

Applications of the Collinear Anomaly: q_T Resummation and Jet Broadening

Matthias Neubert
Johannes Gutenberg University Mainz
Continuous Advances in QCD IX
Minneapolis, 12-15 May 2011

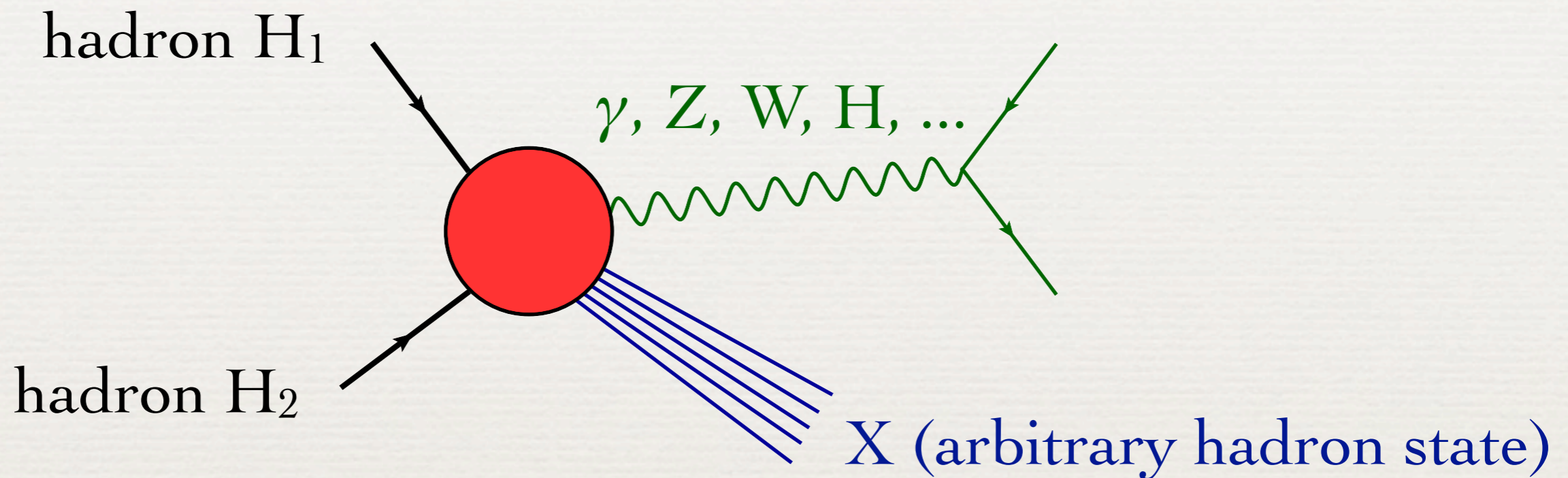


Based on:



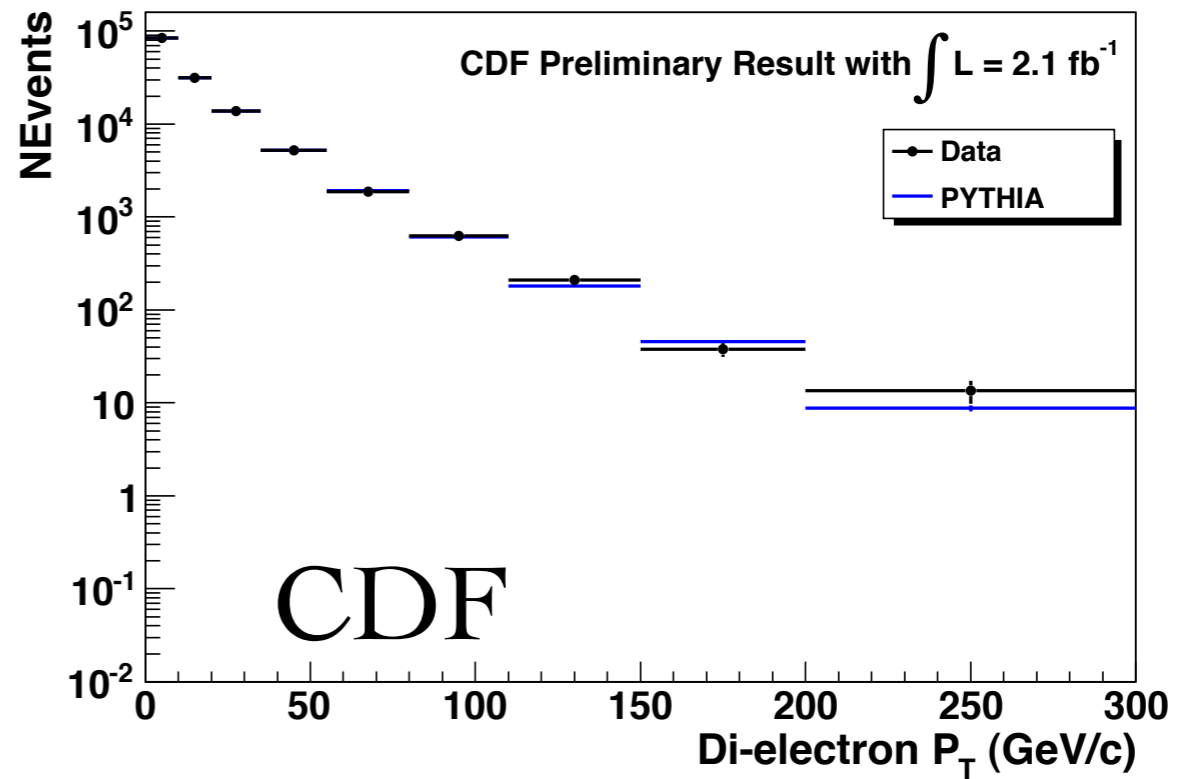
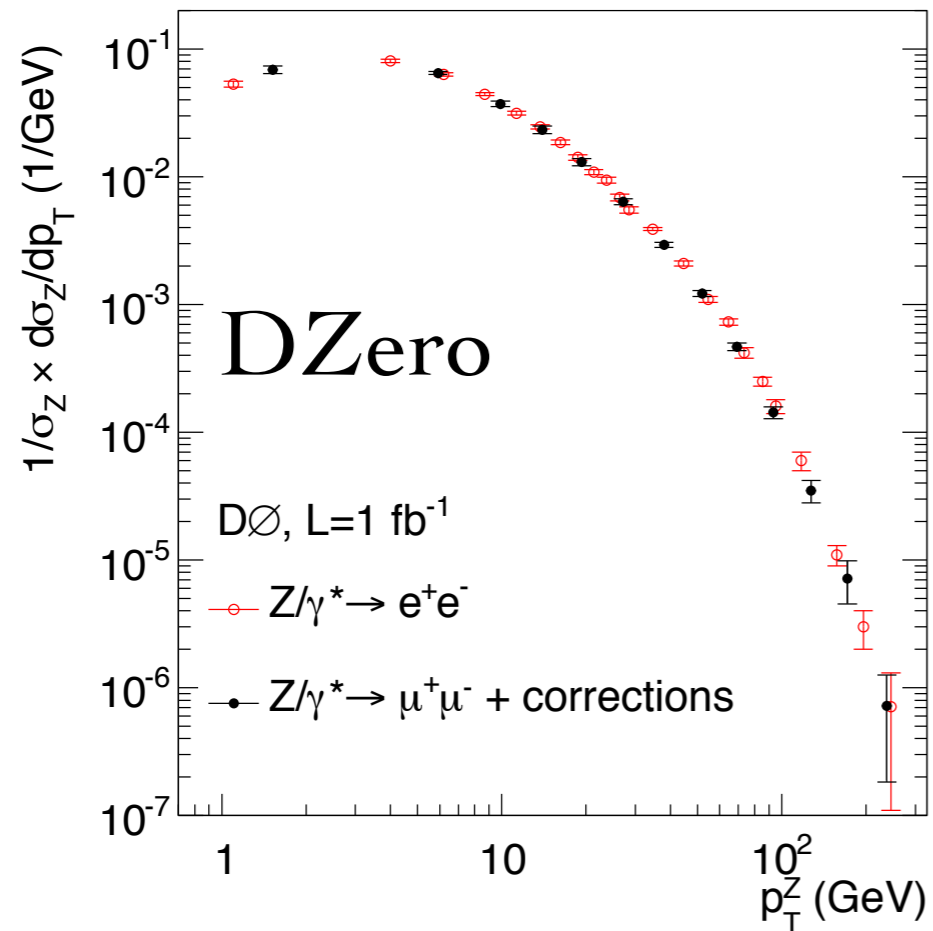
- ♦ T. Becher and M. Neubert:
Drell-Yan production at small q_T , transverse
parton distributions and the collinear anomaly
[arXiv:1007.4005](#) (to appear in EPJC)
- ♦ T. Becher, G. Bell and M. Neubert:
Factorization and resummation for jet broadening
[arXiv:1104.4108](#) (submitted to PLB)

Drell-Yan processes

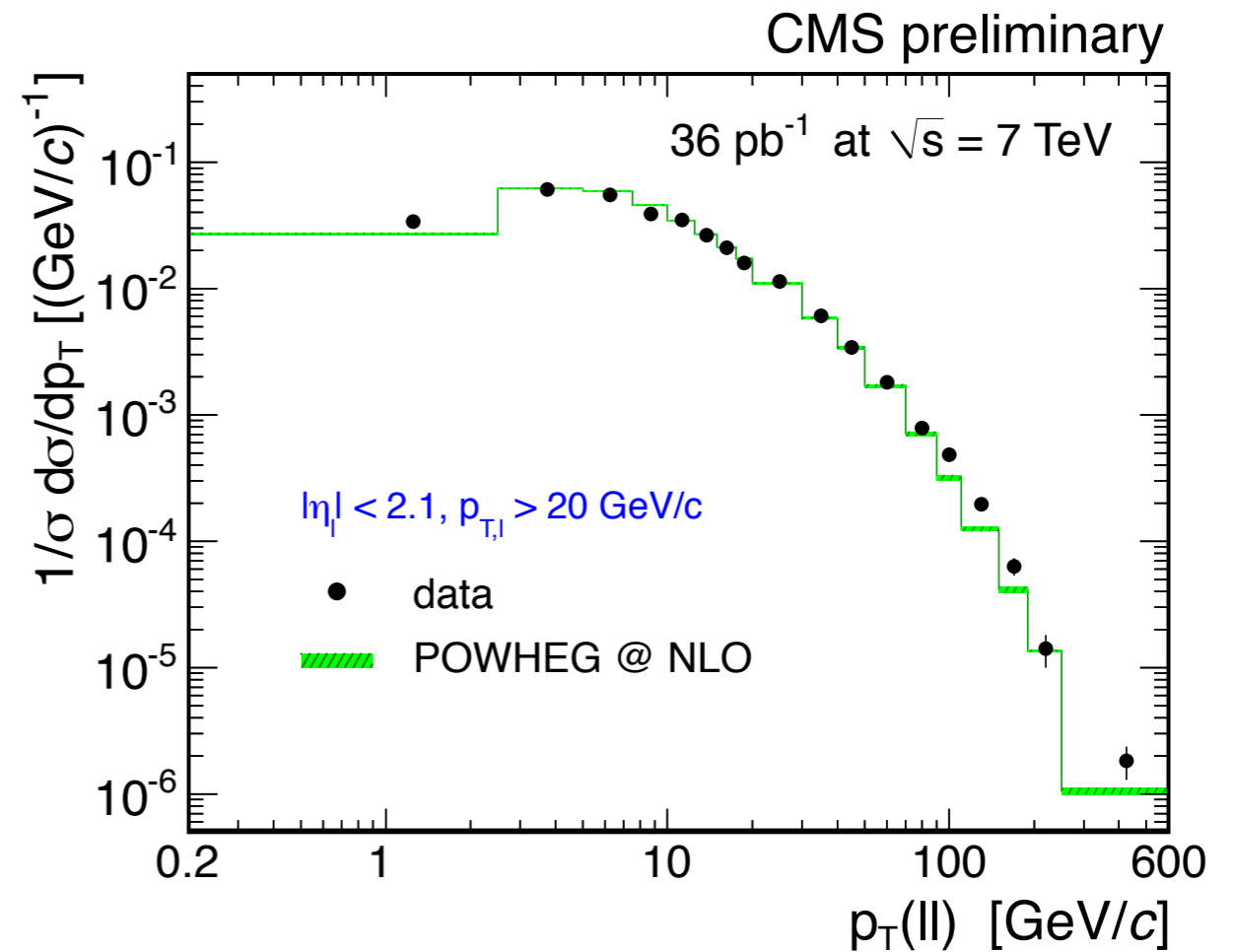
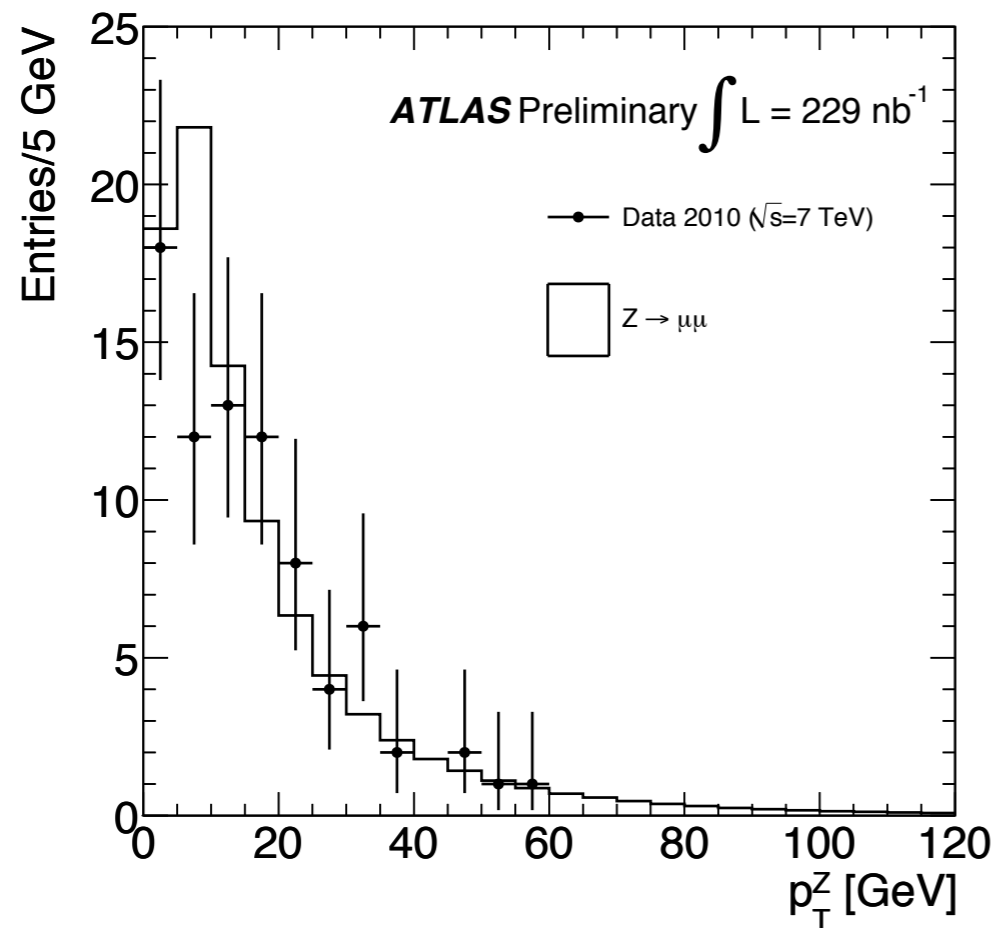


- ♦ Used for measurement of W -boson mass and width, PDF determinations, Higgs discovery, background to New Physics searches
- ♦ Region of small $q_T \ll M$ particularly relevant to extraction of W mass and reduction of background to Higgs searches

Z-boson production at Tevatron ...

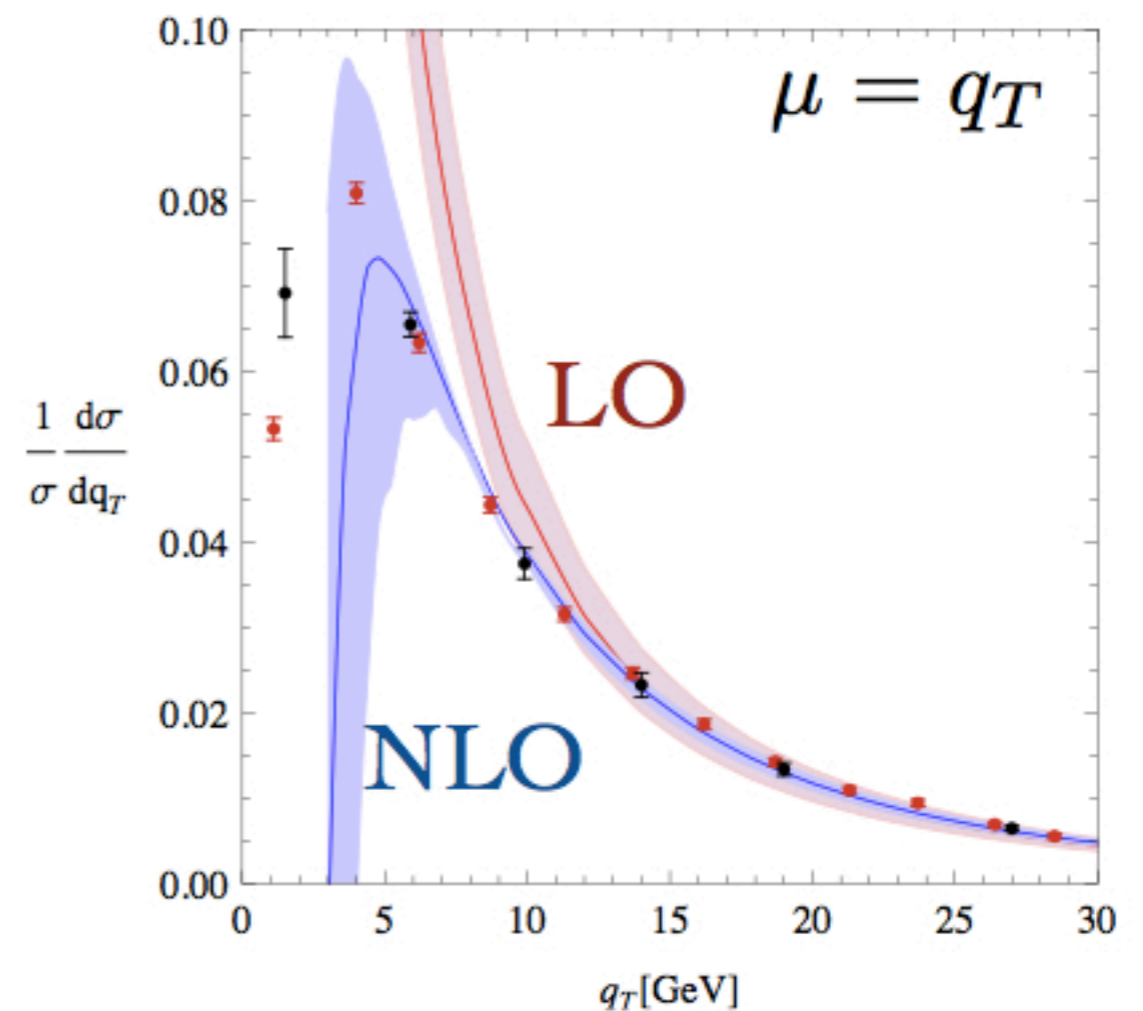
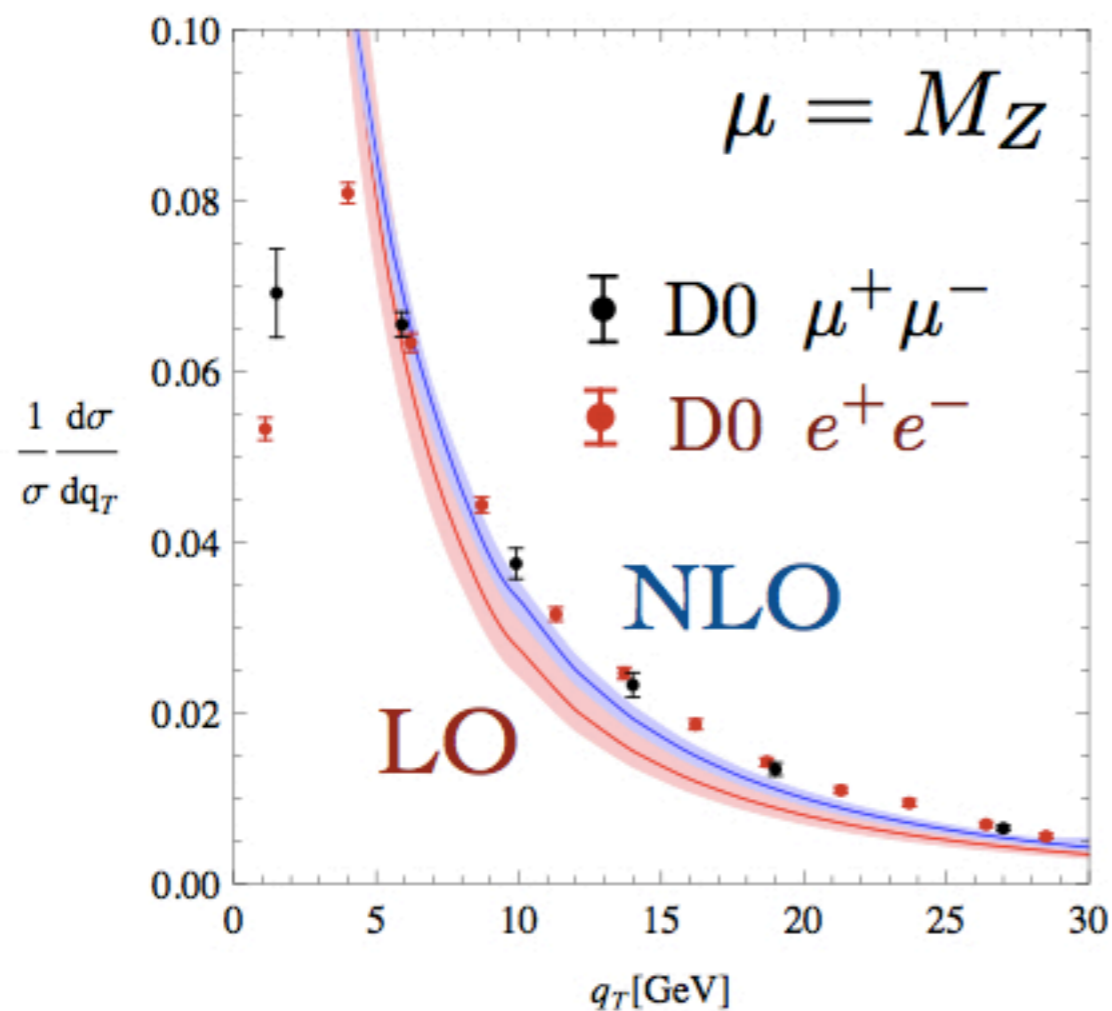


... and at LHC



Drell-Yan processes

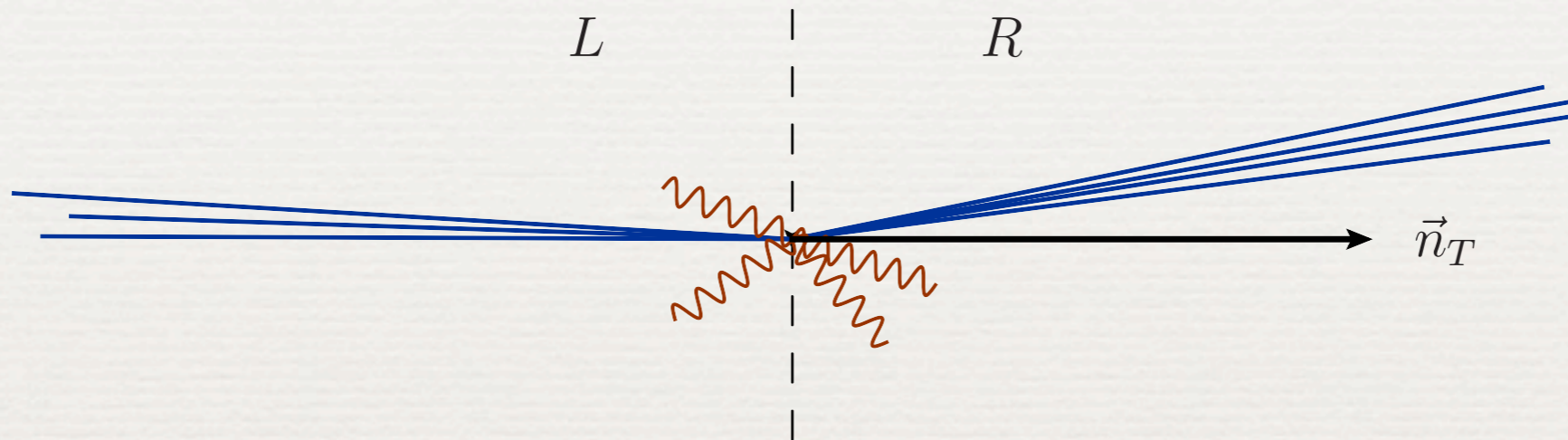
- ♦ Classical two-scale problem ($q_T \ll M$), for which large Sudakov logarithms $\sim (\alpha_s \ln^2 M/q_T)^n$ arise that must be resummed



Drell-Yan processes

- ◆ Transverse momentum of Drell-Yan object (W, Z, H) due to initial-state radiation (ISR) off collinear partons
- ◆ Simple example of **beam jets** described by **beam functions** in SCET Stewart, Tackmann, Waalewijn 2009
- ◆ Yet many surprises and subtleties arise, which may be relevant also for other applications of beam functions in jet processes

Jet broadening in e^+e^- annihilation



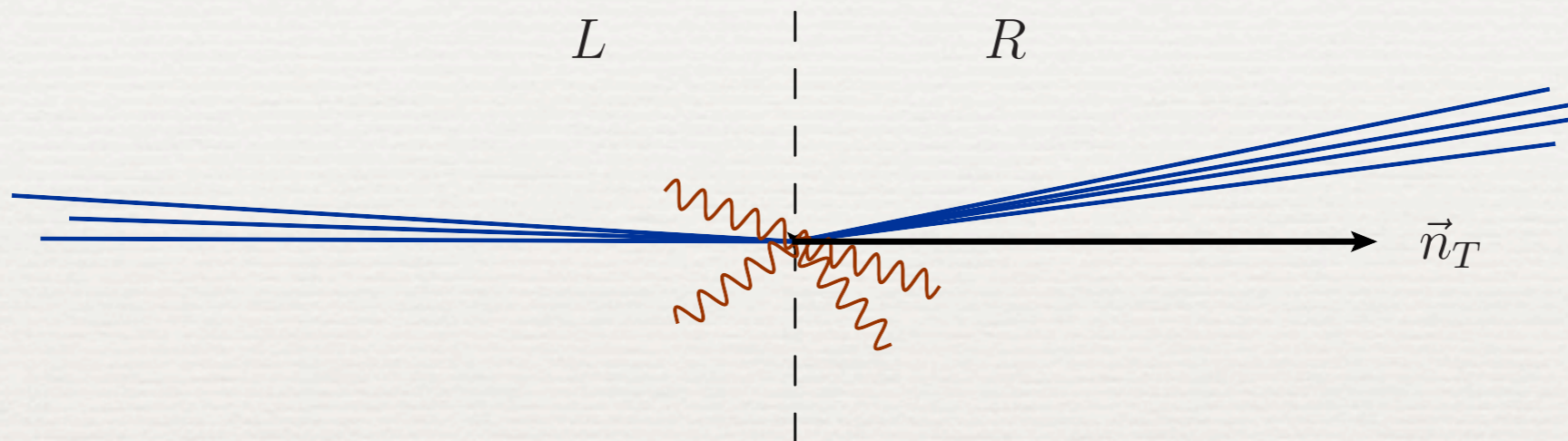
- ♦ Broadening measures transverse momenta relative to thrust axis:

$$b_L = \frac{1}{2} \sum_i |\vec{p}_i^\perp| = \frac{1}{2} \sum_i |\vec{p}_i \times \vec{n}_T|$$

- ♦ Total and wide broadening defined as:

$$b_T = b_L + b_R; \quad b_W = \max(b_L, b_R)$$

Jet broadening in e^+e^- annihilation



- ♦ Important event shape, relevant for precision determination of α_s
- ♦ Cross section is largest for $b_{L,R} \ll Q = \sqrt{s}$, where resummation of Sudakov logarithms is required for reliable prediction
- ♦ But so far no all-order factorization theorem existed for jet broadening

Collinear Anomaly



Soft-collinear factorization in SCET

- ♦ Common to Drell-Yan at small q_T and jet broadening at small $b_{L,R}$ is that observables select final-state partons with small transverse momenta $p_i^\perp = \lambda M$; $\lambda \ll 1$
- ♦ Partons can be (anti-)collinear, aligned with initial- or final-state jets, or soft
- ♦ Describe these in **soft-collinear effective theory (SCET)** in terms of (anti-)collinear and soft quark and gluon fields

Soft-collinear factorization in SCET

- ◆ Relevant effective theory SCET_{II} contains collinear, anti-collinear, and soft partons with momenta:

$$p_i^c \sim (\lambda^2, 1, \lambda)M$$

$$p_i^{\bar{c}} \sim (1, \lambda^2, \lambda)M$$

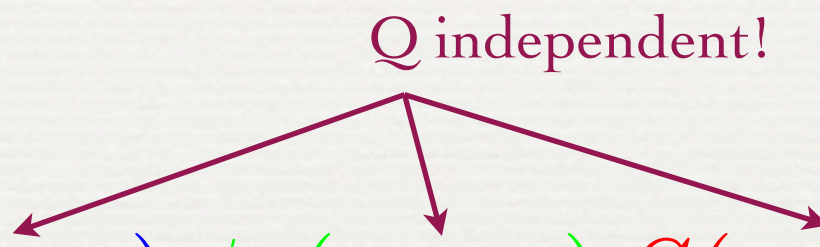
$$p_i^s \sim (\lambda, \lambda, \lambda)M$$

- ◆ Classical effective Lagrangian contains no interactions between different modes, implying a complete factorization:


$$\mathcal{L}_{\text{SCET}_{\text{II}}} = \mathcal{L}_c + \mathcal{L}_{\bar{c}} + \mathcal{L}_s$$

Soft-collinear factorization in SCET

- ◆ If this was true, then:

$$d\sigma \sim H(Q, \mu) \phi_c(q_T, \mu) \phi_{\bar{c}}(q_T, \mu) S(q_T, \mu)$$


- ◆ But RGE for hard function shows that this cannot be correct:

$$\frac{d}{d \ln \mu} H(Q^2, \mu) = \left[2\Gamma_{\text{cusp}}(\alpha_s) \ln \frac{Q^2}{\mu^2} + 4\gamma^q(\alpha_s) \right] H(Q^2, \mu)$$


- ◆ RG invariance of cross section implies that soft-collinear part $\phi_c \phi_{\bar{c}} S$ must carry some hidden (anomalous) dependence on Q

→ not observed in previous SCET papers on q_T resummation:

Gao, Li, Liu 2005; Idilbi, Ji, Yuan 2005; Mantry, Petriello 2009

Soft-collinear factorization in SCET

- ♦ At classical level, the SCET_{II} Lagrangian

$$\mathcal{L}_{\text{SCET}_{\text{II}}} = \mathcal{L}_c + \mathcal{L}_{\bar{c}} + \mathcal{L}_s$$

exhibits certain symmetries, e.g.:

- ♦ \mathcal{L}_c is invariant under rescalings $\bar{p} \rightarrow \bar{\lambda}\bar{p}$ of anti-collinear jet momentum
- ♦ $\mathcal{L}_{\bar{c}}$ is invariant under rescalings $p \rightarrow \lambda p$ of collinear jet momentum
- ♦ This symmetry is **anomalous**, not preserved by regularization (broken to subgroup $\lambda\bar{\lambda} = 1$)

“collinear anomaly”

Soft-collinear factorization in SCET

- ♦ Not an anomaly of QCD, but of the **effective theory** relevant to QCD factorization
- ♦ In a different context (B→π form factor), Beneke called this the “factorization anomaly”
Dubna lectures 2005
- ♦ Fact that **additional Q dependence** arises from a **quantum anomaly** gives rise to stringent constraints, which imply that it exponentiates; e.g. for Drell-Yan production at small q_T:

$$(Q^2 x_T^2)^{-F(x_T^2, \mu)} = \exp \left[-F(x_T^2, \mu) \ln (Q^2 x_T^2) \right]$$

↑
calculable if $x_T \ll \Lambda^{-1}$

↑
single logarithm

Soft-collinear factorization in SCET

- ◆ There exist many ways to regularize the loop graphs giving rise to the anomaly, but dimensional regularization alone is not sufficient
- ◆ Here we use **analytic regularization** Smirnov 1993
- ◆ Other schemes have been proposed, e.g. the “**rapidity RG**”, but their consistency has not yet been demonstrated beyond 1-loop order
Chiu, Jain, Neill, Rothstein 2011; see also: Manohar, Stewart 2006
- ◆ For any consistent scheme, final results will be independent of the regularization procedure

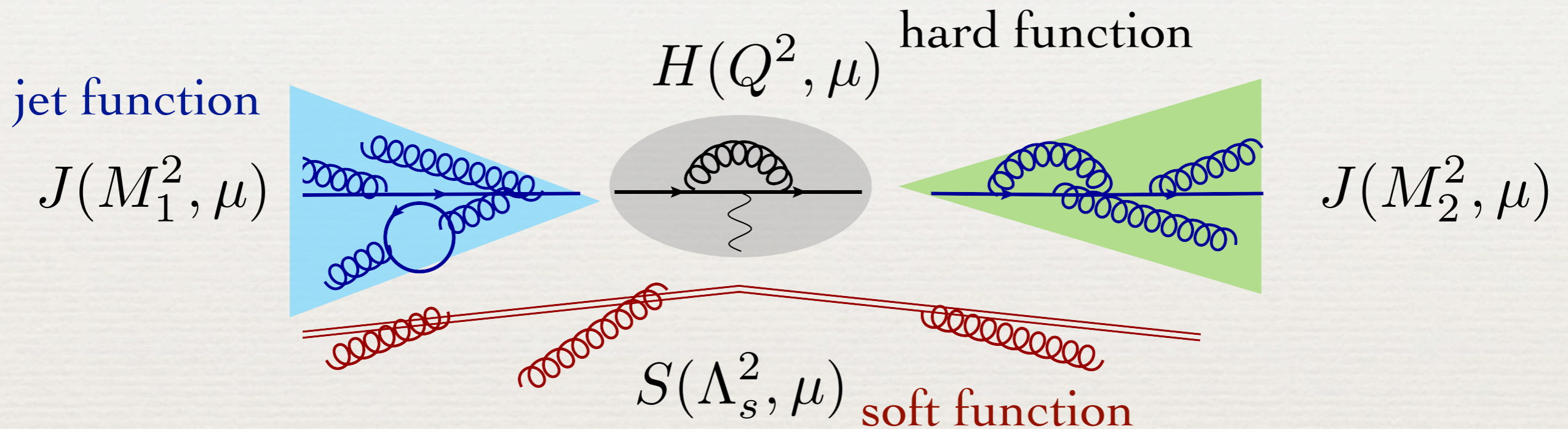


Factorization and Resummation for the Drell-Yan Cross Section at small q_T

(T. Becher, MN, arXiv:1007.4005)

Drell-Yan cross section in SCET

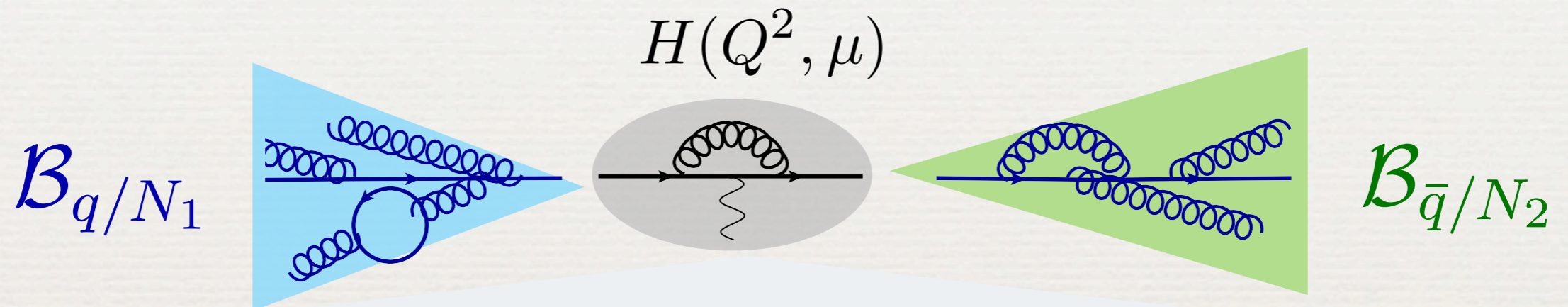
- ◆ Naive soft-collinear factorization:



- ◆ In our regularization scheme the soft contribution in this particular case gives rise to scaleless integrals that vanish

Drell-Yan cross section in SCET

- Remaining naive factorization formula:



“hard function” \otimes “transverse PDF” \otimes “transverse PDF”

- Transverse PDF:

$$\mathcal{B}_{q/N}(z, x_T^2, \mu) = \frac{1}{2\pi} \int dt e^{-izt\bar{n}\cdot p} \langle N(p) | \bar{\chi}(t\bar{n} + x_\perp) \frac{\not{\bar{n}}}{2} \chi(0) | N(p) \rangle$$

This spells trouble: well known that transverse PDF
not well defined without additional regulator

Drell-Yan cross section in SCET

- ◆ Remaining naive factorization formula:

$$\frac{d^3\sigma}{dM^2 dq_T^2 dy} = \frac{4\pi\alpha^2}{3N_c M^2 s} |H(M^2, \mu)| \frac{1}{4\pi} \int d^2x_\perp e^{-iq_\perp \cdot x_\perp} \\ \times \sum_q e_q^2 \left[\mathcal{B}_{q/N_1}(\xi_1, x_T^2, \mu) \mathcal{B}_{\bar{q}/N_2}(\xi_2, x_T^2, \mu) + (q \leftrightarrow \bar{q}) \right] + \mathcal{O}\left(\frac{q_T^2}{M^2}\right)$$

where: $\xi_1 = \sqrt{\tau} e^y$, $\xi_2 = \sqrt{\tau} e^{-y}$, with $\tau = \frac{m_\perp^2}{s} = \frac{M^2 + q_T^2}{s}$

- ◆ Resummation would then be accomplished by solving the RGE for the hard function:

$$\frac{d}{d \ln \mu} H(M^2, \mu) = \left[2\Gamma_{\text{cusp}}^F(\alpha_s) \ln \frac{M^2}{\mu^2} + 4\gamma^q(\alpha_s) \right] H(M^2, \mu)$$

→ see SCET papers by: Gao, Li, Liu 2005; Idilbi, Ji, Yuan 2005; Mantry, Petriello 2009

Drell-Yan cross section in SCET

- ◆ Remaining naive factorization formula:

$$\frac{d^3\sigma}{dM^2 dq_T^2 dy} = \frac{4\pi\alpha^2}{3N_c M^2 s} |H(M^2, \mu)| \frac{1}{4\pi} \int d^2x_\perp e^{-iq_\perp \cdot x_\perp} \times \sum_q e_q^2 \left[\mathcal{B}_{q/N_1}(\xi_1, x_T^2, \mu) \mathcal{B}_{\bar{q}/N_2}(\xi_2, x_T^2, \mu) + (q \leftrightarrow \bar{q}) \right] + \mathcal{O}\left(\frac{q_T^2}{M^2}\right)$$

where: $\xi_1 = \sqrt{\tau} e^y$, $\xi_2 = \sqrt{\tau} e^{-y}$, with $\tau = \frac{m_\perp^2}{s} = \frac{M^2 + q_T^2}{s}$

- ◆ Resummation would then be accomplished by solving the RGE for the hard function:

$$\frac{d}{d \ln \mu} H(M^2, \mu) = \left[2\Gamma_F(\alpha_s) \ln \frac{M^2}{\mu^2} + 4\gamma^q(\alpha_s) \right] H(M^2, \mu)$$

This must be wrong!

→ see SCET papers by: Gao, Li, Liu 2005; Idilbi, Ji, Yuan 2005; Mantry, Petriello 2009

Collinear anomaly

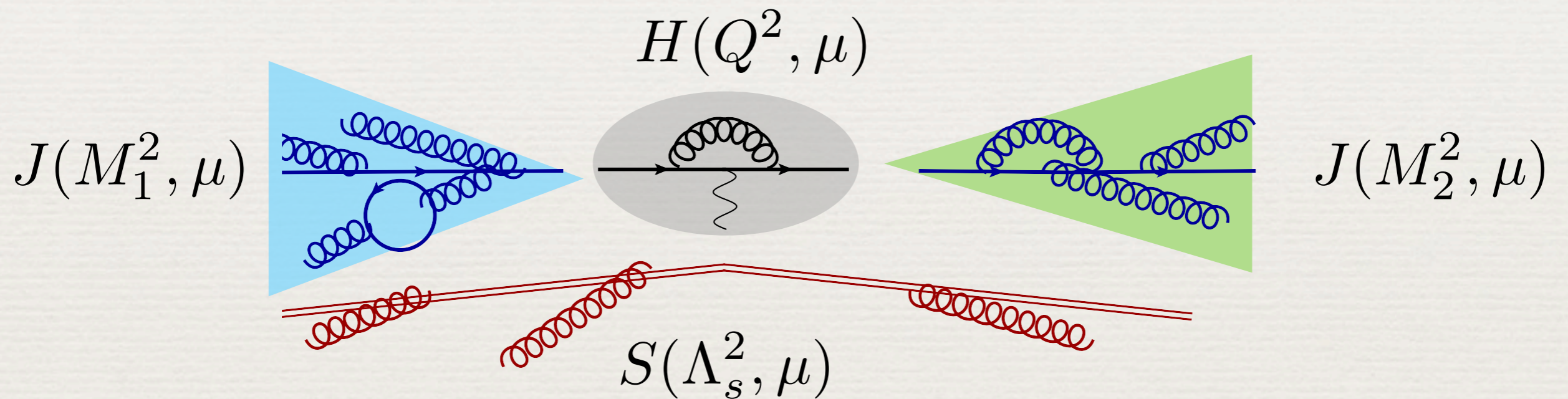
- ◆ RG invariance of the cross section requires that the product $\mathcal{B}_{q/N_1}(\xi_1, x_T^2, \mu) \mathcal{B}_{\bar{q}/N_2}(\xi_2, x_T^2, \mu)$ must contain a hidden M dependence
- ◆ Analyzing the relevant diagrams, we find that an additional regulator is needed to make transverse PDFs well defined; in the product of two PDFs this regulator can be removed, but an anomalous M dependence remains:

$$[\mathcal{B}_{q/N_1}(z_1, x_T^2, \mu) \mathcal{B}_{\bar{q}/N_2}(z_2, x_T^2, \mu)]_{M^2} = \left(\frac{x_T^2 M^2}{4e^{-2\gamma_E}} \right)^{-F_{q\bar{q}}(x_T^2, \mu)} \mathcal{B}_{q/N_1}(z_1, x_T^2, \mu) \mathcal{B}_{\bar{q}/N_2}(z_2, x_T^2, \mu)$$

with:
$$\frac{dF_{q\bar{q}}(x_T^2, \mu)}{d \ln \mu} = 2\Gamma_{\text{cusp}}^F(\alpha_s)$$

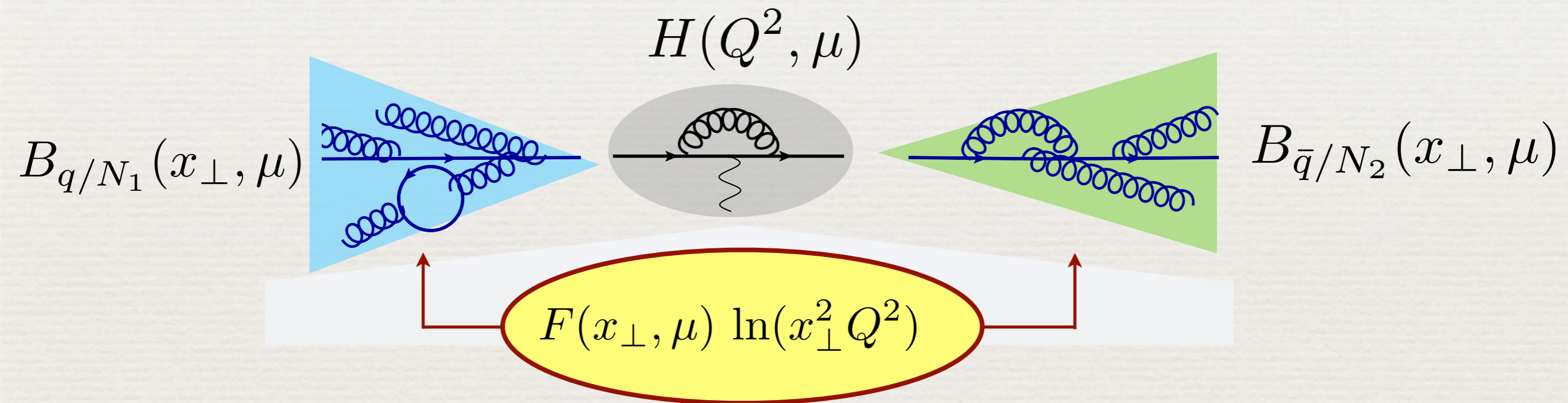
Collinear anomaly

- ♦ Regular soft-collinear factorization:



Collinear anomaly

- ♦ Anomalous soft-collinear factorization:



Transverse PDFs

“What God has joined together, let no man separate...”

- ◆ The “operator definition of TMP PDFs is quite problematic [...] and is nowadays under active investigation” Cherednikov, Stefanis 2009

for a review, see: Collins 2003, 2008

for an elegant recent definition, see: Collins 2011

- ◆ Our result:
Regularization of individual transverse PDFs is delicate, but the product of two transverse PDFs is well defined and has a specific dependence on hard momentum transfer M^2

Comparison with the CSS formula

- ♦ Classic result from **Collins-Soper-Sterman:** 1985

$$\begin{aligned} \frac{d^3\sigma}{dM^2 dq_T^2 dy} &= \frac{4\pi\alpha^2}{3N_c M^2 s} \frac{1}{4\pi} \int d^2x_\perp e^{-iq_\perp \cdot x_\perp} \sum_q e_q^2 \sum_{i=q,g} \sum_{j=\bar{q},g} \int_{\xi_1}^1 \frac{dz_1}{z_1} \int_{\xi_2}^1 \frac{dz_2}{z_2} \\ &\times \exp \left\{ - \int_{\mu_b^2}^{M^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\ln \frac{M^2}{\bar{\mu}^2} A(\alpha_s(\bar{\mu})) + B(\alpha_s(\bar{\mu})) \right] \right\} \\ &\times \left[\bar{\mathcal{P}}_{q/N_1}(\xi_1, x_T, \mu_b) \bar{\mathcal{P}}_{\bar{q}/N_2}(\xi_2, x_T, \mu_b) + (q, i \leftrightarrow \bar{q}, j) \right] \end{aligned}$$

$$\mu_b = \frac{2e^{-\gamma_E}}{x_T}$$

- ♦ Disadvantages compared with our approach:
 - ♦ $\bar{\mu}$ integral hits the **Landau pole of running coupling** and requires PDFs at arbitrarily low scales
 - ♦ practical calculations employ an x_T -space cutoff, which introduces some ad hoc **model dependence**

Comparison with the CSS formula

- Classic result from **Collins-Soper-Sterman**: 1985

$$\frac{d^3\sigma}{dM^2 dq_T^2 dy} = \frac{4\pi\alpha^2}{3N_c M^2 s} \frac{1}{4\pi} \int d^2x_\perp e^{-iq_\perp \cdot x_\perp} \sum_q e_q^2 \sum_{i=q,g} \sum_{j=\bar{q},g} \int_{\xi_1}^1 \frac{dz_1}{z_1} \int_{\xi_2}^1 \frac{dz_2}{z_2}$$

$$\times \exp \left\{ - \int_{\mu_b^2}^{M^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\ln \frac{M^2}{\bar{\mu}^2} A(\alpha_s(\bar{\mu})) + B(\alpha_s(\bar{\mu})) \right] \right\}$$

$$\times \left[\bar{\mathcal{P}}_{q/N_1}(\xi_1, x_T, \mu_b) \bar{\mathcal{P}}_{\bar{q}/N_2}(\xi_2, x_T, \mu_b) + (q, i \leftrightarrow \bar{q}, j) \right]$$

$$\mu_b = \frac{2e^{-\gamma_E}}{x_T}$$

- All-order equivalence to our result, if:

$$A(\alpha_s) = \Gamma_{\text{cusp}}^F(\alpha_s) - \frac{\beta(\alpha_s)}{2} \frac{dg_1(\alpha_s)}{d\alpha_s}, \quad g_1(\alpha_s) = F(0, \alpha_s)$$

$$B(\alpha_s) = 2\gamma^q(\alpha_s) + g_1(\alpha_s) - \frac{\beta(\alpha_s)}{2} \frac{dg_2(\alpha_s)}{d\alpha_s}, \quad g_2(\alpha_s) = \ln H(-\mu^2, \mu)$$

$$\bar{\mathcal{P}}_{i/N}(\xi, x_T) = H(-\mu_b^2, \mu_b) B_{i/N}(\xi, x_T^2, \mu_b)$$

anomaly contributions

Comparison with the CSS formula

- ◆ Only **linear dependence on $\log(Q)$** in exponent can be made consistent with CSS formula!
- ◆ Non-trivial soft function absent in CSS, too!
- ◆ Anomaly implies a non-trivial contribution to A , such that $A(\alpha_s) \neq \Gamma_{\text{cusp}}^F(\alpha_s)$ in this case!
→ missed by all previous SCET analyses:
Gao, Li, Liu 2005; Idilbi, Ji, Yuan 2005; Mantry, Petriello 2009
- ◆ Can predict unknown 3-loop coefficient of A based on known 2-loop result for B :

$$\Gamma_2^F = 538.2 \text{ while } A^{(3)} = -930.8 \quad \rightarrow \text{important effect}$$

Simplification for $x_T \ll \Lambda^{-1}$ (large q_T)

- ◆ Can perform operator product expansion:

$$\mathcal{B}_{i/N}(\xi, x_T^2, \mu) = \sum_j \int_{\xi}^1 \frac{dz}{z} \mathcal{I}_{i \leftarrow j}(z, x_T^2, \mu) \phi_{j/N}(\xi/z, \mu) + \mathcal{O}(\Lambda_{\text{QCD}}^2 x_T^2)$$

- ◆ Only the product of two $\mathcal{I}_{i \leftarrow j}(z, x_T^2, \mu)$ functions is well defined due to the anomaly:

$$[\mathcal{I}_{q \leftarrow i}(z_1, x_T^2, \mu) \mathcal{I}_{\bar{q} \leftarrow j}(z_2, x_T^2, \mu)]_{q^2} = \left(\frac{x_T^2 q^2}{4e^{-2\gamma_E}} \right)^{-F_{q\bar{q}}(x_T^2, \mu)} I_{q \leftarrow i}(z_1, x_T^2, \mu) I_{\bar{q} \leftarrow j}(z_2, x_T^2, \mu)$$

↑ anomalous q^2 dependence

- ◆ Using analytic regulators in the calculation of these functions is very economical, since it does not introduce any new scales

Simplification for $x_T \ll \Lambda^{-1}$ (large q_T)

- ◆ Factorized cross section at small q_T :

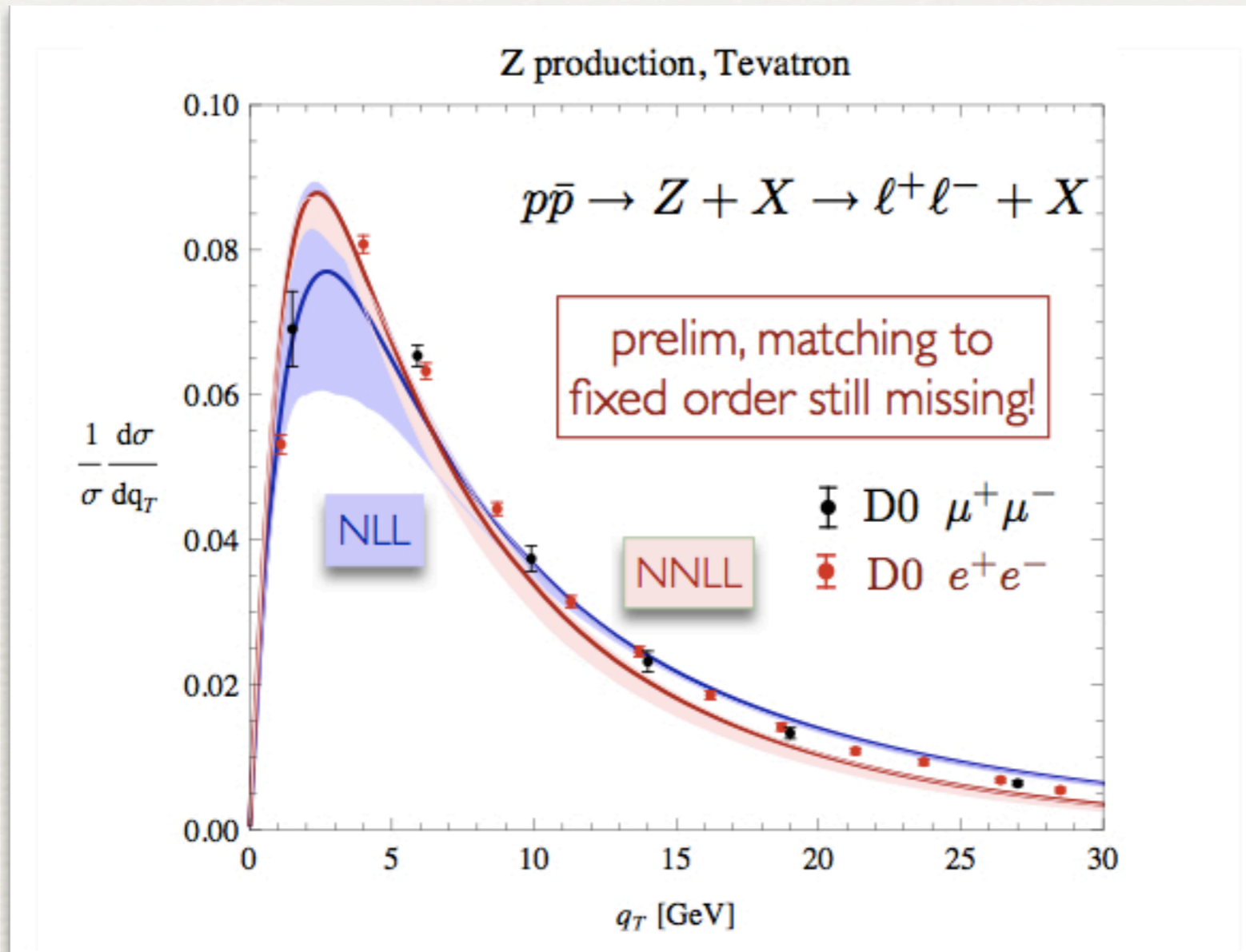
$$\frac{d^3\sigma}{dM^2 dq_T^2 dy} = \frac{4\pi\alpha^2}{3N_c M^2 s} \sum_q e_q^2 \sum_{i=q,g} \sum_{j=\bar{q},g} \int_{\xi_1}^1 \frac{dz_1}{z_1} \int_{\xi_2}^1 \frac{dz_2}{z_2} \times \left[C_{q\bar{q}\rightarrow ij} \left(\frac{\xi_1}{z_1}, \frac{\xi_2}{z_2}, q_T^2, M^2, \mu \right) \phi_{i/N_1}(z_1, \mu) \phi_{j/N_2}(z_2, \mu) + (q, i \leftrightarrow \bar{q}, j) \right]$$

- ◆ Hard-scattering kernels:

$$C_{q\bar{q}\rightarrow ij}(z_1, z_2, q_T^2, M^2, \mu) = H(M^2, \mu) \frac{1}{4\pi} \int d^2x_\perp e^{-iq_\perp \cdot x_\perp} \left(\frac{x_T^2 M^2}{4e^{-2\gamma_E}} \right)^{-F_{q\bar{q}}(x_T^2, \mu)} \times I_{q\leftarrow i}(z_1, x_T^2, \mu) I_{\bar{q}\leftarrow j}(z_2, x_T^2, \mu)$$

- ◆ Two sources of M dependence: **hard function** and **collinear anomaly**

Numerical results (preliminary)





Factorization and Resummation for Jet Broadening in e^+e^- Annihilation

(T. Becher, G. Bell, MN, arXiv:1104.4108)

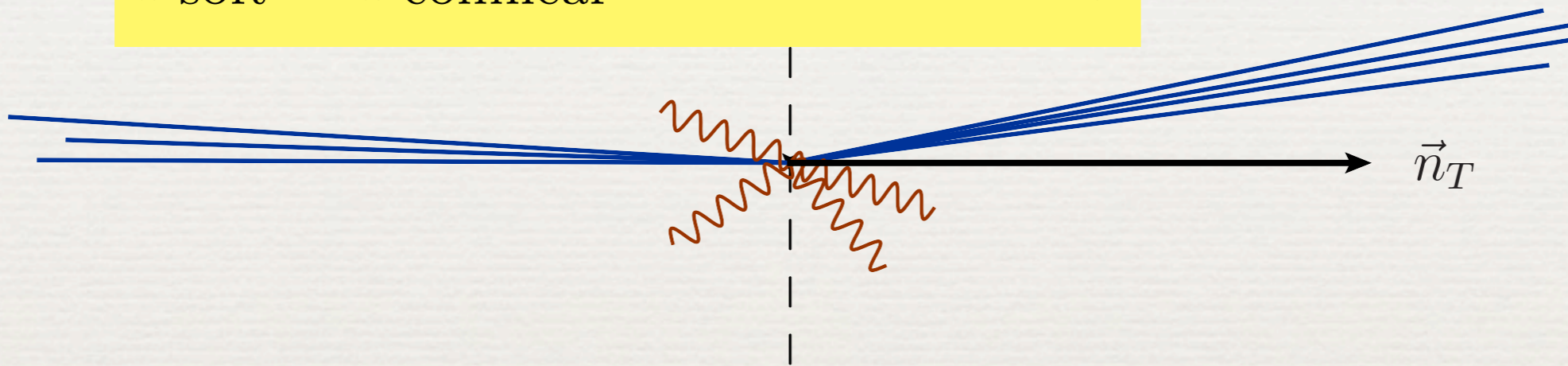
Factorization for jet broadening

Problem that individual jet and soft functions are not well defined without additional regularization also arises in other factorization theorems

- ♦ electroweak Sudakov resummation (and any other process at high Q^2 with small but nonzero masses) Chiu, Golf, Kelley, Manohar 2007
- ♦ other observables sensitive to transverse momenta, such as jet broadening Becher, Bell, MN 2011

Factorization for jet broadening

$$p_{\text{soft}}^\perp \sim p_{\text{collinear}}^\perp \sim b_L \sim b_R \ll Q$$



- ♦ Naive factorization theorem for broadening, (jets recoil against soft radiation):

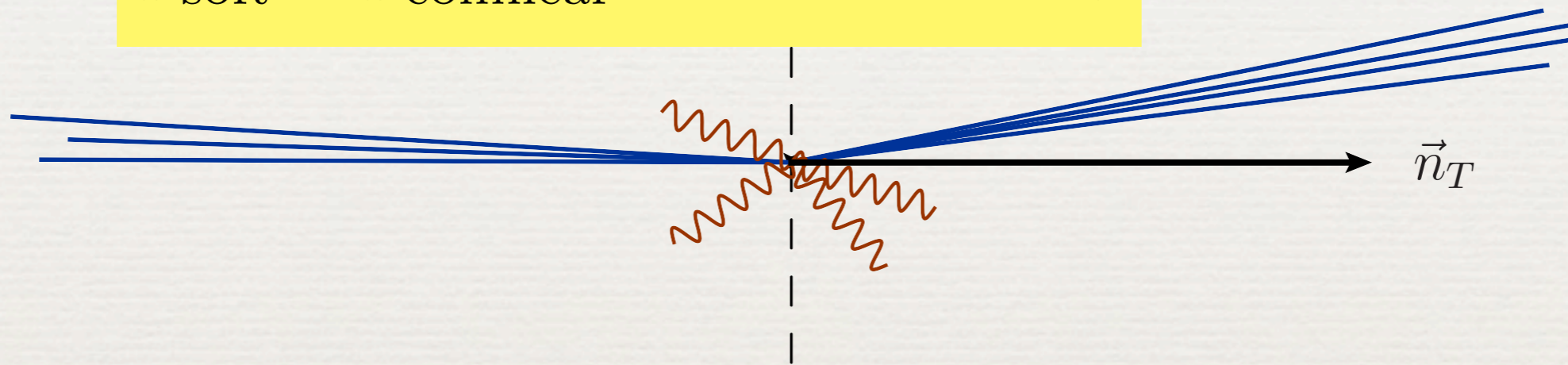
$$\frac{1}{\sigma_0} \frac{d^2\sigma}{db_L db_R} = H(Q^2, \mu) \int db_L^s \int db_R^s \int d^{d-2}p_L^\perp \int d^{d-2}p_R^\perp$$

$$\times \mathcal{J}_L(b_L - b_L^s, p_L^\perp, \mu) \mathcal{J}_R(b_R - b_R^s, p_R^\perp, \mu) \mathcal{S}(b_L^s, b_R^s, -p_L^\perp, -p_R^\perp, \mu)$$

- ♦ Non-trivial soft function arises in this case, since radiation is restricted to hemispheres

Factorization for jet broadening

$$p_{\text{soft}}^\perp \sim p_{\text{collinear}}^\perp \sim b_L \sim b_R \ll Q$$



- ◆ Laplace ($b_{L,R} \rightarrow \tau_{L,R}$) and Fourier transforms ($p_{L,R}^\perp \rightarrow z_{L,R} = 2|x_{L,R}^\perp|/\tau_{L,R}$):

$$\frac{1}{\sigma_0} \frac{d^2\sigma}{d\tau_L d\tau_R} = H(Q^2, \mu) \int_0^\infty dz_L \int_0^\infty dz_R \bar{\mathcal{J}}_L(\tau_L, z_L, \mu) \bar{\mathcal{J}}_R(\tau_R, z_R, \mu) \bar{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R, \mu)$$

- ◆ Jet and soft functions must contain a hidden (anomalous) Q dependence

Anomalous factorization

- ◆ Have derived the Q dependence of product

$$P(Q^2, \tau_L, \tau_R, z_L, z_R, \mu) = \overline{\mathcal{J}}_L(\tau_L, z_L, \mu) \overline{\mathcal{J}}_R(\tau_R, z_R, \mu) \overline{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R, \mu)$$

using invariance under analytic regularization

- ◆ General result:

double logarithm!

single logarithms

$$\ln P = \frac{k_2(\mu)}{4} \ln^2(Q^2 \bar{\tau}_L \bar{\tau}_R) - F_B(\tau_L, z_L, \mu) \ln(Q^2 \bar{\tau}_L^2) - F_B(\tau_R, z_R, \mu) \ln(Q^2 \bar{\tau}_R^2) + \ln W(\tau_L, \tau_R, z_L, z_R, \mu)$$

with:

$$\frac{d}{d \ln \mu} k_2(\mu) = 0, \quad \frac{d}{d \ln \mu} F_B(\tau, z, \mu) = \Gamma_{\text{cusp}}(\alpha_s)$$

Anomalous factorization

◆ General result:

double logarithm!

single logarithms

$$\ln P = \frac{k_2(\mu)}{4} \ln^2(Q^2 \bar{\tau}_L \bar{\tau}_R) - F_B(\tau_L, z_L, \mu) \ln(Q^2 \bar{\tau}_L^2) - F_B(\tau_R, z_R, \mu) \ln(Q^2 \bar{\tau}_R^2) + \ln W(\tau_L, \tau_R, z_L, z_R, \mu)$$

with:

$$\frac{d}{d \ln \mu} k_2(\mu) = 0, \quad \frac{d}{d \ln \mu} F_B(\tau, z, \mu) = \Gamma_{\text{cusp}}(\alpha_s)$$

◆ Perturbative analysis reveals that $k_2 = 0$ (to all orders), and:

$$F_B(\tau, z, \mu) = \frac{C_F \alpha_s}{\pi} \left[\ln(\mu \bar{\tau}) + \ln \frac{\sqrt{1+z^2} + 1}{4} \right] + \mathcal{O}(\alpha_s^2)$$

Anomalous factorization

- ◆ First all-order factorization formula:

$$\frac{1}{\sigma_0} \frac{d^2\sigma}{d\tau_L d\tau_R} = H(Q^2, \mu) \int_0^\infty dz_L \int_0^\infty dz_R (Q^2 \bar{\tau}_L^2)^{-F_B(\tau_L, z_L, \mu)} (Q^2 \bar{\tau}_R^2)^{-F_B(\tau_R, z_R, \mu)} \times W(\tau_L, \tau_R, z_L, z_R, \mu)$$

anomalous Q dependence

- ◆ At NLL order, Mellin inversion can be done analytically:

$$\frac{1}{\sigma_0} \frac{d\sigma}{db_T} = H(Q^2, \mu) \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \frac{1}{b_T} \left(\frac{b_T}{\mu}\right)^{2\eta} I^2(\eta)$$

with:

$$I(\eta) = \int_0^\infty dz \frac{z}{(1+z^2)^{3/2}} \left(\frac{\sqrt{1+z^2}+1}{4}\right)^{-\eta}, \quad \eta \equiv \frac{C_F \alpha_s(\mu)}{\pi} \ln \frac{Q^2}{\mu^2}$$

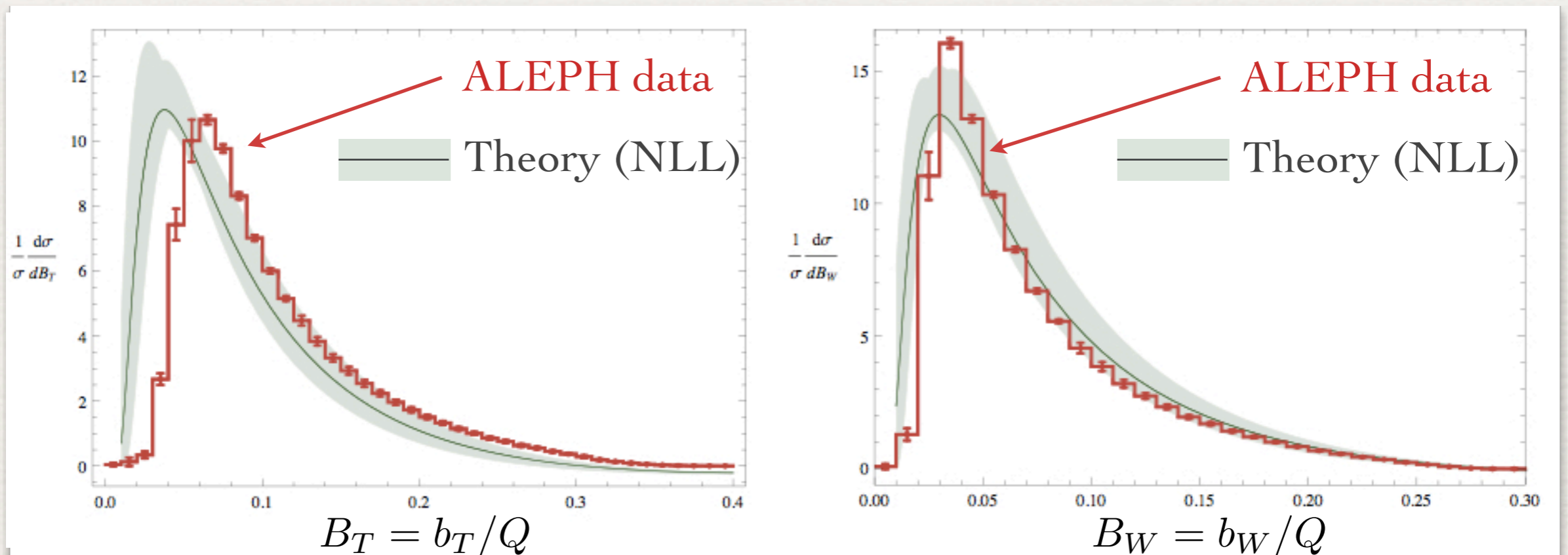
→ equivalent to: Dokshitzer, Lucenti, Markesini, Salam 1998

[correcting Catani, Turnock, Webber 1992, who missed the $I^2(\eta)$ term]

→ $I^2(\eta)$ term also missed in: Chiu, Jain, Neill, Rothstein 2011

Numerical results (preliminary)

- ◆ Comparison with ALEPH data ($Q=91.2$ GeV)
- ◆ Theory predictions at NLL order, still without matching to NLO



- ◆ Calculation of NNLL terms desired!

Extension to NNLL?

- ◆ Have operator definitions of jet and soft functions, e.g.:

$$\frac{\pi}{2} (\not{n})_{\alpha\beta} \mathcal{J}_L(b, p^\perp, \mu) = \sum_X (2\pi)^d \delta(\bar{n} \cdot p_X - Q) \delta^{d-2}(p_X^\perp - p^\perp) \\ \times \delta\left(b - \frac{1}{2} \sum_{i \in X} |p_i^\perp|\right) \langle 0 | \chi_\alpha(0) | X \rangle \langle X | \bar{\chi}_\beta(0) | 0 \rangle$$

- ◆ For NNLL accuracy we need one-loop jet and soft functions (latter is known) and two-loop anomaly function $F_B(\tau, z, \mu)$
- ◆ Appears doable and worthwhile

Conclusions

- ♦ Have derived **all-order resummed expression** for Drell-Yan cross section at small $q_T \ll M$
 - ♦ Naive factorization broken by **collinear anomaly**
 - ♦ Correct SCET analysis reproduces CSS formula with a **nontrivial relation** between A and Γ_{cusp} ; predicted $A^{(3)}$, last missing ingredient for NNLL
 - ♦ Transverse PDFs **do not exist** as individual objects; ^{*}) only products of two PDFs are well defined, and carry an **anomalous M dependence**
- ^{*}) They are gauge dependent in the standard treatment and affected by (dim. unregularized) “rapidity divergences”

Conclusions

- ◆ Extending these methods, we have derived the **first all-order resummation formula** for jet broadening in e^+e^- annihilations
- ◆ Features non-trivial **anomalous Q dependence** due to anomaly
- ◆ NLL results agree with (the correct) known expressions in literature
- ◆ Calculations necessary to achieve NNLL resummation appear feasible
- ◆ Phenomenology in progress

BACKUP SLIDES:

Analytic regulators at work

Drell-Yan cross section in SCET

Side remark:

- ◆ Absence of soft contributions $k \sim (\lambda, \lambda, \lambda)$ follows after proper multipole expansion using that $x \sim (1, 1, \lambda^{-1})$, which implies:

$$(p - k) \cdot x = p \cdot x - k_{\perp} \cdot x_{\perp} + \mathcal{O}(\lambda)$$

- ◆ Relevant loops integrals such as

$$\int d^d k \frac{1}{(n \cdot k - i\epsilon)^{1+\alpha}} \frac{1}{(\bar{n} \cdot k - i\epsilon)^{1+\beta}} \delta(k^2) \theta(k^0) e^{ip \cdot x - ik_{\perp} \cdot x_{\perp}}$$

are scaleless and vanish in dimensional regularization

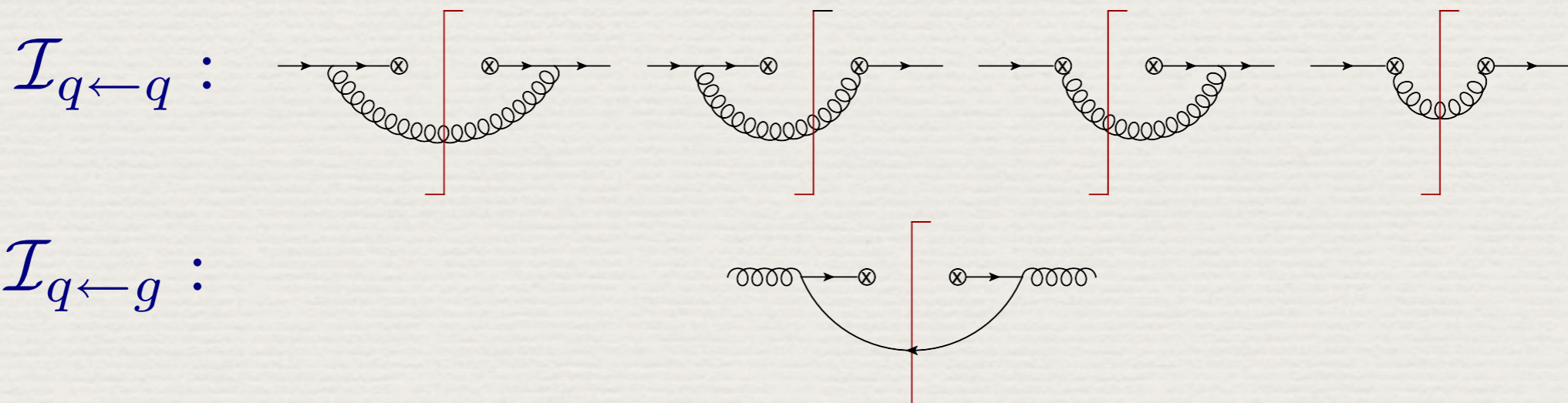
Short-distance expansion for $x_T \ll \Lambda_{\text{QCD}}^{-1}$

- Generalized PDFs at small transverse separation can be expanded in usual PDFs:

$$\mathcal{B}_{i/N}(\xi, x_T^2, \mu) = \sum_j \int_\xi^1 \frac{dz}{z} \mathcal{I}_{i \leftarrow j}(z, x_T^2, \mu) \phi_{j/N}(\xi/z, \mu) + \mathcal{O}(\Lambda_{\text{QCD}}^2 x_T^2)$$

$$B_{i/N}(\xi, x_T^2, \mu) = \sum_j \int_\xi^1 \frac{dz}{z} I_{i \leftarrow j}(\xi/z, x_T^2, \mu) \phi_{j/N}(z, \mu) + \mathcal{O}(\Lambda_{\text{QCD}}^2 x_T^2)$$

- Expansion kernels are obtained from matching calculation

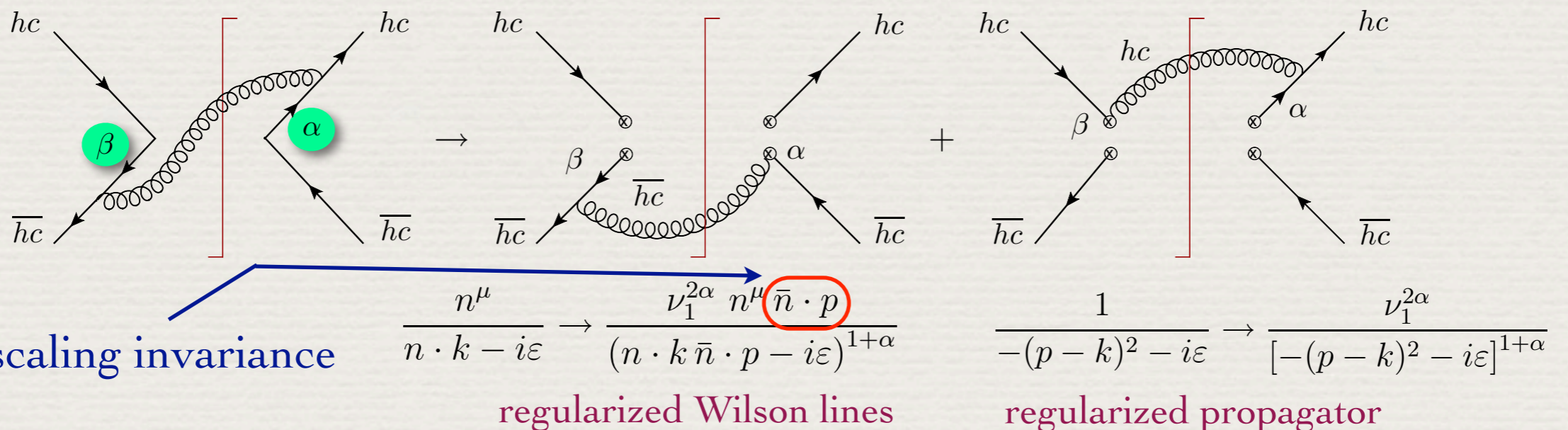


Short-distance expansion for $x_T \ll \Lambda_{\text{QCD}}^{-1}$

- ♦ Collinear loops are not defined and require a regulator beyond dimensional regularization
- ♦ Most economic possibility is to use **analytic regularization scheme**: Smirnov 1993

$$\frac{1}{-(p-k)^2 - i\varepsilon} \rightarrow \frac{\nu_1^{2\alpha}}{[-(p-k)^2 - i\varepsilon]^{1+\alpha}}$$

- ♦ Adaption to SCET collinear propagators:



Short-distance expansion for $x_T \ll \Lambda_{\text{QCD}}^{-1}$

- ♦ Introducing analogous regulator β in anti-collinear sector, we find:

$$\mathcal{I}_{q \leftarrow q}(z, x_T^2, \mu) \Big|_{\alpha \text{ reg.}} = -\frac{C_F \alpha_s}{2\pi} \left\{ \left(\frac{1}{\epsilon} + L_\perp \right) \left[\left(\frac{2}{\alpha} - 2 \ln \frac{\mu^2}{\nu_1^2} \right) \delta(1-z) + \frac{1+z^2}{(1-z)_+} \right] + \delta(1-z) \left(-\frac{2}{\epsilon^2} + L_\perp^2 + \frac{\pi^2}{6} \right) - (1-z) \right\}.$$

$$L_\perp = \ln \frac{x_T^2 \mu^2}{4e^{-2\gamma_E}}$$

$$\mathcal{I}_{q \leftarrow q}(z, x_T^2, \mu) \Big|_{\beta \text{ reg.}} = -\frac{C_F \alpha_s}{2\pi} \left\{ \left(\frac{1}{\epsilon} + L_\perp \right) \left[\left(-\frac{2}{\beta} + 2 \ln \frac{q^2}{\nu_2^2} \right) \delta(1-z) + \frac{1+z^2}{(1-z)_+} \right] - (1-z) \right\}$$

- ♦ The product of **two** such functions is regulator independent:

$$\begin{aligned} & [\mathcal{I}_{q \leftarrow q}(z_1, x_T^2, \mu) \mathcal{I}_{\bar{q} \leftarrow \bar{q}}(z_2, x_T^2, \mu)]_{q^2} \\ &= \delta(1-z_1) \delta(1-z_2) \left[1 - \frac{C_F \alpha_s}{2\pi} \left(2L_\perp \ln \frac{q^2}{\mu^2} + L_\perp^2 - 3L_\perp + \frac{\pi^2}{6} \right) \right] \\ & \quad - \frac{C_F \alpha_s}{2\pi} \left\{ \delta(1-z_1) \left[L_\perp \left(\frac{1+z_2^2}{1-z_2} \right)_+ - (1-z_2) \right] + (z_1 \leftrightarrow z_2) \right\} + \mathcal{O}(\alpha_s^2) \end{aligned}$$

anomalous hard logarithm

Short-distance expansion for $x_T \ll \Lambda_{\text{QCD}}^{-1}$

- From previous result we read off:

$$\begin{aligned}
 F_{q\bar{q}}(L_\perp, \alpha_s) &= \frac{C_F \alpha_s}{\pi} L_\perp + \mathcal{O}(\alpha_s^2) \\
 I_{q \leftarrow q}(z, L_\perp, \alpha_s) &= \delta(1-z) \left[1 + \frac{C_F \alpha_s}{4\pi} \left(L_\perp^2 + 3L_\perp - \frac{\pi^2}{6} \right) \right] \\
 &\quad - \frac{C_F \alpha_s}{2\pi} \left[L_\perp P_{q \leftarrow q}(z) - (1-z) \right] + \mathcal{O}(\alpha_s^2) \\
 I_{q \leftarrow g}(z, L_\perp, \alpha_s) &= -\frac{T_F \alpha_s}{2\pi} \left[L_\perp P_{q \leftarrow g}(z) - 2z(1-z) \right] + \mathcal{O}(\alpha_s^2)
 \end{aligned}$$

Altarelli-Parisi splitting functions

- Two-loop result for $F_{q\bar{q}}(L_\perp, \alpha_s) = \sum_{n=1}^{\infty} d_n^q(L_\perp) \left(\frac{\alpha_s}{4\pi} \right)^n$:

$$d_2^q(L_\perp) = \frac{\Gamma_0^F \beta_0}{2} L_\perp^2 + \Gamma_1^F L_\perp + d_2^q, \quad d_2^q = C_F C_A \left(\frac{808}{27} - 28\zeta_3 \right) - \frac{224}{27} C_F T_F n_f$$