Large Nc QCD with adjoint fermions on the lattice

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Action on a single site lattice

- Gauge degrees of freedom: U_μ — SU(N) group — $\mu = 1,2,3,4$ — Number of dimensions
- Fermion degrees of freedom: $\Psi_k, \bar{\Psi}_k$ — su(N) algebra — $k = 1, \cdots, f$ — Number of Dirac flavors

Gauge action

$$S^g = -bN \sum_{\mu\neq\nu=1}^{4} \mathrm{Tr}\left[U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger\right]$$

Lattice Gauge Coupling

$$b = \frac{1}{\lambda} = \frac{1}{g^2 N} = \frac{\beta}{2N^2}$$

't Hooft coupling

Standard lattice coupling

$$b = 0.35$$

$$\beta = 2.8, N = 2$$

$$\beta = 6.3, N = 3$$

Fermion action

$$S_f = \mathrm{Tr}\left[\bar{\Psi}_k D_o(m_q)\Psi_k\right] \qquad D_o(m_q) = \frac{1}{2}\left[(1+m_q) + (1-m_q)V\right]$$

Overlap Dirac operator

fermion mass

Unitary operator

$$V = \gamma_5 \epsilon(H)$$

sign function

$$\epsilon(H) = \sum_{k=1}^{n} \frac{r_k H}{H^2 + p_k}; \quad 0 < p_1 < p_2 \cdots < p_n,$$

Wilson-Dirac operator

$$H = \begin{pmatrix} 4 - m - \frac{1}{2}\sum_\mu (V_\mu + V_\mu^t) & \frac{1}{2}\sum_\mu \sigma_\mu (V_\mu - V_\mu^t) \\ -\frac{1}{2}\sum_\mu \sigma_\mu^\dagger (V_\mu - V_\mu^t) & -4 + m + \frac{1}{2}\sum_\mu (V_\mu + V_\mu^t) \end{pmatrix} \quad m \in [0,2]$$

$$= (4-m)\gamma_5 - \sum_\mu \left(w_\mu V_\mu + w_\mu^\dagger V_\mu^t\right)$$

Wilson "regulator" mass

$$w_\mu = \frac{1}{2}\begin{pmatrix} 1 & -\sigma_\mu \\ \sigma_\mu^\dagger & -1 \end{pmatrix}$$

$$V_\mu \Psi = U_\mu \Psi U_\mu^\dagger \qquad V_\mu^t \Psi = U_\mu^\dagger \Psi U_\mu$$

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Weak Coupling Perturbation Theory

Polyakov loop eigenvalues

$$D_\mu^{ij} = e^{i\theta_\mu^i}\delta_{ij}$$

Perturbation

$$U_\mu = e^{ia_\mu} D_\mu e^{-ia_\mu}$$

Leading order result

$$S_g = \sum_{i\neq j} \ln\left[\sum_\mu \sin^2 \frac{1}{2}\left(\theta_\mu^i - \theta_\mu^j\right)\right]$$

$$S_f = -2\sum_{i\neq j}\ln\left[\frac{1+m_q^2}{2} + \frac{1-m_q^2}{2}\,\frac{2\sum_\mu \sin^2\frac{\theta_\mu^i-\theta_\mu^j}{2} - m}{\sqrt{\left(2\sum_\mu \sin^2\frac{\theta_\mu^i-\theta_\mu^j}{2} - m\right)^2 + \sum_\mu \sin^2(\theta_\mu^i - \theta_\mu^j)}}\right]$$

If all $\theta_\mu^i = 0$ then $S_g \to -\infty$

But, if $m_q = 0$ then $S_f \to \infty$

Single site Polyakov loop eigenvalues and momenta on the infinite lattice

At lowest order in weak coupling perturbation theory
adjoint fermions on a single site lattice see momentum modes

$$e^{i(\theta^i_\mu - \theta^j_\mu)} \quad 1 \leq i,j \leq N$$

The N^2-1 angles, $\theta^i_\mu - \theta^j_\mu$, approach a continuum of momenta, p_μ, as N approaches infinity

We want the measure to be $\int \prod_\mu dp_\mu$ in order to reproduce correct infinite volume perturbation theory

<u>Naive fermions</u>

$$S_g = \sum_{i\neq j} \ln \left[\sum_\mu \sin^2 \frac{1}{2} \left(\theta^i_\mu - \theta^j_\mu\right) \right] \qquad S_f = \sum_{i\neq j} \ln \left[\sum_\mu \sin^2(\theta^i_\mu - \theta^j_\mu) \right]$$

Momenta, $p_\mu \in [\frac{\pi}{2}, \pi]$, will spoil the uniform measure in the large N limit.

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Massless overlap fermions

$$S_f = -2\sum_{i\neq j} \ln\left[\frac{1}{2} + \frac{1}{2}\frac{2\sum_\mu \sin^2\frac{\theta^i_\mu - \theta^j_\mu}{2} - m}{\sqrt{\left(2\sum_\mu \sin^2\frac{\theta^i_\mu-\theta^j_\mu}{2} - m\right)^2 + \sum_\mu \sin^2(\theta^i_\mu - \theta^j_\mu)}}\right]$$

Unlike naive fermions, momenta $p_\mu \in [0, \frac{\pi}{2}]$ and $p_\mu \in [\frac{\pi}{2}, \pi]$ are not identified

We need $m > 2\sum_\mu \sin^2\frac{\theta^i_\mu - \theta^j_\mu}{2}$ for the mode corresponding to that momentum to be massless

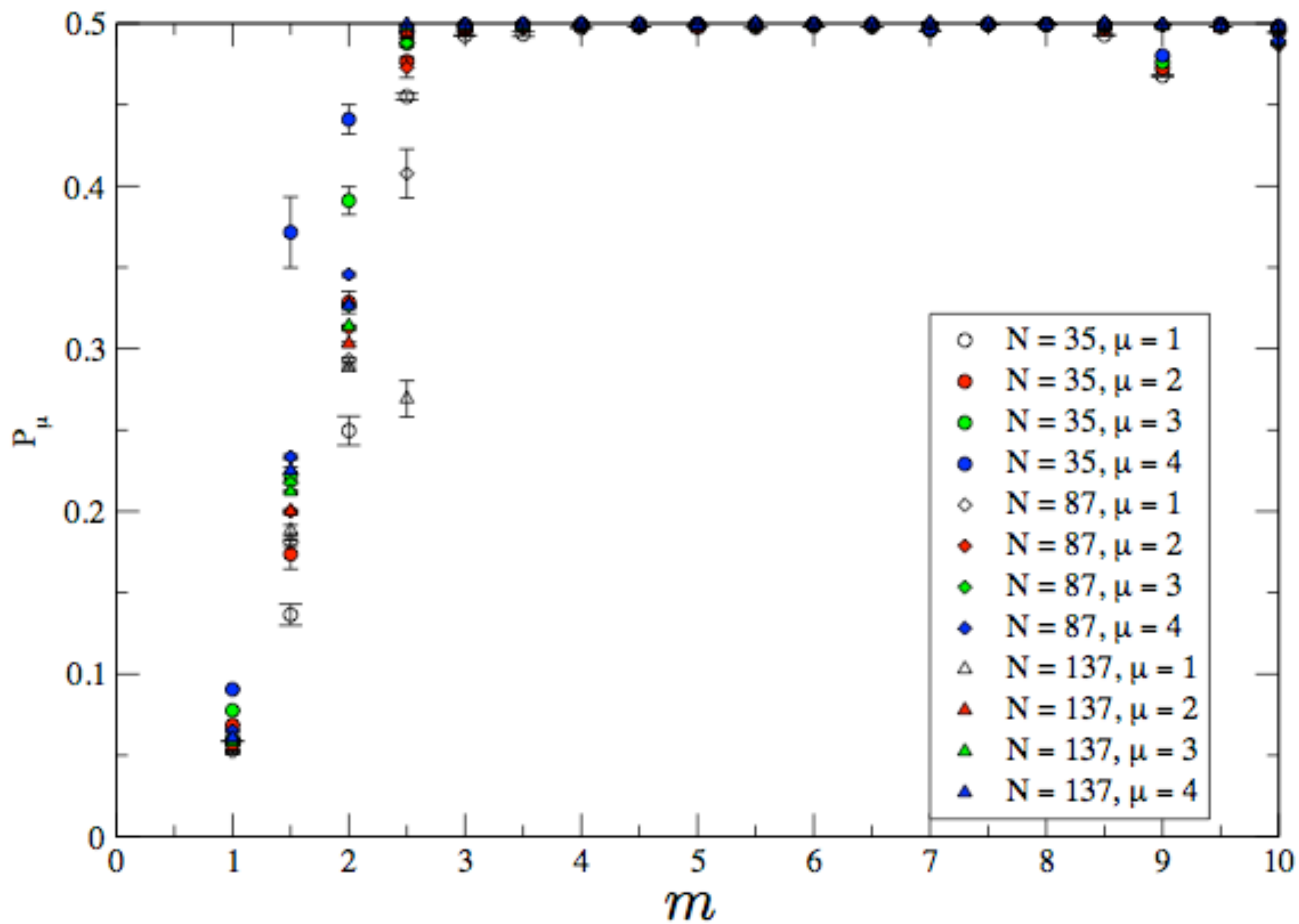
$m \to \infty$ is the naive fermion limit and so we cannot make it too large

Making it too small will restrict the region inside the Brillouin zone, $p_\mu \in [-\pi, \pi]$

where we have massless fermions and therefore the correct momentum measure

Distribution of Polyakov loop eigenvalues

$$P_\mu = \frac{1}{2}\left(1 - \frac{1}{N^2}|\mathrm{Tr}U_\mu|^2\right) = \frac{1}{N^2}\sum_{i,j}\sin^2\frac{1}{2}\left(\theta^i_\mu - \theta^j_\mu\right)$$



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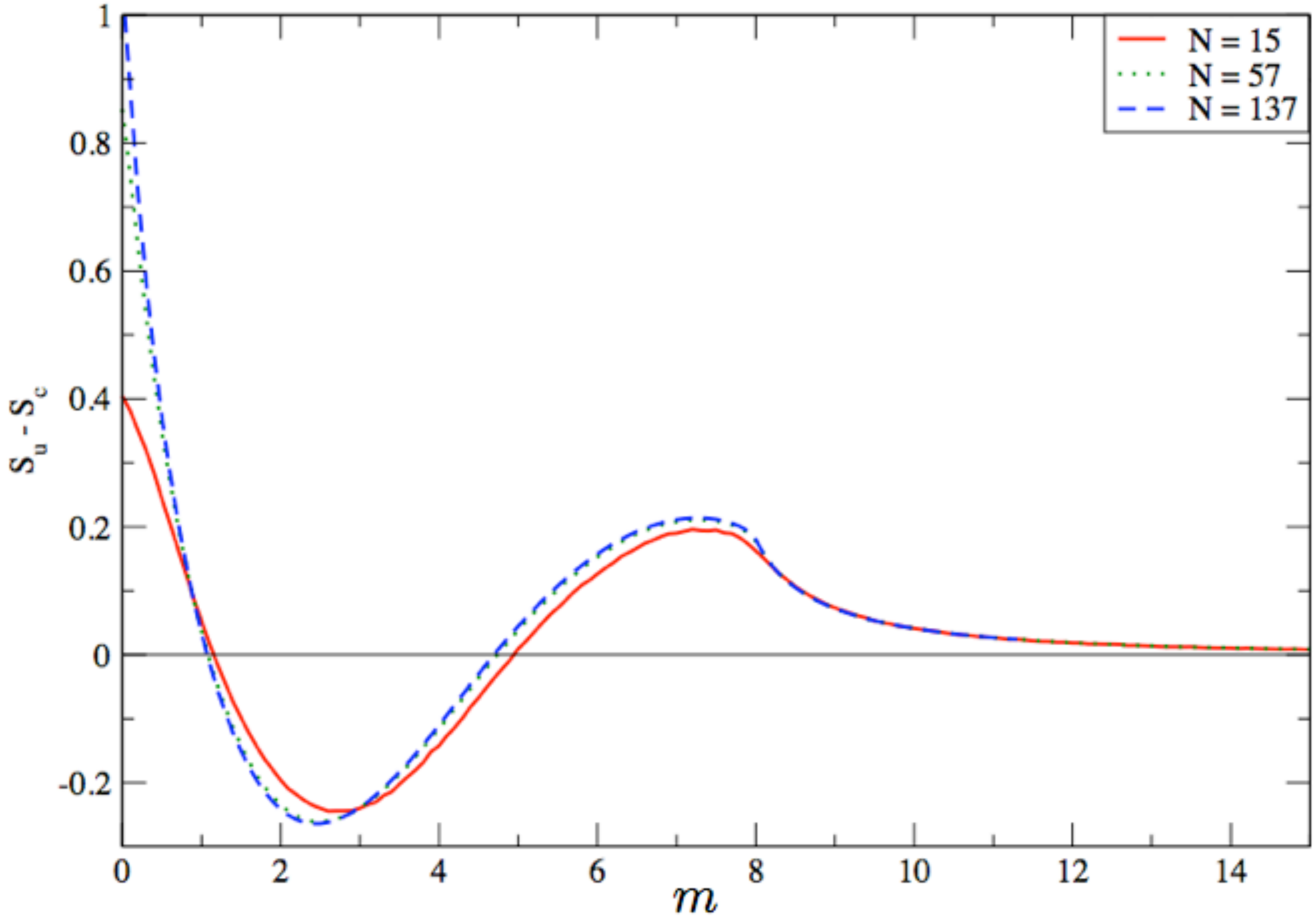
Correlated versus uncorrelated momenta

Correlated momenta $\theta^j_\mu = \frac{2\pi j}{N}$ $j = 1 \cdots, N$ in all four directions S_c: action

Uncorrelated momenta $\theta^j_\mu = \frac{2\pi\pi^\mu_j}{N}$ π^μ is a permutation of $j = 1 \cdots, N$ S_u: action



$m \in [3.5, 4.5]$ is a good choice

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Fermion momenta

The single site model stays in the correct continuum infinite volume phase if we replace

$$V_\mu \rightarrow V_\mu e^{iq_\mu} \quad q_\mu \approx 0$$

We should be able to compute flavor non-singlet meson propagators directly in momentum space

A numerical proposal

- Use Hybrid Monte Carlo Algorithm with Pseudo-fermions.
  - Works for integer number of Dirac flavors.
  - A direct fermion HMC algorithm for non-integer Dirac flavors.
- Pick N and b such that we are in the large N limit for that b.
  - We expect N to increase as b increases.
- Pick a quantity to set the scale.
  - Lowest positive eigenvalue of the overlap Dirac operator.
  - Strong to weak coupling transition.
- Find the region of b where we observe scaling.
- Measure physically interesting quantities.

We will report on progress toward this goal by showing some results with

$$f = 1$$

$$N = 18$$

$$b = 0.32, 0.35, 0.40$$

$$m_q = 0.05, 0.1$$

$$m = 4$$

Strong to weak coupling transition

Consider eigenvalues of Wilson loop operator

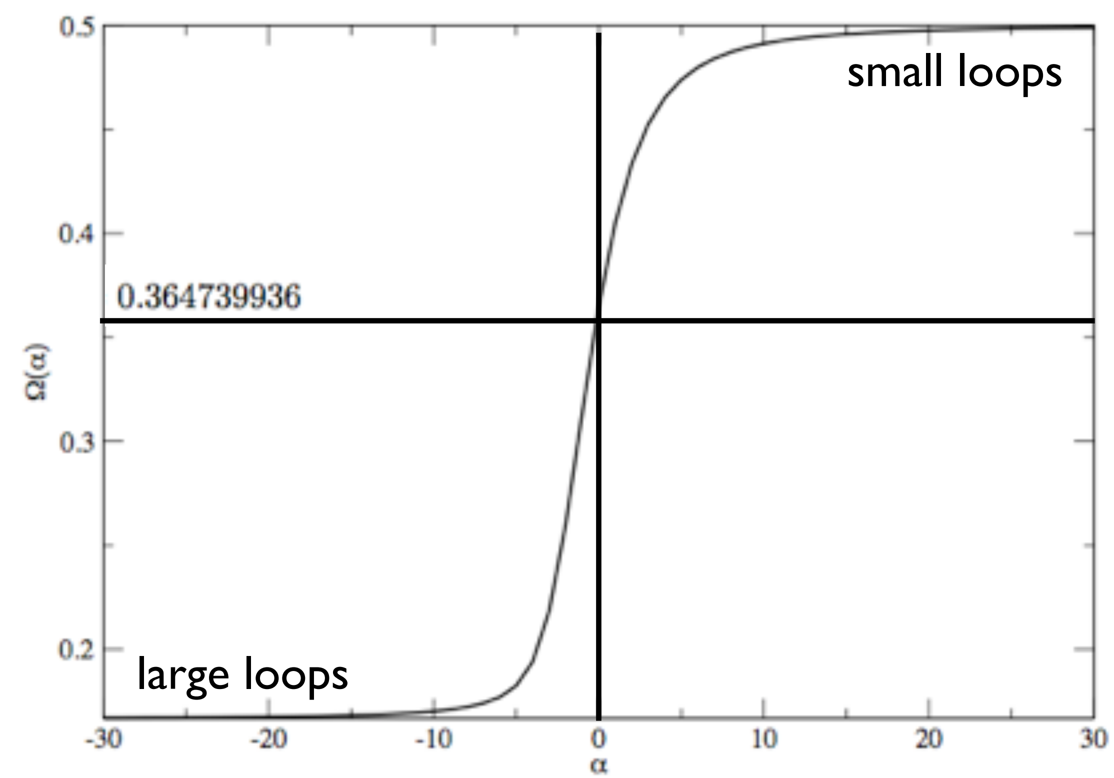
- Gauge invariant
- Distributed on a unit circle
- Sharply peaked for loops with small area (distribution has a gap at infinite N)
- Uniform for very large area (distribution has no gap at infinite N)
- Deviation from uniform distribution gives the string tension
- Phase transition from small area to large area in the limit of large N (Durhuus-Olesen transition)
- Universal behavior in the double scaling limit where the area becoming critical and the gap closes

$$O_N(y,b) = \left\langle \det\left(e^{\frac{y}{2}} + e^{-\frac{y}{2}} W\right)\right\rangle$$

$$O_N(y,b) = C_0(b,N) + C_1(b,N)y^2 + C_2(b,N)y^4 + \cdots$$

$$\Omega(b,N) = \frac{C_0(b,N)C_2(b,N)}{C_1^2(b,N)}$$

$$b = b_c(L,N)\left[1 + \frac{\alpha}{\sqrt{3N}a_2(L,N)}\right]$$



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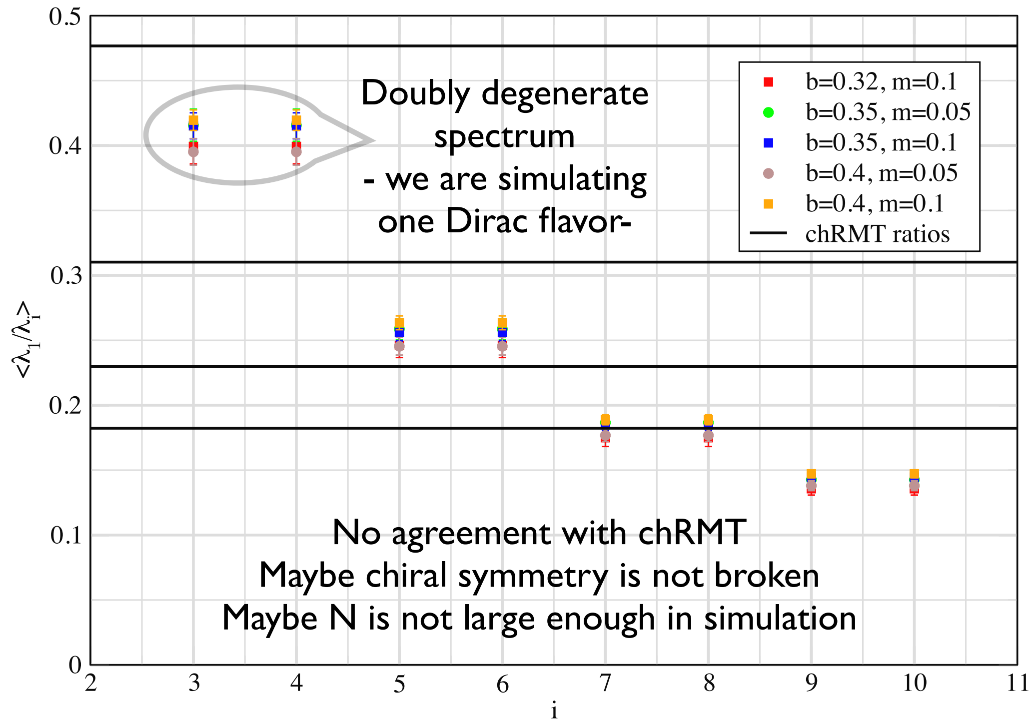
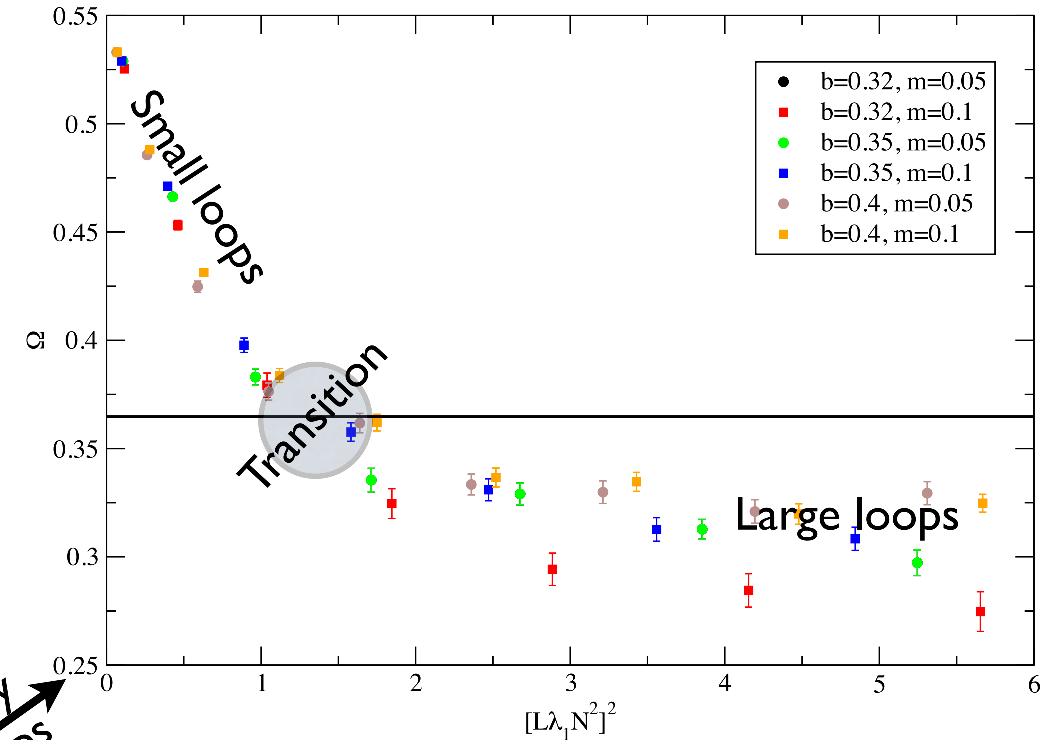
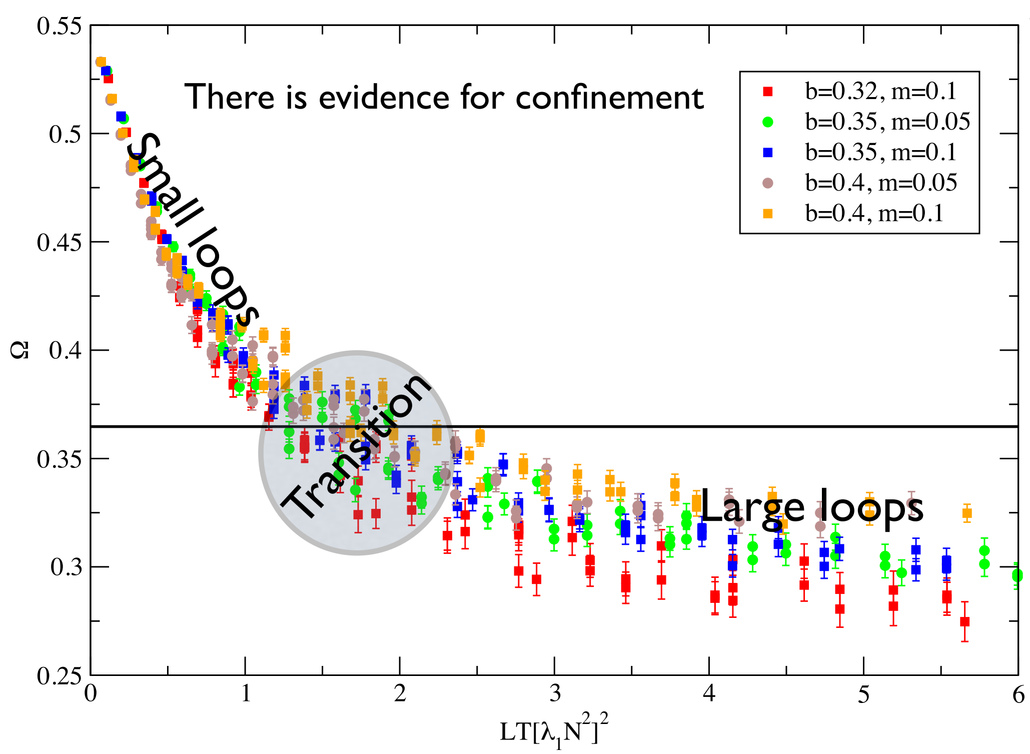
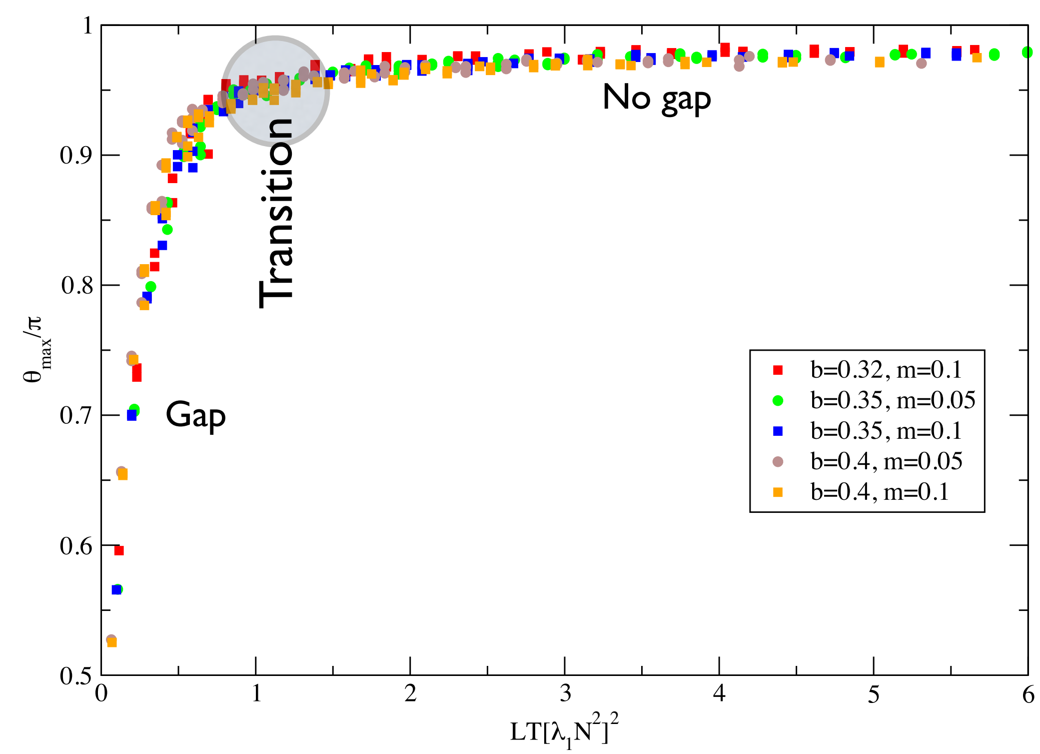
Numerical tests of scaling





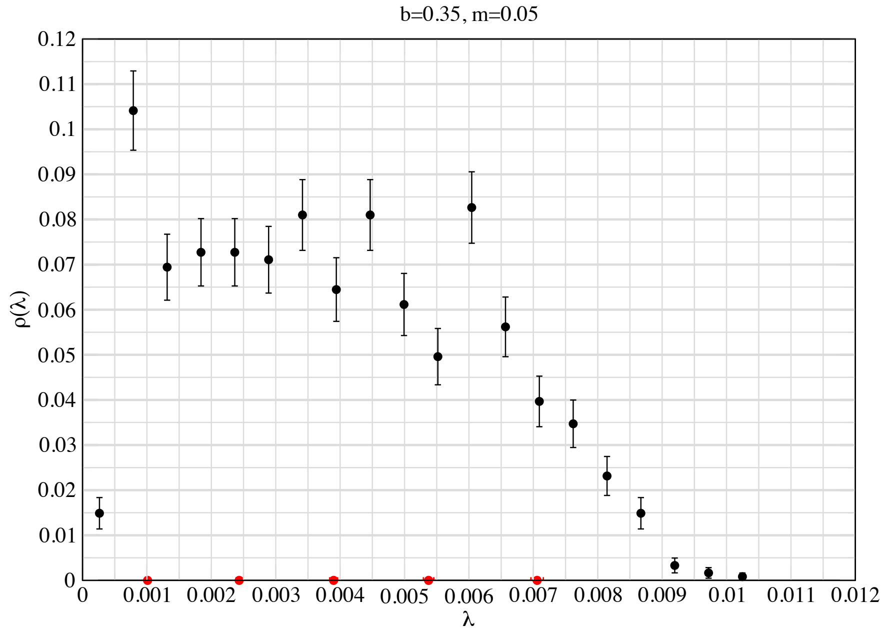
look at only square loops

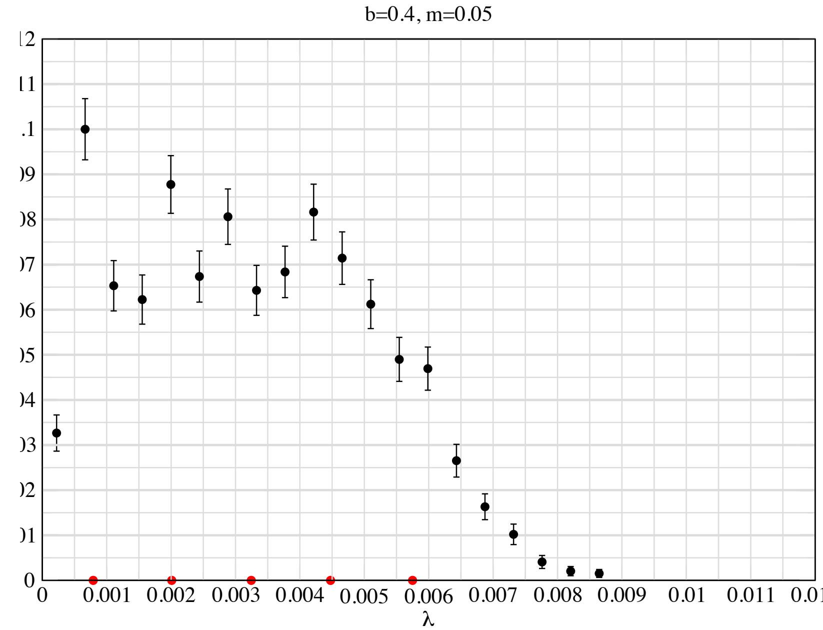


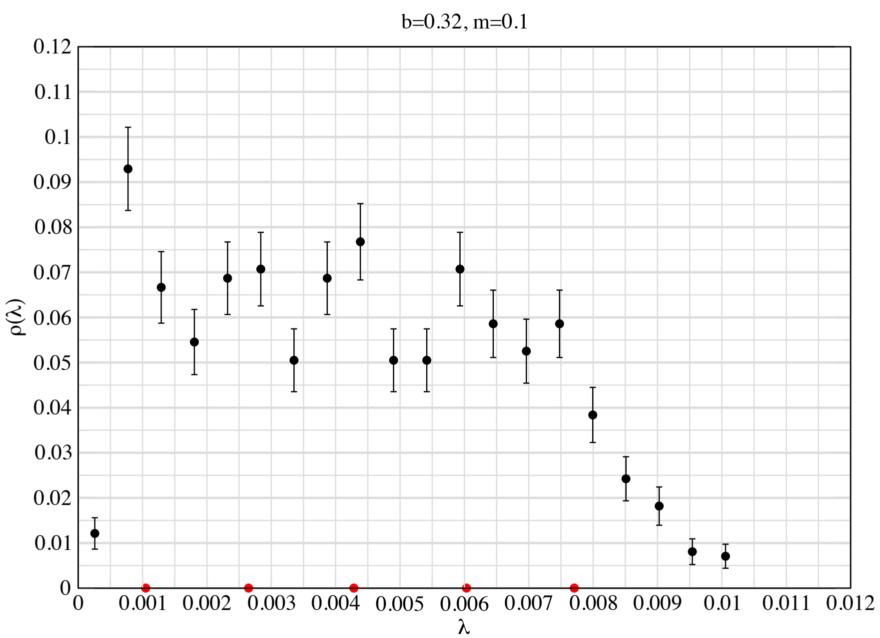


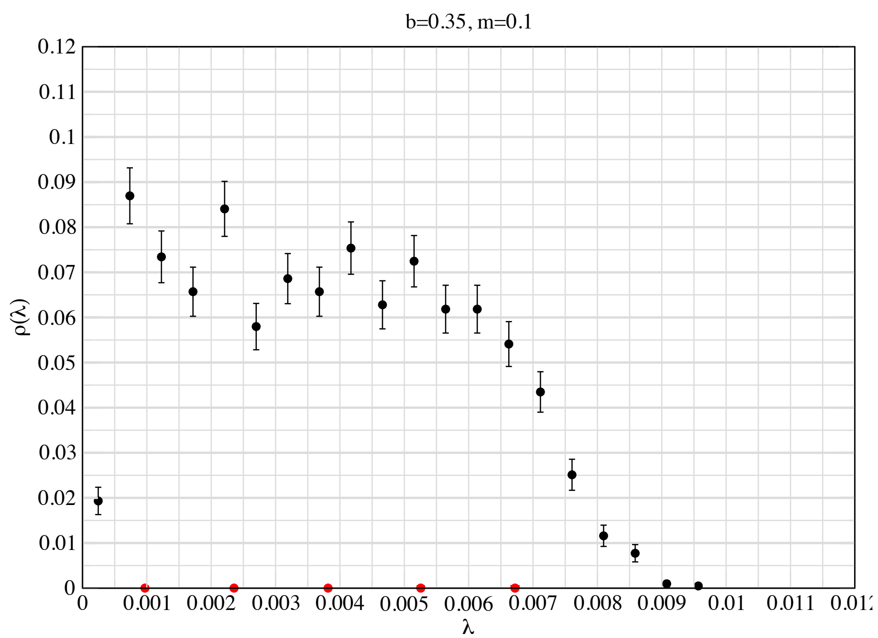
Is chiral symmetry broken?

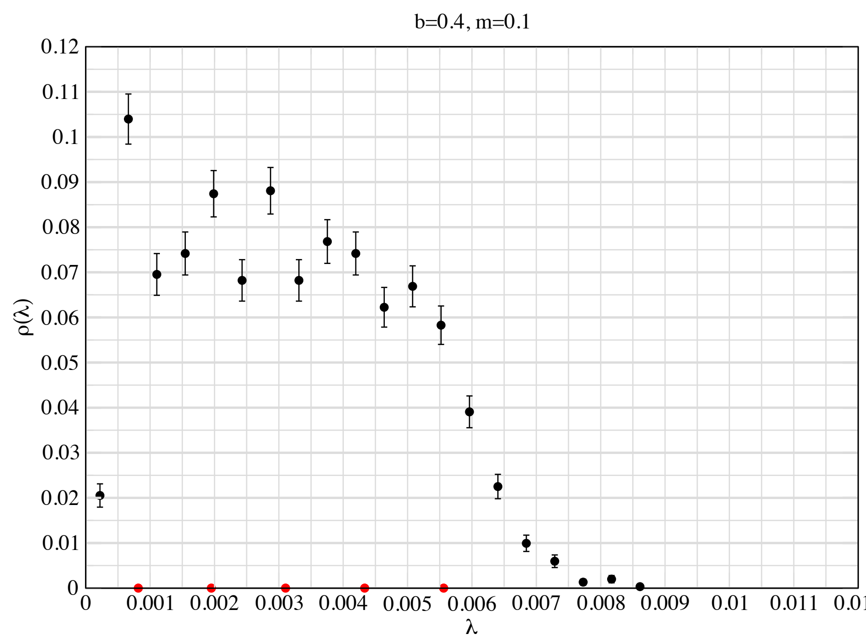
- Look at the distribution of lowest eigenvalues of the adjoint overlap Dirac operator
- Red dots are the averages of the five lowest distinct eigenvalues
- Not enough statistics to see clean peaks in the distribution associated with the five eigenvalues
- Is there a flattening of the distribution as one approaches zero eigenvalue? May be.
- But ratios of eigenvalues do not match predictions from chiral random matrix theory.











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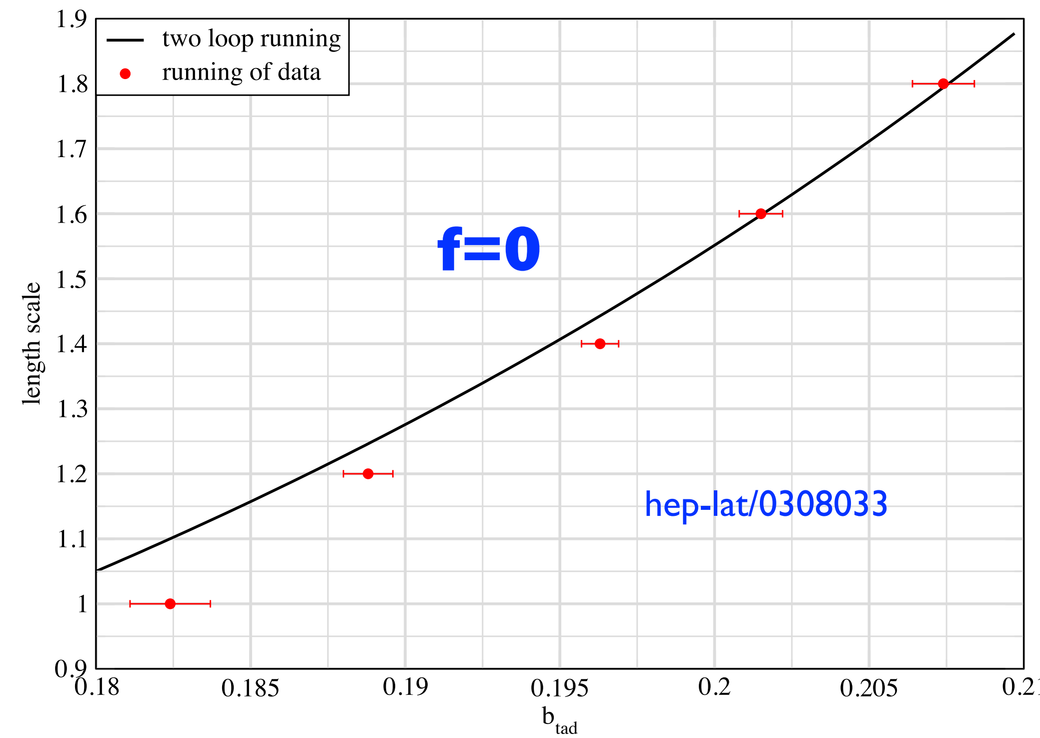
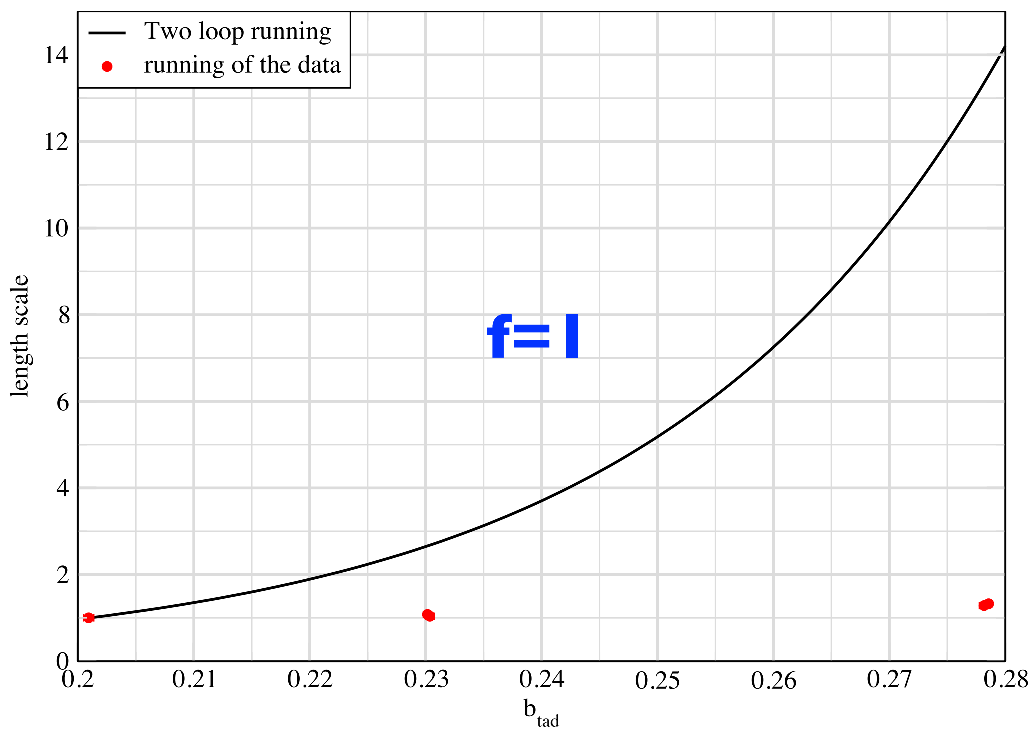
Does the theory with f=1 walk or run?

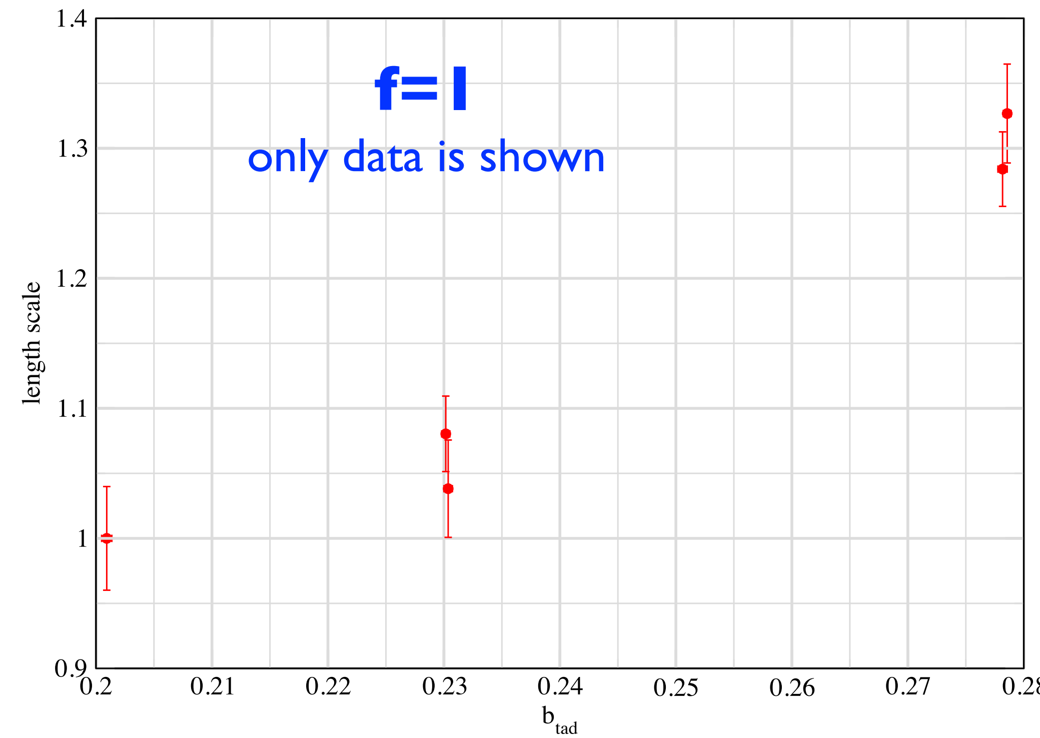
Two loop beta function

$$\alpha = \frac{1}{16\pi^2 b}$$

$$a\frac{d\alpha}{da} = 2\left(\frac{11}{3} - \frac{4}{3}f\right)\alpha^2 + 2\left(\frac{34}{3} - \frac{32}{3}f\right)\alpha^3 + \cdots$$







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