Holography and colliding gravitational shock waves

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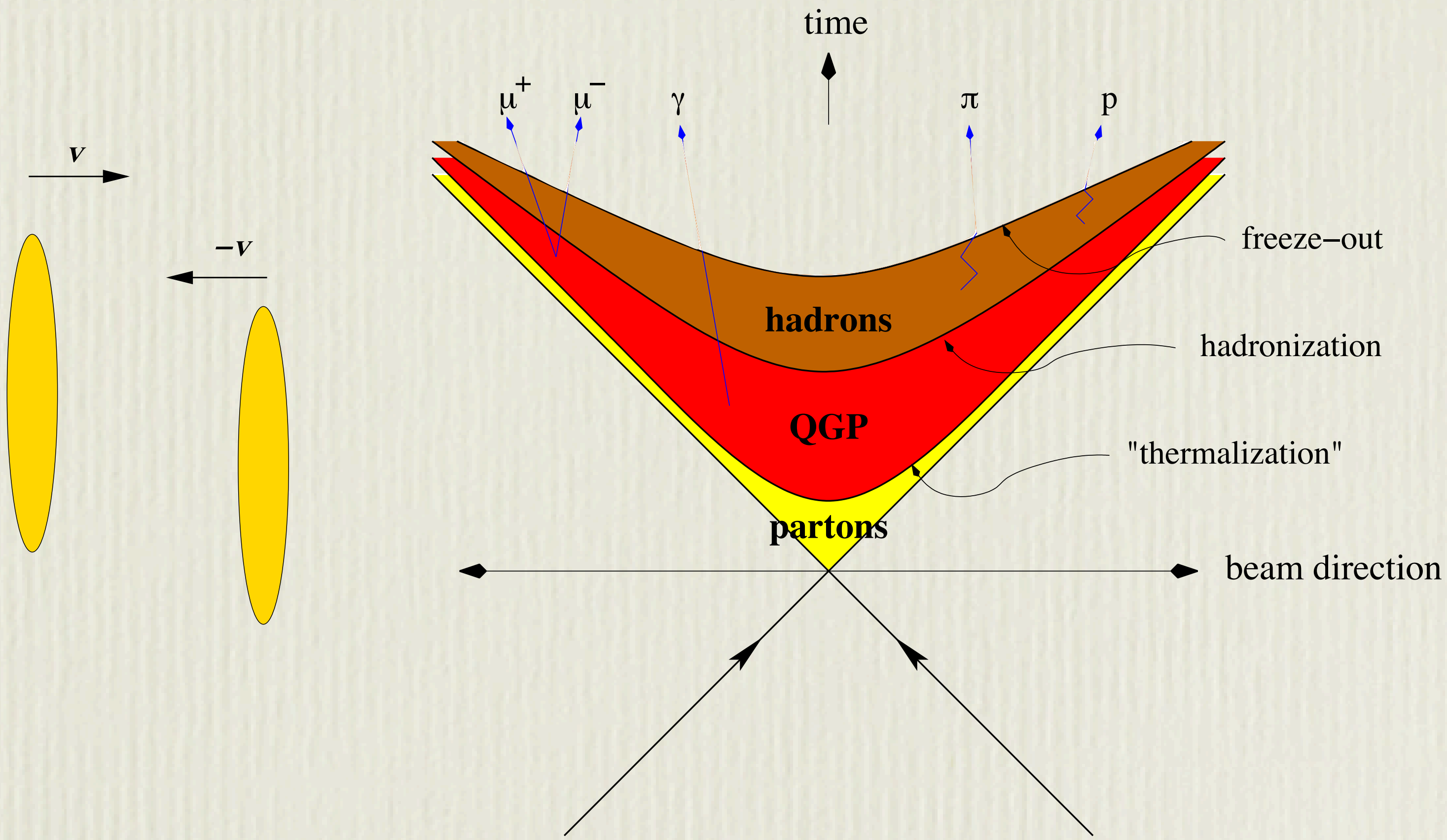
Colliding heavy ions / colliding shocks



Idealize:

Large, highly Lorentz contracted nuclei ➡ infinite planar null shocks

$SU(3)$ gauge field + fundamental quarks ➡ $SU(N_c)$ gauge field + adjoint matter

strongly coupled QGP ➡ strongly coupled, large-N_c maximally supersymmetric Yang-Mills ($\mathcal{N}$=4 SYM)

➡ Study colliding shocks in $\mathcal{N}$=4 SYM using gauge/gravity duality

Gauge/gravity duality

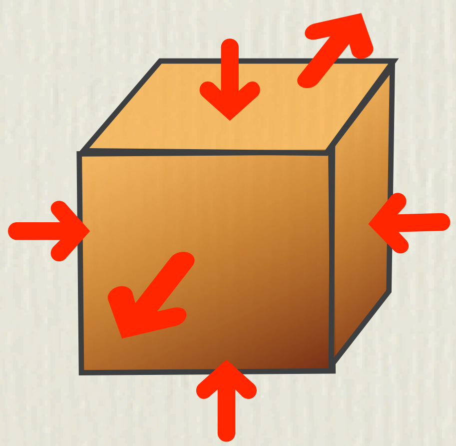
$\mathcal{N}$=4 Super-Yang-Mills = string theory on $AdS_5 \times S^5$

large N_c, $\lambda \equiv g^2 N_c \gg 1$: string theory ➡ classical (super)gravity

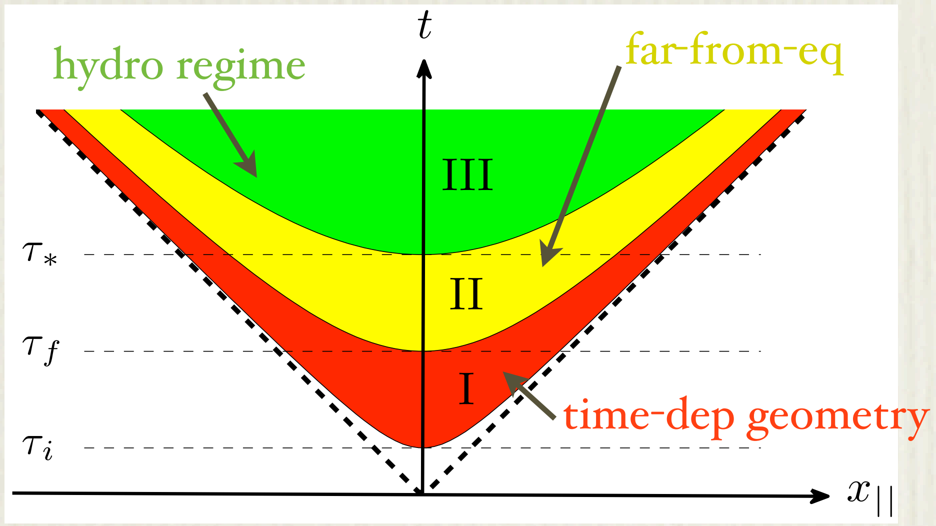
Non-equilibrium dynamics ➡ 5D gravitational initial value problem

Previous work: 3D spatial homogeneity ➡ reduction to 1+1D PDEs

Isotropization



Boost invariant expansion



Colliding planar shocks: transverse homogeneity ➡ reduction to 2+1D PDEs

Colliding shock geometry

- Metric ansatz: $ds^2 = -A\,dv^2 + \Sigma^2\left[e^{B} d\boldsymbol{x}_\perp^2 + e^{-2B} dz^2\right] + 2dv\,(dr + F dz)$

  time coord. collision axis 5D radial coord.

  4 unknown functions of (v,r,z)

  $v =$ const.: null hypersurface

  $r =$ affine parameter along infalling null geodesics

  $r = \infty$: holographic boundary

- Residual diffeomorphism freedom: $r \to r + \xi(v,z)$

- Boundary asymptotics: $A = r^2\left[1 + \frac{2\xi}{r} + \frac{\xi^2 - 2\partial_v \xi}{r^2} + \frac{a_4}{r^4} + O(r^{-5})\right],$

$$B = \frac{b_4}{r^4} + O(r^{-5})\,, \quad \Sigma = r + \xi + O(r^{-7})\,, \quad F = \partial_z \xi + \frac{f_2}{r^2} + O(r^{-3})$$

- Holographic mapping: $\mathcal{E} \equiv \frac{2\pi^2}{N_c^2}\,T^{00} = -\frac{3}{4}a_4\,, \quad \mathcal{P}_\parallel \equiv \frac{2\pi^2}{N_c^2}\,T^{zz} = -\frac{1}{4}a_4 - 2b_4\,,$

$$\mathcal{S} \equiv \frac{2\pi^2}{N_c^2}\,T^{0z} = -f_2\,, \quad \mathcal{P}_\perp \equiv \frac{2\pi^2}{N_c^2}\,T^{\perp\perp} = -\frac{1}{4}a_4 + b_4$$

Einstein equations

Einstein equations

$$0 = \Sigma'' + (B')^2\,\Sigma \tag{1}$$

$$0 = \Sigma^2\left[F'' - 2(d_3B)' - 3B'd_3B\right] + 4\Sigma' d_3\Sigma - \Sigma\left[3\Sigma' F' + 4(d_3\Sigma)' + 6B'd_3\Sigma\right], \tag{2}$$

$$\begin{aligned}0 = {} & 6\Sigma^3(d_+\Sigma)' + 12\Sigma^2(\Sigma' d_+\Sigma - \Sigma^2) - e^{2B}\big\{2(d_3\Sigma)^2 \\ & + \Sigma^2\left[\tfrac{1}{2}(F')^2 + (d_3F)' + 2F'd_3B - \tfrac{7}{2}(d_3B)^2 - 2d_3^2B\right] + \Sigma\left[(F' - 8d_3B)\,d_3\Sigma - 4d_3^2\Sigma\right]\big\}.\end{aligned} \tag{3}$$

$$\begin{aligned}0 = {} & 6\Sigma^4(d_+B)' + 9\Sigma^3(\Sigma' d_+B + B'd_+\Sigma) + e^{2B}\big\{\Sigma^2[(F')^2 + 2(d_3F)' + F'd_3B - (d_3B)^2 - d_3^2B] \\ & + 4(d_3\Sigma)^2 - \Sigma\left[(4F' + d_3B)\,d_3\Sigma + 2d_3^2\Sigma\right]\big\},\end{aligned} \tag{4}$$

$$\begin{aligned}0 = {} & \Sigma^4\left[A'' + 3B'd_+B + 4\right] - 12\Sigma^2\Sigma' d_+\Sigma \\ & + e^{2B}\big\{\Sigma^2\left[(F')^2 - \tfrac{7}{2}(d_3B)^2 - 2d_3^2B\right] + 2(d_3\Sigma)^2 - 4\Sigma\left[2(d_3B)d_3\Sigma + d_3^2\Sigma\right]\big\},\end{aligned} \tag{5}$$

$$\begin{aligned}0 = {} & 6\Sigma^2 d_+^2\Sigma - 3\Sigma^2 A'd_+\Sigma + 3\Sigma^3(d_+B)^2 - e^{2B}\big\{(d_3\Sigma + 2\Sigma d_3B)(2d_+F + d_3A) \\ & + \Sigma\left[2d_3(d_+F) + d_3^2A\right]\big\},\end{aligned}$$

$$\begin{aligned}0 = {} & \Sigma\left[2d_+(d_3\Sigma) + 2d_3(d_+\Sigma) + 3F'd_+\Sigma\right] + \Sigma^2\left[d_+(F') + d_3(A') + 4d_3(d_+B) - 2d_+(d_3B)\right] \\ & + 3\Sigma\left(\Sigma d_3B + 2d_3\Sigma\right)d_+B - 4(d_3\Sigma)d_+\Sigma\end{aligned}$$

$$h' \equiv \partial_r h\,, \quad d_+h \equiv \partial_v h + A\,\partial_r h\,, \quad d_3h \equiv \partial_z h - F\,\partial_r h$$

Einstein equations

$$
\begin{aligned}
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&\quad \left. + \Sigma^2\left[\tfrac{1}{2}(F')^2 + (d_3F)' + 2F'd_3B - \tfrac{7}{2}(d_3B)^2 - 2d_3^2B\right] + \Sigma\left[(F' - 8d_3B)\,d_3\Sigma - 4d_3^2\Sigma\right]\right\}. && (3)\\
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&\quad \left. + 4(d_3\Sigma)^2 - \Sigma\left[(4F' + d_3B)\,d_3\Sigma + 2d_3^2\Sigma\right]\right\}, && (4)\\
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&\quad + 3\Sigma\left(\Sigma d_3B + 2d_3\Sigma\right)d_+B - 4(d_3\Sigma)d_+\Sigma &&
\end{aligned}
$$

$$h' \equiv \partial_r h\,, \quad d_+h \equiv \partial_v h + A\,\partial_r h\,, \quad d_3 h \equiv \partial_z h - F\,\partial_r h$$

Nested linear radial ODEs! ➡ simple time evolution procedure:

Given $B(v,z,r)$ at time v_0: (1) → Σ, (2) → F, (3) → $d_+\Sigma$, (4) → d_+B, (5) → A ➡ $\partial_v B$ ➡ $B(v_0+\delta v,z,r)$

Initial conditions

- Single shock: analytic solution in Fefferman-Graham coordinates

$$ds^2 = r^2[-dx_+dx_- + d\boldsymbol{x}_\perp^2] + \frac{1}{r^2}\,[dr^2 + h(x_\pm)\,dx_\pm^2]$$

Janik & Peschanski

- Choose Gaussian profile with width w, surface energy density μ^3:

$$h(x_\pm) \equiv \mu^3\,(2\pi w^2)^{-1/2}\,e^{-\frac{1}{2}x_\pm^2/w^2}$$

- Single shock, our coordinates: must solve for diffeomorphism numerically

- Superpose single shocks to generate incoming two-shock initial data

Numerical issues

- Computational domain:

  Impose periodic boundary conditions in z

  Excise geometry inside apparent horizon, $r < r_{\mathrm{h}}(v,z)$

- Residual diffeomorphism freedom:

  Fix apparent horizon at $r = r_{\mathrm{h}} = \text{const.}$ ➡ $0 = 3\Sigma^2\, d_+\Sigma - \partial_z(F\,\Sigma\, e^{2B}) + \frac{3}{2}F^2\,\Sigma' e^{2B}$

- Singular point at $r = \infty$:

  Use (pseudo)spectral methods: Fourier (z) & Chebyshev (r) basis expansion

- Precision loss due to very rapid growth of A, F deep in bulk:

  Add small background energy density ➡ $a_4 \to a_4 - \delta$

- Short wavelength instabilities induced by discretization:

  Introduce tiny numerical viscosity

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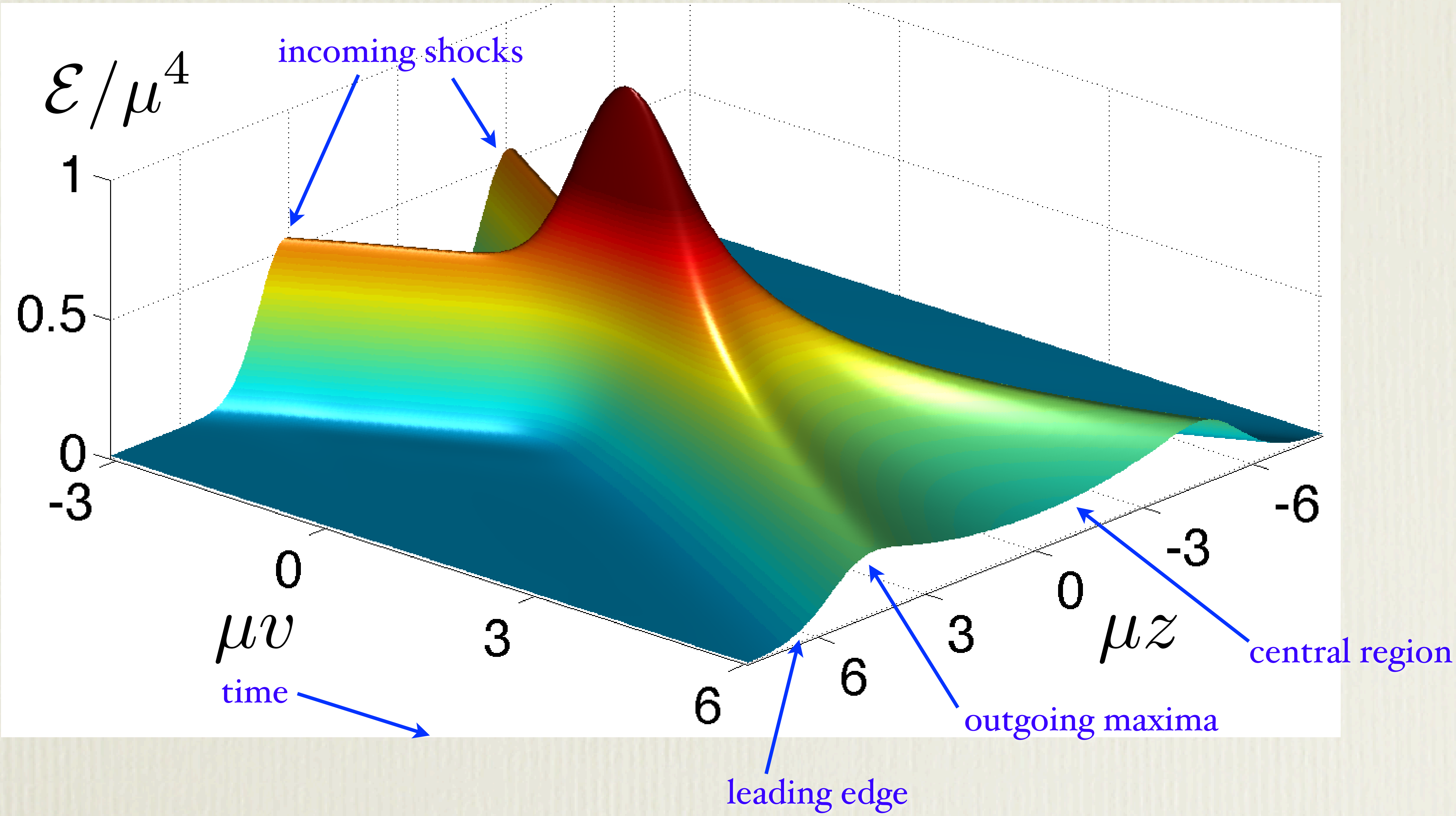
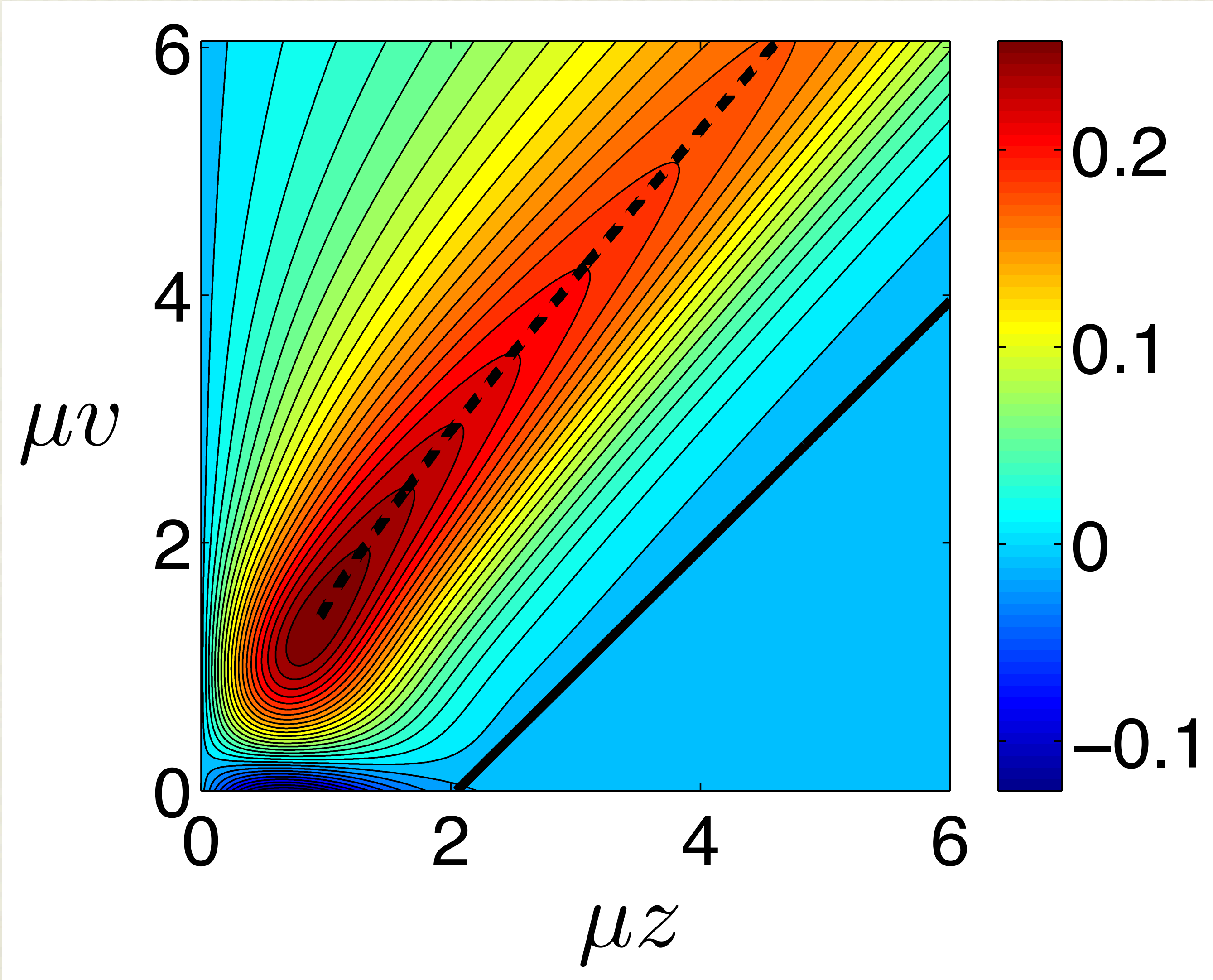
Can achieve stable evolution

Results: energy density



$w = 0.75/\mu,\ \delta = 0.014\ \mu^4$

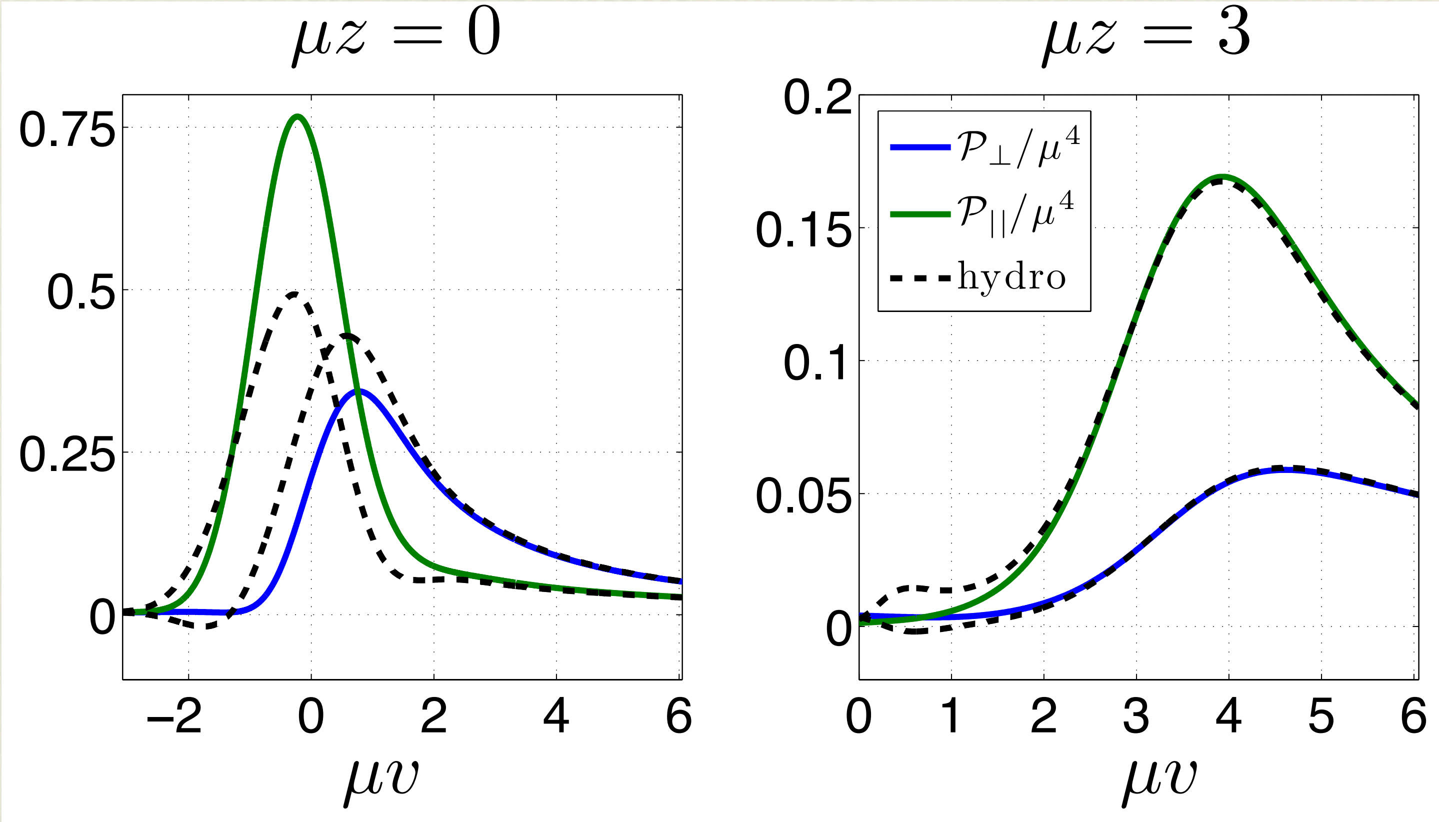
Results: outgoing energy flux



- Outgoing maxima move at speed $v \approx 0.86$ c

Not an artifact of background energy density

Results: pressures & hydro validity



- Early times: large anisotropy, far from local equilibrium
- Late times: accurate agreement with hydro constitutive relations
- Central region: onset of hydro validity ≈ 4/μ after initial interaction

  μ ≈ 2.3 GeV for modeling RHIC ➡ $\tau_{hydro} \approx 0.35$ fm/c

- Near outgoing maxima & leading edges: fortuitous agreement with 1st order hydro: big difference between 1st and 2nd order hydro

Future projects:

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- Asymmetric shocks
- Shocks with non-zero charge density (Einstein-Maxwell)
- Shocks in non-conformal theories with dual description
- Shocks with finite transverse extent (3+1 PDEs)

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Plenty of opportunities for more people!