Tilings, Dimers, and Quiver Gauge Theories

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Continuous Advances in QCD

Thanks to:

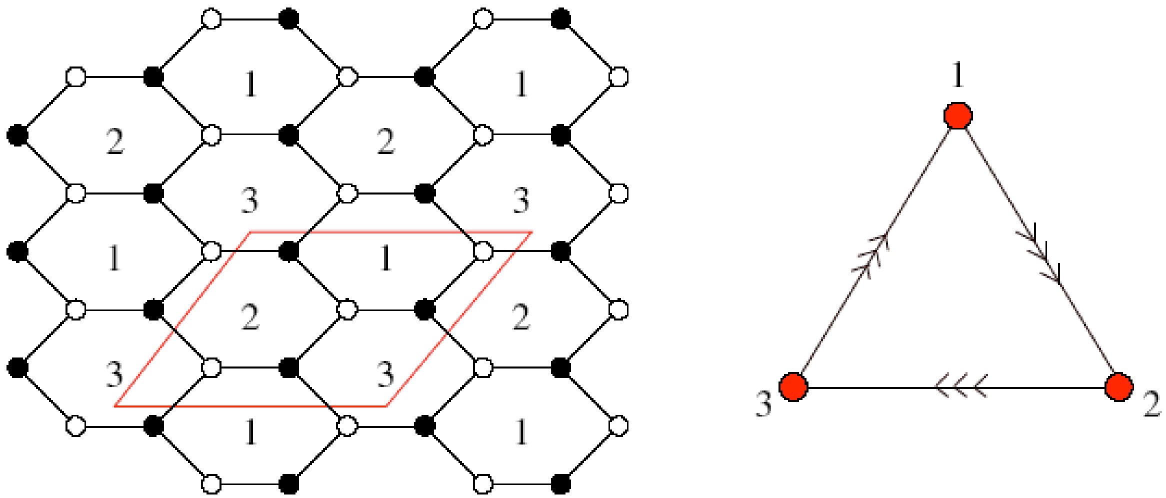
Benvenuti, Franco, Kazakopoulos, Kennaway, Martelli, Sparks, Uranga, Vegh, Wecht

Central Question

- What is the gauge theory living on a D3 brane that probes a non-compact singular CY manifold?
- The construction we will see today solves this long standing problem for the case of toric CY singularities
- Get the spectrum of BPS KK states on any toric SE manifold

Motivation for study

- New tools to get exact results on $\mathcal{N}$=1 supersymmetric gauge theories
- Generically theories one studies are chiral - as in real world..
- More examples of SCFTs in 4 dimensions
- Get information on string backgrounds using D brane probes - what is a D brane?

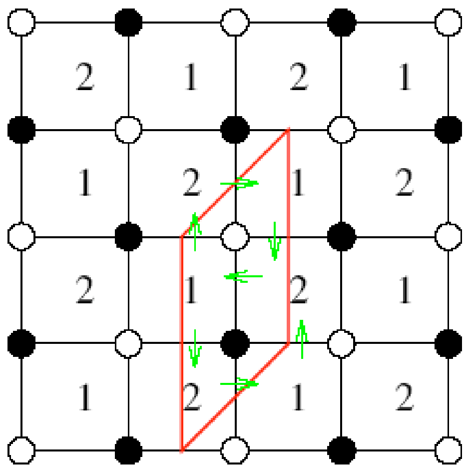


Periodic bipartite tiling

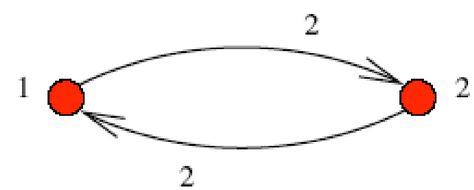
Tiling - Quiver dictionary

- 2n sided face - U(N) Gauge group with nN flavors
- Edge - A bi-fundamental chiral multiplet charged under the two gauge groups corresponding to the faces it separates.
- k valent node - A k-th order interaction term in the superpotential

$$CY_6 = conifold$$



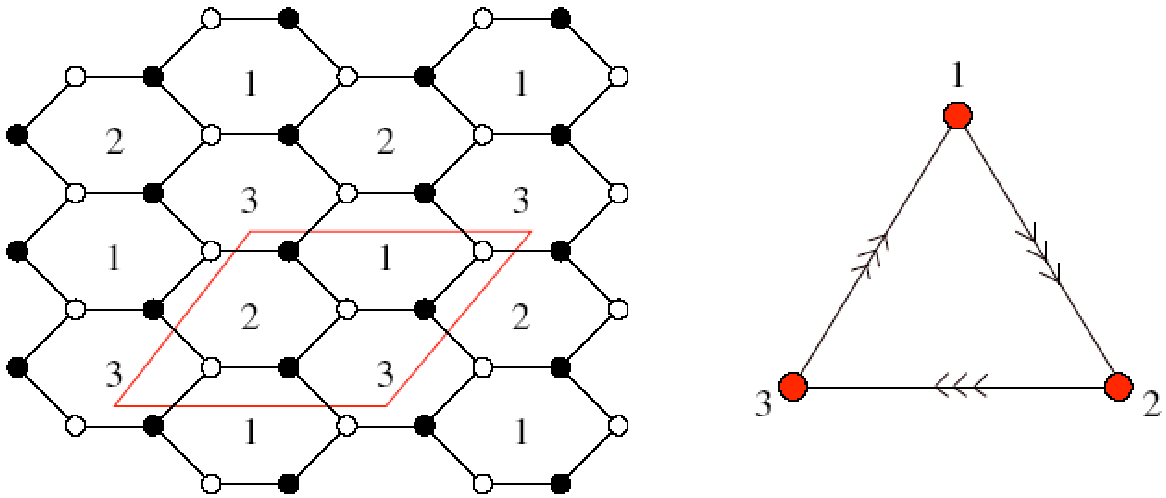
brane tiling



quiver

$$W = X_{12}^{(1)}X_{21}^{(1)}X_{12}^{(2)}X_{21}^{(2)} - X_{12}^{(1)}X_{21}^{(2)}X_{12}^{(2)}X_{21}^{(1)}$$

Example: Conifold



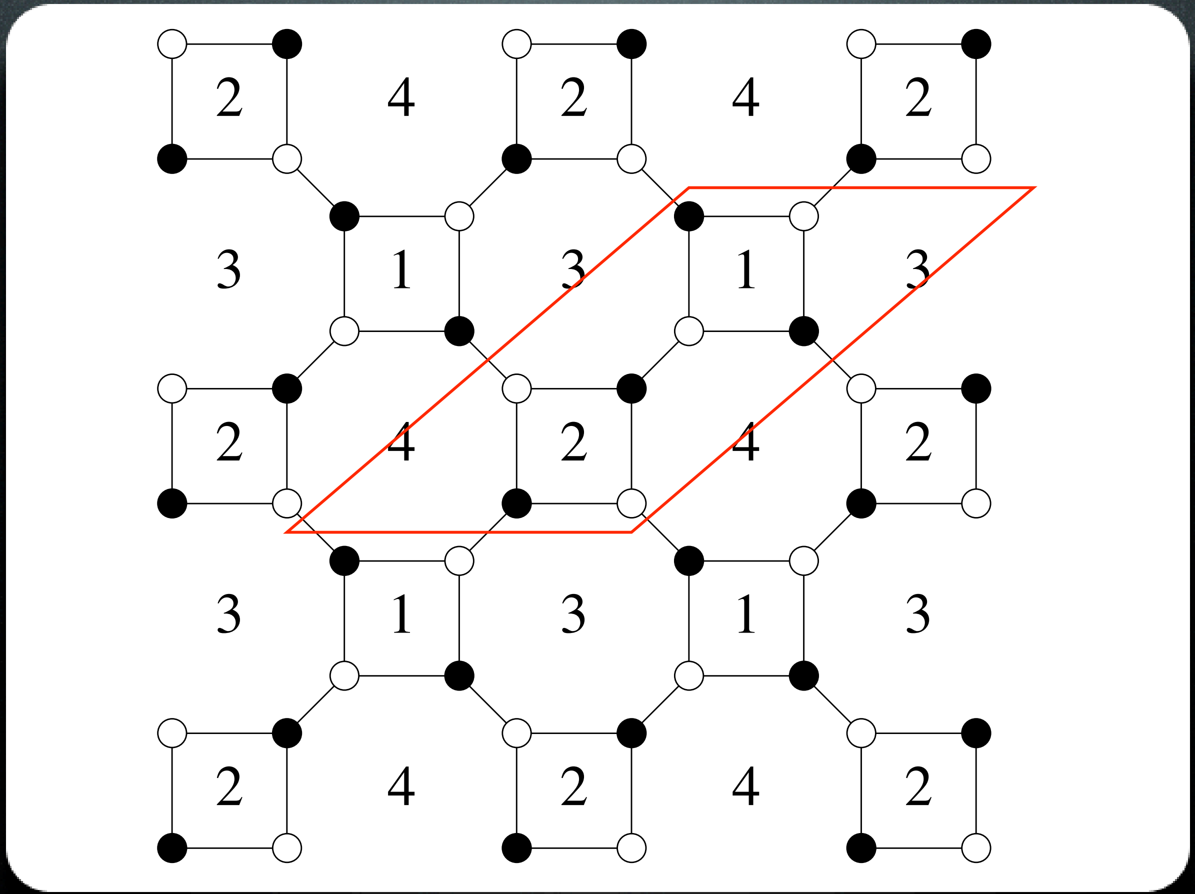
Z_3 orbifold of C^3

Comments

- Arrows are oriented in an alternating fashion
- Graph is bi-partite: Nodes alternate between clockwise (white) and counterclockwise (black) orientations of arrows
- black (white) nodes connected to white (black) only

Comments

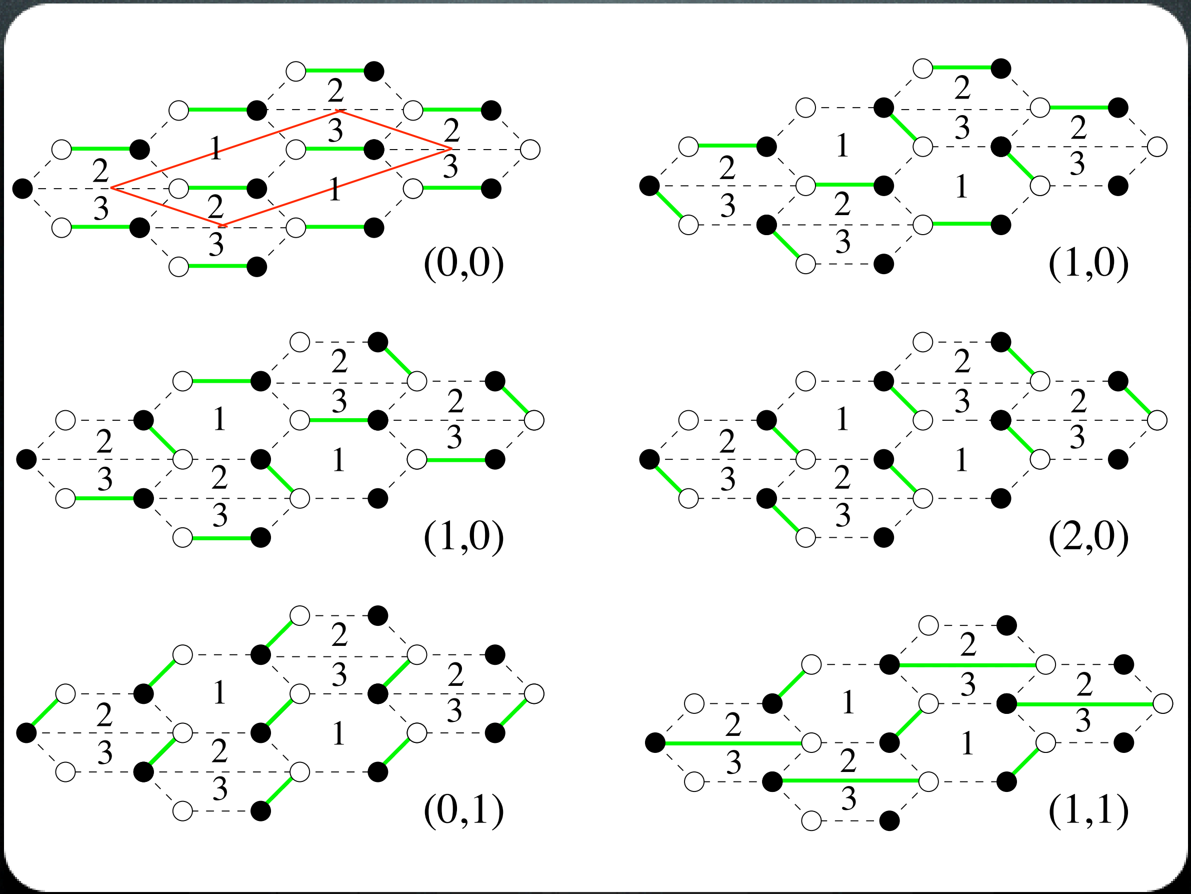
- odd sided faces are forbidden by anomaly cancellation condition
- white nodes with + sign in the superpotential
- black nodes with - sign in the superpotential
- These rules define a unique Lagrangian



Tiling for F_0 $(P^1 \times P^1)$

Dimers

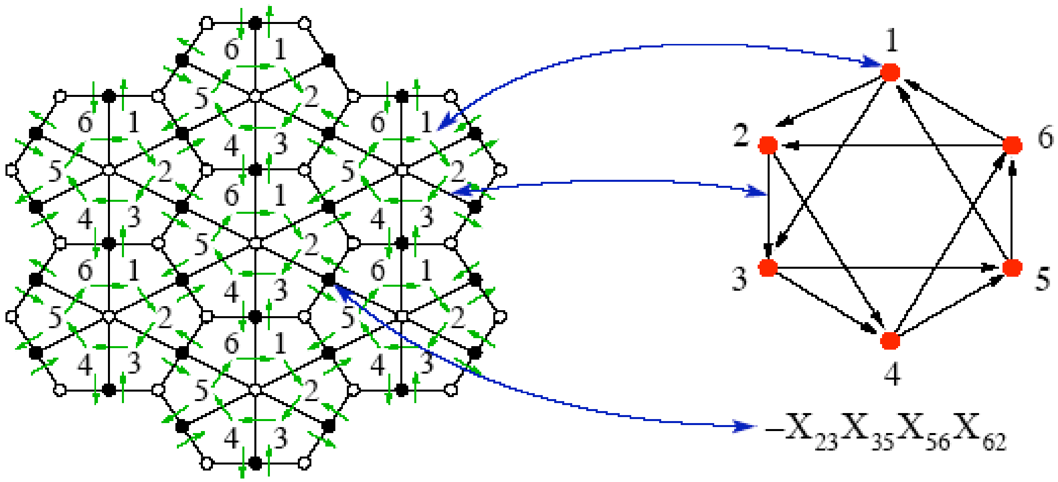
- and now for “Dimer” techniques
- Dimer - a line connecting 2 nodes
- Perfect matching - a collection of dimers such that every node is covered precisely once
- Adjacency matrix between white & black nodes - Kasteleyn matrix



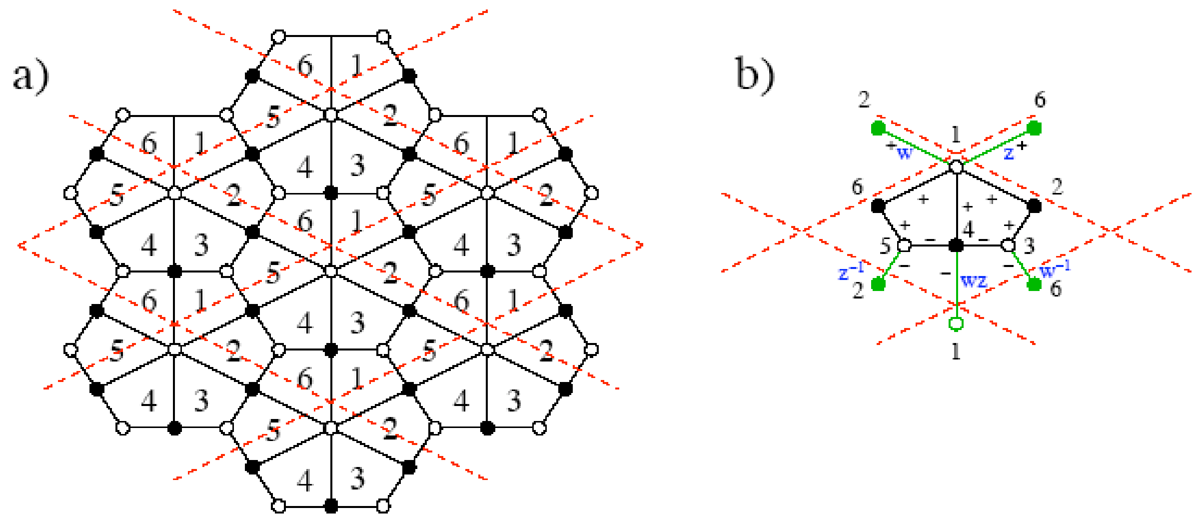
Perfect matchings SPP

Combinatorial Problem

- Given a Tiling, how many perfect matchings can one write down?
- Solved by writing the Adjacency (Kasteleyn) Matrix



Example: Del Pezzo 3, Model I



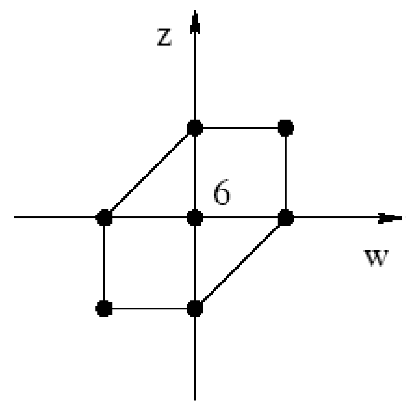
$$K = \begin{array}{c|ccc} & 2 & 4 & 6 \\ \hline 1 & 1+w & 1-zw & 1+z \\ 3 & 1 & -1 & -w^{-1} \\ 5 & -z^{-1} & -1 & 1 \end{array}$$

Kasteleyn matrix

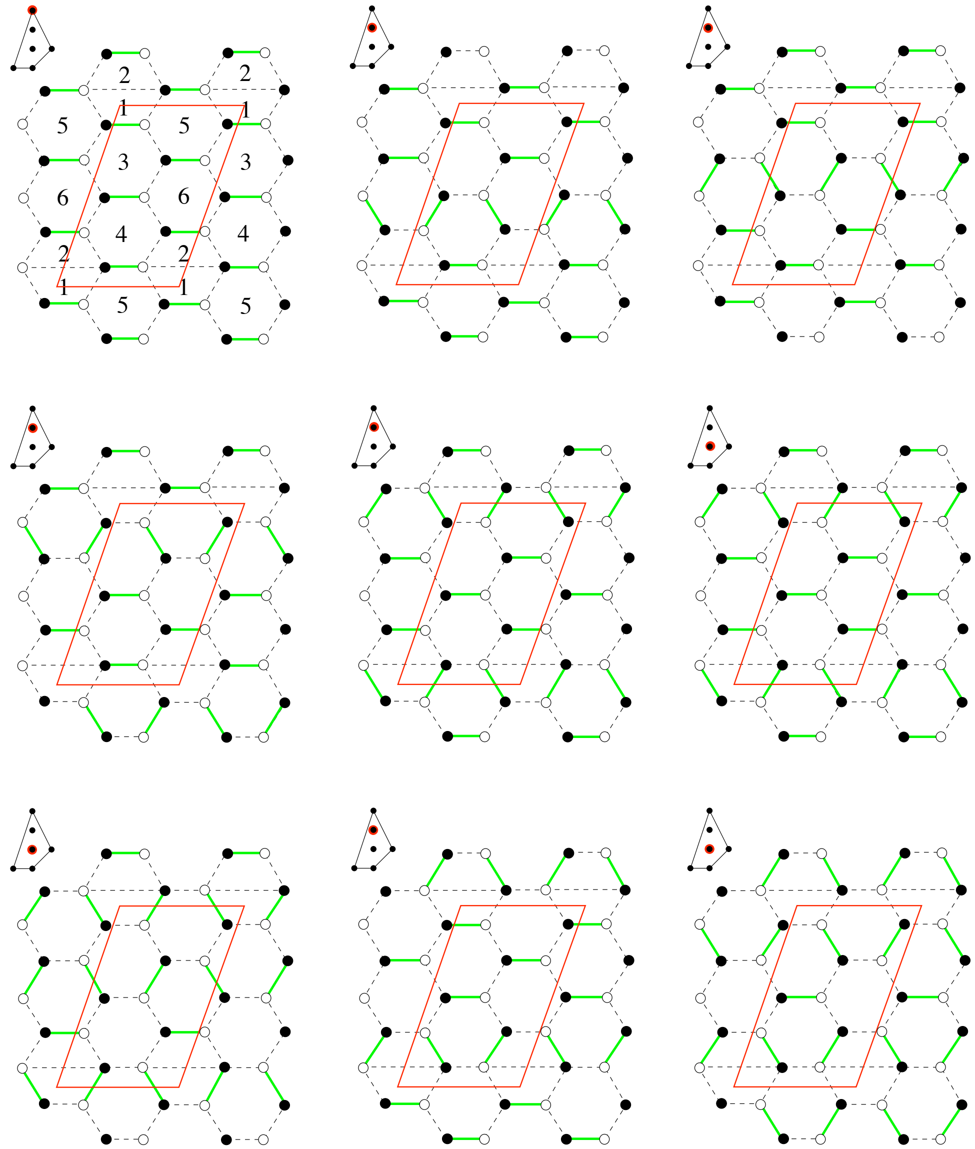
R. Kenyon

$$P(z,w) = \det K$$

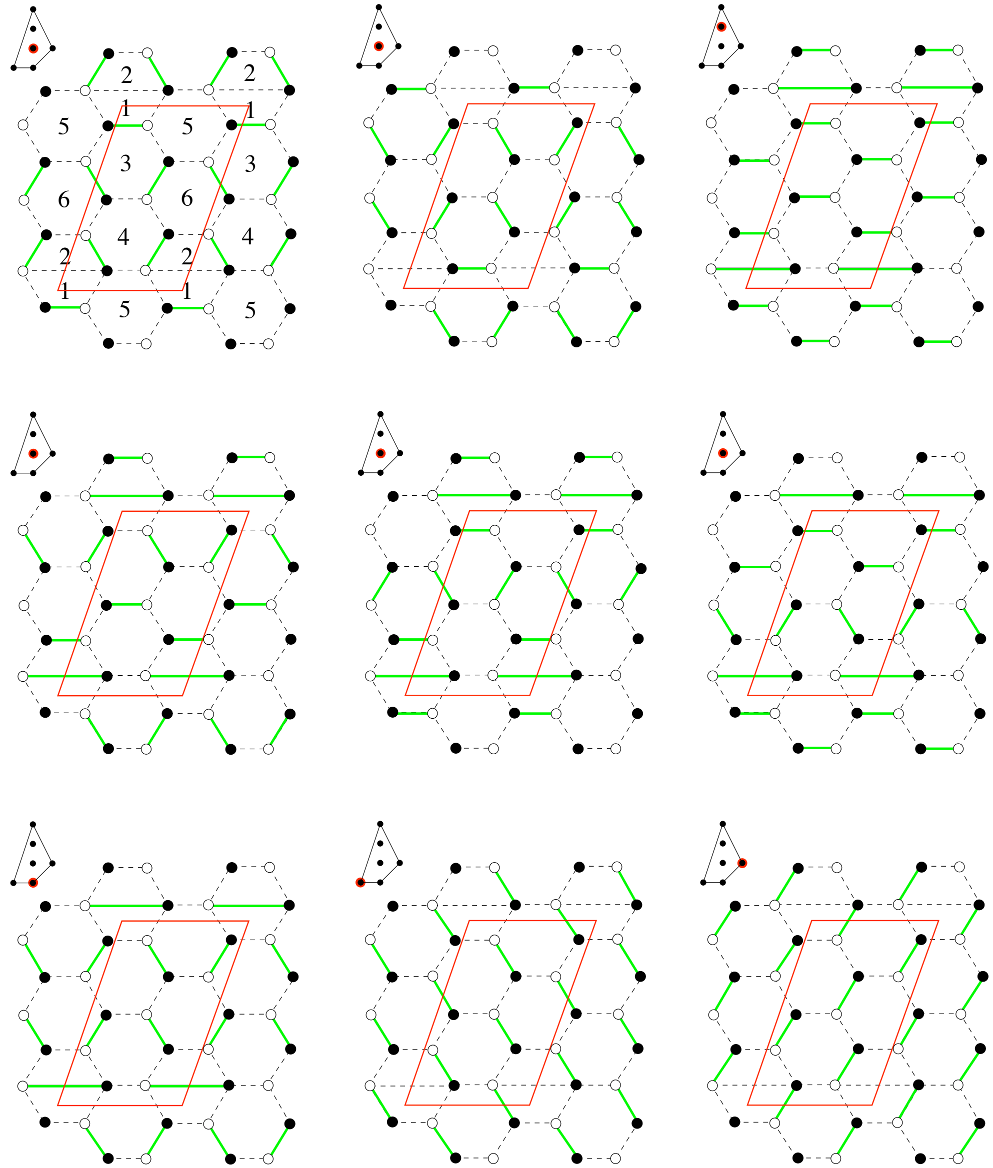
$$P(z,w) = w^{-1}z^{-1} - z^{-1} - w^{-1} - 6 - w - z + wz.$$



Toric diagram of dP3I



Perfect matching Y^{32} I



Perfect matching Y^{32} II

Moduli Space of Vacua

- All quiver theories arising from periodic bipartite tilings have toric noncompact CY as their moduli space of vacua
- Computed using the Kasteleyn matrix:
- Adjacency matrix between white and black nodes
- det K gives a convex polygon on 2d lattice

homology from toric diagram

- Given toric diagram set
- I = # internal nodes, E = # external nodes
- #4 cycles = I
- #2 cycles = I + E - 3
- 2 Area = 2I + E - 2 (Pick's Theorem)
- # Gauge Groups = #4 + #2 + #0 = 2 Area

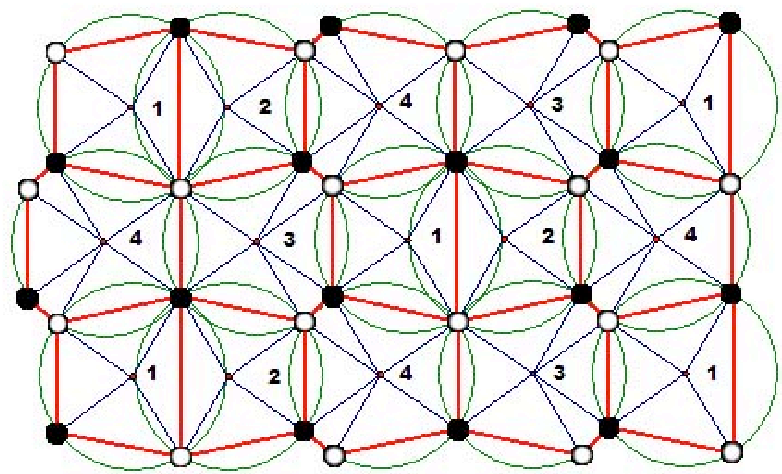
$$\sum_{i \in edges\ around\ node} R_i = 2 \qquad \text{for each node}$$

$$\sum_{i \in edges\ around\ face} (1 - R_i) = 2 \qquad \text{for each face}$$

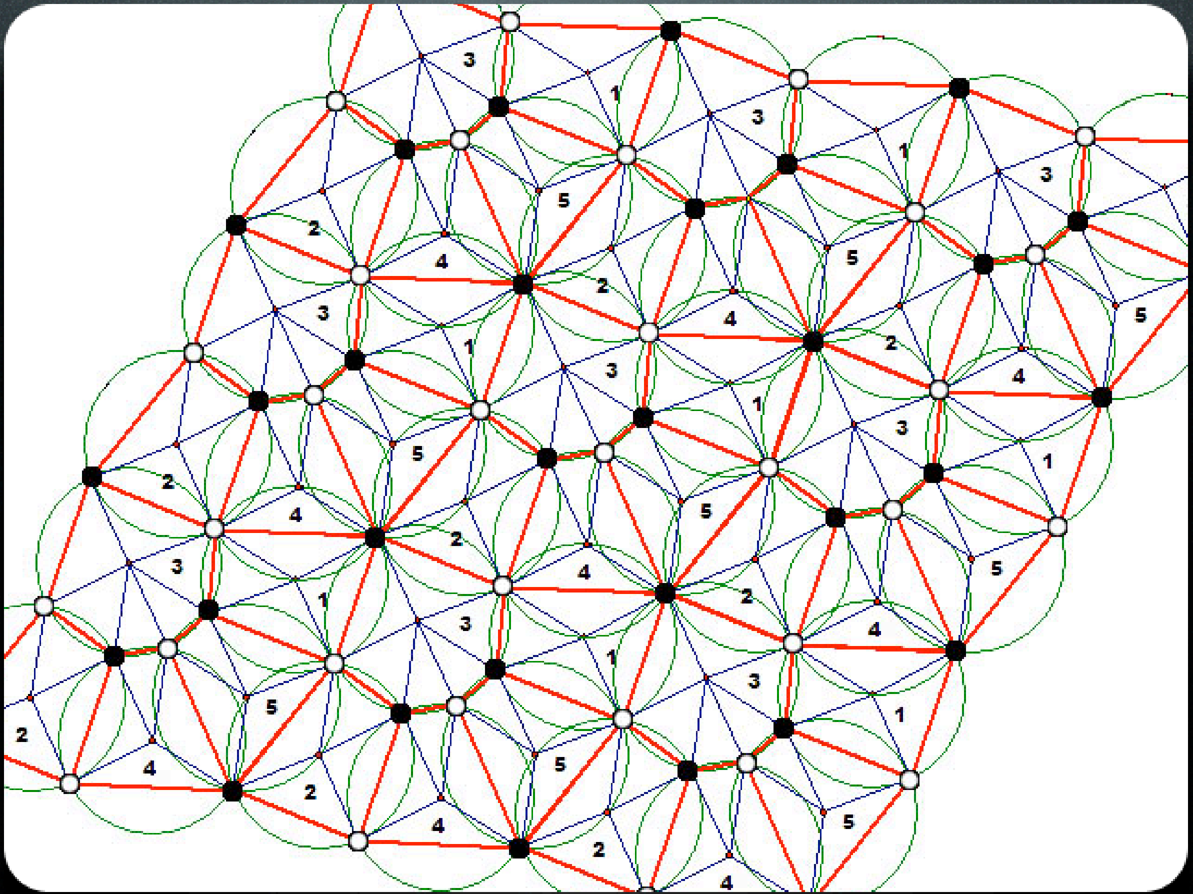
$$\sum_{i \in edges\ around\ node} (\pi R_i) = 2\pi \qquad \text{for each node}$$

$$\sum_{i \in edges\ around\ face} (\pi R_i) = (\#edges - 2)\pi \qquad \text{for each face}$$

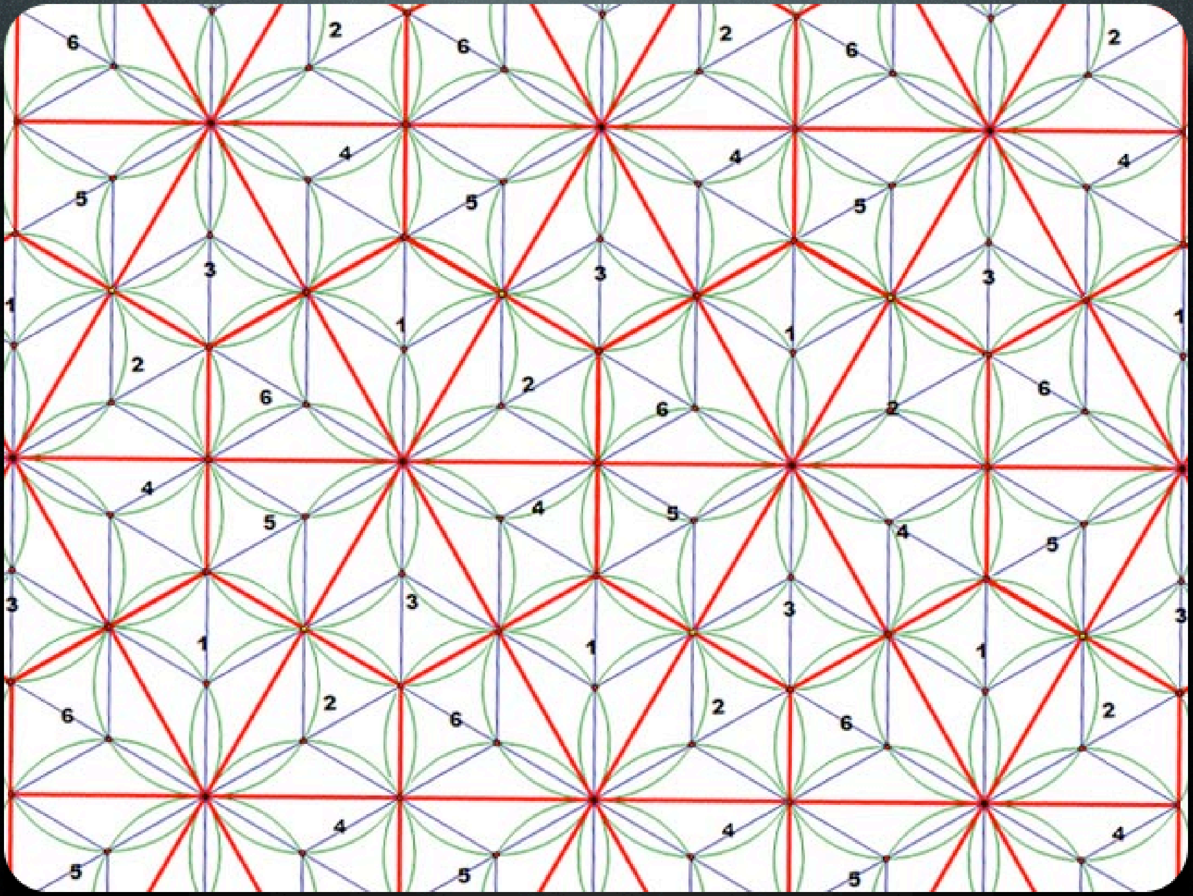
IR fixed point



Isoradial embed. dP1



dP2 II


6
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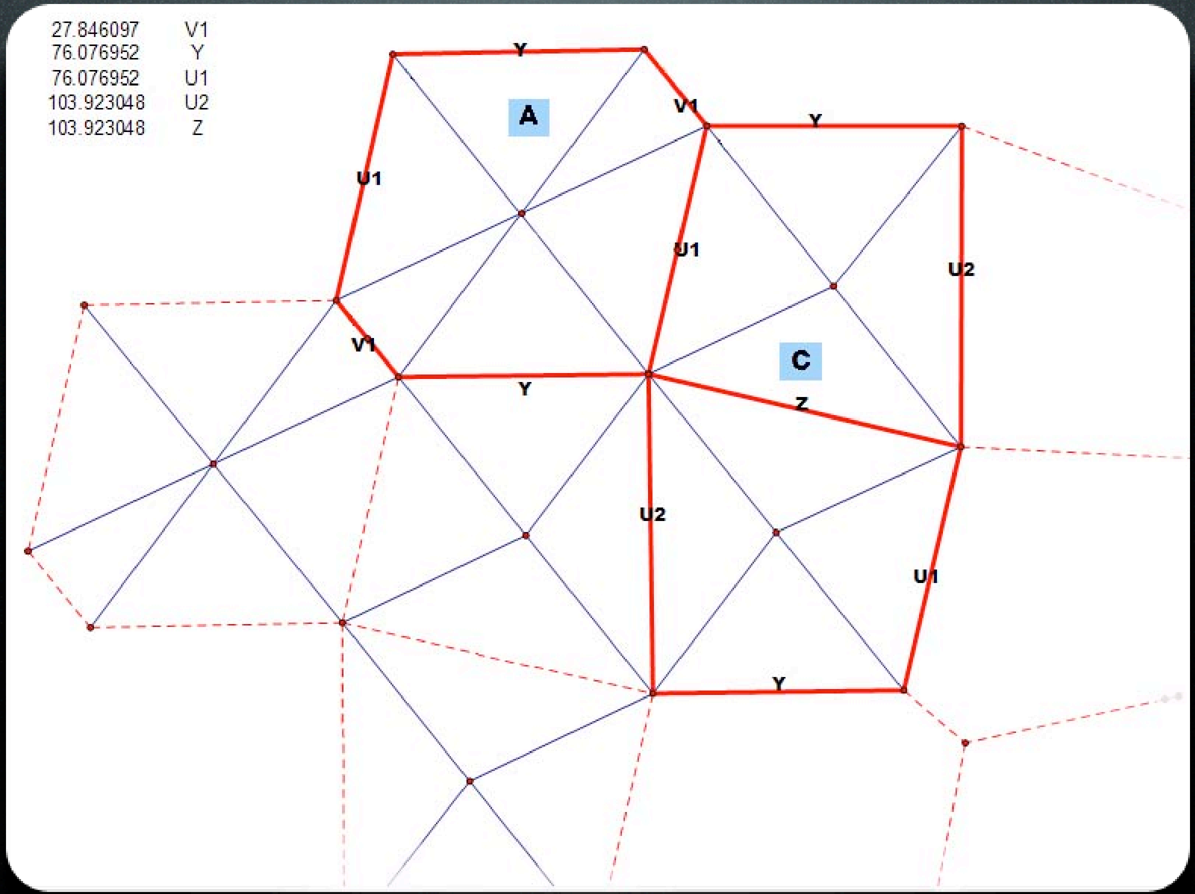
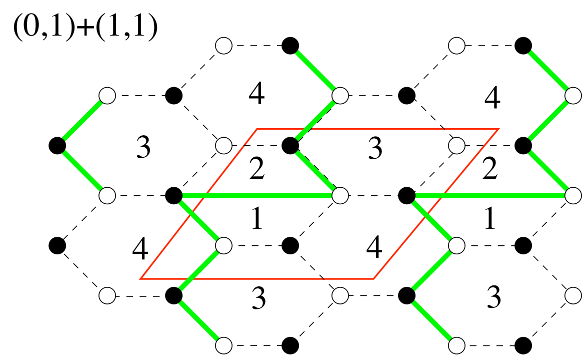
5

dP3 I


27.846097 V1
76.076952 Y
76.076952 U1
103.923048 U2
103.923048 Z
A
C
Y
V1
U1
U2
Z

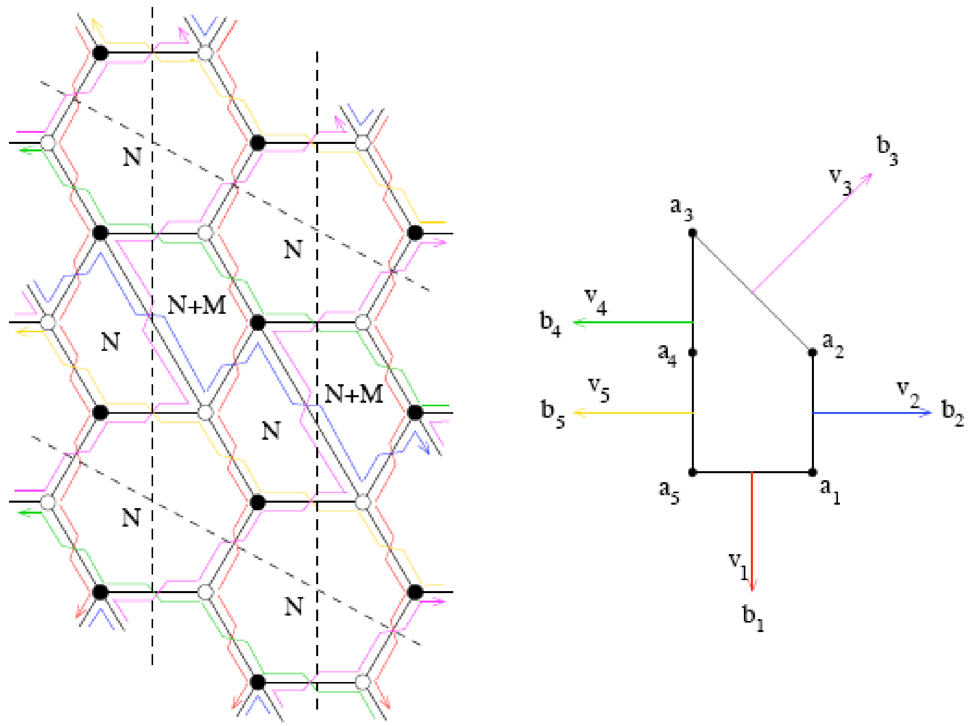
SPP tiling



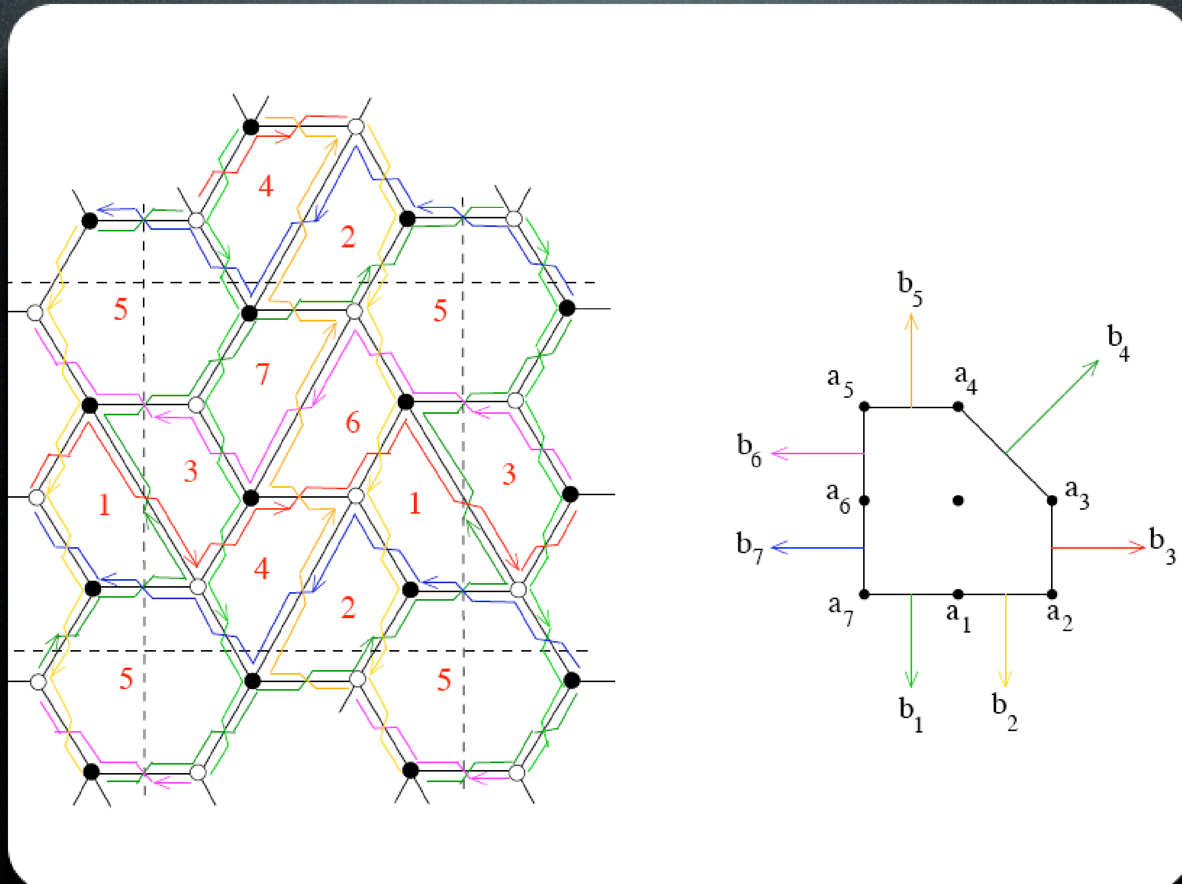
Zig-Zag path (dP1)

properties of zig-zag paths

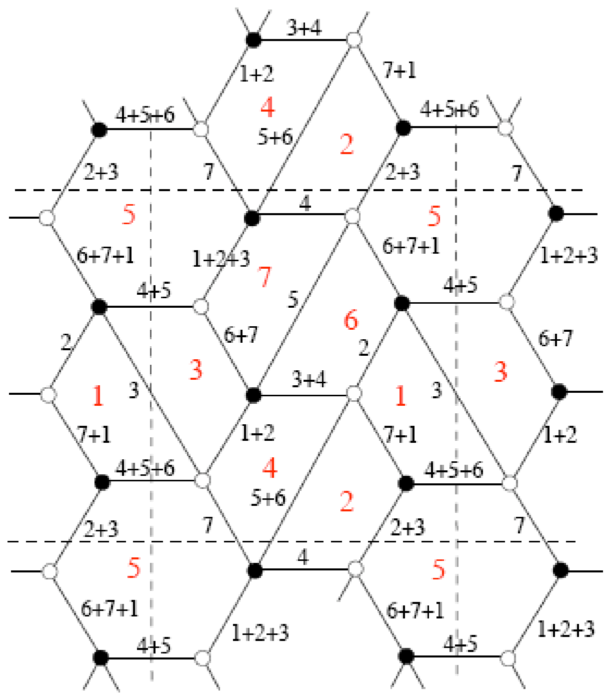
- each edge has precisely two paths going through it
- each path corresponds to an external leg in the (p,q) web dual to the toric diagram
- important for computing R charges & a-maximization
- following taken from Butti & Zaffaroni

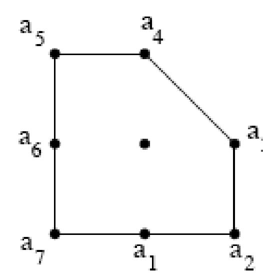


Zigzag paths SPP



Zigzag paths for PdP4





charges PdP4

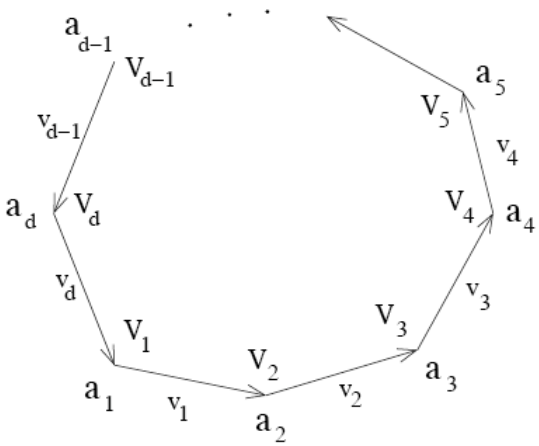


Figure 1: The convex polygon P.

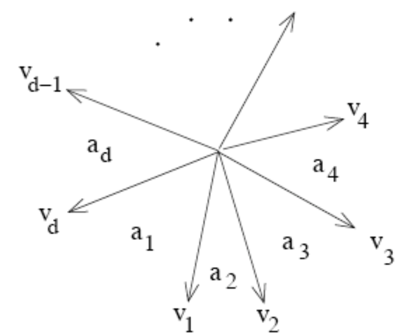


Figure 2: The (p, q) web for P.

General toric diagram

a maximization

$$a = \frac{9}{32}\,\mathrm{tr}\,R^3 = \frac{9}{32}\left(V + \sum_{(i,j)\in C} |\langle v_i, v_j\rangle|\ (a_{i+1} + a_{i+2} + \ldots a_j - 1)^3 \right)$$

$$\textstyle\sum_{i=1}^{d} a_i \;=\; 2$$

$$\langle w_i, w_j\rangle \equiv \det(w_i, w_j)$$

Spectrum of BPS KK states on toric SE manifold

$$\Psi(X) = \sum_{i=1}^{d} c_i a_i$$

- Set PM_i to be the perfect matching corresponding to external node i
- c_i are given by #intersections of PM_i with the path associated to gauge invariant X
- Exact spectrum of R charges in the IR for any quiver described by tiling

Conclusions

- Periodic tilings of 2d plane - N=1 SCFT's
- compute properties of quiver gauge theories using dimer techniques
- Solved a long standing problem - computing superpotetials for D3 branes probing singular CY's - exact BPS spectra
- Construct infinite families of quiver gauge theories ($Y^{p,q}$ $L^{a,b,c}$ ...)