Reheating of the Universe after Inflation with f(ϕ)R Gravity

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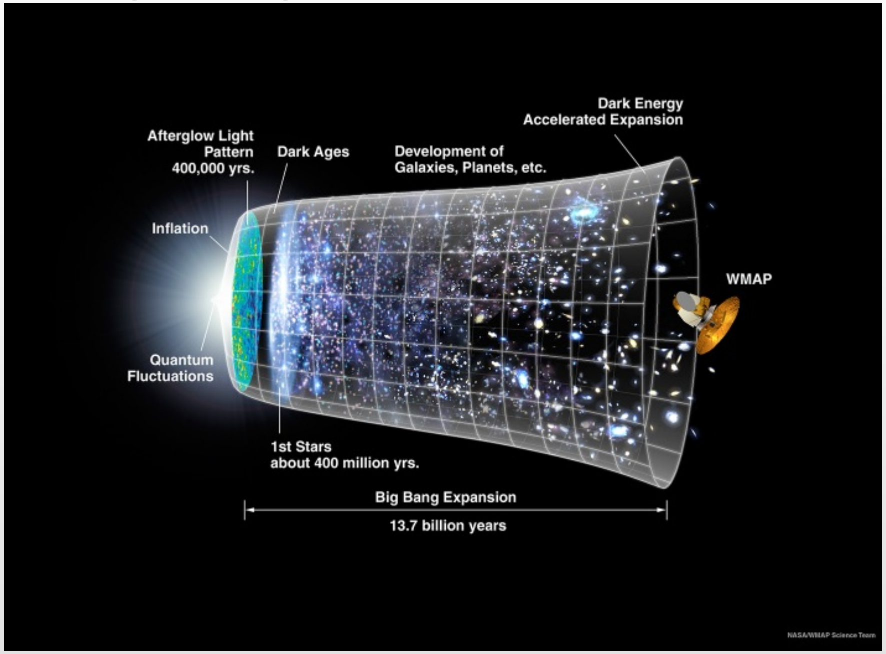
Why study reheating?

Inflation saves the Big Bang model.

By exponentially expanding a small region, inflation solves several problems not addressed by the Big Bang model:

- The isotropy of the CMB radiation.
- The origin of the cosmic structure, $\delta T/T \sim 10^{-5}$.
- The flatness of the universe, $\Omega_{tot}=1$.



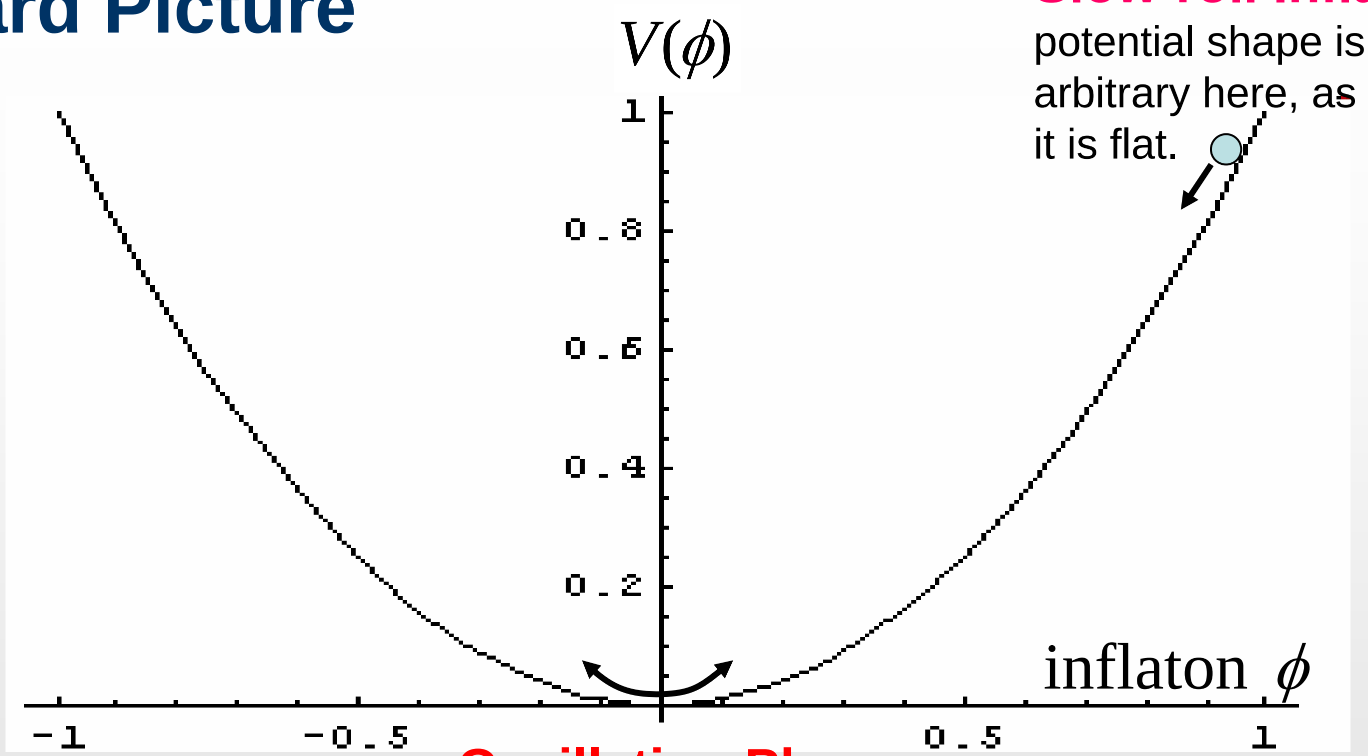
The universe is left cold and empty after inflation…

⇨ **The universe must reheat to have a hot Big Bang cosmology.**

Energy in inflaton must transfer to radiation, and heat the universe to at least ~1 MeV for successful nucleosynthesis.

Standard Picture

Slow-roll Inflation:
potential shape is arbitrary here, as long as it is flat.



Oscillation Phase:
around the potential minimum at the end of inflation

Energetics:

$$\rho_{rad} \sim T_{rh}^4$$
$$\sim g^4 V_{\inf}(\phi) \sim g^4 M_{Pl}^2 H_{\inf}^2$$
$$\Rightarrow T_{rh} \sim \sqrt{g^2 M_{Pl} H_{\inf}}$$

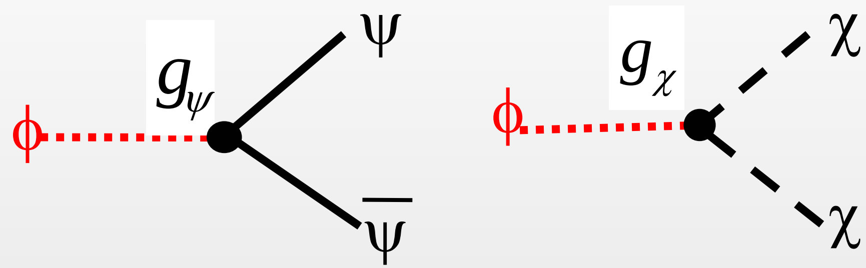
What determines *"energy-conversion efficiency factor", g?*

Perturbative Reheating

Dolgov & Linde (1982); Abbott, Farhi & Wise (1982); Albrecht et al. (1982)

Inflaton decays and thermalizes through the tree-level interactions like:

$$L_{\text{int}} = -\left(g_\psi \phi \overline{\psi} \psi + g_\chi \phi \chi^2 + \frac{1}{4} \lambda \phi^4 + \cdots \right)$$



Inflaton can decay if allowed kinematically with the widths given by

$$\Gamma_{\phi\bar{\psi}\psi} = \frac{g_\psi^2 m_\phi}{8\pi} \left(1 - \frac{4m_\psi^2}{m_\phi^2} \right)^{3/2} \tanh\left(\frac{m_\phi}{4T} \right) C_\psi$$

$$\Gamma_{\phi\chi\chi} = \frac{g_\chi^2}{8\pi m_\phi} \left(1 - \frac{4m_\chi^2}{m_\phi^2} \right)^{1/2} \coth\left(\frac{m_\phi}{4T} \right) C_\chi,$$

Pauli blocking

Bose condensate

Thermal medium effect

Reheating Temperature from Energetics

$$\ddot{\phi} + (3H + \Gamma_{tot})\dot{\phi} + m_{\phi}^2 \phi = 0 \qquad H_{\inf} >> H_{osc} \propto a^{-3/2}$$

$3H_{osc} > \Gamma_{tot} \Rightarrow$ Inflaton dominates the energy density.

$3H_{osc} < \Gamma_{tot} \Rightarrow$ Decay products dominate the energy density.

$$\rho_{rad}(t_{rh}) = 3M_{Pl}^2 H_{osc}^2 = \frac{M_{Pl}^2 \Gamma_{tot}^2}{3} = \frac{\pi^2}{30} g_*(T_{rh}) T_{rh}^4$$

$$T_{rh} = \frac{\sqrt{M_{Pl}\Gamma_{tot}}}{(10\pi^2)^{1/4}} \left(\frac{g_*(T_{rh})}{100} \right)^{-1/4}$$

Coupling constants determine the decay width, Γ.

But, what determines coupling constants?

Fine-tuning Problem?

$$\rho_{rad} \sim T_{rh}^4$$

$$\sim g^4 V_{\inf}(\phi) \sim g^4 M_{Pl}^2 H_{\inf}^2$$

$$\Rightarrow T_{rh} \sim \sqrt{g^2 M_{Pl} H_{\inf}}$$

If $T_{rh} = 10^{-10} M_{Pl}$ and $H_{\inf} = 10^{-6} M_{Pl}$,

then $\underline{g \sim 10^{-7}}$.

To relax fine-tuning, one needs:

(a) High reheat temperature

- ➢ unwanted relics (e.g., gravitinos),

(b) Very low-scale inflation ($H \sim 10^{-18}\ M_{pl} \sim 10$ GeV)

- ➢ worse fine-tuning, or

(c) Natural explanation for the smallness of g.

What are coupling constants?
Problem: arbitrariness of the nature of inflaton fields

- Inflation works very well as a concept, but we do not understand the nature (including interaction properties) of inflaton.

  e.g. Higgs-like scalar fields, Axion-like fields, Flat directions, RH sneutrino, Moduli fields, Distances between branes, and many more…

- Arbitrariness of inflaton = Arbitrariness of couplings
- Can we say anything generic about reheating? Universal reheating? **Universal coupling?**

Gravitational coupling is universal

➢too weak to cause reheating with GR.

In the early universe, however, GR would be modified.

⇒ **What happens to "gravitational decay channel", when GR is modified?**

Conventional Einstein gravity during inflation

$$\mathcal{L} = \sqrt{-g}\left[\underline{\frac{1}{2}M_{Pl}^2 R} - \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right] + \sqrt{-g}L_{matt}$$

Einstein-Hilbert term generates GR.

Inflaton minimally couples to gravity.

$$L_{\text{int}} = -\left(g_\psi \phi\bar{\psi}\psi + g_\chi \phi\chi^2 + \lambda\phi^2\chi^2 + \cdots\right)$$

Conventionally one introduced explicit couplings between inflaton and matter.

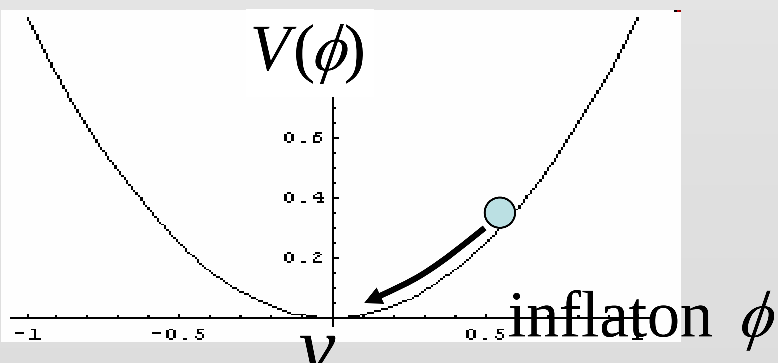
Modifying Einstein gravity during inflation

Instead of introducing explicit coupling by hand,

$$\cancel{L_{\text{int}} = -\left(g_\psi \phi \bar{\psi}\psi + g_\chi \phi \chi^2 + \lambda \phi^2 \chi^2 + \cdots\right)}$$

$$\mathcal{L} = \sqrt{-g}\left[\underline{\frac{1}{2}f(\phi)R} - \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right] + \sqrt{-g}L_{matt}$$

Non-minimal gravitational coupling: common in effective Lagrangian from extra dimensional theories
In order to ensure GR after inflation, $f(v) = M_{Pl}^2$



Matter (everything but gravity and inflaton) completely decouples from inflaton and minimally coupled to gravity as usual.

Field equations: GR

$$M_{Pl}^2 G_{\mu\nu} = T^{\mathrm{m}}_{\mu\nu} + \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} (\partial\phi)^2 - g_{\mu\nu} V(\phi)$$

Linearized field equation:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$\frac{M_{\mathrm{Pl}}^2}{2} \left[-\Box h_{\mu\nu} + \cdots \right] = T^{\mathrm{m}}_{\mu\nu}$$

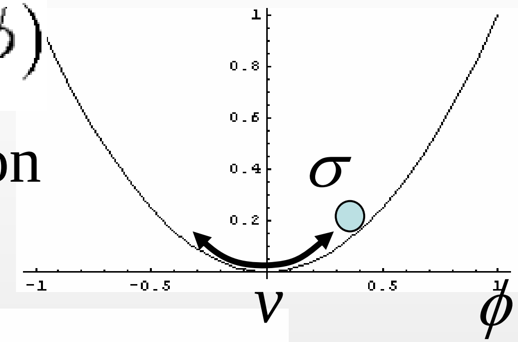
Wave modes are gravitational waves. To identify the wave modes, we usually define $\tilde{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} h$

$$\Box \tilde{h}_{\mu\nu} - 2\bar{R}_{\mu\lambda\rho\nu} \tilde{h}^{\lambda\rho} - 2\bar{R}_{\rho(\mu} \tilde{h}^{\rho}{}_{\nu)} - \bar{g}_{\mu\nu} \bar{R}_{\lambda\rho} \tilde{h}^{\lambda\rho} + \bar{R} \tilde{h}_{\mu\nu} = -\frac{2}{M_{\mathrm{Pl}}^2} T^{\mathrm{m}}_{\mu\nu}$$

Harmonic (Lorenz) gauge: $\nabla_\lambda \tilde{h}^{\lambda}{}_{\nu} = 0$

Field equations: f(ϕ)R gravity

$$f(\phi)G_{\mu\nu} = T^{\mathrm{m}}_{\mu\nu} + \nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2 - g_{\mu\nu}V(\phi)$$
$$-(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f(\phi)$$



Linearized field equation during coherent oscillation

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \qquad \phi = v + \sigma$$

$$\frac{M_{\mathrm{Pl}}^2}{2}\left[-\Box h_{\mu\nu} + \cdots\right] + f'(v)(\bar{g}_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\sigma = T^{\mathrm{m}}_{\mu\nu}$$

Wave modes are mixed up. To extract “true” gravitational degrees of freedom, we define

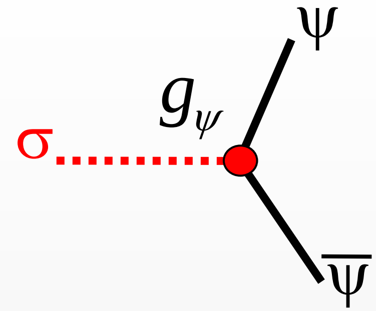
$$\tilde{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}h - \frac{f'(v)}{M_{\mathrm{Pl}}^2}\bar{g}_{\mu\nu}\sigma$$

$$\Box\tilde{h}_{\mu\nu} - 2\bar{R}_{\mu\lambda\rho\nu}\tilde{h}^{\lambda\rho} - 2\bar{R}_{\rho(\mu}\tilde{h}^{\rho}{}_{\nu)} - \bar{g}_{\mu\nu}\bar{R}_{\lambda\rho}\tilde{h}^{\lambda\rho} + \bar{R}\tilde{h}_{\mu\nu} = -\frac{2}{M_{\mathrm{Pl}}^2}T^{\mathrm{m}}_{\mu\nu}$$

Harmonic (Lorenz) gauge: $\nabla_\lambda\tilde{h}^{\lambda}{}_{\nu} = 0$

New decay channel through “scalar gravity waves”

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \tilde{h}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\tilde{h} - \frac{f'(v)}{M_{\rm Pl}^2}\bar{g}_{\mu\nu}\sigma$$



Fermionic (spinor) matter field:

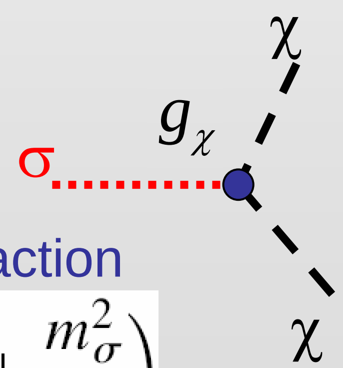
Yukawa interaction

$$\mathcal{L}_\psi \simeq -\bar{e}\,\bar{\psi}[\bar{e}^{\mu\alpha}\gamma_\alpha D_\mu + m_\psi]\psi + \frac{1}{2}\bar{e}\,\bar{\psi}[\bar{e}^{\nu\alpha}\gamma_\alpha(\tilde{h}^\mu_\nu + \frac{1}{2}\tilde{h}\delta^\mu_\nu)D_\mu + \tilde{h}m_\psi]\psi + \bar{e}\frac{f'(v)m_\psi}{2M_{\rm Pl}^2}\sigma\bar{\psi}\psi$$

$$\frac{f'(v)m_\psi}{2M_{\rm Pl}^2} = g_\psi$$

Bosonic (scalar) matter field:

$$\mathcal{L}_\chi \simeq \sqrt{-\bar{g}}\Big[-\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - U(\chi) + \frac{1}{2}\tilde{h}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + \frac{1}{2}\tilde{h}U(\chi) + \frac{f'(v)}{2M_{\rm Pl}^2}(4\sigma U(\chi) + \sigma\bar{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi)\Big],$$



Three-legged interaction

$$g_\chi \equiv \frac{f'(v)}{2M_{\rm Pl}^2}\left(m_\chi^2 + \frac{m_\sigma^2}{2}\right)$$

$$U(\chi) = m_\chi^2\chi^2/2$$

Magnitude of Yukawa coupling

$$g_\psi \equiv \frac{f'(v) m_\psi}{2M_{\rm Pl}^2}$$

- For $f(\phi)=\xi\phi^2$, $g=\xi(v/M_{pl})(m_\psi/M_{pl})$
  - Natural to obtain $g\sim 10^{-7}$ for e.g., $m_\psi \sim 10^{-7} M_{pl}$
- The induced Yukawa coupling vanishes for massless fermions: conformal invariance of massless fermions.

- Massless, minimally-coupled scalar fields are not conformally invariant. Therefore, the three-legged interaction does not vanish even for massless scalar fields:

$$g_\chi \equiv \frac{f'(v)}{2M_{\rm Pl}^2}\left(\cancel{m_\chi^2} + \frac{m_\sigma^2}{2}\right)$$

Breaking of conformal invariance

$$g_{\mu\nu}(x) \to \hat{g}_{\mu\nu}(x) = \Omega^2(x) g_{\mu\nu}(x) \approx g_{\mu\nu} + g_{\mu\nu}\frac{f'(v)\sigma}{M_{Pl}^2}$$

$$\Omega^2 = \frac{f(\phi)}{M_{\rm Pl}^2} = 1 + \frac{f'(v)\sigma}{M_{\rm Pl}^2}$$

Conformal invariance: local scale invariance at every spacetime point

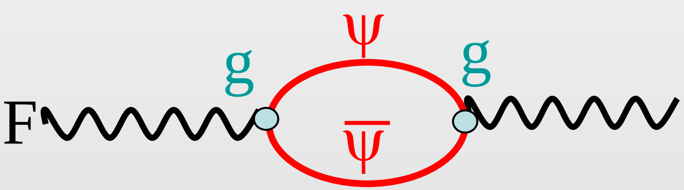
Mass term obviously fixes the energy scale, and breaks conformal invariance.

$$T^{\mu}_{m\ \mu}[g_{\mu\nu}] = -\frac{\Omega}{\sqrt{-g}}\frac{\delta S_m[\hat{g}_{\mu\nu}]}{\delta\Omega}\bigg|_{\Omega=1}$$

If matter action is invariant under conformal transformations, then the stress-energy tensor is traceless.

Conformally invariant fields:

- Massless spin-½ fields
- Conformally coupled massless spin-0 fields
- Gauge fields (classical level)



Energy evolution parameter: t

$$\frac{dg}{dt} = \beta(g) = -\frac{g^3}{(4\pi)^2}\left(\frac{11}{3}n - \frac{2}{3}n_f\right)$$

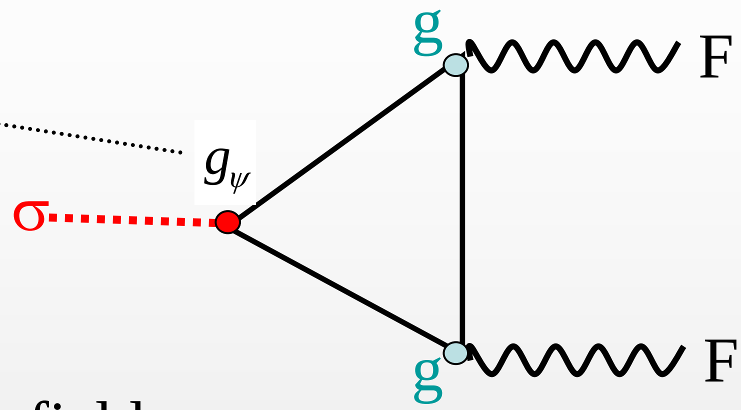
$$\mathcal{L}_{\rm int} = -\sqrt{-g}\,\frac{f'(v)\sigma}{2M_{Pl}^2}T^{\mu}_{m\ \mu}$$

$$T^{\mu}_{m\mu} = \underbrace{\sum_f m_f\bar{\psi}_f\psi_f + \sum_s\left(g^{\mu\nu}\partial_\mu\chi_s\partial_\nu\chi_s + 4U(\chi_s)\right) + \sum_V m_V^2 V_\mu V^\mu}_{\text{at the classical level}} + \frac{\beta(g)}{2g}F^a_{\mu\nu}F^{a\mu\nu}$$

Conformal anomaly: Lowest order decay channel to gauge fields

(c.f.) two-photon decay of the Higgs

$$g_\psi \equiv \frac{f'(v) m_\psi}{2 M_{\rm Pl}^2}$$



Inflaton -> 2 gauge fields

$$\begin{aligned}
\Gamma(\sigma \to FF) &= \frac{1}{(2\pi)^2}\frac{1}{2\mu}\frac{1}{2}\int \frac{d^3k}{2\omega_k}\int \frac{d^3k'}{2\omega_{k'}}\delta^4(q-k-k')\sum_{\lambda\lambda'}|\mathcal{M}|^2 \\
&= \frac{1}{32\pi\mu}\sum_{\lambda\lambda'}|\mathcal{M}|^2 \\
&= \frac{1}{32\pi\mu}\frac{g^4 g_\psi^2 \mu^4}{2(2\pi)^4 m_\psi^2}\left|I\left(\frac{\mu^2}{m_\psi^2}\right)\right|^2 \\
&= \frac{\alpha^2 g_\psi^2 \mu^3}{64\pi^3 m_\psi^2}\left|I\left(\frac{\mu^2}{m_\psi^2}\right)\right|^2 \\
&= \frac{\alpha^2 [f'(v)]^2 \mu^3}{256\pi^3 M_{\rm Pl}^4}\left|I\left(\frac{\mu^2}{m_\psi^2}\right)\right|^2,
\end{aligned}$$

$$\mathcal{M} = i g^2 g_\psi \epsilon_\mu^*(k,\lambda)\epsilon_\nu^*(k',\lambda')\frac{-i}{(2\pi)^2 m_\psi}\left(k'^\mu k^\nu - \frac{\mu^2}{2}g^{\mu\nu}\right) I\left(\frac{\mu^2}{m_\psi^2}\right)$$

$$I(\xi) \equiv \int_0^1 dx \int_0^{1-x} dy \frac{1-4xy}{1-\xi xy}$$

$I(\xi) \to 1/3$ as $\xi \to 0$

$I(\xi) \to 0$ as $\xi \to \infty$

Decay Width Summary

Fermions

$$\Gamma = \frac{[f'(v)]^2 m_\psi^2 m_\sigma}{32\pi M_{pl}^4}$$

Scalar Bosons

$$\Gamma = \frac{[f'(v)]^2 \left(m_\chi^2 + \frac{m_\sigma^2}{2} \right)^2}{32\pi M_{pl}^4 m_\sigma}$$

Probably the most dominant decay channels

Gauge Bosons

$$\Gamma = \frac{\alpha^2 [f'(v)]^2 m_\sigma^3}{256\pi^3 M_{pl}^4} \times (\#\ \text{of internal fermions etc})$$

Constraint on f(ϕ)R gravity from reheating

$$T_{rh} = \frac{\sqrt{M_{Pl}\Gamma_{tot}}}{(10\pi^2)^{1/4}}\left(\frac{g_*(T_{rh})}{100}\right)^{-1/4}$$

$$\Gamma_{\rm tot} > \Gamma_{\sigma\bar{\psi}\psi} + \Gamma_{\sigma\chi\chi} + \Gamma_{\sigma FF}$$

$$\boxed{|f'(v)| < 8\pi(40)^{1/4} T_{\rm rh}\left(\frac{M_{\rm Pl}}{m_\sigma}\right)^{3/2}\left(\frac{g_*(T_{\rm rh})}{100}\right)^{1/4}}$$

e.g. $f(\phi) = M^2 + \xi\phi^2$

$$|\xi| < 4\pi(40)^{1/4} \times \frac{T_{rh}}{v}\left(\frac{M_{\rm Pl}}{m_\sigma}\right)^{3/2}\left(\frac{g_*(T_{\rm rh})}{100}\right)^{1/4}$$

Constraints from chaotic inflation

$\xi > -10^{-3}$

Futamase & Maeda(1989)

$|\xi| > 5\times 10^4\sqrt{\lambda}$

Komatsu & Futamase(1999)

Conclusions

- A natural mechanism for reheating after inflation with f(ϕ)R gravity: *Why natural?*
  - Inflaton quanta decay spontaneously into any matter fields (spin-0, ½, 1) without explicit interactions in the original Lagrangian
  - Conformal invariance must be broken at the tree-level or by loops
  - Reheating spontaneously occurs in any theories with f(ϕ)R gravity
- Predictability
  - All the decay widths are related through a single function, f(ϕ).
- A constraint on f(ϕ) from the reheat temperature can be found
  - A possible limit on the reheat temperature can constrain the form of f(ϕ), or vice versa.
  - These constraints on f(ϕ) are totally independent of the other constraints from inflation and density fluctuations

Further Study…

- Preheating? F(ϕ,R) gravity?