Identifying Conformal Gauge Theories (CGT) on the lattice

Roman Zwicky (Southampton)

14.5.11, CAQCD Minneapolis

Overview

- ☆ Motivation & introduction conformal window studies

- ☆ Identification on lattice
  - observables in CGT and mass-deformed CGT
  - hyperscaling laws of hadronic observables

Del Debbio & RZ
PRD'10 & PLB'11

- ☆ Brief overview lattice results

- ☆ Backup slides (further topics), some relevant references
  (Incomplete) list of contributers to non-SUSY conformal window

DelDebbio, Picca, Catterral, Sannino, DeGrand, Appelquist, A.Hasenfratz, Frandsen, Rago, Patella, Kuti Tepper, Ryttov, Dietrich, Rebbi, Giedt, Pickup, Keegan, Lucini, Holland, Fodor, Fleming, Neil, Holland, Pallante Svetitsky, Gardi, Grunberg, Miransky, Lombardo, Deuzman, Schroder, Nogradi, Hayakawa, Osaki, Takeda, Uno, Ishikawa, Schneible, Tuominen, Hietanen, Rummukainen, Kerrane, Meurice et al

+ functional RGE + SUSY + AdS/CFT

Scaling dimension $\Delta_{\mathcal{O}} = d_{\mathcal{O}} + \gamma_{\mathcal{O}}$; naive plus anomalous dimension

$\gamma_m = -\gamma_{\bar{q}q}$, denoted by γ_* at fixed-point $\Rightarrow \Delta_{\bar{q}q} = 3 - \gamma_*$

Motivations for studying "gauge theory space"

gauge theories are interesting per se (applications in mathematical physics)

gauge theories are associated with forces in nature (LHC?)

prototype: **Technicolor** *(Susskind/Weinberg '79)*

- Higgs sector ⇨ strongly coupled gauge theory

- Electroweak symmetry breaking due to chiral symmetry breaking - $M_W = g f_\pi^{(TC)}$

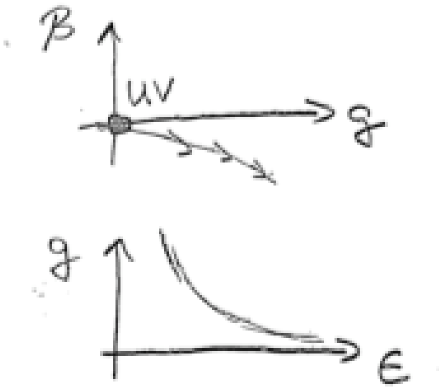
- Phenomenologically (constraints) advantageous are:
  Walking technicolour *(Holdom '81)*
  Conformal technicolour *(Luty, Okui '04)*

What is meant by: "gauge theory space"

- ☆ Adjustable: (1) gauge group e.g. SU(**N_c**)
  (2) **N_f** (massless) fermions
  (3) fermion **representation**
- ☆ Focus on asymptotically free theories (not many representations)

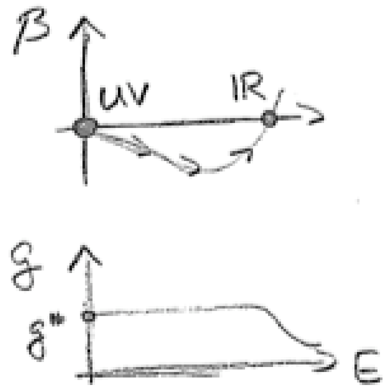
  - well defined on lattice
  - chance for unification in TC

QCD-like



Cartoon

IR-conformal



Distinction of these two cases goal of this talk

A few more remarks

☆ "Bad language": 1) What is **IR-conformal** we called conformal gauge theories
2) Conformal and scale invariant not distinguished here
(Truly conformal gauge theories so far in SUSY e.g. N=4)

☆ **Unitarity bound** CFT from SO(4,2) representation theory:
Scaling dimension Δ_O for a *scalar* *operator O:*

$$\boxed{\Delta_O \geq 1 \qquad \text{(=1 then free field)}}$$

Mack'77

Major goals (not exhaustive)

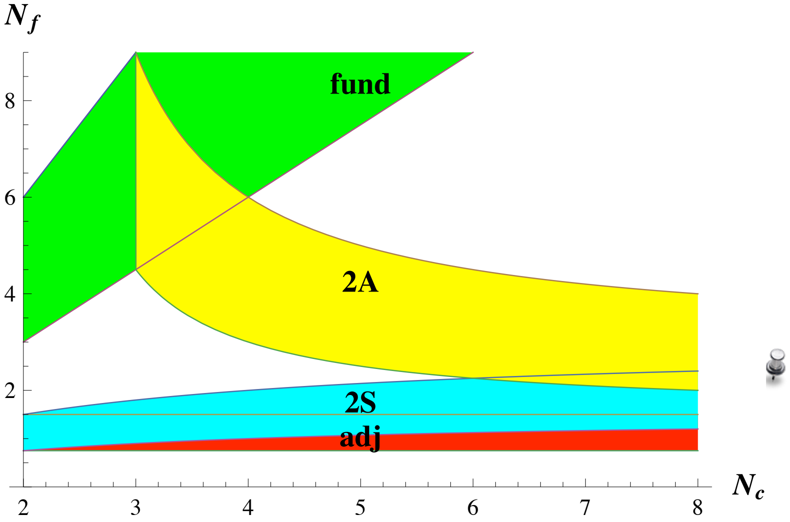
1. How many CGT? size of conformal window
2. Anomalous dimension γ_m^*: $\Delta_{qq} = 3-\gamma_m^* \geq 1$ -- (γ_m^* large favorable for WTC)
3. The electroweak S-parameter -- (has discredited QCD-like TC)

N.B. unitarity bound: $\gamma_m^* \leq 2$

Conformal window (the picture)

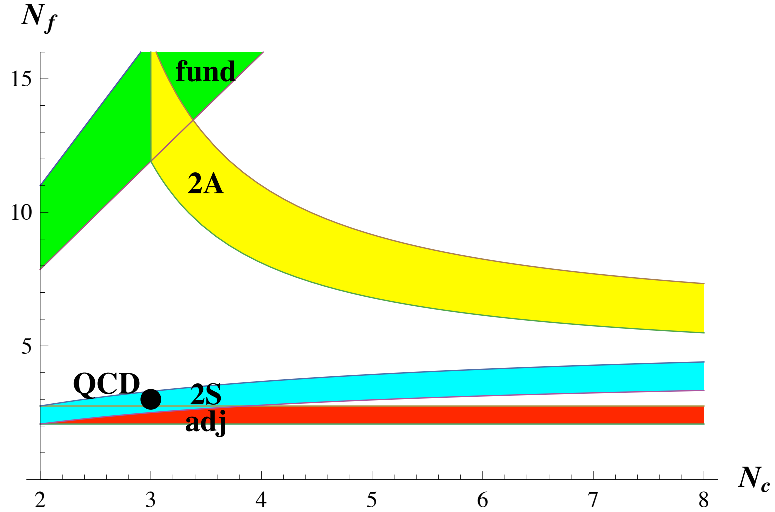
“SU(N)”

N=1 SUSY



- upper line AF

non-SUSY



- just below pert. BZ/BM fixed pt:

weak coupling

$$\beta_0 \text{ tuned small } \frac{\alpha_s^*}{2\pi} = \frac{\beta_0}{-\beta_1} \ll 1$$

- lower line BZ/BM fixed pt “electromagnetic dual”

strong coupling

- lower line Dyson-Schwinger eqs predict chiral symmetry breaking (lattice results later ...)

- assume in between conformal use $\beta_{NSVZ}(\gamma^*)= 0$ to get γ^*

- $\gamma^*|_{strong} = 1$ (unitarity bound QQ state)

- $\gamma^*|_{strong} \approx 1$ DS eqs ladder

Observables in a CFT?

Or how to identify a CFT

1. Observables: vanishing β-function & $\langle O(x)O(0)\rangle \sim (x^2)^{-\Delta}$; $\Delta = d + \gamma_*$
2. Lattice computation finite m_{quark} (& volume) anyway

⇒ look mass-deformed conformal gauge theories (mCGT)*

$$\mathcal{L} = \mathcal{L}_{\mathrm{CGT}} - m\bar{q}q$$

* hardly related to 2D CFT mass deformation a part of algebra and 'therefore' integrability is maintained

Observables in mCGT

- Goal: analytic guidance for lattice (parametric laws)
- finite m_q ; quarks decouple ➪ pure YM confines (string tension confirmed lattice)
  (*Miransky*'s picture '98 resembles heavy quark theory ..later)
- ➪ hadronic spectrum ➪ beloved hadronic observables

signature of such a theory: each hadronic observable

$$\mathcal{O} \sim m^{\eta_{\mathcal{O}}} \,, \qquad \eta_{\mathcal{O}} > 0 \,, \eta = f(\gamma_*)$$

Hyperscaling laws

physical states
no anomalous dim.

Consider matrix element: $\mathcal{O}_{12}(g,\hat{m},\mu) \equiv \langle \varphi_2|\mathcal{O}|\varphi_1\rangle$

RG -transformation*
$\mu = b\mu'$

1. $\mathcal{O}_{12}(g,\hat{m},\mu) = b^{-\gamma_{\mathcal{O}}}\mathcal{O}_{12}(g',\hat{m}',\mu')\ ,$

$$g' = b^{y_g} g \quad \hat{m}' = b^{y_m}\hat{m}\ , \quad y_m = 1+\gamma_*\ , \quad y_g < 0 \ (\text{irrelevant})$$

change
physical units

2. $\mathcal{O}_{12}(\hat{m}',\mu') = b^{-(d_{\mathcal{O}}+d_{\varphi_1}+d_{\varphi_2})}\mathcal{O}_{12}(\hat{m}',\mu)$

3. Choose b s.t. $\hat{m}' = 1$

Hyperscaling
relations

$$\Rightarrow \quad \boxed{\mathcal{O}_{12}(\hat{m},\mu) \sim (\hat{m})^{(\Delta_{\mathcal{O}}+d_{\varphi_1}+d_{\varphi_2})/y_m}}$$

* From Weinberg-like RNG eqs on correlation functions (widely used in critical phenomena)

Applications:

$$\eta_{\mathcal{O}_{12}} = (\Delta_{\mathcal{O}} + d_{\varphi_1} + d_{\varphi_2})/y_m$$

- vacuum condensates: $\langle \bar{q}q \rangle \sim m^{\frac{3-\gamma_*}{1+\gamma_*}}$, $\langle G^2 \rangle \sim m^{\frac{4}{1+\gamma_*}}$

more later on...

- decay constants:
  $|\varphi\rangle = |H(\text{adronic})\rangle$

  N.B. ($\Delta_H = d_H = -1$ choice)

$\mathcal{O}$	def	$\langle 0\vert\mathcal{O}\vert J^{P(C)}(p)\rangle$	$J^{P(C)}$	$\Delta_{\mathcal{O}}$	$\eta_{G[F]}$
S	$\bar{q}q$	G_S	0^{++}	$3-\gamma_*$	$(2-\gamma_*)/y_m$
S^a	$\bar{q}\lambda^a q$	G_{S^a}	0^+	$3-\gamma_*$	$(2-\gamma_*)/y_m$
P^a	$\bar{q}i\gamma_5 q$	G_{P^a}	0^-	$3-\gamma_*$	$(2-\gamma_*)/y_m$
V	$\bar{q}\gamma_\mu q$	$\epsilon_\mu(p)M_V F_V$	1^{--}	3	$1/y_m$
V^a	$\bar{q}\gamma_\mu\lambda^a q$	$\epsilon_\mu(p)M_V F_{V^a}$	1^-	3	$1/y_m$
A^a	$\bar{q}\gamma_\mu\gamma_5\lambda^a q$	$\epsilon_\mu(p)M_A F_{A^a}$	1^+	3	$1/y_m$
		$ip_\mu F_{P^a}$	0^-	3	$1/y_m$

- masses from **trace anomaly**:

  Adler et al, Collins et al
  N.Nielsen '77 Fujikawa '81

$$\theta_\alpha{}^\alpha|_{\text{on-shell}} = \tfrac{1}{2}\beta G^2 + N_f m(1+\gamma_m)\bar{q}q$$

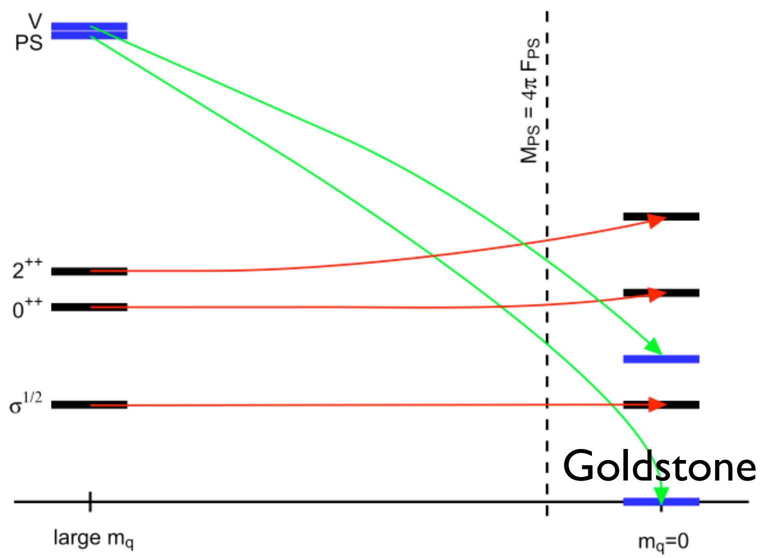
$$\beta = 0 \quad \& \quad \langle H(p)|H(k)\rangle = 2E_p\delta^{(3)}(p-k) \Rightarrow$$

$$\boxed{2M_h^2 = N_f(1+\gamma_*)m\langle H|\bar{q}q|H\rangle} \quad \sim m^{\frac{2}{(1+\gamma_*)}}$$

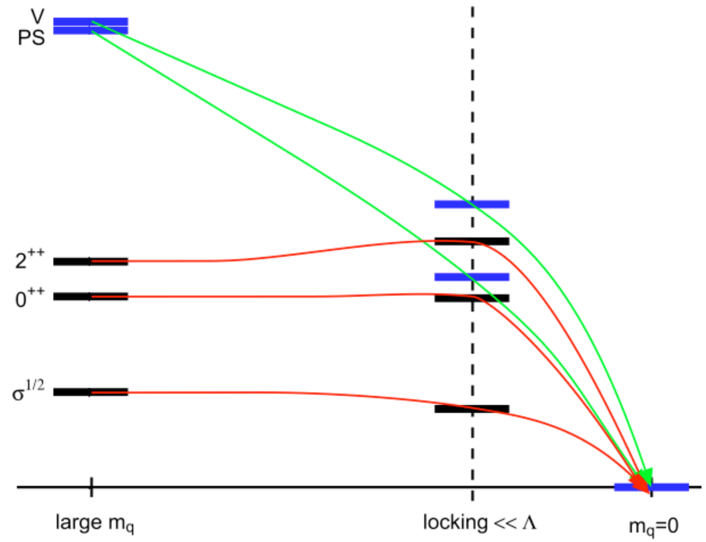
relation reminiscent GMOR-relation

Let's pause: comparison with QCD-like spectrum

QCD-like (chiral symmetry broken)



mCGT spectrum



- No SSB of χ-symmetry breaking (no goldstone boson) since condensate triggered by explicit χ-breaking $L = m q_L q_R$ +h.c.

⇨ **no chiral perturbation theory** (no parametric suppression of the "pion mass")

- A point that can be clarified: $M_H \sim m^{1/(1+\gamma^*)}$ looks a bit like heavy quark physics
  Settled by observing: $f_{P(\text{B-meson})} \sim m^{-1/2}$ whereas $f_{P(\text{mCGT})} \sim m^{(2-\gamma^*)/(1+\gamma^*)}$

Mass scaling without RG

Del Debbio, RZ Sep'10

Hellmann-Feynman-Thm

$$\boxed{\frac{\partial E_\lambda}{\partial \lambda} = \langle \psi(\lambda) | \frac{\partial \hat{H}(\lambda)}{\partial \lambda} | \psi(\lambda) \rangle}$$

idea: $\frac{\partial \langle \psi(\lambda) | \psi(\lambda) \rangle}{\partial \lambda} = 0$

- applied to our case: $$m \frac{\partial M_h^2}{\partial m} = N_f m \langle H | \bar{q} q | H \rangle$$

- combined with GMOR-like .. $$m \frac{\partial M_H}{\partial m} = \frac{1}{1+\gamma_*} M_H$$

  $2M_H^2 = N_f (1+\gamma_*) m \langle H | \bar{q} q | H \rangle$

$$\boxed{M_H \sim m^{\frac{1}{1+\gamma_*}}}$$

scaling law
without using RG!

Generalized Banks-Casher relation

- Banks & Casher '80 a la Leutwyler & Smilga 92':

Green's function: $\langle q(x)\bar{q}(y)\rangle = \sum_n \frac{u_n(x)u_n^\dagger(y)}{m-i\lambda_n}$, where $\not{D}u_n = \lambda_n u_n$

$$\langle \bar{q}q\rangle_V = \frac{1}{V}\int dx\,\langle \bar{q}(x)q(x)\rangle = -\frac{2m}{V}\sum_{\lambda_n>0}\frac{1}{m^2+\lambda_n^2} \overset{V\to\infty}{=} -2m\int_0^\infty d\lambda\frac{\rho(\lambda)}{m^2+\lambda^2}$$

- UV-divergences later -- focus IR-physics

$$\langle \bar{q}q\rangle \overset{m\to 0}{\sim} m^{\eta_{\bar{q}q}} \quad \Leftrightarrow \quad \rho(\lambda) \overset{\lambda\to 0}{\sim} \lambda^{\eta_{\bar{q}q}}$$

- QCD : $\eta_{\bar{q}q} = 0 \Rightarrow \rho(0) = -\pi\langle \bar{q}q\rangle$

  mCGT: another way to measure anomalous dimension

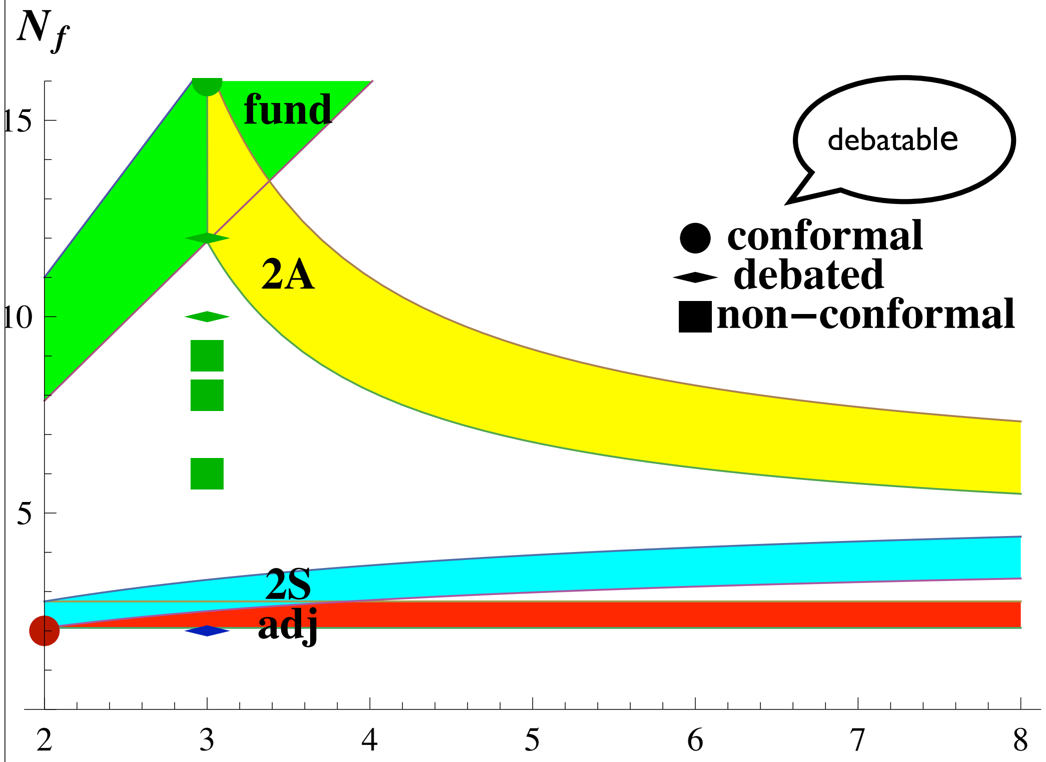
Banks, Casher'80

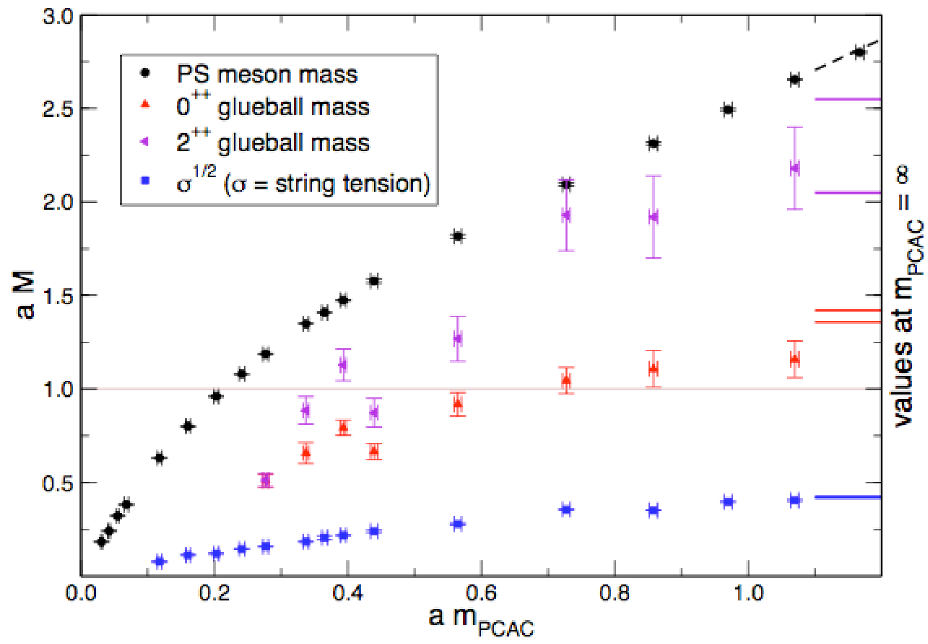
DeGrand'09

DelDebbio RZ'10 May

Lattice results

control systematics $a \to 0$, $V \to \infty$ and scaling m
$\Rightarrow$ challenging





Del Debbio et al SU(2) adj

Methods:

- spectral studies (scaling laws)
- β-fct through stepsize scaling
- enhancement of <qq>/f_π^3 for WTC (no parametric control)

- SU(3) fund. (code exists) — (trend towards parity doubling for $N_F = 6$)
- SU(2) adj=2S MWTC — consensus $0.05 < \gamma_* < 0.56$ glueballs lighter mesons
- SU(3) 2S (hextet) NMWTC — maybe conformal $\gamma_* < 0.6$

Epilogue

- People have accepted that identification of conformal window is more difficult than anticipated (need more CPU-time)

- Some tools e.g. hyperscaling laws are there but more would be better in order to settle debates
  "The more cross checks the more robust numerical results become"

- Conformal gauge theories have been identified -- qualitatively confirm conformal window plots from DS-eqns

- Anomalous mass dimension significantly below $\gamma^* < 1$
  (unitarity bound of $\gamma^* < 2$ not nearly reached so far!)

- Numerical results for S-parameter are eagerly awaited.
  First study has been reported by DeGrand'10

Thanks for your attention!

Backup slides …

Some relevant/useful references

- Miransky hep-ph/9812350 spectrum (with mass) as signal of conformal window works with pole mass -- weak coupling regime $\Lambda_{YM} \cong$ m Exp[-1/b_{YM} α^*]
  ➯ glueballs lighter than mesons

- Luty Okui JHEP'96 conformal technicolor propose spectrum as signal of cw

- Dietrich/Sannino PRD'07 conformal window SU(N) higher representation using Dyson-Schwinger techniques known from WTC

- Sannino/RZ PRD'08 <qq> done heuristically IR and UV effects understood 0905

- DelDebbio et al ArXiv 0907 Mass lowest state from RGE equation

- DeGrand scaling <qq> stated ArXiv 0910

- DelDebbio RZ ArXiv 0905 scaling of vacuum condensates, all lowest lying states

- DelDebbio RZ ArXiv 0909 scaling extended to entire spectrum and all local matrix elements

Del Debbio, RZ Sep'10

remarks S-parameter

Analytical guidance S-parameter: $S = 4\pi\Pi_{V-A}(0)$ − pion pole

$$(q^\mu q^\nu - q^2 g^{\mu\nu})\delta_{ab}\Pi_{V-A}(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T\left(V_a^\mu(x)V_b^\nu(0) - (V\leftrightarrow A)\right)|0\rangle$$

$$\Pi_{V-A}(q^2) \simeq \frac{f_V^2}{m_V^2 - q^2} - \frac{f_A^2}{m_A^2 - q^2} - \frac{f_P^2}{m_P^2 - q^2} + \dots .$$

(conspiracy) cancellations
modulo
improve on ...

$$\Pi_{V-A}^{\mathrm{W-TC}}(0) \sim O(m^{-1})$$

$$\Pi_{V-A}^{\mathrm{mCGT}}(0) \sim O(m^{0})$$

$$\Pi_{V-A}^{\mathrm{mCGT}}(q^2) \sim \frac{m^{2/y_m}}{q^2}$$

for $-q^2 \gg (\Lambda_U)^2$

Sannino'10 free theory

⇒ lattice determination coming soon (already some market)

Another look at the β-function

- Consider the again the trace (scale) anomaly:

$$\theta_\alpha{}^\alpha|_{\rm on-shell} = \frac{1}{2g}\beta G^2 + N_f m(1+\gamma_m)\bar{q}q$$

- Evaluate it on any hadronic state |H> and solve for β:

$$\boxed{\beta = \frac{A_H + \gamma_m B_H}{G_H}}$$

$$A_H = 2M_H^2 - mN_f\langle H|\bar{q}q|H\rangle\,,$$
$$B_H = mN_f\langle H|\bar{q}q|H\rangle\,,$$
$$G_H = \langle H|G^2|H\rangle\,.$$

  - Ratios of A_H/G_H & B_H/G_H independent

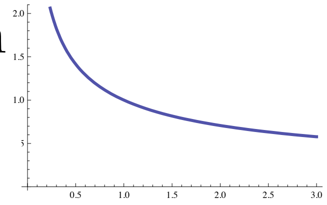
  - Form β-function close to NSVZ β (for N=1 SUSY gauge theories)

$$\boxed{\beta_{NSVZ} = -\frac{1}{16\pi^2}\frac{3T_G - T_R(1+\gamma)}{1 - T_G/(8\pi^2)}}$$

Heuristic look

- Deconstruct the continuous spectrum of a two point function — Stephanov'07

  Infinite sum of adjusted particles can mimick continuous spectrum



$$\bar{q}q(x) \sim \sum_n f_n \varphi_n(x)\,; \qquad \langle \varphi_n | \bar{q}q | 0 \rangle \sim f_n\,, \qquad \begin{cases} f_n^2 = \delta^2 \left(M_n^2\right)^{\Delta_{\bar{q}q}-2} \\ M_n^2 = n\delta^2 \end{cases}$$

- Adding mass term looks like tadpole.

  ⇒ find new minimum -- add M_n to potential

$$\mathcal{L} = -m \sum_n f_n \varphi_n - 1/2 \sum_n M_n^2 \varphi_n^2$$

Delgado, Espinoso, Quiros'07

- Solve $\quad m f_n + M_n^2 \varphi_n = 0 \quad \Rightarrow \quad \langle \varphi_n \rangle = -m f_n / M_n^2$ and reinsert:

$$\langle \bar{q}q \rangle \sim \sum_n f_n \langle \varphi_n \rangle = -m \sum_n \frac{f_n^2}{M_n^2} \overset{\delta \to 0}{\to} -m \int_{\Lambda_{\mathrm{IR}}^2}^{\Lambda_{\mathrm{UV}}^2} s^{\Delta_{\bar{q}q}-3} ds$$

  - Λ_{UV}: $\Delta_{qq} = 3$ find quadratic divergence known from Leutwyler-Smilga rep.
  - Λ_{IR}: 1) $\Lambda_{IR} \sim M_H \sim m^{1/(1+\gamma)}$ or use $(M_{dyn})^{\Delta qq} \sim$ <qq> generalizing Politzer OPE. and confirm $\eta_{qq} = \Delta_{qq}/(1+\gamma)$!

Mass & decay constant trajectory

- At large-N_c neglect width $\qquad g_{H_n} \equiv \langle 0|\mathcal{O}|H_n\rangle$ (decay constant)

$$\Delta(q^2) \sim \int_x e^{ixq}\langle 0|\mathcal{O}(x)\mathcal{O}(0)|0\rangle = \sum_n \frac{|g_{H_n}|^2}{q^2+M^2_{H_n}}$$

- In limit m → 0 (scale invariant correlator)

$$\Delta(q^2) = \int_0^\infty \frac{ds\, s^{1-\gamma_*}}{q^2+s} + \text{s.t} \propto (q^2)^{1-\gamma_*}$$

- Solution are given by:

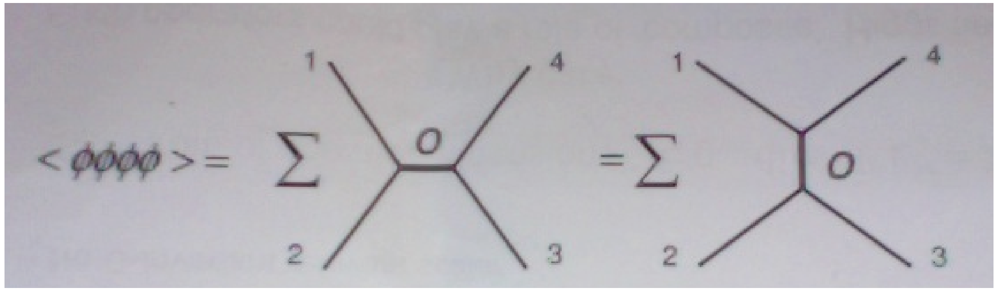
$$\boxed{M^2_{H_n} \sim \alpha_n m^{\frac{2}{1+\gamma_*}}\ , \quad g^2_{H_n} \sim \alpha'_n(\alpha_n)^{1-\gamma_*} m^{\frac{2(2-\gamma_*)}{1+\gamma_*}}}$$

where α_n arbitrary function (corresponds freedom change of variables in ∫)

- QCD expect $\alpha_n \sim n$ (linear radial Regge-trajectory) (few more words)
- For those who know: resembles deconstruction Stephanov'07
  difference physical interpretation of spacing due to scaling spectrum

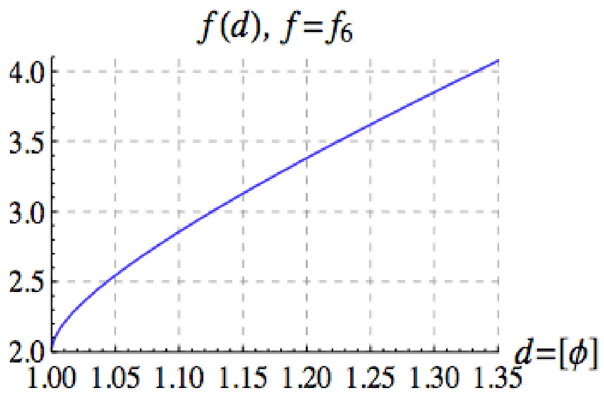
Addendum (bounds scaling dimension)

- assume add L = mqq (N.B. not a scalar under global flavour symmetry!)
- using bootstrap ('associative' OPE on 4pt function) possible to obtain <u>upper</u>-bound on scaling dimension Δ of lowest operator in OPE



non-singlet

Rattazzi, Rychkov Tonni & Vichi'08



1.35 still rather close to unitarity bound

singlet $\Delta \leq 4$

allows for Δ_{qq} to be:

Rattazzi, Rychkov & Vichi '10

G	$U(1) \equiv SO(2)$	$SO(3)$	$SO(4)$	$SU(2)$	$SU(3)$
d_*	1.063 ($k=2$) 1.12 ($k=4$)	1.032 ($k=2$) 1.08 ($k=4$)	1.017 ($k=2$) 1.06 ($k=4$)	1.016 ($k=2$)	1.003 ($k=2$)

very close to unitarity bound!

good news for Luty's conformal TC