Baryons on the Light Cone:

Theory and Phenomenology of Baryon Distribution Amplitudes

Vladimir M. Braun

University of Regensburg

Outline

Motivation:

- Hard inclusive vs. exclusive reactions
- Hadron distribution amplitudes

Theory:

- Operator mixing and conformal symmetry
- Exact integrability
- Diquarks?

Phenomenology:

- Experimental highlights
- Soft vs. hard contributions
- Status of perturbative factorization
- Light-Cone Sum Rules

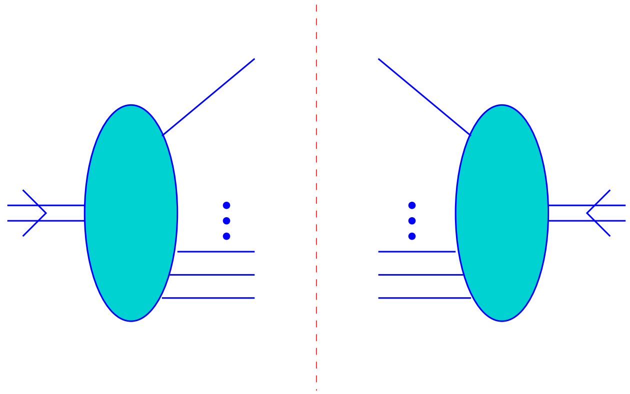
Outlook

Hard inclusive vs. exclusive reactions

Concept: Parton distributions

are essentially one-particle densities:

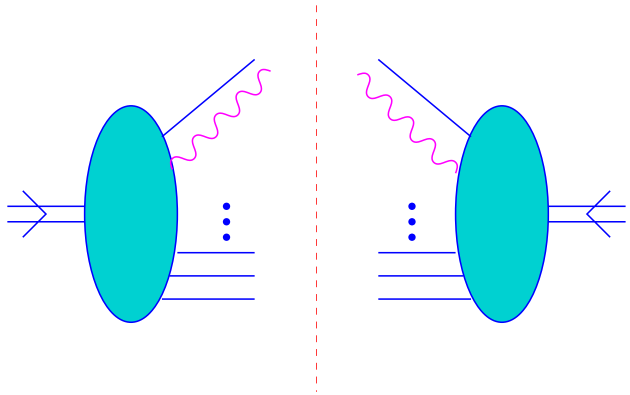
Fock-Space: $|\mathrm{Proton}\rangle = \begin{cases} qqq \\ qqqG \\ qqq\bar{q}q \\ \ldots \end{cases}$



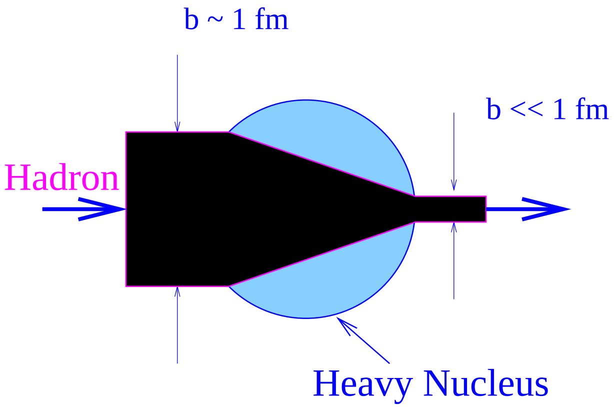
$$F_2(x) \simeq \sum_{qqq\ldots} \int dx_i \, dk_i^{\perp} \; |\Psi_{qqq\ldots}(x_i, k_i^{\perp})|^2 \delta(x_q - x)$$

Multiparton (higher twist) distributions

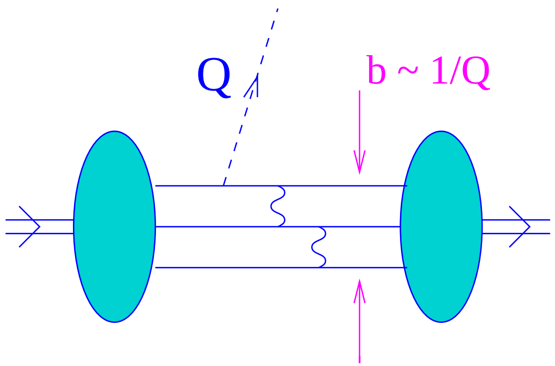
describe correlations and interference effects



How to isolate different components?



Color filtering



Large momentum transfer

Concept: **Distribution amplitudes**

Wave functions at small transverse separations

$$\varphi_N(x_1, x_2, x_3) \simeq \int dk_i^{\perp} \; \Psi_{qqq}(x_i, k_i^{\perp})$$

Future of Hadronic Physics?

Baryon distribution amplitudes $B = N, \Delta, \ldots$

quark fields “live” on a light-ray $z^2 = 0$

$$\langle 0|q(z_1)q(z_2)q(z_3)|B(p,\lambda)\rangle \quad = \quad \ldots \int_0^1 dx_1 dx_2 dx_3\, \delta(\textstyle\sum x_i - 1)\, e^{-ip(x_1 z_1 + x_2 z_2 + x_3 z_3)} \varphi_B(x_i, \mu^2)$$

$$\boxed{\begin{aligned} q^\uparrow q^\uparrow q^\uparrow \quad &\Rightarrow \quad \varphi_\Delta^{\lambda=3/2}(x_i,\mu^2)\,, \\ q^\uparrow q^\downarrow q^\uparrow \quad &\Rightarrow \quad \begin{cases} \varphi_N^{\lambda=1/2}(x_i,\mu^2) \\ \varphi_\Delta^{\lambda=1/2}(x_i,\mu^2) \end{cases} \end{aligned}}$$

- **Limit of zero transverse separation is singular; traded for the scale dependence**

Moments of distribution amplitudes $\Leftrightarrow$ local operators:

$$\varphi(x_i) \to \varphi(k_i) \quad = \quad \int \mathcal{D}x_i\, x_1^{k_1} x_2^{k_2} x_3^{k_3}\, \varphi(x_i,\mu^2)$$

$$q(z_1)q(z_2)q(z_3) \quad \to \quad (D_+^{k_1} q)\,(D_+^{k_2} q)\,(D_+^{k_3} q)$$

Nucleon Distribution Amplitudes - *continued*

Twist-3

$$\langle 0|\,\varepsilon^{ijk}\left(u_i^\uparrow(a_1 z) C \not{z} u_j^\downarrow(a_2 z)\right)\not{z} d_k^\uparrow(a_3 z)\,|N(P,\lambda)\rangle = -2pz\not{z}N^\uparrow(P)\int \mathcal{D}\xi e^{-ipz\sum \xi_i a_i}\Phi_3(\xi_i)$$

Twist-4

$$\begin{aligned}
\langle 0|\,\varepsilon^{ijk}\left(u_i^\uparrow(a_1 z) C \not{z} u_j^\downarrow(a_2 z)\right)\not{p} d_k^\uparrow(a_3 z)\,|N(P,\lambda)\rangle &= -2pz\not{p}N^\uparrow(P)\int \mathcal{D}\xi e^{-ipz\sum \xi_i a_i}\Phi_4^{\parallel}(\xi_i)\\
\langle 0|\,\varepsilon^{ijk}\left(u_i^\uparrow(a_1 z) C \gamma_\perp u_j^\downarrow(a_2 z)\right)\gamma_\perp \not{z} d_k^\downarrow(a_3 z)\,|N(P,\lambda)\rangle &= 4m_N\not{z}N^\uparrow(P)\int \mathcal{D}\xi e^{-ipz\sum \xi_i a_i}\Phi_4^{\perp}(\xi_i)\\
\langle 0|\,\varepsilon^{ijk}\left(u_i^\uparrow(a_1 z) C \not{p}\not{z} u_j^\downarrow(a_2 z)\right)\not{z} d_k^\uparrow(a_3 z)\,|N(P,\lambda)\rangle &= 2m_N pz\not{z}N^\uparrow(P)\int \mathcal{D}\xi e^{-ipz\sum \xi_i a_i}\Phi_4^{T}(\xi_i)
\end{aligned}$$

- **Momentum fraction distributions of quarks in the state with minimum number of Fock constituents at small (zero) transverse separations**
- **Complementary to parton distributions that include summation of all Fock states**
- **Largerly unknown**

Separation of coordinates

In Quantum Mechanics:

O(3) rotational symmetry $\Rightarrow$ Angular vs. radial dependence

$$\left[-\frac{\hbar^2}{2m}\Delta + V(|r|)\right]\Psi = E\Psi \quad \Rightarrow \quad \Psi(\vec{r}) = R(r)Y_{lm}(\theta,\phi)$$

$\heartsuit$

$Y_{lm}(\theta,\phi)$ are eigenfunctions of $L^2 Y_{lm} = l(l+1)Y_{lm}$, $[\mathcal{H}, L^2] = 0$.

Lorentz boosts + Dilatations + Inversion $x \to 1/x$ form a group of symmetry

In Quantum Chromodynamics:

SL(2,R) conformal symmetry $\Rightarrow$ Longitudinal vs. transverse dependence

Separation of coordinates -*continued.*

- **Dependence on transverse coordinates is traded for the scale dependence of the DAs:**

$$\mu^2\frac{d}{d\mu^2}\,\phi_\pi(u,\mu) = \int_0^1 dw\,\mathcal{H}(u,w)\,\phi_\pi(u,\mu) \Rightarrow \begin{array}{l} \phi_\pi(u,\mu) = 6u(1-u)\left[1+\phi_\pi^2(\mu)C_2^{3/2}(2u-1)+\ldots\right] \\ \phi_\pi^n(\mu) = \phi_\pi^n(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_n/b_0} \end{array}$$

- **Summation goes over possible values of the conformal spin:** $j = j_q + j_{\bar q} + n = 1 + 1 + n$
- **Gegenbauer Polynomials** $C_n^{3/2}$ **are 'spherical harmonics' of the** $SL(2,R)$ **group**
- **Exact analogy: Partial wave expansion in Quantum Mechanics**

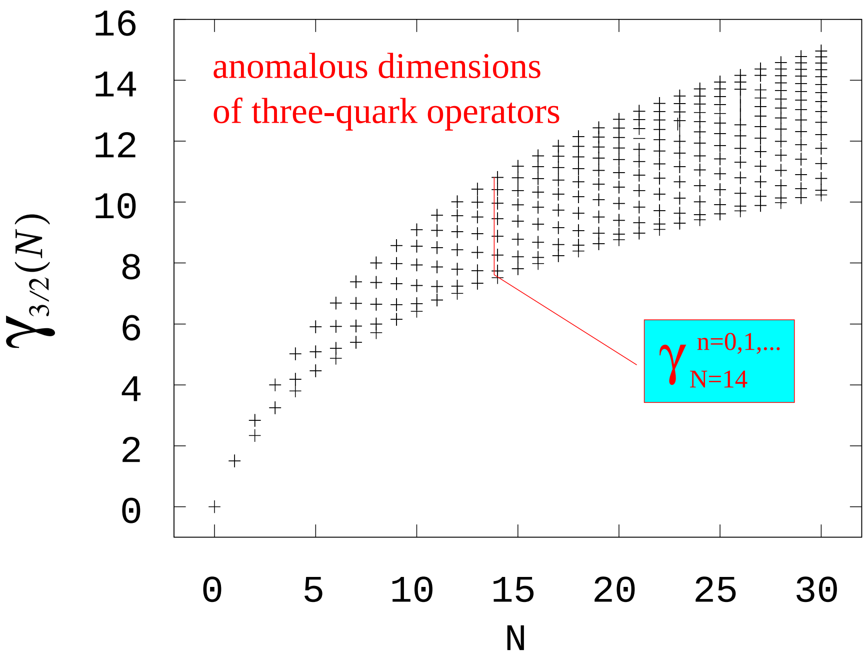
For Baryons: Summation of three conformal spins

$$\begin{aligned} \phi_N(u_i) &= 120u_1u_2u_3\sum_{J=3}^{\infty}\sum_{j=2}^{J-1}\phi_N^{J,j}(\mu)\,Y_{J,j}^{(12)3}(u) \\ J &= 3+N, N=0,1,\ldots \quad \text{total spin of three quarks} \\ j &= 2+n, n=0,1,\ldots \quad \text{spin of the (12)-quark pair} \end{aligned}$$

Scale Dependence

The three quarks in the nucleon with the given spin J can still be in different parton states

$\Rightarrow$ a nontrivial mixing matrix:



The lowest anomalous dimension is separated from the rest by a finite 'mass gap'. This is due to 'binding' of the scalar diquark and in a different context may lead to color superconductivity

- **Evolution equations for the operators with maximum helicity are completely integrable**
- **Large part of the two-loop kernel calculated (Belitsky, Korchemsky, Mueller '05)**

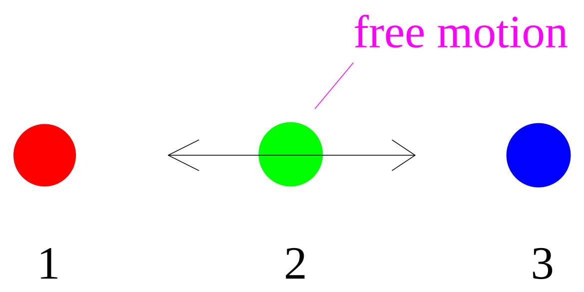
Breakdown of integrability: Mass gaps and bound states —*"interpretation"*

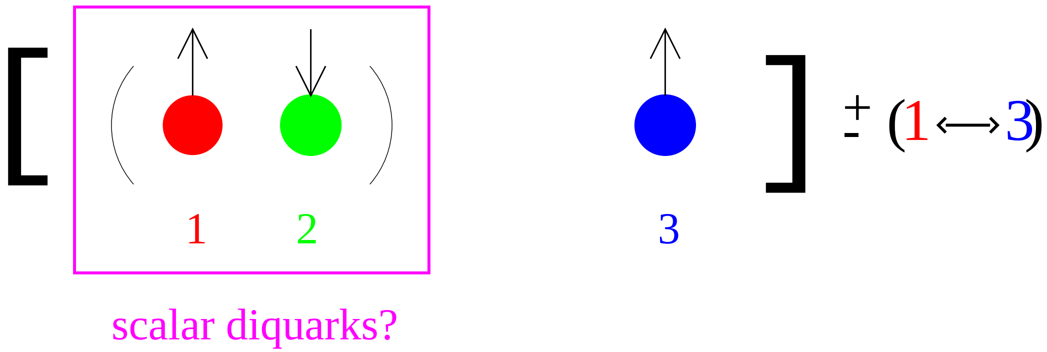
$\Delta^{\lambda=3/2}$ wave function

Nucleon and $\Delta^{1/2}$ wave functions

$$\varphi_{\Delta^{3/2}}(x_i)^{\mu} = \sum_{N=0}^{\infty} \varphi_{N,n=0}^{\mu_0} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_{N,n=0}} \Big\{ x_1(1-x_1)C_{N+1}^{3/2}(1-2x_1) + x_2(1-x_2)C_{N+1}^{3/2}(1-2x_2) + x_3(1-x_3)C_{N+1}^{3/2}(1-2x_3) \Big\}$$

$$\varphi_{\Delta^{1/2}}(x_i)^{\mu} = x_1 x_2 x_3 \sum_{N=0}^{\infty} \varphi_{N,n=0}^{\mu_0} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_{N,n=0}} \Big\{ P_N^{(1,3)}(1-2x_3) \pm P_N^{(1,3)}(1-2x_1) \Big\}$$





To summarize:

Theory of DAs is based on

- QCD factorization
- Conformal symmetry of the QCD Lagrangian

Principal physics questions:

- Is conformal spin a good quantum number for realistic Q^2?
- What physics is beyond exact integrability?

Challenges:

- Rare processes; difficult to observe
- Difficult to observe a DA directly; rather convolution integrals

typically $\int_0^1 \frac{du}{1-u} \phi_\pi(u,\mu) \sim 1 + \sum_{n=2,4,\ldots} \phi_n(\mu)$

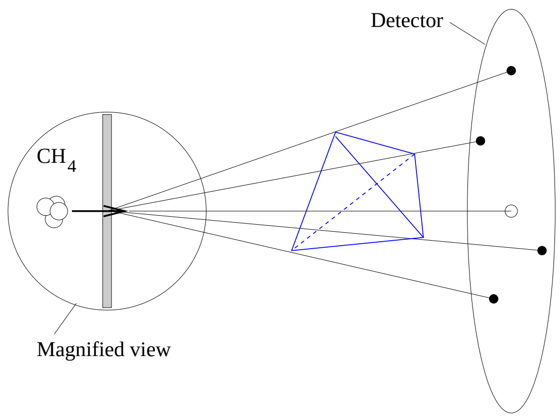
- In many cases, suspect large corrections

Therefore, assume rapid convergence of the expansion and use models

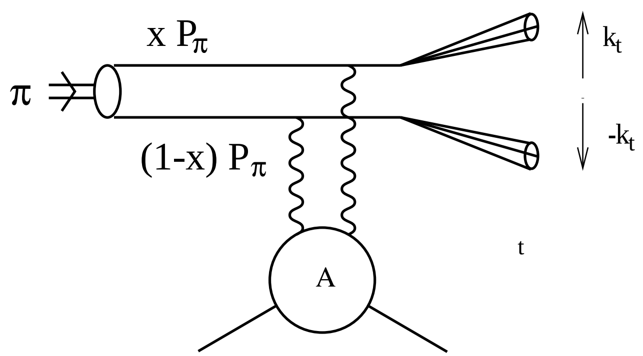
$$\boxed{\begin{aligned} \phi_\pi(u) &= 6u(1-u)\left[1 + \phi_\pi^2(\mu) 6(1-5u+5u^2)\right] \\ \phi_N(u_i) &= 120 u_1 u_2 u_3 \left[\phi_N^0 + \phi_N^-(u_1-u_2) + \phi_N^+(1-3u_3)\right] \end{aligned}}$$

Can one measure DAs directly?

Coulomb Explosion Imaging of Small Molecules

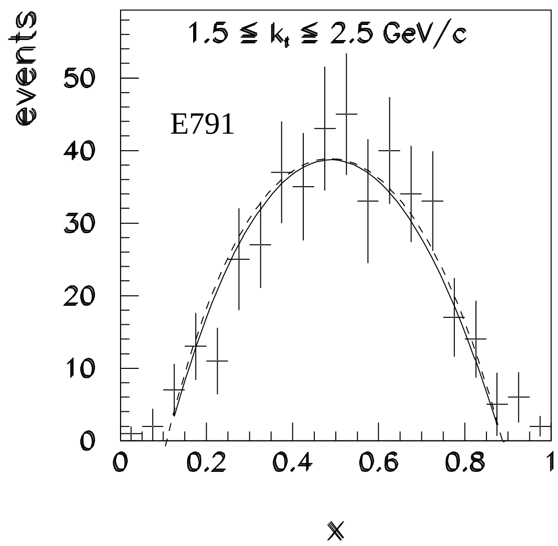


Color Explosion Imaging of the Pion DA



- In the exclusive dijet production, quark fraction x corresponds to the ratio of the jet momenta:

$$x_{\text{measured}} \simeq \frac{p_{\text{jet1}}}{p_{\text{jet1}} + p_{\text{jet2}}}$$

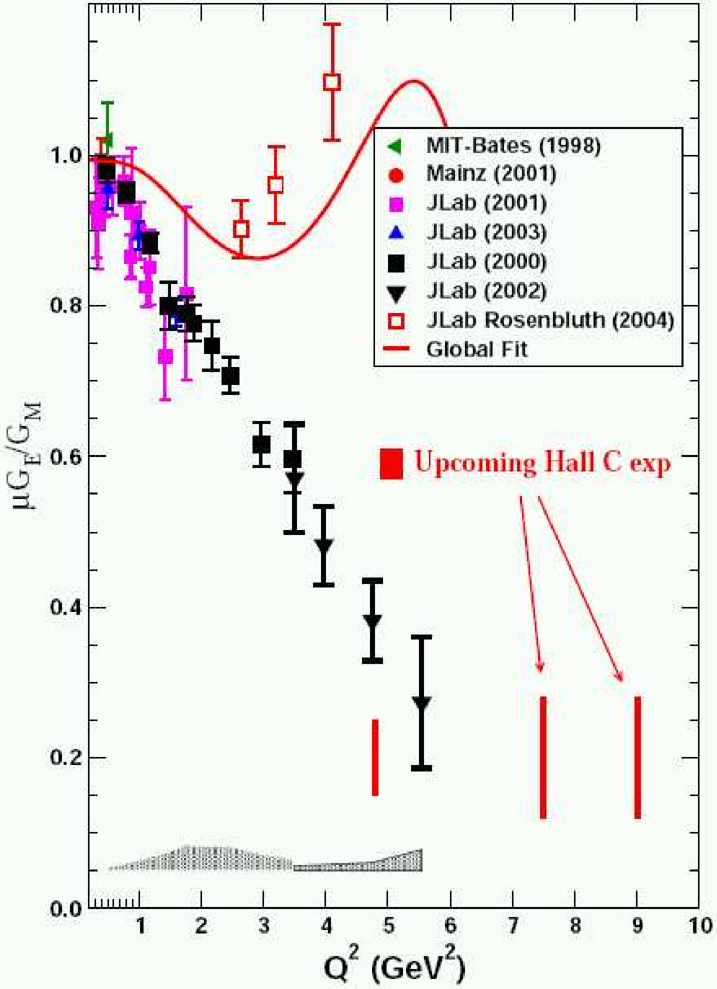


- Baryons: Forward detector at the LHC?
- Problem: Collinear factorization broken
- Interpretation not straightforward

Present and . . .

Future G_E/G_M at JLab in Hall C

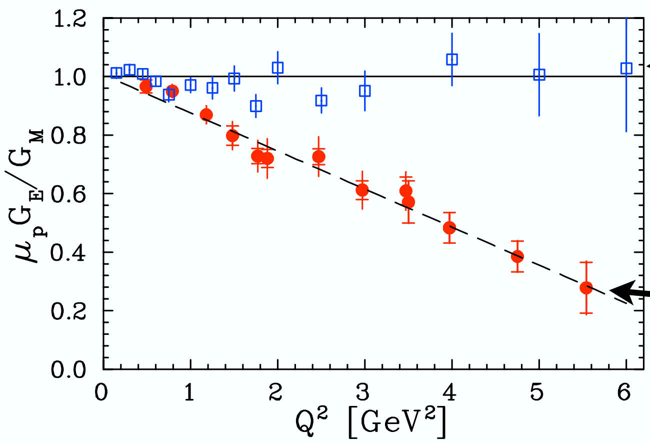
- FPP has been built at Dubna and will be installed in the HMS
- A large calorimeter has been assembled with help of IHEP and Yerevan.
- Scheduled to run in 2007.
- With the 12 GeV upgrade at JLab can reach Q^2 = 14



M.Jones, 12.10.05

Nucleon05 – p.15/26

Proton G_E/G_M Ratio



Rosenbluth (Longitudinal-Transverse) Separation

Polarization Transfer

LT

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

$$\tau = Q^2/4M^2$$

$$\varepsilon = \left[1 + 2(1+\tau)\tan^2\theta/2\right]^{-1}$$

G_E/G_M from slope in ε plot

PT

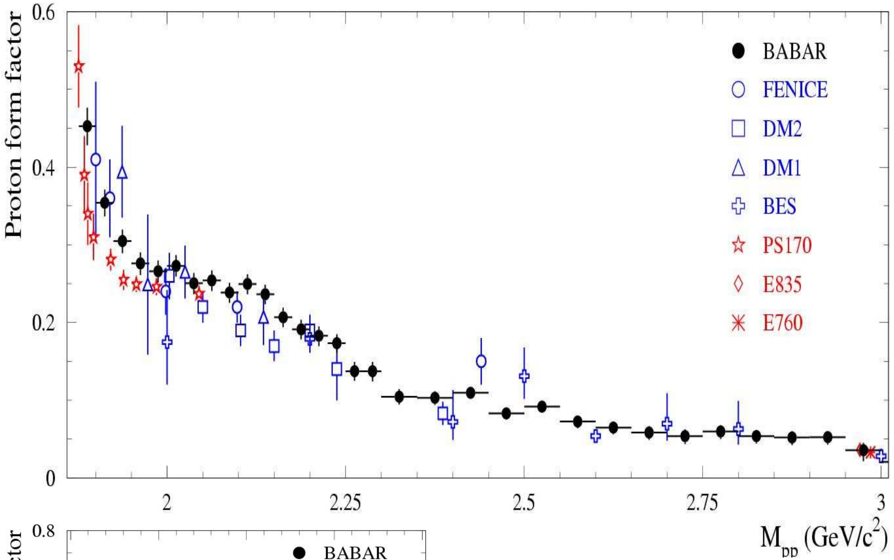
$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\frac{P_T}{P_L}$$

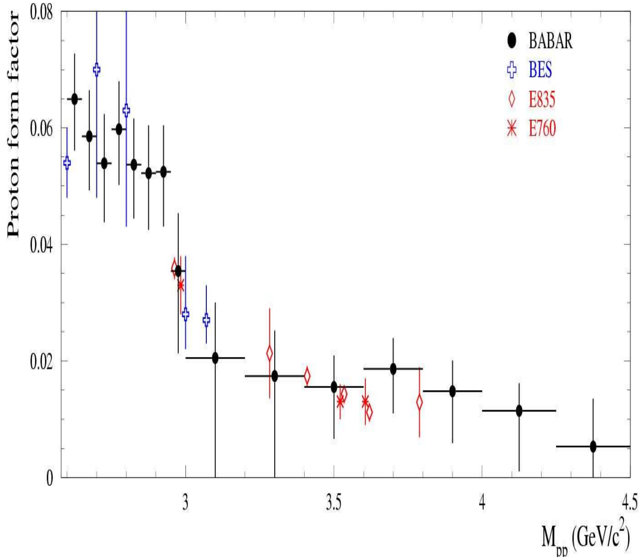
$P_{T,L}$ polarization of recoil proton

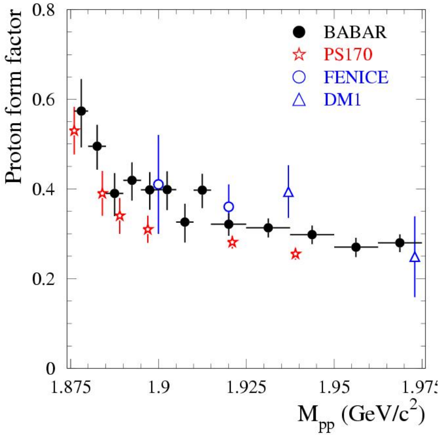
W.Melnitchouk, 12.10.05

Effective proton form factor

V.Druzhinin, 13.10.05







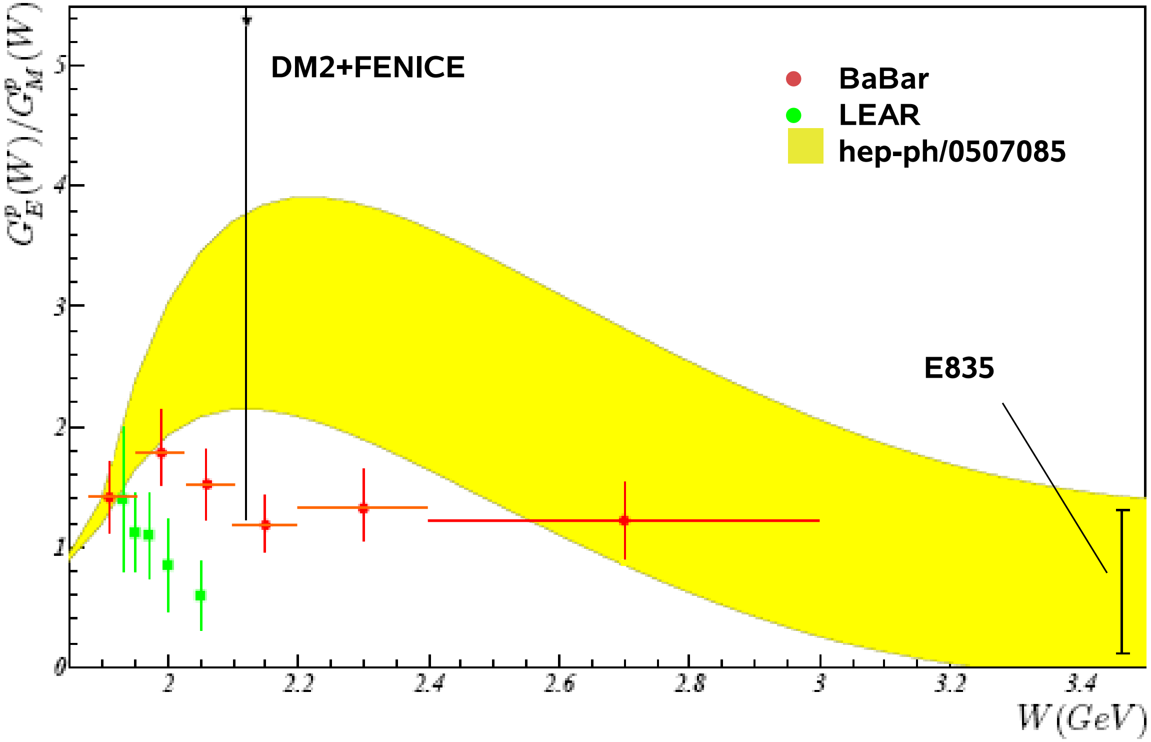
- *Steep behaviour at threshold confirmed*
- *Similar behaviour at threshold is seen in other processes with different quantum numbers*

$$BES \quad J/\psi \to \gamma\ p\bar{p}$$
$$BABAR \quad B^{+} \to K^{+} p\bar{p},\ B^{0} \to D^{0} p\bar{p}$$

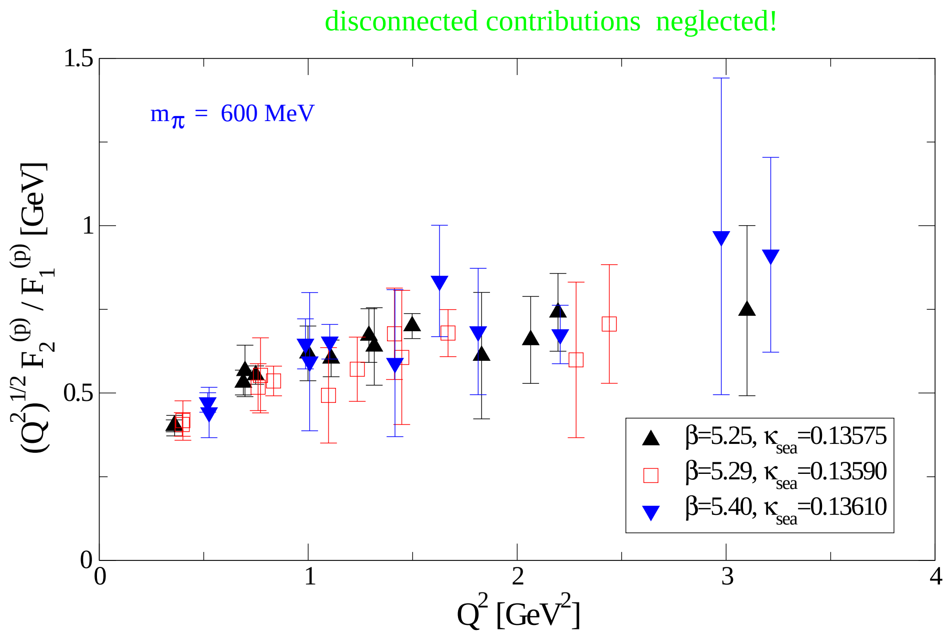
$|GE/GM|$ ratio

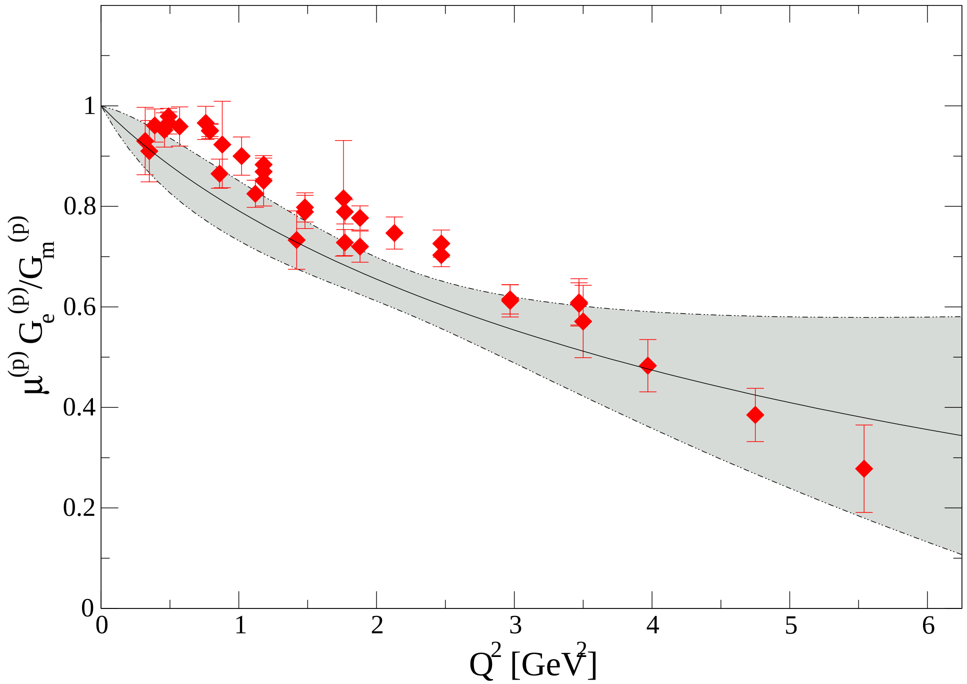
V.Druzhinin, 13.10.05

BaBar $|G_E/G_M|$ measurements vs previous ones and dispersion relation prediction (yellow) based on JLab space-like G_E/G_M and analyticity



- Nonperturbatively $O(a)$ improved Wilson (clover) fermions
- $N_f = 2$ dynamical configurations
- $N_f = 2$ lattice spacing $a = 0.07 - 0.11$ fm; Length of the spacial box $L = 1.4 - 2$ fm
- $N_f = 2$ dynamical configurations



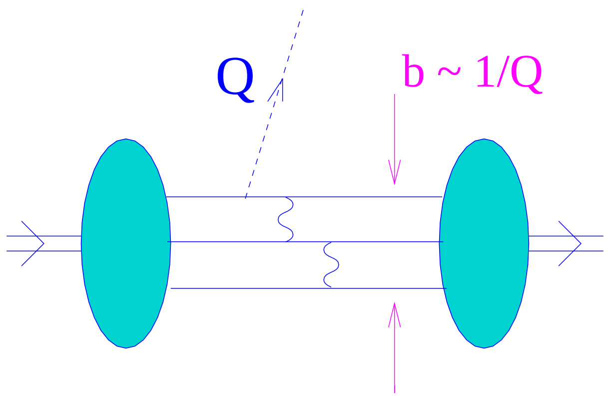


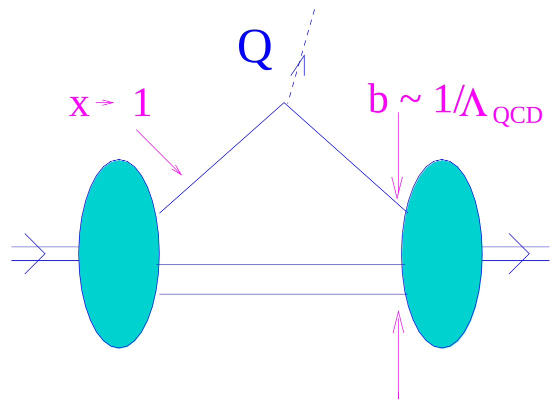
- chiral extrapolation compared with JLab data

M.Goeckeler, 14.10.05

Also N to Δ transition form factors, C. Alexandrou *et al.* PRL94(2005)021601

Paradigm: Soft vs. Hard





Hard rescattering:

Small b
Average $0 < x < 1$

Soft (Feynman):

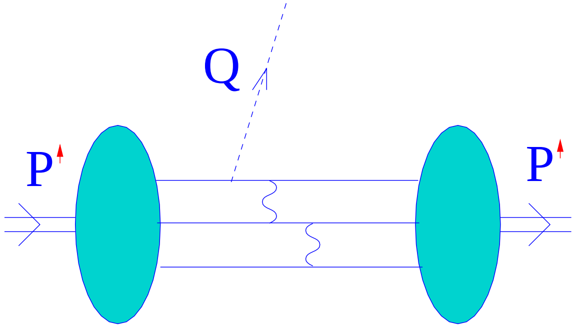
Average b
Large $x \to 1$

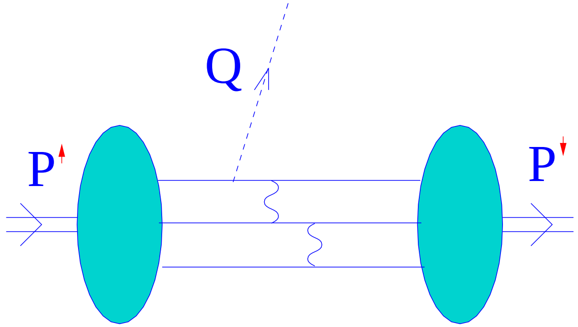
- Dominance of hard rescattering is only true for simplest reactions
- Soft constributions enter at the same power in $1/Q^2$ as higher-twist hard contributions
- Separation of soft and hard contributions is nontrivial and not unique

- Estimates of soft terms require a nonperturbative approach that would be explicitly consistent with perturbative QCD factorization

PQCD: Collinear factorization

Classical Brosky-Lepage framework: two hard gluon exchanges





$$F_1^{p,n} \sim \frac{\alpha_s^2(Q)}{Q^4}$$

$$F_2^{p,n} \sim \frac{\alpha_s^2(Q)}{Q^6} \ln^2 Q^2/\mu_{IR}^2 \equiv \frac{1}{Q^6}$$

- Pauli form factor corresponds to the twist-3 — twist-4 inteference;

  Explicit calculation (Belitsky, Ji, Yuan '03) confirms that collinear factorization is broken

- The experimental behaviour $F_2(Q^2)/F_1(Q^2) \sim 1/Q$ is not supported by QCD
- Reason for the large difference between space-like and time-like form factors unclear

? Applicability at realistic values of Q^2 Wide-spread sceptizism

? Possible and necessary to calculate the radiative correction (new color structures)

PQCD: Modified (k_t) factorization

Sterman, Li, '92: make use of the Sudakov suppression of large transverse separations

- **Introduce transverse-momentum dependent pion wave functions**

$$F_\pi(Q^2) = \int dx_1 dx_2 \int d^2b \phi_\pi(x_1, b)\phi_\pi(x_2, b) e^{-S(x_i,b)} T_H(x_i, Q, b)$$

- **Retain the transverse-momentum dependence of the hard kernel**

$$T_H^{LO}(x_i, Q, b) = \frac{4g^2 C_F}{x_1 x_2 Q^2 + (k_{1\perp} + k_{2\perp})^2}$$

- **Sudakov suppression factors:** $b \sim 1/Q$

$$e^{-S(x_i,b)} = \exp\left\{-\frac{C_F}{8\pi}\sum_{i=1}^{2} \ln^2 \frac{b^2 x_i^2 Q^2}{b_0^2} + (x_i \leftrightarrow 1 - x_i)\right\}$$

? **Exponentiated π^2-terms: difference between space-like and time-like form factors**

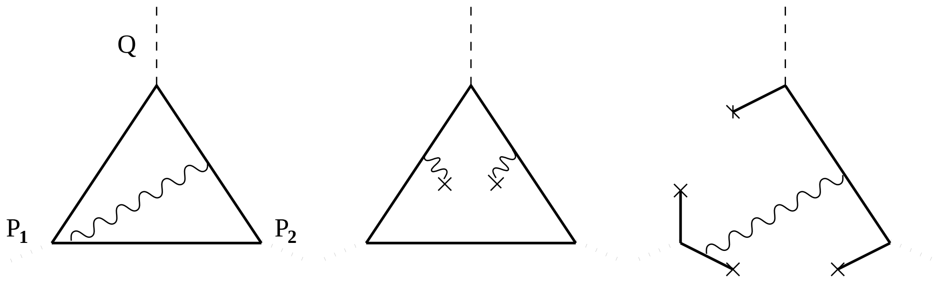
◊ **Sudakov suppression too weak: "Intrinsic" transverse-separation dependence of the wave functions cannot be ignored**

◊ **Considerable model dependence (e.g. Bolz *et al.* '95)**

QCD Sum Rules

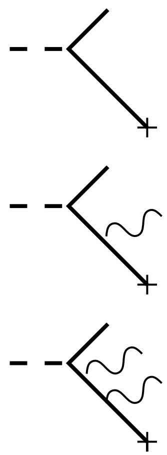
QCD Sum Rules

Nesterenko, Radyushkin '82; Ioffe, Smilga '82



$$= \left(\frac{1}{Q^4} + \frac{\alpha_s(Q)}{Q^2}\right) + \mathrm{const}\cdot\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + Q^2 \cdot \langle \bar{q} q \rangle^2 + \dots$$

- **Power counting in Q^2 is not consistent with OPE ?**



$\longrightarrow \delta(1-x)$

$\longrightarrow \delta'(1-x)$

$\longrightarrow \delta''(1-x)$

Expansion goes in derivatives of the delta-function!

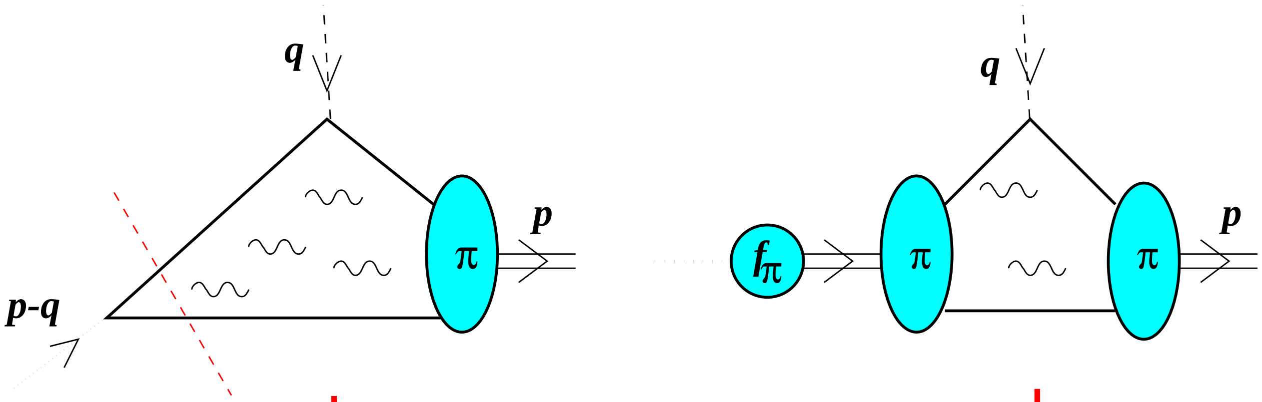
Sum of all orders: $\longrightarrow$ ***const (1-x)***

Light-Cone Sum Rules

Example: Pion Form Factor

$$T_{\mu\nu}(p,q) = i\int dx\, e^{-iqx}\langle 0|T\{j_\nu^{\rm A}(0) j_\mu^{\rm em}(x)\}|\pi(p)\rangle$$

duality:



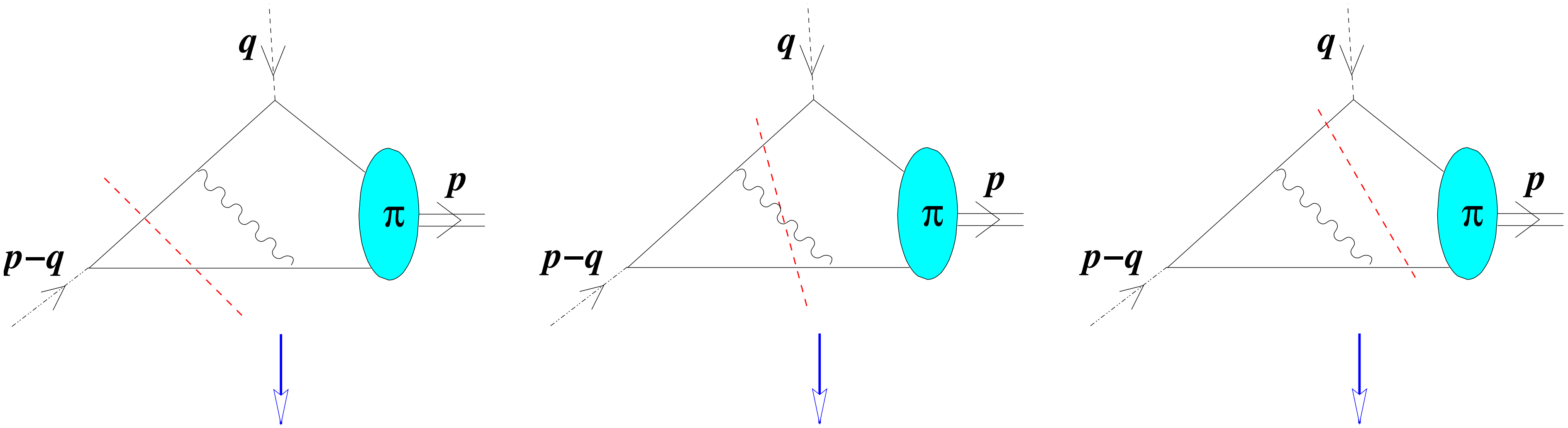
$$\int_0^{s_0} ds\, \frac{\mathrm{Disc}_{(p-q)^2} T(p,q)}{s-(p-q)^2} \overset{p^2\sim -1\mathrm{GeV}^2}{=} f_\pi \cdot \frac{1}{m_\pi^2 - (p-q)^2}\cdot F_\pi(Q^2)$$

- $T(p,q)$ is calculated in terms of distribution amplitudes of increasing twist
- Dispersion relation in one variable

Braun, Halperin, '94

Braun, Khodjamirian, Maul, '99

different dispersion parts:



hard rescattering qGq component in the WF   initial state interaction

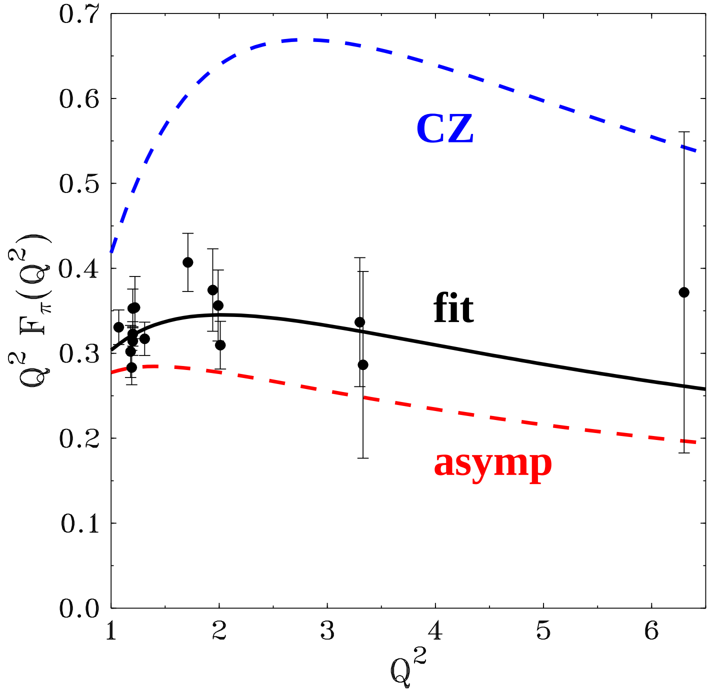
♡ **LCSR are fully consistent with pQCD**

- **a complicated interplay of soft and hard contributions;**

  explicit calculation suggests significant cancellations
  between soft and hard higher-twist corrections

General features of LCSRs

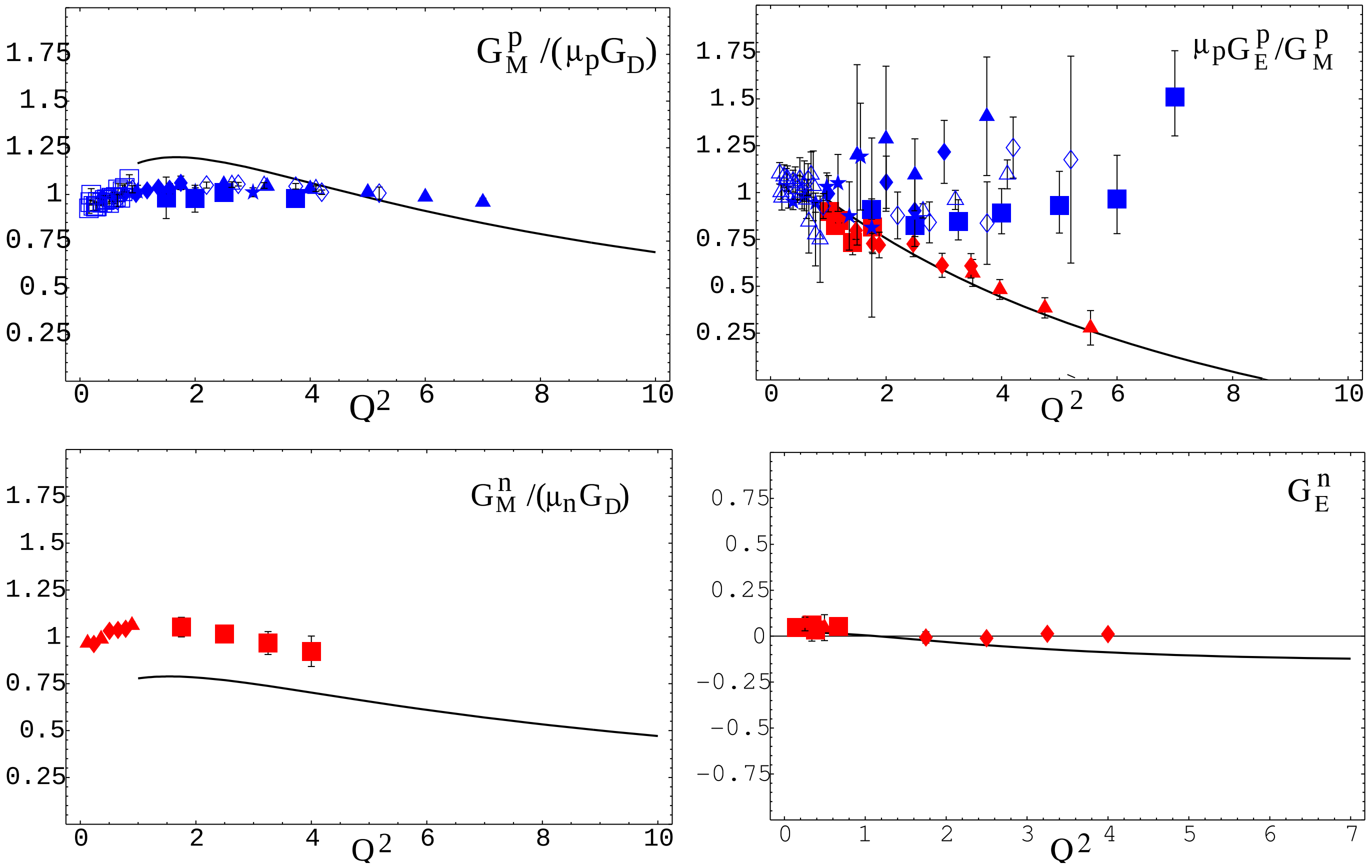
- Hadron distribution amplitudes provide the principal input
- both ’soft’ and ’hard’ contributions are included
- ’hard’ constribution is the usual pQCD, it appears as a part of the radiative correction
- ’soft’ contribution is modelled as a sum of contributions of DAs of increasing twist
- higher twist DAs contribute significantly to the ’soft’ part; the twist expansion goes here in powers of the Borel parameter and not powers of Q^2



LCSRs provide one with the most direct relation between form factors and hadron distribution amplitudes that is available at present, with no other parameters

Nucleon electromagnetic form factors

$$\langle N(P')|j_\mu^{em}(0)|N(P)\rangle = \bar{N}(P')\left[\gamma_\mu F_1(Q^2) - i\frac{\sigma_{\mu\nu}q^\nu}{2m_N}F_2(Q^2)\right]N(P)$$



proton

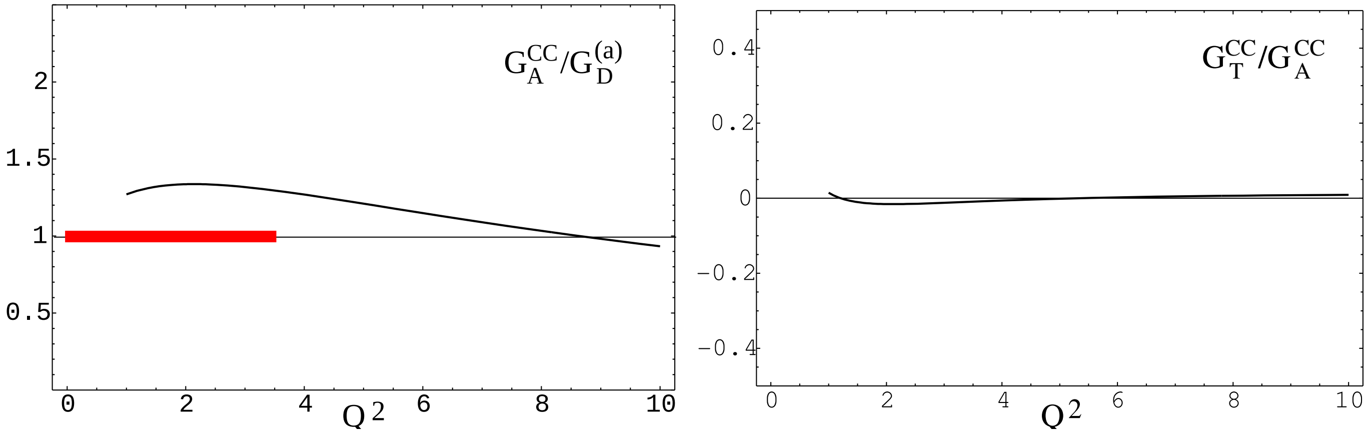
neutron

- **Leading order LCSR, BLW distribution amplitudes**

Braun, Lenz, Wittmann; hep-ph/0604050

Nucleon axial vector form factors

$$\langle N(P')|A_\mu(0)|N(P)\rangle = \bar{N}(P')\left[\gamma_\mu G_A(Q^2) - \frac{q_\mu}{2m_N}G_P(Q^2) - i\frac{\sigma_{\mu\nu}q^\nu}{2m_N}G_T(Q^2)\right]\gamma_5 N(P)$$



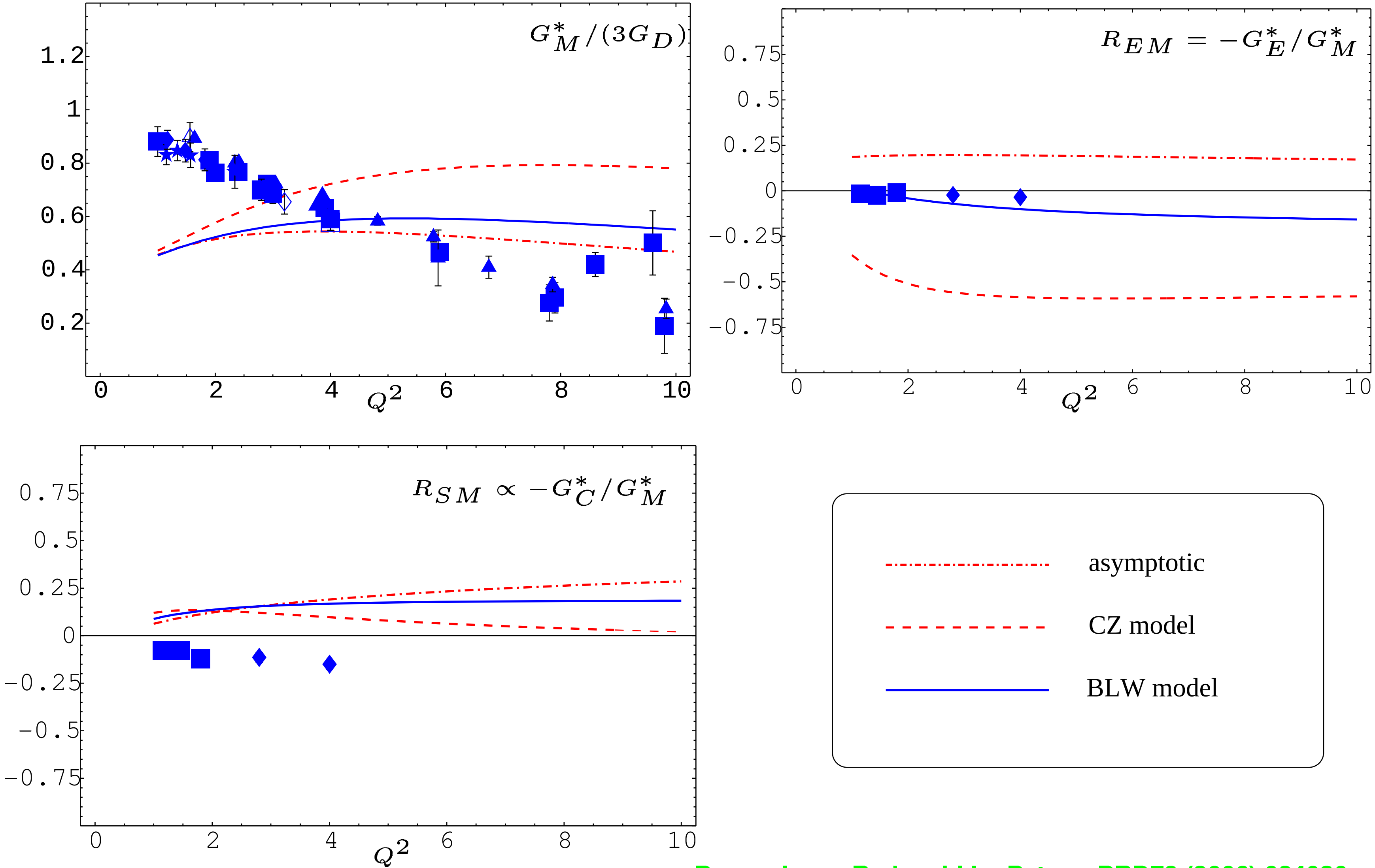
charged current

- **Leading order LCSR, BLW distribution amplitudes**

Braun, Lenz, Wittmann; hep-ph/0604050

$N\Delta\gamma$ transition form factors

- **magnetic, electric and quadrupole form factors exist:**



Braun, Lenz, Radyushkin, Peters; PRD73 (2006) 034020

Electroproduction of soft pions

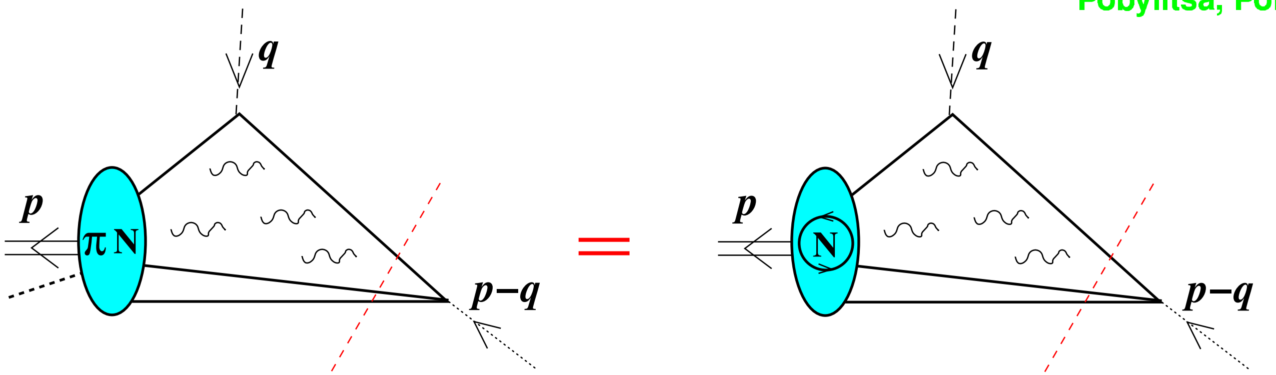
- $Q^2 \to \infty$ **does not commute with the chiral limit** $m_\pi \to 0$**:** $Q^2 \ll \Lambda^3/m_\pi$ **vs.** $Q^2 \gg \Lambda^3/m_\pi$

at the threshold

$$\langle N\pi(P-q)|j_\mu^{\rm em}|N(P)\rangle = -\frac{i}{f_\pi}\bar{N}(P-q)\gamma_5\left\{(\gamma_\mu q^2 - q_\mu \not{q})G_A^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}G_T^{\pi N}(Q^2)\right] N(P)$$

chiral rotation:

Pobylitsa, Polyakov, Strikman '01



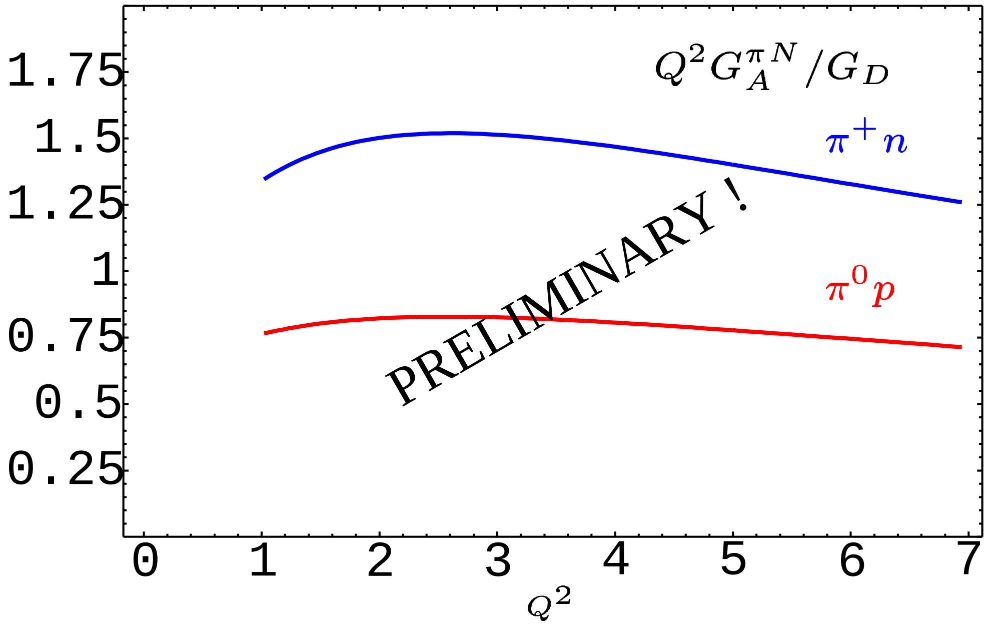
$$|p\uparrow\rangle = \frac{\phi_s(x)}{\sqrt{6}}|2u_\uparrow d_\downarrow u_\uparrow - u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle + \frac{\phi_s(x)}{\sqrt{2}}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

$$|p\uparrow\pi^0\rangle = \frac{\phi_s(x)}{2\sqrt{6}f_\pi}|6u_\uparrow d_\downarrow u_\uparrow + u_\uparrow u_\downarrow d_\uparrow + d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_s(x)}{2\sqrt{2}f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

$$|n\uparrow\pi^+\rangle = \frac{\phi_s(x)}{\sqrt{12}f_\pi}|2u_\uparrow d_\downarrow u_\uparrow - 3u_\uparrow u_\downarrow d_\uparrow - 3d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_s(x)}{2f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

Braun, Ivanov, Lenz, Peters *work in progress*

- **Tree-level light-cone sum rules, preliminary:**



- **solid curves:** *BLW distribution amplitudes*

Braun, Ivanov, Lenz, Peters; work in progress

Outlook

- many new experimental data; more to come
- Numerical experiment increasing in importance

- Understanding form factors:
  - ? Interplay of soft and hard mechanisms
  - ? Time-like vs. Space-like

- To be done:
  - Two-loop PQCD
  - radiative corrections to LCSR
  - soft-pion emission in hard processes

- Mid-term goal: information on nucleon distribution amplitide, in particular average momentum fraction carried by the three valence quarks to 5-7% precision

	$u^{\uparrow}$	$u^{\downarrow}$	$d^{\uparrow}$
asymptotic	33%	33%	33%
CZ model	58%	19%	23%
BLW model	41%	28%	30%