The Pomeron and Gauge/String Duality

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The Pomeron:
a word that provokes excitement in a few,
fear in some,
and confusion in many...

The subject is inherently murky:

- mysterious resummed perturbation on the one hand [BFKL]
- nonperturbative string-like phenomena on the other hand

We will see that in gauge theories with string-theoretical dual descriptions, the Pomeron emerges swiftly and without confusion from tree-level string scattering amplitudes.

Both the IR Pomeron and the UV Pomeron are dealt with, and this is accomplished in a single unified step.

Questions:

- What is the Pomeron?

The Pomeron is the universal, colorless, flavorless coherent gluonic excitation that dominates hadronic elastic scattering

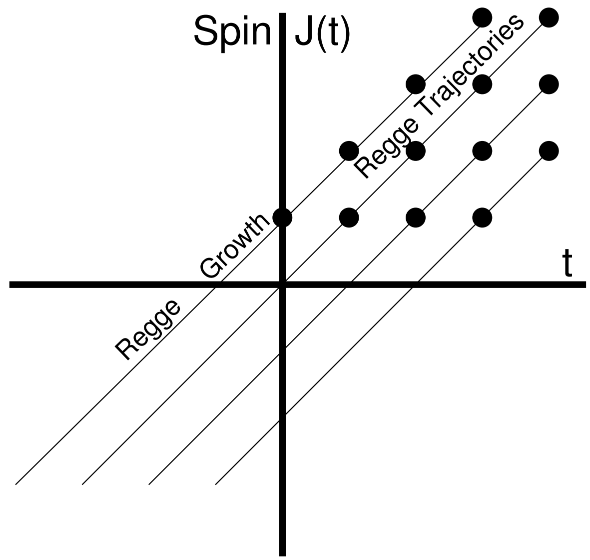
... *at very large s, small t, and very large N* ...

After Mellin transform, it contributes the leading singularity in the angular momentum ("j") plane.

Existence of Pomeron is universal in gauge theory;
intimately tied to gluons and to $T_{\mu\nu}$

Suitable for study at large λ in non-QCD theories.

String amplitudes $\rightarrow$ Regge behavior $\mathcal{A}(s,t) \sim s^{\alpha_0 + \alpha' t}$



Leading singularity at every t is the leading Regge trajectory, which includes the graviton (lightest spin-two particle)

Pomeron dominates scattering is at $t < 0$, large s.

At $t > 0$, leading trajectory of hadron resonances: lowest is lightest spin-two state.

Questions

- What is the Pomeron as a function of s (or j), t (or $x_\perp$), N, and the 't Hooft coupling $\lambda \propto \alpha_s N$?
- Is there only one? or is there an IR Pomeron and a UV Pomeron representing separate phenomena?
- What is the asymptotic (in s) behavior of the Pomeron? Are there computable subleading effects?
- What is the Pomeron in a conformally invariant theory?
- What is the effect of a running coupling on the UV Pomeron?
- What is the effect of confinement on the Pomeron?

- Why does the Pomeron give Regge phenomena, analogous to string theory, in the IR?
- And what happens to this relation in the UV?
- What is the relationship of the Pomeron to integrability?
- What is the technical relation between BFKL and DGLAP in general?
- What is the technical relation between BFKL and DGLAP in a theory with a running coupling and confinement?
- What happens beyond the leading Pomeron approximation?
- How is unitarity realized?
- How is the Froissart bound realized?

String theory provides a unified context to rephrase these questions, offer new perspectives, eliminate some proposals, reduce confusions, and even (in the large λ, large N regime) provide some answers.

- String theory is useful in a class of gauge theories with $1 \ll \lambda \ll N$ (but N finite)

- String answers are resummmed to all orders in $\lambda \ln s$ (like BFKL) and *to all orders in* λ (unlike BFKL!)

- It can handle any t and a range of s in a uniform manner; thus it can give fully analytic results that unify the UV and IR, the timelike and the spacelike

Emphasize: This is not about making another phenomenological model for gauge theory!

We obtain a concrete technically-controlled approximation scheme for computations in theories which are QCD-like: $d = 4$, gluons, matter, continuum, Minkowski, conformal, running, confining. Expansion in $1/N$, $1/\sqrt{\lambda}$.

We will find definite answers that are universal to large-λ theories and can plausibly be analytically continued from large λ to small λ.

Only one sticking point: it relies upon a conjecture relating string theory and gauge theory (but one which is ever-increasingly believable).

Work of Brower, Polchinski, MJS, Tan 06

See intuition of Levin Tan 93

other attempts
Janik and Peschanski 99 ; Janik 00 ; Andreev and Seigel 04

Field Theory side: many works starting with
Balitsky and Lipatov, Faddeev and Kuraev;
. . .
Kotikov Lipatov 00, 02
Kotikov Lipatov Onishchenko Velizhanin 04

First I will show some graphical results.

Two-hadron elastic-scattering amplitude has an analytic structure that depends on the Pomeron propagator*

**the hadron-independent part of the amplitude... caution because I am ignoring the hadron wave functions for today's discussion – many interesting transient effects depend on their details.*

It is the analytic structure of this Pomeron in the j plane, as a function of t, that we will seek to understand.

- Pomeron in QCD? too hard.
- Pomeron in large-N QCD? still a bit too hard.
- Pomeron in a large-N theory which is UV conformal and IR confining? Maybe we can do this.

Example: $\mathcal{N}=4$ *YM broken to* $\mathcal{N}=0$ *or* $\mathcal{N}=1$ *YM*

This theory has a conformal ultraviolet with a constant λ and confinement at low energies with scale Λ.

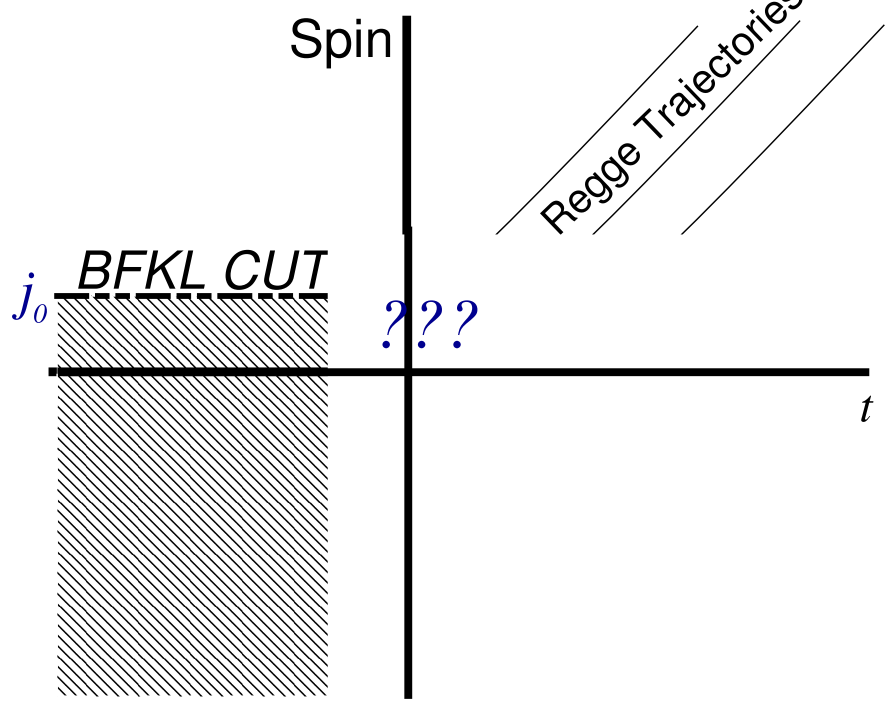
We will focus on this theory

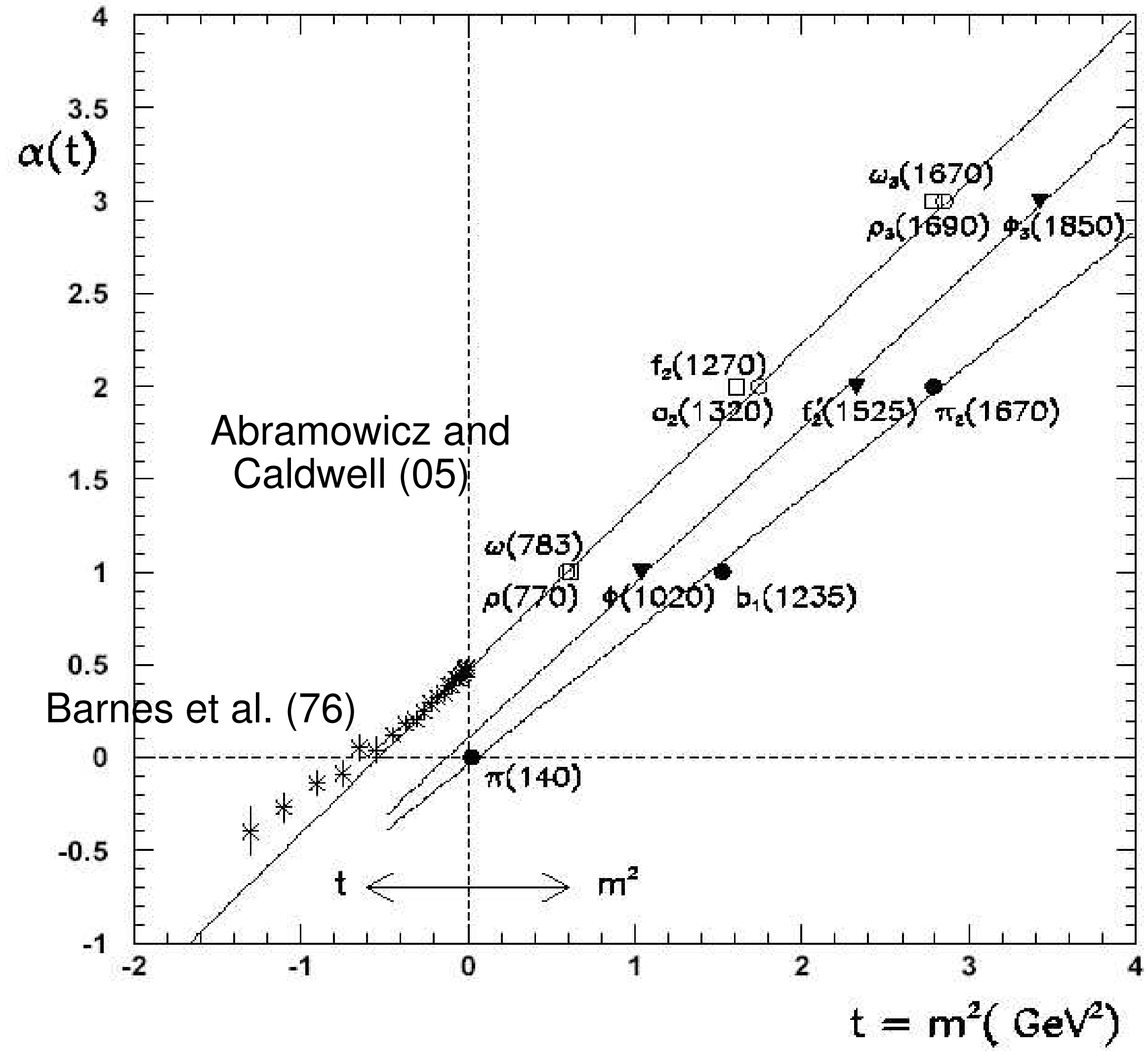
... but the results below are much more general ...

What do we think we know?

Analytic Structure of Pomeron Propagator

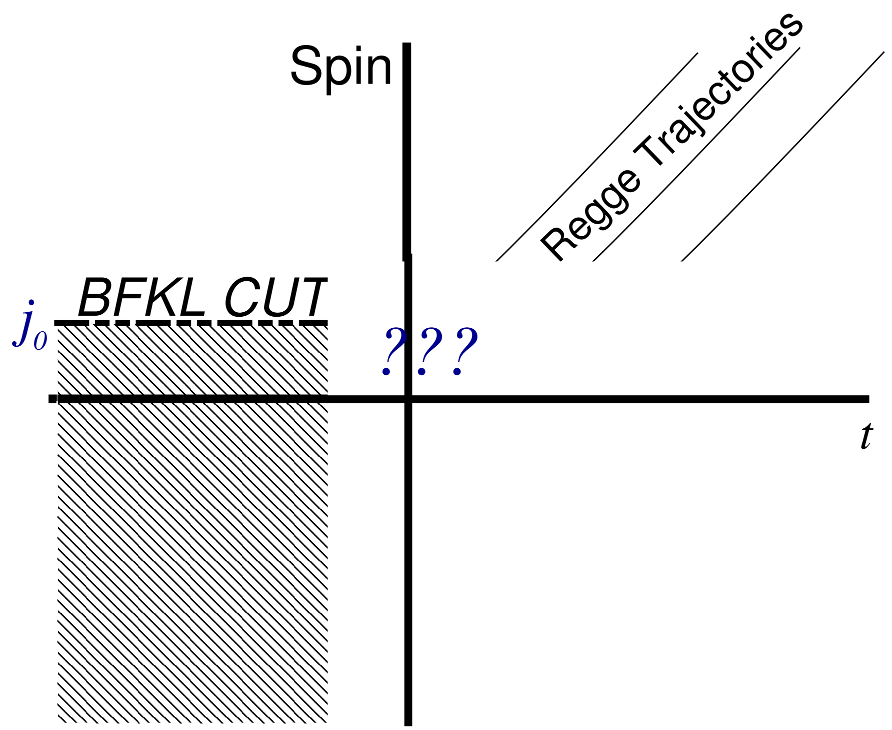
Conformal UV, Confining IR



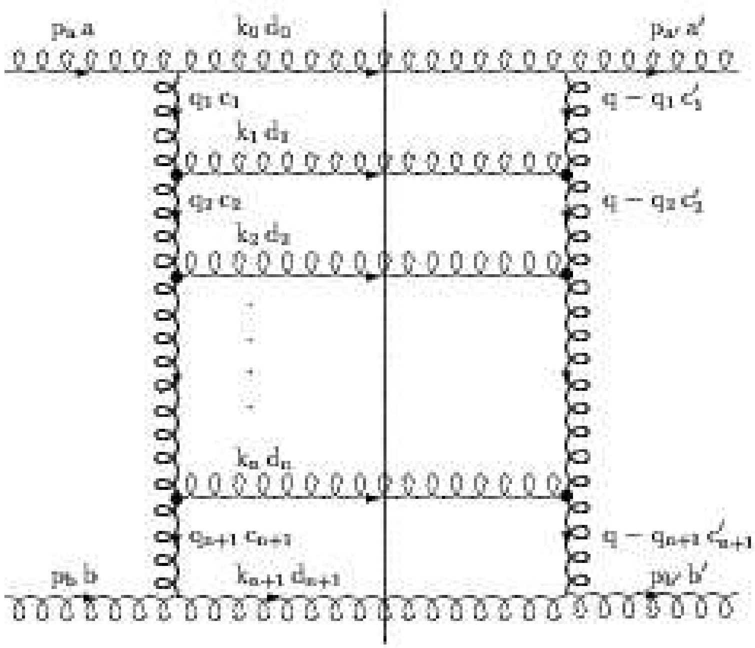

α(t)
ω3(1670)
ρ3(1690) φ3(1850)
f2(1270)
a2(1320) f2'(1525) π2(1670)
Abramowicz and Caldwell (05)
ω(783)
ρ(770) φ(1020) b1(1235)
Barnes et al. (76)
π(140)
t
m²
t = m²(GeV²)

Analytic Structure of Pomeron Propagator

Conformal UV, Confining IR (large N)



BFKL Resummation



BFKL at $t = 0$

$$\mathcal{A}_{2\to 2} = \int \frac{dk_\perp}{k_\perp} \int \frac{dk'_\perp}{k'_\perp} \Phi_1(k_\perp) K(s; k_\perp, k'_\perp) \Phi_2(k'_\perp)$$

After much work, s very large, obtain power of s times a *diffusion kernel* with space = $\ln k_\perp$, time = $\ln s$

$$K(s, k_\perp, k'_\perp) \approx s^{j_0} \frac{e^{-\left[(\ln[k'_\perp/k_\perp])^2/4\mathcal{D}\ln s\right]}}{\sqrt{4\pi\mathcal{D}\ln s}}$$

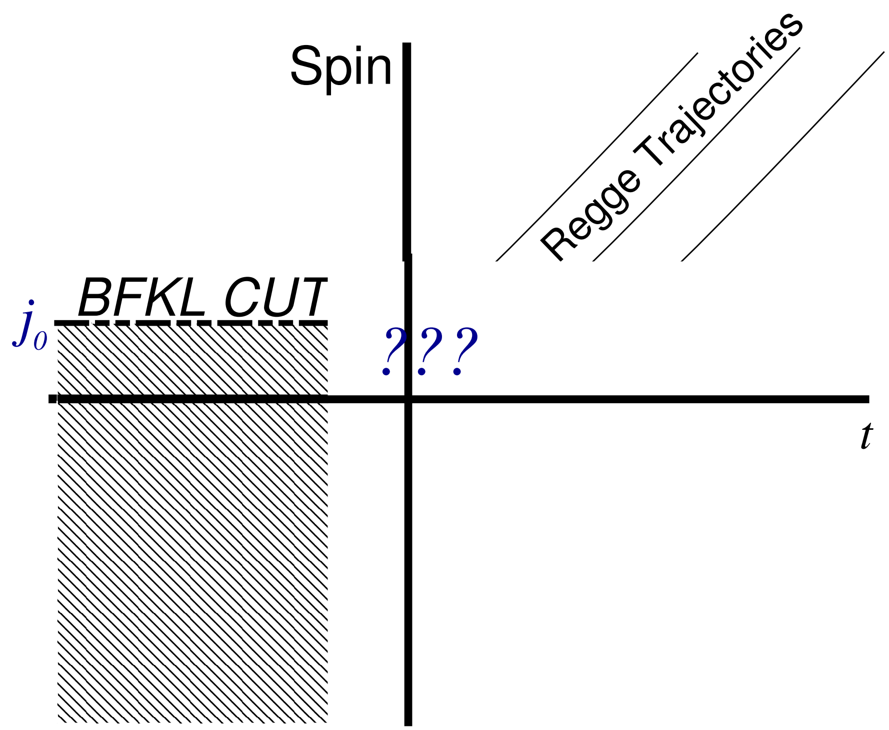
where

$$j_0 = 1 + \frac{4\ln 2}{\pi}\alpha N \ , \quad \mathcal{D} = \frac{7\zeta(3)}{\pi}\alpha N \ .$$

Unfortunately not reliable at $t = 0$ but j_0 independent of t in UV conformal case, so j_0 and cut same at all t.

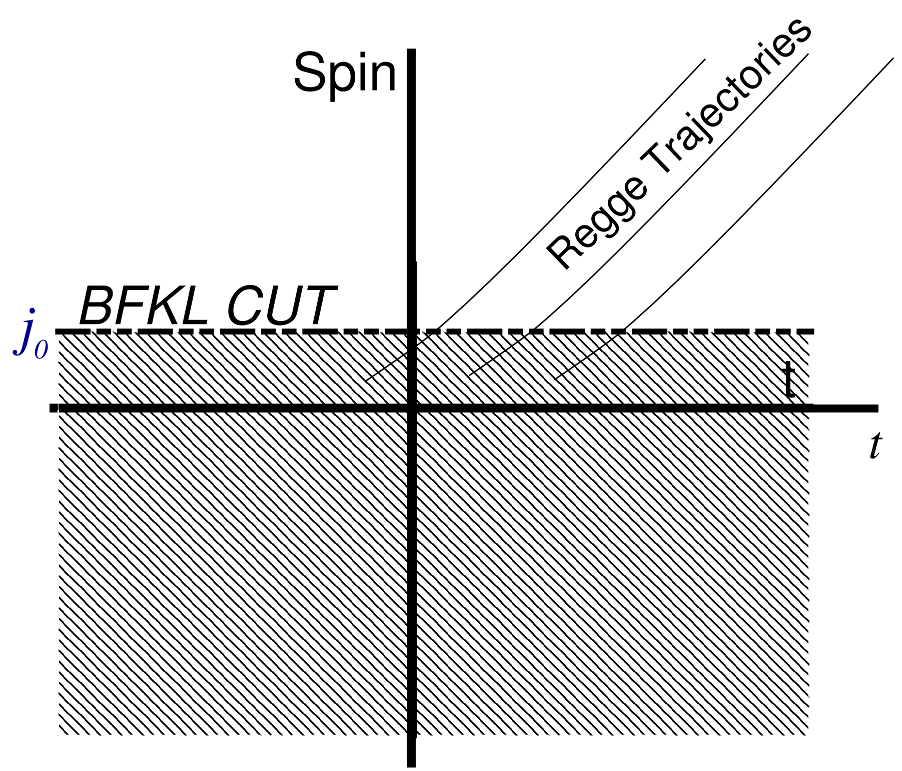
Analytic Structure of Pomeron Propagator

Conformal UV, Confining IR (large N)



Analytic Structure of Pomeron Propagator

Conformal UV, Confining IR (large N)



The last figure is schematic; will be more precise below.

Suppose true — a cut starting at j_0 for any t.

BFKL give j_0 at leading order in $\lambda \propto \alpha_s N$.

What happens at higher orders in λ?

Is the cut present at all orders? Yes, CFT argument.

Ok – *What is $j_0(\lambda)$?*

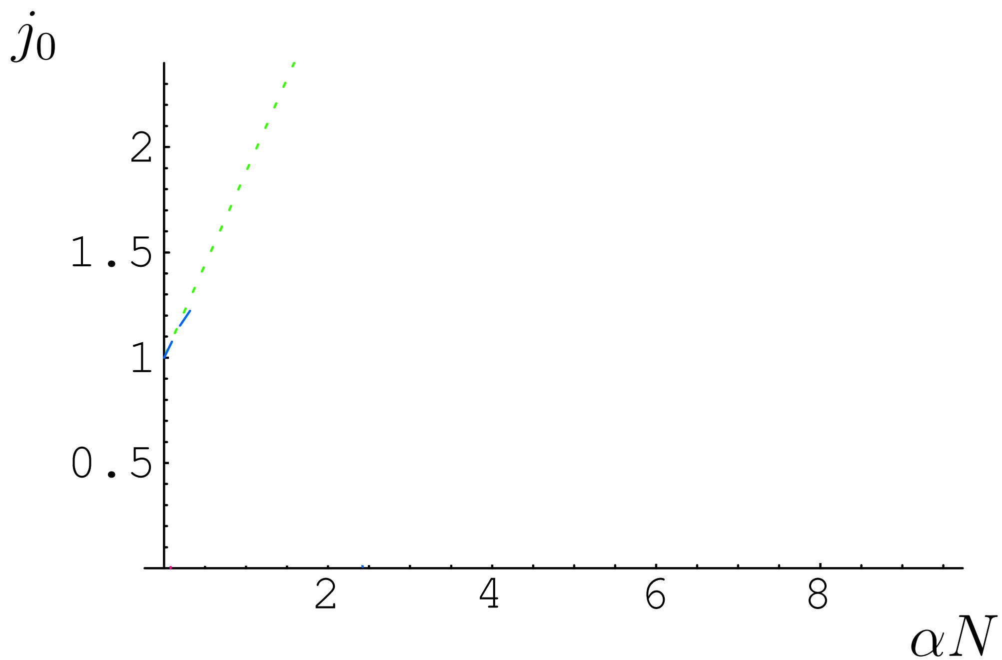
- in QCD, NLO correction negative, LARGE, runs
- in $\mathcal{N} = 4$, NLO correction is negative, moderate, constant

We obtain a result valid for large λ in $\mathcal{N} = 4$

$$j_0 = 2 - \frac{2}{\sqrt{\lambda}}$$

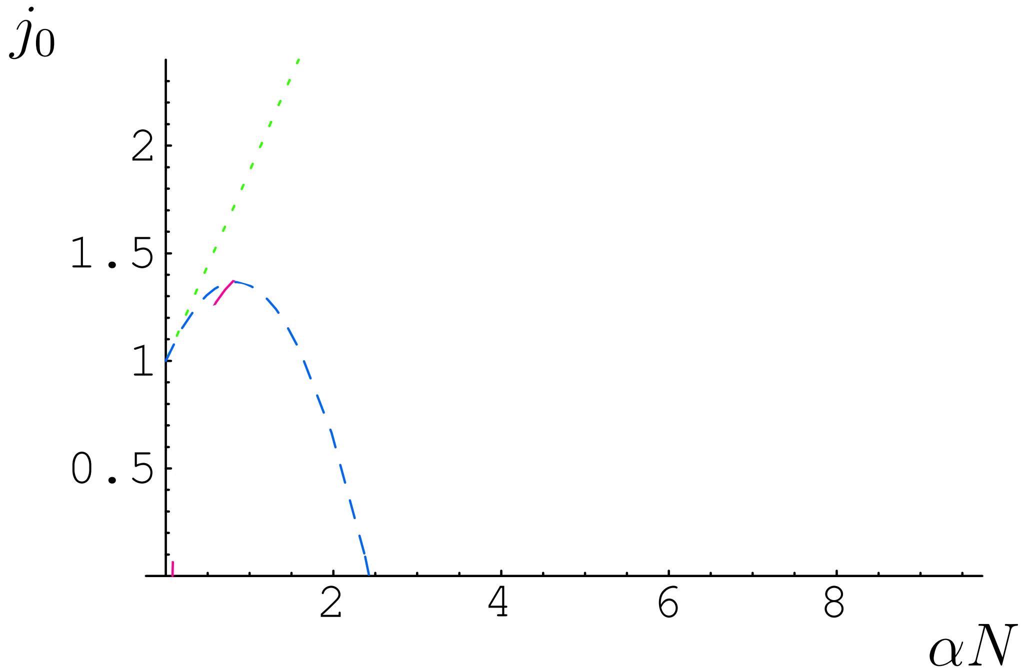
[obtained subsequently using BFKL-DGLAP relation by Kotikov Lipatov 0404092 v5]

$j_0(\lambda)$ ($\mathcal{N} = 4$ **YM**)



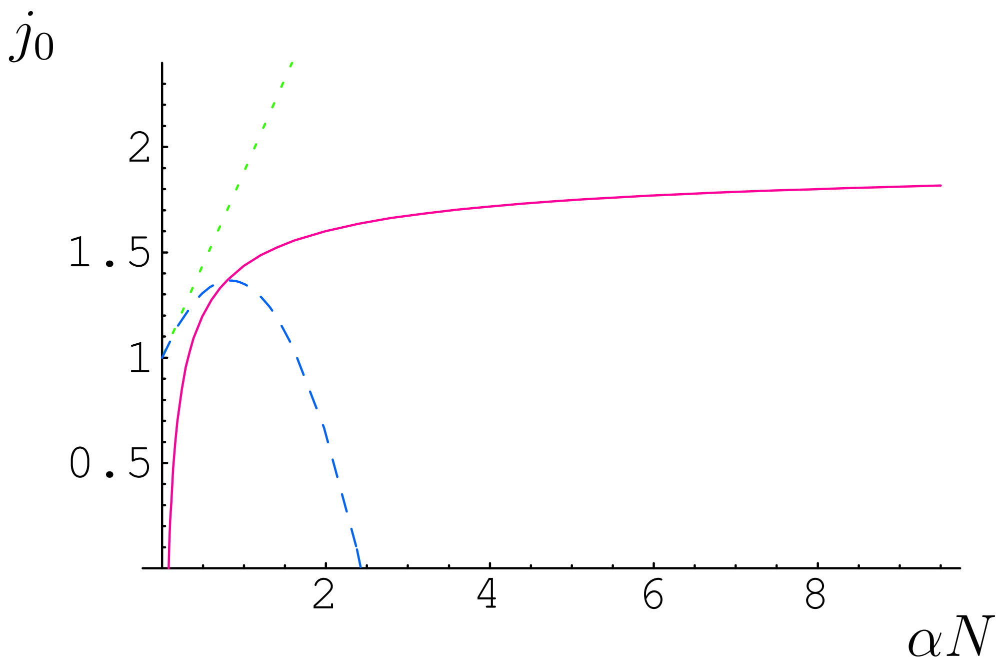
BFKL

$j_0(\lambda)$ ($\mathcal{N} = 4$ **YM**)



Kotikov & Lipatov 02; Kotikov et al. 04

$j_0(\lambda)$ **($\mathcal{N} = 4$ YM)**



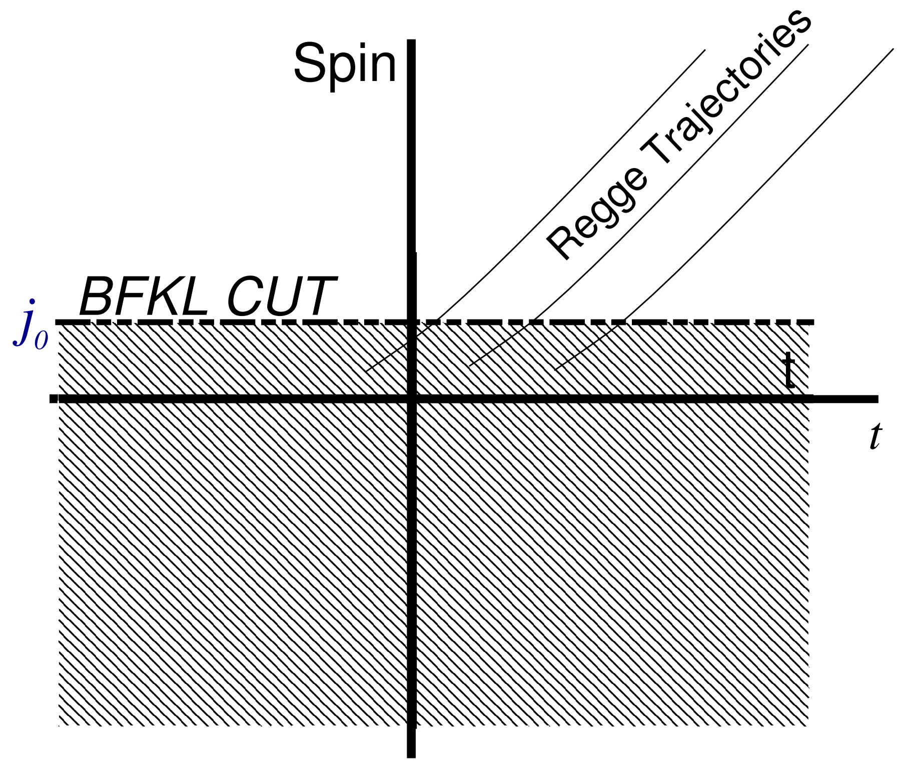
our result

BFKL starts at 1, rises.

Our result starts at 2, interpolates with NLO result.

BFKL λ expansion is poor, but not the general picture.

Claim: the analytic structure

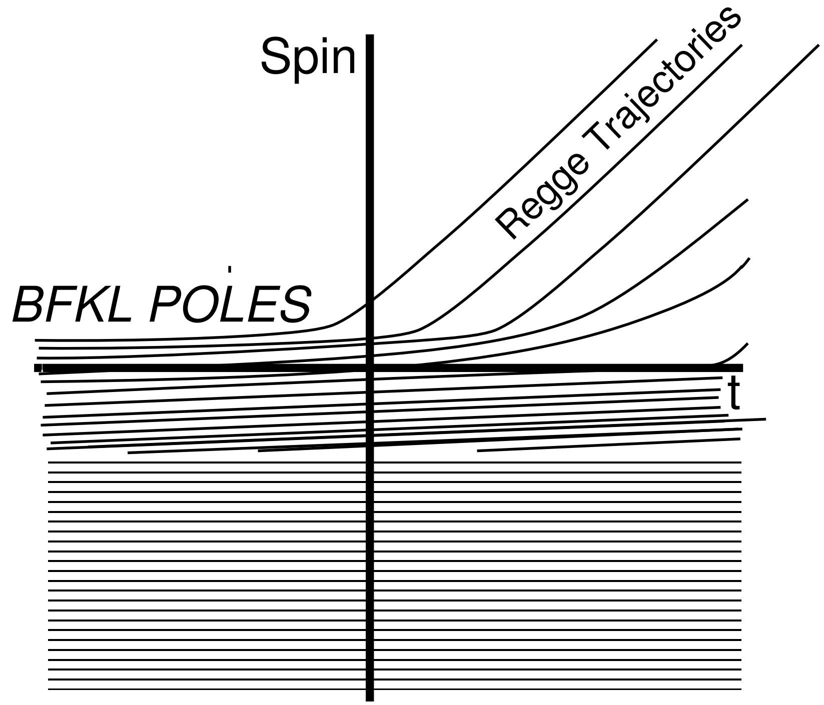


is valid qualitatively at all λ

We will obtain a precise picture at large λ.

What will we find for a running coupling?

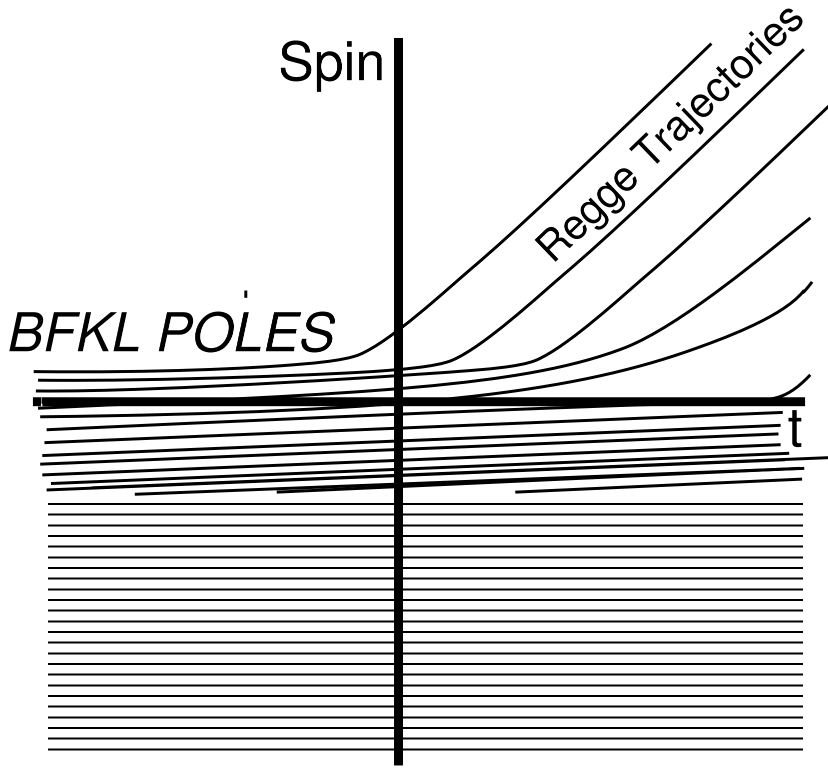
Running UV, Confining IR (large N)



The hadronic spectrum is little changed, as expected.

The BFKL cut turns into a set of poles, as expected.

Running UV, Confining IR



Near $t = 0$: **BFKL poles** $\leftrightarrow$ **Regge trajectories**

Again we *expect* this is true at all λ.
We will actually *obtain* this result at large λ, $\beta_\lambda \sim 1/N$, and $|t|$ not too large.

Back to Conformal Case

At very small **constant** λ, the BFKL calculation is *universal*: independent of N, matter content, etc.

So can recalculate it in a theory with a string description, large N, adjustable λ

- At small λ, compute kernel from BFKL resummation — infinite set of Feynman diagrams.

- At large λ, compute kernel using string theory — single tree-level $2 \to 2$ string diagram, calculated on curved $3 + 1 + \mathbf{1}$ $[+5]$ dimensional space.

What is the curved space?

Maldacena: UV (large r) is (almost) an $AdS_5 \times X$ space

$$ds^2 = r^2 dx_\mu dx^\mu + \frac{dr^2}{r^2} + ds_X^2$$

Captures QCD's approximate UV conformal invariance

$$x \to \zeta x \ , \ \ r \to \frac{r}{\zeta} \qquad\qquad (\text{recall } r \sim \mu)$$

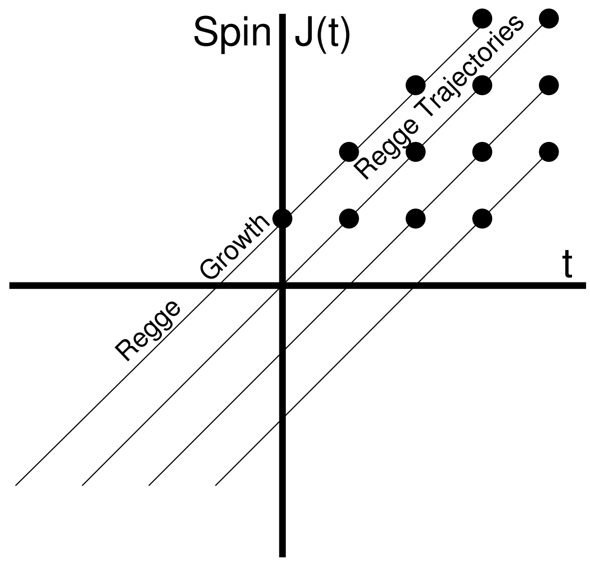
Confinement: IR (small r) is cut off in some way

$$r \sim \mu > r_{min} \sim \Lambda_{QCD}$$

For Pomeron: *string theory* on cut-off AdS_5 (X plays no role)

String amplitudes $\to$ Regge behavior $\mathcal{A} \sim \sum_i s^{J_i(t)}$

$[J(t) = \alpha(t) = \alpha_0 + \alpha' t]$

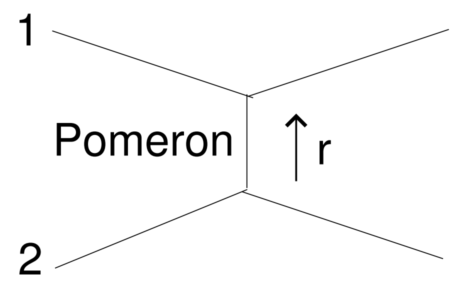


- t negative; Fourier transform momentum space $\longrightarrow$ position space

$$J(t) \sim \alpha_0 + \alpha' t \Rightarrow \mathcal{A} \sim s^{\alpha_0} \, \frac{\exp\left[-|\vec{x}|^2/\alpha' \ln s\right]}{\sqrt{\ln s}}$$

Strings grow: $\langle |\vec{x}|^2 \rangle \sim \ln s$

(random-walk diffusion, with $\tau \sim \ln s$)



$$\begin{aligned}\mathcal{A} \sim s^{J(t)} &= s^{2+\alpha' t/2} \quad \text{(flat space)} \\ &\to s^{2+\alpha'\nabla^2/2} \quad \text{(curved space)} \\ &= s^2 e^{(\alpha' \ln s)\nabla^2/2} \equiv s^2 e^{-H\tau}\end{aligned}$$

where $\tau \propto \ln s$ is again a diffusion time, and

$$H \propto -\nabla^2 = -\frac{1}{r^2}\nabla_{3+1} - \nabla^{\mathbf{2}}_{\mathbf{r}} + 0 = -\partial_u^2 + (4 - e^{-2u} t/t_0)$$

where $u = \ln r$

A Schrödinger operator with potential $V(u;t) = 4 - e^{-2u} t/t_0$

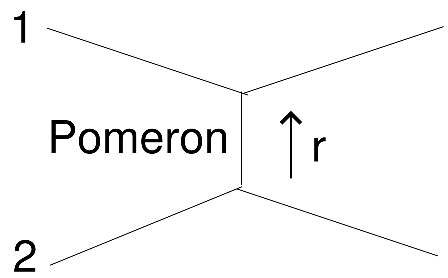
The argument given on the previous slide was heuristic.

The result is not.

A detailed derivation from string theory has been obtained, but requires some small technical developments in string theory.

The result follows on earlier work on deep inelastic scattering at large λ.

Polchinski and MJS 02



$$
\begin{aligned}
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&\rightarrow s^{2+\alpha' \nabla^2/2} \quad \text{(curved space)} \\
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$$
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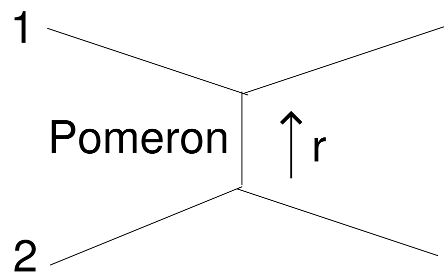
A Schrödinger operator with potential $V(u;t) = 4 - e^{-2u} t/t_0$



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where $\tau \propto \ln s$ is again a diffusion time, and for $t = 0$,

$$H \propto -\nabla^2 = -\frac{1}{r^2}\nabla_{3+1} - \nabla^{\mathbf{2}}_{\mathbf{r}} + 0 = -\partial_u^2 + 4$$

where $u = \ln r$

A Schrödinger operator with potential $V(u;t) = 4$

$$\mathcal{A} \sim s^2 e^{-H\tau} \sim s^{j_0} e^{-\mathcal{D}\tau[-\partial_u^2]} \quad , \quad j_0 = 2 - \frac{2}{\sqrt{\lambda}} \; , \; \mathcal{D} = \frac{1}{2\sqrt{\lambda}}$$

$$\mathcal{A} \sim s^2 e^{-H\tau} \sim s^{j_0} e^{-\mathcal{D}\tau[-\partial_u^2]} \quad , \quad j_0 = 2 - \frac{2}{\sqrt{\lambda}} \, , \; \mathcal{D} = \frac{1}{2\sqrt{\lambda}}$$

Sandwiching this differential operator between the two scattering hadrons, writing the kernel explicitly, and recalling $\tau \propto \ln s$, $u = \ln r$,

$$\mathcal{A} \sim \int \frac{dr}{r} \int \frac{dr'}{r'} \, \Phi_1(r) \;\; s^{j_0} \frac{e^{-\left[(\ln[r'/r])^2/4\mathcal{D}\ln s\right]}}{\sqrt{4\pi\mathcal{D}\ln s}} \;\; \Phi_2(u')$$

$$j_0 = 2 - \frac{2}{\sqrt{\lambda}} \, , \; \mathcal{D} = \frac{1}{2\sqrt{\lambda}}$$

Same form as the BFKL kernel for $t = 0$:

$$\mathcal{A} = \int \frac{dk_\perp}{k_\perp} \int \frac{dk'_\perp}{k'_\perp} \Phi_1(k_\perp) \quad s^{j_0} \frac{e^{-\left[(\ln[k'_\perp/k_\perp])^2/4\mathcal{D}\ln s\right]}}{\sqrt{4\pi\mathcal{D}\ln s}} \quad \Phi_2(k'_\perp)$$

$$j_0 = 1 + \frac{4\ln 2}{\pi}\alpha N \ , \quad \mathcal{D} = \frac{7\zeta(3)}{\pi}\alpha N \ .$$

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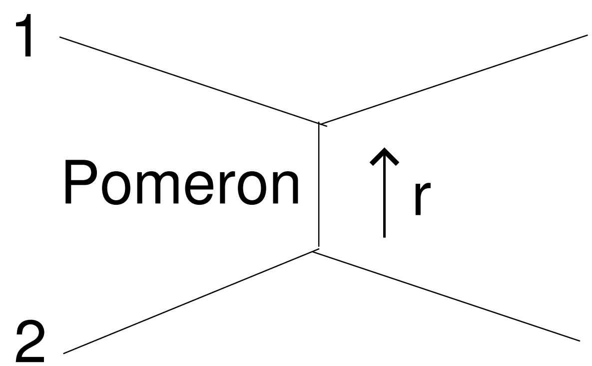
$$j_0 = 2 - \frac{2}{\sqrt{\lambda}} \ , \ \mathcal{D} = \frac{1}{2\sqrt{\lambda}}$$

In this string calculation, the exchanged Pomeron — the graviton trajectory — propagates in the curved 5th dimension!



- BFKL result = Regge behavior in 5 curved dims.
- Amplitude takes BFKL *form*, with $k_\perp \to r$.
- BFKL diffusion is Regge diffusion (space $= \ln r$, time $= \ln s$).
- Coefficients differ (as expected; λ is different)
- Form of answer follows from conformal invariance.

Now a theory with confinement; but philosophy first:

We are not seeking quantitative predictions for QCD.

We are only trying to learn clear but qualitative lessons.

Our goal here is to extract *universal* features of large N, large λ theories

The idea is that what is universal at large λ may in some cases, by analyticity in λ, be universal at all λ.

Therefore we need a method to identify what is universal.

This means that any reasonable metric will do as long as we can see what does and does not depend on our choice of metric.

What makes a metric "reasonable"

The theory *must* be asymptotically four-dimensional in the UV.

Otherwise the structure of the UV Pomeron won't even vaguely resemble QCD.

This rules out most existing solutions, e.g. Witten's black hole.

The $\mathcal{N} = 1^*$ theory would be perfect if we knew the metric exactly. But we don't.

Even if we did, the calculation would at best be challenging, intricate, and its details unilluminating; for the price of great pain and suffering, we'd obtain the exact answer for a theory that isn't QCD.

So now what?

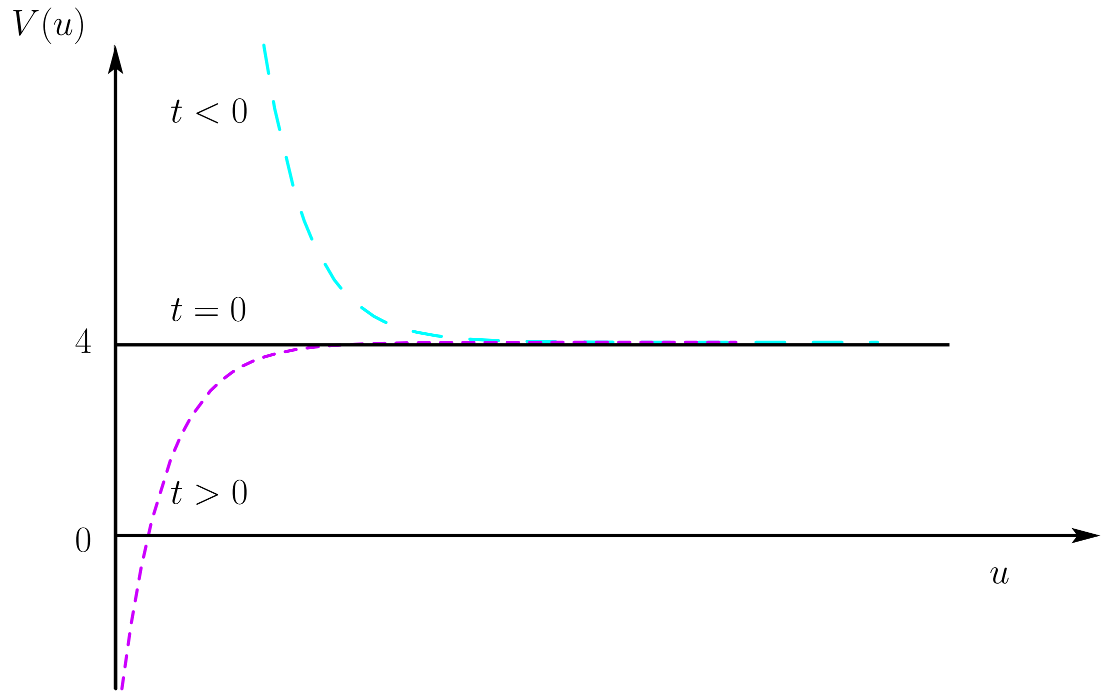
For this reason, we choose the hard-wall *model* — as an AdS_5 space with a cutoff at $r = r_0$ ($u = 0$) — since what is universal becomes clear immediately.

You will see that the model actually tells us what we want to know, and saves from computing details that we don't want to know anyway.

A change in the boundary conditions or of the metric would not change the answer (as you will see) except in subleading corrections.

Now a theory with UV conformal invariance, IR confinement

The potential $V(u;t) = 4 - e^{-2u}t/\Lambda^2$, wall at $u = 0$

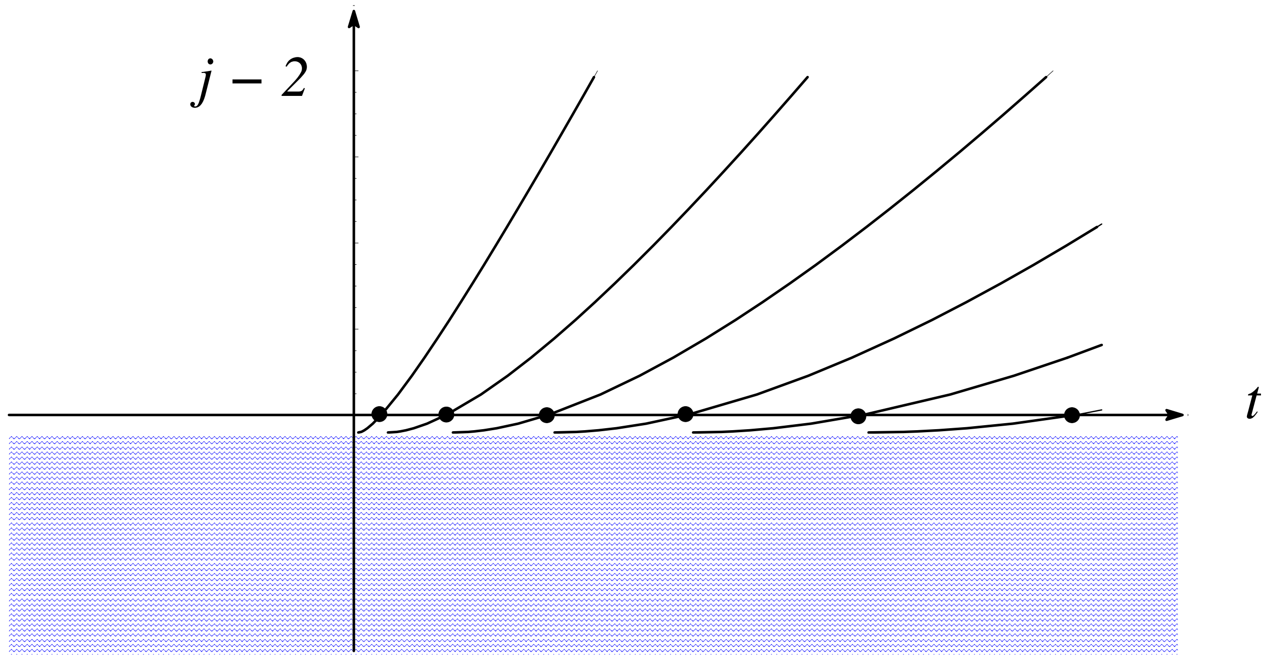


For $t = 0$, there is a continuum starting at $E = 4$.

For $t < 0$, there is a continuum starting at $E = 4$.

For $t > 0$, bound states appear.

The analytic structure in $j = 2 - E/2\sqrt{\lambda}$.



For $t = 0$, there is a continuum starting at $E = 4$.

For $t < 0$, there is a continuum starting at $E = 4$.

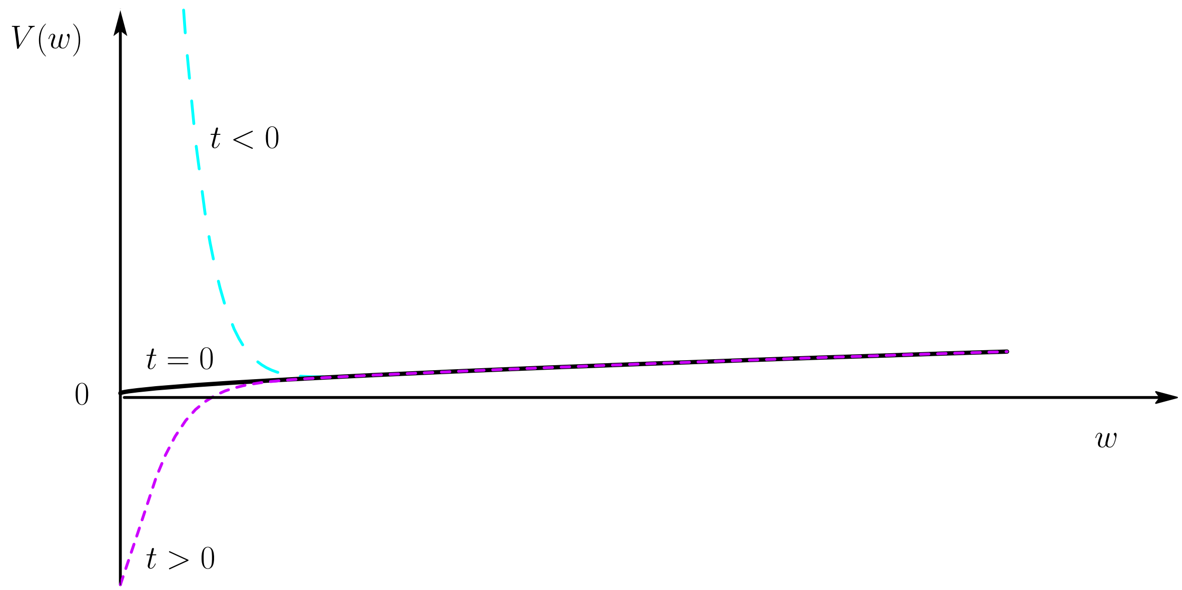
For $t > 0$, bound states appear.

Spin-2 glueballs where bound states hit $E = 0$, $j = 2$.

Brower,Mathur,Tan; Constable Myers

Now add a running coupling (small β_λ), IR confinement

The potential in a confining theory, for various t.

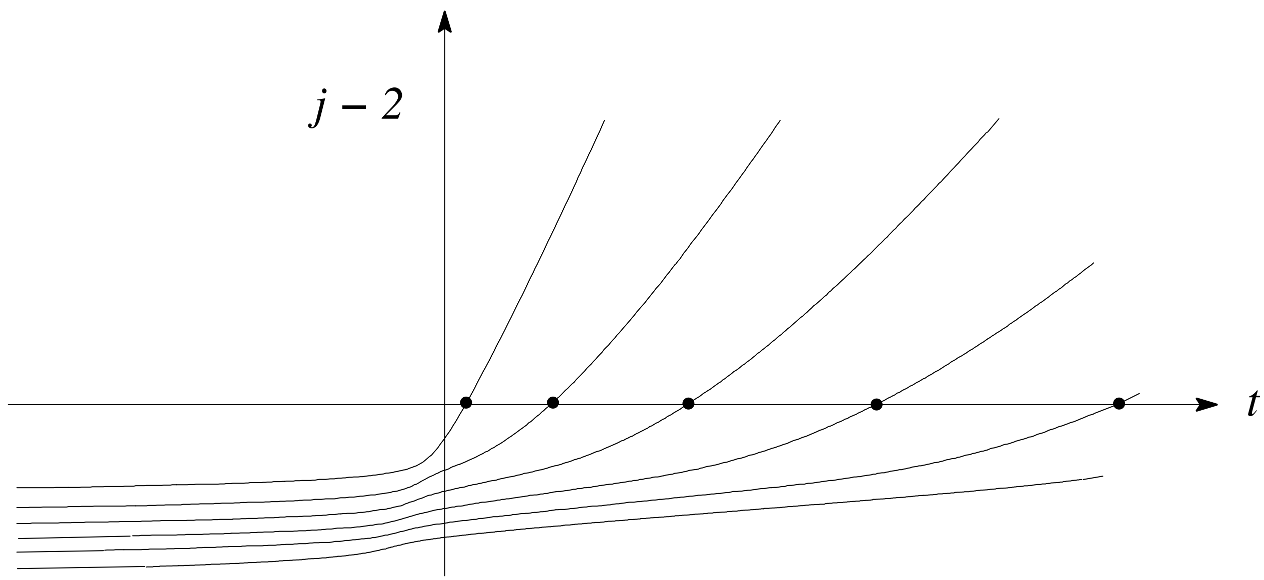


For $t > 0$, there are bound states little changed from before.

For $t < 0$, there is now a closely-spaced set of bound states where previously there was a continuum.

Note region near $t = 0$ is maximally model-dependent!

The analytic structure in $j = 2 - E/2\sqrt{\lambda}$.



For $t > 0$, there are bound states little changed from before.

For $t < 0$, there is now a closely-spaced set of bound states where previously there was a continuum.

Glueball states where bound states hit at $E = 0$, $j = 2$.

Note our formalism argues against j_0 appearing in small-x deep inelastic scattering

In DIS one has a γ off-shell by Q^2 and forward γp scattering $(t = 0)$ at small $x = Q^2/s$

But $t = 0$ *region is very sensitive, not universal.*

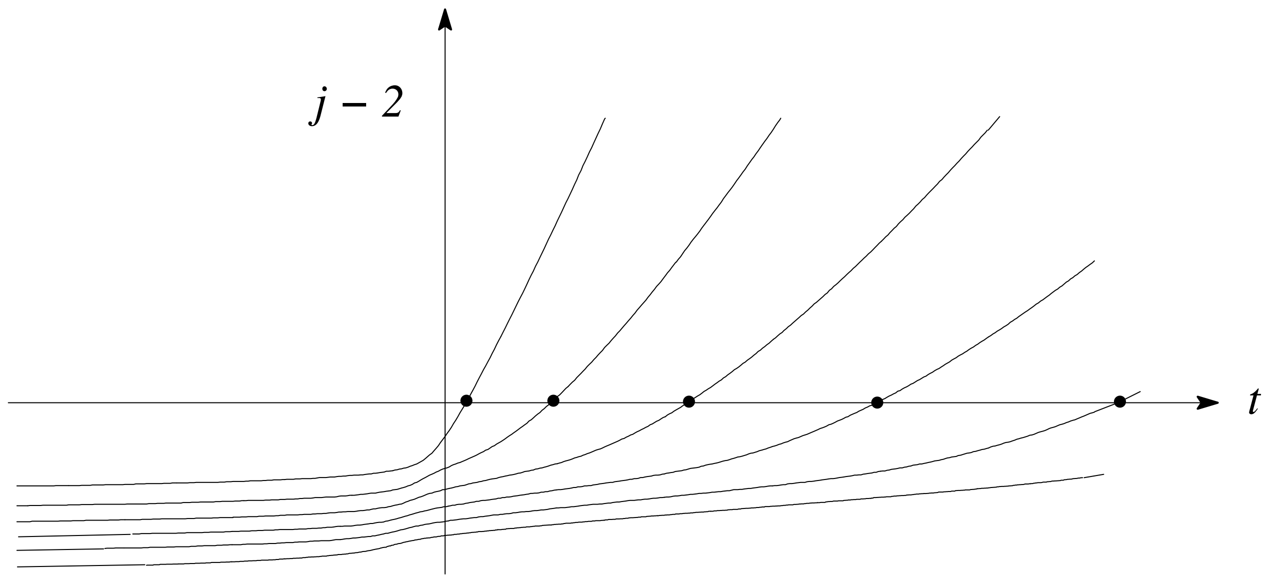
Even if we take Q^2 very large, either we

• take s extremely large, then get exponent from position of leading trajectory (very sensitive to confinement!), or

• take s not too large, then get a transient effect which is sensitive to details of hadron wave functions

$$\mathcal{A}_{2\to 2} = \int \frac{dk_\perp}{k_\perp} \int \frac{dk'_\perp}{k'_\perp} \Phi_1(k_\perp) K(s; k_\perp, k'_\perp) \Phi_2(k'_\perp)$$

The analytic structure in $j = 2 - E/2\sqrt{\lambda}$.



Notice that j_0 always increases with t (because the potential is always decreasing with t.)

This is interesting...

Estimates from HERA/BFKL–related discussions of j_0?

I have heard people say "probably $j_0 \sim 1.3$"

referring to the UV or "hard" Pomeron

presumably this refers to large negative t

But Domanchie and Landshoff fit pp, $p\bar{p}$, πp scattering data

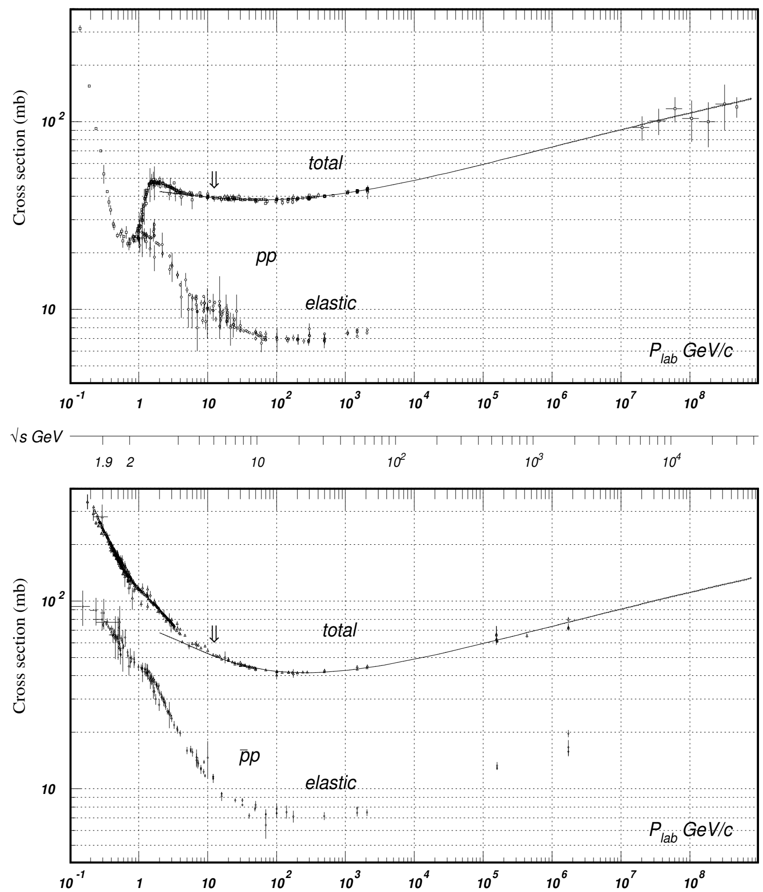


Figure 40.11: Total and elastic cross sections for pp and $\overline{p}p$ collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at http://pdg.lbl.gov/xsect/contents.html (Courtesy of the COMPAS group, IHEP, Protvino, August 2003)

Estimates from HERA/BFKL–related discussions about j_0?

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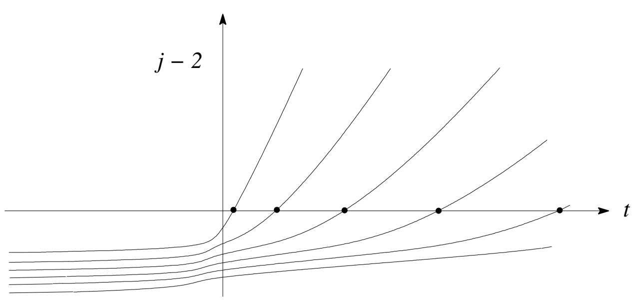
presumably this refers to large negative t

But Domanchie and Landshoff fit pp, $p\bar{p}$, πp scattering data with $j_0 = 1.08$

this is dominated by the IR or "soft" Pomeron at $t \to 0$

But we find j_0 increases with t, so how can this be?



This represents the next frontier for gauge/string duality:

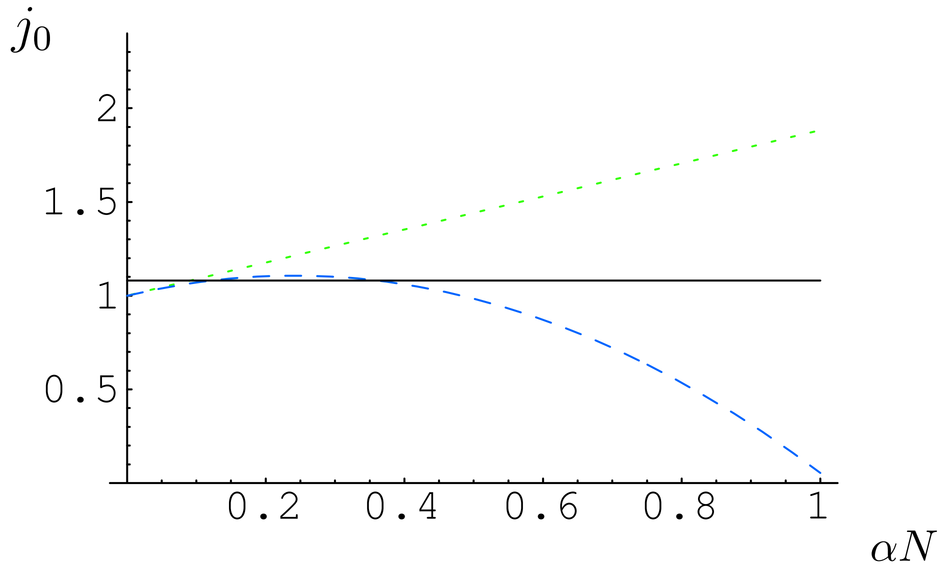
What happens near $t = 0$?

Well-known that multi-Pomeron exchange is important at $t = 0$ for high-enough s; needed for unitarity, Froissart bound.

Is it true that *multi-Pomeron exchange* has already kicked in and is already suppressing the amplitude below the single-Pomeron exchange prediction?

We see no clear resolution at present.

Actually there is no evidence in BFKL for large j_0 in real QCD.



Fadin Lipatov 98; Ciafaloni Camici 98

So there isn't an evident problem.

Even if there is, there could be a resonance on the second sheet of the j plane that sits at $j = 1.08$ and dominates the amplitudes currently, makes apparent j_0 small.

Or maybe multi-Pomeron exchange really is important;
- is there any regime where eikonal approximation valid?
- something more drastic, such as nonperturbative string effects? (Black holes for ultra-high s Giddings, but much may happen before that point.)

We don't know how to resolve these issues.

We do hope to make contact with the Balitsky-Kovchegov equation and with other methods to learn about the stringy view of the small-x regime, saturation, etc.

Perhaps this will move us in the right direction

Summary

- Gauge/string duality allows us to compute the analytic structure of the Pomeron propagator in gauge theories with large 't Hooft coupling and large N.
- The results are given as an expansion in $1/\sqrt{\lambda}$.
- We are able to obtain this for all t; we can see
- the glueballs and their trajectories at $t > 0$,
- the soft Pomeron at $|t| < \Lambda$, (4d Regge physics)
- the BFKL hard Pomeron (5d Regge physics)
- We see the BFKL cut and the Regge trajectories in theories with a conformal UV and IR confinement.
- Adding a running coupling, we see the BFKL poles moving continuously as a function of t and becoming the Regge trajectories at $t > 0$

- A Pomeron vertex operator is introduced into string theory and proves useful for the computations.
- We are able to see the analytic relation between DGLAP and BFKL operators in $\mathcal{N} = 4$ SYM, clarifying the $\Delta(j)$ relation, its λ dependence, and its appearance in string theory.
- Many applications remain to explore; many questions remain to be answered.