

# Flavor physics from lattice QCD

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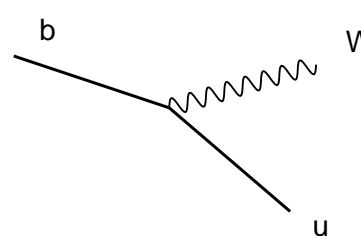
## Apologies:

- to my continuum colleagues for not having the time nor space to systematically acknowledge their important contributions
- to my discrete colleagues for not being able to cover all interesting lattice work in flavor physics

# Motivation

Test SM paradigm of **quark flavor mixing** and **CP violation** and look for **new physics**

## Unitary CKM matrix



$$\sim V_{ub} \rightarrow V = \begin{matrix} & d & s & b \\ \begin{matrix} u \\ c \\ t \end{matrix} & \begin{pmatrix} 1 - \frac{\lambda}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} & \end{matrix} + \mathcal{O}(\lambda^4)$$

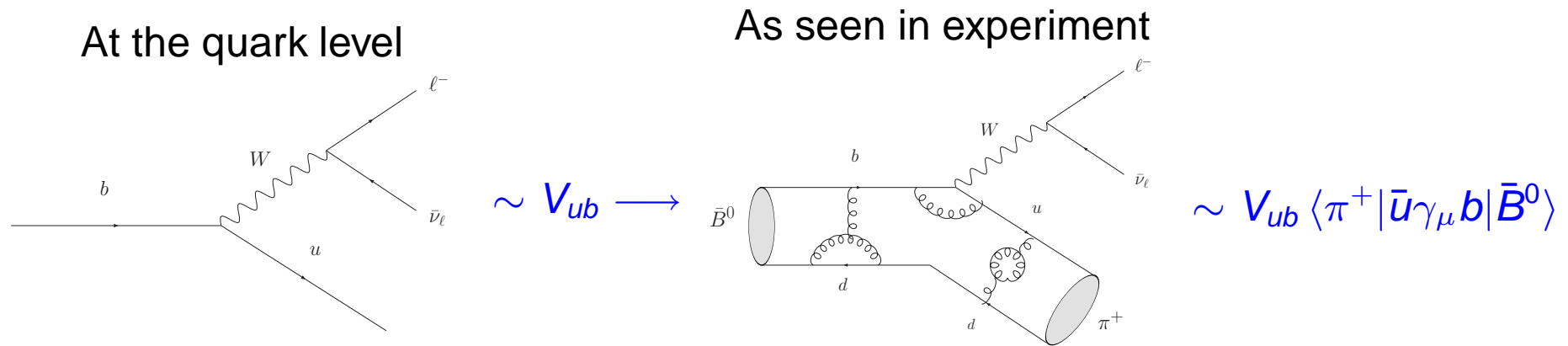
$$\lambda = \sin \theta_c = 0.2272(10) [0.4\%] \quad A = 0.809(14) [1.7\%] \quad \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = 0.391(15) [3.8\%] \quad (\text{CKM Fitter '06})$$

## Strategy

- Measure **CKM element magnitudes** with **CP conserving** processes
- Measure **CKM element phases** with **CP violating** processes
- Impose **unitarity** conditions and **look for inconsistencies**

# The need for a non-perturbative QCD tool

E.g.: exclusive semileptonic  $b$  quark decay



To get model-independent information about  $|V_{ub}|$  from these decays:  
must evaluate **non-perturbative strong interaction corrections** in  
fundamental theory

$\Rightarrow$  **Lattice QCD**

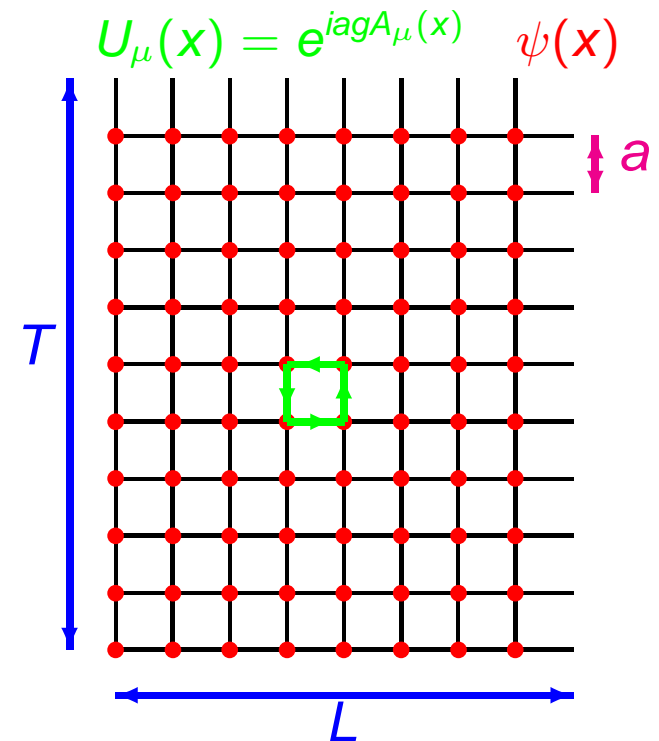
# What is lattice QCD (LQCD)?

Lattice gauge theory  $\longrightarrow$  mathematically sound definition of NP QCD:

- UV (and IR) cutoffs and a well defined path integral in Euclidean spacetime:

$$\begin{aligned}\langle O \rangle &= \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-S_G - \int \bar{\psi} D[M] \psi} O[U, \psi, \bar{\psi}] \\ &= \int \mathcal{D}U e^{-S_G} \det(D[M]) O[U, \psi, \bar{\psi}]_{\text{Wick}}\end{aligned}$$

- $e^{-S_G} \det(D[M]) \geq 0$  and finite # of dof's  
 $\longrightarrow$  evaluate numerically using stochastic methods



NOT A MODEL: LQCD is QCD when  $a \rightarrow 0$ ,  $V \rightarrow \infty$  and stats  $\rightarrow \infty$

*In practice, limitations ...*

# Fermions on the lattice

- All discretizations of vector gauge theories have:
  - **gauge invariance** at finite  $a$
  - **Poincaré invariance** when  $a \rightarrow 0$
- **Fermion doubling**  $\longrightarrow$  very difficult to preserve chiral flavor symmetries
  - $\Rightarrow$  break axial symmetries: Wilson fermions, domain-wall (DW) w/ small  $N_5$
  - $\Rightarrow$  break some vector and axial symmetries: staggered, twisted-mass fermions

Symmetries recovered when  $a \rightarrow 0$  (4 *tastes* per flavor for staggered)

- **Full chiral flavor symmetry** at finite  $a$  only recently
  - $\rightarrow$  **Ginsparg-Wilson fermions (GWF)** (domain-wall with large  $N_5$ , Neuberger ...)
  - (G & W '82, Kaplan '92, Narayanan & Neuberger '93-'95, Neuberger '98, Hasenfratz et al '98, Lüscher '98, ...)
- Faster simulations if we relinquish global symmetries
- GWF numerically more expensive (typically 50 times): use when chiral symmetry is important

# Limitations: statistical and systematic errors

Limited computer resources  $\rightarrow a$ ,  $L$  and  $m_q$  are compromises and statistics finite

Associated errors:

- **Statistical:**  $1/\sqrt{N_{conf}}$ ; eliminate with  $N_{conf} \rightarrow \infty$
- **Discretization:**  $a\Lambda_{QCD}$ ,  $am_q$ ,  $a|\vec{p}|$ , with  $a^{-1} \sim 2 - 4 \text{ GeV}$   
 $1/m_b < a < 1/m_c \Rightarrow b$  quark cannot be simulated directly  
 $\Rightarrow$  EFTs: HQET, FNAL, NRQCD and possibly interpolation with charm  
Eliminate with  $a \rightarrow 0$
- **Chiral extrapolation:**  $m_q \rightarrow m_u, m_d$   
 $\rightarrow$  use  $\chi$ PT  
Requires  $m_q \sim m_s/4 \rightarrow m_s/8$
- **Finite volume:** for simple quantities, single  $L$  large enough ...  
Eliminate with  $L \rightarrow \infty$  ( $\chi$ PT)
- **Renormalization:** LQCD gives bare quantities  $\rightarrow$  must renormalize: best done non-perturbatively

# Limitations: quenching

- Large overhead for computing  $\det(D[M])$   
→ **quenched approximation** ( $N_f = 0$ ):  
sea quarks are treated as mean field

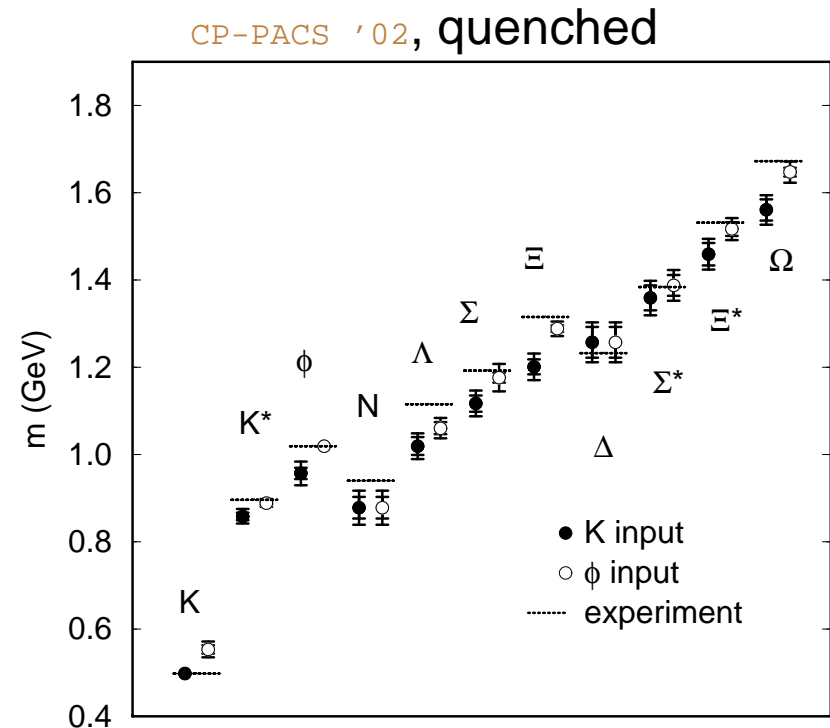
$$\langle O \rangle \approx \int \mathcal{D}U e^{-S_G} \det(D[M]) [O]_{\text{Wick}}$$

→ commonly used in past; now used for testing new methods

- Not a systematic approximation  
Errors  $\sim 10 - 20\%$   
→ *Not* QCD but  $\sim$  good model
- Must be eliminated in tests of SM

**Partial quenching:** valence and sea quark masses different

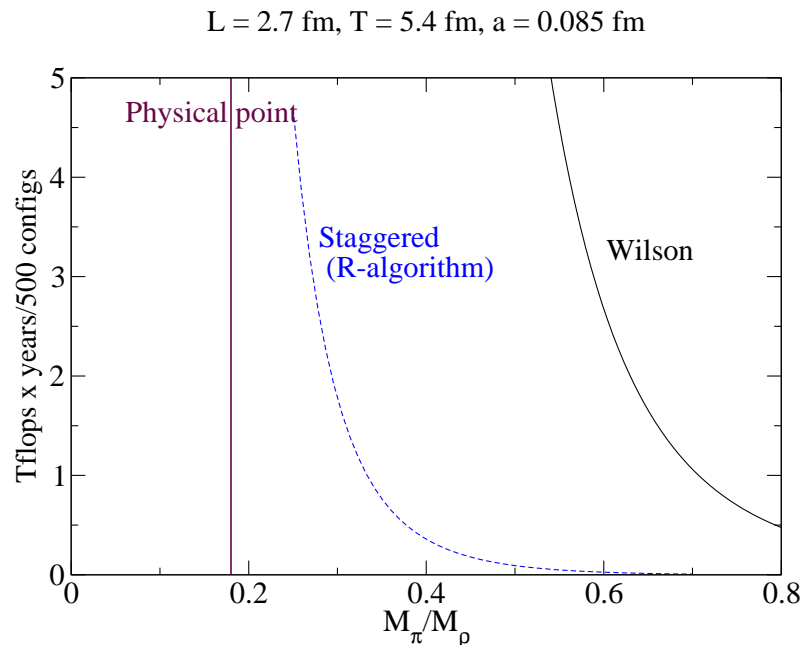
- $N_f = 2$  ( $m_s^{\text{sea}} = \infty$ ): better than quenched
- $N_f = 2 + 1$ : contains QCD for  $E \lesssim m_c$  ( $m^{\text{val.}} = m^{\text{sea}}$ ); better than real world!?





# Limitations: the Berlin wall

Unquenched calculations are numerically very demanding: # of d.o.f.  $\sim \mathcal{O}(10^9)$  and calculation of  $\det(D[M])$



Staggered and Wilson with “traditional” unquenched algorithms ( $\leq 2004$ )

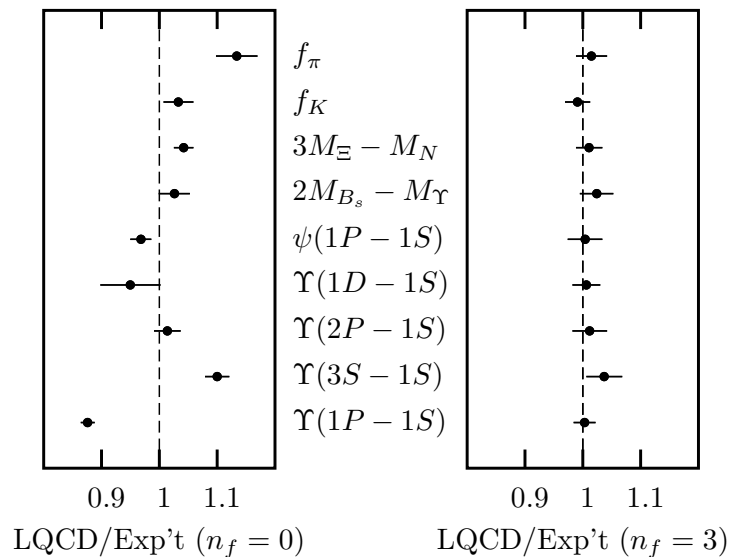
- $\text{cost} \sim N_{\text{conf}} V^{5/4} m_q^{-2.5 \rightarrow 3} a^{-7}$  (Gottlieb '02, Ukawa '02)
- Both formulations have a cost wall
- Wall appears for much lighter quarks w/ staggered, but algorithm not exact

→ MILC has gone for the gusto:  $N_f = 2 + 1$  simulations with  $m_q \gtrsim m_s/8!$

- Impressive effort: many quantities studied
- Detailed study of chiral extrapolation with staggered  $\chi$ PT (masses small enough)

# Are we there?

Certainly looks like it!



(Davies et al '04)

And growing non-perturbative evidence against disaster (Dürr et al '04, ...)

**Devil's advocate!** → potential problems:

- $\det(D[M])_{N_f=1} \equiv \det(D[M]_{\text{stagg}})^{1/4}$  to get rid of spurious “tastes”  
⇒ no local action known to give that determinant
- algorithm used not exact
- at current  $a$ , significant lattice artefacts
- renormalizability of staggered fermions not shown to all orders in PT (however, see Giedt's contribution)

⇒ **unknown systematic error** (is it QCD?)

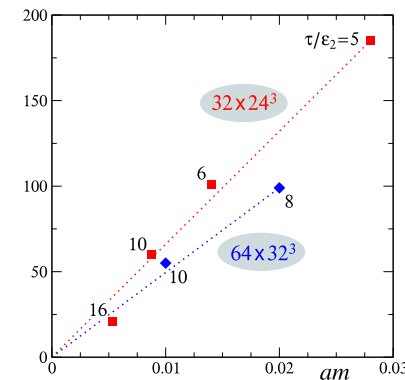
⇒ *cannot be final word* unless approach is put on firmer ground

# The fall of the Berlin wall

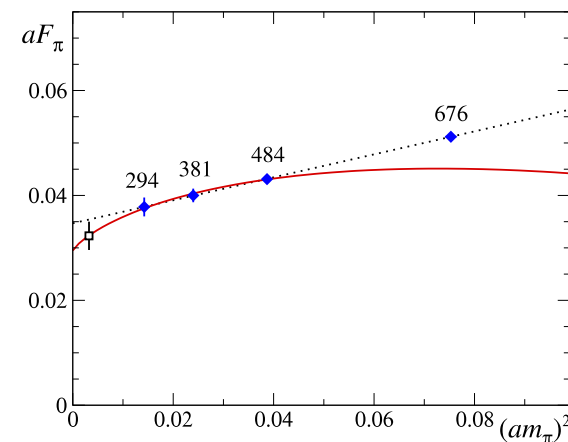
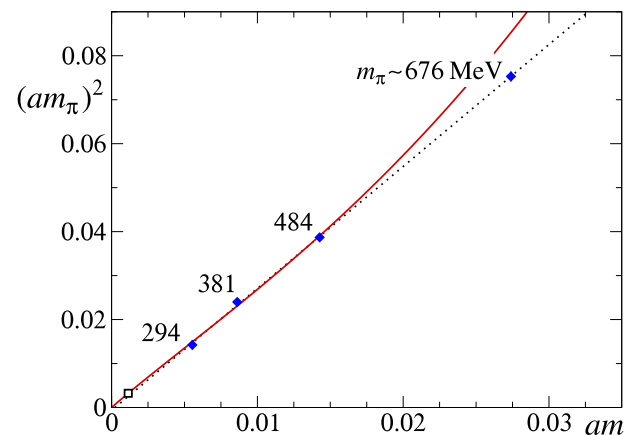
Conceptually sound Wilson fermions are back in the race ( $\geq 2004$ ):

- improved algorithms and actions  
(Lüscher '05, Urbach et al '06, ...)
- improved understanding of optimal simulation parameter ranges (Del Debbio et al '06, Jansen et al '04-'05)

# of  $24^3 \times 32$  ( $32^3 \times 64$ ) confs accepted per day on 8 (32) PC nodes (Lüscher '05)



(Lüscher '05, Del Debbio et al)



# Limitations: gold-plated quantities

Quantities which the lattice should be able to compute to **a few %** in coming years:

- At most one hadron in initial and final state, e.g.:

$$\sum_{\vec{x}} \langle [\bar{d}\gamma_\mu\gamma_5 u](x) [\bar{u}\gamma_5 d](0) \rangle \xrightarrow{0 \ll t_x \ll T} \langle 0 | \bar{d}\gamma_\mu\gamma_5 u | \pi^+(\vec{0}) \rangle \langle \pi^+(\vec{0}) | \bar{u}\gamma_5 d | 0 \rangle e^{-M_\pi t_x}$$

$$\sum_{\vec{x}, \vec{y}} e^{i\vec{q}\cdot\vec{y} - i\vec{p}\cdot\vec{x}} \langle [\bar{d}\gamma_5 u](y) [\bar{u}\gamma_\mu b](0) [\bar{b}\gamma_5 d](x) \rangle \xrightarrow{0 \ll t_y \ll T/2 \ll t_x \ll T} \langle 0 | \bar{d}\gamma_5 u | \pi^+(\vec{q}) \rangle \times$$

$$\times \langle \pi^+(\vec{q}) | \bar{u}\gamma_\mu b | \bar{B}^0(\vec{p}) \rangle \langle \bar{B}^0(\vec{p}) | \bar{b}\gamma_5 d | 0 \rangle e^{-E_B(T-t_x) - E_\pi t_y}$$

- Stable hadrons not near thresholds  $\rightarrow \rho, K^*, \dots$  difficult
- Disconnected graphs are difficult  $\rightarrow$  no  $\eta'$  (and  $\eta$  due to mixing)
- No high momenta:  $a|\vec{p}| \ll 1 \Rightarrow |\vec{p}| \lesssim 1 \text{ GeV}$
- If hadron with momentum requires chiral extrapolation,  $|\vec{p}|$  even more limited

$|V_{us}|$  from the lattice

# $|V_{us}|$ from $K \rightarrow \pi \ell \nu$ : introduction

- Measurement of  $|V_{us}|$  requires theoretical determination of  $f_+(0)$ :

$$\langle \pi^+(p') | \bar{u} \gamma_\mu s | \bar{K}^0(p) \rangle \longrightarrow f_+(q^2), f_0(q^2) \quad q = p - p'$$

- Need  $\delta f_+(0) < 0.37\%$  to fully exploit new results from KTeV, KLOE and NA48 (CKM'05)!
- Theoretical framework:  $\chi PT$  (Leutwyler & Roos '84, Gasser & Leutwyler '85)

$$f_+(0) = 1 + f_2 + f_4 + \dots$$

- $f_2$ : Ademollo-Gatto thm  $\rightarrow$  chiral logs determined in terms of  $M_K$ ,  $M_\pi$  and  $F_\pi$
- $f_4$ : computed in  $\chi PT$  (Post & Schilcher '02, Bijmans & Talavera '03), but requires  $O(p^6)$  LECs (determined from slope and curvature of  $f_+(q^2)$ , but need more precise experiments)

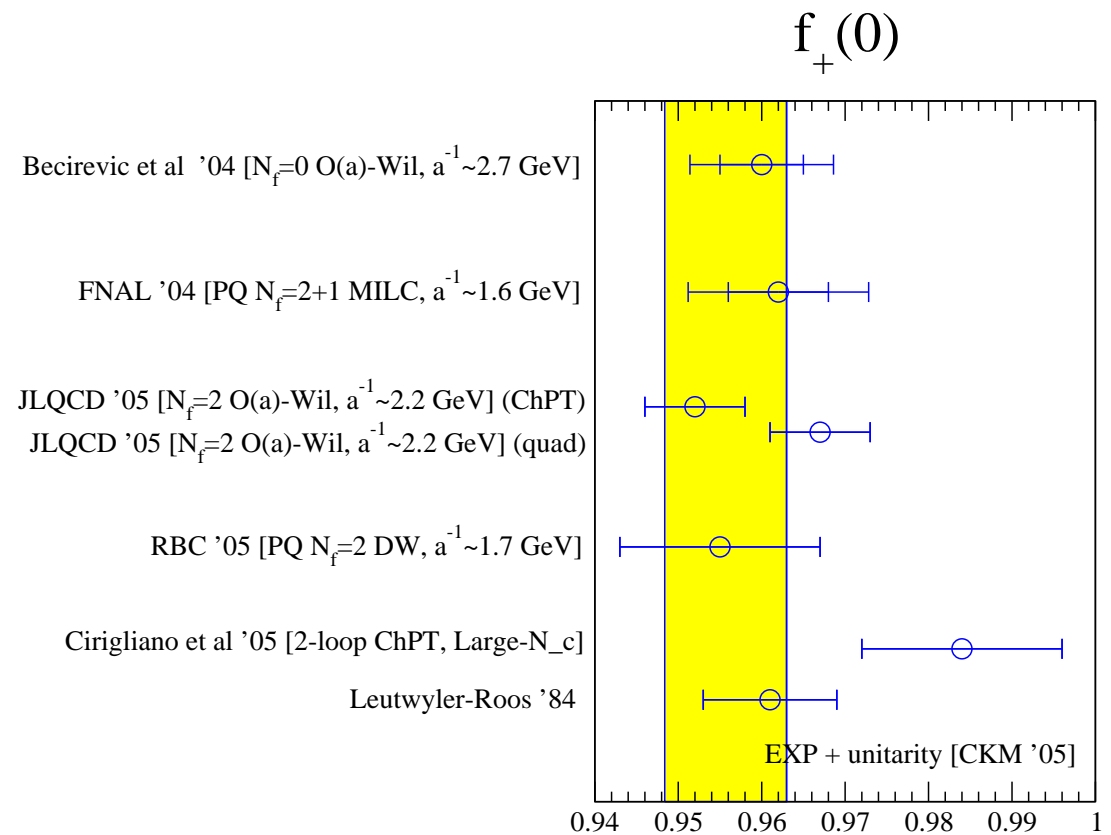
$\Rightarrow$  need a precise calculation of

$$\Delta f \equiv 1 + f_2 - f^+(0)$$

# $|V_{us}|$ from $K \rightarrow \pi \ell \nu$ : lattice results

Becirevic et al '04: calculate  $\Delta f$  on the lattice

- 1 Calculate  $f_0(q_{max}^2)$ , using double ratio trick in LQCD (statistical error  $< 0.1\%$ !)
- 2 Interpolate to  $q^2 = 0$  and get  $f_+(0) = f_0(0)$  using  $q^2$ -dependence of  $f_+(q^2)$  and ansatz
- 3 Extrapolate polynomially to physical isospin averaged  $u$  and  $d$  masses
- 4 Get  $\delta^{lat} f_+(0) \simeq 1\%$



# $|V_{us}|$ from $K \rightarrow \pi \ell \nu$ : conclusions

- “Preliminary” lattice results are in good agreement with each other and Leutwyler-Roos
- $2\sigma$  violation in PDG'04 of unitarity relation

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

has disappeared thanks to new measurements

- Discretization errors induce mass dependence of form  $a^2(m_s - \hat{m})^2 \rightarrow a^{-1}$  must be much larger than chiral scale
- Chiral extrapolations are from above  $M_K$
- Would have to be able to identify 1 and 2-loop logs in the data to be fully convincing  $\rightarrow$  simulations with much lighter quarks
- As one approaches physical masses, interpolation from  $q_{max}^2$  to  $q^2 = 0$  becomes more difficult
- Much hard work necessary to make significant improvements
- Partially quenched 1-loop chiral logs have been calculated very recently

(Becirevic et al '05)



# $|V_{us}|$ from $K \rightarrow \mu\bar{\nu}$ : introduction

Marciano '04: window of opportunity

$$\frac{\Gamma(K \rightarrow \mu\bar{\nu}(\gamma))}{\Gamma(\pi \rightarrow \mu\bar{\nu}(\gamma))} \longrightarrow \frac{|V_{us}|^2 f_K^2}{|V_{ud}|^2 f_\pi^2} = 0.07602(23)[\text{expt.}](27)[\text{rad.corr.}] \quad [2 \times 0.23\%]$$

and  $|V_{ud}| = 0.9738(3)$  [0.03%] (CKM '05)

Competitive with  $K \rightarrow \pi\ell\nu$ !

Need:

- $f_K/f_\pi$  to 1% to match  $K \rightarrow \pi\ell\nu$  determination
- $f_K/f_\pi$  to 0.23% to match experimental error in  $K \rightarrow \mu\bar{\nu}(\gamma)/\pi \rightarrow \mu\bar{\nu}(\gamma)$

Also:

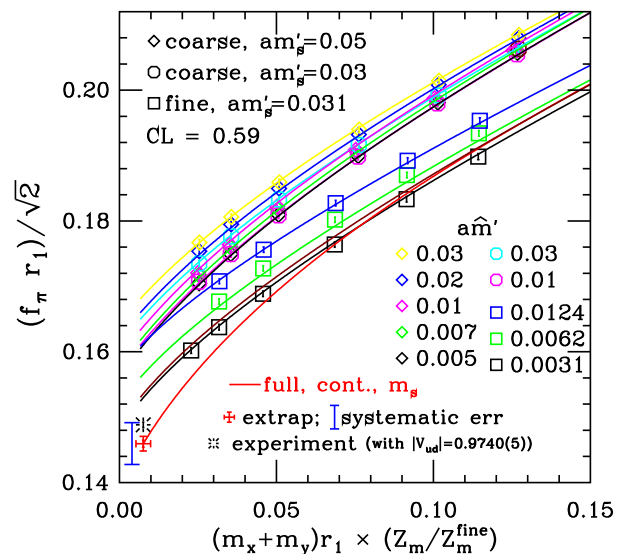
- $f_K/f_\pi$  not protected from leading  $SU(3)$ -flavor corrections by Ademollo-Gatto thm
- $f_K/f_\pi$  significantly more simple to calculate than  $f_+(0)$  on the lattice

# $|V_{us}|$ from $K \rightarrow \mu \bar{\nu}$ : $f_K/f_\pi$ from the lattice

MILC '03-'05:  $f_K/f_\pi$  from  $N_f = 2 + 1$ , staggered fermion calculation with  $\hat{m}_{sea} \rightarrow m_s/8$ !

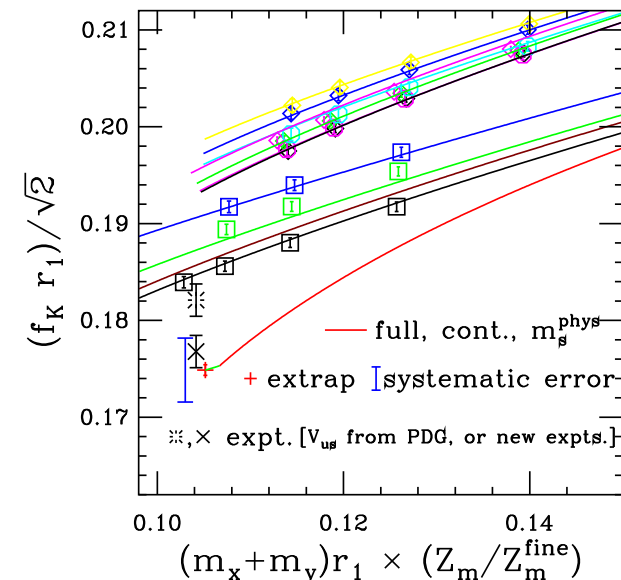
Fit  $M^2$  and  $f$  computed at  $a^{-1} \simeq 1.6$  and  $2.3 \text{ GeV}$  (from  $\Upsilon 2P-1S$ ), as fn of valence and sea masses, to partially quenched, staggered  $\chi$ PT (pqS $\chi$ PT) at NLO ++

→ 56 parameters of which 20 account for lattice artefacts (simultaneous chiral and continuum extrapolation)



A):  $f_\chi$ ,  $\Sigma$  and  $L_i$  from “data” w/  $m_x + m_y \lesssim 0.6m_s$  and  $\hat{m}^{sea} \lesssim 0.8m_s$

B):  $f_K$ ,  $m_s$  from “data” w/  $m_x + m_y \lesssim 1.4m_s$  (602 points)



Statistical errors 0.1% - 0.4% on  $f$  and 0.1% - 0.7% on  $M^2$

Real tour de force. Lattice '05 results:

$$\begin{aligned} f_\pi &= 128.1 \pm 0.5 \pm 2.8 \text{ MeV} \\ f_K &= 153.5 \pm 0.5 \pm 2.9 \text{ MeV} \end{aligned}$$

$$\frac{f_K}{f_\pi} = 1.198 \pm 0.003_{-0.005}^{+0.016} \quad [0.9\%]$$

- Matches accuracy of  $K \rightarrow \pi \ell \nu$  determination of  $|V_{us}|$
- Dominant known systematic on  $f_{\pi,K}$  is  $\sim 2\%$  scale uncertainty  
 $\leftrightarrow O(10\%)$  + quenching error, minimum systematic in a quenched calculation!
- Spin-offs:  $m_s$ ,  $\hat{m}$  and ChPT LECs  $L_4$ ,  $L_5$ ,  $L_6$  and  $L_8$

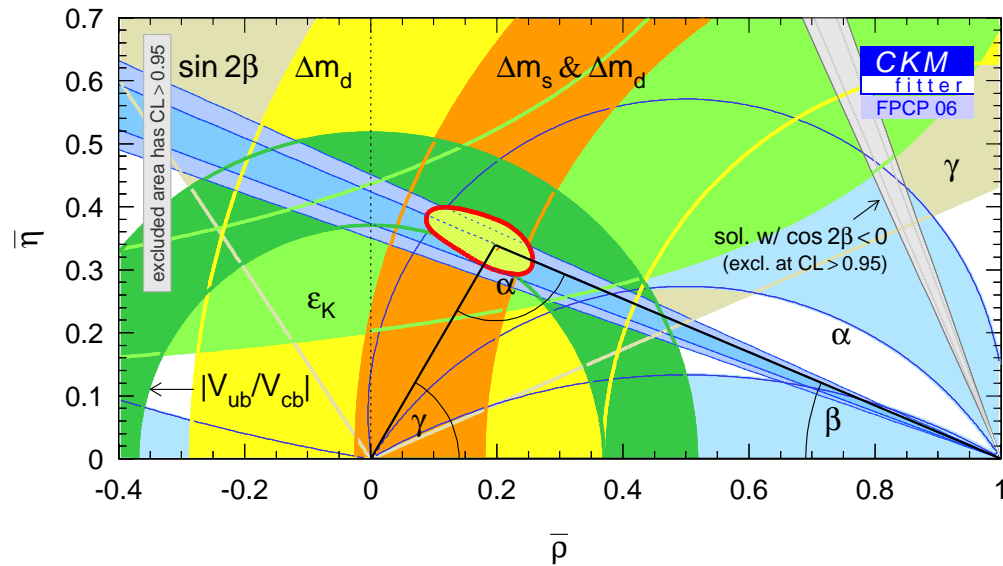
In addition to potential pbs w/ simulation itself (staggered  $\sqrt[4]{\det}$ -trick):

- Complex fitting procedure  $\rightarrow$  stability? (though some parameters are constrained)
- Validity of  $\chi$ PT expansion in presence of taste partners with masses up to  $\sim 750 \text{ MeV}$ ?

Less mature calculations:

- RBC-UKQCD '05: exploratory  $N_f = 2 + 1$  DWF calculations on  $16^3 \times 38 \times 8$  lattices with  $a^{-1} \approx 1.4 \rightarrow 2 \text{ GeV}$  and  $\hat{m}^{sea} \gtrsim m_s/4$
- QCDSF-UKQCD '05:  $N_f = 2$ , NP  $O(a)$ -improved Wilson on lattice with  $a^{-1} \approx 1.8 \rightarrow 2.8 \text{ GeV}$  and light sea quarks corresponding to  $M_\pi \approx 0.6 \rightarrow 1.2 \text{ GeV}$ .

# Lattice QCD for the unitarity triangle



- Only  $\sin 2\beta$  is free from hadronic uncertainties
- $|V_{ub}|/|V_{cb}|$  and  $\gamma$  (tree-level decays) should have no new physics contributions
- $\Rightarrow |V_{ub}|/|V_{cb}|$  and  $\gamma$  fix the summit and other constraints probe of new phys.

$$\Delta M_d = C_B M_{B_d} f_{B_d}^2 \hat{B}_{B_d} A^2 \lambda^6 [(1 - \bar{\rho})^2 + \bar{\eta}^2]$$

$$\frac{\Delta M_d}{\Delta M_s} = \frac{M_{B_d}}{M_{B_s}} \xi^{-2} \lambda^2 [(1 - \bar{\rho})^2 + \bar{\eta}^2]$$

$$|\epsilon_K| = C_K \hat{B}_K A^2 \lambda^6 \bar{\eta} [A^2 \lambda^4 (1 - \bar{\rho}) S_{tt} + S_{tc}]$$

$$\left| \frac{V_{ub}}{V_{cb}} \right| = \lambda / (1 - \lambda^2/2) \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$$

$$a_{\psi K_S}(t) = \sin(2\beta) \sin(\Delta M_d t)$$

Lattice QCD  $\rightarrow f_{B_d} \sqrt{\hat{B}_{B_d}}, \xi, \hat{B}_K$

$|V_{cb}|$  can be obtained from  
 $B \rightarrow D^*(D)l\nu$

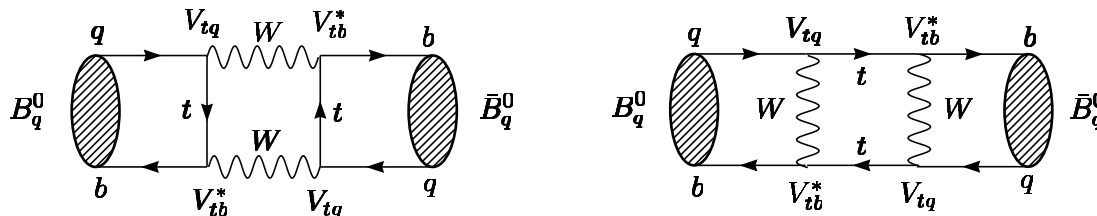
$|V_{ub}|$  from  $B \rightarrow \pi(\rho)l\nu$

Lattice QCD  $\rightarrow$  form factors

All are gold-plated quantities on the lattice

# $B_{(d,s)}^0 - \bar{B}_{(d,s)}^0$ mixing

In Standard Model :

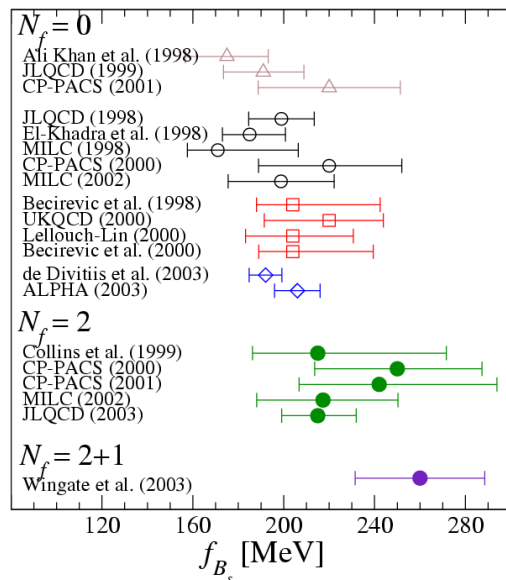


$$\begin{aligned} \Delta M_q &\simeq \frac{G_F^2}{8\pi^2} M_W^2 |V_{tq} V_{tb}^*|^2 \eta_B S_0(x_t) c_B(\mu) \frac{|\langle \bar{B}_q | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A}(\mu) | B_q \rangle|}{2M_{B_d}} \\ &= 0.504(4) \text{ ps}^{-1} \quad [0.8\%] \quad \text{for } q = d \\ &= 17.3_{-2}^{+4} \text{ ps}^{-1} \quad [1.7\%] \quad \text{for } q = s \quad (\text{CDF '06, D0 '06}) \end{aligned}$$

$$c_B(\mu) \langle \bar{B}_q^0 | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A}(\mu) | B_q^0 \rangle = \frac{8}{3} M_{B_q}^2 f_{B_q}^2 \hat{B}_{B_q}$$

Consider  $f_{B_q} = \langle 0 | \bar{b} \gamma_0 \gamma_5 q | B_q(\vec{0}) \rangle / M_{B_q}$  and  $B_{B_q} = \frac{3 \langle \bar{B}_q | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A}(\mu) | B_q \rangle}{8 \langle \bar{B}_q | \bar{b} \gamma_\mu \gamma_5 q | 0 \rangle \langle 0 | \bar{b} \gamma_\mu \gamma_5 q | B_q \rangle}$  separately, because systematics very different

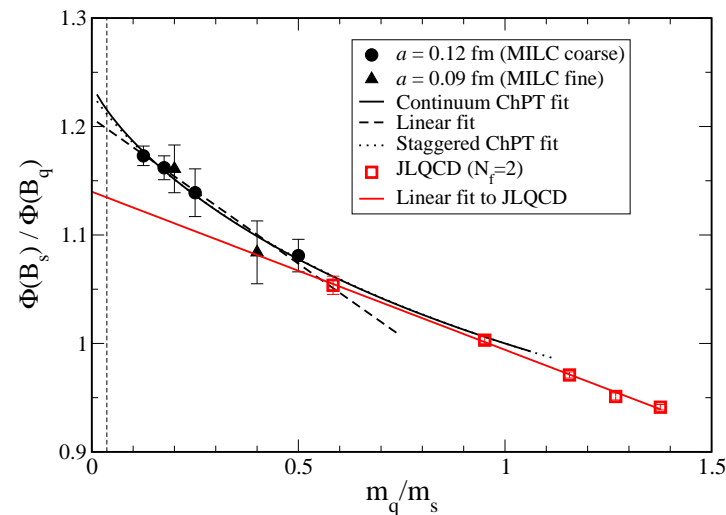
# $B_{(d,s)}^0 - \bar{B}_{(d,s)}^0$ mixing: decay constants



(Hashimoto '04)

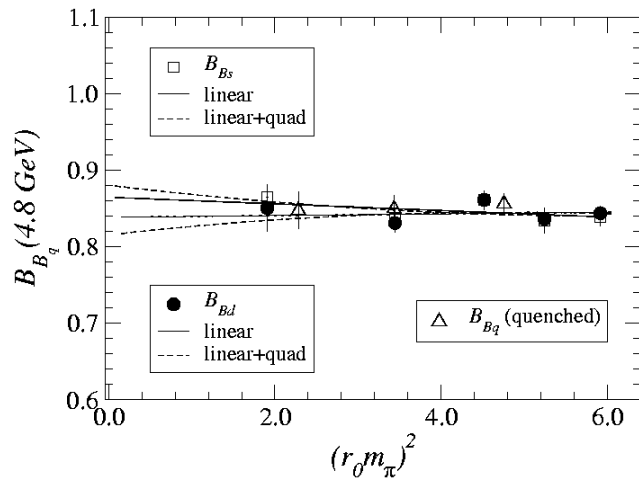
- Dependence of  $f_{B_s}$  on  $m_q$  only through sea  $\rightarrow$  weak
- De Divitiis et al '03 and ALPHA '03 perform continuum extrap.
- Different HQ approaches give consistent results in quenched approx.
- $N_f = 2 + 1$  result slightly large, but only 1-loop matching

- Result significantly higher than predicted by naïve extrapolation of older  $m_q/m_s > 0.5$  results
- Dependence on form of chiral extrap is mild
- Can consider  $(f_{B_s}/f_B)/(f_K/f_\pi)$  (Becirevic et al '03) or  $(f_{B_s}/f_B)/(f_{D_s}/f_D)$  (Grinstein '93) (+ CLEO-c) where logs partially cancel



(Okamoto '05)

# $B_{(d,s)}^0 - \bar{B}_{(d,s)}^0$ mixing: $B$ parameters and summary



$N_f = 2, m_q \gtrsim m_s/2$  (JLQCD '03)

- Chiral log coefficients small
- Quenching effects appear small, but await  $N_f = 2 + 1$  w/  $m_q \lesssim m_s/4$  results

$$\bullet \frac{\Delta M_s}{\Delta M_d} = \left| \frac{V_{ts}}{V_{td}} \right|^2 \frac{M_{B_s}}{M_B} \xi^2, \quad \xi = \frac{f_{B_s} \sqrt{B_{B_s}}}{f_B \sqrt{B_B}}$$

Many lattice errors cancel, but not chiral log

- Take HPQCD '04-'05 for  $f_{B_{(d,s)}}$  and JLQCD '03 for  $B_{B_{(d,s)}}$

- $\delta \xi \simeq 3\%$ ,  $\delta[f_B \sqrt{\hat{B}_B}] \simeq 11\%$ ,  $\delta[f_{B_s} \sqrt{\hat{B}_{B_s}}] \simeq 16\%$

- CKM fit gives  $f_B = 183(10) \text{ MeV}$  (CKM Fitter '06), and Belle '06 gives  $f_B = 176_{-23}^{+28+20} \text{ MeV}$

- Non-lattice errors in relation of  $\sqrt{\Delta M_s / \Delta M_d}$  and  $\sqrt{\Delta M_d}$  to  $(\bar{\rho}, \bar{\eta})$  are  $\simeq 1.5\%$  and  $\simeq 3\%$ , respectively

→ error goals for lattice calculations of  $\xi$  and  $f_B \sqrt{\hat{B}_B}$

Summary [in MeV]	
$f_B$	216(9)(20)
$f_{B_s}$	260(7)(20)
$f_B \hat{B}_B^{1/2}$	$245_{-26}^{+25}$
$f_{B_s} \hat{B}_{B_s}^{1/2}$	$297_{-33}^{+33}$
$f_{B_s} / f_B$	1.20(3)(1)
$\xi$	$1.21_{-3}^{+5}$



# $|V_{cb}|$ from $B \rightarrow D^* \ell \nu$

$|V_{cb}|$  plays important rôle in constraining UT  $\rightarrow$  must be determined precisely

- Can extract from differential rate extrapolated to  $w = V_B \cdot V_{D^*} = 1$  (Neubert '91)

$$\left. \frac{d\Gamma}{d\omega}(B \rightarrow D^* \ell \nu) \right|_{w=1} \propto |V_{cb}|^2 |\mathcal{F}_{D^*}(1)|^2$$

- HQET and Luke's theorem give:  $\mathcal{F}_{D^*}(1) = 1 + \mathcal{O}(1/m_{c,b}^2)$ , but precise measurement of  $|V_{cb}|$  requires reliable determination of  $\mathcal{F}_{D^*}(1) - 1$
- Double ratios in an  $N_f = 0$  calculation (3  $a$ 's) (Kronfeld et al '01, CKM '03):

$$\mathcal{F}_{D^*}(1) = 0.913_{-17-30}^{+24+17} \longrightarrow 0.91(4) \quad [4.4\%]$$

- compare w/  $\delta_{excl}^{expt} |V_{cb}| \simeq 2.1\%$ ,  $\delta_{incl}^{thy} |V_{cb}| \simeq 1.4\%$  and  $\delta_{incl}^{expt} |V_{cb}| \simeq 1.1\%$

Preliminary  $N_f = 2 + 1$  (MILC) result for  $B \rightarrow D$  form factor gives

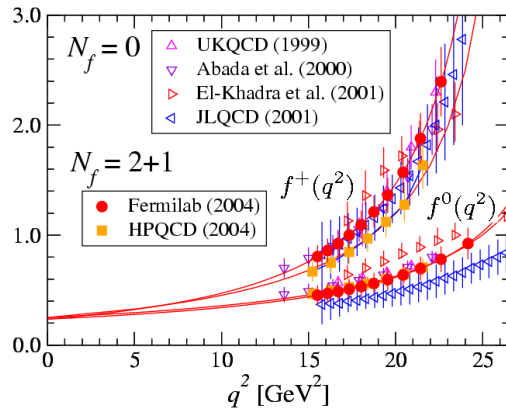
$$\mathcal{F}_D(1) = 1.074(18)(16) \text{ (FNAL '04-'05)}, \text{ consistent with } N_f = 0 \text{ result } \mathcal{F}_D(1) = 1.058_{-17}^{+20}$$

(FNAL '99)

# $|V_{ub}|$ from $B \rightarrow \pi \ell \nu$

In future, best measurement of  $|V_{ub}|$  likely to come from exclusive  $B \rightarrow X_{u\ell\nu}$

$$\langle \pi^+(\vec{k}) | \bar{u} \gamma_\mu b | \bar{B}^0(\vec{p}) \rangle \longrightarrow f^+(q^2), f^0(q^2)$$



(Hashimoto '04)

- Good consistency on dominant  $f^+(q^2)$  from different heavy quark approaches
- Errors  $\sim O(15\%)$
- $N_f = 2 + 1$  (MILC) vs  $N_f = 0 \Rightarrow$  quenching effects not significant *but* chiral log not yet studied (will limit  $q^2$  more)!

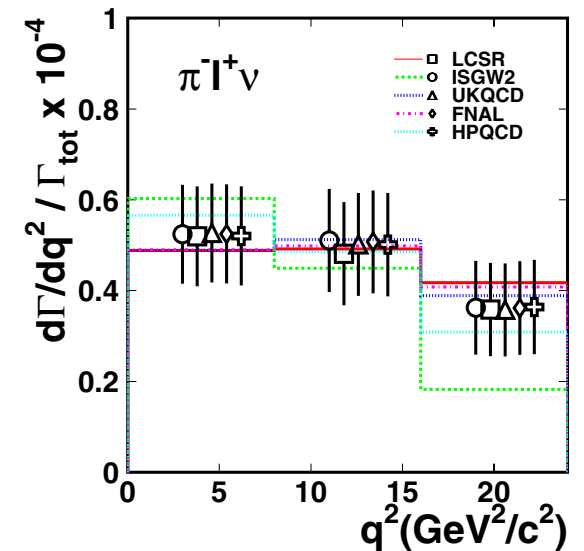
- No model-dependence if directly from differential rate in lattice  $q^2$  range (Flynn et al '96)

- HFAG '06 with rate from FNAL '04 in  $q^2 \geq 16 \text{ GeV}^2$  bin:

$$|V_{ub}| = (3.75 \pm 0.26_{-0.43}^{+0.65}) \times 10^{-3}$$

- $\delta_{excl}^{lat} |V_{ub}| \simeq 14\%$  vs.  $\delta_{incl}^{thy} |V_{ub}| \simeq 9\%$  and  $\delta_{both}^{expt} |V_{ub}| \simeq 5 \div 7\%$

(Belle '06)



# Non-leptonic weak decays of kaons

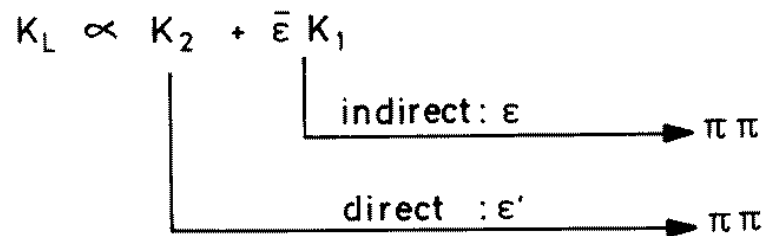
(Unreferenced results are from *Babich, Garron, Hoelbling,  
Howard, LL & Rebbi '06*)

# $K \rightarrow \pi\pi$ decays: phenomenology

$$\begin{aligned}
 -iT[K^0 \rightarrow \pi^+\pi^-] &= \sqrt{\frac{1}{3}}A_0e^{i\delta_0} + \sqrt{\frac{1}{6}}A_2e^{i\delta_2} & -iT[K^+ \rightarrow \pi^+\pi^0] &= \frac{\sqrt{3}}{2}A_2e^{i\delta_2} \\
 -iT[K^0 \rightarrow \pi^0\pi^0] &= -\sqrt{\frac{1}{3}}A_0e^{i\delta_0} + \sqrt{\frac{2}{3}}A_2e^{i\delta_2}
 \end{aligned}$$

CP violation implies  $A_i^* \neq A_i$

$$\Delta M_K = M_{K_L} - M_{K_S} \simeq 2 \operatorname{Re}M_{12}$$



$$\epsilon \equiv \frac{T[K_L \rightarrow (\pi\pi)_{I=0}]}{T[K_S \rightarrow (\pi\pi)_{I=0}]} \simeq \frac{1}{\sqrt{2}} e^{i\pi/4} \frac{\operatorname{Im}M_{12}}{\Delta M_K}$$

$$\epsilon' \simeq \frac{1}{\sqrt{2}} e^{i\pi/4} \operatorname{Im} \left( \frac{A_2}{A_0} \right)$$

Experimentally:

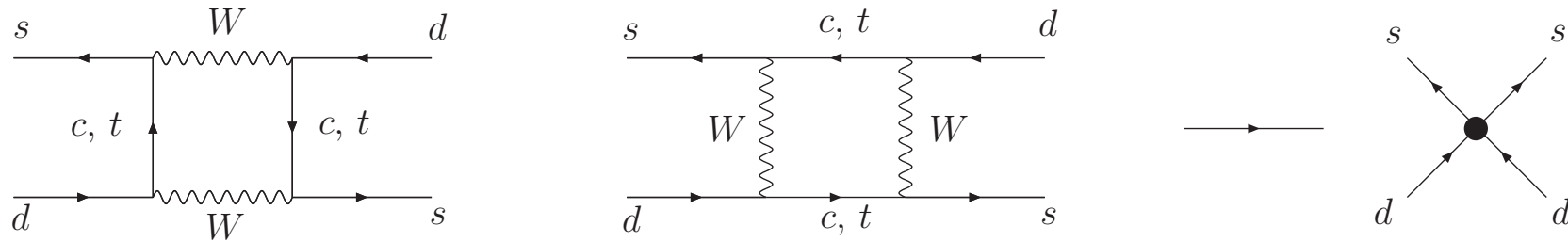
$$\Delta M_K = (3.483 \pm 0.006) \times 10^{-12} \text{ MeV} \quad [0.2\%]$$

$$|A_0/A_2| \simeq 22.2 \quad (\Delta I = 1/2 \text{ rule})$$

$$|\epsilon| = (2.282 \pm 0.017) \cdot 10^{-3} \quad [0.7\%]$$

$$\operatorname{Re}(\epsilon'/\epsilon) = (16.7 \pm 2.3) \cdot 10^{-4} \quad [14\%]$$

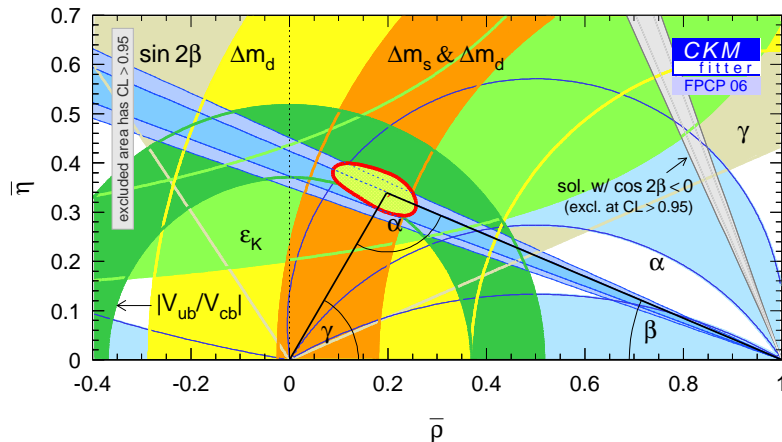
# $K^0-\bar{K}^0$ mixing in the SM: $B_K$



$$2M_K M_{12}^* = \langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 \rangle = C_1^{\text{SM}}(\mu) \langle \bar{K}^0 | O_1(\mu) | K^0 \rangle$$

$$O_1 = (\bar{s}d)_{V-A}(\bar{s}d)_{V-A} \quad \langle \bar{K}^0 | O_1(\mu) | K^0 \rangle = \frac{16}{3} M_K^2 F_K^2 B_K(\mu)$$

Constraint  $\text{Im}\lambda_t^2$ ,  $\text{Im}\lambda_c^2$  and  $\text{Im}\lambda_t\lambda_c$ , with  $\lambda_q = V_{qs}^* V_{qd}$



SM UT fit gives

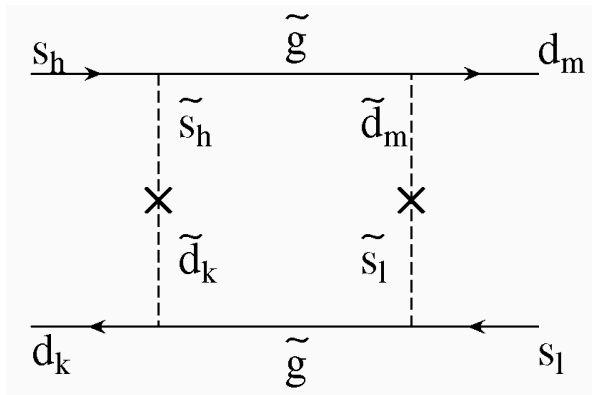
$$B_K^{\text{NDR}}(2 \text{ GeV}) = 0.54_{-6}^{+19} \quad [23\%]$$

and non- $B_K$  error in  $\epsilon$  is  $\sim 9\%$

→ lattice can have significant impact

# $\Delta S = 2$ processes beyond the SM

e.g. gluino mediated FCNC in mass insertion approximation (Ciuchini et al '99)



$$+ \dots \xrightarrow{OPE} \begin{cases} O_1 & = & [\bar{s}d]_{V-A}[\bar{s}d]_{V-A} \\ O_{2,3} & = & [\bar{s}d]_{S-P}[\bar{s}d]_{S-P} & (\text{unmix, mix}) \\ O_{4,5} & = & [\bar{s}d]_{S-P}[\bar{s}d]_{S+P} & (\text{unmix, mix}) \end{cases}$$

$$\mathcal{H}_{\text{eff,BSM}}^{\Delta S=2} = \sum_{i=1}^5 C_i^{\text{BSM}}(\mu) O_i(\mu) + \sum_{i=1}^3 \tilde{C}_i^{\text{BSM}}(\mu) \tilde{O}_i(\mu)$$

- $\tilde{O}_i$  are parity transformed operators
- $C_i^{\text{BSM}}(\mu)$ ,  $\tilde{C}_i^{\text{BSM}}(\mu)$  short distance coefficients  $\supset$  flavor mixing parameters of BSM model
- $\epsilon$  and  $\Delta M_K$  impose constraints on **Im** and **Re** of these parameters

# $\Delta S = 2$ processes (continued)

op.	$SU(3)_L \times SU(3)_R$	LO (chiral)	renorm.
$O_1$	$(27, 1)$	$p^2 / \Lambda_\chi^2$	mult.
$O_{2,3}$	$(6, \bar{6})$	1	mix
$O_{4,5}$	$(8, 8)$	1	mix

→ BSM contributions are chirally enhanced

→ in  $SU(3)_L \times SU(3)_R$  limit, relation to EW penguins discussed below:

$$\langle \pi^+ | Q_{7,8}^{3/2} | K^+ \rangle = \frac{1}{2} \langle \bar{K}^0 | O_{5,4} | K^0 \rangle$$

In SM or beyond, need non-perturbative tool to compute

$$\langle \bar{K}^0 | O_i | K^0 \rangle$$

→ lattice

# Neuberger fermions

Overlap action ( $\bar{a} = a/\rho$ ) (Neuberger '98)

$$S = a^4 \sum_x \bar{q} D[m_q] q = a^4 \sum_x \bar{q} \left[ \left( 1 - \frac{1}{2} \bar{a} m_q \right) D[0] + m_q \right] q$$

w/ Neuberger-Dirac operator

$$D[0] = \frac{1}{\bar{a}} \left( 1 + X / \sqrt{X^\dagger X} \right) \quad \text{and} \quad X = D_W - 1/\bar{a}$$

$$D_W = \frac{1}{2} \left[ \gamma_\mu (\nabla_\mu + \nabla_\mu^*) - a \nabla_\mu^* \nabla_\mu \right]$$

$\sqrt{\quad}$   $\Rightarrow$  high numerical cost  $\rightarrow$  **quenched** results

Satisfies Ginsparg-Wilson relation (Ginsparg et al '82)

$$\gamma_5 D[0] + D[0] \gamma_5 = \bar{a} D[0] \gamma_5 D[0]$$

$\Rightarrow$  violation of continuum chiral symmetry is local (doesn't contribute to on-shell Green's functions)



⇒ lattice chiral symmetry (Lüscher '98)

$$\delta\bar{q} = \bar{q}\gamma_5 T^a, \quad \delta\hat{q} = T^a\gamma_5\hat{q}, \quad \text{w/} \quad \hat{q} = (1 - \bar{a}D/2)q$$

⇒ multiplicative mass renormalization → can go to much lower quark masses (e.g. Hernández et al '99)

⇒ continuum mixing for composite operators → no chiral/flavor “tuning”

⇒ can attack problems where chiral-flavor symmetry is critical, e.g.  $\Delta I = 1/2$  rule or direct CPV in  $K \rightarrow \pi\pi$

⇒ NP  $O(a)$ -improvement

⇒ Index thm: (Hasenfratz et al '98, Lüscher '98)

$$Q = \frac{\bar{a}}{2} \text{Tr}[\gamma_5 D] = n_- - n_+$$

Potential nuisance →  $\frac{1}{\sqrt{m_q}}$  inchirality flipping part of quark propagators  
→ can eliminate, in some cases, with chirality

Perform calculations at two values of the lattice spacing:  $a^{-1} = 2.18(8) \text{ GeV}$  ( $\beta = 6.0$ )  
and  $a^{-1} = 1.50(4) \text{ GeV}$  ( $\beta = 5.85$ )

# $\Delta S = 2$ matrix elements

$\langle \bar{K}^0 | O_1 | K^0 \rangle$  best given in terms of

$$B_1(\mu) = B_K(\mu) = \frac{3}{16} \frac{\langle \bar{K}^0 | O_1(\mu) | K^0 \rangle}{F_K^2 M_K^2}$$

Also ( $i = 2, \dots, 5$ )

$$B_i(\mu) = \frac{1}{N_i} \frac{\langle \bar{K}^0 | O_i(\mu) | K^0 \rangle}{\langle \bar{K}^0 | \bar{s} \gamma_5 d(\mu) | 0 \rangle \langle 0 | \bar{s} \gamma_5 d(\mu) | K^0 \rangle}$$

but  $|\langle 0 | \bar{s} \gamma_5 d | K^0 \rangle| = \sqrt{2} F_K M_K^2 / (m_s + \hat{m})$  poorly known

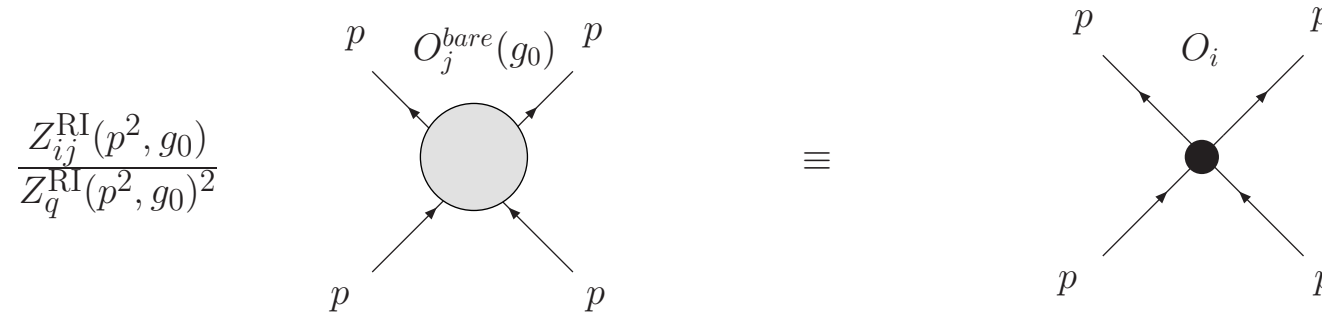
$\Rightarrow$  Define ( $i = 2, \dots, 5$ ) (see also Donini et al '99)

$$R_i^{\text{BSM}}(\mu, M^2) \equiv \left[ \frac{F_K^2}{M_K^2} \right]_{\text{expt}} \times \left[ \frac{M^2}{F^2} \frac{\langle \bar{P}^0 | O_i(\mu) | P^0 \rangle}{\langle \bar{P}^0 | O_1(\mu) | P^0 \rangle} \right]_{\text{lat}}$$

Advantages:  $[ \ ]_{\text{lat}}$  is dimensionless and should have smooth chiral behavior;  
 $R_i^{\text{BSM}}(M_K^2/F_K^2)$  is ratio of BSM to SM contribution

# RI/MOM NP renormalization (Martinelli et al '95)

Compute amputated, forward quark Green's functions in Landau gauge  
 $(\Lambda_{\text{QCD}}^2 \ll p^2 \ll (\pi/a)^2)$



We define the ratio  $(Z_A(g_0))^2$  obtained from AWI

$$\mathcal{R}_{ij}^{\text{RI}}(m_q, p^2, g_0) \equiv Z_A(g_0)^2 \Gamma_V(m, p^2, g_0)^2 \frac{1}{\Gamma_O^{ij}(m_q, p^2, g_0)}$$

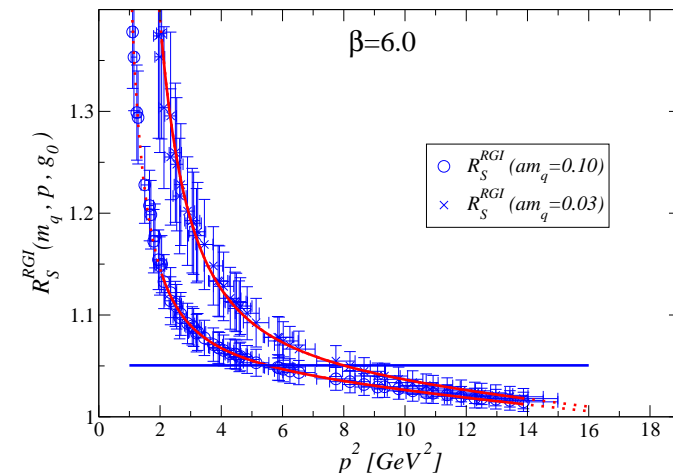
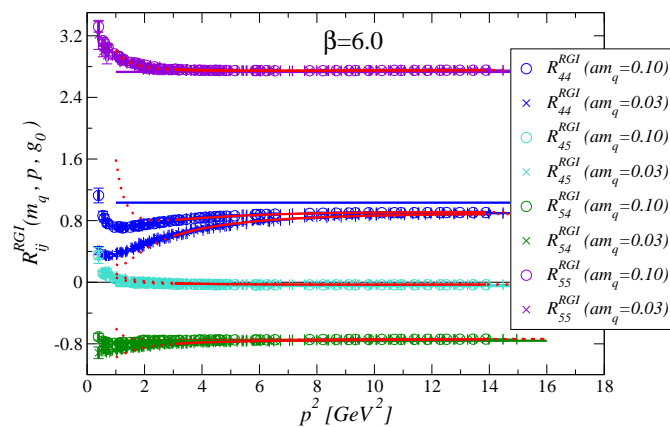
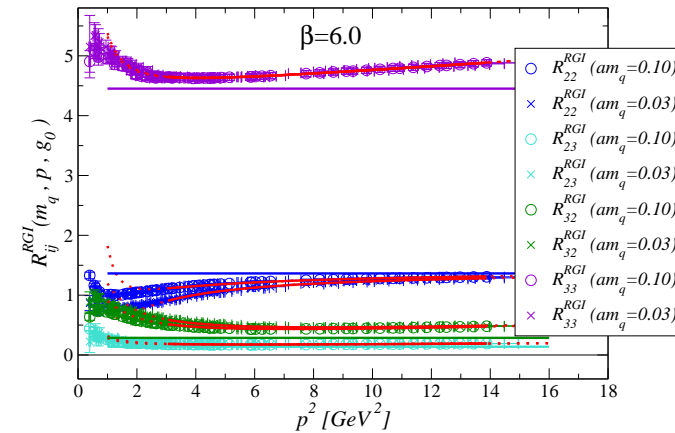
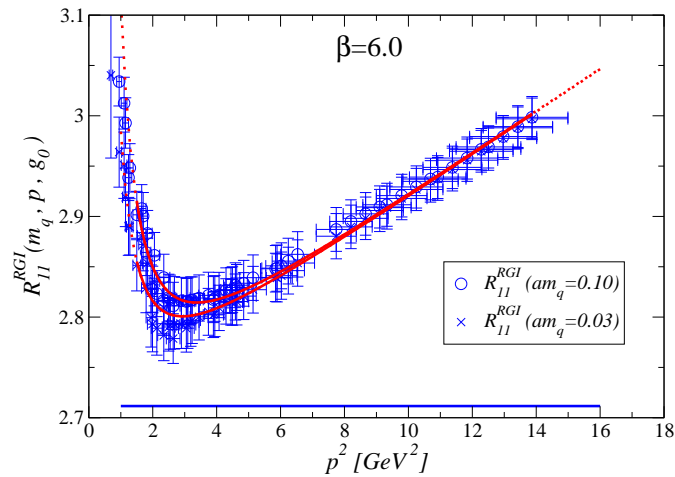
Fit to OPE form  $(m_q^2 \ll p^2)$

$$\mathcal{R}_{ij}^{\text{RI}}(m_q, p^2, g_0) = \dots + \frac{A_{ij}}{p^2} + U_{ik}^{\text{RI}}(p^2) Z_{kj}^{\text{RGI}}(g_0) + B_{ij}(ap)^2 + \dots$$

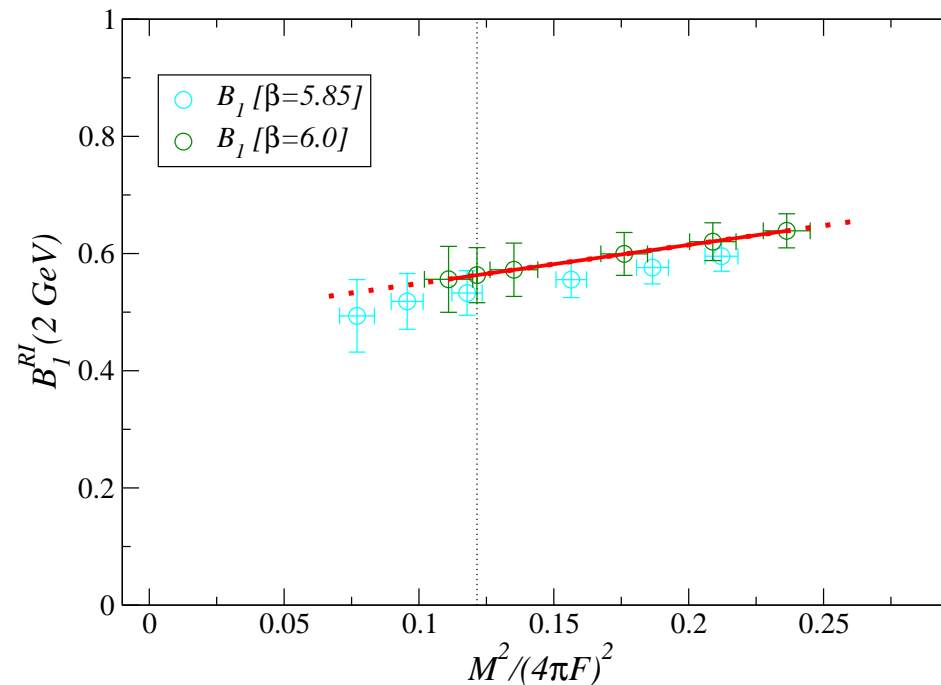
where  $U_{ik}^{\text{RI}}(p^2)$  describes the running of  $Z_{ij}^{\text{RI}}(p^2, g_0)$ , implemented at 2-loops w/  $N_f = 0$   
 from Ciuchini et al '98 and  $\alpha_s$  from Capitani et al '99

# RI/MOM renormalization for $O_i$ and $P$ at $\beta = 6.0$

Combined fit of lightest and heaviest quark results to OPE form in range  $3. \text{ GeV}^2 \leq p^2 \leq 14. \text{ GeV}^2$  (systematics range  $7.5 \text{ GeV}^2 \leq p^2 \leq 14. \text{ GeV}^2$ )



# Result for $B_K$



- Linear interpolation to the kaon:

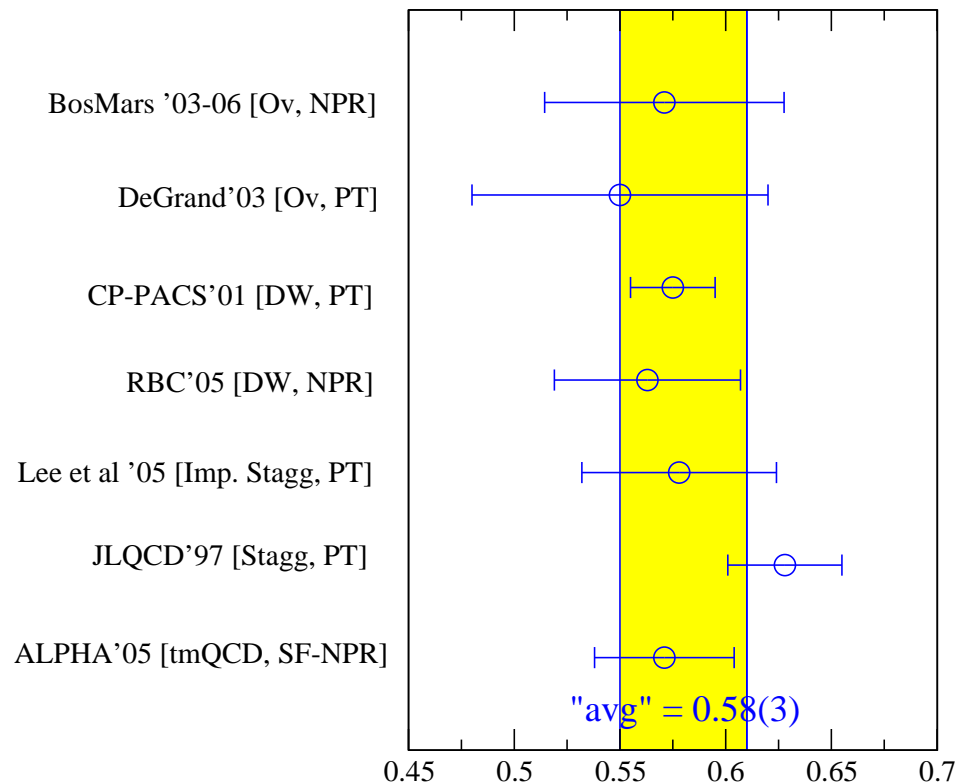
$$B_K^{\text{NDR}}(2 \text{ GeV}) = 0.571(48)(30)$$

where central value and statistical error from  $\beta = 6.0$  simulation and second error comes from systematic in  $Z_{11}$  added in quadrature to difference from  $\beta = 5.85$  result

- $B_K^{\text{NDR}}(2 \text{ GeV}) = 0.63(6)(1)$  on  $16^3 \times 32$  lattice at  $\beta = 6.0$  (Garron et al '04)  
→ no significant finite volume effect

# $K^0-\bar{K}^0$ mixing in the SM: quenched summary

Quenched  $B_K^{NDR}(2\text{ GeV})$



Unquenched results are beginning to appear:

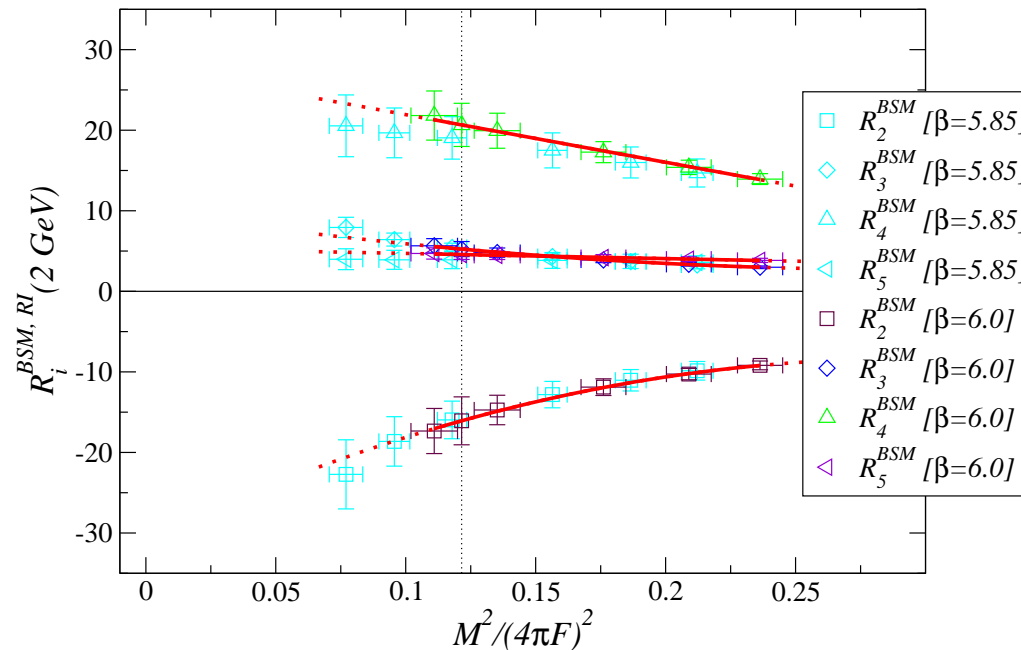
(UKQCD '04, RBC '04-'05, Baez et al '05)

- Improved staggered on MILC  
 $N_f = 2 + 1$  coarse ensembles (Gamiz et al '06)  
 $\rightarrow B_K^{NDR}(2\text{ GeV}) = 0.618(18)_{stat}(19)_\chi(30)_{a^2}(130)_{\alpha_s^2}$
- $m_{u,d}^{lat} \gtrsim m_s^{phys}/2$  in most cases
- Use  $N_f = 2$  and  $N_f = 0$  from RBC '05 at fixed lattice spacing to estimate quenching error
- Add in quadrature w/ 5%  $SU(3)$  breaking error

$$B_K^{NDR}(2\text{ GeV}) = 0.58(3)(6) \leftrightarrow \hat{B}_K^{N_f=3} = 0.78(4)(9) \quad [13\%]$$

# Results for BSM $\Delta S = 2$ matrix elements

Results at 2 GeV in RI/MOM scheme for BSM ratios  $R_i^{BSM}$



Interpolate polynomially to physical point

Do the same for  $B$ -parameters

# Results for BSM $\Delta S = 2$ matrix elements (2)

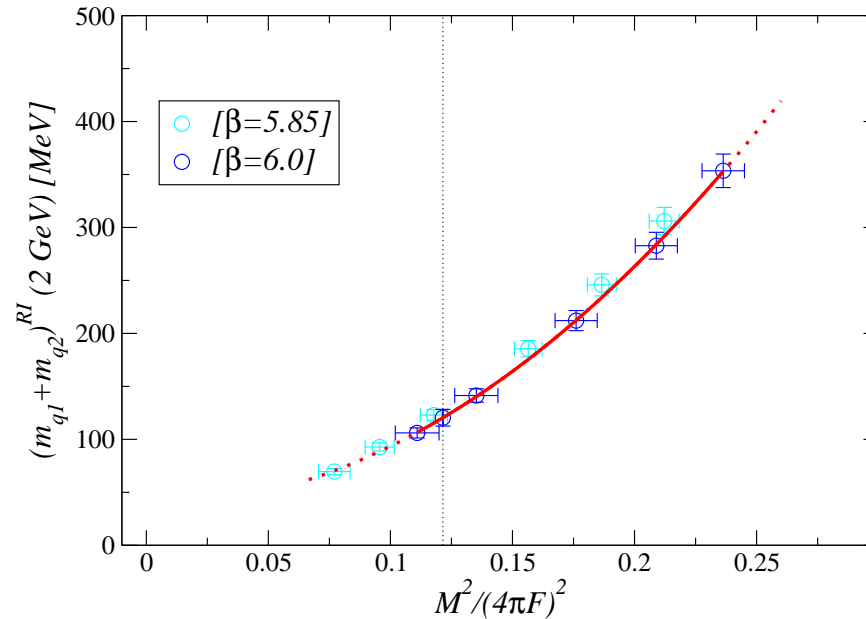
Results at 2 GeV in RI/MOM scheme

$i$	$B_i$		$R_i$	
	This work	Donini et al '99	This work	Donini et al '99
1	0.563(47)(30)	0.69(21)	1	1
2	0.865(72)(35)	0.70(9)	-16.1(30)(6)	-6.4(18)
3	1.41(10)(13)	1.1(1)	5.24(93)(16)	2.0(6)
4	0.938(48)(36)	1.1(1)	20.7(27)(18)	12.(3)
5	0.616(51)(59)	0.77(11)	4.57(60)(68)	2.8(8)

- BSM matrix elements are enhanced at  $M_K$  by a factor of  $\simeq 5 \div 20$
- Enhancement significantly larger than in Donini et al '99
- Only partially due to our smaller  $B_1$
- BSM  $B$ -parameters agree much better than  $R_i^{\text{BSM}} \rightarrow$  we already disagree on  $\langle 0 | \bar{s} \gamma_5 d | K^0 \rangle$
- $\langle 0 | \bar{s} \gamma_5 d | K^0 \rangle = \sqrt{2} F_K M_K^2 / (m_s + \hat{m}) \rightarrow$  we must find smaller  $(m_s + \hat{m}_d)$



# Result for $(m_s + \hat{m})$



Obtain by interpolation and conversion (errors as above)

$$(m_s + \hat{m})^{\overline{\text{MS}}}(2 \text{ GeV}) = 102.(7)(6) \text{ MeV}$$

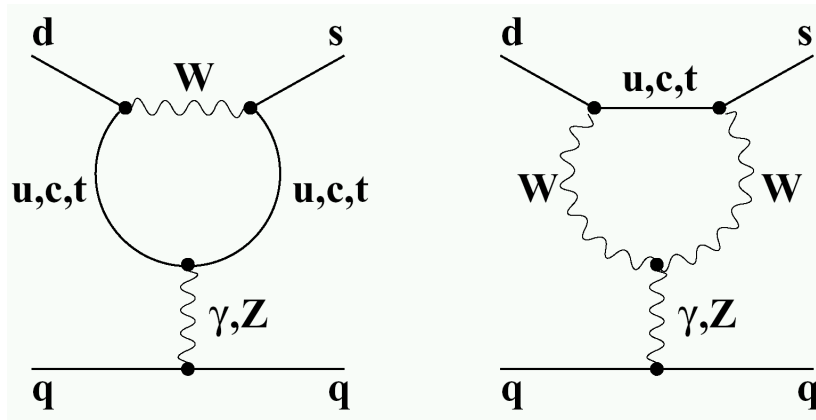
in excellent agreement with quenched continuum limit result [ALPHA '00](#)

Compare with  $(m_s + \hat{m})^{\overline{\text{MS}}}(2 \text{ GeV}) \approx 130(2)(18) \text{ MeV}$  as found with Wilson fermions at couplings used by [Donini et al '99](#)

→ due to large discretization errors absent in our calculation?

# $\epsilon'$ and EW penguins

EW penguins give dominant  $\Delta I = 3/2$  contribution to  $\epsilon'$  (Flynn et al '87)



$$Q_{7,8}^{3/2} = \frac{1}{2} [(\bar{s}d)_{V-A}(\bar{u}u)_{V+A} + (\bar{s}u)_{V-A}(\bar{u}d)_{V+A} - (\bar{s}d)_{V-A}(\bar{d}d)_{V+A}]$$

$Q_7^{3/2}$  color diagonal and  $Q_8^{3/2}$  color mixed

- In limit  $m_u = m_d$ , no mixing w/ lower dim. ops  $\Rightarrow$  much simplified lattice calculation
- Compute  $\langle \pi^+ | Q_{7,8}^{3/2} | K^+ \rangle$  and obtain physical amplitude at LO in  $\chi$ PT through

$$\langle (\pi\pi)_{I=2} | Q_{7,8} | K^0 \rangle \propto \frac{1}{F} \langle \pi^+ | Q_{7,8}^{3/2} | K^+ \rangle$$

- NPR is identical to that for BSM  $\Delta S = 2$  operators  $O_{5,4}$

# Electroweak penguins: quenched mass-dependence

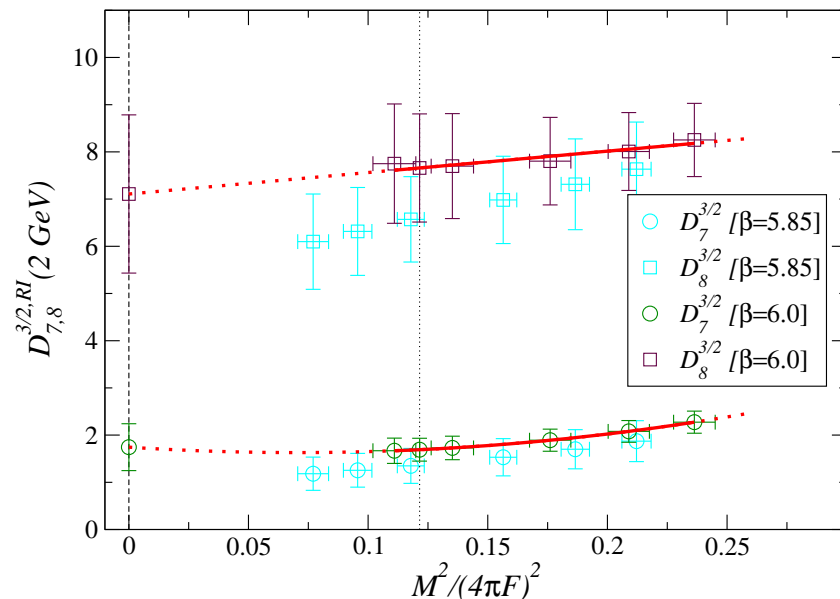
- Look for quantity which has same chiral behavior in  $N_f = 0$  and 3  $\chi$ PT
- log coefficient small and mass-dependence weak  $\Rightarrow$  reasonable chiral extrapolations give

$$\left. \frac{\langle \pi^+ | Q_{7,8}^{3/2} | K^+ \rangle}{F^2} \right|_{\chi} \approx \left. \frac{\langle \pi^+ | Q_{7,8}^{3/2} | K^+ \rangle}{F^2} \right|_{M_K^2}$$

- Take 
$$\left. \frac{\langle \pi^+ | Q_{7,8}^{3/2} | K^+ \rangle}{F} \right|_{\chi} = \left. \frac{\langle \pi^+ | Q_{7,8}^{3/2} | K^+ \rangle}{F^2} \right|_{M_K^2} \times \frac{F_{\chi} + F_K}{2}$$
 with systematic given by difference

Using Gasser & Leutwyler '85, Bernard & Golterman '92, Becirevic & Villadoro '04 and Villadoro '04:

$$\frac{\langle \pi^+ | Q_{7,8}^{3/2} | K^+ \rangle}{F^2} = D_{7,8}^{\chi} \left\{ 1 - \left[ \frac{M}{4\pi F} \right]^2 \times \ln \left[ \frac{M}{4\pi F \alpha_{7,8}} \right]^2 + O(p^4) \right\}$$



# Electroweak penguins: results in the chiral limit

Summary of results, in the  $\chi$ ral limit, in the  $\text{NDR} - \overline{\text{MS}}$  scheme at  $2 \text{ GeV}$ , in units of  $\text{GeV}^3$

All lattice results quenched

Ref.	Action	$\langle \pi^+   Q_7^{3/2}   K^+ \rangle / F$	$\langle \pi^+   Q_8^{3/2}   K^+ \rangle / F$
This work	Neuberger	0.211(20)(42)	0.93(13)(18)
CP-PACS'01	Domain-Wall	0.220(6)(??)	0.92(3)(??)
RBC'02	Domain-Wall	0.255(12)(??)	1.02(4)(??)
SPQcdR'04	Wilson	$\sqrt{2/3} \times 0.16(3)$	$\sqrt{2/3} \times 0.82(15)$
Bijnens et al '01		0.24(3)	1.2(7)
Cirigliano et al '02		0.22(5)	1.50(27)
Friot et al '04		0.12(2)	2.00(36)
Knecht et al '01		0.11(3)	3.5(1.1)
Narison '00		0.21(5)	1.40(35)

Need unquenched results and control on final state interactions ( $K \rightarrow \pi$  vs  $K \rightarrow \pi\pi$ )

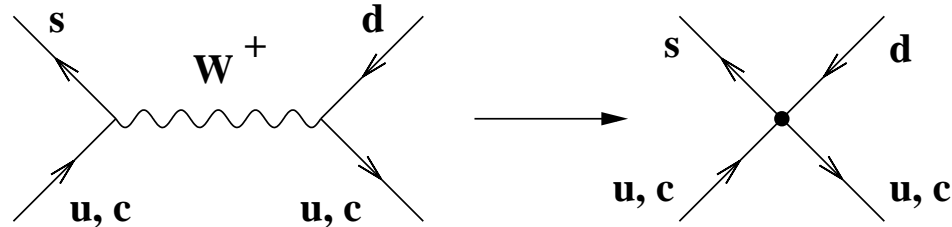
SPQcdR '04 obtain  $K \rightarrow \pi\pi$  MEs at NLO (rescaled to agree with above normalization)

$$\langle Q_7^{3/2} \rangle = \sqrt{2/3} \times 0.12(1)(1)(1) \text{ GeV}^3 \quad \langle Q_8^{3/2} \rangle = \sqrt{2/3} \times 0.68(6)(4)(5) \text{ GeV}^3$$

# $\Delta I = 1/2$ rule with an active charm

First step toward a calculation of  $\epsilon'$  from first principles

OPE (CP-conserving  $\Delta S = 1$  transitions)



$$+\text{rad.corr.} \longrightarrow \mathcal{H}_{\text{eff}}^{\Delta S=1} = \frac{G_F}{2\sqrt{2}} V_{ud} V_{us}^* \sum_{i=\pm} C_i(\mu, M_W) \mathcal{O}_i(\mu)$$

$$\mathcal{O}_{\pm} = [(\bar{s}d)_{V-A}(\bar{u}u)_{V-A} \pm (\bar{s}u)_{V-A}(\bar{u}d)_{V-A}] - [u \rightarrow c]$$

- $\mathcal{O}_-$  is pure  $I = 1/2$
- $\mathcal{O}_+$  has both  $I = 1/2, 3/2$

Short distance enhancement?

$$|C_-(M_W)/C_+(M_W)| = 1 + \mathcal{O}(\alpha_s(M_W)) \longrightarrow |C_-(2\text{GeV})/C_+(2\text{GeV})| \sim 2$$

(Gaillard et al '74; Altarelli et al '74; Shifman et al '75-'77)

# $\Delta I = 1/2$ rule with an active charm (2)

Most of enhancement must come from long distance QCD effects in

$$\frac{\langle (\pi\pi)_{I=0} | C_+ \mathcal{O}_+ + C_- \mathcal{O}_- | K^0 \rangle}{\langle (\pi\pi)_{I=2} | C_+ \mathcal{O}_+ | K^0 \rangle}$$

Some evidence from quenched lattice studies of  $K \rightarrow \pi$  w/ integrated charm (CP-PACS '01; RBC '01; Pekurovsky et al '01) and in unphysical  $SU(4)_f$  limit (Giusti et al '05)

LQCD w/  $\chi$ ral symmetry  $\Rightarrow$  renormalized, continuum operators are given by:

$$\hat{\mathcal{O}}_{\pm}(\mu) = Z_{\pm}(\mu a, g_0) \tilde{\mathcal{O}}_{\pm}(g_0) + \mathcal{O}(a^2)$$

$$\tilde{\mathcal{O}}_{\pm} = \mathcal{O}_{\pm} + (m_c^2 - m_u^2) C_{\pm}^m \mathcal{O}_m \quad \text{and} \quad \mathcal{O}_m = (m_s + m_d) \bar{s}d - (m_s - m_d) \bar{s}\gamma_5 d$$

$\Rightarrow$  no power divergent subtractions w/ GW fermions, even when simpler  $K \rightarrow \pi$  transitions are studied (e.g. Capitani et al '01)

$\Rightarrow$  GW fermions provide a unique opportunity to elucidate  $\Delta I = 1/2$  puzzle

# Conclusion

- Large range of quantities of central importance to particle physics is being computed w/ lattice QCD, many of which could not be presented here . . .
- . . . e.g. leptonic and semileptonic  $D$  decays with  $N_f=2+1$  staggered MILC configs in good agreement with FOCUS and CLEO-c measurements (Fermilab/MILC '04-'06)
- More and more  $N_f=2+1$  results with *controlled* chiral extrapolation available . . .
- . . . albeit mostly on staggered MILC configurations ( $\det(D[M]_{\text{stagg}})^{1/4}$ , inexact algorithm, correlations . . .)
- Projects with light seas of theoretically sound Wilson quarks are in progress
- Presented quenched calculation of BSM  $\Delta S = 2$  and  $\Delta I = 3/2$  matrix elements with chirally symmetric fermions, NPR and at two values of  $a$ 
  - BSM matrix elements are significantly more enhanced than found previously
  - new possibilities for the calculation of weak matrix elements associated with the  $\Delta I = 1/2$  rule and  $\epsilon'$  (cf DWF calculations by CP-PACS and RBC '01; analytical work by Capitani et al '00, '01)
- Chirally symmetric fermions work; improved algorithms allow quarks light enough to match onto ChiPT; and new computers are coming on line
  - expect many exciting results in the coming months/years

# Conclusion (2)

Given  $O(5 - 10)$  Tflops, we should be able simulate full QCD with

$$L = 2.0 - 2.5 \text{ fm} \quad m_q = (0.1 - 0.5)m_s \quad a = 0.05 - 0.10 \text{ fm}$$

and can hope for:

Qty	current err.	goal	projected err.
$f_{B_{d,s}} \sqrt{\hat{B}_{B_{d,s}}}$	15%	3%	5%
$\xi$	3%	1.5%	1.5%
$B \rightarrow D^* \ell \nu$	4.4%	1.8%	3%
$B \rightarrow \pi \ell \nu$	15%	5%	10%
$f_K / f_\pi$	0.9%	0.25%	0.9%
$f_{K\pi}^+(0)$	1.1%	0.5%	0.7%
$B_K$	13%	10%	5%

“Just” need the computer and the man power!