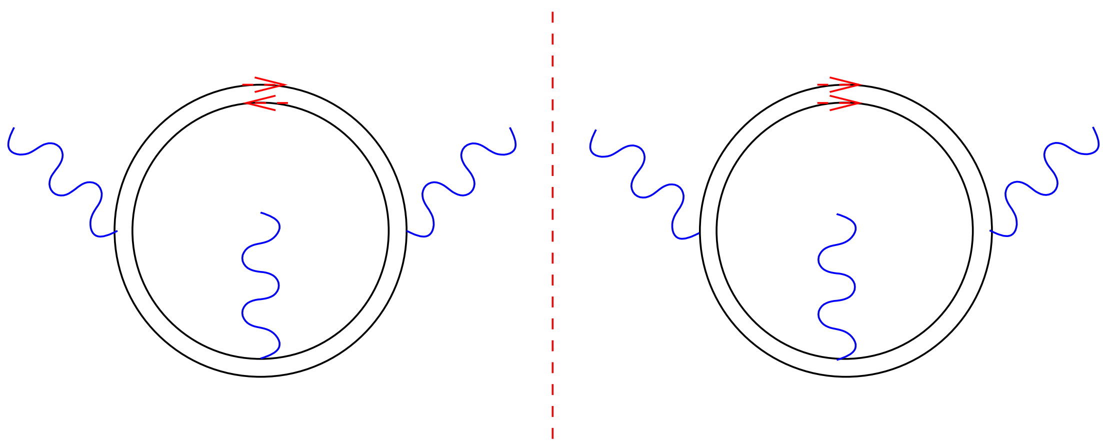
Planar Equivalence



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Continuous Advances in QCD 2006

FTPI, May 11-14, Minneapolis

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Background (Orbifold field theories)

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Related Works

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Introduction

It is extremely difficult to make reliable calculations in QCD at the strong coupling regime, since this regime is controlled by non-perturbative physics.

In supersymmetric theories the situation is better due to holomorphicity.

The idea behind planar equivalence is to approximate QCD by a supersymmetric theory !

Following Kachru, Silverstein and Strassler we (A.Armoni, M. Shifman, G. Veneziano) proposed an equivalence between planar $SU(N)$ $\mathcal{N}=1$ Super Yang-Mills and a non-supersymmetric "orientifold field theory".

By using this equivalence we were able to derive analytic non-perturbative results in QCD !

Planar-Equivalence

At $N \to \infty$ a bosonic sector of

$$SU(N)\ \mathcal{N}=1\ \textit{Super Yang-Mills}$$

$$=$$

$$SU(N)\ \textit{Yang-Mills} + \begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}\ \textit{Dirac fermion}$$

Moreover,

Since for $SU(3)$ $\overline{\square} \sim \begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}$ we can use planar equivalence to calculate (with a $1/N$ error) non-perturbative quantities in one-flavor QCD by copying their values from Super Yang-Mills.

Plan of the talk

- Introduction
- Proof of planar equivalence
- Results
- The Orientifold large-N expansion
- Applications to one-flavor QCD
- Applications to three-flavors QCD
- String theory: Sagnotti's model
- Remarks on gauge/gravity correspondence
- Related works
- Conclusions

Perturbative proof

At the perturbative level the planar "orientifold field theory" is equivalent to planar $\mathcal{N} = 1$ SYM.

The planar equivalence follows from the Feynman rules of the two theories

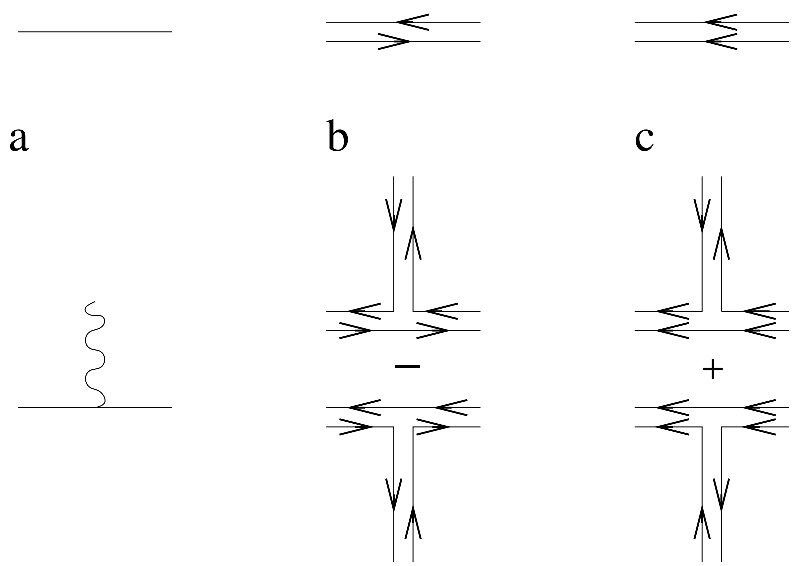


Figure 1: a. The fermion propagator and the fermion-gluon vertex. b. For $\mathcal{N} = 1$ SYM. c. For the orientifold theory.

An example: a planar contribution to the vacuum polarisation.

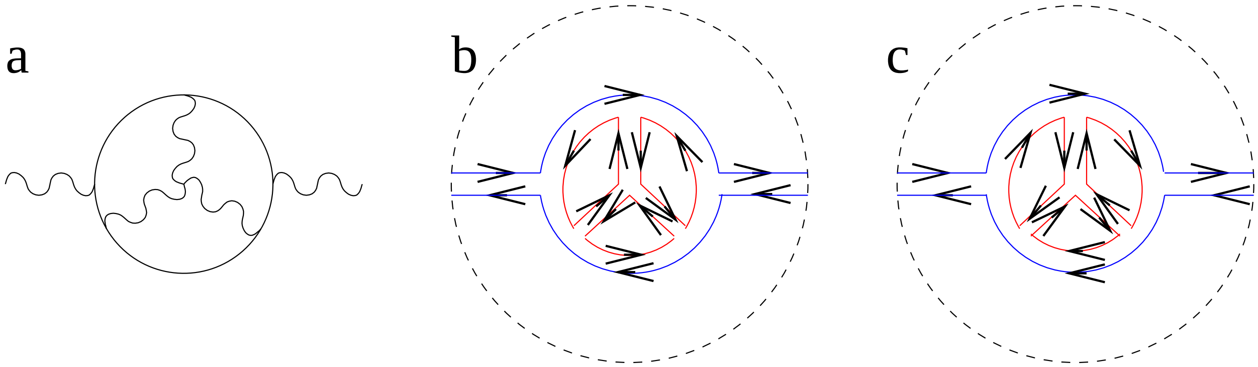


Figure b. is for $\mathcal{N} = 1$ SYM. Figure c. is for the orientifold theory.

In order to pass from b. to c., one has to reverse the orientation of the red lines.

For planar graphs (drawn on the sphere) the red lines and the blue lines do not intersect. It is as if we had two types of gluons in the theory: a red gluon and a blue gluon. The two types of gluons do not communicate to each other. This is why we can safely reverse theorientation of the arrows on the red lines, without changing the value of the Feynman graph.

Non-Perturbative proof

Since the orientifold theory and $\mathcal{N}=1$ SYM theory are similar to each other, but differ in their fermion field representation, we wish to compare their partition function after an integration over the fermion field.

We claim that after integration the two partition functions coincide at large N.

The partition function of $\mathcal{N}=1$ SYM is

$$\mathcal{Z}_0 = \int \mathcal{D}A \, \exp\left(iS[A]\right) \det\left(\not\partial + \not A^a T^a_{\mathrm{adj}}\right)$$

Next, we will make use of the fact that

$$T^a_{\mathrm{adj}} \sim T^a_{\Box\times\overline{\Box}} = T^a_{\Box} \otimes 1 + 1 \otimes T^a_{\overline{\Box}},$$

to write the partition function as follows

$$\mathcal{Z}_0 = \int \mathcal{D}A \, \mathcal{D}B \, \delta(A-B) \, \exp -\tfrac{1}{2}\left(S[A] + S[B]\right)$$
$$\det\left(\not\partial + \not A^a (T^a_{\Box} \otimes 1) + \not B^a (1 \otimes T^a_{\overline{\Box}})\right)$$

The determinant is gauge invariant and hence it can be written formally by using Wilson loops (the detailed expansion can be found in ASV, hep-th/0412203, see also A. Patella, hep-lat/0511037.)

$\mathcal{Z}_0 = \int \mathcal{D}A\, \mathcal{D}B\, \delta(A - B) \exp -\frac{1}{2}(S[A] + S[B])$
$\sum \prod \mathcal{W}(A)\mathcal{W}(B)$.

Or $\mathcal{Z}_0 = \sum \langle \prod \mathcal{W}\mathcal{W} \rangle$

The partition function of the "orientifold field theory" is similar, but with reversed orientation for the red Wilson loop $\mathcal{W}^\star$.

At large-N gauge invariant amplitudes factorise. For example

$\langle \mathcal{W}(A)\mathcal{W}(B) \rangle = \langle \mathcal{W}(A)\mathcal{W}^\star(B) \rangle = \langle \mathcal{W} \rangle \langle \mathcal{W} \rangle$

(since reversing the orientation is as replacing a quark by an anti-quark and it does not change the expectation value of the Wilson loop).

More generally, we proved that at large-N
$\langle \prod \mathcal{W}_{\text{adj.}} \rangle_{\text{conn.}} = \langle \prod \mathcal{W}_{\text{anti-symm.}} \rangle_{\text{conn.}}$

So, the partition functions of the two theories coincide.

Main results (large-N only)

Due to the relation with $\mathcal{N}=1$ SYM we arrive to the following results for the *planar* 'orientifold field theory':

1. A zero cosmological constant. It follows from the relation $-i\rho V = \log \mathcal{Z}$ and the equality of the two partition functions.

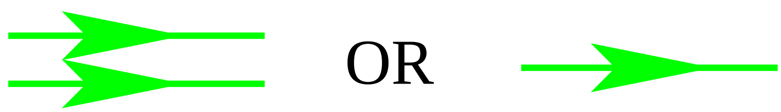
2. Matching of bosonic color singlets. The reason is that masses correspond to poles in amplitudes. One can add a source J to the gauge field. The partitions functions in the presence of the source $\mathcal{Z}_{J(x)}$ are also the same. Then, all bosonic amplitudes should be the same in the two theories. The spectrum of the "orientifold field theory", however, is purely bosonic.

3. We can couple a source $J(x)$ to $\bar{\Psi}_L \Psi_R(x)$ and integrate over the fermions to find a matching of the gluino condensate with the quark condensate in the 'orientifold field theory'.

The Orientifold Large-N Expansion

The results that were obtained so far for the large-N theory can be applied to QCD.

For $SU(3)$ with N_f quarks in the fundamental representation, we have the option of thinking about the quarks as if they transform in the antisymmetric representation



Let us choose the latter option. The idea is then to generalise to $SU(N)$ with N_f flavors in the antisymmetric representation. Finally, one can take the large-N limit, while keeping N_f as well as the 't Hooft coupling fixed.

This is a new kind a large-N expansion where quarks loops count like gluons loops.

Applications to one-flavor QCD

For the specific case of 1-flavor QCD, the large N theory becomes supersymmetric (in the bosonic sector).

One, thus, can copy results from $\mathcal{N} = 1$ SYM to 1-flavor QCD, with an expected error of $1/N = 1/3$.

We expect to have a degeneracy between the odd parity hadrons and the even parity hadrons.

In particular, for the η' and the σ (the lowest 'glueballs' in the tower) we expect

$$M_{\eta'}^2 / M_\sigma^2 = 1 + O(1/N)$$

(actually, in this specific case, $1/N$ corrections are expected to be large).

The quark condensate in 1-flavor QCD

An important application of our result is the calculation of the quark condensate in 1-flavor QCD. We can simply copy the value from SUSY Yang-Mills.

For the SUSY theory the value of the RGI gluino condensate is $\langle\lambda\lambda\rangle_{\rm RGI} = -\frac{N^2}{2\pi^2}\Lambda^3$

The $1/N$ corrections are expected to be large in the "orientifold" theory. An important hint is that for $SU(2)$ the condensate should vanish, since the antisymmetric representation is equivalent to the singlet. Therefore we expect $\langle\bar{\Psi}_L\Psi_R\rangle \sim -(1-\frac{2}{N})\frac{N^2}{2\pi^2}\Lambda^3$

Thus we find
$\langle\bar{\Psi}_L\Psi_R\rangle_{2\ \rm GeV} = -(0.6\ \text{ to }\ 1.1)\Lambda^3_{\bar{\rm MS}}$

In a nice agreement with lattice results
$\langle\bar{\Psi}_L\Psi_R\rangle_{2\ \rm GeV} = -(0.4\ \text{ to }\ 0.9)\Lambda^3_{\bar{\rm MS}}$

Applications to three-flavors QCD

Recently (A.A., G. Shore and G. Veneziano) we generalised our analysis to three-flavors QCD !

The idea is to consider an $SU(N)$ theory with one-flavor in the antisymmetric representation and two additional flavors in the fundamental representation.

This model reduces to three flavors QCD for $SU(3)$, becomes SUSY in the bosonic sector in the large N limit, but differs from the previous model by the $1/N$ corrections.

Our result for the QCD quark condensate is

$$\langle \bar{\Psi}\Psi \rangle_{2\,\mathrm{GeV}} = -(317 \pm 30 \pm 36\ \mathrm{MeV})^3$$

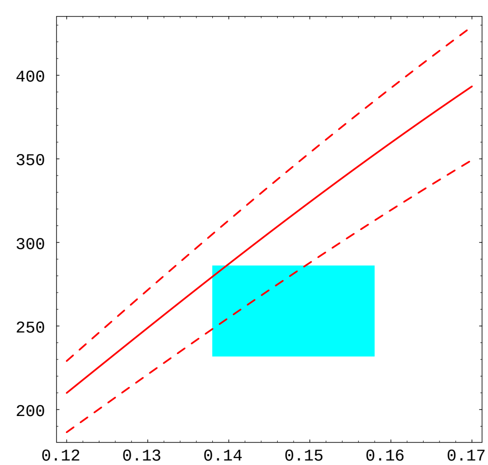


Figure 2: $\langle \bar{\Psi}\Psi \rangle$ as a function of λ

Sagnotti's model

The "orientifold field theory" has a nice embedding in string theory.

Consider first type 0B string theory. It is a bosonic closed string theory (with worldsheet supersymmetry) which contains the following low-energy modes

NS-NS: $\Phi, G_{\mu\nu}, B_{\mu\nu}, T$

R-R: $2 \times\ C, C_{\mu\nu}, C_{\mu\nu\rho\lambda}$

Sagnotti's model is obtained by adding a special orientifold $\Omega' = \Omega(-1)^{f_R}$ and 32 $D9$ branes.

The resulting theory on the D9-branes is a $U(32)$ gauge theory with an antisymmetric fermion.

The closed string spectrum of Sagnotti's model is simply the bosonic (NS-NS and R-R) truncation of the type II string (the closed string tachyon of the type 0 string is projected out).

Gauge/Gravity correspondence

String theory (the Gauge/Gravity correspondence) might be useful to estimate certain $1/N$ corrections. The 'orientifold field theory' has a realisation in type 0'B string theory on a stack of wrapped D5-branes.

In such setups the total RR flux becomes $N-2$ due to the presence of the orientifold 5-plane. Certain quantities, which are 'sensitive' to the background RR flux, such as the chiral anomaly, are shifted $N \to N-2$.

With E. Imeroni, we used it to make a couple of predictions about the finite-N theory. A concrete example is

$$M_{\eta'}/M_\sigma \sim C_-/C_+ \sim (N-2)/N$$

This result is consistent with the result of Sannino and Shifman who used an effective action approach (VY like) analysis to calculate the same glueball ratio.

Related works

Major related works

- Di Vecchia et.al.: realisation of 'orientifold field theory' on $R^4 \times C^3/Z_2 \times Z_2$ orbifold singularity in type 0'B string theory.
- Sannino and Collaborators: using ('orientifold') theories with symmetric fermions for technicolor models. (more about it in Sannino's talk in this conference).
- Patella: An analytic lattice proof of planar equivalence and lattice simulations.

Conclusions

Exact non-perturbative results can be copied from the supersymmetric theory to the non-supersymmetric "orientifold field theory" (at large N).

Planar equivalence is useful for QCD !

In addition, wide range of applications including string theory, technicolor, lattice, ...

Other aspects that I haven't discussed in detail are:

- Generalisations: $\mathcal{N}=1$ SYM with matter (Seiberg duality for a non-supersymmetric theory ?) $\mathcal{N}=2$ and $\mathcal{N}=4$ SYM.
- The AdS/CFT correspondence.
- Further applications for QCD.