On Heterotic Noncompact Nonlinear Sigma Models

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Outline

- N=2 SQCD in 4d & Sigma models in 2d
- GLSM as a tool to find BPS spectrum
- Heterotic deformation and Large-N solution - beyond BPS sector
- Worldsheet effective theory as reduction from 4d [see T. Fulimori, A. Yung's and W. Vinci's talks]

4d SQCD vs 2d sigma models

4d / 2d duality

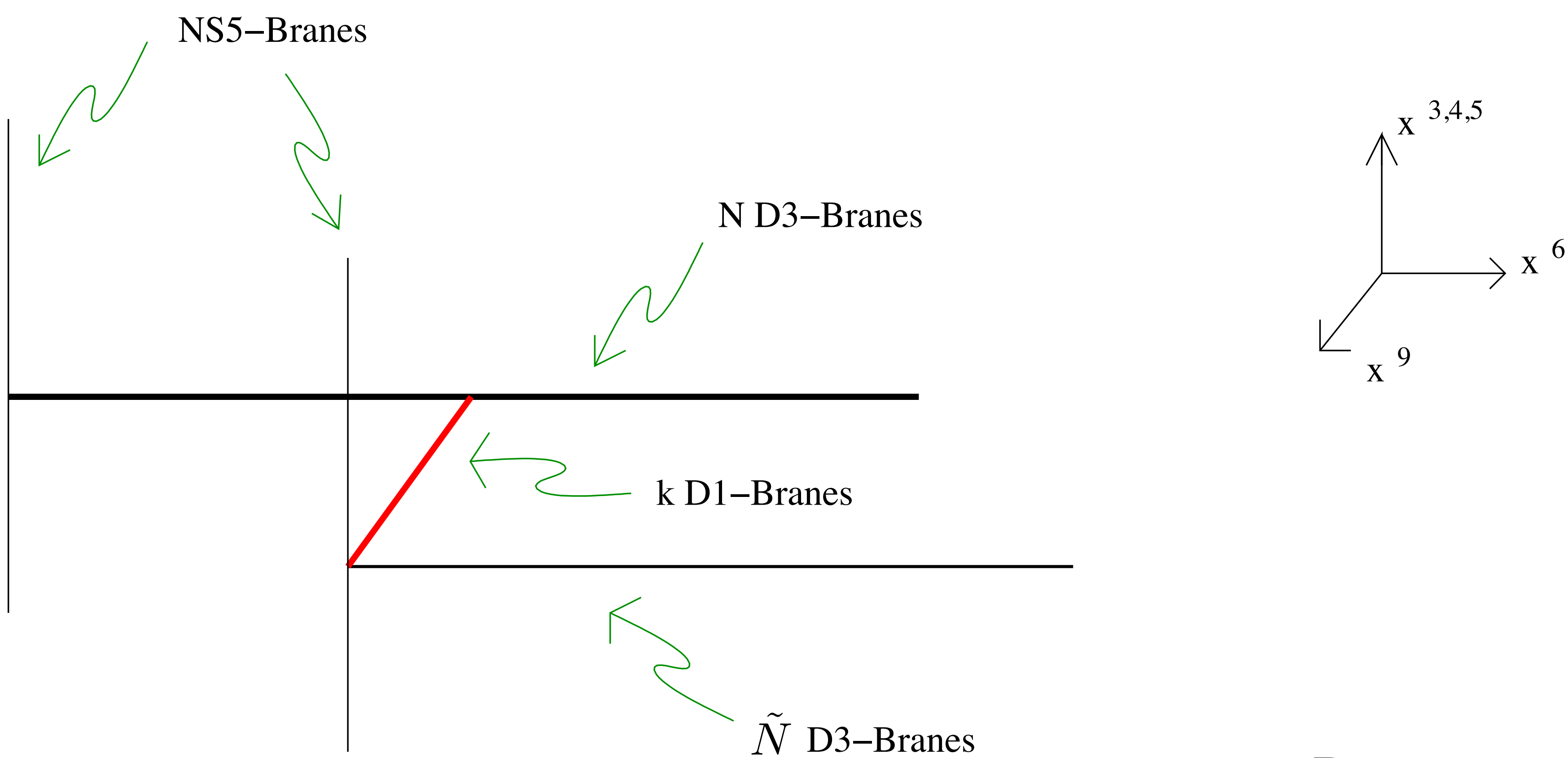
[Dorey Hollowood, Tong]

$\mathcal{N}=2$ $SU(N)$ SQCD	(2,2) $U(1)$ GLSM e
$N_f = N + \tilde{N}$ fund hypers	N chiral +1 $\tilde{N}$ chiral -1
w/ masses	w/ *twisted* masses
$m_1, \ldots, m_N$ $\mu_1, \ldots, \mu_{\tilde{N}}$	$m_1, \ldots, m_N$ $\mu_1, \ldots, \mu_{\tilde{N}}$
$\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$	$\tau = ir + \frac{\theta}{2\pi}$
on baryonic Higgs branch	vortex moduli space
BPS dyons	kinks interpolating between different vacua

BPS spectra (as functions of masses, Lambda) are the same

Brane construction

[Hanany Tong]



take k=1 to get U(1) theory on D1 (semi-local vortex)

D-term

$$|Q_i|^2 - |\tilde{Q}_j|^2 = r$$

FI term -- separation of NS5s in x6

will refer to as HT model

$U(N_c)$ $\mathcal{N}=2$ $d=4$ **SQCD** N_f **quarks**

$$\mathcal{L} = \mathrm{Im}\left[\tau\int d^4\theta\,\mathrm{Tr}\left(Q^{i\,\dagger}e^V Q_i + \tilde{Q}^{i\,\dagger}e^V\tilde{Q}_i + \Phi^\dagger e^V\Phi\right)\right]$$

$$+\mathrm{Im}\left[\tau\int d^2\theta\left(\mathrm{Tr}W^{\alpha\,2} + m^i_j\tilde{Q}_iQ^j + Q_i\Phi\tilde{Q}^i\right)\right]$$

bosonic part

FI term

$$S = \int d^4x\,\mathrm{Tr}\left\{\frac{1}{2g^2}F_{\mu\nu}^2 + \frac{1}{g^2}|D_\mu\Phi|^2 + |\nabla_\mu Q|^2 + \frac{g^2}{4}(Q\bar{Q}-\xi)^2 + |\Phi Q + QM|^2\right\}$$

Nf=Nc color-flavor locked phase

single SUSY vacuum

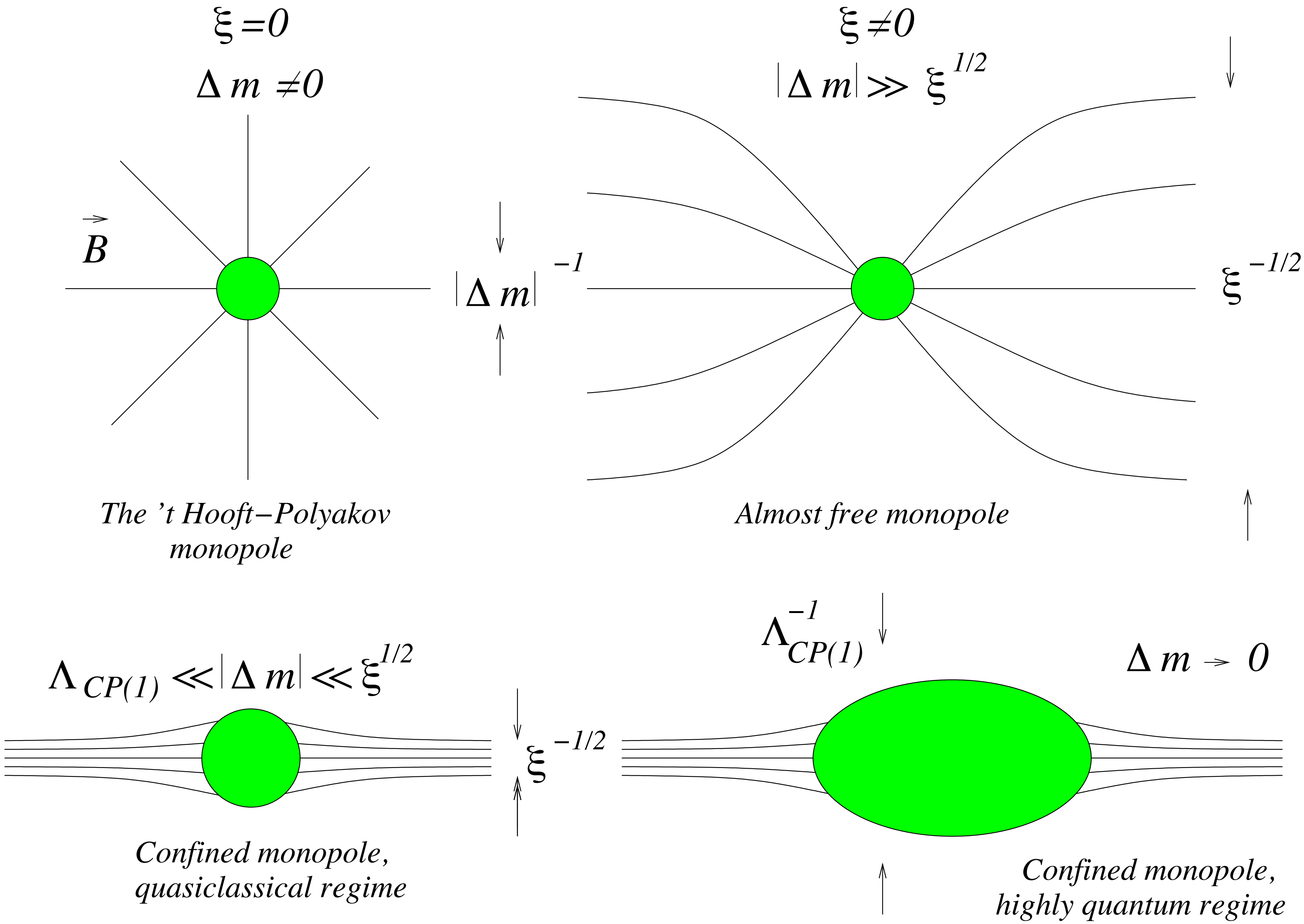
$$U(N_c)\times SU(N_f)\to SU(N)$$

on straight string

[see T. Fujimori's talk]

$$\frac{SU(N)}{SU(N-1)\times U(1)} = \mathbb{CP}^{N-1}$$

Confined monopoles



$\frac{(\Delta m)^2}{\xi}$ becomes 2d FI term r

Hanany-Tong model as U(1) GLSM

$$\mathcal{L} = \int d^4\theta \left[\sum_{i=1}^{N_c} \Phi_i^\dagger \, \mathrm{e}^{\mathcal{V}} \Phi_i + \sum_{i=1}^{\tilde{N}} \widetilde{\Phi}_i^\dagger \, \mathrm{e}^{-\mathcal{V}} \widetilde{\Phi}_i - r\mathcal{V} + \frac{1}{2e^2} \Sigma^\dagger \Sigma \right]$$

Limit $e \to \infty$ defines vacuum manifold $\mathcal{O}(-1)^{\otimes \tilde{N}}$ over $\mathbb{CP}^{N-1}$

perturbation theory is subtle

$$\beta_{i\bar{\jmath}} = a^{(1)} R_{i\bar{\jmath}} \log \frac{M}{\mu} + \ldots \qquad R_{i\bar{\jmath}} = \frac{N - \tilde{N}}{r} g_{i\bar{\jmath}} + \mathcal{O}(\frac{1}{r^2})$$

however there are nonperturbative corrections

One loop twisted effective superpotential is exact in (2,2)

$$\widetilde{\mathcal{W}}_{eff} = \frac{N - \tilde{N}}{4\pi} \sigma \left(\log \frac{\sigma}{\Lambda} - 1 \right)$$

$$V = \theta^+ \bar{\theta}^+ (A_0 + A_3) + \theta^- \bar{\theta}^- (A_0 - A_3) - \theta^- \bar{\theta}^+ \sigma - \theta^- \bar{\theta}^+ \bar{\sigma} + \bar{\theta}^2 \theta \lambda + \theta^2 \bar{\theta} \bar{\lambda} + \bar{\theta} \theta \bar{\theta} \theta D$$

gives vacua of the theory and its BPS spectrum !!

Heterotic deformation

Mass deformation

[Gorsky Shifman Yung]

In 4d introduce masses $\int d^2\theta\, \mu^2 (\Phi^a)^2$

breaks $\mathcal{N}=2$ to $\mathcal{N}=1$

On the flux tube $(2,2) \longmapsto (0,2)$ obtain heterotic sigma model

[Edalati Tong]
[Shifman Yung]

(0,2) Sigma models -very generic construction

[Distler Kachru]

(0,2) theory

[see X. Cui's talk]

$$\mathcal{L} = \int d^4\theta \left(\Phi_i^\dagger e^V \Phi^i - rV - \mathcal{B}V \right)$$

$$\mathbb{CP}^{N-1} \times \mathbb{C}$$

B-right handed superfield

can be treated as model w/ field dependent FI term

$$K = (r + \mathcal{B}) \log(1 + |\phi^i|^2)$$

Geometry becomes non-Kahler
due to generation of H field (field dependent
theta term)

What to do in neutral case? [PK Monin]

$O(N)$ sigma model

effective theory on flux tubes in O(N+1)
bulk theory with neutral flavors (?)

use real valued Majorana spinors

$$\mathcal{L} = \int d^2\theta \left[\frac{1}{4}\,\varepsilon_{\alpha\beta}\mathcal{D}_\beta\mathcal{N}^i\mathcal{D}_\alpha\mathcal{N}_i + \frac{1}{4e_0^2}\,\varepsilon_{\alpha\beta}\mathcal{D}_\beta\mathcal{S}\,\mathcal{D}_\alpha\mathcal{S} + \frac{i}{2}\mathcal{S}\left(\mathcal{N}^2 - r_0\right)\right]$$

Isovector $\quad \mathcal{N}^i = n^i + \bar\theta\psi^i + \frac{1}{2}\bar\theta\theta F^i$

Constraint $\quad \mathcal{S} = \sigma + \bar\theta\lambda + \frac{1}{2}\bar\theta\theta D.$

Heterotic deformation $\quad \Delta\mathcal{L} = \int d^2\theta\ \left[\frac{1}{4}\,\varepsilon_{\alpha\beta}\mathcal{D}_\beta\mathcal{B}\,\mathcal{D}_\alpha\mathcal{B} - i\gamma\mathcal{S}\mathcal{B}\right]$

$(1,1) \longmapsto (0,1)$ $\quad \mathcal{B} = \bar\theta\zeta + \frac{1}{2}\bar\theta\theta G$

constraint becomes $\quad \mathcal{N}^i\mathcal{N}_i = r_0 + 2\gamma\mathcal{B}$

Can solve at large-N

$$\mathcal{L}_{eff} = \frac{1}{2e_\sigma^2}\left(\partial_\mu\sigma\right)^2 + \frac{i}{2e_\lambda^2}\bar\lambda\gamma^\mu\partial_\mu\lambda - V_{eff}(\sigma) + i\zeta_L\partial_R\zeta_L + \frac{1}{2}\Gamma\sigma\bar\lambda\lambda + i\gamma\lambda_R\zeta_L$$

Can do CP(N-1) as well

[PK Monin]

$$\mathcal{L}_{\mathbb{CP}^N} = \int d^2\theta \left[\tfrac{1}{2}\varepsilon_{\beta\alpha}(\mathcal{D}_\alpha + i\mathcal{A}_\alpha)\mathcal{N}_i^\dagger(\mathcal{D}_\beta - i\mathcal{A}_\beta)\mathcal{N}_i + i\mathcal{S}(\mathcal{N}_i^\dagger\mathcal{N}_i - r_0)\right.$$
$$\left. + \tfrac{1}{4}\varepsilon_{\beta\alpha}\mathcal{D}_\alpha\mathcal{B}^\dagger\,\mathcal{D}_\beta\mathcal{B} + \left(i\,\omega\,\mathcal{B}(\mathcal{S} - \tfrac{i}{2}\overline{\mathcal{D}}\gamma^5\mathcal{A}) + \text{H.c.}\right)\right],$$

Isovector $\mathcal{N}^i = n^i + \bar\theta\xi^i + \frac{1}{2}\bar\theta\theta F^i,$

Spinor $\mathcal{A}_\alpha = -i(\gamma^\mu\theta)_\alpha A_\mu + \sqrt{2}(\gamma^5\theta)_\alpha\sigma_2 + \sqrt{2}\bar\theta\theta\, v_\alpha,$

Constraint $\mathcal{S} = \sqrt{2}\sigma_1 + \sqrt{2}\bar\theta u + \frac{1}{2}\bar\theta\theta D$

complex fields $\sigma = \sigma_1 + i\sigma_2\,, \quad \lambda_\alpha = u_\alpha + iv_\alpha$

if negatively charged fields are included

$$\begin{aligned}
\mathcal{L}^{\rm w}_{\mathbb{CP}^N} &= |\nabla_\mu n_i|^2 + |\nabla_\mu \rho_i|^2 + i\bar\xi^i_L\nabla_R\xi^i_L + i\bar\xi^i_R\nabla_L\xi^i_R + i\bar\eta^i_L\nabla_R\eta^i_L + i\bar\eta^i_R\nabla_L\eta^i_R \\
&- 2|\sigma|^2|n_i|^2 - 2|\sigma|^2|\rho_i|^2 - D\left(|n_i|^2 - |\rho_i|^2 - r_0\right) - 4|\omega|^2|\sigma|^2 \\
&+ \left[i\sqrt{2}\bar n_i\left(\lambda_L\xi^i_R - \lambda_R\xi^i_L\right) - i\sqrt{2}\sigma\bar\xi^i_R\xi^i_L + \text{H.c.}\right] \\
&+ \left[-i\sqrt{2}\bar\rho_i\left(\bar\lambda_L\eta^i_R - \bar\lambda_R\eta^i_L\right) + i\sqrt{2}\bar\sigma\bar\eta^i_R\eta^i_L + \text{H.c.}\right] \\
&+ \tfrac{i}{2}\bar\zeta_R\partial_L\zeta_R - \left[i\sqrt{2}\omega\lambda_L\zeta_R + \text{H.c.}\right],
\end{aligned}$$

(0,2) 'weighted' model

[PK Monin Vinci]

$$\int d^4\theta \left[\sum_{i=1}^{N_c} \Phi_i^\dagger \, e^V \Phi_i + \sum_{i=1}^{N_c - N_f} \tilde{\Phi}_i^\dagger \, e^{-V} \tilde{\Phi}_i - (r + \mathcal{B})V + \frac{1}{2e^2} \Sigma^\dagger \Sigma \right]$$

$$\Phi^i = n^i + \bar{\theta}\xi^i + \theta\bar{\xi}^i + \bar{\theta}\theta F^i \,, \quad i = 1, \ldots, N_c$$
$$\widetilde{\Phi}^j = \rho^j + \bar{\theta}\eta^j + \theta\bar{\eta}^j + \bar{\theta}\theta \tilde{F}^j \,, \quad j = 1, \ldots, \tilde{N}$$

$$\Sigma = \sigma + i\theta^+ \bar{\lambda}_+ - i\bar{\theta}^- \lambda_- + \theta^+ \bar{\theta}^- (D - iF_{01})$$

$$\mathcal{B} = \omega(\bar{\theta}\zeta_R + \bar{\theta}\theta \bar{\mathcal{F}} \mathcal{F})$$

deformation adds

$$\mathcal{L}^{het} = \mathcal{L} + \bar{\zeta}_R \partial_L \zeta_R - |\omega|^2 |\sigma|^2 - [i\omega\lambda_L \zeta_R + \mathrm{H.c.}]$$

Not enough SUSY to fix superpotential

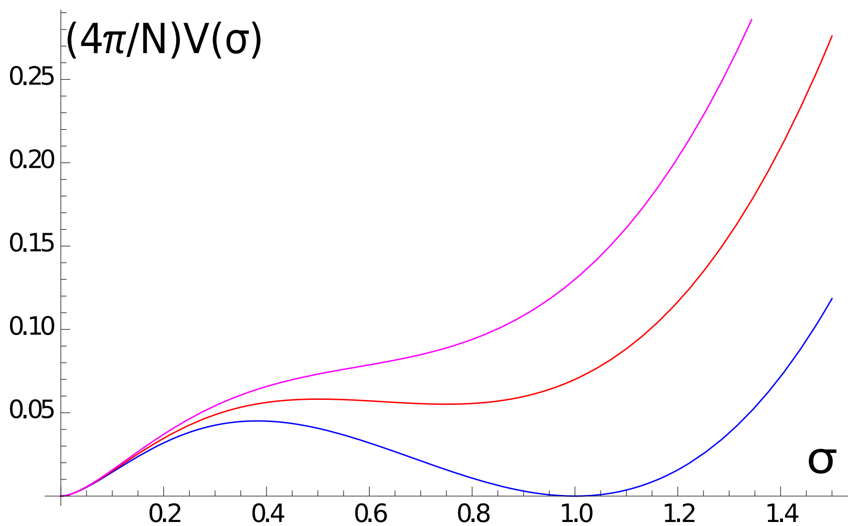
Have to dwell on large-N approach

Large-N solution of (0,2)

$$V_{1-loop} = \frac{1}{4\pi}\sum_{i=1}^{N-1}\left(-\left(D+|\sigma-m_i|^2\right)\log\frac{|\sigma-m_i|^2+D}{\Lambda^2}+|\sigma-m_i|^2\log\frac{|\sigma-m_i|^2}{\Lambda^2}\right)$$
$$-\frac{1}{4\pi}\sum_{j=1}^{\tilde{N}-1}\left(-\left(D-|\sigma-\mu_j|^2\right)\log\frac{|\sigma-\mu_j|^2-D}{\Lambda^2}-|\sigma-\mu_j|^2\log\frac{|\sigma-\mu_j|^2}{\Lambda^2}\right)$$
$$+\frac{N-\tilde{N}}{4\pi}D\,.$$

$$V_{eff} = V_{1-loop} + \left(|\sigma-m_0|^2+D\right)|n_0|^2 + \left(|\sigma-\mu_0|^2-D\right)|\rho_0|^2 + \frac{uN}{4\pi}|\sigma|^2$$

for zero masses



Symmetric masses

$$m_k = m\, e^{2\pi i \frac{k}{N}}\,, \quad k=0,\dots,N-1\,,$$
$$\mu_l = \mu\, e^{2\pi i \frac{l}{\tilde{N}}}\,, \quad l=0,\dots,\tilde{N}-1\,.$$

Vacuum equations

$$\left(|\sigma - m_0|^2 + D\right) n_0 = 0\,, \quad \left(|\sigma - \mu_0|^2 - D\right) \rho_0 = 0\,,$$

$$\frac{1}{4\pi}\sum_{i=1}^{N-1} \log \frac{|\sigma - m_i|^2 + D}{\Lambda^2} - \frac{1}{4\pi}\sum_{j=1}^{\tilde{N}-1} \log \frac{|\sigma - \mu_j|^2 - D}{\Lambda^2} = |n_0|^2 - |\rho_0|^2\,,$$

$$\frac{1}{4\pi}\sum_{i=1}^{N-1} (\sigma - m_i) \log \frac{|\sigma - m_i|^2 + D}{|\sigma - m_i|^2} + \frac{1}{4\pi}\sum_{j=1}^{\tilde{N}-1} (\sigma - \mu_j) \log \frac{|\sigma - \mu_j|^2 - D}{|\sigma - \mu_j|^2} =$$

$$= (\sigma - m_0)\,|n_0|^2 + (\sigma - \mu_0)\,|\rho_0|^2 + \frac{uN}{4\pi}\,\sigma\,.$$

Phases

Phase transitions -- artifact of large-N

$$\left(|\sigma - m_0|^2 + D\right) n_0 = 0\,, \quad \left(|\sigma - \mu_0|^2 - D\right) \rho_0 = 0$$

Higgs in n (Hn)

$$\rho_0 = 0 \quad D = -|\sigma - m|^2$$

$$r = \begin{cases} \frac{N-\tilde{N}}{2\pi} \log \frac{m}{\Lambda}\,, & \mu < m \\ \frac{N}{2\pi} \log \frac{m}{\Lambda} - \frac{\tilde{N}}{2\pi} \log \frac{\mu}{\Lambda}\,, & \mu > m\,. \end{cases}$$

Higgs in rho (Hrho)

$$n_0 = 0 \quad D = |\sigma - \mu|^2$$

$$r = \begin{cases} \frac{N-\tilde{N}}{2\pi} \log \frac{\mu}{\Lambda}\,, & \mu > m \\ \frac{N}{2\pi} \log \frac{m}{\Lambda} - \frac{\tilde{N}}{2\pi} \log \frac{\mu}{\Lambda}\,, & \mu < m \end{cases}$$

Coulomb (C)

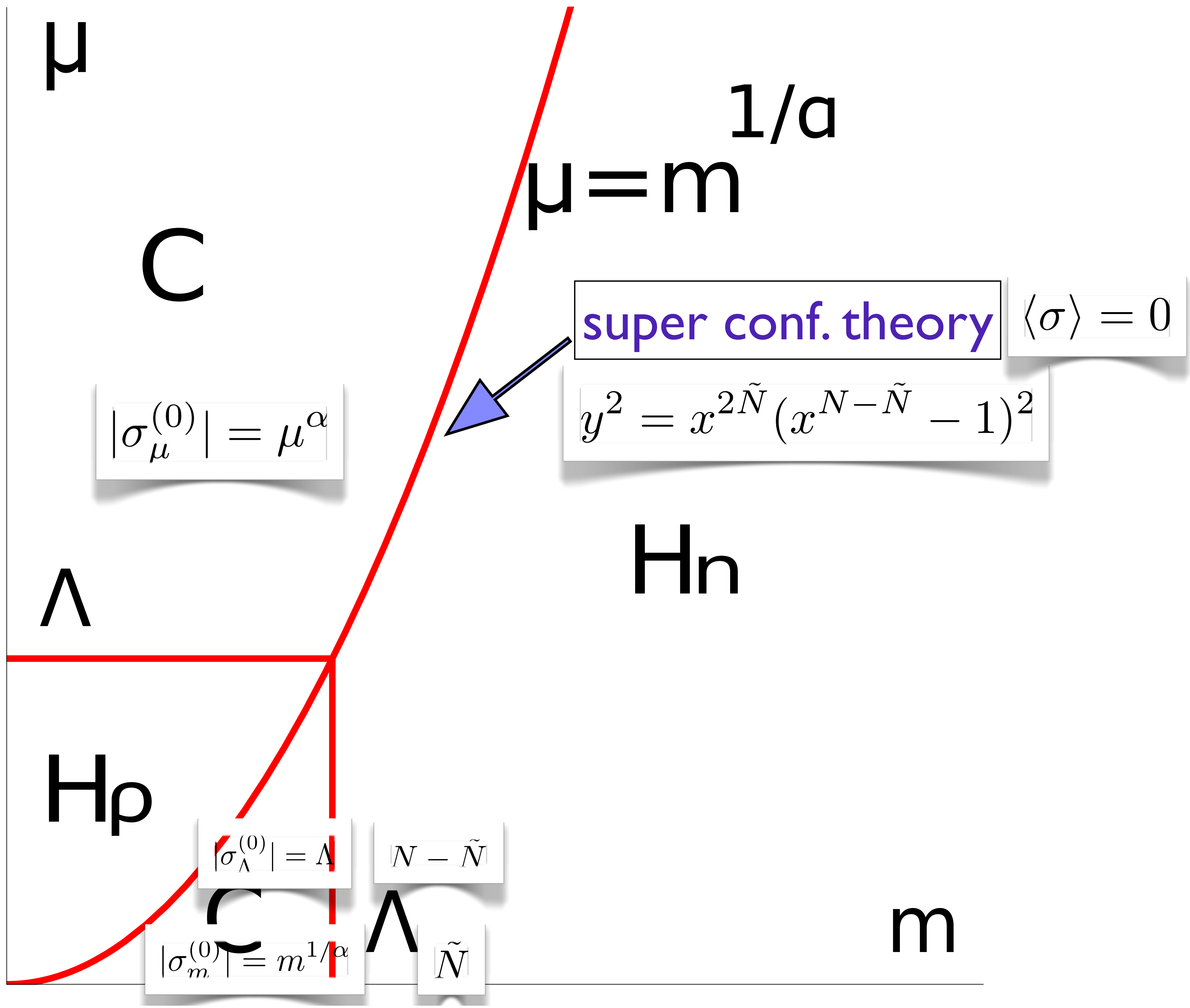
$$n_0 = \rho_0 = 0$$

renormalized FI term vanishes in C phase

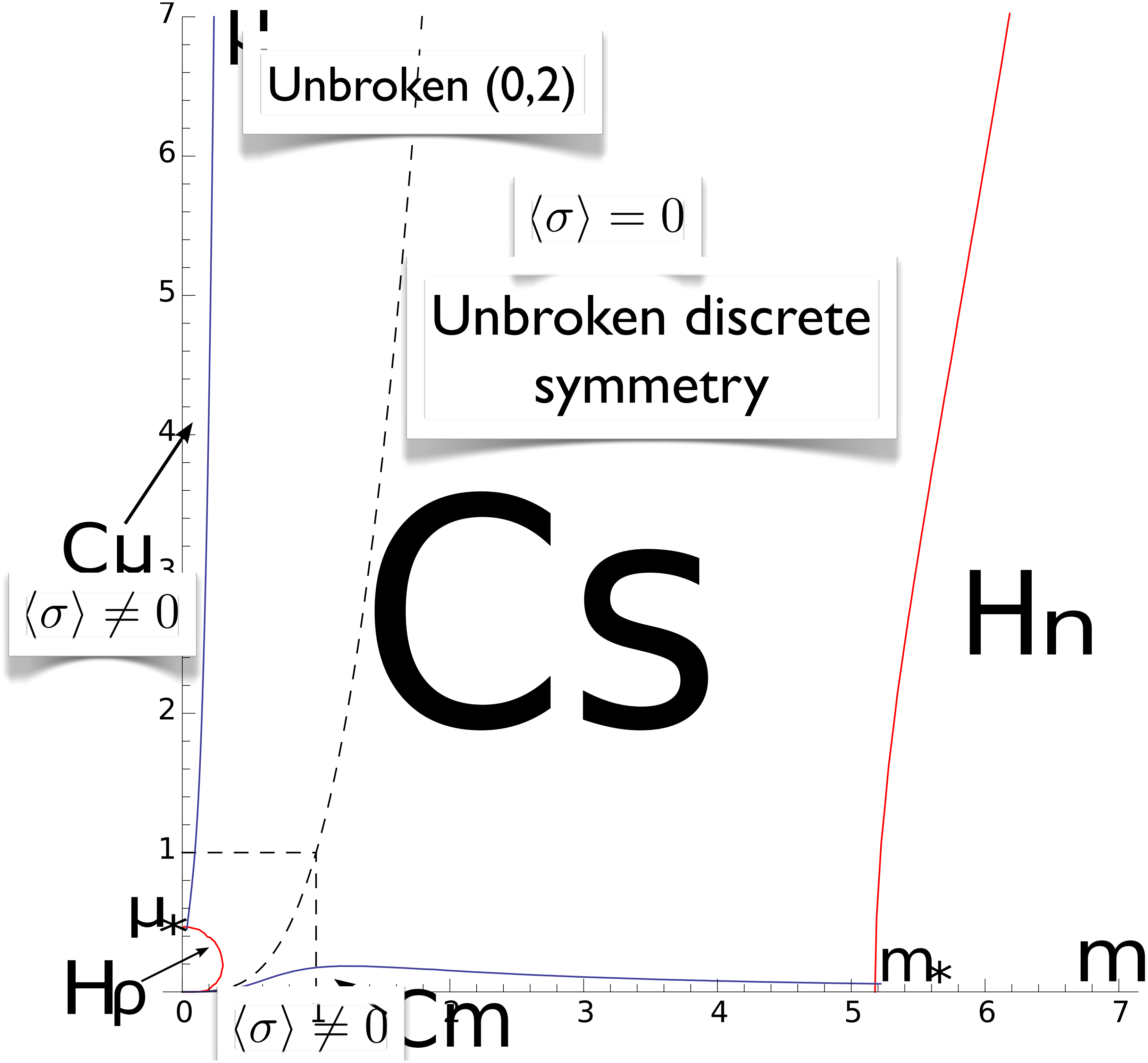
in (2,2) from exact superpotential

$$\frac{\prod_i (\sigma - m_i)}{\prod_i (\sigma - \mu_j)} = \Lambda^{N-\tilde{N}} \qquad \sigma = 0$$

is one of the solutions...



μ
1/α
μ=m
C
$|\sigma_\mu^{(0)}| = \mu^\alpha$
super conf. theory
$\langle\sigma\rangle = 0$
$y^2 = x^{2\tilde{N}}(x^{N-\tilde{N}} - 1)^2$
Hn
Λ
Hp
$|\sigma_\Lambda^{(0)}| = \Lambda$
$N - \tilde{N}$
C
Λ
$|\sigma_m^{(0)}| = m^{1/\alpha}$
$\tilde{N}$
m



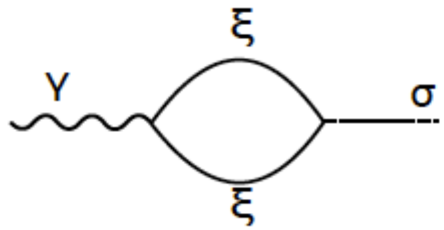
Unbroken (0,2)
$\langle\sigma\rangle = 0$
Unbroken discrete symmetry
Cs
Hn
$\langle\sigma\rangle \neq 0$
$\langle\sigma\rangle \neq 0$
Hp
Cm
m
m_*
μ_*

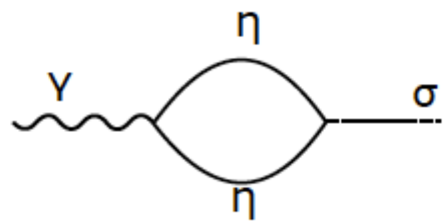
Spectrum

[Bolokhov Shifman Yung]
[PK Monin Vinci]

$$\mathcal{L} = -\frac{1}{4e_\gamma^2}F_{\mu\nu}^2 + \frac{1}{e_{\sigma\,1}^2}(\partial_\mu \mathfrak{Re}\,\sigma)^2 + \frac{1}{e_{\sigma\,2}^2}(\partial_\mu \mathfrak{Im}\,\sigma)^2 + i\mathfrak{Im}(\bar{b}\,\delta\sigma)\epsilon_{\mu\nu}F^{\mu\nu} - V_{\text{eff}}(\sigma) + \text{Fermions}$$

Anomaly





$$b = \frac{N}{4\pi}\left(\frac{1}{N}\sum_{i=1}^{N-1}\frac{1}{\bar{\sigma}_0 - \bar{m}_i} - \alpha\frac{1}{\tilde{N}}\sum_{i=1}^{\tilde{N}-1}\frac{1}{\bar{\sigma}_0 - \bar{\mu}_i}\right)$$

$$m_\gamma = e_{\sigma\,2}e_\gamma|b|$$

Photon becomes massless in Cs phase!!

Note that Lambda vacua disappear at large deformations

Need to sit in zero-vacua

e.g. in Cm phase

$$m_\gamma = \sqrt{6}\,\Lambda\left(\frac{\Lambda}{m}\right)^{1/\alpha}\left(\left(\frac{m}{\Lambda}\right)^{2/\alpha} - \left(\frac{\mu}{\Lambda}\right)^2 e^{u/\alpha}\right)\mathrm{e}^{-\frac{u}{2\alpha}}$$

Massless goldstino in fermionic sector

Worldsheet sigma model from the 4d theory

Derivation from 4d theory

[Shifman Vinci Yung 2011]

Brane construction is not sensitive to IR physics

Blind to deformations within the same universality class

Need to know explicit metric on the vacuum manifold

in order to go beyond BPS sector

1/2 BPS vortices

$$
\begin{aligned}
S &= \int d^4x \, \mathrm{Tr} \left\{ \frac{1}{g^2} \left(F_{12} + \frac{g^2}{2}(Q\bar{Q} - \xi) \right)^2 + \right. \\
&+ |\nabla_1 Q + i \nabla_2 Q|^2 + |\Phi Q + QM|^2 + \xi \, F_{12} + \\
&+ \left. \frac{1}{g^2}(F_{ik})^2 + (\nabla_k Q)^* (\nabla_k Q) + \frac{1}{g^2}(F_{kl})^2 \right\}, \\
& i = 1, 2, \quad k, l = 0, 3 \, .
\end{aligned}
$$

String tension

$$
T = \xi \int d^2x \, \mathrm{Tr} \, F_{12} = 2\pi\xi
$$

ansatz

$$
Q_0 = \left(\begin{array}{cccc|c}
\phi_1(r) & 0 & 0 & 0 & 0 \\
0 & \ddots & \vdots & \vdots & \vdots \\
\vdots & \dots & \phi_1(r) & 0 & 0 \\
0 & \dots & 0 & \phi_2(r) e^{i\theta} & \phi_3(r)
\end{array} \right)
\qquad
A_{0,i} = \epsilon_{ij} \frac{x_j}{r^2} f(r) \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \end{pmatrix}
$$

Bogomol'ny equations

reduce to Abelian Higgs

$$r\phi_1'(r) = 0\,,$$

$$r\phi_2'(r) - f(r)\,\phi_2(r) = 0\,,$$

$$r\phi_3'(r) - (f(r) - 1)\phi_3(r) = 0\,,$$

$$\frac{1}{r}f'(r) + \frac{g^2}{2}(\phi_2^2(r) + |\phi_3(r)|^2 - \xi) = 0\,.$$

modulus

$$\phi_1(r) = \sqrt{\xi}, \quad \phi_3 = \frac{\rho}{r}\phi_2$$

can solve the rest of equations analytically provided that

$$\frac{1}{g\sqrt{\xi}|\rho|} << 1$$

e.g. gauge field

$$A_k = -i\Big(\partial_k n\, n^* - n\,\partial_k n^* - 2n\,n^*(n^*\partial_k n)\Big)\omega(r) \\ -i\,n\,n^*\Big(\rho^*\partial_k\rho - \rho\partial_k\rho^* + 2|\rho|^2(n^*\partial_k n)\Big)\gamma(r)$$

after some work [Shifman Vinci Yung] we get...

Effective action

$$\mathcal{L}_{\text{eff}} = \pi\xi\left(\ln\frac{L^2}{|\rho|^2}\right)\left|\partial_k(\rho\, n)\right|^2 - \pi\xi|\partial_k\rho + \rho\,(n^*\partial_k n)|^2$$
$$+ \frac{2\pi}{g^2}\left[\partial_k n^*\partial_k n + (\partial_k n^* n)^2\right].$$

for large L can insert Log under derivative

already includes
subleading corrections

$$z = \rho\left[2\pi\xi\,\ln\frac{L}{|\rho|}\right]^{1/2} \qquad L \sim |\Delta m|^{-1}$$

Arrive to a new model (ZN) with Kahler potential

$$K = |\mathcal{M}|^2(1+|\phi_i|^2)(1+|\tilde{\phi}_i|^2) + r\log(1+|\phi_i|^2)$$

ZN model vs HT model

$$K=\sqrt{r^2+4|r||\mathcal{M}|^2\left(1+\sum_{i=2}^{N_c}|\phi_i|^2\right)\left(1+\sum_{i=2}^{\tilde{N}}|\tilde{\phi}_i|^2\right)}$$
$$-\,r\log\left(r+\sqrt{r^2+4|r||\mathcal{M}|^2\left(1+\sum_{i=2}^{N_c}|\phi_i|^2\right)\left(1+\sum_{i=2}^{\tilde{N}}|\tilde{\phi}_i|^2\right)}\right)$$
$$+\,r\log\left(1+\sum_{i=2}^{N_c}|\phi_i|^2\right)\,,$$

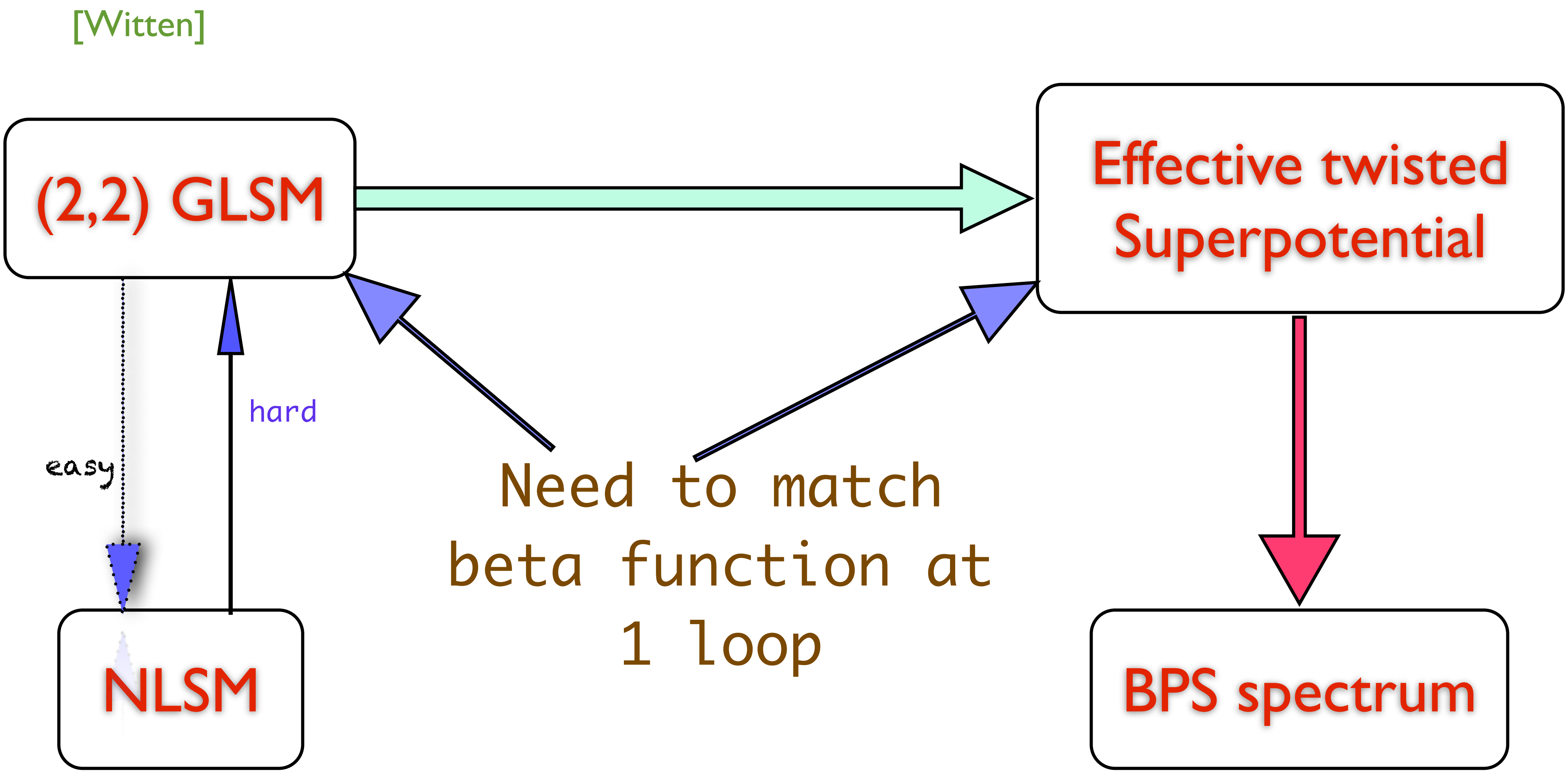
small |M|

$$K=|\mathcal{M}|^2\left(1+\sum_{i=2}^{N_c}|\phi_i|^2\right)\left(1+\sum_{i=2}^{\tilde{N}}|\tilde{\phi}_i|^2\right)+r\log\left(1+\sum_{i=2}^{N}|\phi_i|^2\right)$$

Small |M| implies semiclassical regime

Question: how ZN model behaves at strong coupling?

Possible ways to solve



work in progress...

Conclusions and open questions

- Study BPS (and beyond) spectrum of SQCD can effectively be done using 2d NLSM (and GLSM)
- Rich variety of phases in (0,2) model at strong coupling
- Other heterotic deformations

$$\bar{D}\Phi_{+} \sim \bar{D}\Phi_{-}$$

- Are there flux tubes in theories without FI term? (e.g. SU(N)) Omega deformed 4d theory may have such solutions...
- Connections to integrable systems in 2d...
- Fixed point in metric flow...