Quantum fields close to black hole horizons

Kinematics

Accelerated scalar fields and inertial forces

Photons in Rindler space vs thermal photons

Interactions

"Static" interactions of scalar, electric and gravitational charges

Instability of matter

Symmetry breakdown close to horizons ?

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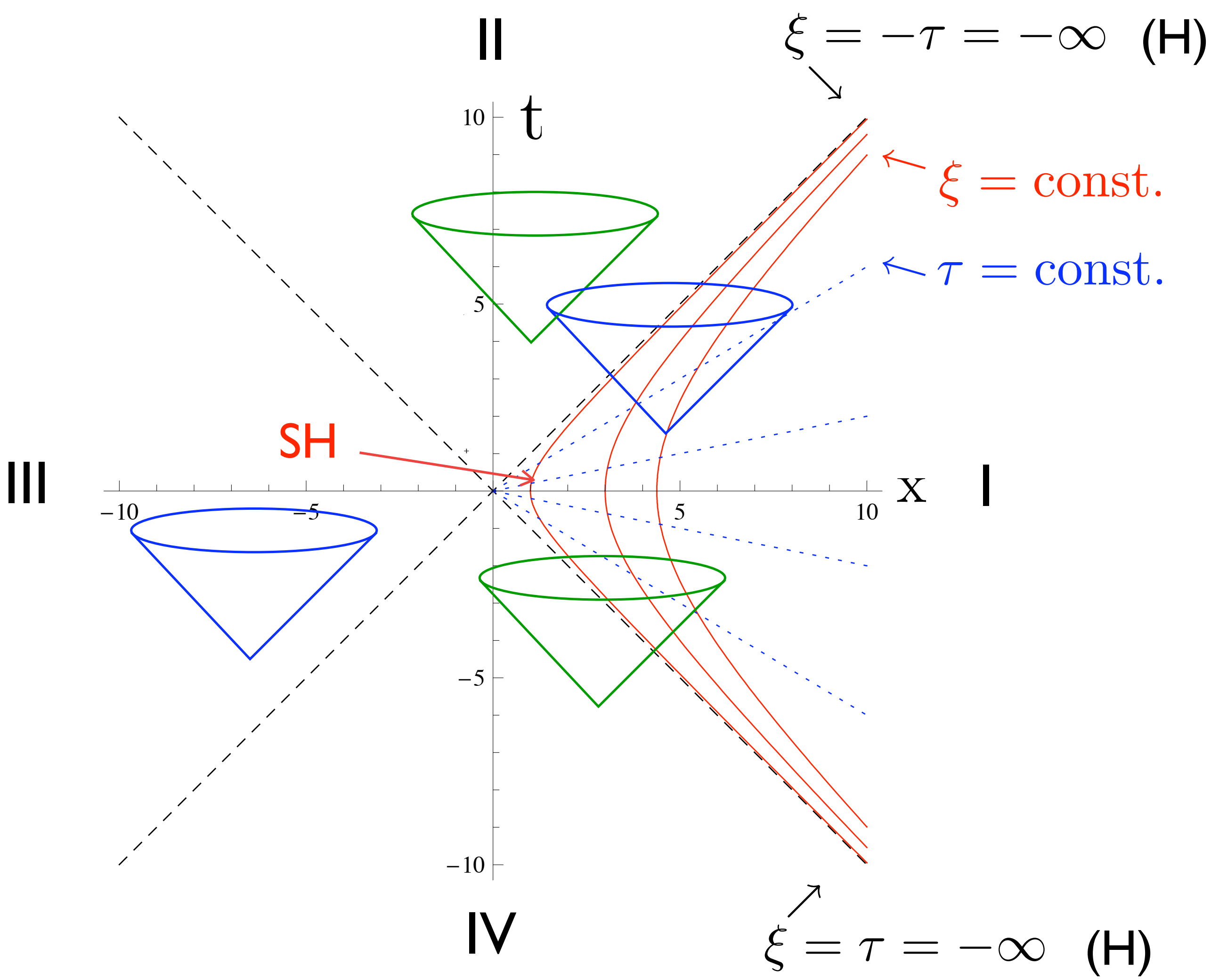
Uniformly accelerated observer in Minkowski space - Kinematics

Transformation to observer's rest frame

$$t, x, x_\perp \to \tau, \xi, x_\perp$$

$$t(\tau,\xi) = \frac{1}{a}\, e^{a\xi} \sinh a\tau\,, \quad x(\tau,\xi) = \frac{1}{a}\, e^{a\xi} \cosh a\tau$$



$$x^2 - t^2 = \frac{1}{a^2}\, e^{2a\xi} \qquad \frac{t}{x} = \tanh a\tau$$

$$x \geq |t|$$

$$-\infty < \tau < \infty, \quad -\infty < \xi < \infty$$

$$ds^2 = \left(1 - \frac{R}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{R}{r}} - r^2 d\Omega^2\,, \quad R = 2MG$$

$$\to ds_R^2\,, \quad e^{2a\xi} = \frac{r-R}{R} \ll 1$$

Stretched horizon (SH) close to mathematical horizon (H)

Scalar Fields in Rindler Spaces

Rindler metric $$ds^2 = dt^2 - dx^2 - d\mathbf{x}_\perp^2 = e^{2a\xi}(d\tau^2 - d\xi^2) - d\mathbf{x}_\perp^2$$

Action

$$S = \frac{1}{2}\int d\tau\, d\xi\, d^2x_\perp \{(\partial_\tau\phi)^2 - (\partial_\xi\phi)^2 - (m^2\phi^2 + (\partial_\perp\phi)^2)\, e^{2a\xi}\}$$

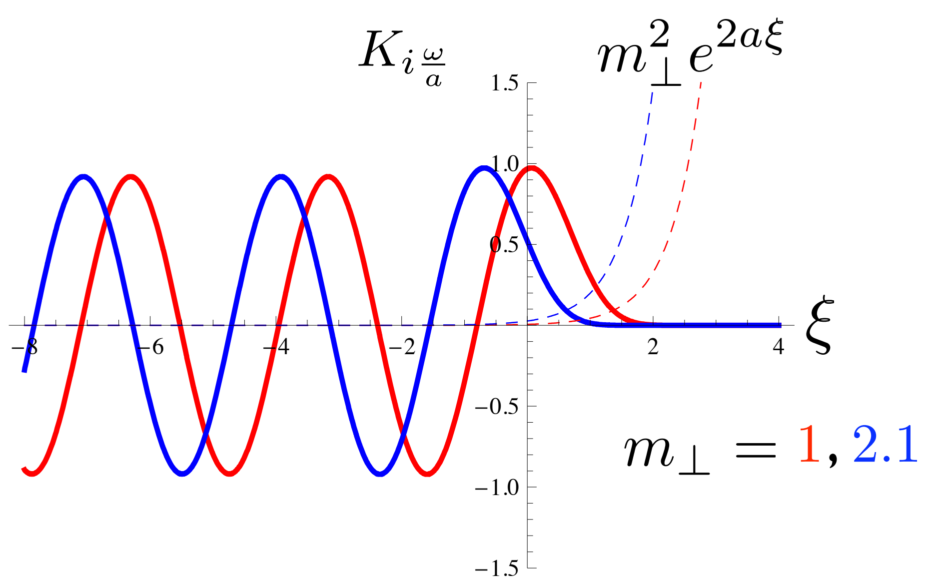
Wave equation

$$\left[\partial_\tau^2 - \partial_\xi^2 + (m^2 - \boldsymbol{\nabla}_\perp^2)\, e^{2a\xi}\right]\phi = 0, \quad \phi = e^{-i\omega\tau}\, e^{i\mathbf{k}_\perp \mathbf{x}_\perp}\, \varphi(\xi)$$

$$\left[-\frac{d^2}{d\xi^2} + m_\perp^2 e^{2a\xi} - \omega^2\right]\varphi(\xi) = 0, \quad \varphi(\xi) = K_{\frac{i\omega}{a}}(m_\perp e^{a\xi}), \;\; m_\perp^2 = (m^2 + \mathbf{k}_\perp^2)/a^2$$

The inertial force: Exponentially growing potential

$$\left[-\frac{d^2}{d\xi^2} + m_\perp^2 e^{2a\xi} - \omega^2\right]K_{\frac{i\omega}{a}}(m_\perp e^{a\xi}) = 0$$



$$\frac{\partial\omega}{\partial\mathbf{k}_\perp} = \frac{\partial\omega}{\partial m} = 0$$

Hamiltonian $$H_a = \int d^2k_\perp \int_0^\infty d\omega\, \omega a^\dagger(\omega, \mathbf{k}_\perp) a(\omega, \mathbf{k}_\perp)$$

Degeneracy: consequence of generalized scale invariance

$$\partial_\tau = a\left(x\partial_t + t\partial_x\right)$$ Boosts and Dilations commute

Unruh Temperature

Distance of two spcace-time points in Rindler and Minkowski coordinates

$$(x-x')^2 = \frac{2e^{a(\xi+\xi')}}{a^2}\left[\cosh a(\tau-\tau') - \cosh\eta\right] \qquad \cosh\eta = 1 + \frac{\left(e^{a\xi} - e^{a\xi'}\right)^2 + a^2\left(\mathbf{x}_\perp - \mathbf{x}'_\perp\right)^2}{2e^{a(\xi+\xi')}}$$

2-point function $$i\langle 0_M \big| T\left[\phi(\tau,\xi,\mathbf{x}_\perp)\phi(0,\xi',\mathbf{0}_\perp)\right]\big| 0_M\rangle = D\big((x-x')^2\big)$$

$$= D\left(\frac{2e^{a(\xi+\xi')}}{a^2}\left[\cosh a(\tau-\tau') - \cosh\eta\right]\right)$$

Transition to imaginary Rindler time $\tau \to -i\tau$

$$D\big((x_E - x'_E)^2\big) = D\left(\frac{2e^{a(\xi+\xi')}}{a^2}\left[\cos a(\tau-\tau') - \cosh\eta\right]\right)$$

Periodic time dependence $\beta = \dfrac{2\pi}{a} = \dfrac{1}{T}$ Partition function $Z = \left(\mathrm{tr}\, e^{-\beta H_a}\right)_{\beta = 2\pi/a}$

With acceleration a, temperature T and Hamiltonian H_a change

Accelerated and thermal photons

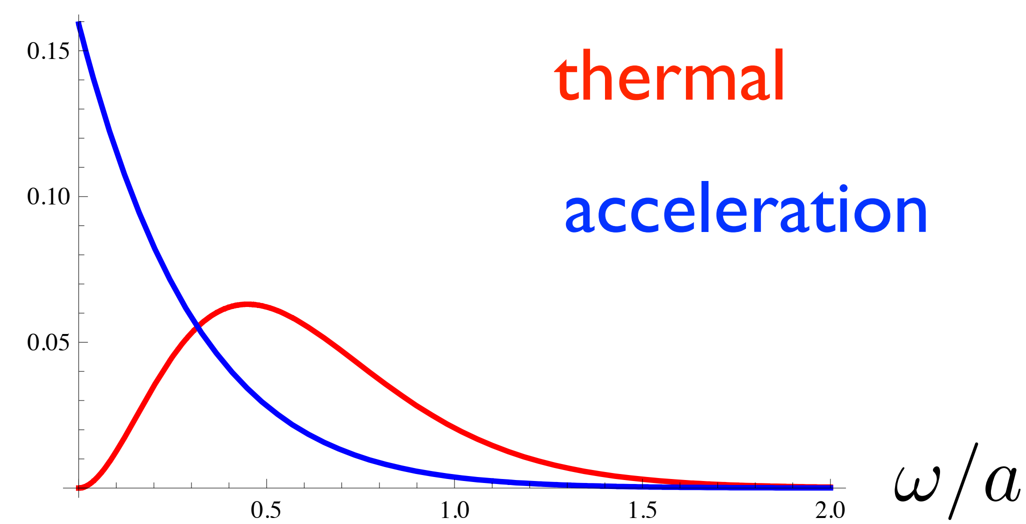
- Modified Planck's formula $T = \dfrac{a}{2\pi}$ $T_\xi = e^{-a\xi} T$

$$\frac{1}{\sqrt{|g(\xi,\mathbf{x}_\perp)|}}\langle 0_M| : \mathcal{H}_E(\xi,\mathbf{x}_\perp) + \mathcal{H}_B(\xi,\mathbf{x}_\perp) : |0_M\rangle = \frac{1}{\pi^2} e^{-4a\xi} \int \omega d\omega \frac{\omega^2 + 4\pi^2 T^2}{e^{\frac{\omega}{T}} - 1} = \frac{11\pi^2}{15} T_\xi^4$$

Density of states

$$\omega \ll T \quad \sim \quad T^3/\omega, \qquad T\omega$$

$$\omega \gg T \quad \sim \quad \omega^2 e^{-\omega/T}, \qquad \omega^2 e^{-\omega/T}$$



Energy density varies in space

- Tolman's law $T_\xi \sqrt{g_{00}} = \text{const.}$ is satisfied.

Temperature T_ξ diverges when approaching horizon

Propagators in Rindler space

Free field Propagators

Propagator of massless scalar particles

$$D(x,x') = D(\tau,\xi,\xi',\mathbf{x}_\perp) = i\langle 0_M \big| T\big[\phi(\tau,\xi,\mathbf{x}_\perp)\phi(0,\xi',\mathbf{0}_\perp)\big]\big|0_M\rangle = \frac{1}{4i\pi^2\big((x-x')^2 - i\delta\big)}$$

$$= \frac{a^2 e^{-a(\xi+\xi')}}{8i\pi^2}\frac{1}{\cosh a\tau - \cosh\eta - i\delta}$$

$$\tilde{D}(\xi,\xi',\mathbf{x}_\perp) = \int d\tau D(\tau,\xi,\xi',\mathbf{x}_\perp) = \frac{ae^{-a(\xi+\xi')}}{4\pi}\frac{1}{\sinh\eta}[1+\frac{i}{\pi}\eta]$$

$$\big(\partial_\xi^2 + e^{2a\xi}\partial_\perp^2\big)\tilde{D}(\xi,\xi',\mathbf{x}_\perp) = \delta(\xi-\xi')\delta(\mathbf{x}_\perp)$$

$\tilde{D}$ satisfies Poisson equation, imaginary part (homogeneous) Laplace equation

$\mathrm{Im}\,\tilde{D} =$ superposition of zero energy modes: $\omega = 0\,, \quad -\infty \le k_2,\, k_3 \le \infty$

Imaginary part of self-energy of a static charge is given by the total rate of Bremsstrahlung in Minkowski space

$$\mathrm{Im}\,\Sigma = \frac{\kappa^2}{2}\mathrm{Im}\,\tilde{D}(\xi,\xi,0) = \frac{\kappa^2}{8\pi^2}a$$

Required by energy conservation in Rindler space, Bremsstrahlungs photons observed in Minkowski space appear as zero energy, transverse photons (with finite transverse momentum) in Rindler space

Interaction of static charges

Scalar interaction energy

$$V_{\rm s}(s) = -\kappa^2 e^{a(\xi+\xi')}\tilde{D}(\xi,\xi',\mathbf{x}_\perp)$$

$$D_{00}(\tau,\xi,\xi',\mathbf{x}_\perp) = (\partial_\tau t\,\partial_{\tau'}t' - \partial_\tau x\,\partial_{\tau'}x')\,D(\tau,\xi,\xi',\mathbf{x}_\perp)$$

Electrostatic interaction energy

$$V_{\rm v}(s) = -e^2\tilde{D}(\xi,\xi',\mathbf{x}_\perp)$$

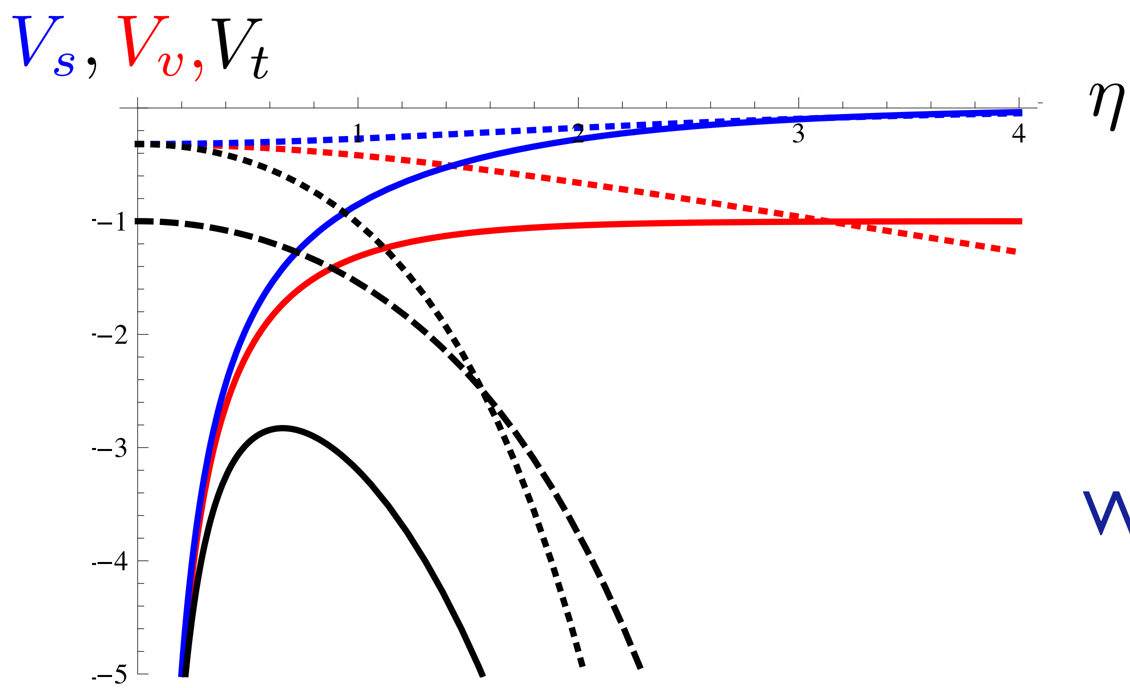
$$V_s = -\frac{\kappa^2}{4\pi}\,v(\eta),$$

$$V_v = -\frac{e^2}{4\pi}\,v(\eta)\cosh\eta,$$

$$v(\eta) = \frac{a}{\sinh\eta}\left(1+\frac{i\eta}{\pi}\right)$$

$$\cosh\eta = 1 + \frac{\left(e^{a\xi}-e^{a\xi'}\right)^2 + a^2\left(\mathbf{x}_\perp - \mathbf{x}'_\perp\right)^2}{2e^{a(\xi+\xi')}}$$

$$V_t = -\frac{GM_1M_2}{4\pi}\left[v(\eta)(2\cosh^2\eta - 1) + \frac{i}{\pi}\tau_{max}\cosh\eta\right]$$



With $\cosh\eta, V_{s,v,t}$ invariant under scale transformations

$$\xi,\,\xi' \to \xi+\xi_0,\,\xi'+\xi_0\,,\quad \mathbf{x}_\perp,\,\mathbf{x}'_\perp \to e^{a\xi_0}\mathbf{x}_\perp,\,e^{a\xi_0}\mathbf{x}'_\perp$$

The instability of atoms in Rindler space

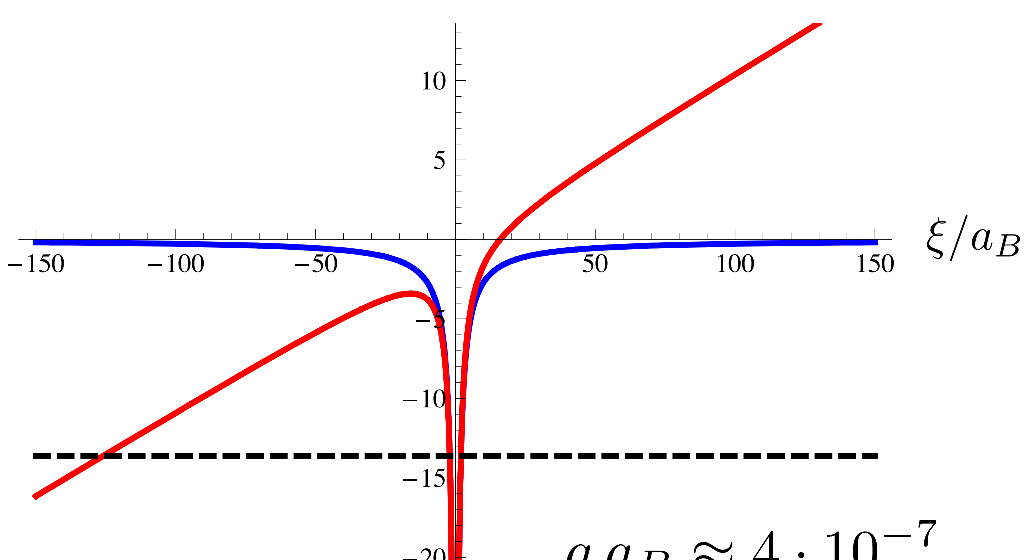
Electron coupled to a static charge located in Rindler space at

$$\xi = \xi_0, \quad \mathbf{x}_\perp = \mathbf{x}_{\perp\,0}$$

Weak acceleration $$a_B\, e^{-2a\xi_0} a = e^{-2a\xi_0} \frac{a}{me^2} \ll 1$$

$$H \approx -\frac{1}{2m}\left(\partial_\xi^2 + \partial_\perp^2\right) + ma(\xi - \xi_0) - \frac{e^2}{4\pi} \frac{1}{\sqrt{(\xi - \xi_0)^2 + (\mathbf{x}_\perp - \mathbf{x}_{0\,\perp})^2}}$$

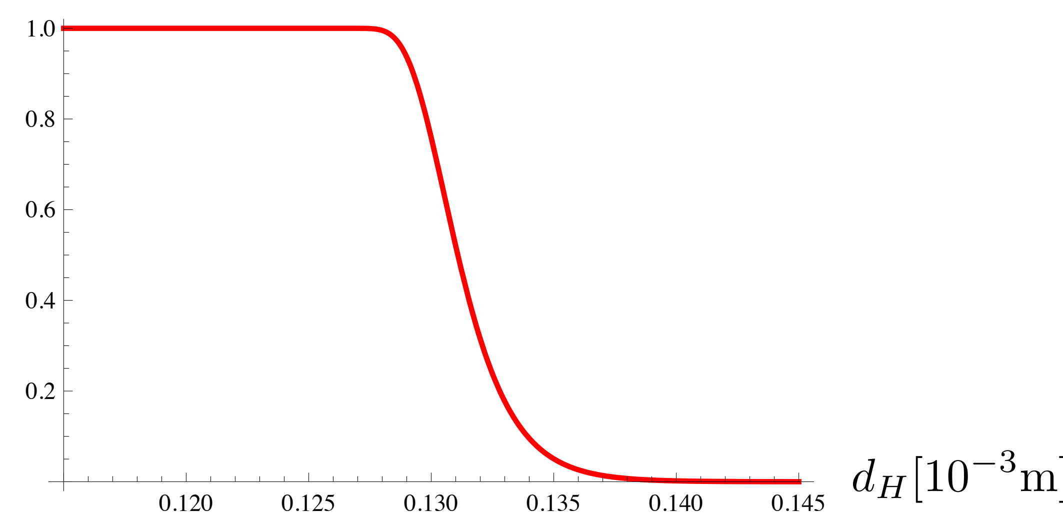
In the weak acceleration limit inertial force acts like an external electric field



$a\, a_B \approx 4 \cdot 10^{-7}$

$t_H \approx 10^{10} y$

$a \approx 7 \cdot 10^{19} g$



Ionization probability of hydrogen

$$d_H = \frac{1}{a} e^{a\xi_0}$$

Spontaneous symmetry breaking close to horizons ?

$$H_a = \frac{1}{2}\int d\xi d^2x_\perp \left\{\pi^2 + (\partial_\xi\phi)^2 + e^{2a\xi}\big((\boldsymbol{\partial}_\perp\phi)^2 + V(\phi)\big)\right\}$$

Hamiltonian depends on acceleration **a**

ϕ^4- model, discrete symmetry $\quad \phi \longrightarrow -\phi$

$$V(\phi) = \frac{\lambda}{8}\phi^2(\phi^2 - 2\phi_0^2) \approx -\frac{\lambda}{8}\phi_0^4 + \frac{m^2}{2}\varphi^2\,, \quad m^2 = \lambda\phi_0^2$$

Symmetry restoration if energy density of fluctuations

$$\epsilon(\xi) = \frac{e^{-2a\xi}}{2}\langle 0_M| : \pi^2 + (\partial_\xi\varphi)^2 + e^{2a\xi}\big((\boldsymbol{\partial}_\perp\varphi)^2 + m^2\varphi\big) : |0_M\rangle$$

of the order of the energy gain by symmetry breakdown

$$\epsilon(\xi) \approx \frac{\lambda}{8}\phi_0^4$$

$$\epsilon(d_H) \approx \frac{11}{480\pi^2 d_H^4} \qquad (m d_H \le 0.2) \qquad d_H \approx \Big(\frac{11}{60\,\pi^2\lambda}\Big)^{\frac{1}{4}}\frac{1}{\phi_0}$$

Higgs model parameters $\quad d_H \approx 3\cdot 10^{-19} \qquad T_{\text{Tolman}} \approx 90\,\text{GeV}$

Summary

- Kinematics
  - On safe grounds - rewriting Minkowski space propagators in terms of Rindler coordinates
  - Rindler particles $\neq$ Minkowski particles
  - accelerating $\neq$ heating
- Dynamics
  - Indications of significant differences in Minkowski and Rindler space dynamics - new type of interactions mediated by zero energy excitations
  - Hints for symmetry restoration by acceleration

    deconfinement ?
  - Influence of the zero-mode radiation field $\mathrm{Im}D(\xi, \xi', \mathbf{x}_\perp)$ on the structure of stretched horizons ?