CAQCD - University of Minnesota - May 2011

Hall Transport at Strong Coupling and Axions in AdS

Adam Ritz

University of Victoria

With K. Jensen, M. Kaminski, P. Kovtun, R. Meyer, A. Yarom

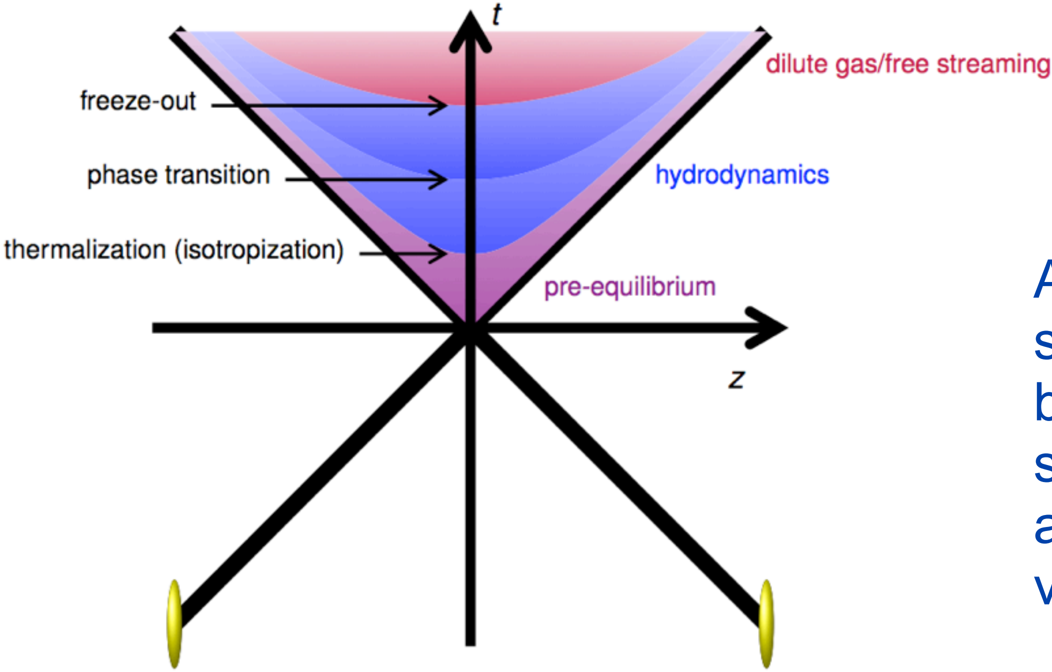
To appear

Strong (hydro)dynamics

Hydrodynamics is a universal low energy effective theory for scales » 1/T (with many dofs) over which an interacting system can achieve local thermal equilibrium

But its application to strongly-interacting theories is nontrivial

➠ requires **thermodynamic functions** & ***transport coefficients***.

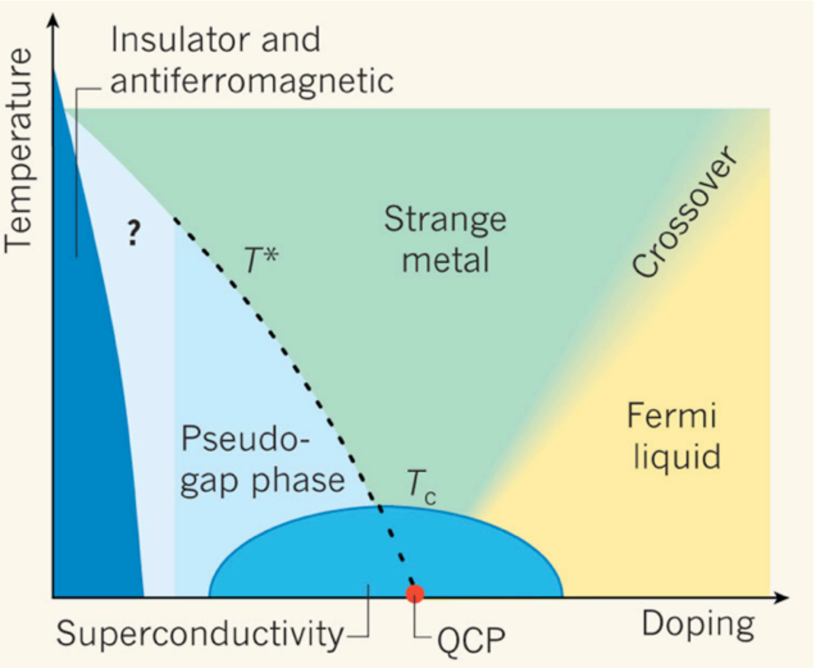


AdS/CFT has provided strong-coupling "data" for both, e.g. for models of the sQGP which appears to be a (nearly) perfect fluid with very low η/s.

Strong (hydro)dynamics

Hydrodynamics is a universal low energy effective theory for scales » 1/T (with many dofs) over which an interacting system can achieve local thermal equilibrium

But its application to strongly-interacting theories is nontrivial
➠ requires **thermodynamic functions** & ***transport coefficients***.



We can ask related questions about the hydrodynamic regime of strongly-correlated systems in 2+1D (e.g. strange metal phase).

➠ Focus of this talk is 2+1D hydrodynamics, emphasizing the role of *parity*$^{(\#)}$, with applications to (anomalous) Hall transport.

(#) In 2+1D, P: $x^1 \to -x^1$, analogous to T, and pseudoscalars like magnetic field or vorticity can play an important role.

Outline

1. P-odd relativistic hydrodynamics in 2+1D
   - constructing the constitutive relations
     - ➪ linearized hydro and Kubo formulae
     - ➪ new P-odd transport coefficients

2. Computing the transport coefficients
   - weak coupling ➙ free fields
   - strong coupling ➙ AdS/CFT
     - ➪ link to the membrane paradigm

Ideal hydrodynamics

- Conservation laws:

$$\partial_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho \qquad \partial_\mu J^\mu = 0$$

In 2+1D, 3 eqns
in 6 unknowns

- Constitutive relations (*assuming* local equilibrium):

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + p P^{\mu\nu} \qquad J^\mu = \rho u^\mu$$

In 2+1D, now 3
unknowns: p(ε), u^1, u^2

$$\mathrm{NB}: \quad u^\mu u_\mu = -1, \qquad P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$$

Systematic Corrections

Near equilibrium *iff* $t_{therm} \ll \Delta t, \Delta l/v \Rightarrow$ derivative expansion

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{eq}} + \pi^{\mu\nu} \qquad J^{\mu} = J^{\mu}_{\mathrm{eq}} + \nu^{\mu}$$

$$\text{Landau frame}: u_\mu \pi^{\mu\nu} = u_\mu \nu^\mu = 0$$

Dissipative corrections (*assuming P invariance!*)

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \zeta P^{\mu\nu}\partial\cdot u \qquad \sigma^{\mu\nu} \equiv \partial^{<\mu}u^{\nu>}$$

$$\nu^\mu = \sigma P^{\mu\nu}\left(E_\mu - T\partial_\mu \frac{\mu}{T}\right)$$

Systematic Corrections

Near equilibrium *iff* $t_{therm} \ll \Delta t, \Delta l/v \Rightarrow$ derivative expansion

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \pi^{\mu\nu} \qquad J^{\mu} = J^{\mu}_{\rm eq} + \nu^{\mu}$$

$$\text{Landau frame}: u_\mu \pi^{\mu\nu} = u_\mu \nu^\mu = 0$$

Dissipative corrections (*assuming P invariance!*)

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \zeta P^{\mu\nu}\partial\cdot u \qquad \sigma^{\mu\nu} \equiv \partial^{<\mu}u^{\nu>}$$

$$\nu^\mu = \sigma P^{\mu\nu}\left(E_\mu - T\partial_\mu \frac{\mu}{T}\right)$$

Transport coefficients,
positive for $\partial\cdot S \geq 0$

P(T) Violation in 1st-order hydrodynamics

Additional terms are allowed with microscopic P-violation

➠ Strategy to fix the constitutive relations:

- Write down a 1-derivative tensor basis
- Use ideal hydro to remove redundancies
- Use T-covariance (Onsager) constraints on 2-pt correlators in linear theory
- Require the existence of an entropy current with a positive divergence

$$TS^{\mu} = pu^{\mu} - \mu J^{\mu} - u_{\nu} T^{\mu\nu} + \mathcal{O}(\partial)$$

NB: P-odd terms must be *dissipationless*, so this gives strong constraints.

P(T) Violation in 1st-order hydrodynamics

NB: We assume background fields E, B are O(∂), and subtract magnetization/polarization effects

$$\pi^{\mu\nu}_{\mathrm{P.O.}} = \eta_H \epsilon^{\rho\sigma(\mu} u_\rho \sigma^{\nu)}_\sigma + \zeta_H P^{\mu\nu} w + \mathcal{O}(P^{\mu\nu}, F_{\mu\nu})$$

$$\nu^\mu_{\mathrm{P.O.}} = \sigma_H \epsilon^{\mu\nu\rho} u_\nu \left(E_\rho - T\partial_\rho \frac{\mu}{T} \right) + \sigma_1 \epsilon^{\mu\nu\rho} u_\nu E_\rho + \sigma_2 \epsilon^{\mu\nu\rho} u_\nu \partial_\rho T + \mathcal{O}(P^{\mu\nu})$$

$$\mathrm{NB:}\quad E_\mu = F_{\mu\nu} u^\nu, \;\; B = -\epsilon^{\mu\nu\rho} u_\mu F_{\nu\rho}/2, \quad w = \epsilon^{\mu\nu\rho} u_\mu \partial_\nu u_\rho$$

P(T) Violation in 1st-order hydrodynamics

NB: We assume background fields E, B are O(∂), and subtract magnetization/polarization effects

$$\pi^{\mu\nu}_{\mathrm{P.O.}} = \eta_H \epsilon^{\rho\sigma(\mu} u_\rho \sigma^{\nu)}_\sigma + \zeta_H P^{\mu\nu} w + \mathcal{O}(P^{\mu\nu}, F_{\mu\nu})$$

Hall (shear) viscosity
[Son & Saremi ’11; Nicolis & Son ’11; Avron et al ’95]

$$\nu^\mu_{\mathrm{P.O.}} = \sigma_H \epsilon^{\mu\nu\rho} u_\nu \left(E_\rho - T\partial_\rho \frac{\mu}{T} \right) + \sigma_1 \epsilon^{\mu\nu\rho} u_\nu E_\rho + \sigma_2 \epsilon^{\mu\nu\rho} u_\nu \partial_\rho T + \mathcal{O}(P^{\mu\nu})$$

$$\mathrm{NB:} \quad E_\mu = F_{\mu\nu} u^\nu, \quad B = -\epsilon^{\mu\nu\rho} u_\mu F_{\nu\rho}/2, \quad w = \epsilon^{\mu\nu\rho} u_\mu \partial_\nu u_\rho$$

P(T) Violation in 1st-order hydrodynamics

NB: We assume background fields E, B are O(∂), and subtract magnetization/polarization effects

$$\pi^{\mu\nu}_{\text{P.O.}} = \eta_H \epsilon^{\rho\sigma(\mu} u_\rho \sigma^{\nu)}_\sigma + \zeta_H P^{\mu\nu} w + \mathcal{O}(P^{\mu\nu}, F_{\mu\nu})$$

Hall (shear) viscosity
[Son & Saremi '11; Nicolis & Son '11; Avron et al '95]

$$\nu^\mu_{\text{P.O.}} = \sigma_H \epsilon^{\mu\nu\rho} u_\nu \left(E_\rho - T\partial_\rho \frac{\mu}{T} \right) + \sigma_1 \epsilon^{\mu\nu\rho} u_\nu E_\rho + \sigma_2 \epsilon^{\mu\nu\rho} u_\nu \partial_\rho T + \mathcal{O}(P^{\mu\nu})$$

contributions to Hall conductivity

$$\text{NB}: \quad E_\mu = F_{\mu\nu} u^\nu, \quad B = -\epsilon^{\mu\nu\rho} u_\mu F_{\nu\rho}/2, \quad w = \epsilon^{\mu\nu\rho} u_\mu \partial_\nu u_\rho$$

Kubo Formulae

With a background metric, the linearized conservation equations relate fluctuations $\delta X=(\delta T,\delta\mu,\delta u^{\mu},\ldots)$ to background fields:

$$\mathbf{M}(\omega,\mathbf{k})\delta\vec{X} = \mathbf{S}(A_\mu, h_{\mu\nu})$$

The constitutive relations then express linear response, and by differentiation we obtain retarded 2-pt functions, and Kubo formulae for the transport coefficients:

$$\langle J^\mu J^\nu\rangle^{\mu=0}_{\text{P.O.}} = \left.\frac{\delta J^\mu}{\delta A_\nu}\right|^{\mu=0}_{\text{P.O.}} = -i\epsilon^{\mu\nu\rho}q_\rho\left(\frac{\sigma_H\omega}{\omega+iDk^2}+\sigma_1\right), \qquad q^\mu=(\omega,\mathbf{k})$$

Kubo Formulae

With a background metric, the linearized conservation equations relate fluctuations $\delta X=(\delta T,\delta\mu,\delta u^{\mu},\ldots)$ to background fields:

$$\mathbf{M}(\omega,\mathbf{k})\delta\vec{X} = \mathbf{S}(A_{\mu},h_{\mu\nu})$$

The constitutive relations then express linear response, and by differentiation we obtain retarded 2-pt functions, and Kubo formulae for the transport coefficients:

$$\langle J^{\mu}J^{\nu}\rangle^{\mu=0}_{\mathrm{P.O.}} = \left.\frac{\delta J^{\mu}}{\delta A_{\nu}}\right|^{\mu=0}_{\mathrm{P.O.}} = -i\epsilon^{\mu\nu\rho}q_{\rho}\left(\frac{\sigma_H\omega}{\omega+iDk^2}+\sigma_1\right), \qquad q^{\mu}=(\omega,\mathbf{k})$$

NB: σ_H is a transport coefficient, as it requires taking $\lim(\omega\to 0)$ at $k=0$.

NB: When $\mu=B=0$, $\sigma_1 = \partial\rho/\partial B$ is a thermodynamic quantity.

Interpreting Hall transport

On a New Action of the Magnet on Electric Currents.

BY E. H. HALL, *Fellow of the Johns Hopkins University.*

SOMETIME during the last University year, while I was reading Maxwell's Electricity and Magnetism in connection with Professor Rowland's lectures, my attention was particularly attracted by the following passage in Vol. II, p. 144:..............

[Am J. Math, 1879]

Interpreting Hall transport

Boost invariance in the presence of *nonzero* ρ, B implies:

$$J_i = \sigma_{ij} E_j, \qquad \sigma_{xy} = \frac{\rho}{B}$$

NB: Many (e.g. ferromagnetic) systems dont follow this scaling,
and have an "Anomalous Hall Effect" [Hall, 1881].
Cases are also known where $\sigma_{xy} \neq 0$ when B=0.

We find in the hydro regime at B=0 (given another source of T-violation):

$$\nu^\mu_{\mathrm{P.O.}} = (\overbrace{\sigma_H + \sigma_1}^{\sigma_{xy}})\epsilon^{\mu\nu\rho} u_\nu E_\rho + \cdots$$

transport thermodynamic

Transport Coefficients

Computing the transport coefficients via Kubo relations

- E.G.(1) - Free Dirac fermions in a U(1) background

$$\langle J^\mu J^\nu \rangle_{\mathrm{P.O.}} = -i\epsilon^{\mu\nu\rho} q_\rho \frac{\mathrm{sgn(m)}}{4\pi} \tanh\left(\frac{|m|}{2T}\right) + \mathcal{O}(q^2)$$

$$\Downarrow$$

$$\sigma_1 = \frac{\mathrm{sgn(m)}}{4\pi} \tanh\left(\frac{|m|}{2T}\right), \qquad \sigma_H = 0$$

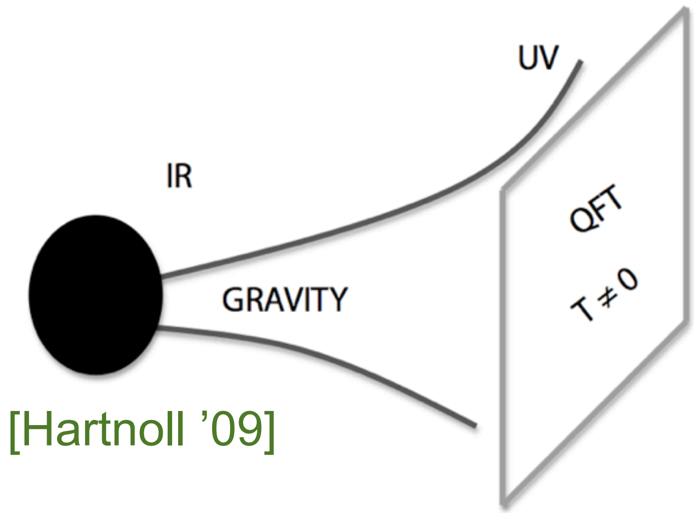
CS coefficient at $T \neq 0$
[Ishikawa et al ’87; Babu et al ’87]

no hydrodynamics
in a free theory

- E.G.(2) - Strongly-interacting matter in a U(1) background?

➠ Use the AdS/CFT correspondence

Axions in AdS/CFT



$$T_{\mu\nu} \Leftrightarrow h_{\mu\nu}$$

$$J_\mu \Leftrightarrow A_\mu$$

- To break parity and conformal invariance, we add a bulk axion with a nontrivial profile.

$$\mathcal{L}^{\text{bulk}}_{(A)} = -\frac{1}{4e^2}\sqrt{-g}F^2 + \frac{1}{64\pi^2}\theta(z)F\tilde{F}$$

- For simplicity, consider μ=0, where the EM fluctuations decouple from the metric

$$ds^2 = \frac{1}{z^2}\left(-f(z)dt^2 + d\mathbf{x}^2 + g(z)^2\frac{dz^2}{f(z)^2}\right)$$

Axions in AdS/CFT & Hall transport at B=0

Equations for A_μ solved by matching:

1. Expansion about horizon in $(z-z_h)$
2. Expansion in ω and k

AdS/CFT dictionary [Son & Starinets '02] $\Rightarrow \langle JJ \rangle$ and we find (at B=0 !)

$$\sigma_1 = \frac{d\rho}{dB} = \frac{1}{8\pi^2} \frac{\int_0^{z_h} dz\, g(z)\theta(z)}{\int_0^{z_h} dz\, g(z)} \qquad \sigma_H = \frac{\theta(z_h)}{8\pi^2} - \frac{d\rho}{dB}$$

nonzero transport
at strong coupling

Alternatively, the currents can be computed directly using the nonlinear fluid-gravity correspondence [Bhattacharyya et al '07], which is useful when $\mu \neq 0$.

Membrane conductivity

Recall, via the Kubo formulae or the constitutive relations, that the Hall conductivity is:

$$\sigma_{xy} = \sigma_H + \sigma_1 = \frac{\theta(z_h)}{8\pi^2}$$

BH membrane paradigm?

[Thorne et al. '86]

Adding a dilaton $e^2(z) \Rightarrow \sigma_{xx} = 1/e^2(z_h)$ [cf. Herzog et al '07; Iqbal & Liu '08]

➠ complex conductivity admits an action of "membrane" S-duality!

$$\tau(z_h) = \frac{\theta(z_h)}{2\pi} + \frac{4\pi i}{e^2(z_h)} = 4\pi i(\sigma_{xx} - i\sigma_{xy})$$

[cf. Witten '03, Hartnoll & Herzog '08]

Concluding Remarks

Summary

- P-violation in 2+1D allows for new (Hall) transport coefficients in 1st order relativistic hydrodynamics

  - Dissipationless!
  - computable in a class of strong-coupling theories via AdS/CFT with axions in the bulk
  - Allows for Hall transport in the absence of external fields (analogy with anomalous Hall effect).

In progress...

- Full analysis at finite chemical potential

  - AdS computations using nonlinear fluid/gravity correspondence
  - Phenomenology - structure of spectral functions at finite ω, ...
  - implications of bulk S-duality, etc.