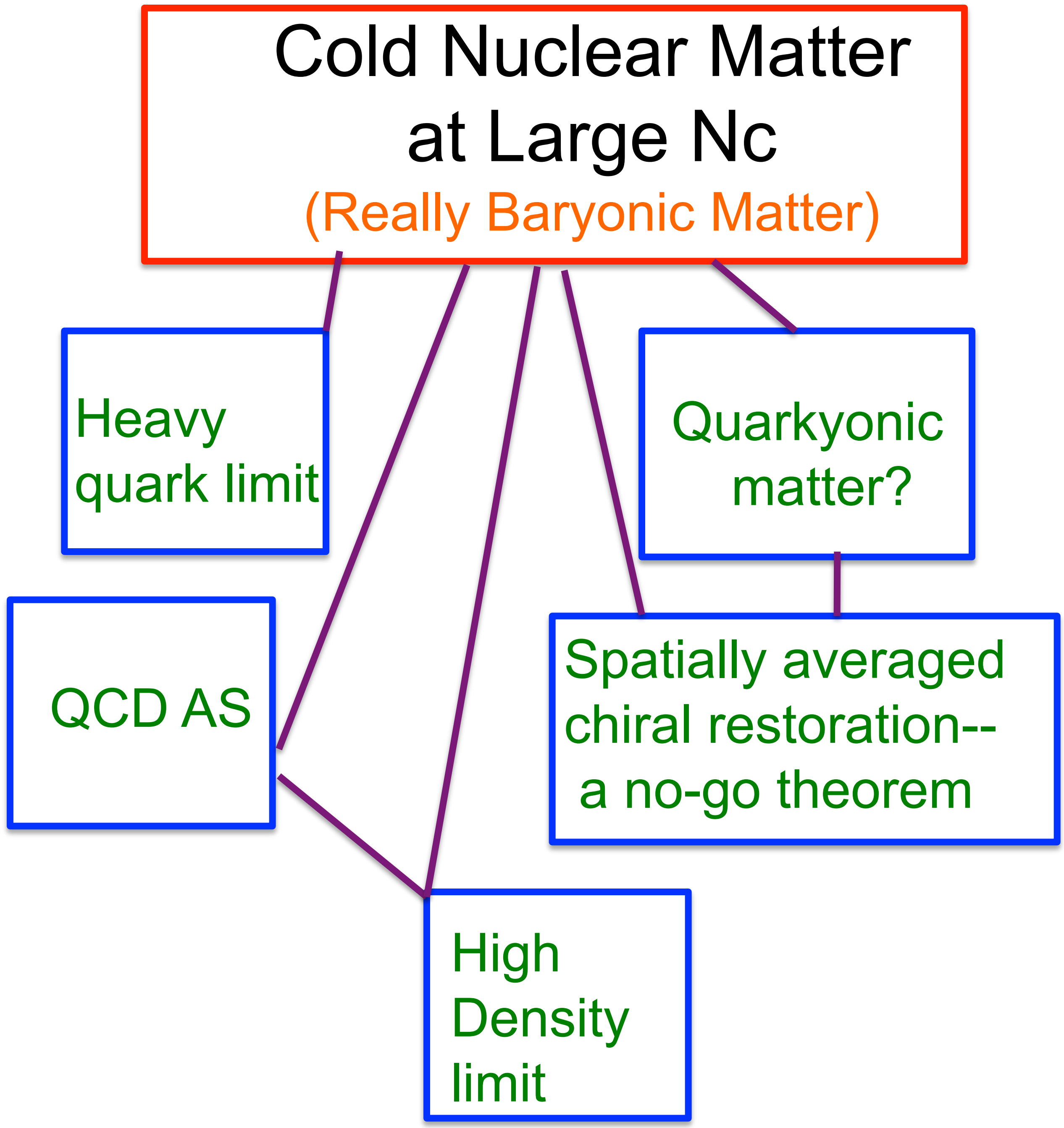
Cold Nuclear Matter in Large Nc QCD

“Continuous Advances in QCD 2011”

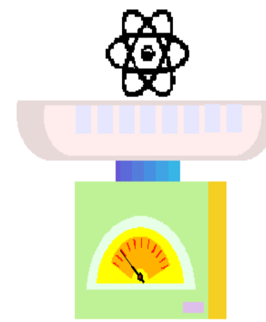
An overview

Elliott Hall
Northrop
Wulling Hall
Arlington St
Johnston Hall
Morrill Hall
Rapson Hall
Fraser Hall
Walter Library
Tate Lab of Physics
University of Minnesota
Appleby Hall
Vincent Hall
Lind Hall
Smith Hall
Amundson Hall
St & Ss Building(2010)
Ford Hall
East River Pkwy
Washington Ave SE
The Cube(Union Entrance)
Weisman Art Museum
Molecular and Cellular Biology
Coffman Memorial Union
Church St SE
Mayo Memorial Auditorium
Comstock Hall
Delaware St SE
Mark G. Yudof Hall
Mayo Memorial Bldg
Children's Rehab
Variety Club Research Center
S 1st St
S 2nd St
19th Ave S
20th Ave S
21st Ave S
22nd Ave S
W River Pkwy
Bluff St S
10th Ave
Walter F. Mondale Hall
Elmer L. Andersen Library
West Bank Program Hall
Anderson Hall
Blegen Hall
Hubert H. Humphrey Center
Heller Hall
Ferguson Hall
John R. Borchert Map Library
Rarig Center
Ted Mann Concert Hall


Cold Nuclear Matter
at Large Nc
(Really Baryonic Matter)
Heavy quark limit
QCD AS
High Density limit
Quarkyonic matter?
Spatially averaged chiral restoration-- a no-go theorem

- Can we understand cold nuclear matter from QCD?
  - Lattice is useless at present: fermion sign problem.

- Can we use the 1/Nc Expansion?
  - Scales suggest that 1/Nc Expansion is hopeless.
  - Nuclear scales are radically smaller than typical hadronic scales for essentially unknown reason---reasons that have nothing to do with Nc.



- *Eg. Binding energy per nucleon is formally of order N_c^1 and is 16 MeV ; The N-Δ mass splitting is N_c^{-1} and is 300 MeV .*

- *Eg. Nuclear matter: NN potential is order N_c^1 , characteristic spacing is order N_c^0 and nucleon mass is N_c^1: potential energy ,U, dominates over nucleon kinetic energy, T, by order N_c^2 . Real nuclear matter computations have $T \gg |U-T|$. Real nuclear matter behaves as a fluid, not a crystal.*

- ***Clearly large scales and small scales from large Nc are mixed; there is no clean scale separation based on large Nc.***

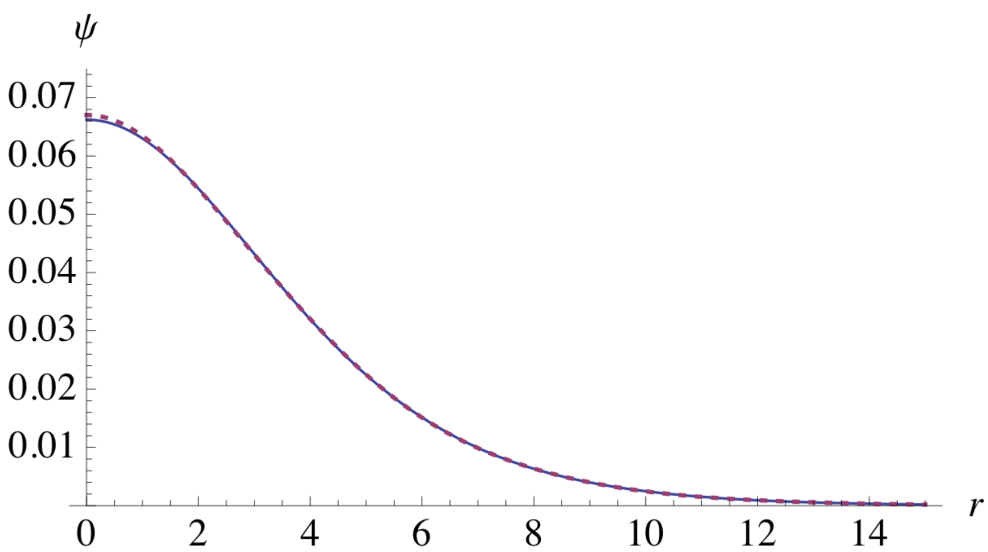
Thus the problem may be only theoretical as opposed to phenomenological interest. However, it *is* theoretically interesting.

While in general we do not how to solve the baryonic matter problem at large Nc we can at low density in the “Witten limit” where all baryon species are heavy and of degenerate mass. (TDC, N. Kumar,K. Ndousse 2011)000

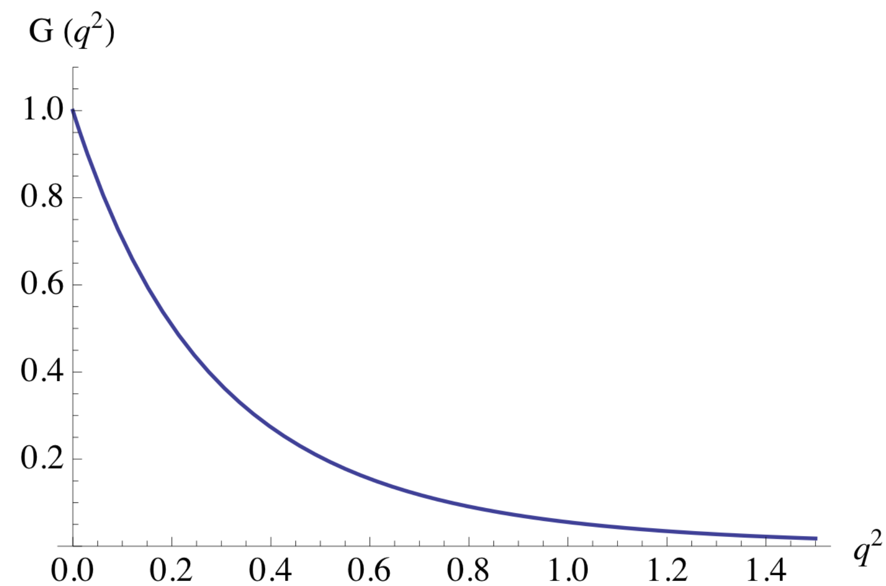
As a first step one needs to find the properties of single baryons by solving the mean-field equations of motion.

$$M_{Baryon} = N_c M_Q \left(1 - 0.05426 (N_c \alpha_s)^2\right)$$

$$\langle r^2 \rangle_{Baryon} = \frac{21.5}{\left(M_Q (N_c \alpha_s)\right)^2}$$



Mean-field wave function; distance in units of $((N_c\alpha_s)M_Q)^{-1}$



Baryon form factor; momentum in units of $((N_c\alpha_s)M_Q)$

To go further, one needs to formulate that the many-body problem. Again we use Witten's mean-field ansatz. In the case of infinitely low density it will simply give widely separated baryons.

One can include perturbatively the effects of interactions between these baryons. These come from two sources---the Pauli principle at the quark level and color-coulomb interactions.

It is easy to show that at low densities color-coulomb effects are down parametrically by a factor of order $\rho^{1/3} /(M_Q\ (Nc\ \alpha_s))$

Thus at low density here will be a mean-field solution which is purely repulsive and requires external pressure. The system them forms a closely-packed crystal (FCC or Hexagonal close packed). The energy density is given by

$$\frac{\text{Interaction energy}}{\text{baryon}} \approx .00042858\, N_c\, M_Q (N_c \alpha_s)^2\, \tilde{\rho}^{2.3369} \exp\left(-2.0332\, \tilde{\rho}^{-1/3}\right)$$

$$\text{with } \tilde{\rho} \equiv \frac{\rho}{2N_f \left(M_Q (N_c \alpha_s)\right)^3}$$

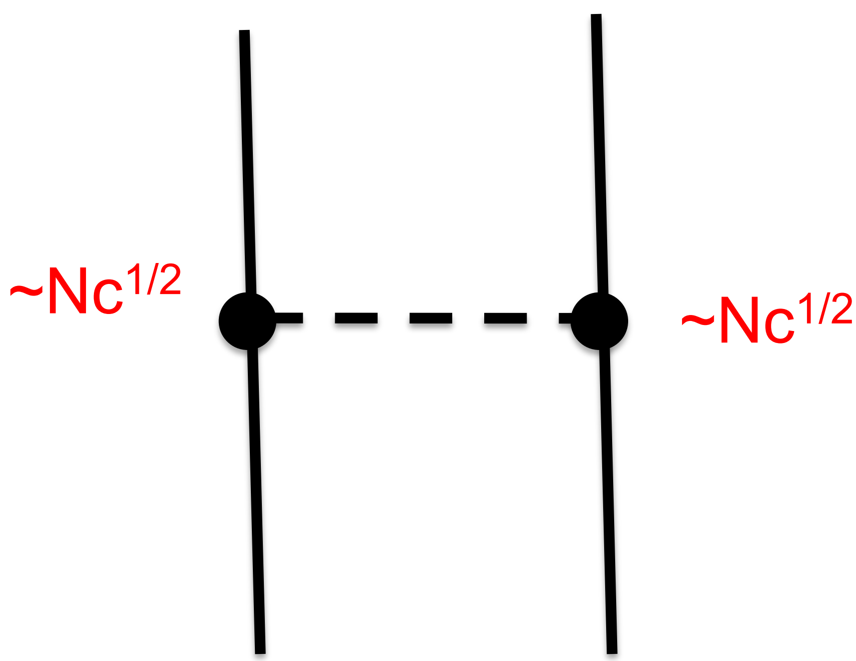
It is easy to show that such a field configuration corresponds to other the ground or a very long lived metastable state τ~exp(const Nc).

- Note that if this state is the true ground state, then baryonic matter does not “saturate” (i.e. is not self-bound) in this limit.
- However if $1/N_c$ effects are included the picture changes
  - Leading order effect purely repulsive is large ($\sim N_c$) but short-ranged ($r \sim (MQ(N_c\,\alpha_s)^{-1})$)
  - Longest-range effect ($r \sim \Lambda_{QCD}^{-1}$) is one scalar-glueball exchange. It is attractive but weak (N_c^0).
    - Despite its weakness it will denominate at very low densties
  - Baryonic matter will saturate since the longest range attraction will bind heavy baryons but the binding energy is subleading in the 1/Nc approximation and the saturation density goes to zero as Nc goes to infinity.

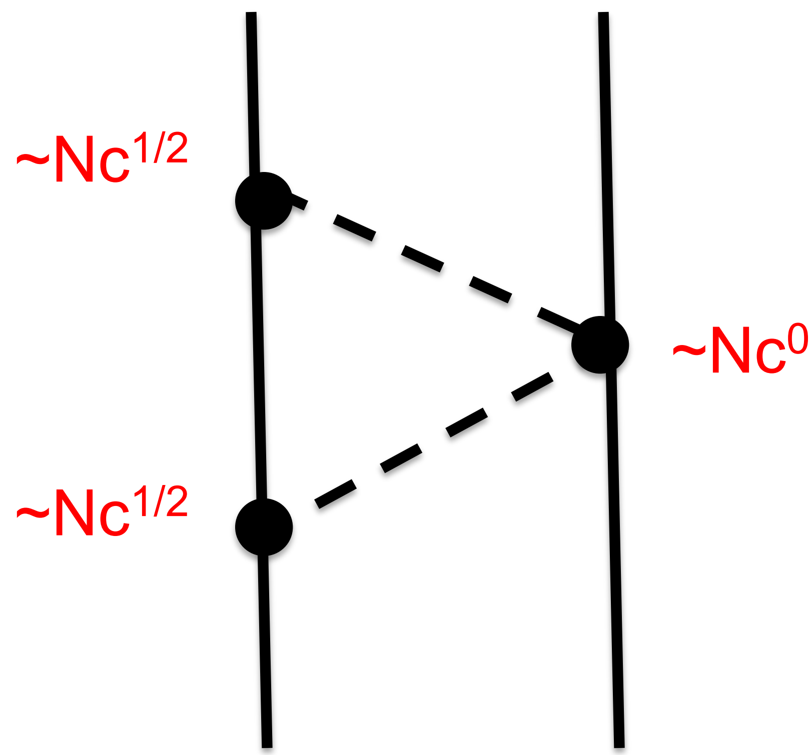
What do we know about nuclear matter at large Nc in general away from the heavy quark limit?

- Nucleon-Nucleon forces are strong in large Nc limit
- $V_{NN} \sim Nc$

Easily seen via a meson exchange picture



- Nucleon-Nucleon forces include dynamics of multi-meson exchanges at leading order in 1/Nc

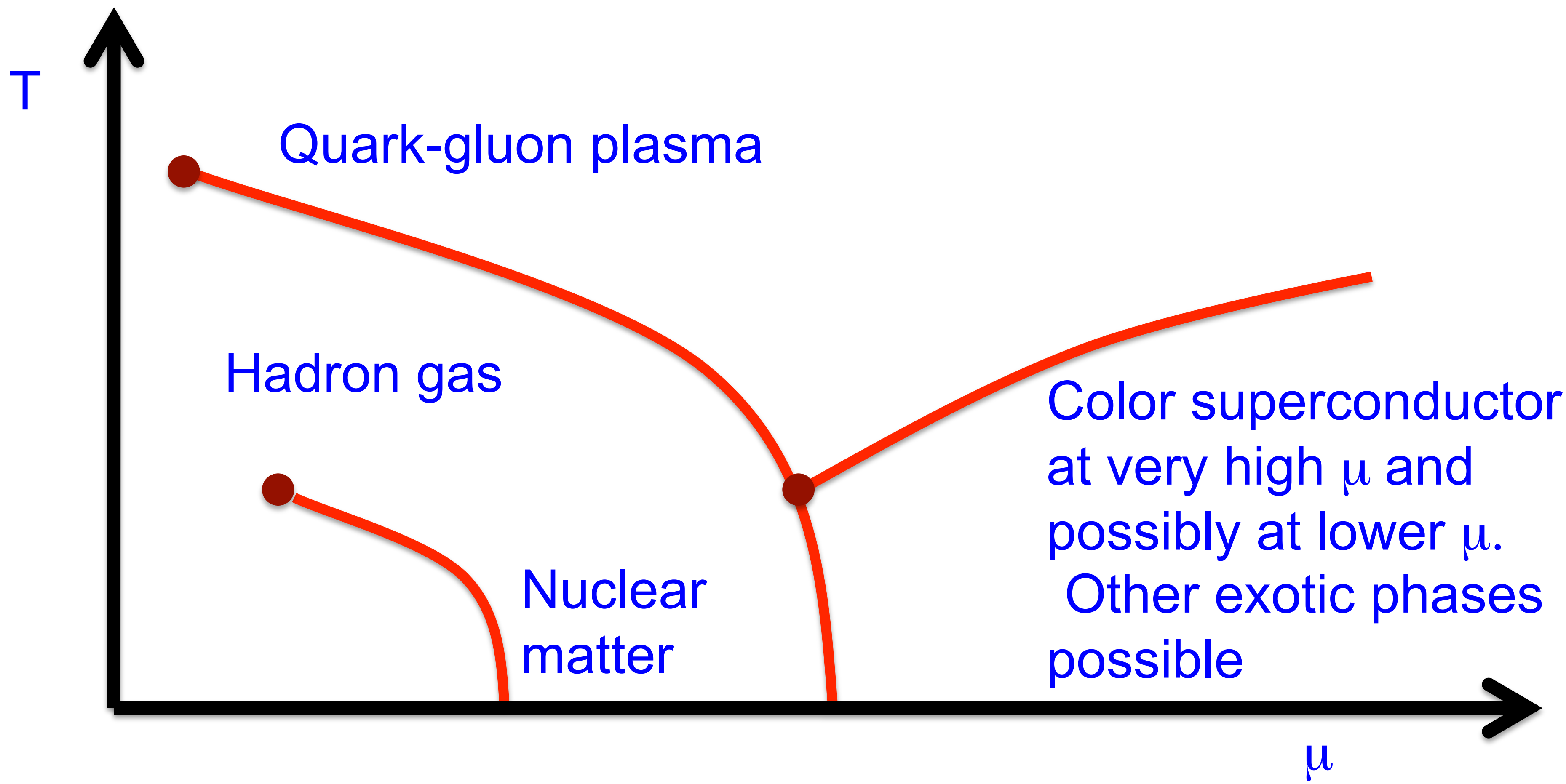


Overall contribiution is
$V_{NN} \sim Nc$
This is leading order scaling and is correctly captured by sensible large Nc model

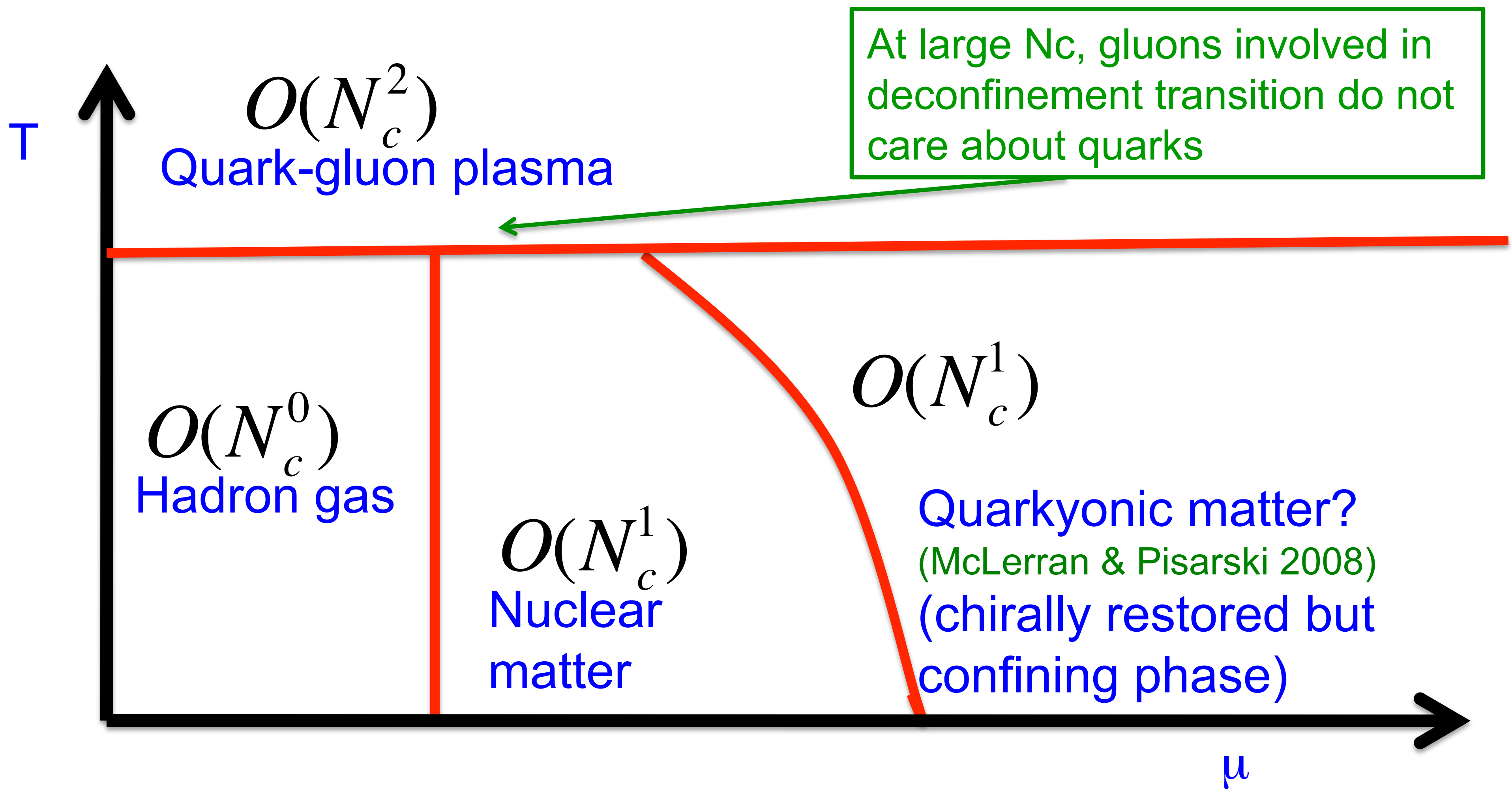
Note that this physics is absent in the Sakai-Sugimoto treated as an instanton

- Nuclear matter is crystalline and saturates in the large Nc limit
  - $\rho_{sat} \sim Nc^0$ $B \sim Nc^1$
  - Pion exchange is dominant long range interaction and has an attractive channel. Any attractive quantum system with parametrically strong forces or heavy mass will become arbitrarily well localized around the classical minimum
- This scaling is expected to yield an equation of state which is VERY different than Nc=3

The Nc=3 QCD Phase Diagram:
A Cartoon



QCD (F) Phase Diagram at Large Nc : A Cartoon



Large Nc behavior for dense matter with $\mu \sim N_c^0$ looks completely different from Nc=3!!

Quarkyonic Matter

- The idea at large Nc with $\mu \sim N_c^{0}$ is that while nothing can happen to confinement--- which is a gluonic property---chiral symmetry can be restored.
- Why would one expect chiral restoration?
  - It happens at high T
  - Models suggest this happens
    - Original paper cites some Skyrme model calculations from the late 80's and early 90's which indicate chiral restoration.

- Skyrme Models????
- This seems crazy---the models are built on non-linear sigma models which *always* spontaneously breaks chiral symmetry

$$\overset{\leftrightarrow}{\vec{U}}^{+}(\vec{x})\overset{\leftrightarrow}{\vec{U}}(\vec{x}) = \overset{\leftrightarrow}{1}$$

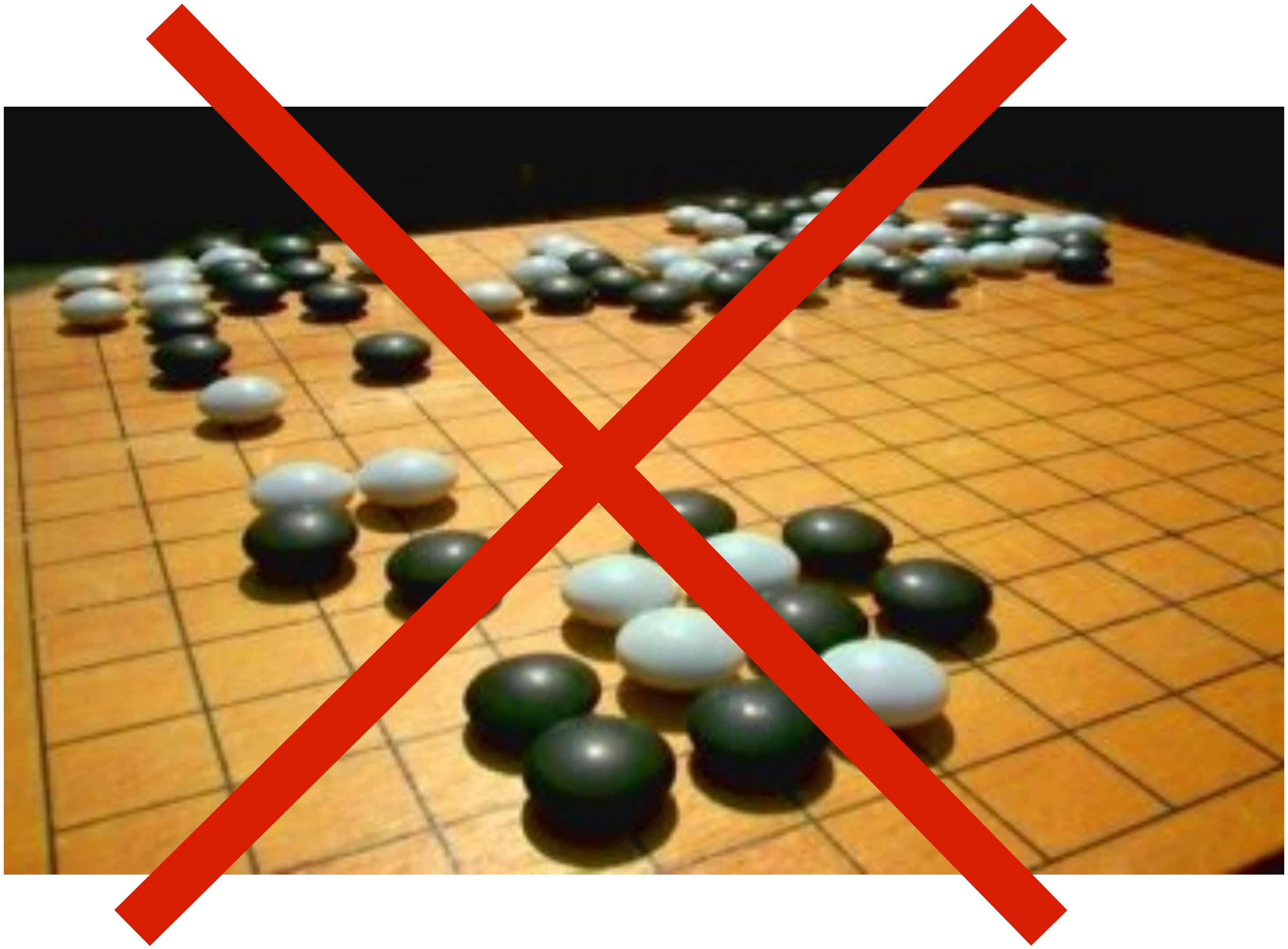
- However, beyond some critical density the system goes into a phase for which the spatially average chiral order parameter goes to zero:

$$\frac{\int d^3x \, Tr\left(\overset{\leftrightarrow}{\vec{U}}(\vec{x})\right)}{Vol} = 0$$

- Note that there are two types of spontaneous symmetry breaking at issue: chiral symmetry and translational symmetry.
- One could imagine that the two get intertwined.
  - It is useful to study aspects of chiral symmetry and its spontaneous breaking which are independent of translational symmetry breaking.
  - Spatially averaged chiral order parameters are of this type.

- Note that if spatially averaged chirally order parameters are not zero for infinite nuclear matter in the Skyrme model, then there must be 6 zero frequency modes---3 translational and 3 chiral (pionic)
  - If spatially averaged chiral order paramters are zero and the system is chiral restored in some spatially averaged sense, one expects only three zero modes.
  - The zero modes for “1/2 Skyrmion” configurations” with vanishing average chiral condensate were never computed.

- However for the tractable case where unit cells Skyrmion crystals are approximated (in an ad hoc way) by a single Skyrmion on a hypersphere, in the phase where the spatially average chiral condensate vanishes, the 6 zero modes associated with translations and chiral rotations collapse into 3.
- Suggests that a vanishing spatially averaged chiral condensate signals an effective restoration of chiral symmetry
  - Helps justify the argument for a chiral restored phase but…



Recently a "no-go" theorem was proved (TDC,P. Adhikari, M. Strother & R. Ayyagari 2011) showing that it is impossible in Skyrme trype models for chrial symmetry to be restored in the spatially averaged sense.

- This theorem states that for any model based on a non-linear sigma model treated classically, regardless of dynamics no configurations can exist which correspond to chiral restoration in the spatially averaged sense.
  - That is there must be some spatially averaged chiral order parameters which do not vanish
  - Proof is indirect. It assumes that all chiral order parameters vanish undr spatially averaging and then shows a contradiction emerge.

- Proof has three parts:
  - *U*(*x*) is mapping from configuration space to internal space. If all spatially averaged chiral order parameters vanish, it must be a mapping which covers the internal space with a constant density.
  - Given the previous result, it can be shown that in for all spatially averaged chiral order parameters vanish, all chiral singlet operators must be constant in space.
  - One can show that if all spatially averaged chiral order parameters vanish, then one can explicitly construct a chirally operator operator which cannot be constant in space.

The proof was for Skyrme type models built on non-linear sigma model. However, it applies to large Nc QCD as well.

Any configuration with a non-vanishing chiral condensate on a point-to-point basis cannot have all chiral order parameters vanish under spatial integration

At large Nc there are no quantum correlations but there can be spatial ones. Trick: define an observable which acts like U in the Skyrme model

$$\overleftrightarrow{V}(\vec{x}) = \overleftrightarrow{1}(\overline{q}(x)q(x)) + \sum_k \overleftrightarrow{\tau}_k(\overline{q}(x)i\gamma_5\overleftrightarrow{\tau}_k q(x))$$

$$\overleftrightarrow{V}(\vec{x}) = v(\vec{x})U(\vec{x}) \quad \text{with} \quad U^+(\vec{x})U(\vec{x}) = \overleftrightarrow{1}$$

The same proof basically goes through; in any putative chirally restored phase *v* which is a chiral singlet must be constant and the system reduces to a nonlinear σ−model

Some comments

- This no-go theorem shows that if chiral restoration occurs at large Nc it must occur at every point—not just in a spatially averaged sense.
- How is the no-go theorem consistent with vanishing zero-modes on hypercube
  - Theorem doesn't apply---only works for flat space
  - Hypercube is an ad hoc approximation. It happens that the unphysical geometry chosen is the same as the internal geometry for the σ-model

$$x^2 + y^2 + z^2 + w^2 = L^2 \qquad \text{hypersphere}$$

$$\pi_x^2 + \pi_y^2 + \pi_z^2 + \sigma^2 = f_\pi^2 \qquad \sigma - \text{model}$$

The chirally restored phase on the hypersphere is simply the mapping from a point in space directly to the analogous point internal space. This has no analog in flat space

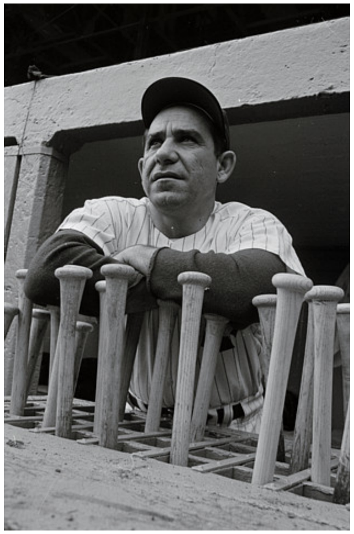
- How is the vanishing of the spatially averaged chiral condensate in the “1/2 Skyrmion” configuration consistent with no-go theorem.
  - Theorem states that some spatially averaged chiral order parameters must be non-zero. Not that *all* order parameters vanish.
  - Thus even if the chiral condensate vanishes upon spatial average the system need not be---and indeed is not---chirally restored in the spatially averaged sense.

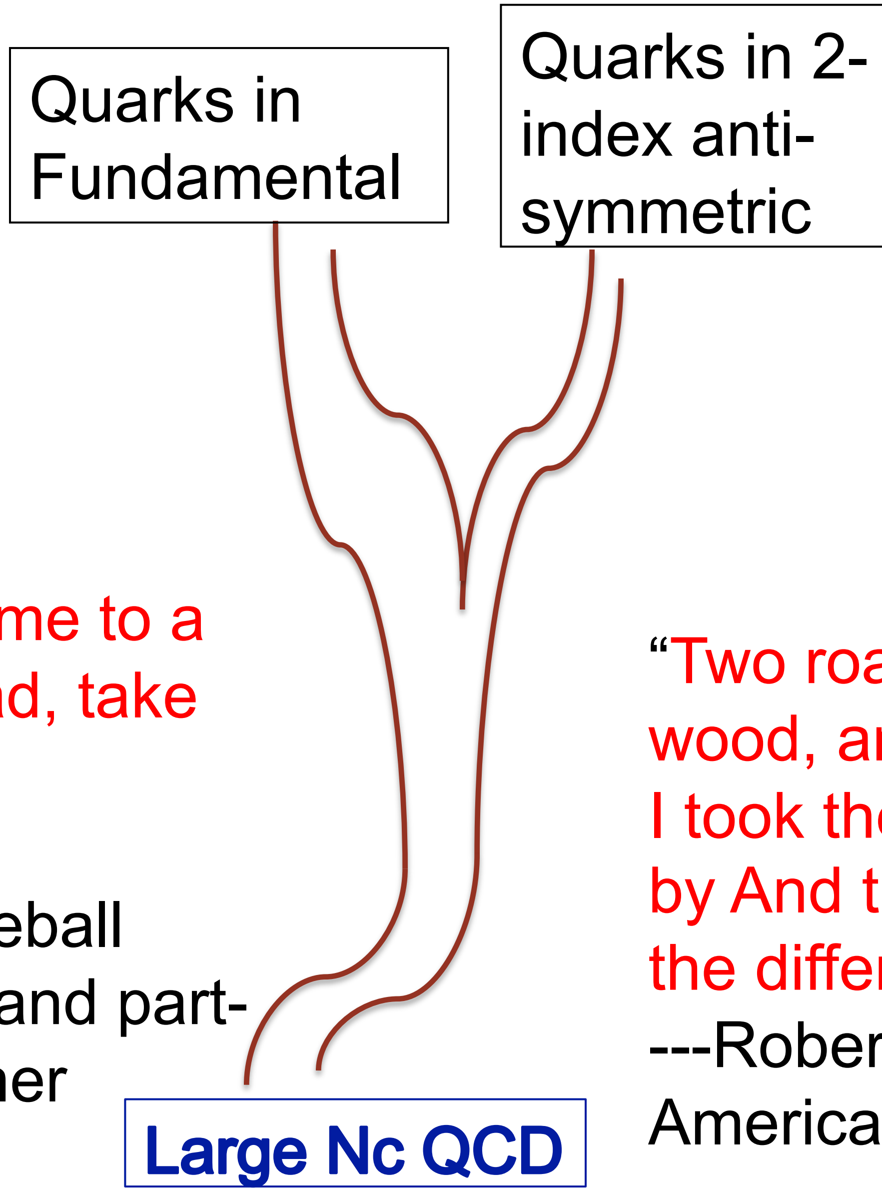
- It seems implausible that some chiral order parameters vanish will others do not. Never-the-less it happens.

- It appears to be inconsistent with a rigorous result (Kogan,Kovner, Shifman 1998) based on QCD inequalities that it is not possible in QCD for the chiral condensate to vanish without chiral symmetry being restored.

    - However, that derivation used the fact the functional determinant in the QCD path integral is real and non-negative. This is not when a chemical potential is present and thus there is no inconsistency.

Another large Nc limit :





"When you come to a fork in the road, take it."
---Yogi Berra,
American baseball player, coach and part-time philosopher

"Two roads diverged in a wood, and I—
I took the one less traveled by And that has made all the difference."
---Robert Frost,
American poet

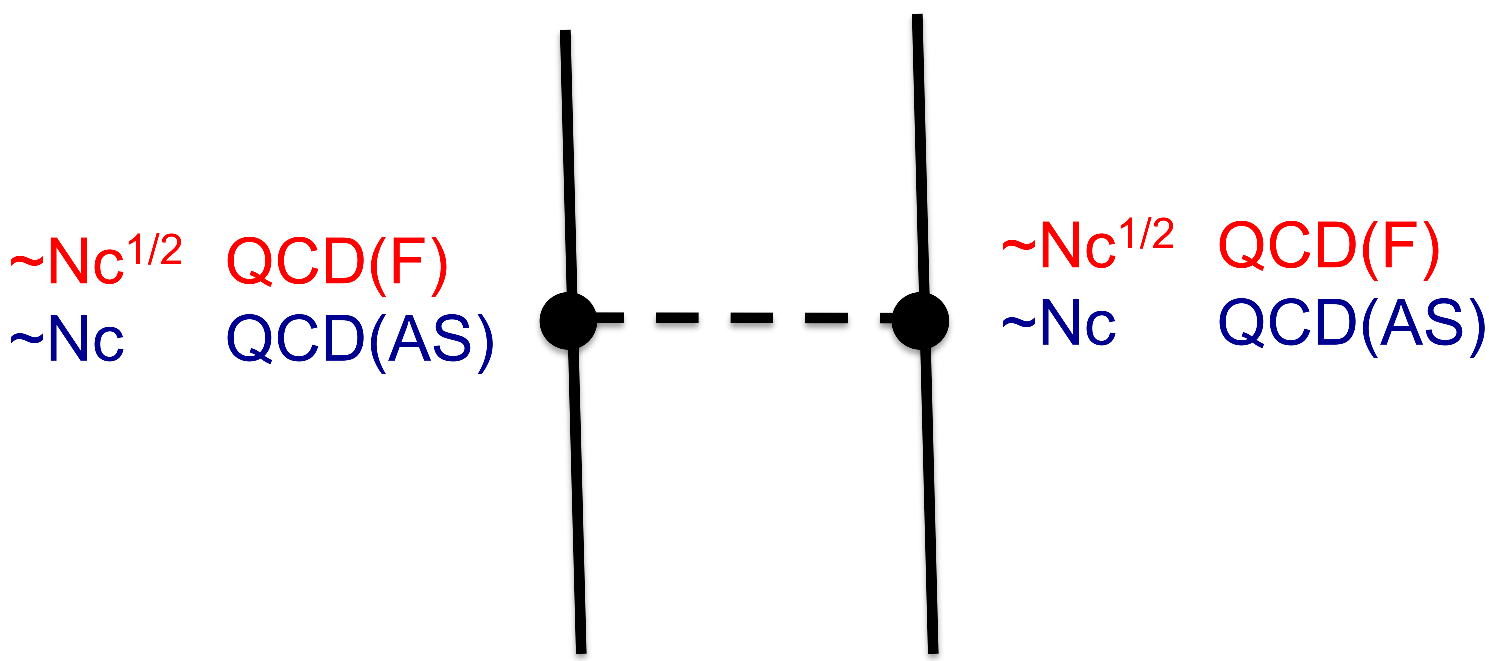
QCD and its large Nc limits:

- The large Nc limit of QCD is not unique
  - However for quarks, we can choose different representations of the gauge group
  - Note that Nc=3 quarks in the two index antisymmetric (AS) representation are indistinguishable from the (anti-)fundamental.
- However quarks in the AS and F extrapolate to large Nc in different ways.
  - The large Nc limits are physically different
  - The 1/Nc expansions are different.
  - A priori it is not obvious which expansion is better
  - It may well depend on the observable in question

What do we know about nuclear matter in each of the two large Nc limits?

- Nucleon-Nucleon forces are strong in both large Nc limits
  - QCD(F) $V_{NN} \sim Nc$
  - QCD(AS) $V_{NN} \sim Nc^2$

Easily seen via a meson exchange picture

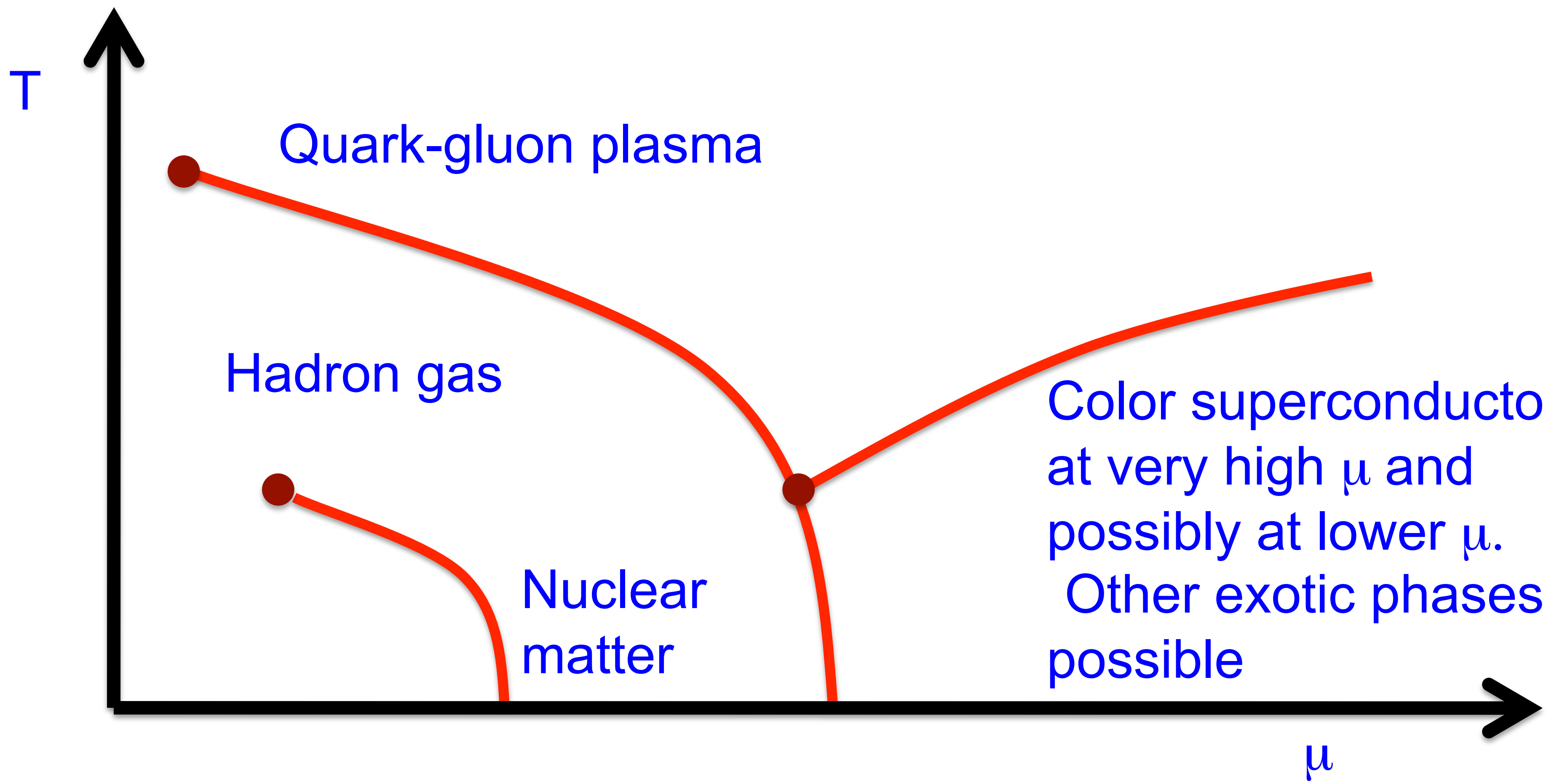


- Nuclear matter is crystalline and saturates in both large Nc limits
  - QCD(F) $\rho_{sat}\sim Nc^0$ $B\sim Nc^1$
  - QCD(AS) $\rho_{sat}\sim Nc^0$ $B\sim Nc^2$

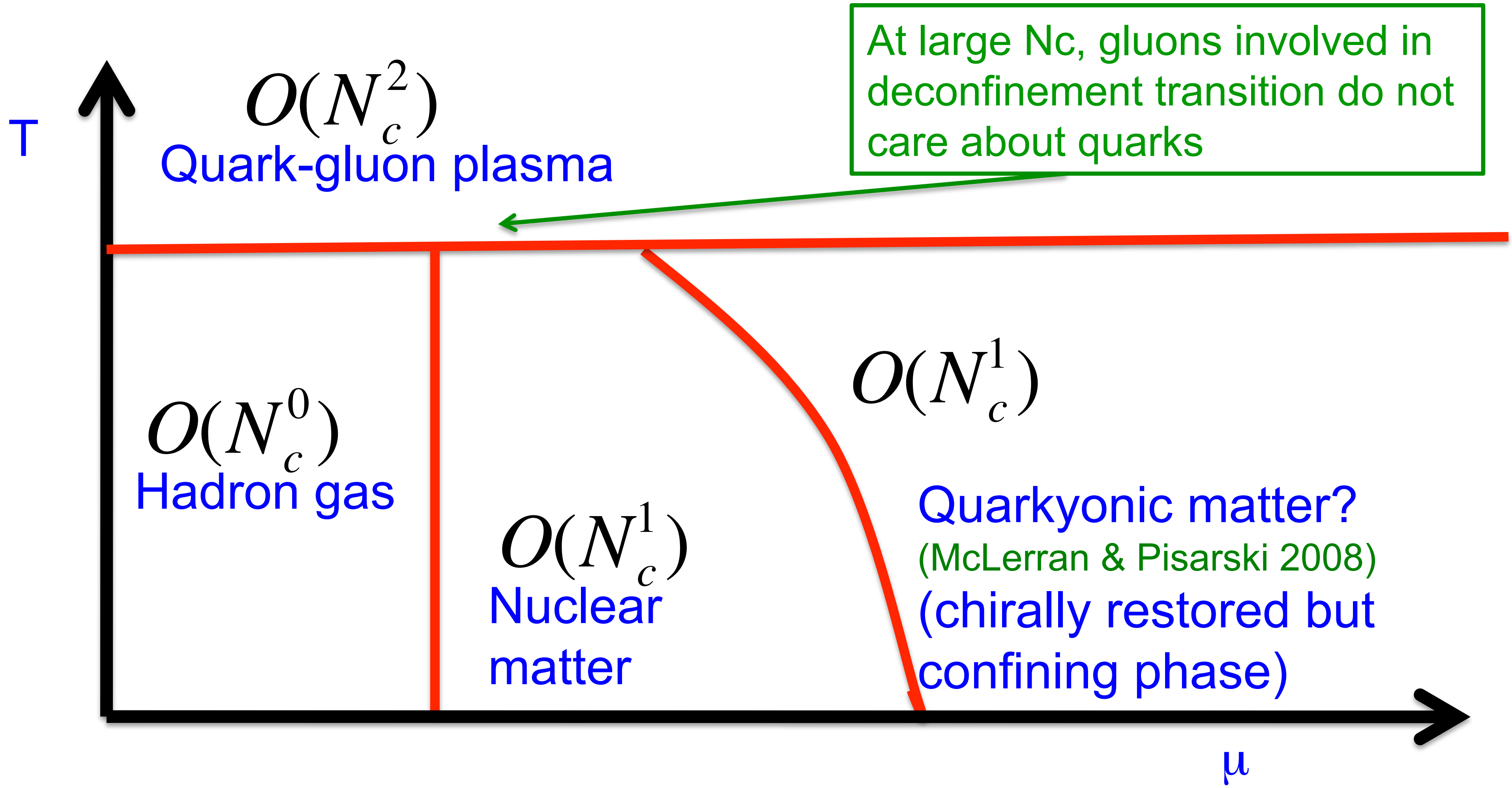
  - Pion exchange is dominant long range interaction and has an attractive channel. Any attractive quantum system with parametrically strong forces or heavy mass will become arbitrarily well localized around the classical minimum

- While both limits are similar in this respect there equations of state are expected to qualitatively different. Consider $T,\mu\sim Nc^0$

The Nc=3 QCD Phase Diagram:
A Cartoon

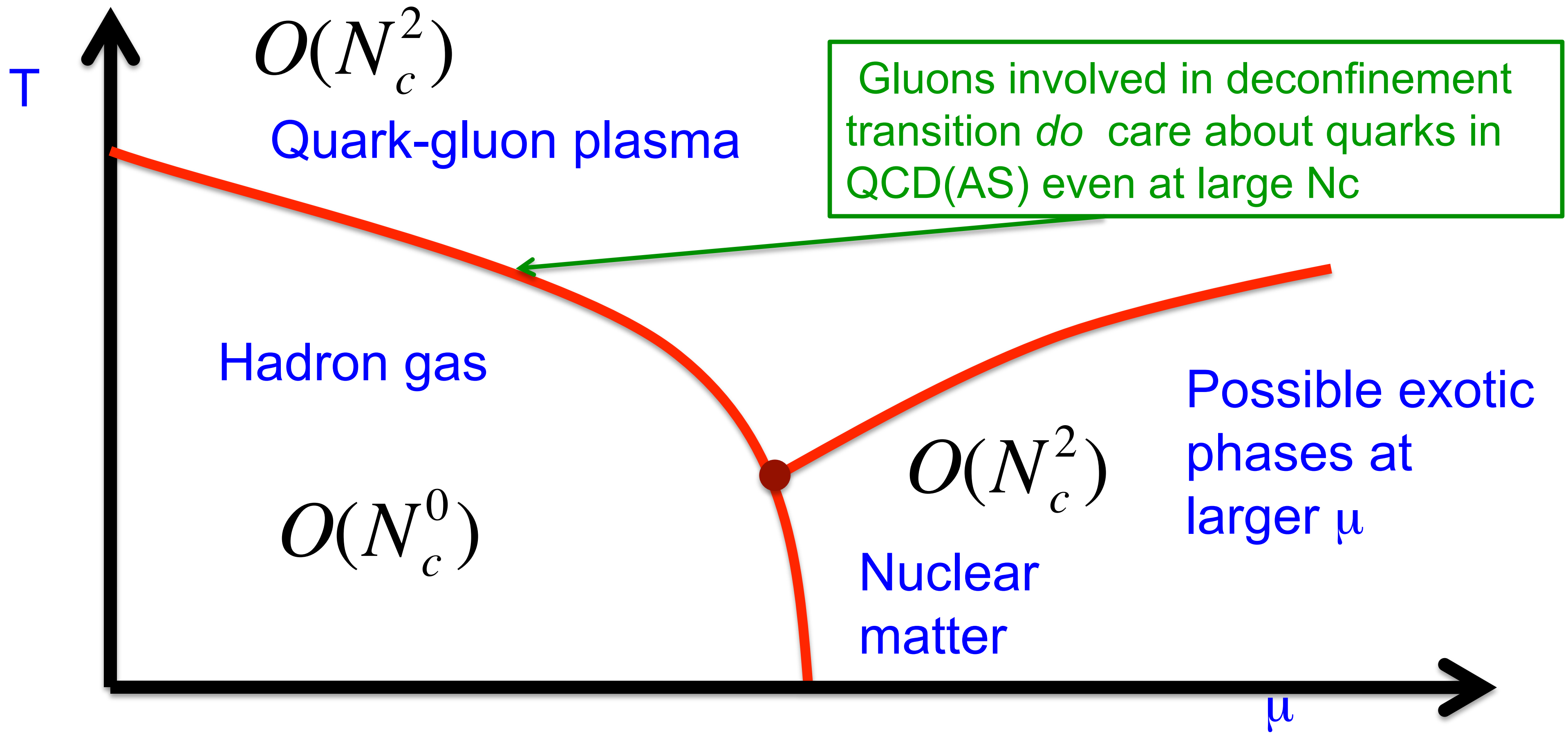


QCD (F) Phase Diagram at Large Nc : A Cartoon



Large Nc behavior for dense matter with $\mu \sim N_c^0$ looks completely different from Nc=3 and QCD(AS)!!!

QCD (AS) Phase Diagram at Large Nc : A Cartoon



Large Nc behavior for dense matter with $\mu \sim N_c^0$ in QCD(AS) looks qualitatively somewhat different from real world and very different from QCD(F).

What about asymptotically high densities?

- Characteristic momenta are small interactions via 1-gluon exchange; nonperturbative effects through infrared enhancement of effects with perturbative kernal.
- Nc=3 : As noted by Son (1999) there is strong evidence for color superconductivity; BCS instability in RG flow; BCS gap given parametrically by

$$\Delta_{\mathrm{BCS}} \sim \mu g^5 \exp\left(\frac{-\sqrt{6}\pi^2}{g}\right)$$

Note *1/g* not *1/g*^{2} in exponential

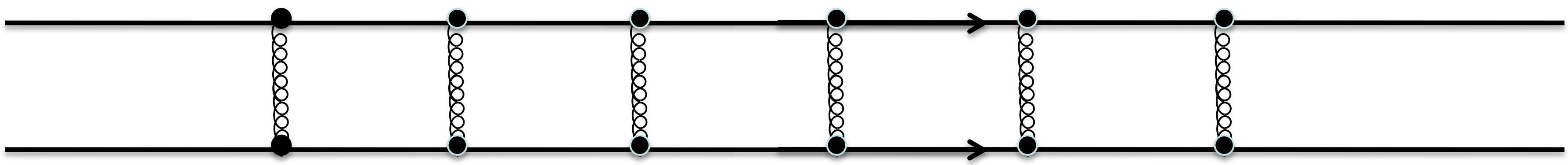
- Nc → ∞: $g = \sqrt{\frac{\lambda}{N_c}}$ where λ, the `t Hooft coupling, is independent of Nc

$$\Delta_{\mathrm{BCS}} \sim \mu \left(\frac{\lambda}{N_c} \right)^{\frac{5}{2}} \exp\left(-\sqrt{6}\pi^2 \sqrt{\frac{N_c}{\lambda}} \right)$$

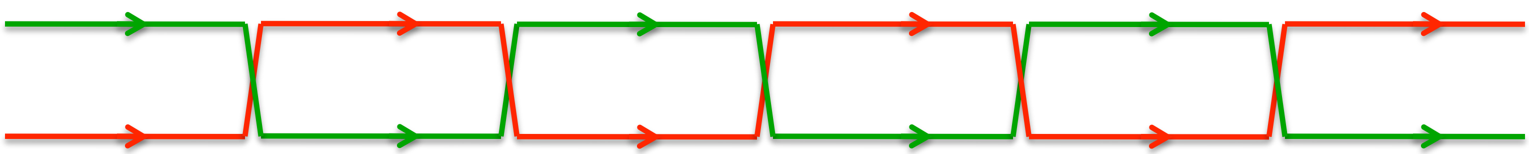
  - **The gap is exponentially suppressed at large Nc!!**
  - Note that color-nonsinglet condensates such as BCS depend on g^2 not $N_c\, g^2$. This is why the effect is exponentially small.

However, this exponential suppression in Nc is not present for color-singlet condensates. Thus ***IF*** an instability towards a color-singlet condensate exists at large *Nc* it will occur rather than the BCS phase.

Ladders are key ingredient

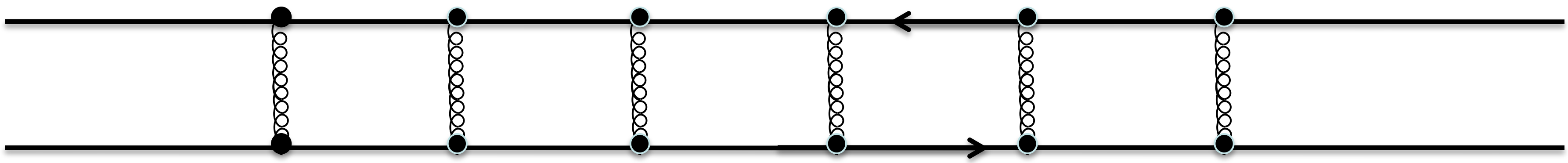


Look at color flow for BCS case ('t Hooft diagrams with gluons carrying color-anticolor)

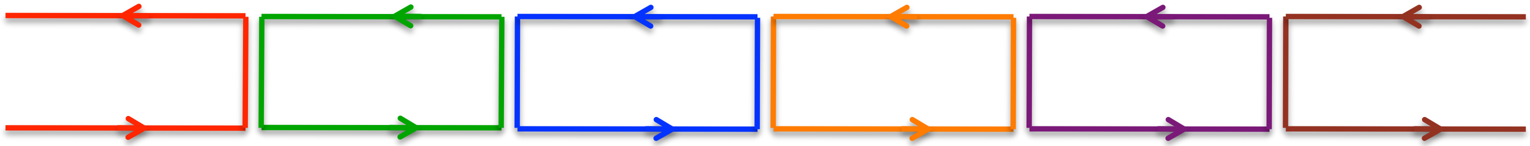


Note factors of couplings cost 1/Nc but no loop factors counteract it. The color just bounces back and forth.

The situation is quite different with instabilities towards condensates which are color singlets (although not necessarily gauge invariant), *eg*. some type of possibly nonlocal $\langle \bar{q} q \rangle$ condensate.



Look at color flow ('t Hooft diagrams with gluons carrying color-anticolor)



Note for singlets factors of couplings cost 1/Nc but are compensated by color loop factors. The relevant combination is $N_c\, g^2 = \lambda$. Thus, effects should not be exponentially down in N_c.

Thus ***IF*** an instability towards a color-singlet condensate exists at large *Nc* it will occur rather than the BCS phase.

Son and Shuster (1999) showed that that such a condensate exists in standard ‘t Hooft-Witten large Nc limit.

It is a spatially varying chiral condensate of the Deryagin, Grigoriev, and Rubakov (DGR) type:

$$\left\langle \overline{q}(x')q(x)\right\rangle = e^{i\vec{P}\cdot(\vec{x}'+\vec{x})}\int d^4q\, e^{-iq(\vec{x}-\vec{x}')} f(q) \qquad |\vec{P}|=\mu$$

The DGR instability can only be reliably computed for $\mu \gg \Lambda_{QCD}$ (perturbatively large) and only occurs for $\mu<\mu_{crit}$.

The reason that μ_{crit} exists is that at sufficiently high values of μ, the Debye mass cuts off the RG running before the instability sets in.

$$\mu_{crtit} \sim \Lambda_{QCD} \exp\left(\gamma \log^2(N_c)\right) \qquad \gamma \approx .02173$$

As $Nc \rightarrow \infty$, $\mu_{crit} \rightarrow \infty$ and the DGR instability exists for all pertubative values of μ. For Nc=3, however, DGR instabiliy does not occur & BCS wins.

Moreover as expected its scale is NOT exponentially down in *Nc*

$$\Delta_{\mathrm{DGR}} \sim \mu \exp\left(-\underbrace{\frac{4\pi^3}{g^2 N_c}}_{\lambda}\right)$$

Thus, the DGR instability is much stronger than the BCS instability. The system will form a DGR phase rather than a BCS phase when possible and at large *Nc* it is always possible since μ_{crit} goes to infinity.

The bottom line: At sufficiently large Nc the DGR phase exists. **The large Nc world for QCD(F) at high density is qualitatively different from Nc=3**

What happens in QCD(AS)?

Both the BCS and DGR instabilities using were studied by standard means Buchoff, Cherman, TDC (2010) :

An RG equation was set up for excitations near the Fermi surface. Now if the Fermi surface is unstable the coupling strength will diverge as one integrates out the contributions of everything except a small shell near the Fermi surface.

The gap is determined qualitatively from the position at which the divergence occurs.

For QCD(AS) we found that

$$\Delta_{\mathrm{BCS}}^{(AS)} \sim \mu \frac{\lambda^{5/2}}{N_c^3} \exp\left(-\pi^2 \sqrt{\frac{3N_c}{2\lambda}}\right)$$

As with QCD(F) the gap is exponentially down in Nc.

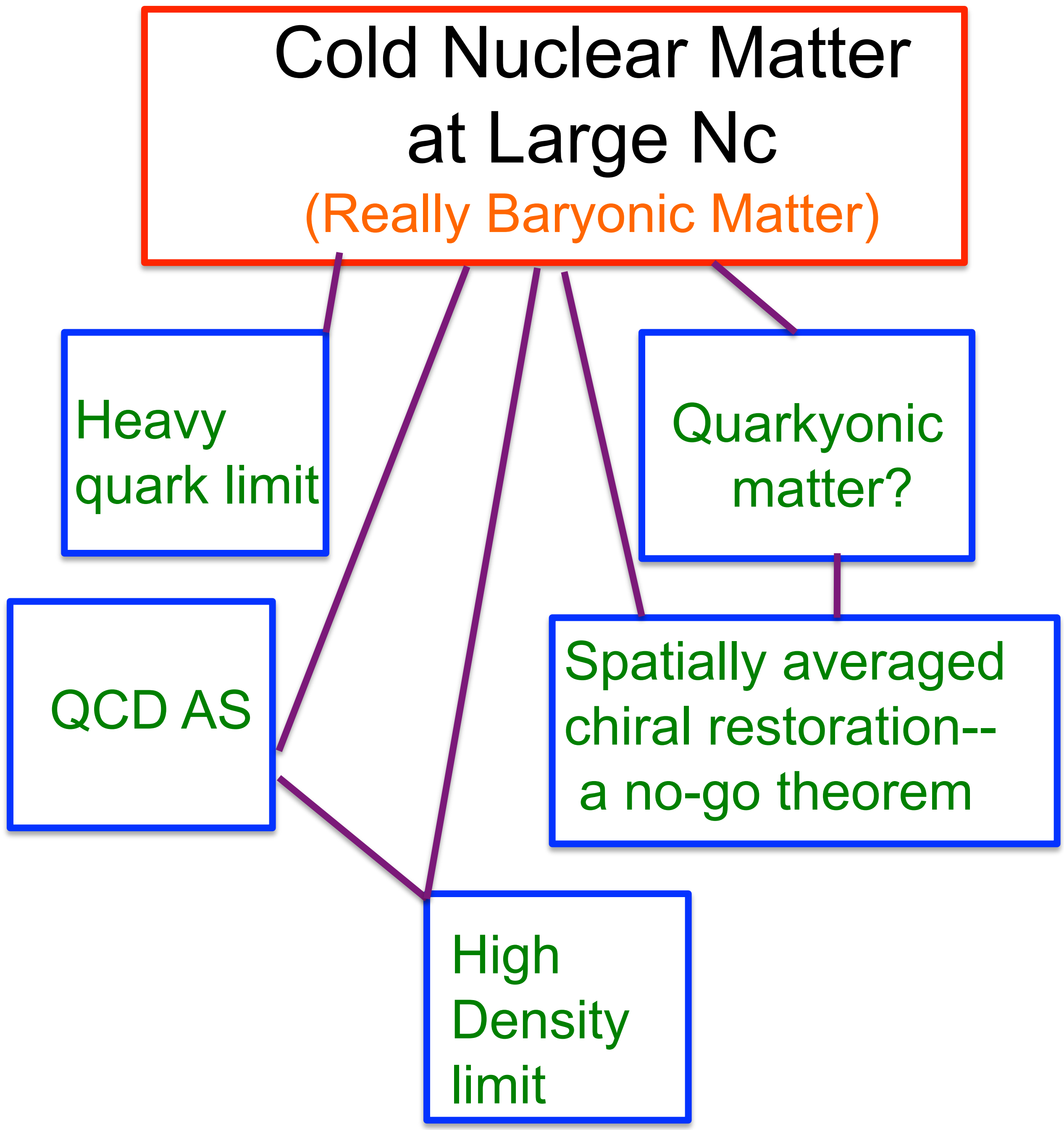
Thus we again expect that the DGR instability will win as it is a color singlet, **provided that it occurs.**

Does it?

NO!!

The RG analysis is done using the same effective 1-d theory near the Fermi surface as was done for QCD(F). However, in QCD(AS) the RG running is affected by quark loops. These serve to screen the gluons and cutoff the RG flow before the instability is reached.

Thus QCD(AS) at very high densities is qualitative different QCD(F) at large Nc. As for the case of Nc=3 it is likely to be in a BCS phase and is certainly not in a DGR



Cold Nuclear Matter
at Large Nc
(Really Baryonic Matter)
Heavy quark limit
Quarkyonic matter?
QCD AS
Spatially averaged chiral restoration-- a no-go theorem
High Density limit

From McLerran & Pisarski(2008)

Abstract:

In the limit of a large number of colors, Nc, we suggest that gauge theories can exhibit several distinct phases at nonzero temperature and quark density. Two are familiar: a cold, dilute phase of confined hadrons, where the pressure is ~ 1, and a hot phase of deconfined quarks and gluons, with pressure ~ Nc^2. When the quark chemical potential μ ~ 1, the deconfining transition temperature, Td, is independent of μ. For T < Td, as μ increases above the mass threshold, baryons quickly form a dense phase where the pressure is ~ Nc. *As illustrated by a Skyrme crystal, chiral symmetry can be both spontaneously broken, and then restored, in the dense phase. While the pressure is ~ Nc, like that of (non-ideal) quarks, the dense phase is still confined, with interactions near the Fermi surface those of baryons, and not of quarks. Thus in the chirally symmetric region….*