Recent Progress on Massive Gravity

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(C. de Rham, GG, 2010)

(C. de Rham, GG, L. Heisenberg, D. Pirtskhalava, 2010)

(C. de Rham, GG, A.J. Tolley 2010)

(works in progress)

Motivation:

Construct a classical ghost-free massive extension of General Relativity (fundamental question of FT)

Mass = Hubble parameter today; technically natural; study cosmic self-acceleration due to graviton mass

Screening of large scale sources such as vacuum energy, the cosmological constant problem (evades S. Weinberg's no-go theorem).

History of the subject:

A linear theory of massive spin-2 with 5 dof (Fierz and Pauli, 1939)

No continuous limit of FP to massless theory (van Dam, Veltman; Zakharov, 1970)

Continuity can be restored due to nonlinearities (Vainshtein, 1972)

The nonlinearities introduce the sixth dof (ghost) (Boulware, Deser, 1972)

Recent context:

(Dvali, GG, Porrati, 2000; Deffayet, Dvali, GG, Vainshtein, 2001)

(Arkani-Hamed, Georgi, M. Schwartz, 2002)

(Luty, Porrati, Rattazzi, 2003) other works 2008 ->

Massive gravity beyond Fierz-Pauli (FP) action:

$$\mathcal{L} = \sqrt{g}R -$$

$$m^2 \left(h_{\mu\nu}^2 - h^2 + c_1 h^3 + c_2 h h_{\mu\nu}^2 + c_3 h_\mu^\nu h_\nu^\alpha h_\alpha^\mu \cdots \right)$$

A guiding principle for the coefficients of the nonlinear terms?

Are these infinite number of terms an expansion of something?

Anything interesting for cosmology?

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A guiding principle for the coefficients of the nonlinear terms?

YES: absence of ghost in the decoupling limit (C. de Rham, GG)

Are these infinite number of terms an expansion of something?

YES: resummation once the above condition satisfied (C. de Rham, GG, A.J. Tolley)

Anything interesting for cosmology?

We think so (C. de Rham, GG, L. Heisenberg, D. Pirtskhalava), also works in progress

A toy model: massive Abelian gauge field

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{m^2}{2}B_\mu^2$$

No symmetry. Very naively, 4 dof from 4 components of B; however, no time derivative of B_0, hence, only 3 dof

The 3 dof can be identified in either the Lagrangian or Hamiltonian formalisms; we use the Stuckelberg trick:

$$B_\mu = A_\mu - \frac{\partial_\mu \pi}{m}$$

The Lagrangian is invariant w.r.t. simultaneous transformations of A and pi. Gauge fixing A leaves 2 dof in it, pi ads as 3rd, helicity-0, dof

In the decoupling limit this is more manifest:

$$m \longrightarrow 0 \qquad \frac{m}{e} = v \qquad \text{fixed}$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}(\partial_\mu \pi)^2$$

two helicity-1 states are decoupled from helicity-0
(the latter decouples from conserved sources)

Covariant formulation of massive gravity

Introduce four scalar fields

$$H_{\mu\nu} = g_{\mu\nu} - \eta_{ab}\partial_\mu\varphi^a\partial_\nu\varphi^b$$

In the Unitary Gauge:

$$\varphi^a = \delta^a_\mu X^\mu \qquad H_{\mu\nu} = h_{\mu\nu}$$

Illustrative terms for helicity-0 of massive GR

A generic choice of the nonlinear terms: ill-defined Cauchy problem due to the cubic term → BD ghost on local backgrounds

$$(\sqrt{g}R)^L + \pi\Box\pi + \frac{(\Box\pi)^3}{\Lambda_5^5} \qquad \Lambda_5^5 \equiv m^4 M_{\rm Pl}$$

A special choice of the nonlinear terms well-defined Cauchy problem → no BD ghost on local backgrounds (e.g., in DGP)

$$(\sqrt{g}R)^L + \pi\Box\pi + \frac{(\Box\pi)(\partial\pi)^2}{\Lambda_3^3} \qquad \Lambda_3^3 \equiv m^2 M_{\rm Pl}$$

The exact Lagrangian in the decoupling limit:

$$\mathcal{L} = GR_{lin} + h^{\mu\nu}\left(X^{(1)}_{\mu\nu} + a_2 X^{(2)}_{\mu\nu} + a_3 X^{(3)}_{\mu\nu}\right)$$

$$\Pi_{\alpha\beta} = \partial_\alpha \partial_\beta \pi$$

$$X^{(1)}_{\mu\nu} = \epsilon_{\mu\alpha\rho\gamma}\epsilon_{\nu\alpha\rho\delta}\Pi^{\gamma\delta}$$

$$X^{(2)}_{\mu\nu} = \epsilon_{\mu\alpha\rho\gamma}\epsilon_{\nu\alpha\sigma\delta}\Pi^{\rho\sigma}\Pi^{\gamma\delta}$$

$$X^{(3)}_{\mu\nu} = \epsilon_{\mu\alpha\rho\gamma}\epsilon_{\nu\beta\sigma\delta}\Pi^{\alpha\beta}\Pi^{\rho\sigma}\Pi^{\gamma\delta}$$

The Lagrangian is exactly invariant under linear diffeomorphisms, as well as under the shift and Galilean transformations of the helicity-0

Resummation of infinite number of terms:

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\sqrt{-g}\left(R - \frac{m^2}{4}\mathcal{U}(g,H)\right)$$

$$\sqrt{-g}\,\mathcal{U}(g,H)\Big| = \text{function of } \partial\partial\pi = \text{total derivative}$$

$$\begin{aligned}
\mathcal{L}^{(2)}_{\rm der}(\Pi) &= [\Pi]^2 - [\Pi^2]\,, \\
\mathcal{L}^{(3)}_{\rm der}(\Pi) &= [\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]\,, \\
\mathcal{L}^{(4)}_{\rm der}(\Pi) &= [\Pi]^4 - 6[\Pi^2][\Pi]^2 + 8[\Pi^3][\Pi] + 3[\Pi^2]^2 - 6[\Pi^4]
\end{aligned}$$

No higher (n>4) total derivatives exist in 4D

C. de Rham, GG, Tolley

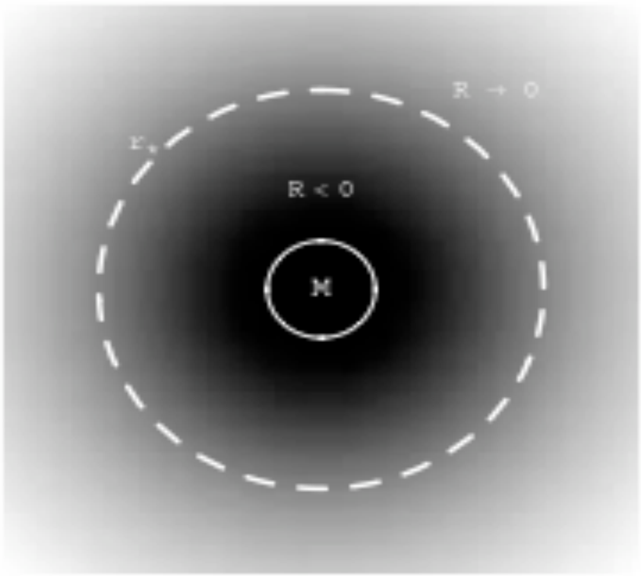
$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\sqrt{-g}\left(R + m^2 \sum_{n\geq 2} \alpha_n \, \mathcal{L}_{\rm der}^{(n)}(\mathcal{K})\right)$$

$$\mathcal{K}^\mu_\nu(g, H) \;=\; \delta^\mu_\nu - \sqrt{\delta^\mu_\nu - H^\mu_\nu} = \delta^\mu_\nu - \sqrt{\partial^\mu\phi^a\partial_\nu\phi^b\eta_{ab}}$$

Only quadratic, cubic and quartic terms in K in the action

Simplest example: only quadratic terms in K

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\sqrt{-g}\,(R - m^2(\mathcal{K}^\mu_\nu\mathcal{K}^\nu_\mu - (\mathcal{K}^\alpha_\alpha)^2)$$



Some recent results:

Vainshtein recovery of GR works fine:

C. de Rham, GG, (in the decoupling limit)

Koyama, Niz, Tasinato, (exact theory)

G. Chkareuli, Pirtskhalava

Interesting exact cosmological solutions:

C. de Rham, GG, L. Heisenberg, Pirtskhalava, (dec limit)

Koyama, Niz, Tasinato, (exact theory)

T. Nieuwenhuizen (exact theory)

Cosmology in the decoupling limit: scales <<1/H

Modified Einstein equation:

$$G^{L}_{\mu\nu} + X^{(1)}_{\mu\nu} + a_2 X^{(2)}_{\mu\nu} + a_3 X^{(3)}_{\mu\nu} = 0$$

Equation for the helicity-0 mode (schematic form)

$$\partial\partial h(1 + a_2\partial\partial\pi + a_3(\partial\partial\pi)^2) = 0$$

A solution: $$h_{\mu\nu} \propto \eta_{\mu\nu} x^2_\alpha, \quad \pi \propto x^2_\alpha$$

This is selfaccelerated solution: restoring powers of m at scales <<1/H

$$ds^2 = [1 - \frac{1}{2}H^2 x^\alpha x_\alpha]\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu.$$

where H is proportional to m

Introducing matter one gets the Friedmann equation:

$$H^2 = m^2 + \frac{8 \cdot 3.14 \cdot G_N}{3}\rho_m$$

In this approximation identical to LambdaCDM

What is the exact cosmology of the resummed theory?

Works in progress

G. D’Amico, C. de Rham, S. Dubovsky, GG, D. Pirtskhalava, A.J. Tolley

There is no exactly homegeneous and isotropic cosmology:

Full theory (the resummed Lagrangian)

$$ds^2 = -N^2(\tau)d\tau^2 + a^2(\tau)d\vec{x}^2$$

$$\mathcal{L} = -\frac{6a(a')^2}{N} + Nm^2 f(a) + m^2 g(a)$$

Variations of the above Lagrangian w.r.t. N and a necessarily lead to:

$$g(a) = consts. \quad hence \quad a = const.$$

Fundamental reason: absence of BD ghost imposes a constraint which is not compatible with exactly FRW metric. However, evolution can be approximately FRW before reaching large scales.

So far we've discussed hard mass. Dynamically generated mass:

$$\mathcal{L} = \sqrt{g}\left(R - f(\sigma)(K_{\mu\nu}^2 - K^2)\right)$$
$$\sqrt{g}\left(-\frac{1}{2}(\partial_\mu\sigma)^2 - V(\sigma)\right)$$

VEV of sigma determines the mass (V could even be zero)

There exist a choice of f and V for which the sigma fluctuations are also Galileons (resulting in two Galileons coupled to a tensor field)

FRW solutions possible, the constraint determines sigma, however, the evolution never transitions to that of the hard mass theory, the dynamical mass theories are very different.

Conclusions:

Strong theoretical evidence that Massive Gravity exists as a consistent ghost-free classical covariant theory (decoupling limit Lagrangian; Hamiltonian constraint in quartic order).

Interesting mathematical structure of the Lagrangian

It has ghost-free selfaccelerated solution in the decoupling limit; also has a degravitating solution.

Novel cosmology (inhomogeneous and/or anisotropic) at very large scales for hard graviton mass.