

# Light-Front Holographic QCD: an Overview

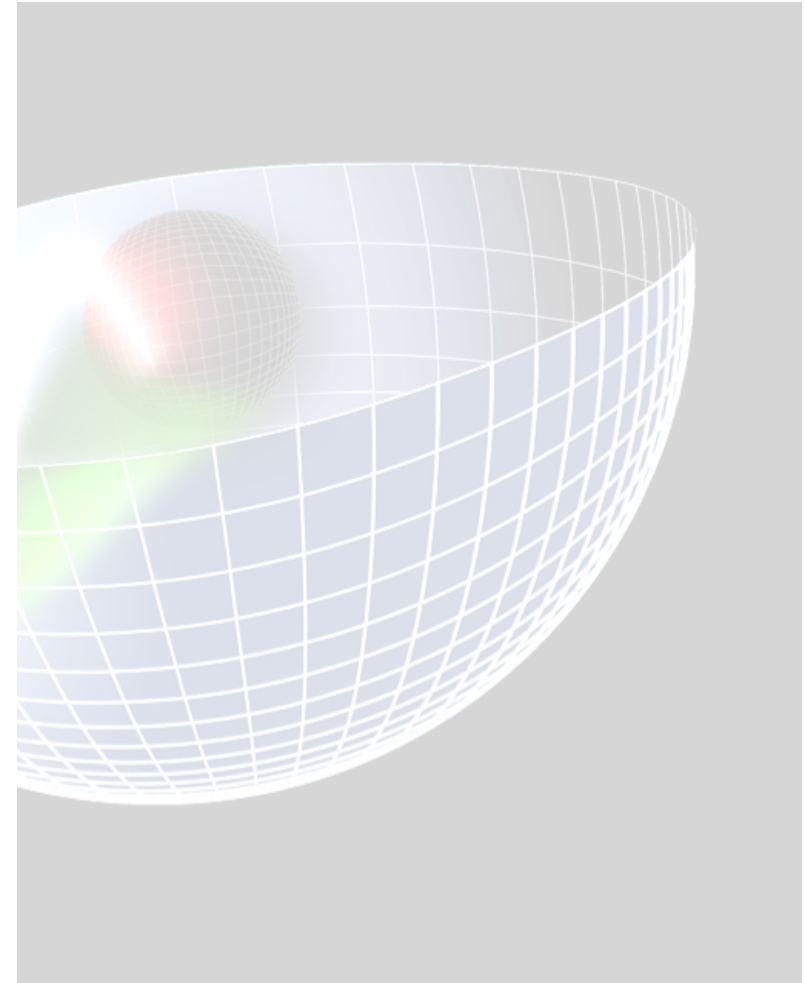
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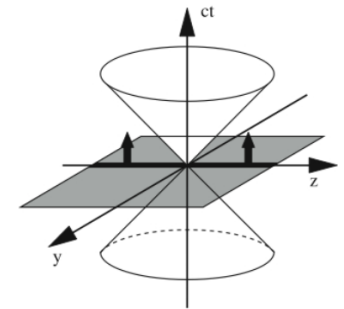
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# 1 Light Front Dynamics

- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different “times” and has its own Hamiltonian, but should give the same physical results
- *Instant form*: hypersurface defined by  $t = 0$ , the familiar one

- *Front form*: hypersurface is tangent to the light cone at  $\tau = t + z/c = 0$



$$x^+ = x^0 + x^3 \quad \text{light-front time}$$

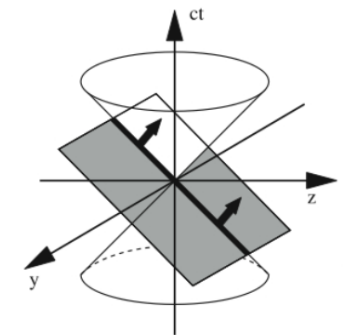
$$x^- = x^0 - x^3 \quad \text{longitudinal space variable}$$

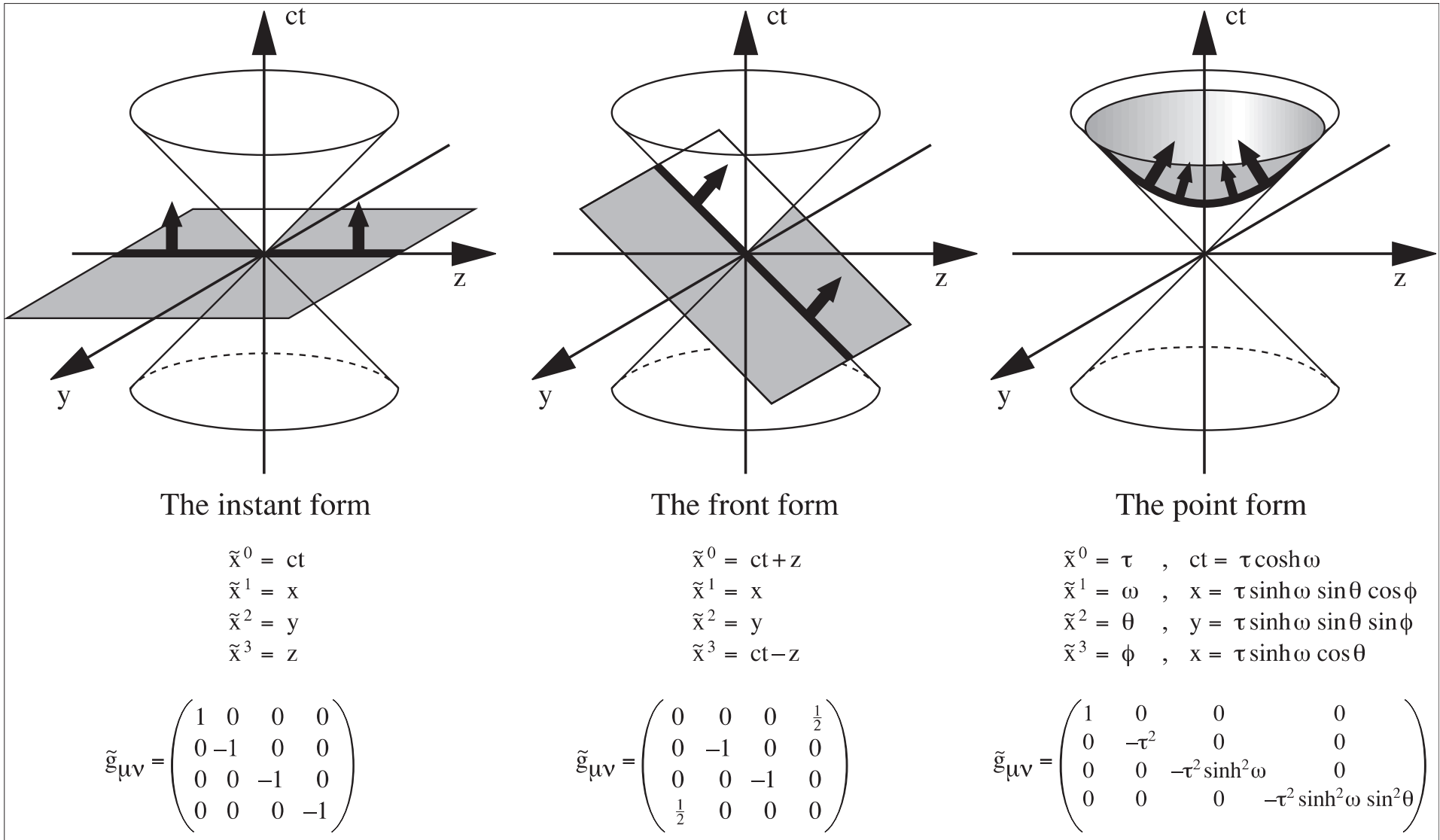
$$k^+ = k^0 + k^3 \quad \text{longitudinal momentum} \quad (k^+ > 0)$$

$$k^- = k^0 - k^3 \quad \text{light-front energy}$$

$$k \cdot x = \frac{1}{2} (k^+ x^- + k^- x^+) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$$

On shell relation  $k^2 = m^2$  leads to dispersion relation  $k^- = \frac{\mathbf{k}_\perp^2 + m^2}{k^+}$





- Forms of Relativistic Dynamics: dynamical vs. kinematical generators [Dirac (1949)]

- Instant form

$H, \mathbf{K}$  dynamical

$\mathbf{L}, \mathbf{P}$  kinematical

- Point form

$P^\mu$  dynamical

$M^{\mu\nu}$  kinematical

- Front form

$P^-, L^x, L^y$  dynamical

$P^+, \mathbf{P}_\perp, L^z, \mathbf{K}$  kinematical

- LF quantization is the ideal framework to describe hadronic structure in terms of quarks and gluons: simple vacuum structure allows unambiguous definition of partonic content of a hadron
- Calculation of a matrix elements  $\langle P + q | J | P \rangle$  requires boosting the hadronic bound state from  $|P\rangle$  to  $|P + q\rangle$ : extremely complicated in the instant form, whereas  $\mathbf{K}$  is trivial in the LF
- Invariant Hamiltonian equation for bound states similar structure of AdS equations of motion: direct connection of QCD and AdS/CFT possible

- QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4g^2} \text{Tr} (G^{\mu\nu} G_{\mu\nu}) + i\bar{\psi} D_\mu \gamma^\mu \psi + m\bar{\psi}\psi$$

- LF Momentum Generators  $P = (P^+, P^-, \mathbf{P}_\perp)$  in terms of dynamical fields  $\psi, \mathbf{A}_\perp$

$$P^- = \frac{1}{2} \int dx^- d^2 \mathbf{x}_\perp \bar{\psi} \gamma^+ \frac{(i\nabla_\perp)^2 + m^2}{i\partial^+} \psi + \text{interactions}$$

$$P^+ = \int dx^- d^2 \mathbf{x}_\perp \bar{\psi} \gamma^+ i\partial^+ \psi$$

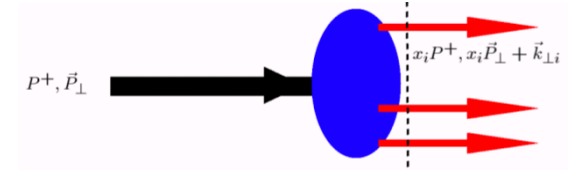
$$\mathbf{P}_\perp = \frac{1}{2} \int dx^- d^2 \mathbf{x}_\perp \bar{\psi} \gamma^+ i\nabla_\perp \psi$$

- LF Hamiltonian  $P^-$  generates LF time translations

$$[\psi(x), P^-] = i \frac{\partial}{\partial x^+} \psi(x)$$

and the generators  $P^+$  and  $\mathbf{P}_\perp$  are kinematical

## Light-Front Fock Representation



- Dirac field  $\psi$ , expanded in terms of ladder operators on the initial surface

$$P^- = \sum_{\lambda} \int \frac{dq^+ d^2 \mathbf{q}_{\perp}}{(2\pi)^3} \left( \frac{\mathbf{q}_{\perp}^2 + m^2}{q^+} \right) b_{\lambda}^{\dagger}(q) b_{\lambda}(q) + \text{interactions}$$

- Construct LF Lorentz invariant Hamiltonian equation for the relativistic bound state

$$P_{\mu} P^{\mu} |\psi(P)\rangle = (P^- P^+ - \mathbf{P}_{\perp}^2) |\psi(P)\rangle = \mathcal{M}^2 |\psi(P)\rangle$$

- State  $|\psi(P)\rangle$  is expanded in multi-particle Fock states  $|n\rangle$  of the free LF Hamiltonian

$$|\psi\rangle = \sum_n \psi_n |n\rangle, \quad |n\rangle = \{ |uud\rangle, |uudg\rangle, |uud\bar{q}q\rangle, \dots \}$$

with  $k_i^2 = m_i^2$ ,  $k_i = (k_i^+, k_i^-, \mathbf{k}_{\perp i})$ , for each constituent  $i$  in state  $n$

- Fock components  $\psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i^z)$  independent of  $P^+$  and  $\mathbf{P}_{\perp}$  and depend only on relative partonic coordinates: momentum fraction  $x_i = k_i^+ / P^+$ , transverse momentum  $\mathbf{k}_{\perp i}$  and spin  $\lambda_i^z$

$$\sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n \mathbf{k}_{\perp i} = 0.$$

## Semiclassical Approximation to QCD in the Light Front

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Compute  $\mathcal{M}^2$  from hadronic matrix element  $\langle \psi(P') | P_\mu P^\mu | \psi(P) \rangle = \mathcal{M}^2 \langle \psi(P') | \psi(P) \rangle$
- Find

$$\mathcal{M}^2 = \sum_n \int [dx_i] [d^2\mathbf{k}_{\perp i}] \sum_\ell \left( \frac{\mathbf{k}_{\perp \ell}^2 + m_\ell^2}{x_\ell} \right) |\psi_n(x_i, \mathbf{k}_{\perp i})|^2 + \text{interactions}$$

- LFWF  $\psi_n$  represents a bound state which is off the LF energy shell  $\mathcal{M}^2 - \mathcal{M}_n^2$

$$\mathcal{M}_n^2 = \left( \sum_{a=1}^n k_a^\mu \right)^2 = \sum_a \frac{\mathbf{k}_{\perp a}^2 + m_a^2}{x_a}$$

with  $k_a^2 = m_a^2$  for each constituent

- Invariant mass  $\mathcal{M}_n^2$  key variable which controls the bound state: LFWF peaks at the minimum  $\mathcal{M}_n^2$
- Semiclassical approximation to QCD:

$$\psi_n(k_1, k_2, \dots, k_n) \rightarrow \phi_n \left( \underbrace{(k_1 + k_2 + \dots + k_n)^2}_{\mathcal{M}_n^2} \right), \quad m_q \rightarrow 0$$



- In terms of  $n - 1$  independent transverse impact coordinates  $\mathbf{b}_{\perp j}, j = 1, 2, \dots, n - 1,$

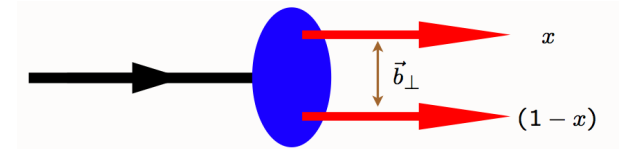
$$\mathcal{M}^2 = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \psi_n^*(x_i, \mathbf{b}_{\perp i}) \sum_{\ell} \left( \frac{-\nabla_{\mathbf{b}_{\perp \ell}}^2 + m_{\ell}^2}{x_{\ell}} \right) \psi_n(x_i, \mathbf{b}_{\perp i}) + \text{interactions}$$

- Relevant variable conjugate to invariant mass in the limit of zero quark masses

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|$$

the  $x$ -weighted transverse impact coordinate of the spectator system ( $x$  active quark)

- For a two-parton system  $\zeta^2 = x(1-x)\mathbf{b}_{\perp}^2$



- To first approximation LF dynamics depend only on the invariant variable  $\zeta$ , and hadronic properties are encoded in the hadronic mode  $\phi(\zeta)$  from

$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

factoring angular  $\varphi$ , longitudinal  $X(x)$  and transverse mode  $\phi(\zeta)$

( $P^+, \mathbf{P}_{\perp}$  and  $J_z$  commute with  $P^-$ )

- Ultra relativistic limit  $m_q \rightarrow 0$  longitudinal modes  $X(x)$  decouple ( $L = L^z$ )

$$\mathcal{M}^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

where the confining forces from the interaction terms are summed up in the effective potential  $U(\zeta)$

- LF eigenvalue equation  $P_\mu P^\mu |\phi\rangle = \mathcal{M}^2 |\phi\rangle$  is a LF wave equation for  $\phi$

$$\left( \underbrace{-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2}}_{\text{kinetic energy of partons}} + \underbrace{U(\zeta)}_{\text{confinement}} \right) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$



- Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable
- Eigenmodes  $\phi(\zeta)$  determine the hadronic mass spectrum and represent the probability amplitude to find  $n$ -massless partons at transverse impact separation  $\zeta$  within the hadron at equal light-front time
- Semiclassical approximation to light-front QCD does not account for particle creation and absorption

## 2 Light-Front Holographic Mapping

### Higher Spin Modes in AdS Space

- Description of higher spin modes in AdS space (Fronsdal, Fradkin and Vasiliev)
- Action for spin- $J$  field in  $\text{AdS}_{d+1}$  in presence of dilaton background  $\varphi(z)$  ( $x^M = (x^\mu, z)$ )

$$S = \frac{1}{2} \int d^d x dz \sqrt{g} e^{\varphi(z)} \left( g^{NN'} g^{M_1 M'_1} \dots g^{M_J M'_J} D_N \Phi_{M_1 \dots M_J} D_{N'} \Phi_{M'_1 \dots M'_J} - \mu^2 g^{M_1 M'_1} \dots g^{M_J M'_J} \Phi_{M_1 \dots M_J} \Phi_{M'_1 \dots M'_J} + \dots \right)$$

where  $D_M$  is the covariant derivative which includes parallel transport

$$[D_N, D_K] \Phi_{M_1 \dots M_J} = -R^L_{M_1 N K} \Phi_{L \dots M_J} - \dots - R^L_{M_J N K} \Phi_{M_1 \dots L}$$

- Physical hadron has plane-wave and polarization indices along  $3+1$  physical coordinates

$$\Phi_P(x, z)_{\mu_1 \dots \mu_J} = e^{-iP \cdot x} \Phi(z)_{\mu_1 \dots \mu_J}, \quad \Phi_{z \mu_2 \dots \mu_J} = \dots = \Phi_{\mu_1 \mu_2 \dots z} = 0$$

with four-momentum  $P_\mu$  and invariant hadronic mass  $P_\mu P^\mu = M^2$

- Construct effective action in terms of spin- $J$  modes  $\Phi_J$  with only physical degrees of freedom  
[ H. G. Dosch, S. J. Brodsky and GdT (in preparation)]
- Introduce fields with tangent indices

$$\hat{\Phi}_{A_1 A_2 \dots A_J} = e_{A_1}^{M_1} e_{A_2}^{M_2} \dots e_{A_J}^{M_J} \Phi_{M_1 M_2 \dots M_J} = \left( \frac{z}{R} \right)^J \Phi_{A_1 A_2 \dots A_J}$$

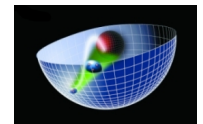
- Find effective action

$$S = \frac{1}{2} \int d^d x dz \sqrt{g} e^{\varphi(z)} \left( g^{NN'} \eta^{\mu_1 \mu'_1} \dots \eta^{\mu_J \mu'_J} \partial_N \hat{\Phi}_{\mu_1 \dots \mu_J} \partial_{N'} \hat{\Phi}_{\mu'_1 \dots \mu'_J} - \mu^2 \eta^{\mu_1 \mu'_1} \dots \eta^{\mu_J \mu'_J} \hat{\Phi}_{\mu_1 \dots \mu_J} \hat{\Phi}_{\mu'_1 \dots \mu'_J} \right)$$

upon  $\mu$ -rescaling

- Variation of the action gives AdS wave equation for spin- $J$  mode  $\Phi_J = \Phi_{\mu_1 \dots \mu_J}$

$$\left[ -\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left( \frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \left( \frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$



with  $\hat{\Phi}_J(z) = (z/R)^J \Phi_J(z)$  and all indices along 3+1

## Dual QCD Light-Front Wave Equation

$$\Phi_P(z) \Leftrightarrow |\psi(P)\rangle$$

- LF Holographic mapping found originally matching expressions of EM and gravitational form factors of hadrons in AdS and LF QCD [Brodsky and GdT (2006, 2008)]
- Upon substitution  $z \rightarrow \zeta$  and  $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2} \Phi_J(\zeta)$  in AdS WE

$$\left[ -\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left( \frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \left( \frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$

find LFWE ( $d = 4$ )

$$\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

with

$$U(\zeta) = \frac{1}{2} \varphi''(z) + \frac{1}{4} \varphi'(z)^2 + \frac{2J - 3}{2z} \varphi'(z)$$

and  $(\mu R)^2 = -(2 - J)^2 + L^2$

- AdS Breitenlohner-Freedman bound  $(\mu R)^2 \geq -4$  equivalent to LF QM stability condition  $L^2 \geq 0$
- Scaling dimension  $\tau$  of AdS mode  $\hat{\Phi}_J$  is  $\tau = 2 + L$  in agreement with twist scaling dimension of a two parton bound state in QCD and determined by QM stability condition

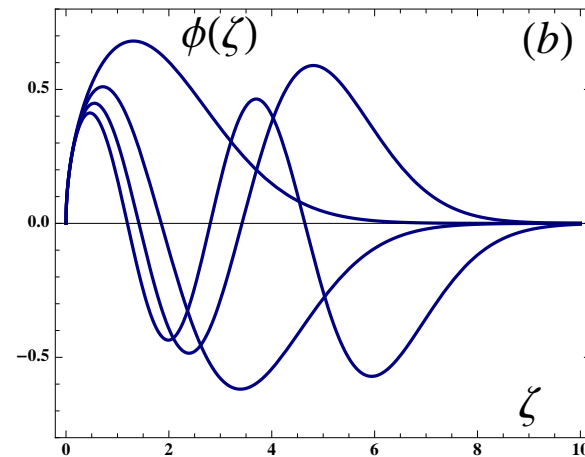
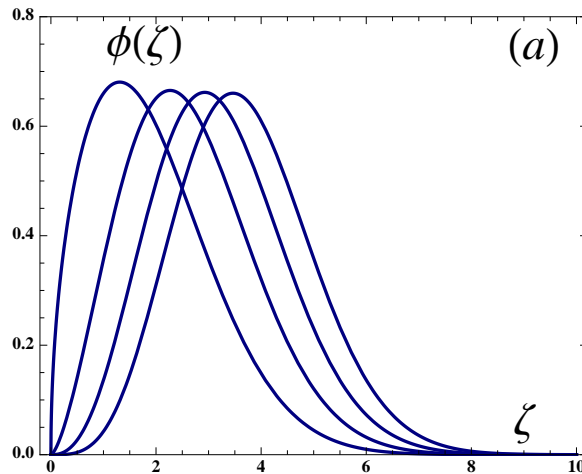
## Bosonic Modes and Meson Spectrum

Soft wall model [Karch, Katz, Son and Stephanov (2006)]

- Dilaton  $\varphi(z) = +\kappa^2 z^2$  (Minkowski metrics),  $\varphi(z) = -\kappa^2 z^2$  (Euclidean metrics)
- Effective potential  $U(z) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$
- Normalized eigenfunctions  $\langle \phi | \phi \rangle = \int d\zeta |\phi(z)|^2 = 1$

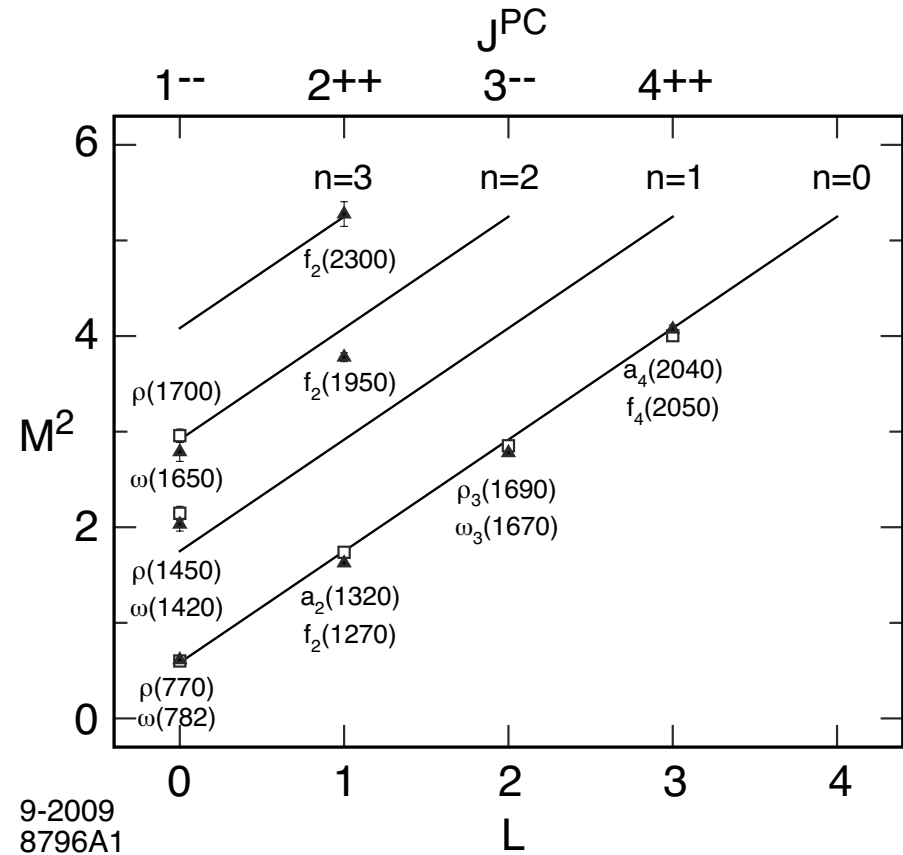
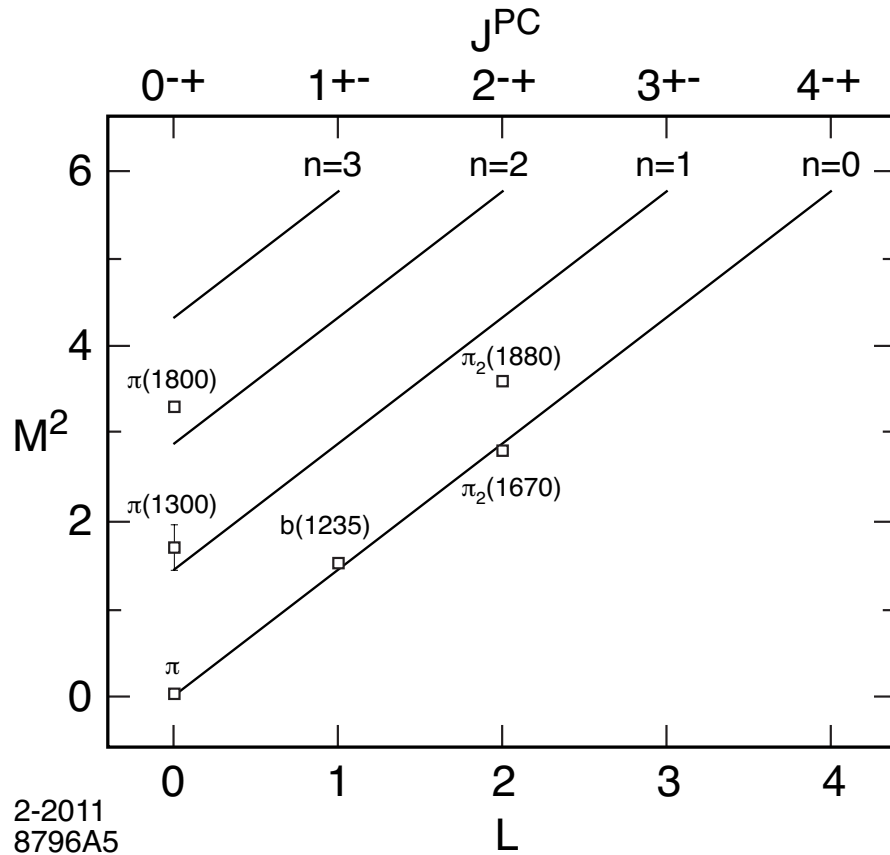
$$\phi_{nL}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues  $\mathcal{M}_{n,L,S}^2 = 4\kappa^2 (n + L + S/2)$



LFWFs  $\phi_{n,L}(\zeta)$  in physical space time for dilaton  $\exp(\kappa^2 z^2)$ : a) orbital modes and b) radial modes

$4\kappa^2$  for  $\Delta n = 1$   
 $4\kappa^2$  for  $\Delta L = 1$   
 $2\kappa^2$  for  $\Delta S = 1$

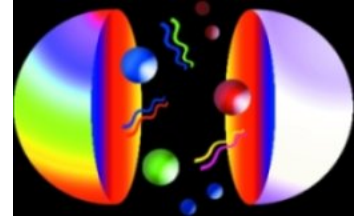


Regge trajectories for the  $\pi$  ( $\kappa = 0.6$  GeV) and the  $I = 1$   $\rho$ -meson and  $I = 0$   $\omega$ -meson families ( $\kappa = 0.54$  GeV)

## Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{3/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+1}(\kappa^2 \zeta^2)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L} \frac{1}{\sqrt{n+L+2}} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{5/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+2}(\kappa^2 \zeta^2)$$

- Normalization

$$\int d\zeta \psi_+^2(\zeta) = \int d\zeta \psi_-^2(\zeta) = 1$$

- Eigenvalues

$$\mathcal{M}_{n,L,S=1/2}^2 = 4\kappa^2 (n+L+1)$$

- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

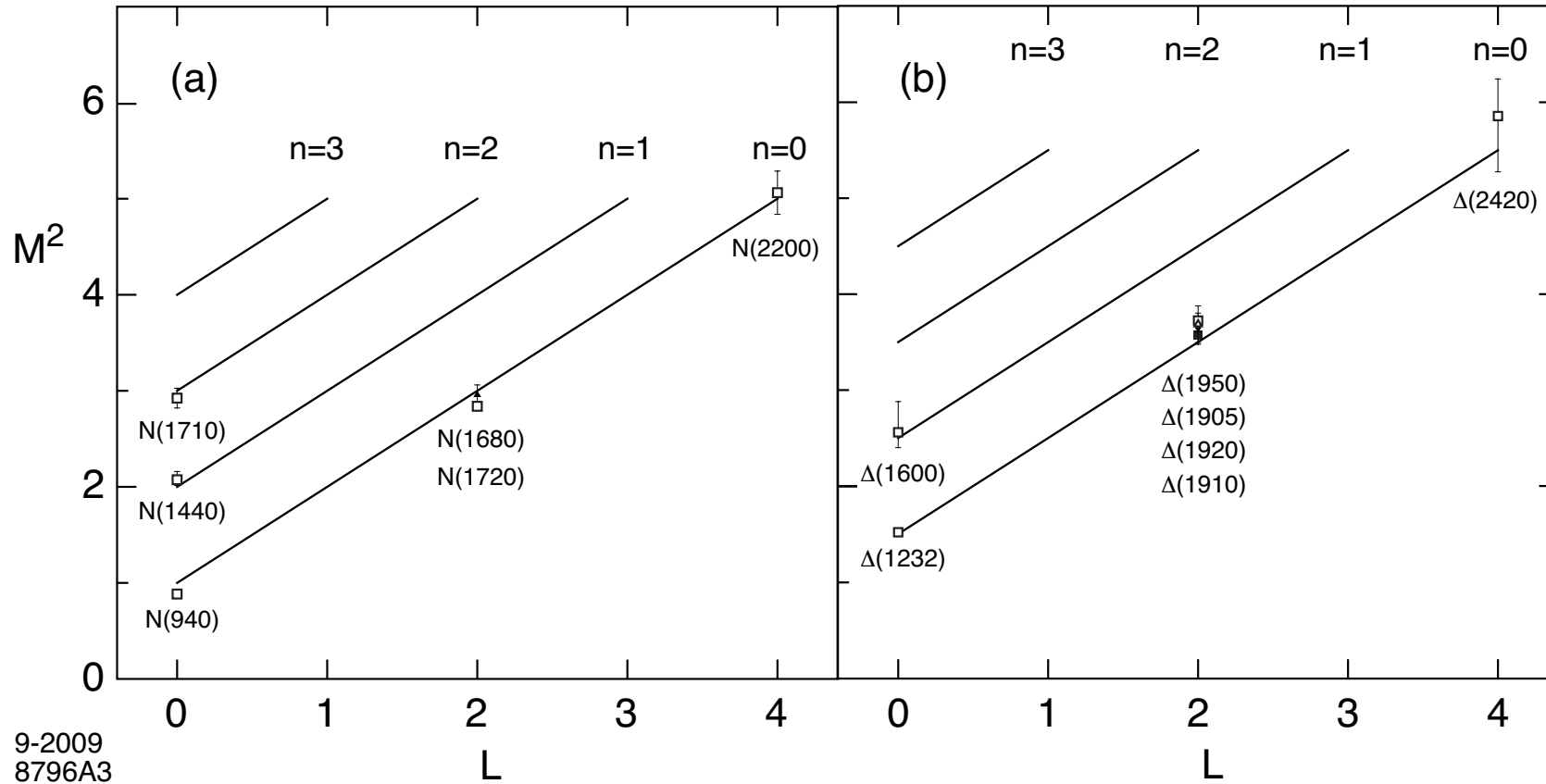


Same multiplicity of states for mesons and baryons!

$$4\kappa^2 \text{ for } \Delta n = 1$$

$$4\kappa^2 \text{ for } \Delta L = 1$$

$$2\kappa^2 \text{ for } \Delta S = 1$$



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Regge trajectories for positive parity  $N$  and  $\Delta$  baryon families ( $\kappa = 0.5$  GeV)

### 3 Light-Front Holographic Mapping of Current Matrix Elements

[S. J. Brodsky and GdT, PRL **96**, 201601 (2006)], PRD **77**, 056007 (2008)]

- EM transition matrix element in QCD: local coupling to pointlike constituents

$$\langle \psi(P') | J^\mu | \psi(P) \rangle = (P + P')^\mu F(Q^2)$$

where  $Q = P' - P$  and  $J^\mu = e_q \bar{q} \gamma^\mu q$

- EM hadronic matrix element in AdS space from non-local coupling of external EM field propagating in AdS with extended mode  $\Phi(x, z)$

$$\int d^4x dz \sqrt{g} e^{\varphi(z)} A^M(x, z) \Phi_{P'}^*(x, z) \overleftrightarrow{\partial}_M \Phi_P(x, z) \\ \sim (2\pi)^4 \delta^4(P' - P) \epsilon_\mu (P + P')^\mu F(Q^2)$$

where  $A_\mu(x, z) = \epsilon_\mu(q) e^{-iq \cdot x} V(q, z)$ ,  $A_z = 0$ ,  $Q^2 = -q^2 > 0$

- How to recover hard pointlike scattering at large  $Q$  out of soft collision of extended objects?

[Polchinski and Strassler (2002)]

- Mapping of  $J^+$  elements at fixed light-front time:  $\Phi_P(z) \Leftrightarrow |\psi(P)\rangle$

## Electromagnetic Form-Factor

- Drell-Yan-West electromagnetic FF in impact space [Soper (1977)]

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_q e_q \exp\left(i\mathbf{q}_{\perp} \cdot \sum_{k=1}^{n-1} x_k \mathbf{b}_{\perp k}\right) |\psi_n(x_j, \mathbf{b}_{\perp j})|^2$$

- Consider a two-quark  $\pi^+$  Fock state  $|u\bar{d}\rangle$  with  $e_u = \frac{2}{3}$  and  $e_{\bar{d}} = \frac{1}{3}$

$$F_{\pi^+}(q^2) = \int_0^1 dx \int d^2\mathbf{b}_{\perp} e^{i\mathbf{q}_{\perp} \cdot \mathbf{b}_{\perp}(1-x)} \left| \psi_{u\bar{d}/\pi}(x, \mathbf{b}_{\perp}) \right|^2$$

with normalization  $F_{\pi^+}(q=0) = 1$

- Integrating over angle

$$F_{\pi^+}(q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \left| \psi_{u\bar{d}/\pi}(x, \zeta) \right|^2$$

where  $\zeta^2 = x(1-x)\mathbf{b}_{\perp}^2$

- Compare with electromagnetic FF in AdS space [Polchinski and Strassler (2002)]

$$F(Q^2) = R^3 \int \frac{dz}{z^3} V(Q, z) \Phi_{\pi^+}^2(z)$$

where  $V(Q, z) = zQK_1(zQ)$

- Use the integral representation

$$V(Q, z) = \int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right)$$

- Find

$$F(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} J_0 \left( zQ \sqrt{\frac{1-x}{x}} \right) \Phi_{\pi^+}^2(z)$$

- Compare with electromagnetic FF in LF QCD for arbitrary  $Q$ . Expressions can be matched only if LFWF is factorized

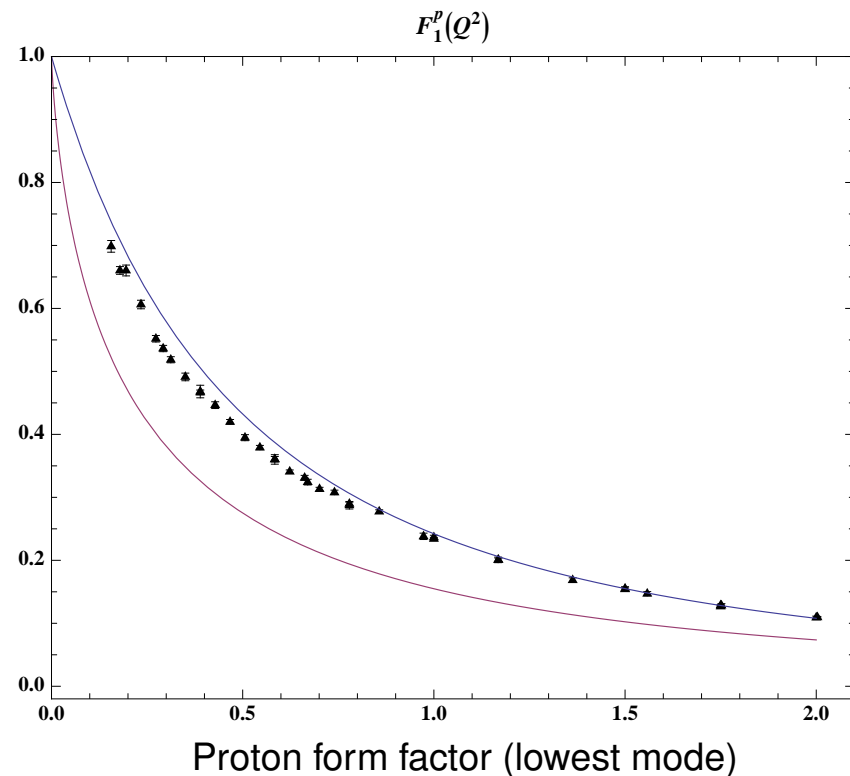
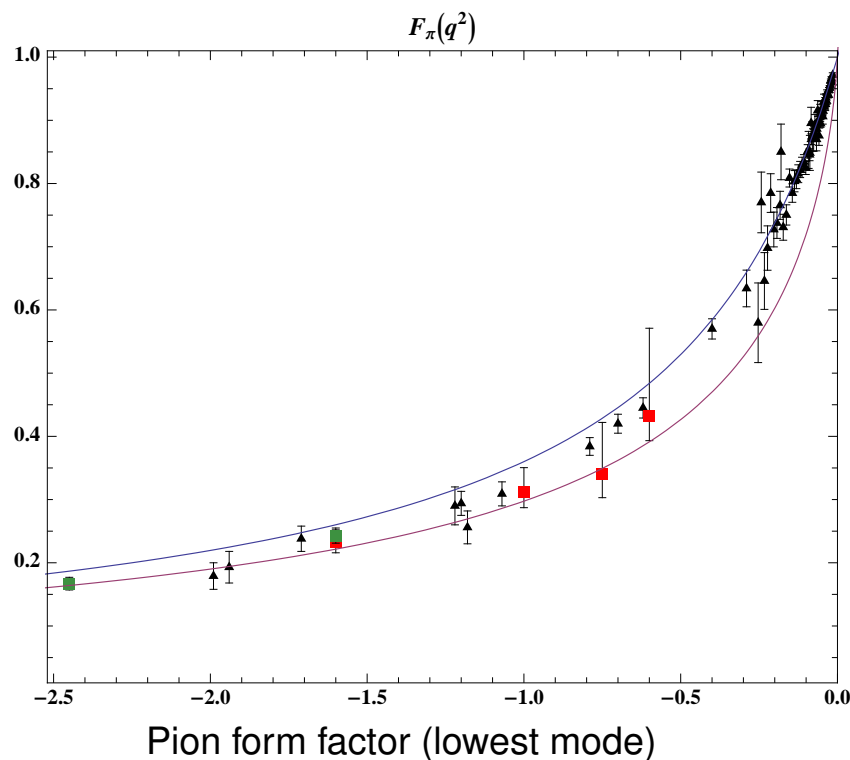
$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

- Find

$$X(x) = \sqrt{x(1-x)}, \quad \phi(\zeta) = \left( \frac{\zeta}{R} \right)^{-3/2} \Phi(\zeta), \quad z \rightarrow \zeta$$

- “Free current”  $V(Q, z) = zQK_1(zQ) \rightarrow$  infinite radius (mauve), no pole structure in time-like region
- “Dressed current” non-perturbative sum of an infinite number of terms  $\rightarrow$  finite radius (blue)
- Form factor in soft-wall model expressed as  $N - 1$  product of poles along vector radial trajectory [Brodsky and GdT (2008)]  $(\mathcal{M}_\rho^2 \rightarrow 4\kappa^2(n + 1/2))$

$$F(Q^2) = \left[ \left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \cdots \left(1 + \frac{Q^2}{\mathcal{M}_{\rho^{N-2}}^2}\right) \right]^{-1}$$

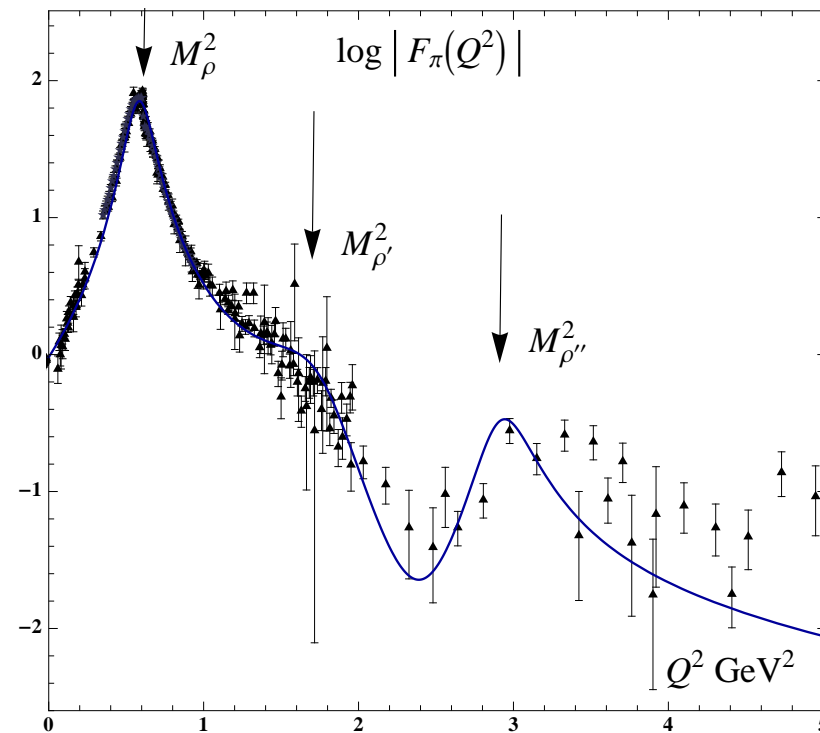
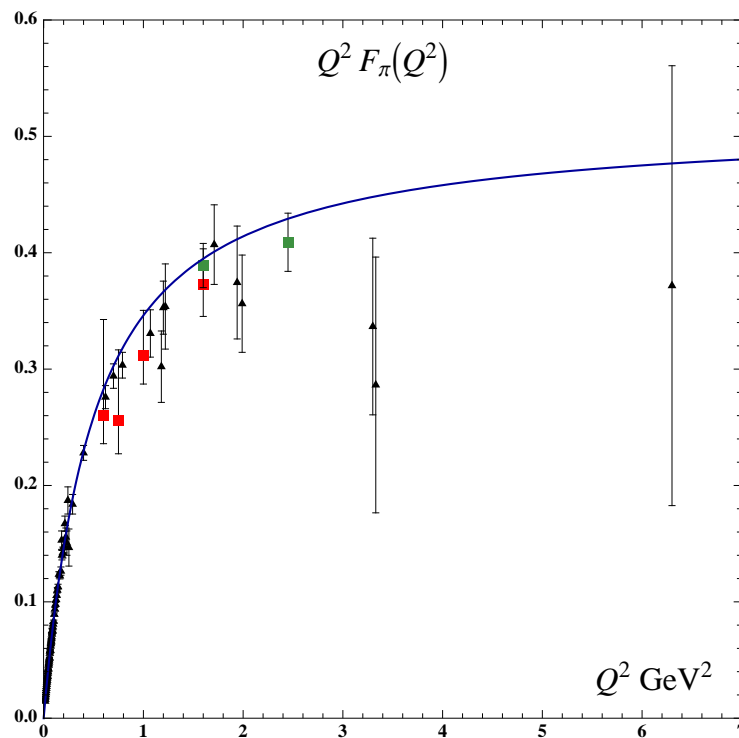


- Higher Fock components in pion LFWF

$$|\pi\rangle = \psi_{q\bar{q}/\pi}|q\bar{q}\rangle_{\tau=2} + \psi_{q\bar{q}q\bar{q}/\pi}|q\bar{q}q\bar{q}\rangle_{\tau=4} + \dots$$

- Expansion of LFWF up to twist 4 (monopole + tripole)

$$\kappa = 0.54 \text{ GeV}, \Gamma_{\rho} = 130, \Gamma_{\rho'} = 400, \Gamma_{\rho''} = 300 \text{ MeV}, P_{q\bar{q}q\bar{q}} = 13\%$$



- Only interaction in LF holographic semiclassical approx is the confinement potential: create Fock states with extra quark-antiquark pairs, no dynamical gluons

## Gravitational or Energy-Momentum Form-Factor

[S. J. Brodsky and GdT, PRD **78**, 025032 (2008)]

- Gravitational form factor of composite hadrons in QCD: local coupling to pointlike constituents

$$\langle P' | \Theta_{\mu}^{\nu} | P \rangle = (P^{\nu} P'_{\mu} + P_{\mu} P'^{\nu}) A(Q^2)$$

where  $Q = P' - P$  and

$$\Theta_{\mu\nu} = \frac{1}{2} \bar{q} i (\gamma_{\mu} D_{\nu} + \gamma_{\nu} D_{\mu}) q - g_{\mu\nu} \bar{q} (i \not{D} - m) q - G_{\mu\lambda}^a G_{\nu}^{a\lambda} + \frac{1}{4} g_{\mu\nu} G_{\lambda\sigma}^a G^{a\lambda\sigma}$$

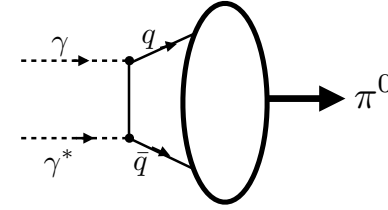
- Hadronic matrix element of energy-momentum tensor from perturbing static AdS metric: non-local coupling of external graviton propagating in AdS with extended mode  $\Phi(x, z)$  [Abidin and Carlson (2008)]

$$\int d^4x dz \sqrt{g} h_{\ell m} \left( \partial^{\ell} \Phi_{P'}^* \partial^m \Phi_P + \partial^m \Phi_{P'}^* \partial^{\ell} \Phi_P \right)$$

- Are the transition amplitudes related ?
- Mapping of  $\Theta^{++}$  elements at fixed LF time: Identical mapping  $\Phi_P(z) \Leftrightarrow |\psi(P)\rangle$  as EM FF

## Meson Transition Form-Factors

[S. J. Brodsky, Fu-Guang Cao and GdT (in preparation)]



- Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]

$$\int d^4x \int dz \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$$

$$\sim (2\pi)^4 \delta^{(4)}(p_\pi + q - k) F_{\pi\gamma}(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q) (p_\pi)_\nu \epsilon_\rho(k) q_\sigma$$

- Take  $A_z \propto \Phi_\pi(z)/z$ ,  $\Phi_\pi(z) = \sqrt{2P_{q\bar{q}}} \kappa z^2 e^{-\kappa^2 z^2/2}$ ,  $\langle \Phi_\pi | \Phi_\pi \rangle = P_{q\bar{q}}$
- Find  $\phi(x) = \sqrt{3} f_\pi x(1-x)$ ,  $f_\pi = \sqrt{P_{q\bar{q}}} \kappa / \sqrt{2\pi}$

$$Q^2 F_{\pi\gamma}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \frac{\phi(x)}{1-x} \left[ 1 - e^{-P_{q\bar{q}} Q^2 (1-x) / 4\pi^2 f_\pi^2 x} \right]$$

normalized to the asymptotic DA  $[P_{q\bar{q}} = 1 \rightarrow \text{Musatov and Radyushkin (1997)}]$

- Large  $Q^2$  TFF is identical to first principles asymptotic QCD result  $Q^2 F_{\pi\gamma}(Q^2 \rightarrow \infty) = 2f_\pi$
- The CS form is local in AdS space and projects out only the asymptotic form of the pion DA



