Relic Anisotropy as a Source for CMB Anomalies

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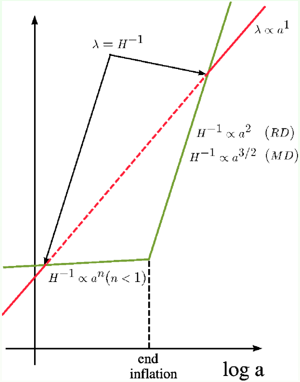
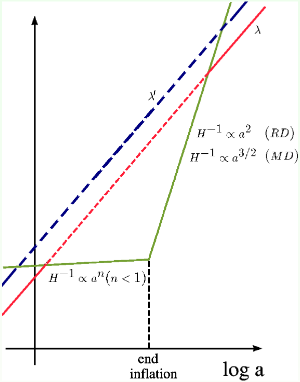
Inflation and Beyond

- Inflation is a solution of horizon, flatness, monopole, ... problems.
- As a bonus, it provides the origin of the structure in the universe.
- It wipes away all information on previous conditions. Only possible exception: largest observable scales, provided that the inflation is not too long.

- Modes leave the horizon during inflation, and re-enter much later during radiation or matter dominated eras.



- The larger the mode, the earlier it exits the horizon during inflation.
- Larger modes are those mostly affected by details of pre-inflationary cosmology (the smaller ones are still deeply inside the horizon).
- If inflation too long, those modes are inflated at sizes larger than what we can observe. Need only a minimal amount of inflation.



Large Scale Anomalies in CMB

With the advent of satellite CMB measurements, we are now able to look at large scales. Despite their nearly perfect agreement with the concordance model, they show some anomalies

- Missing Power at angles $> 60°$,

  $\Rightarrow 1/20$ Tegmark et al.(2003),Contaldi et al.(2003),Efstathiou et al.(2004),de Oliveira-Costa et al.(2004)
- $\ell = 2$ and $\ell = 3$ are planar, and appear to be correlated,

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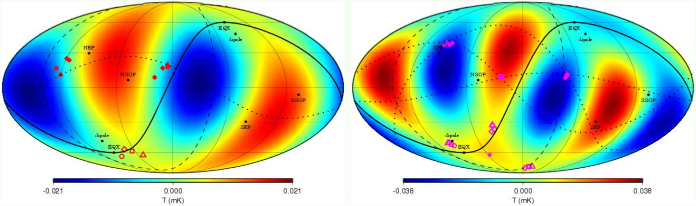

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- Hemispherical asymmetries Eriksen et al.(2004, 2007)

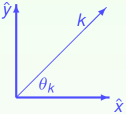
Attempts to explain the anomalies with foregrounds and systematics put forward, but not yet conclusive. With the lack of a convincing explanation, it is legitimate to ask if they have a cosmological origin.

Cosmological Perturbations Bardeen (1980), Sasaki (1986), Mukhanov (1988)

- Background metric $ds^2 = a^2\left(-d\eta^2 + dx^2 + dy^2 + dz^2\right)$, with scalar field $\mathcal{L}_\phi = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$. $(d\eta = dt/a)$
- Introduce scalar field perturbations $\phi = \phi^0 + \delta\phi$, and cosmological perturbations in longitudinal gauge

$$\begin{aligned} \delta g_{00} &= -2\,a^2\,\Phi\,, \\ \delta g_{0i} &= a^2 B_i\,, \\ \delta g_{ij} &= a^2\left(-2\,\Psi\delta_{ij} + h_{ij}\right)\,. \end{aligned}$$

- Constraining the momentum k to x–y plane, and defining $k_{\|} \equiv k\,\cos\theta_k$, $k_\perp \equiv k\,\sin\theta_k$, the quadratic action can be written as



$$S = \frac{1}{2}\int d\eta\, d^3k\,\left[|v'|^2 - \left(k^2 - \frac{z''}{z}\right)|v|^2 + |h'_+|^2 - \left(k^2 - \frac{a''}{a}\right)|h_+|^2 + |h'_\times|^2 - \left(k^2 - \frac{a''}{a}\right)|h_\times|^2\right]\,, \qquad z = \frac{a^2\,\phi'}{a'}$$

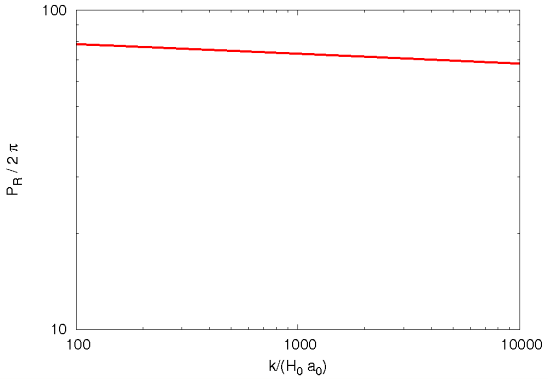
with

$$v \equiv a\left(\delta\phi + \frac{a\,\phi'}{a'}\Psi\right)\,, \qquad h_+ \equiv \frac{M_p}{\sqrt{2}}\frac{a\,k^2}{k_\perp^2}h_{11}\,, \qquad h_\times \equiv \frac{M_p}{\sqrt{2}}\frac{a\,k}{k_\perp}h_{13}\,.$$

Cosmological Perturbations

- The power spectrum for $\mathcal{R} = -v/z$ and $h = \sqrt{2}\, h_{+,\times}/(a\, M_p)$ $\quad (P_X(\vec{k}) = k^3 |X|^2)$

$$P_{\mathcal{R}} = \frac{a'^2\, k^3}{\phi'^2\, a^4} |v|^2 \,, \qquad P_h = \frac{2\, k^3}{M_p^2\, \pi\, a^2} |h_{+,\times}|^2 \,.$$



(evolved from $\phi_0 = 17\, M_p$ to $\phi = M_p$)

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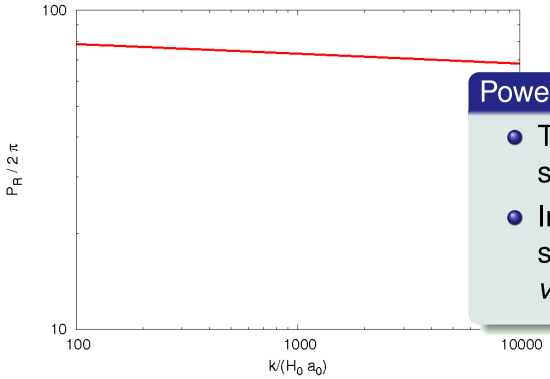

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Power Spectrum

- The power spectrum is almost scale invariant, $P_{\mathcal{R}} \sim k^{n_s - 1}$
- Initially, sub-horizon modes start with $v\,,\ h_{+\,,\times} = \frac{1}{\sqrt{2\,k}} \mathrm{e}^{-i\,k \int d\eta}$

Background

We consider the simplest extension of the standard inflationary picture by taking the a Bianchi-I background, driven by a one field chaotic inflation. We assume large initial anisotropy in expansion rates ($H_a \gg H_b$).

$$ds^2 = a^2 \left(-d\eta^2 + dx^2\right) + b^2 \left(dy^2 + dz^2\right) \, ,$$

equations of motion

- $2\, H_a\, H_b + H_b^2 = \frac{1}{2}\, \dot{\phi}^2 + V \, ,$
- $2\, \dot{H}_b + 3\, H_b^2 \quad = -\frac{1}{2}\, \dot{\phi}^2 + V \, ,$
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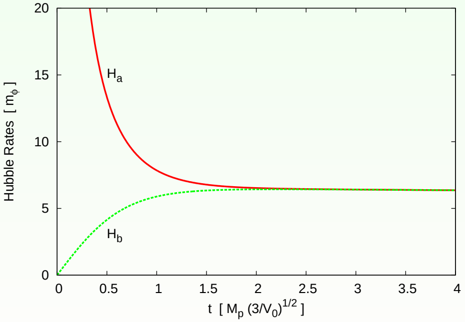

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Early on, $H_a \sim 1/t$ *and* $H_b \sim t$

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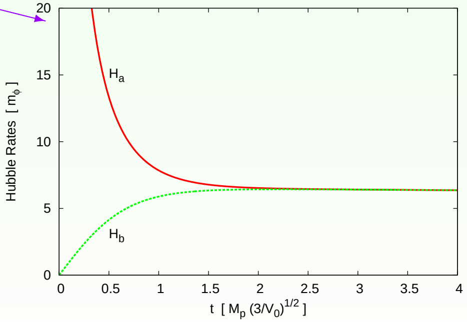

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$$= -\frac{1}{2}\, \dot{\phi}^2 + V \,.$$

Procedure

- The only recipe we have is the theory of cosmological perturbations in the isotropic background. Need to extend it to more complicated backgrounds.
- After introducing the perturbations, writing down the Einstein equations is straight-forward. However, initial conditions are highly nontrivial. First, we need to identify the physical modes (dynamical, quadratic action diagonal). We chose a convenient, but non-standard gauge (with all $\delta g_{0\mu} \neq 0$) so that the non-dynamical modes can be most easily determined and integrated out. Then, we look whether the physical modes can start in an adiabatic vacuum (namely, if $\omega' \ll \omega^2$).

Perturbations

- Introduce metric perturbations classified according to spatial transformation properties (Constrained the comoving momenta to x–y plane).

$$
\begin{aligned}
g_{\mu\nu} &= \begin{pmatrix} -a^2\,(1+2\,\Phi) & a\,\partial_{\|}\chi & a\,\partial_{\perp}B & b^2\,B_3 \\ & a^2\,(1-2\,\Psi) & 0 & b^2\,\partial_{\|}\tilde{B}_3 \\ & & b^2 & 0 \\ & & & b^2 \end{pmatrix} \\
\phi &= \phi^{(0)} + \delta\phi
\end{aligned}
$$

- Start with 10+1 dof $\Longrightarrow$ eliminate 4 with gauge choice.
- 4 dof in the top row are non-dynamical. Only three physical modes remain.

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The final form of the action is

$$S = \frac{1}{2}\int d\eta\, d^3k\, \{|H'_\times|^2 - \omega_\times^2 |H_\times|^2 + \left[|V'|^2 + |H'_+|^2 - (V^\star\,,\,H_+^\star)\;\Omega^2 \begin{pmatrix} V \\ H_+ \end{pmatrix}\right]\}$$

with,

$$\Omega^2 = \begin{pmatrix} \omega_{11}^2 & \omega_{12}^2 \\ \omega_{12}^2 & \omega_{22}^2 \end{pmatrix}$$

$$H_\times \equiv \frac{M_p}{\sqrt{2}}\,\frac{k_{\|}\,k_\perp}{\sqrt{k_{\|}^2 + \frac{a^2}{b^2}k_\perp^2}}\,\tilde{B}_3\,, \qquad H_+ \equiv \frac{\sqrt{2}\,b\,M_p\,p_\perp^2\;H_b}{H_a\,p_\perp^2 + H_b\left(2p_{\|}^2 + p_\perp^2\right)}\Psi\,,$$

$$V \equiv b\left[\delta\phi + \frac{p_\perp^2\;\dot{\phi}}{H_a\,p_\perp^2 + H_b\left(2p_{\|}^2 + p_\perp^2\right)}\Psi\right]$$

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with,

1xTensor : Becomes $h_\times$

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1xScalar & 1xTensor : Become v & h_+

$$H_\times \equiv \frac{M_p}{\sqrt{2}} \frac{k_\parallel\, k_\perp}{\sqrt{k_\parallel^2 + \frac{a^2}{b^2} k_\perp^2}} \tilde{B}_3\,, \qquad H_+ \equiv \frac{\sqrt{2}\, b\, M_p\, p_\perp^2\, H_b}{H_a\, p_\perp^2 + H_b\left(2p_\parallel^2 + p_\perp^2\right)} \Psi\,,$$

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(Physical momenta: $p_\parallel \equiv k_\parallel / a$ and $p_\perp \equiv k_\perp / b$)

The “mass” terms

$$
\begin{aligned}
\omega_\times^2 &= a^2 \left[p_\parallel^2 + p_\perp^2 - 2\,H_a\,H_b + \frac{\dot\phi^2}{2M_p^2} + (H_a - H_b)^2\,\frac{p_\perp^2\left(2\,p_\parallel^2 - p_\perp^2\right)}{\left(p_\parallel^2 + p_\perp^2\right)^2} \right] \\
\left(\frac{\omega_{11}}{a}\right)^2 &= \left(p_\parallel^2 + p_\perp^2 - 2\,H_a\,H_b + \frac{3\,\dot\phi^2}{2\,M_p^2} + \frac{2\,H_a}{H_b}\frac{\dot\phi^2}{M_p^2} - \frac{1}{2\,H_b^2}\frac{\dot\phi^4}{M_p^4} + \frac{2}{H_b}\frac{\dot\phi\,V'}{M_p^2} + V'' \right) \\
&\quad + \frac{p_\perp^2\,(H_a - H_b)}{\left[2\,H_b\,p_\parallel^2 + (H_a + H_b)\,p_\perp^2\right]}\frac{\dot\phi}{M_p}\left[-\frac{4\,\dot\phi}{M_p} - \frac{2\,H_a}{H_b}\frac{\dot\phi}{M_p} + \frac{1}{H_b^2}\frac{\dot\phi^3}{M_p^3} - \frac{2}{H_b}\frac{V'}{M_p} \right. \\
&\quad \left. - \frac{p_\perp^2\,(H_a - H_b)}{\left[2\,H_b\,p_\parallel^2 + (H_a + H_b)\,p_\perp^2\right]}\frac{\dot\phi}{M_p}\left(1 + \frac{1}{2\,H_b^2}\frac{\dot\phi^2}{M_p^2}\right) \right] \\
\left(\frac{\omega_{22}}{a}\right)^2 &= p_\parallel^2 + p_\perp^2 - 2\,H_a\,H_b + \frac{\dot\phi^2}{2\,M_p^2} \\
&\quad + \frac{p_\perp^2\,(H_a - H_b)^2}{\left[2\,H_b\,p_\parallel^2 + (H_a + H_b)\,p_\perp^2\right]}\left[4\,H_b - \frac{p_\perp^2\left(2\,H_b^2 + \frac{\dot\phi^2}{M_p^2}\right)}{\left[2\,H_b\,p_\parallel^2 + (H_a + H_b)\,p_\perp^2\right]} \right] \\
\frac{1}{\sqrt{2}}\left(\frac{\omega_{12}}{a}\right)^2 &= \frac{p_\perp^2\,(H_a - H_b)}{\left[2\,H_b\,p_\parallel^2 + (H_a + H_b)\,p_\perp^2\right]}\left[-3\,H_b\frac{\dot\phi}{M_p} + \frac{1}{2\,H_b}\frac{\dot\phi^3}{M_p^3} - \frac{V'}{M_p} \right. \\
&\quad \left. - \frac{p_\perp^2\,(H_a - H_b)}{\left[2\,H_b\,p_\parallel^2 + (H_a + H_b)\,p_\perp^2\right]}\frac{\dot\phi}{M_p}\left(H_b + \frac{1}{2\,H_b}\frac{\dot\phi^2}{M_p^2}\right) \right]
\end{aligned}
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$p_\parallel \equiv k_\parallel / a$

$p_\perp \equiv k_\perp / b$

$k_\parallel \equiv k \cos\theta_k$

$k_\perp \equiv k \sin\theta_k$

Initial Conditions

- Initial behavior of BG quantities known.
- Using these in the “mass” terms,

$$\omega_\times \, , \, \omega_{11} \, , \, \omega_{22} \simeq k_\parallel \text{ and } \omega_{12} \simeq 0$$

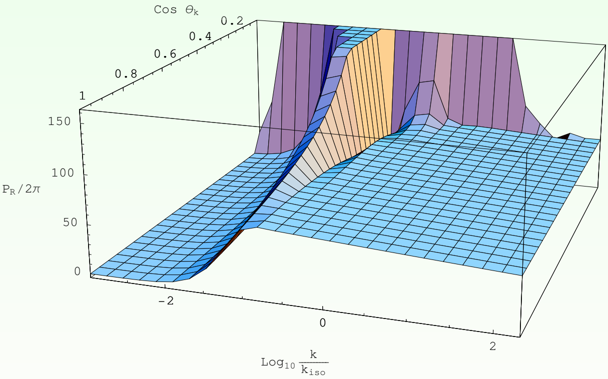
- Also make sure that ωs do not change much initially

$$\frac{\omega'_\times}{\omega_\times^2} \, , \, \frac{\omega'_{11}}{\omega_{11}^2} \, , \, \frac{\omega'_{22}}{\omega_{22}^2} \ll 1$$

- These are satisfied as long as we start the inflation early enough, ie, bigger $H_a - H_b$.
- This way, the initial conditions for the modes are

$$V \, , \, H_{+\,,\times} = \frac{1}{\sqrt{2\,k_\parallel}} \mathrm{e}^{-i\,k_\parallel \int d\eta}$$

Power Spectrum
for Comoving Curvature Perturbations



1. Isotropic at large k
2. Cutoff at small k
3. Oscillations with large increase of power at small $\cos\theta_k$

k_{iso} : Momentum of the mode leaving the horizon when universe becomes isotropic

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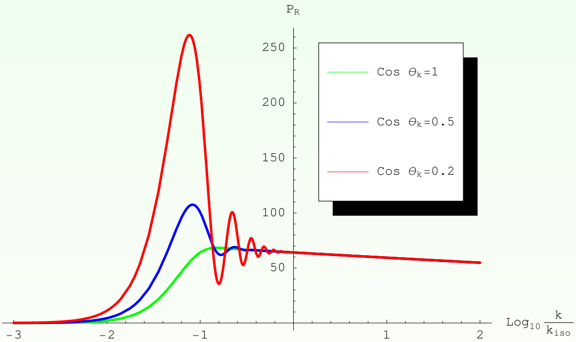


The spectrum diverges as $1/\cos\theta_k$. As a consequence, two-point correlation functions will have log divergence.

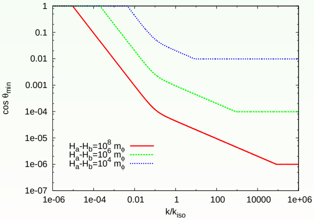
$k_{\rm iso}$: Momentum of the mode leaving the horizon when universe becomes isotropic

Cutoff

- Even for this simplest extension of the standard inflation, the power spectrum diverges . In standard case,

$$|\delta g_{\mathrm{adia}}| = \frac{1}{\sqrt{2\,(k/a)}} = \sqrt{\frac{a}{2k}}$$

  suppressed by the fact that $a \to 0$ as $t \to 0$. In the present case, for $\cos\theta = 0$, k is oriented in the direction where the scale factor is initially constant. This results in a much bigger amplitude in this limit.
- Finite answer possible if one considers full history before anisotropic inflationary stage. This requires a major extension of the model considered here.
- Practical approach: fix initial time (ie, $H_a - H_b$) in the background described here. Keep only modes for which $\omega' < \omega^2$ at this time . The other modes are sensitive to the previous history, which we do not know.



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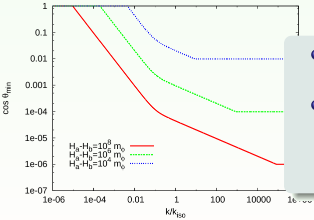

- The divergent integral is: $\int d\cos\theta P_{\mathcal{R}}(k, \cos\theta)$
- Excluded region small in linear scale. Small contribution to the integral as long as the spectrum does not blow up in this limit.

Conclusions-To Do

- We constructed and solved the linear cosmological perturbations in a Bianchi-I background. For small scales ($k > k_{\rm iso}$), the isotropic spectrum with correct spectral index is recovered;
- Consistent extension to the theory of cosmological perturbations beyond exactly isotropic background.
- Phenomenological signature:

$$\langle a_{\ell m}\, a^{\star}_{\ell' m'} \rangle \propto \delta_{mm'} \quad \text{but not} \quad \propto \delta_{\ell\ell'}$$

  where $m = m'$ due to the residual $2d$ isotropy. Comparison with data in progress;
- Negative aspect: stronger sensitivity to initial conditions than in standard inflationary scenarios (need to know the previous history ? Non adiabatic initial vacuum ?)