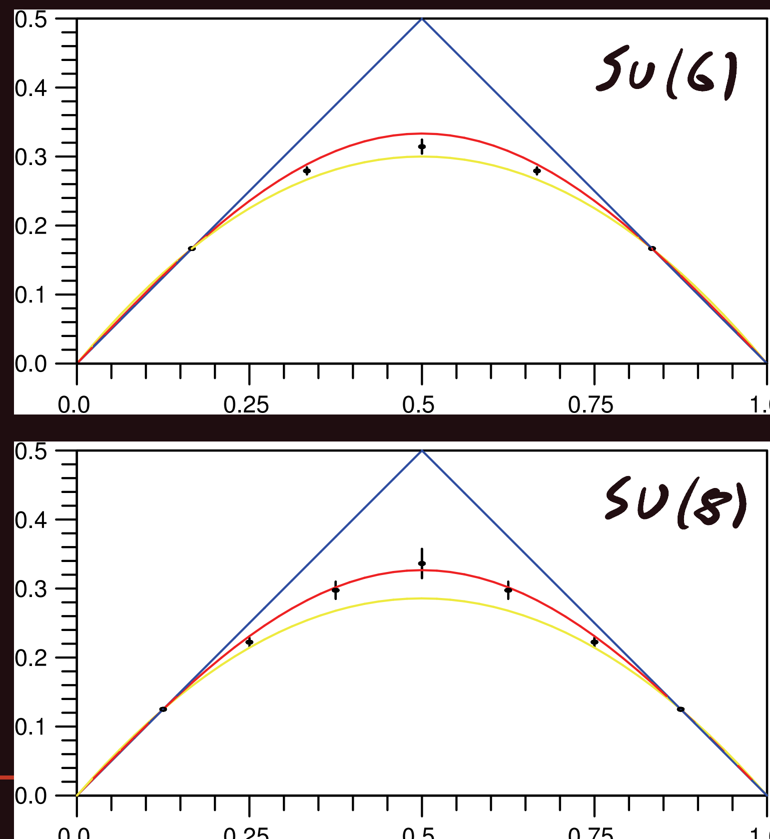
Unique up to gauge invariace

Consequence in the saturation limit

$N \to \infty$ $\frac{K}{N} = x$ fixed

$$R(x,N) = \frac{1}{N}\,\frac{T(K,N)}{T(1,N)}$$

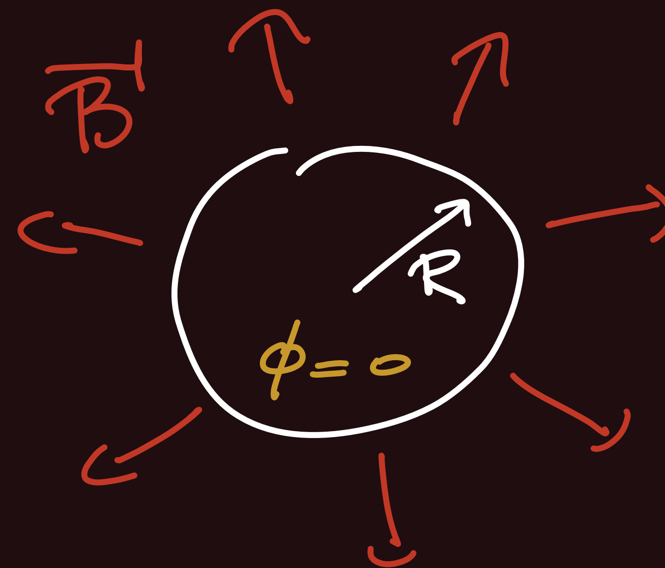
$\min(x, 1-x) + \ldots$



Teper&Lucini

Magnetic Bag

$$E(B) = \int_R^\infty \frac{B^2}{2} = \frac{2\pi n^2}{R}$$



Tail of the Higgs field

$$E(\phi) = \int_R^\infty \frac{\vec{\nabla}\phi^2}{2} = 2\pi v^2 R$$

$$M(R) = \frac{2\pi n^2}{R} + 2\pi v^2 R$$

The minimization gives: $\begin{cases} R_m = \dfrac{n}{v} \\ M_m = 4\pi n v \end{cases}$

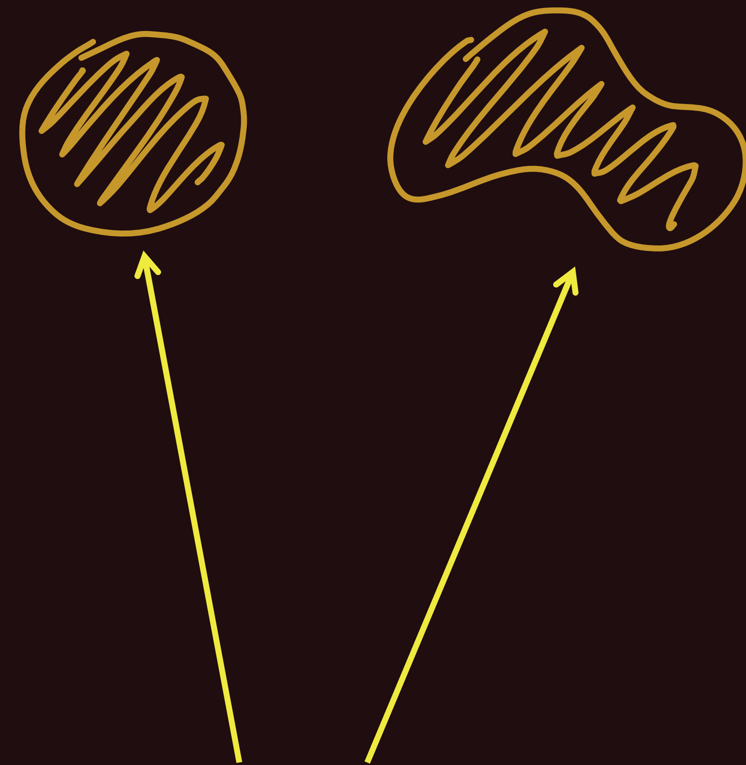
It saturates exactly the BPS bound

Moduli Space

Multi-vortices

$$T(R) = \frac{\phi_B^2}{2\pi R^2} + \varepsilon_0 \pi R^2$$

$$\boxed{T(A) = \frac{\phi_B^2}{2A} + \varepsilon_0 A}$$

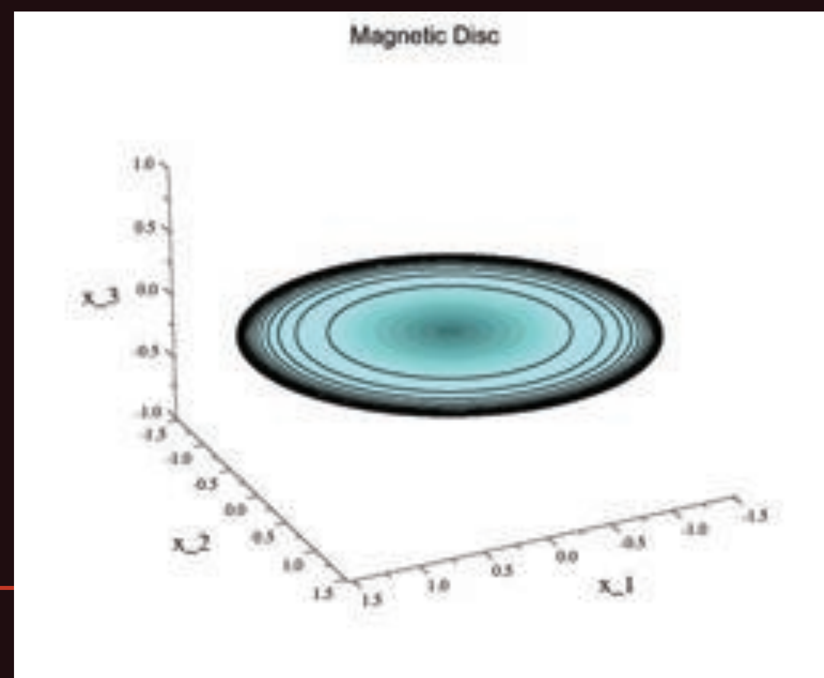


The moduli space is the closed surfaces with constant area!

Axial Symmetric multi-monopole

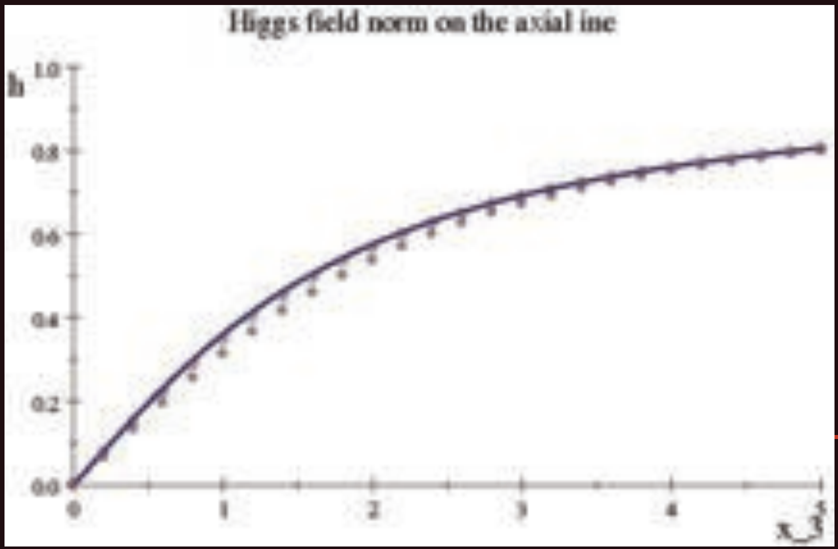
$S^2 \quad |r| \to \infty$ $\qquad$ $S^2 \quad |\phi| = v$

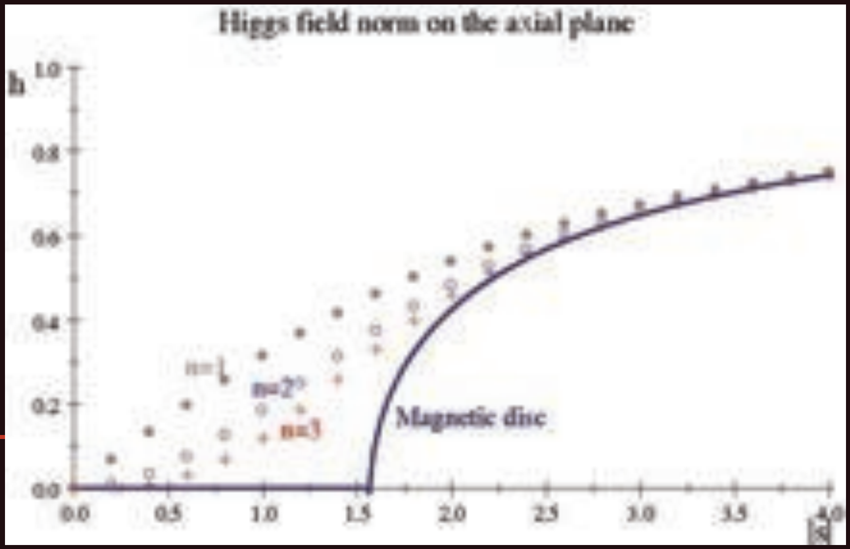
$$\begin{cases} \phi_3 \sim z \\ \phi_1 + i\phi_2 \sim (x+iy)^n \end{cases}$$



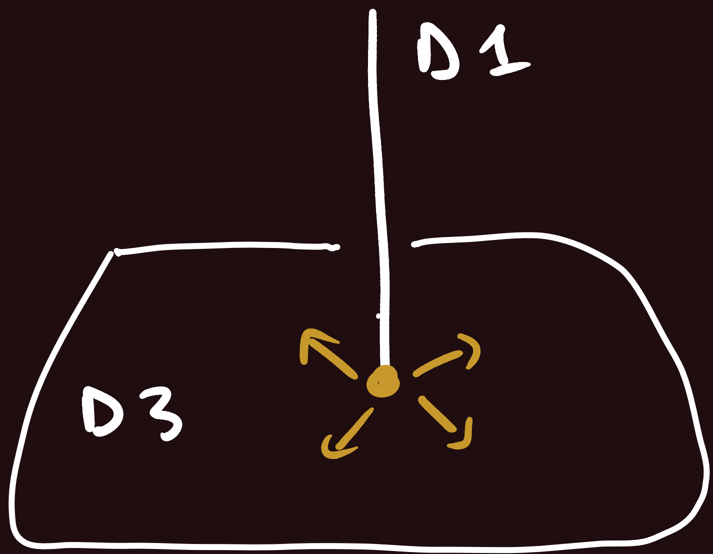
Ward-Prasad-Rossi VS Magnetic Disc

$$\phi(r,\theta) = \begin{cases} 1 - \dfrac{1}{r} \sum_{\ell} \dfrac{(-1)^{\ell}}{2\ell+1} \left(\dfrac{\pi}{2r}\right)^{2\ell} P_{2\ell}(\cos\theta) & r \geq \dfrac{\pi}{2} \\[2ex] \dfrac{2}{\pi} \sum_{\ell} \dfrac{(-1)^{\ell}}{2\ell+1} \left(\dfrac{2r}{\pi}\right)^{2\ell+1} P_{2\ell+1}(\cos\theta) & r \leq \dfrac{\pi}{2} \end{cases}$$

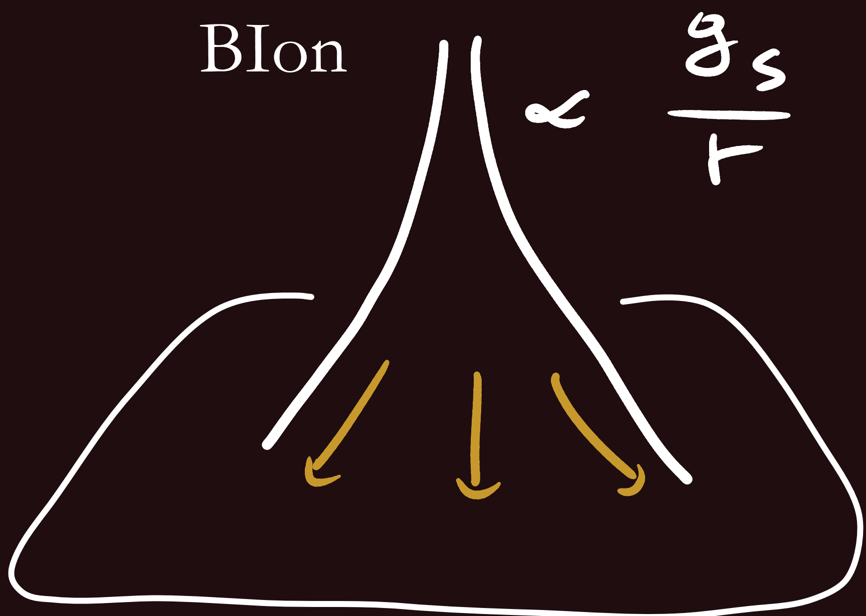




String Theory



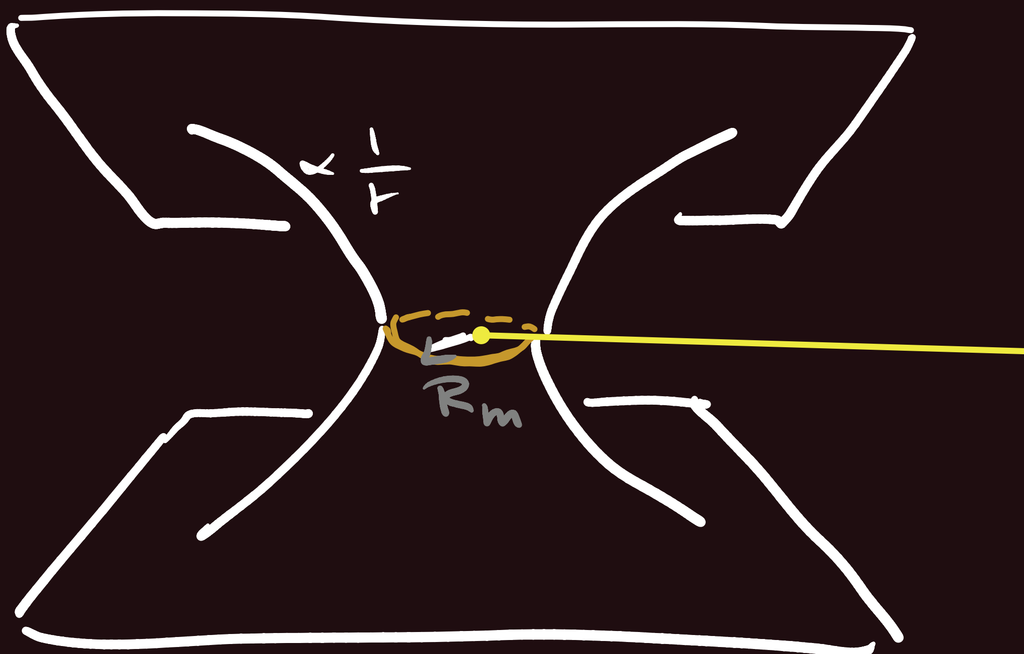
Monopole



BIon

The BIon is a solution of the abelian DBI action

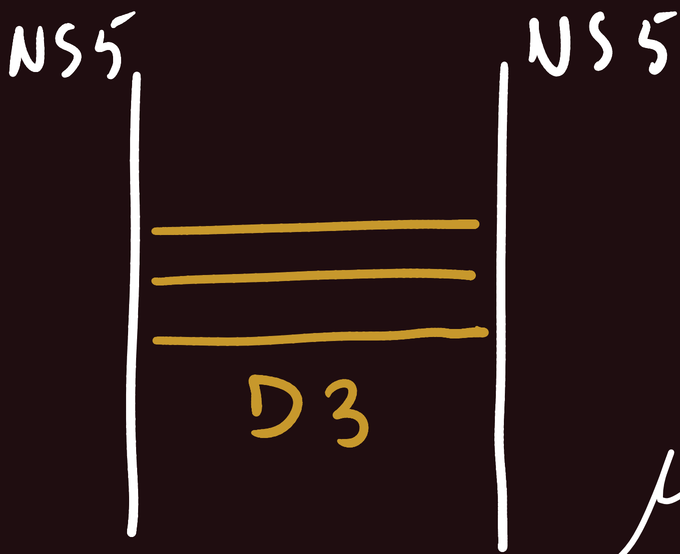
Magnetic bag in string theory



Inside the bag the D3 branes are overlapped

It is like two “cutted” abelian BIons

‘t Hooft and multi-monopole large n



Low energy theory

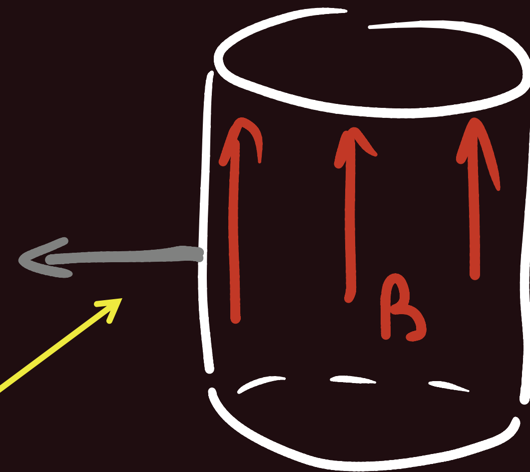
$U(n)$ $\mathcal{N} = 4$ $d = 2+1$

$$\mathcal{M}_{\text{Coulomb branch}} = \mathcal{M}_{\text{Multi-monopoles}}$$

$$\frac{1}{g_{YM}^2} = \frac{v}{g_s} \qquad R_m = \frac{n}{v}$$

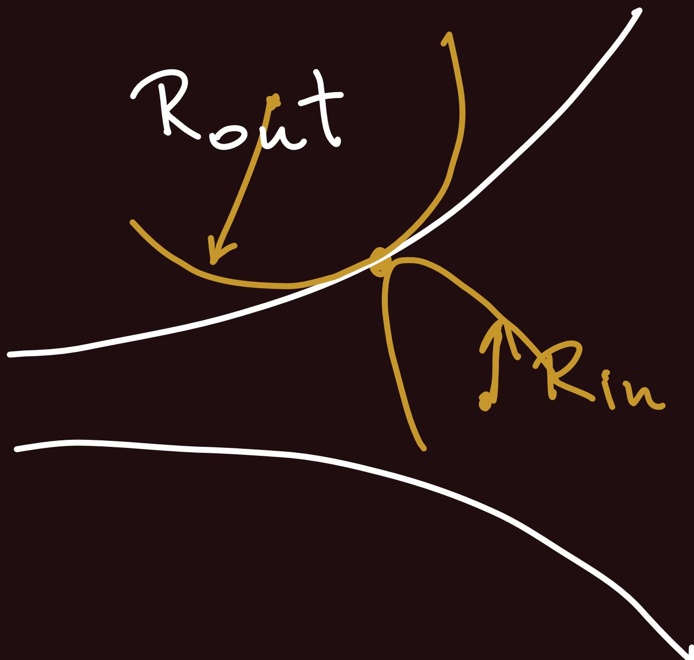
$\boxed{\text{‘t Hooft rescaling = Length rescaling}}$

Soliton Junctions



$$\boxed{\frac{T'}{2\pi R} = -\frac{B^2}{2} + \frac{T_W}{R}}$$

Master Equation



$$\vec{B} = \vec{\nabla}\varphi$$

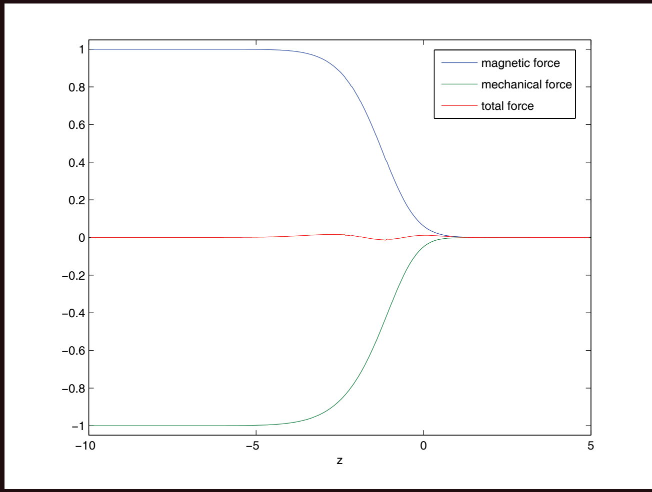
$$\nabla^2 \varphi = 0$$

+ Neuman

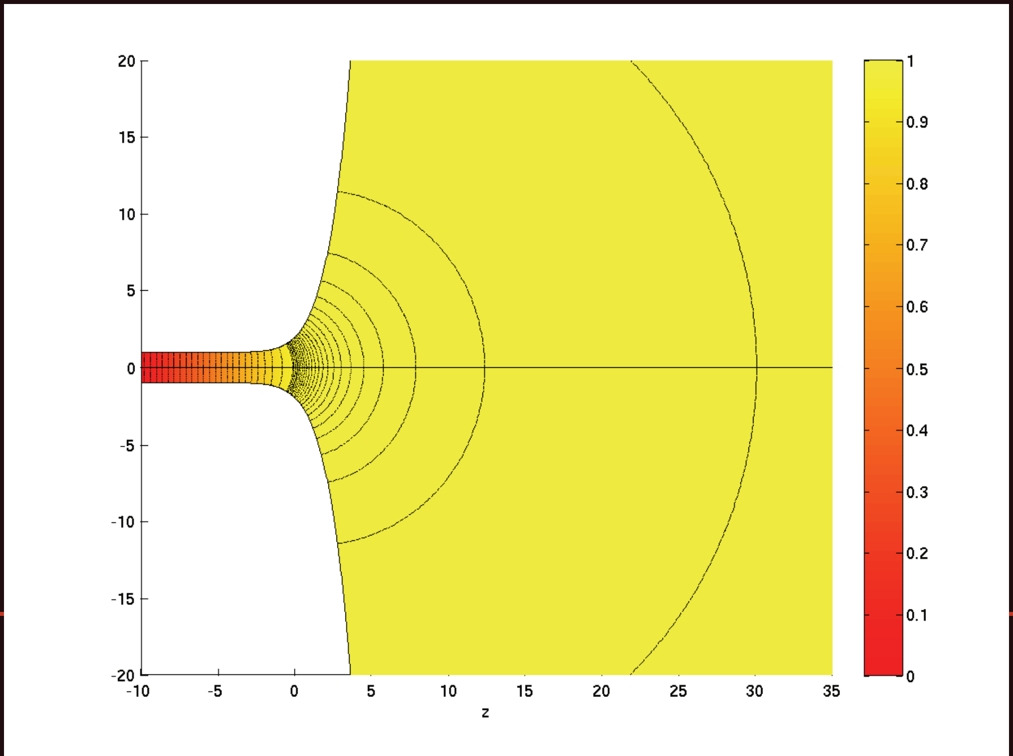
$$-\frac{B^2}{2} + T_W\left(\frac{1}{R_{in}} - \frac{1}{R_{out}}\right) = 0$$

Numerical Analisis

Forces



The Junction



Conclusions

- Large n limit of topbological solitons
- Bag models description
- Simple and General

- Chern-Simons vortices (FQHE)
- Large n limit of textures (Skyrmions)
- …