Analyst Forecasts:
The Roles of Price Impact and Reputational Ranking

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Dedication

To my Baba and Ma who always loved me unconditionally. This is specially for you, Dad!

Abstract

In this dissertation, I develop analytical models to examine how price impact and reputational ranking incentives influence the forecasting behavior of sell-side analysts. First, I analyze the equilibrium behavior of an analyst when he is concerned with maximizing his short-term price impact and long-term reputational value in the market. Second, I discuss how the equilibrium forecasting behavior of an analyst changes in the presence of a second analyst, when each analyst is concerned with maximizing not only his own reputation, but also his relative reputation (*i.e.*, reputational ranking) compared to the other analyst. Two key results emerge : (i) *Positive role of short-term price-impact.* Short-term price-impact motives, together with reputational concerns, provide better incentives for honest forecasting than reputational concerns or price-impact profits alone; (ii) *Convexity of reputational ranking payoff function decreases the information content of analyst forecasts.* The greater the ratio of reputational reward for being ranked higher to reputational penalty for being ranked lower is, the lesser the information content of analyst forecasts.

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Chapter 1

Introduction

This dissertation consists of two models of analyst forecasts. In the first model, I discuss the equilibrium forecasting behavior of an analyst when he is concerned with maximizing his short-term price impact and long-term reputational value in the labor market. In the second model, I discuss how the equilibrium forecasting behavior of an analyst changes in the presence of a second analyst, when he is concerned with maximizing not only his own reputation but also his reputation relative to the other analyst. There is no short-term price impact motive for the analysts in the second model. This chapter presents the background of the research question, preview of the models, main results and contribution of the research.

1.1 Background

Financial analysts play a crucial intermediary role between investors and the companies traded in the capital market. Analysts collect information about a company from multiple sources, analyze the information and make forecasts about various financial indicators of the company. Investors use these forecasts when deciding whether to buy, sell or hold the company's stock. Sell-side analysts, in particular, are employed by investment advisory firms and provide forecasts to institutional and retail investors. In finance and accounting empirical studies, the earnings forecasts of sell-side analysts are typically used as proxies for investors' earnings expectations. An implicit assumption underlying such research is that an analyst seeks to minimize the mean squared forecast error, and thus, truthfully reveals his private information to the investors. Although mean squared error is a useful statistical concept and its minimization is probably the appropriate objective function in a Robinson Crusoe economy, it is unclear that a strategic analyst adopts such an objective function. Indeed, the forecasting strategy of a strategic analyst is an open question that begs investigation.

One specific question concerns identifying a plausible objective function of a strategic analyst. The popular press and empirical literature suggest that sell-side analysts typically have three main incentives. First, they want to maximize the trading volume of the stocks they cover (*e.g.*, Jackson 2005; Beyer and Guttman 2011), thereby generating higher trading commissions. Second, analysts are concerned with managing their relationships with company executives – to obtain either investment banking business or access to private information – by issuing favorable (*e.g.*, Lim 2001) or beatable (*e.g.*, Richardson *et al.* 2004) forecasts. Finally, analysts are concerned with enhancing their reputations for making accurate forecasts. A sell-side analyst's reputational value in the labor market depends on the relative ranking of his reputation as measured against those of his peer group. Reputational ranking is important because highly reputed analysts have a greater impact on future price movements (Stickel 1992; Park and Stice 2000). More highly reputed analysts also receive better compensation and have greater job mobility than their less highly regarded counterparts (Mikhail, Walther and Willis 1999; Leone and Wu 2002; Hong and Kubik 2003).

In this dissertation, I take the view that sell-side analysts are concerned with maximizing their impacts on the capital market. It is unlikely that a sell-side analyst would be employed by any brokerage firm, investment bank or even independent equity research group if the market ignored the analyst's earnings forecasts or stock recommendations. Correspondingly, I assume that a sell-side analyst issues an earnings forecast to maximize his lifetime price impact – the extent to which the stock price of a company moves after he makes an earnings forecast – on the capital market. Furthermore, I assume that a sell-side analyst's reputational value in the market is an abstraction of his benefits from future price impacts. Therefore, an analyst's lifetime benefits from his price impact can be expressed as the sum of his benefits from the current period and his reputational ranking in the market.

In the first model, I consider a strategic sell-side analyst who is making a forecast of a company's earnings with the objective of maximizing his current-period price impact and reputational value in the labor market. Casual empiricism suggests that a sell-side analyst trades off between these two incentives when deciding upon an earnings forecast: maximizing the short-term profits from the price impact of his forecast versus maximizing his long-term reputational value in the labor market. On one hand, an analyst has a strong incentive to move the price of the company's stock to the maximum extent possible, perhaps by misrepresenting his information, in order to maximize his short-term profits. On the other hand, an analyst's desire to generate price-impact profits today is disciplined by his incentive to build a reputation for the future. Consequently, an analyst faces the classic short- versus long-term tradeoff - providing forecasts that enhance his reputational value for future benefits versus potentially misleading investors with forecasts that generate high current-period profits from price impact.

In the second model, I consider two analysts, each making simultaneous forecasts about the earnings of a company, with an objective of having a higher reputational ranking (than the other analyst) in the market. In order to focus primarily on the strategic interaction between the two analysts, I consider only the reputational ranking motive in this model; there is no short-term price impact motive for the analysts. Each analyst faces a tradeoff while making an earnings forecast. On one hand, to maximize the likelihood of being ranked higher in the market, he wants to differentiate himself from the other analyst by issuing a forecast that is different from that of the other analyst. On the other hand, to minimize the odds of being ranked lower, the analyst will tend to move with the other analyst by issuing the same forecast as that of the other analyst. On balance, each analyst's optimal forecasting strategy depends on the relative payoffs of being ranked higher or lower.

1.2 Preview of the Models

In the *first model*, there are two players: one analyst and the capital market. At $t = 0$, the analyst receives a private signal about the earnings of a company that he is covering. The company's earnings are assumed to be either high or low. The analyst's signal and forecast are also represented using binary values. An analyst can have either good or bad talent. A good analyst receives a more precise signal than a bad one. Neither the analyst nor the market knows the talent of the analyst; they only know the prior distribution of an analyst's talent.

At $t = 1$, the analyst makes a forecast about the earnings of the company, and the market prices the company's stock based on the analyst forecast. At $t = 2$, the company's earnings are publicly reported. The market now compares the forecast and the actual realization of earnings and updates the reputation of the analyst. An analyst's reputation is defined as the market's belief about his talent. The objective of the analyst is to maximize both his short-term price-impact (*i.e.*, the extent to which the price moves subsequent to his forecast), and his long-term reputational value in the labor market. The analyst's reputational value is simply his reputation in the market.

In the *second model*, I extend the first model to two analysts. However I concentrate only on the reputational concerns of the analysts – there is no short-term price impact component in this model. There are three players: two analysts and the capital market. At $t = 0$, each analyst receives a private signal about the earnings of a company that both are covering. Like the first model, the company's earnings are assumed to be either high or low. Analyst signals and forecasts are also represented using binary values. An analyst can have either good or bad talent. A good analyst receives a more precise signal than a bad one. Neither the analysts nor the market knows the talent of each analyst; they only know the prior distribution of an analyst's talent. The signals of good analysts are perfectly correlated, conditional on the state (earnings). If either or both of the analysts are bad, their signals are conditionally independent.

At $t = 1$, each analyst makes a forecast about the earnings of the company. At $t = 2$, the company's earnings are publicly reported. The market now compares the forecasts and the actual realization of earnings and updates the reputation of each analyst. The objective of each analyst is to maximize his long-term reputational value in the market. The reputational value – derived from each analyst's reputational ranking payoff – represents the idea that an analyst gains benefits not only from his own reputation but also from his relative reputational value compared to his peer group in the market.

1.3 Preview of the Results

I start with a baseline model, in which a single analyst is making an earnings forecast of a company with the objective of maximizing his expected reputational value in the market. The analysis of this baseline model will help highlight the roles of short-term price impact, and reputational ranking motives in an analyst's forecasting behavior.

In equilibrium in the baseline model, as also discussed in Ottaviani and Sorensen (2001), an analyst can credibly communicate his private information only if the prior of earnings is in the intermediate range (neither very high nor very low). At extreme priors, an analyst with an unlikely signal (one that contradicts the prior) will infer that he is probably a bad type, and, therefore, expects that the communication of his private signal will lead to a downward revision of his reputation in the market. Recall that neither the analyst nor the market knows the analyst's talent. Thus, to appear to be a good type, an analyst will always make a forecast that matches the prior regardless of his signal. The reputational concerns of an analyst thus create a "conformist" bias in an analyst's forecasts, which leads to no information transmission at extreme priors.

When an analyst's sole objective is to maximize his price-impact, he can never fully reveal his private information to the market. His earnings forecast can only partially reveal his private signal. The intuition of this reporting behavior is that the analyst, with his only objective being to maximize his price-impact profits, will forecast in such a way that moves the price to the maximum. In order to move the price, an analyst will tend to forecast against the prior expectations of the market. I call this – the incentive to move against the prior of the market – an analyst's *"contrarian"* incentive, and the bias in an analyst's forecast, due to this incentive, the "contrarian" bias. Thus, when the analyst's signal opposes the prior, his forecast will be consistent with both his private signal and his contrarian incentive. However, when the analyst's signal matches the prior, following his contrarian incentive contradicts his private signal, leading to the loss of information in equilibrium.

The focus of this dissertation, then, is to explore, in the first model, how short-term price-impact profits and reputational incentives, together, influence the forecasting behavior of an analyst and, in the second model, how the analyst's equilibrium forecasting behavior changes in the presence of another analyst, when both are engaged in maximizing their expected reputational ranking payoffs in the market. Two key results emerge:

(i) *Positive role of short-term price-impact.* Price-impact motives, together with reputational concerns, provide better incentives for honest forecasting than reputational concerns or price-impact profits alone.

(ii) *Convexity of reputational ranking payoff function decreases the information content of analyst forecasts.* The greater the ratio of reputational reward for being ranked higher to reputational penalty for being ranked lower is, the lesser the information content of analyst forecasts.

The first key result of this dissertation concerns the potentially positive role played by the short-term price-impact profits in the sense that they, along with reputational concerns, provide better incentives to analysts to reveal their private information than reputational concerns or price-impact profits alone. To develop the intuition of this result, first consider a case that addresses only reputational concerns. Assume that an analyst receives a signal that contradicts the prior of earnings, when the prior is sufficiently precise. Similar to the baseline case, the analyst will tend to follow the prior to appear to be a good type, leading to no information transmission in equilibrium. Now, imagine that the analyst also cares about his short-term benefits from the price impact of his forecasts. The short-term profits incentive will influence the analyst to forecast against the prior to increase price movement. Thus, with the additional incentive of price-impact profits, the analyst will be able to credibly communicate his signal, which was not possible with the reputational incentive alone. This combination of incentives leads to more information revelation in equilibrium.

The intuition of the second result is that if the objective of each analyst is to maximize his expected reputational ranking payoffs in the market, each will tend to maximize the likelihood of being ranked higher and minimize the likelihood of being ranked lower than the other analyst. In order to increase the odds of being ranked higher, each analyst will want to differentiate himself from the other. However, note that the analyst knows neither the private signal of the other analyst nor both his own and the other analyst's talents. The only information each analyst has is his own signal and the possibility that his signal and the other analyst's are conditionally correlated. Given this information, and assuming that the other analyst will forecast his own signal (in a symmetric Nash equilibrium), the best response for each analyst will be to forecast against his own signal. In contrast, to reduce the odds of being ranked lower, each analyst will want to increase the likelihood of moving in conjunction with – rather than differentiating himself from – the other analyst. Again assuming that the other analyst will forecast his own signal, the best response for each analyst will be to forecast his own signal as well.

On balance, if the reputational reward for being ranked higher is more than the reputational penalty for being ranked lower, then each analyst will have a greater incentive to differentiate himself from the other analyst by reporting against his own signal, which leads to less information transmission in equilibrium. On the other hand, if the reward for being ranked higher is less than the penalty for being ranked lower, then each analyst will have a greater incentive to move with the other analyst by reporting his own signal, which leads to a fully informative equilibrium. For example, the popular press suggests that on Wall Street, " All Star" analysts are paid substantially higher than their average counterparts, yet analysts are not penalized comparably if they rank lower (conditional on the analyst not being fired). The second result suggests that such asymmetry in a reputational payoff structure can influence analysts to reveal less information in equilibrium.

1.4 Contribution

My dissertation contributes to the literature regarding the relationship between analyst forecasts and expert reputation in primarily two ways. First, to the best of my knowledge, it is the first study that develops a model of analysts' forecasting behavior using a simple tradeoff: maximizing current-period price-impact profits versus generating future reputational payoffs. Second, the consideration of multiple analysts in the reputational ranking model introduces an element of strategic interaction. On one hand, to increase the odds of being ranked higher in the market, each analyst wants to differentiate himself from the other analyst by issuing a different forecast. On the other hand, to decrease the odds of being ranked lower, the analyst will ensure that his forecast matches with the reported earnings, by issuing the same forecast as that of the other analyst. On balance, each analyst's optimal forecasting strategy depends on the relative payoffs of being ranked higher and lower than the other analyst.

To elaborate further on the first contribution, as mentioned above, there have been several papers, such as Ottaviani and Sorensen (2001, 2006a, 2006b) and Trueman (1994), that develop models in which an expert is maximizing his reputational value; however, none of them address price-impact profits or any other profit motives. The inclusion of a current-period price-impact motive changes some features of the equilibrium, including the increase in the informativeness of equilibrium.

There are also a few papers that model an expert's short- and long-term tradeoffs. In contrast to Prendergast and Stole's (1996) paper, which relies on the difference between actual and expected investments to make inferences about a manager's ability, in my model, the market makes inferences about an expert's talent by updating its belief based on the expert's forecast and the actual realization of the state variable for which the forecast has been made. Dasgupta and Prat (2008) focus on how career concerns reduce information revelation, while in my paper, the focus is on how the addition of the price-impact profits motive improves information revelation.

Jackson's (2005) paper, although somewhat close in setup, differs from mine in two crucial aspects. First, the equilibrium in my model is a function of the prior of the state variable (company's earnings), which is assumed to be half in Jackson's model. It can be easily shown in my model that if the prior is half, there is always a fully informative equilibrium, as in Jackson's model. However, in my model, the equilibrium forecasting behavior of the analysts is interesting when the prior is not half. Second, one major focus of Jackson's paper is to show that on average an analyst's forecast is optimistic in equilibrium, a result that depends primarily on the author's assumption that investors face short-sales constraints. In contrast, the focus of my paper is to show how adding a price-impact profits incentive alleviates – but does not fully mitigate – the conformist bias due to an analyst's reputational concerns.

Finally, to elaborate on the second contribution, by considering the strategic interaction between analysts, my paper contributes to the expert's reputation literature by integrating relative reputational payoff considerations into a model in which experts take simultaneous actions. Two other papers – one by Ottaviani and Sorensen (2006a) and another by Effinger and Polborn (2001) – have considered models on relative reputation. In the first paper, while the authors show that there is no effect of relative reputation when expert signals are conditionally independent, they do not address what happens when there is indeed some correlation between the signals.

The second paper considers a case in which the private signals of two experts are conditionally correlated, and the relative reputation does have an impact. However, the main difference between my (second) model and Effinger and Polborn's is that in my model, the analysts move *simultaneously*, unaware of each other's actions. Moreover, some of the assumptions made by Effinger and Polborn can be very restrictive in the context of analysts' earnings forecasts, the focus of my model. Effinger and Polborn (2001) imply that if the reputational reward for being the only smart agent is sufficiently large, then there is "anti-herding" : the second mover always forecasts against the first mover's action regardless of his own signal. In contrast, if the reward is not that high, the second mover may "herd" with the first mover by reporting in the direction of the first mover's action, ignoring his own signal. In sequential moves, the consideration of relative reputation typically leads to information loss (anti-herding or herding). In my model, while a payoff structure in which the reputational reward for being ranked higher is greater than the reputational penalty for being ranked lower leads to information loss, a payoff structure with a sufficiently high reputational penalty (in comparison to reputational reward) improves information revelation in equilibrium.

1.5 General Outline

Chapters 2 and 3 present the two models of analyst forecasts as discussed in Section 1.2.

Chapter 2 presents a model in which I discuss the equilibrium behavior of an analyst who is concerned with maximizing his short-term price impact and long-term reputational value in the market. Specifically, I characterize the equilibrium and derive comparative statics of the model.

Chapter 3 presents a model in which I discuss the equilibrium forecasting behavior of two analysts when each analyst is simultaneously making an earnings forecast in order to be ranked higher, in terms of his market reputation, than the other analyst. In particular, I derive the equilibrium features and comparative statics of the model.

Chapter 4 presents the overall summary of the dissertation, and a discussion of future research directions.

Chapter 2

The Role of Price Impact

2.1 Introduction

In this chapter, I develop a model in which a sell-side analyst is concerned with maximizing his short-term price impact profits *and* his reputational value in the market. To start with, I assume that a sell-side analyst – while making an earnings forecast – is primarily concerned with maximizing his lifetime impact on the capital market. It is unlikely that a sell-side analyst would be employed by a brokerage firm, an investment bank or an independent equity research company if his earnings reports or stock recommendations have very little impact on the capital market. Also, an analyst's price impact on the market – the extent to which his earnings forecast moves the price of a company's stock – over his lifetime is the sum of his impact from the current and all future periods. An analyst's price impact benefits from future periods is assumed to be his reputational value in the market. Empirical evidence suggests that more highly reputed analysts have a greater impact on future price movements (*e.g.*, Stickel 1992; Park and Stice 2000). As such, a sell-side analyst makes an earnings forecast of a company in order to maximize his current period price impact benefits and his future reputational value in the market.

The main tradeoff of this model is that on one hand, an analyst wants to maximize his current-period price impact benefits by issuing a forecast that moves the company's price as much as possible, perhaps by misrepresenting his private information about the company. On the other hand, his desire to generate price impact profits is somewhat disciplined by his incentive to build his reputation in the market. As it turns out, when an analyst is sufficiently concerned about his reputational concerns, an analyst's short-term price impact motive improves the information content of his forecast.

2.2 Related Literature

My model brings together two important strands of literature: sell-side analyst's forecasting behavior and expert reputation. The first strand focuses on the forecasting strategies of a sell-side analyst under different incentives. For example, Beyer and Guttman (2008) consider a case in which a sell-side analyst's payoff depends on his commission from trade generation as well as his loss from forecast errors. They find a fully separating equilibrium in which the analyst biases his forecast upward (downward) if his private signal reveals relatively good (bad) news. Morgan and Stocken (2003) consider a financial market setting in which the investors are uncertain about the incentives of the security analyst, who makes stock recommendations that the investors use for their investing decisions. The analyst is not concerned about generating trade for his brokerage firm. The authors show that the investors' uncertainty about the analyst's incentives makes full information revelation impossible. There are two classes of equilibria: "partition equilibria", *a la* Crawford and Sobel (1981), and "semiresponsive" equilibria, in which analysts with aligned incentives can effectively communicate only unfavorable information about a company.

The second strand focuses on how reputational concerns influence an expert's professional advice to a decision maker. The expert has an informative signal about the state of the nature. He takes an action, possibly by providing advice or making a forecast about the state, which will be used by an uninformed decision maker. The expert is only concerned with having a reputation for being well-informed. Ottaviani and Sorensen (2001) show that when an expert does not know his talent and is maximizing his expected reputational payoff in the market, then he can credibly communicate his private information to the decision maker only if the prior of the state is in the intermediate range. At an extreme prior, no information is communicated in equilibrium. Trueman (1994) shows that when an analyst knows his talent, a good type always reveals his private signal to the market, while the bad type can do so only in the intermediate range of prior. At an extreme prior, the bad type can credibly communicate only part of his information. None of the studies consider price-impact profits or any other profit motives of an expert in addition to reputational concerns.

There are at least three papers that do consider profit motives. Prendergast and Stole (1996), Dasgupta and Prat (2008) and Jackson (2005) model an expert's short- and long-term tradeoff: current-period profit motives versus future reputation benefits. In Prendergast and Stole (1996), a manager's objective is to maximize both current profits from his investment decisions and end-of-period market perception of his ability. However, in their model, the market never sees the "realized" effects of his decisions. Inferences about the manager's ability - his reputation - are drawn from the difference between actual and expected investment. Dasgupta and Prat (2008) consider a multi-period setting in which traders care about their trading profits as well as their future reputation. However, in Dasgupta and Prat's model, they focus on showing that the career concerns of traders reduce information revelation in equilibrium. Furthermore, when the traders are maximizing only trading profits, there is always a fully informative equilibrium.

Jackson (2005) considers a single-period model in which a sell-side analyst is maximizing a linear combination of price-impact profits and his reputation in the market while making an earnings forecast. He shows, both theoretically and empirically, that on average, an analyst's forecast is optimistic in equilibrium. Optimistic analysts generate more trade, and highly reputed analysts generate higher future trading volume. That the forecast is optimistic in equilibrium results from the combination of two scenarios: if the analyst's reputational concerns are sufficiently high (a "good" analyst), then he can always credibly communicate his private signal to the market; however, if his concerns for future reputation are not that high ("evil" analyst), then he always issues a high forecast in equilibrium, regardless of his signal. It is worth noting that the full-revelation result in the first case crucially depends on the author's assumption that the prior of the state variable (earnings) is half. Furthermore, the result that the analyst's optimism occurs in equilibrium depends primarily on Jackson's assumption that investors face short-sales constraints.

2.3 The Model

There are two players: one analyst, and the market. There are three dates. At $t = 0$, the analyst receives a private signal $s \in S \equiv \{s^h, s^l\}$ of the earnings $x \in X \equiv \{x^H, x^L\}$ of a company. Superscripts $H(h)$ and $L(l)$ can be interpreted as "high" and "low" respectively. Upon observing his signal, at $t = 1$, the analyst makes an earnings forecast, $\hat{s} \in \hat{S} \equiv \{\hat{s}^h, \hat{s}^l\}$. The market observes the forecast $(\hat{s})$, and prices the company's stock as $P(\hat{s}) \equiv E[x|\hat{s}]$. An analyst's talent is $\theta \in \Theta \equiv \{g, b\}$, good or bad. An analyst's talent can be thought of as his type. A good analyst receives a more precise private signal about the earnings of the company than a bad analyst. Neither the market nor the analyst knows θ (this captures the idea that the analyst does not know his talents sufficiently more than the market); the market and the analyst know only the prior distribution, $\Pr(g) \equiv \lambda \in (0, 1)$. Also, everyone knows the prior distribution of earnings, $\Pr(x^H) \equiv q \in (0, 1)$. Finally, at $t = 2$, when earnings x is reported by the company, the market compares the realized x with the analyst forecast $(\hat{s})$, and updates its belief about the analyst's talent, which I define as the analyst's reputation, $r(\hat{s}, x) \equiv \Pr(g|\hat{s}, x)$ The time line of the game is shown in Figure 1, Panel A. All figures are in Appendix C (page 89-93).

Information structure. The precision of the analyst's private signal is

$$\gamma_\theta = \Pr(s = x|x, \theta). \tag{2.1}$$

I assume $1 > \gamma_g > \gamma_b = \frac{1}{2}$. The probability that a good analyst receives a matched signal (*i.e.*, $s = x$), conditional on earnings and his talent, is γ_g, which is higher than that (*i.e.*, γ_b) of a bad analyst. The assumption of $\gamma_b = \frac{1}{2}$ implies that the bad analyst receives a completely noisy signal of the earnings. Since the analyst does not know his talent, he only knows the unconditional probability, which is defined as $\gamma \equiv \Pr(s = x|x) = \lambda\gamma_g + (1 - \lambda)\frac{1}{2} > \frac{1}{2}$. It can also be interpreted as the average signal quality (or precision) of an analyst.

Reputational payoff. I assume that the analyst's reputational payoff is his reputation in the market, which is $\Pr(g|\hat{s}, x)$.

Objective function. The analyst maximizes a linear combination of his current period profits from the price impact of his earnings forecast, and his expected reputational value in the market. The analyst's reputation value in the market captures the analyst's profits from the price impact in future periods. An analyst with a higher reputational value generates more price impact in the future, given that the market uses the analyst's reputation as a prior for his talent in the subsequent period and that a highly reputed analyst generates greater price movements. Accordingly, the analyst's objective function is

$$V(\hat{s}|s) \equiv \alpha\pi(\hat{s}|s) + (1-\alpha)R(\hat{s}|s),\ \alpha \in (0,1) \tag{2.2}$$

where $\pi(\hat{s}|s)$ and $R(\hat{s}|s)$ are the price impact profits and the expected reputational value respectively. The parameter α denotes the relative weight of the price impact profits versus the analyst's reputational value. Price impact profits are defined as

$$\pi(\hat{s}|s) \equiv |P(\hat{s}) - P_0| \tag{2.3}$$

where $P_0 = E[x]$, the price at $t = 0$. The more the price moves subsequent to the analyst forecast, regardless of the direction of the movement, the greater the price impact profits for the analyst. The expected reputational value of the analyst is defined as

$$R(\hat{s}|s) \equiv \mathbb{E}_x[\Pr(g|\hat{s}, x)|s]. \tag{2.4}$$

2.4 Equilibrium Definitions

Let $\sigma : S \to \triangle(\hat{S})$ be the analyst's forecasting strategy, such that σ_j is the probability by which the analyst forecasts $\hat{s}^h$ after receiving a signal s^j where $j \in \{h, l\}$. Specifically

$$\sigma_h = \Pr(\hat{s}^h|s^h) \in [0,1] \text{ and } \sigma_l = \Pr(\hat{s}^h|s^l) \in [0,1]. \tag{2.5}$$

Definition 1 *A* perfect Bayesian equilibrium *consists of analyst's strategy pair* $\{\sigma_h, \sigma_l\}$ *and the market's pricing rule* $P(\hat{s})$ *such that*
(i) for each $s \in \{s^h, s^l\}$*, the analyst maximizes his objective function,*
(ii) for each $\hat{s} \in \{\hat{s}^h, \hat{s}^l\}$*, the market follows the pricing rule* $P(\hat{s}) = E[x|\hat{s}]$
(iii) given $\hat{s}$ *and the realization of* x*, the market's belief about the analyst type,* $Pr(g|\hat{s}, x)$*, is consistent with Bayes' rule.*

Condition (i) states that the analyst maximizes his objective function for each of his signal types, taking the market's pricing rule as given. Condition (ii) says that the market prices the company's stock as an expected value of x conditional on the analyst's forecast. Condition (iii) states that the market's belief about the analyst's type is consistent with Bayes' rule.

Definition 2 *An equilibrium is* natural *if the strategy pair* (σ_h, σ_l) *always satisfies the condition* $\sigma_h \geqslant \sigma_l$. *An equilibrium in which the opposite, i.e.,* $\sigma_h < \sigma_l$, *is true is called a* perverse *equilibrium.*

For every equilibrium with $\sigma_h > \sigma_l$, there is always a "mirror" equilibrium in which $\sigma_h < \sigma_l$. However, the interpretation of an equilibrium with $\sigma_h > \sigma_l$ is more intuitively appealing than the one with $\sigma_h < \sigma_l$. For example, for every equilibrium in which an analyst with a high signal forecasts high and with a low signal forecasts low, there is another equilibrium in which the analyst with a high signal will forecast low and with a low signal will forecast high. In both cases, the analyst fully reveals his private signals. However, in the first equilibrium $\sigma_h = 1 > 0 = \sigma_l$, and in the second equilibrium, $\sigma_h = 0 < 1 = \sigma_l$. Admittedly, the first equilibrium is more intuitively appealing than the second equilibrium. Therefore, I call the first type of equilibrium "natural" and the second "perverse". In this dissertation, for ease of illustration, I focus only on natural equilibria.

Definition 3 *An equilibrium is* noninformative *if the market's posterior of earnings after the analyst's forecast is the same as the prior, before the analyst makes a forecast. This definition is equivalent to* $\sigma_h = \sigma_l$.

Definition 4 *An equilibrium is* informative *if* $\sigma_h \neq \sigma_l$.

Definition 5 *An equilibrium is* fully informative *if the market's posterior of earnings after the analyst's forecast is equal to the posterior under the assumption that the market can directly observe the analyst's private signals. This definition is equivalent to* $\sigma_h = 1$ *and* $\sigma_l = 0$.

Definition 6 *An equilibrium is* partially informative *if it is neither fully informative nor noninformative. This definition is equivalent to any of the following : (i)* $\sigma_h = 1$ *and* $\sigma_l \in (0, 1)$, *(ii)* $\sigma_h \in (0, 1)$ *and* $\sigma_l = 0$,*(iii)* $\sigma_h \in (0, 1)$ *and* $\sigma_l \in (0, 1)$.

2.5 Single Analyst Baseline Reputation Model

I start with the case in which an analyst is concerned with maximizing only his expected reputational value in the market. This analysis will help underscore the differences in equilibrium behavior when (i) there is also a short-term price impact motive, and (ii) there is a second analyst, which introduces an element of strategic interaction (with the first analyst) into the model.

When an analyst's only concern is to maximize his reputation in the labor market, he solves the following problem

$$\max_{\hat{s}} \mathbb{E}_x[Pr(g|\hat{s}, x) \mid s]. \tag{2.6}$$

After the analyst issues an earnings forecast, and the earnings have been reported, the market compares the analyst's forecast with the earnings and updates the analyst's reputation, $Pr(g|\hat{s}, x)$, or the market's belief that the analyst is of good type. Since the market does not have access to the analyst's private signal, the best way to assess the analyst's type is to check whether his forecast and the reported earnings match. If the forecast and the earnings match, the analyst's reputation is favorably updated; if they do not, then his reputation is downgraded. The following property summarizes this intuition.

Property 1

$$\begin{aligned} (i)\, \Pr(g|\hat{s}^h, x^H) &\geqslant \Pr(g|\hat{s}^l, x^H) \\ (ii)\, \Pr(g|\hat{s}^l, x^L) &\geqslant \Pr(g|\hat{s}^h, x^L) \end{aligned} \tag{2.7}$$

where

$$\begin{aligned} \Pr(g|\hat{s}^h, x^H) &= \lambda \left[\frac{\gamma_g\sigma_h + (1-\gamma_g)\sigma_l}{\gamma\sigma_h + (1-\gamma)\sigma_l}\right] \\ \Pr(g|\hat{s}^h, x^L) &= \lambda \left[\frac{(1-\gamma_g)\sigma_h + \gamma_g\sigma_l}{(1-\gamma)\sigma_h + \gamma\sigma_l}\right] \\ \Pr(g|\hat{s}^l, x^H) &= \lambda \left[\frac{\gamma_g(1-\sigma_h) + (1-\gamma_g)(1-\sigma_l)}{\gamma(1-\sigma_h) + (1-\gamma)(1-\sigma_l)}\right] \\ \Pr(g|\hat{s}^l, x^L) &= \lambda \left[\frac{(1-\gamma_g)(1-\sigma_h) + \gamma_g(1-\sigma_l)}{(1-\gamma)(1-\sigma_h) + \gamma(1-\sigma_l)}\right]. \end{aligned} \tag{2.8}$$

Proof. Proofs of all properties are in Appendix A. ■

The intuition is that a forecast consistent with the reported earnings implies that the analyst must have received a very precise private signal, the hallmark of a "good" analyst. Thus, the market updates its belief favorably that the analyst is good. On the other hand, if the analyst's forecast does not match the reported earnings, the market downgrades the analyst's reputation, thinking that his private signal was not precise.

Characterizing the analyst's equilibrium forecasting behavior, the next lemma shows that an analyst can credibly communicate his private information only if the prior (q) is in the intermediate range, *i.e.*, $q \in [1-\gamma, \gamma]$. He cannot reveal any information credibly if the prior is extreme (Ottaviani and Sorensen 2001). The equilibrium regions are shown in Figure 2, Panel A. The intuition is that while maximizing his expected reputation in the market, the analyst's best strategy is to forecast $\hat{s}^h$ if $\Pr(x^H|s) \geq \Pr(x^L|s)$ and $\hat{s}^l$ if $\Pr(x^L|s) \geq \Pr(x^H|s)$. This strategy implies $\hat{s}^h$ if $q \geq 1-\gamma$, and $\hat{s}^l$ if $q \leq \gamma$. Thus, a fully informative equilibrium occurs in the intermediate range of prior, $q \in [1-\gamma, \gamma]$. However, if the prior is extreme, either $q < 1-\gamma$ or $q > \gamma$, then the analyst will not be able to credibly communicate any information to the market, leading to a noninformative equilibrium. The following lemma summarizes this result and has been proved in Ottaviani and Sorensen (2001). I do not prove the lemma here.

Lemma 1 *(Characterization of Equilibrium)*
When an analyst is concerned with maximizing his reputational value in the market, there exists an equilibrium, which can be expressed as follows:
(i) if $q \in [1-\gamma, \gamma]$*, then there is a fully informative equilibrium,*
(ii)if $q \notin [1-\gamma, \gamma]$*, then no information is communicated.*

The intuition for the noninformative region, *i.e.*, $q \notin [1-\gamma, \gamma]$, is as follows. Consider $q > \gamma$, *i.e.*, $\Pr(x^H|s) > \Pr(x^L|s)$ or x^H is more likely than x^L. Now, if the analyst receives a low signal, he infers that he has a higher likelihood of being a bad type since at $q > \gamma$, $\Pr(g|s^l) < \lambda = \Pr(g)$. Thus, to appear to be a good type, and to secure a favorable reputation, the analyst will tend to follow the prior by reporting $\hat{s}^h$ regardless of his private signal. Knowing this, the market will completely ignore whatever the analyst forecasts if $q > \gamma$, and thus no information is transmitted in equilibrium. Similarly, if

$q < 1 - \gamma$, in order to show that he has received a consistent signal, which is s^l, and to appear to be a good type, the analyst will again follow the prior by reporting $\hat{s}^l$ regardless of his private signal and the market will ignore the forecast. The analyst, thus, has a "conformist" bias – conforming to the prior by ignoring his own signal – at either very high or very low priors.

The result that no information can be communicated at extreme priors changes drastically when the talent of an analyst is known to the analyst but not to the market (see Figure 2, Panel B). When the analyst knows his talent, the good type always reveals his private signal, even at extreme priors. The bad type, on the other hand, reveals his private signal only if $q \in [1-\gamma_b, \gamma_b]$; however, if $q < 1-\gamma_b$, he forecasts low if he receives a low signal but strictly randomizes between high and low forecasts if he receives a high signal. Note that in my model, $\gamma_b = \frac{1}{2}$. Also, if $q > \gamma_b$, then the bad type forecasts high if he receives a high signal but strictly randomizes between high and low forecasts if he receives a low signal (Trueman 1994).

The intuition is that when an analyst receives an inconsistent signal at extreme priors, say, a low signal at high prior, then his reporting strategy can depend on another piece of information, his talent, which was missing when he didn't know his type. A good analyst, although having a low signal at extremely high prior, will risk reporting low – his own signal – expecting a huge gain in his reputation if the realization of state is actually low. A bad analyst, on the other hand, cannot risk as much as a good type since the precision of his signal is lower than that of the good analyst. In fact, it can be shown that the expected reputational gain for revealing his own signal is higher for a good type than a bad type. Thus, at extreme priors, whereas a good analyst can credibly communicate his private signals, a bad analyst can only partially do so.

2.6 Price-Impact Motive

When an analyst's objective is to maximize only his price-impact profits, he solves

$$\max_{\hat{s}} |P(\hat{s}) - P_0|. \tag{2.9}$$

His best strategy will then be to move the price $P(\hat{s})$ from the price at $t = 0$ as much as possible to generate maximum profits from price impact. The prices subsequent to high and low forecasts can be expressed as

$$P(\hat{s}^h) = \frac{x^H[\sigma_h\gamma + \sigma_l(1-\gamma)]q + x^L[\sigma_h(1-\gamma) + \sigma_l\gamma](1-q)}{\sigma_h[\gamma q + (1-\gamma)(1-q)] + \sigma_l[(1-\gamma)q + \gamma(1-q)]} \tag{2.10}$$
$$P(\hat{s}^l) = \frac{x^H[(1-\sigma_h)\gamma + (1-\sigma_l)(1-\gamma)]q + x^L[(1-\sigma_h)(1-\gamma) + (1-\sigma_l)\gamma](1-q)}{(1-\sigma_h)[\gamma q + (1-\gamma)(1-q)] + (1-\sigma_l)[(1-\gamma)q + \gamma(1-q)]}$$

The details of these calculations are shown in Appendix A.1. Also, $P_0 = E[x] = x^H q + x^L(1-q)$. Furthermore, the price subsequent to the high forecast is higher than that subsequent to the low forecast

$$P(\hat{s}^h) \geqslant P_0 \geqslant P(\hat{s}^l) \tag{2.11}$$

The analyst's price-impact profits from high and low forecasts are, respectively

$$\pi(\hat{s}^h) = |P(\hat{s}^h) - P_0| = |\frac{(x^H - x^L)q(1-q)(2\gamma-1)(\sigma_h - \sigma_l)}{\sigma_h[\gamma q + (1-\gamma)(1-q)] + \sigma_l[(1-\gamma)q + \gamma(1-q)]}| \tag{2.12}$$
$$\pi(\hat{s}^l) = |P(\hat{s}^l) - P_0| = |\frac{(x^H - x^L)q(1-q)(2\gamma-1)(\sigma_l - \sigma_h)}{(1-\sigma_h)[\gamma q + (1-\gamma)(1-q)] + (1-\sigma_l)[(1-\gamma)q + \gamma(1-q)]}|$$

Equations (2.12) indicate that an analyst's price-impact profits primarily depend on his forecasting strategies (σ), the prior probability of earnings (q), and his average signal precision (γ), which is a function of his prior reputation and the signal precision of the good analyst. The following lemma shows that an analyst's price impact increase with his prior reputation and signal quality.

Lemma 2 *An analyst's price-impact profits ($\pi(\hat{s})$) increase with his prior reputation (λ) and his signal quality (γ).*

Proof. Proofs of all lemmas, propositions and corollaries are in Appendix B. ■

Note also in (2.12) that the price-impact profits do not depend on the analyst's private signal. In fact, if the market (naïvely) believes that the analyst is truthfully revealing his private signal (*i.e.*, $\hat{s} = s$), then the analyst has an incentive to deviate by simply reporting against the prior of the earnings. More specifically, suppose the prior is optimistic, *i.e.*, $q > \frac{1}{2}$. Then given the market's naïve conjecture, the analyst will always issue a low earnings forecast, regardless of his private signal, since a low forecast will generate the maximum price-impact profits given the optimistic prior. Similarly, using the same conjecture, an analyst will always forecast a high forecast when the prior is pessimistic. The following lemma formalizes this intuition.

Lemma 3 *(Impossibility of full revelation) When an analyst is concerned with maximizing* only *price-impact profits, there is no fully informative equilibrium in any interval of* $q \in (0, 1)$. *Specifically, if the market conjectures that the analyst is honestly reporting his private signal, then an analyst will always forecast high if the market's prior is pessimistic, and forecast low if the market's prior is optimistic, regardless of his private signal.*

So, what is an equilibrium when an analyst's only objective is to maximize his price-impact profits? In the following proposition, I show that in equilibrium, an analyst can only partially reveal his private signal. The equilibrium regions are illustrated in Figure 3, Panel B.

Proposition 1 *(Characterization and Informativeness of Equilibrium)*
If an analyst is concerned with maximizing only *price-impact profits, then there exists an equilibrium, which can be expressed as follows:*
(i) if $0 < q < \frac{1}{2}$, *then an analyst with high signal will forecast high; however, an analyst with low signal will strictly randomize between high and low forecasts; the farther* q *is from* $\frac{1}{2}$, *the less information is revealed*
(ii) if $\frac{1}{2} < q < 1$, *then an analyst with low signal will forecast low; however, an analyst with high signal will strictly randomize between high and low forecasts; the farther* q *is from* $\frac{1}{2}$, *the less information is revealed*
(iii) if $q = \frac{1}{2}$, *the analyst fully reveals his private signals.*

The intuition of this result is that the analyst, with his only objective being to maximize his price-impact profits, will tend to forecast so as to move the price to the maximum. If the prior is pessimistic, then a high forecast will move the price more than a low forecast, and thus, will generate higher price-impact profits for the analyst. Similarly, a low forecast in the case of an optimistic prior will generate the maximum price-impact profits. I call this – the incentive to forecast against the market prior – an analyst's "contrarian" incentive, and the bias in his earnings forecast, due to this incentive, the "contrarian" bias.

Suppose the prior of earnings is pessimistic (*i.e.*, $q < \frac{1}{2}$). The analyst's contrarian incentive will induce him to forecast high, regardless of his signal. Now, if the analyst receives a high signal, then a high earnings forecast is consistent with both his private signal and his contrarian incentive. Therefore, the analyst will forecast high with a high signal. However, if the analyst receives a low signal, then a high forecast, although consistent with his contrarian incentive, is not consistent with his private signal, and thus, he will strictly randomize between high and low forecasts. Therefore, for a pessimistic prior, the low forecast has more information content than a high forecast. While a low forecast reveals, unambiguously, that the analyst has received a low signal, a high forecast can be issued by the analyst for both high and low signals.

Note that only at $q = \frac{1}{2}$ can the analyst can fully reveal his private signals. The intuition is that at $q = \frac{1}{2}$, the prior is neither optimistic nor pessimistic; the price moves the same amount regardless of the analyst's forecast, making him indifferent between reporting high and low. In fact, at $q = \frac{1}{2}$, the prior is most diffused; uncertainty about the future values of earnings (x) is at its maximum, decreasing as the prior becomes more precise. It is easy to see that $Var(x)$ is maximum at $q = \frac{1}{2}$ and decreases as q moves farther away from $\frac{1}{2}$.

An analyst will be more likely to move the price to the maximum extent possible at diffused priors, and thus, to generate maximum price-impact profits when the prior hovers around the point $q = \frac{1}{2}$. An analyst's ability to move the price and to generate more profits from price impact diminishes as the prior becomes more precise. This intuition is formalized in the next lemma.

Lemma 4 *An analyst's equilibrium price-impact profits are maximum at $q = \frac{1}{2}$, decrease with q as q moves away from $\frac{1}{2}$, and approach zero as either $q \to 0$ or $q \to 1$.*

2.7 Both Price-Impact and Reputation

In this section, I characterize the equilibrium when an analyst is concerned with *both* the price-impact profits and the reputation motives. The term equilibrium refers to a perfect Bayesian equilibrium.

In order to maximize both his price-impact profits and the expected reputational value in the market, an analyst solves

$$\max_{\hat{s}} \{\alpha\pi(\hat{s}) + (1-\alpha)R(\hat{s}|s)\} \tag{2.13}$$

where $\pi(\hat{s}) = |P(\hat{s}) - P_0|$ and $R(\hat{s} \mid s) = \mathbb{E}_x[Pr(g|\hat{s}, x) \mid s]$. As defined earlier, by (2.2), $V(\hat{s}|s) \equiv \alpha\pi(\hat{s}) + (1-\alpha)R(\hat{s}|s)$. For the ease of illustration, for every $j \in \{h, l\}$, let

$$\triangle V_j \equiv V(\hat{s}^h|s^j) - V(\hat{s}^l|s^j) \tag{2.14}$$

$$\triangle R_j \equiv R(\hat{s}^h|s^j) - R(\hat{s}^l|s^j) \tag{2.15}$$

$$\triangle \pi \equiv \pi(\hat{s}^h) - \pi(\hat{s}^l). \tag{2.16}$$

The term $\triangle V_j$ is the difference in the expected total payoff for the analyst for forecasting $\hat{s}^h$ and $\hat{s}^l$ when he has received a signal s^j. Similarly, $\triangle R_j$ is the difference in the expected reputational value of the analyst for forecasting $\hat{s}^h$ and $\hat{s}^l$ when he has received a signal s^j. Also, $\triangle\pi$ is the difference in the price-impact profits of the analyst for forecasting $\hat{s}^h$ versus $\hat{s}^l$. Note that, by (2.12), the price-impact profits do not depend on the analyst's signal. By definition, for every $j \in \{h, l\}$

$$\triangle V_j = \alpha \triangle \pi + (1-\alpha) \triangle R_j. \tag{2.17}$$

To emphasize the roles of the analyst's strategies (σ), the prior of state (q), and the relative weight of price-impact profits versus reputational payoff (α) in the analyst's expected payoff, I write $\triangle V_j(\sigma_h, \sigma_l, q, \alpha)$ for $\triangle V_j$. Similarly, I use $\triangle\pi(\sigma_h, \sigma_l, q)$ for $\triangle\pi$, and $\triangle R_j(\sigma_h, \sigma_l, q)$ for $\triangle R_j$.

There will be a *fully informative equilibrium* if the following inequalities are satisfied:

$$V(\hat{s}^h|s^h) \geq V(\hat{s}^l|s^h)$$
$$V(\hat{s}^h|s^l) \leq V(\hat{s}^l|s^l)$$

or, equivalently

$$\begin{aligned} \triangle V_h(\sigma_h &= 1, \sigma_l = 0, q, \alpha) \geq 0 \qquad (2.18)\\ \triangle V_l(\sigma_h &= 1, \sigma_l = 0, q, \alpha) \leq 0. \qquad (2.19)\end{aligned}$$

Inequality (2.18) implies that $\alpha \triangle \pi(\sigma_h = 1, \sigma_l = 0, q) + (1-\alpha) \triangle R_h(\sigma_h = 1, \sigma_l = 0, q) \geq 0$, or

$$\alpha \triangle \pi(\sigma_h = 1, \sigma_l = 0, q) \geq -(1-\alpha) \triangle R_h(\sigma_h = 1, \sigma_l = 0, q) \qquad (2.20)$$

which means that for an analyst with a high signal, the optimal forecast will be high if his expected gain in price-impact profits for a high forecast is at least as good as his reputational loss in forecasting high. In other words, each analyst is trading off his short-term gain in price-impact profits with his long-term losses in reputational value in the market. Similarly, inequality (2.19) implies

$$\alpha \triangle \pi(\sigma_h = 1, \sigma_l = 0, q) \leq -(1-\alpha) \triangle R_l(\sigma_h = 1, \sigma_l = 0, q). \qquad (2.21)$$

Proposition 2 characterizes the equilibrium forecasting behavior of an analyst when he is maximizing his price-impact profits and his expected reputational value in the labor market. The equilibrium regions are shown in Figure 3, Panel C.[1]

Proposition 2 *(Characterization of Equilibrium)*
If an analyst's objective is to maximize both *his price-impact profits and reputational value in the market, then there exists an* $\alpha_{\max} \in (0,1)$ *such that for every* $\alpha \leq \alpha_{\max}$*, there exists an equilibrium, which can be expressed as follows:*
(i) if $\underline{q}(\alpha) \leqslant q \leqslant \overline{q}(\alpha)]$, *there is a fully informative equilibrium*
(ii) if either $0 < q < q_{\min}(\alpha)$ *or* $q_{\max}(\alpha) < q < 1$*, then there is a noninformative equilibrium*
(iii) if $q_{\min}(\alpha) \leqslant q < \underline{q}(\alpha)$*, then the equilibrium is partially informative; an analyst with a low signal will forecast low; however, an analyst with a high signal will strictly randomize between high and low forecasts,*
(iv) if $\overline{q}(\alpha) < q \leqslant q_{\max}(\alpha)$*, then the equilibrium is partially informative; an analyst with a high signal will forecast high; however, an analyst with a low signal will strictly*

[1] To avoid notational clutter, α in the parenthesis of q has been dropped in Figure 3, Panel C.

randomize between high and low forecasts,

where,

$\underline{q}(\alpha) = 1 - \overline{q}(\alpha),\ \ q_{\min}(\alpha) = 1 - q_{\max}(\alpha).$

Proposition 2 states that if the relative weight of the price-impact profits component is not very high (which implies that the analyst is sufficiently concerned with his reputational value in the market), then there are three types of equilibrium – fully informative, partially informative and noninformative – at different intervals of the prior of earnings. At intermediate priors, there is a fully informative equilibrium. At sufficiently high priors, the analyst will forecast high if he receives a high signal, and will strictly randomize between high and low forecasts when he receives a low signal. In contrast, at sufficiently low priors, the analyst will strictly randomize if he receives a high signal, but will forecast low when he receives a low signal. At extreme priors, no information is communicated in equilibrium.

Two aspects of this equilibrium are noteworthy. First, in contrast to the model in which the analyst is concerned about maximizing *only* his price-impact profits, in this case, there is a fully informative equilibrium for some interval of priors. Consistent with casual empiricism, reputational concerns do increase the information content of an analyst's forecasts, and thus, the credibility of his forecasts. Second, at extreme priors, no information is communicated in equilibrium. As in the baseline reputation model, at extreme priors – when the prior is either close to one or zero – the public information of earnings is very precise, and thus an analyst with an unlikely signal (for example, a low signal at very high prior or a high signal at very low prior) will tend not to reveal his signal, apprehending that reporting such a signal will negatively impact his reputation in the market. As I have shown in lemma 4, at extreme priors, where the price-impact profits are very small due to less likelihood of price movements, the conformist bias due to the reputational concerns of the analyst persists.

Proposition 3 *(Informativeness of Equilibrium)*
Suppose $\alpha \leq \alpha_{\max}$. *The region of priors for which there is a fully informative equilibrium increases, and the region of priors for which there is a noninformative equilibrium decreases, as the relative weight of price-impact profits increases.*

Formally, if $\alpha_2 > \alpha_1$, *then* $\overline{q}(\alpha_2) \geqslant \overline{q}(\alpha_1)$ *and* $q_{\max}(\alpha_2) \geqslant q_{\max}(\alpha_1)$.

Proposition 3 is the main result of this chapter. It states that, as long as an analyst is sufficiently concerned about his reputation (*i.e.*, $\alpha \leq \alpha_{\max}$), price-impact profits, along with reputational concerns, provide stronger incentive for honest reporting than reputational concerns or price-impact profits alone. Recall that there was no fully informative equilibrium in the case in which the analyst was only concerned with maximizing his short-term price-impact profits. Specifically, the relative weight of price-impact profits increases the region of fully informative equilibrium, and decreases the region of non-informative equilibrium. This result is striking because while the price-impact profits incentive, by itself, discourages the truthful revelation of an analyst's private signal, when added to the reputational concerns incentive, it improves the information content of an analyst's forecast.

The intuition of this result is as follows. Recall my discussions in the baseline model, in which reputational concerns create a conformist bias in an analyst's forecast. At sufficiently high or low priors, an analyst will conform to the prior by ignoring his own private information, and the market, aware of this strategy, will completely ignore the analyst's forecast, leading to no information transmission in equilibrium. To be more specific, suppose the prior of earnings is very pessimistic, *i.e.*, q is sufficiently less than $\frac{1}{2}$. An analyst with only reputational concerns who receives a high signal will have no incentive to forecast high due to his conformist bias. However, with the addition of a price-impact profits motive, the analyst now has a contrarian incentive, which will motivate him to forecast high at a pessimistic prior. Thus, when the two motives are combined, the analyst can credibly communicate his high signal at a very low prior, which was not possible with only a reputational concern.

2.8 Summary of Results

In this chapter, I first discussed the equilibrium forecasting behavior of an analyst in a baseline model in which a single analyst is concerned with maximizing only his expected reputational value in the market. In equilibrium, an analyst can credibly convey his private information only in the intermediate range of priors. At extreme priors, an analyst will ignore his own private information and forecast in the direction of the market's prior. The analyst's reputational concerns create a "conformist" bias in his forecast.

I then considered a case in which an analyst is concerned with maximizing only his short-term price impact profits. I showed that an analyst will never fully reveal his private information, although some information does get communicated in equilibrium. With the objective of moving the price the maximum extent possible, an analyst will tend to forecast against the market's prior expectations. The short-term price impact motive creates a "contrarian" bias in the analyst's forecast.

Finally, I considered a case in which an analyst is concerned with maximizing *both* his short-term price impact profits and the long-term reputational value in the market. The key result in this model is that the price impact motive, together with reputational concerns, provides stronger incentives for honest forecasting than reputational concerns or price impact alone. Adding a short-term price impact motive diminishes – although it does not fully mitigate – the conformist bias in an analyst's forecast due to his reputational concerns.

Chapter 3

The Role of Reputational Ranking

3.1 Introduction

In this chapter, I develop a model in which two analysts are simultaneously making earnings forecasts of a company, each seeking to maximize his own reputational ranking value in the market. The reputational ranking value of an analyst depends on how his reputation in the market compares with that of the other analyst. The popular press and empirical evidence suggest that sell-side analysts' compensations and career paths often depend on their reputational rankings in the market (*e.g.*, annual survey by the U.S. Institutional Investors Magazine). Higher ranked analysts receive better compensation and have greater job mobility than their lower ranked counterparts (Mikhail, Walther and Willis 1999; Leone and Wu 2002; Hong and Kubik 2003).

When analysts are concerned with maximizing their reputational ranking values in the market, each analyst must trade off the odds of his being ranked higher than the other analyst versus the odds of not being ranked lower. On one hand, to improve the odds of being ranked higher, each analyst wants to differentiate himself from the other by making a forecast which is different from that of the other analyst. On the other hand, to minimize the odds of being ranked lower, each analyst will tend to move with the other analyst by making the same forecast as the other analyst. On balance, an analyst's optimal forecasting strategy depends on his relative reputational payoffs of being ranked higher or lower than the other analyst.

As it turns out, the convexity of the reputational ranking payoff function – that is, the extent to which the marginal payoff of being ranked higher is greater than that of being ranked lower – decreases the information content of an analyst's forecast. The popular press suggests that sell-side analysts are often paid very high bonuses when they have higher reputational rankings, but are not penalized equivalently when they are ranked lower. The implication of my result is that such a compensation structure can potentially provide incentives to an analyst to reveal less of his private information to the market.

3.2 Related Literature

Ottaviani and Sorensen (2006a) consider a model of reputational cheap talk in which experts make simultaneous forecasts about some state variable, and are paid based on their reputation relative to their peer group. The authors show that if the expert signals are independent, conditional on the state and expert talent, then each analyst will behave in exactly the same way as a single expert does when he is concerned with maximizing his own reputation. They call this forecasting behavior the "irrelevance of relative reputation." The authors, however, do not discuss what happens when conditional correlation does exist between expert signals.

There are also two papers that discuss the role of relative reputation and relative performance in an expert's behavior when experts move sequentially. Effinger and Polborn (2001) consider a model in which two experts, moving one after the other, are making business decisions about their respective firms. In contrast to the reputational herding literature (*e.g.*, Scharfstein and Stein 1990; Graham 1998), Effinger and Polborn assume that an expert's payoff will depend not only on his own reputation, but also on his relative reputation *vis-a-vis* the other expert. They find that if the value of being the only smart expert is sufficiently large, the second mover always opposes his predecessor's move, regardless of his own signal ("anti-herding"); otherwise, herding may occur. Bernhardt, Campello and Kutsoati (2004) consider a case in which analysts make sequential forecasts, and their compensations are based on both their absolute forecast accuracy and their accuracy relative to other analysts following the same firm.

The authors show that if the relative performance compensation is a convex function, then the last analyst strategically biases his forecast in the direction of his private information. However, a concave function induces the last analyst to bias his forecast toward the consensus.

3.3 The Model

There are three players: two analysts, $i \in \{A, B\}$, and the market. There are three dates. At $t = 0$, each analyst receives a private signal $s_i \in S_i \equiv \{s^h, s^l\}$ of the earnings $x \in X \equiv \{x^H, x^L\}$ of a company. Superscripts $H(h)$ and $L(l)$ can be interpreted as "high" and "low" respectively. Upon observing his signal, at $t = 1$, each analyst makes a forecast, $\hat{s}_i \in \hat{S}_i \equiv \{\hat{s}^h, \hat{s}^l\}$. An analyst's talent is $\theta_i \in \Theta_i \equiv \{g, b\}$, good or bad. A good analyst receives a more precise private signal about the earnings of a company than a bad analyst. Neither the market nor the analysts know θ; they only know the prior distribution, $\Pr(\theta_i = g) \equiv \lambda \in (0, 1)$. Also, everyone knows the prior distribution of earnings, $\Pr(x^H) \equiv q \in (0, 1)$. Finally, at $t = 2$, when earnings x is reported by the company, the market compares the realized x with the analyst forecasts $(\hat{s}_i, \hat{s}_{-i})$, and updates its belief about each analyst's talent, which I define as the analyst's reputation, $r_i(\hat{s}_i, \hat{s}_{-i}, x) \equiv \Pr(\theta_i = g|\hat{s}_i, \hat{s}_{-i}, x)$. The time line of the game is shown in Figure 1, Panel B.

Information structure. As in the model with a single analyst, the precision of each analyst's private signal is

$$\gamma_\theta = \Pr(s = x|x, \theta).$$

I assume $1 > \gamma_g > \gamma_b = \frac{1}{2}$. The probability that a good analyst receives a matched signal (*i.e.*, $s = x$), conditional on earnings and his talent, is γ_g, which is higher than that of a bad analyst (*i.e.*, γ_b). Since analysts do not know their talents, they only know the unconditional probability, which is defined as $\gamma \equiv \Pr(s = x|x) = \lambda\gamma_g + (1-\lambda)\frac{1}{2} > \frac{1}{2}$.

Correlation between signals. I consider two cases. In the first case, analysts' private signals are independent (conditional on earnings); in the second, their signals are conditionally correlated. For the latter case, I assume that the private signals of good analysts are perfectly (positively) correlated conditional on state x and talent θ; however, if one of the analysts is bad, then their signals are conditionally independent.

Thus, the probability that two good analysts observe matched signals is

$$\Pr(s_i = x, s_{-i} = x | x, g_i, g_{-i}) = \gamma_g \tag{3.1}$$

(not γ_g^2), where (g_i, g_{-i}) implies $(\theta_i = g, \theta_{-i} = g)$. This assumption[2] is similar to that made by Scharfstein and Stein (1990). Since in my model analysts do not know their own talents, all my results will be qualitatively similar to those found in a setting in which there is a positive correlation between the private signals of bad analysts; if one of the analysts is good, their private signals are independent. What matters is that there is an *ex-ante* positive correlation between the signals of two analysts.

Reputational payoff. Each analyst's reputational payoff in the market depends not only on his own reputation but also on the reputation of the other analyst. This assumption can be interpreted as an abstraction of the annual rankings of analysts' reputations performed by institutional investors, and linking of the analysts' compensations to their reputational rankings. I define an analyst's reputational ranking payoff function as a mapping of analysts' reputations to some real value, that is, $U : (0, 1) \times (0, 1) \rightarrow \mathbb{R}$. Accordingly, analyst i's reputational ranking payoff function is $U_i(r_i(\hat{s}_i, \hat{s}_{-i}, x), r_{-i}(\hat{s}_i, \hat{s}_{-i}, x))$, where $r_i(\hat{s}_i, \hat{s}_{-i}, x)$ and $r_{-i}(\hat{s}_i, \hat{s}_{-i}, x)$ represent two analysts' reputations. A natural assumption of an analyst's reputational ranking payoff function is that it will increase with the analyst's own reputation and decrease with the other analyst's reputation, that is

$$\frac{\partial U_i}{\partial r_i} > 0 \text{ and } \frac{\partial U_i}{\partial r_{-i}} < 0 \text{ for all } i. \tag{3.2}$$

I will use $U_i(r_i(\hat{s}_i, \hat{s}_{-i}, x), r_{-i}(\hat{s}_i, \hat{s}_{-i}, x))$ (or, $U_i(r_i, r_{-i})$) and $U_i(\hat{s}_i, \hat{s}_{-i}, x)$ interchangeably.

Objective function. Each analyst is maximizing his expected reputational value, which depends on his reputational ranking *vis-a-vis* the other analyst in the market. Accordingly, analyst i's objective function is

$$R_{i,j}(\hat{s}_i \mid s_i) \equiv \mathbb{E}_{x, \hat{s}_{-i}}[U_i(\hat{s}_i, \hat{s}_{-i}, x) \mid s_i \ , \sigma_{-i}] \tag{3.3}$$

[2] Instead of perfect correlation, the level of correlation could have been more general, $\rho \in (0, 1]$ as in Graham (1999). In that case, the probability that two good analysts observe ex-post correct signals is $\rho\gamma_g + (1 - \rho)\gamma_g^2$, a convex combination of two extreme cases when analysts receive perfectly correlated signals (γ_g) and conditionally independent signals (γ_g^2). Similarly, the likelihood that two good analysts receive different signals is $(1 - \rho)\gamma_g(1 - \gamma_g)$. My conjecture is that assuming a more general correlation level will not change the qualitative aspects of my results.

where $U_i(\hat{s}_i, \hat{s}_{-i}, x)$ is analyst i's reputational ranking payoff, given his and the other analyst's forecasts and the realized earnings. The term σ_{-i} is the other analyst's (*i.e.*, -i) strategy. An analyst's strategy is defined as $\sigma_i : S_i \to \triangle(\hat{S}_i)$, a mapping from the analyst i's signal space, S_i, to a probability distribution over his forecast space, $\hat{S}_i$.

3.4 Reputation with Multiple Analysts

In this section, I discuss how each analyst's reputation is updated by the market given both the analysts' forecasts and the reported earnings of the company. Suppose, for example, the reported earnings is x^H, and analysts A and B have made forecasts $\hat{s}^h$ and $\hat{s}^h$ respectively. What will be analyst A's reputation in the market?

$$\Pr(g_A|\hat{s}^h_A, \hat{s}^h_B, x^H) = \frac{\Pr(g_A, g_B|\hat{s}^h_A, \hat{s}^h_B, x^H) + \Pr(g_A, b_B|\hat{s}^h_A, \hat{s}^h_B, x^H)}{\sum_{\theta_A}\sum_{\theta_B} \Pr(\theta_A, \theta_B|\hat{s}^h_A, \hat{s}^h_B, x^H)} \tag{3.4}$$

where each of $\Pr(g_A, g_B|\hat{s}_A, \hat{s}_B, x) = \frac{\Pr(g_A, g_B)\Pr(\hat{s}_A, \hat{s}_B|x, g_A, g_B)}{\Pr(\hat{s}_A, \hat{s}_B|x)}$. Now, similar to the case of a single analyst, let $\sigma_{i,j}$ be the probability that analyst $i \in \{A, B\}$ forecasts $\hat{s}^h$ after receiving a signal s^j where $j \in \{h, l\}$. Specifically

$$\begin{aligned}
\sigma_{A,h} &= \Pr(\hat{s}^h_A|s^h_A) \in [0,1] \text{ and } \sigma_{A,l} = \Pr(\hat{s}^h_A|s^l_A) \in [0,1] \\
\sigma_{B,h} &= \Pr(\hat{s}^h_B|s^h_B) \in [0,1] \text{ and } \sigma_{A,l} = \Pr(\hat{s}^h_B|s^l_B) \in [0,1].
\end{aligned}$$

However, since analysts are playing symmetric strategies

$$\sigma_{A,h} = \sigma_{B,h} \equiv \sigma_h = \Pr(\hat{s}^h|s^h) \in [0,1] \tag{3.5}$$

$$\sigma_{A,l} = \sigma_{B,l} \equiv \sigma_l = \Pr(\hat{s}^h|s^l) \in [0,1]. \tag{3.6}$$

Now, for every θ

$$\begin{aligned}
&\Pr(\hat{s}^h_A, \hat{s}^h_B|x^H, \theta_A, \theta_B) = \Pr(s^h_A, s^h_B|x^H, \theta_A, \theta_B)\sigma_h^2 + \Pr(s^h_A, s^l_B|x^H, \theta_A, \theta_B)\sigma_h\sigma_l + \\
&\Pr(s^l_A, s^h_B|x^H, \theta_A, \theta_B)\sigma_l\sigma_h + \Pr(s^l_A, s^l_B|x^H, \theta_A, \theta_B)\sigma_l^2
\end{aligned} \tag{3.7}$$

and then

$$\begin{aligned}
& \Pr(g_A, g_B)\Pr(\hat{s}^h_A, \hat{s}^h_B|x^H, g_A, g_B) \\
= \; & \lambda^2[\Pr(s^h_A, s^h_B|x^H, g_A, g_B)\sigma_h^2 + \Pr(s^h_A, s^l_B|x^H, g_A, g_B)\sigma_h\sigma_l + \\
& \Pr(s^l_A, s^h_B|x^H, g_A, g_B)\sigma_l\sigma_h + \Pr(s^l_A, s^l_B|x^H, g_A, g_B)\sigma_l^2].
\end{aligned} \tag{3.8}$$

The terms $\Pr(s_A, s_B|x, g_A, g_B)$ are calculated using the assumption that the signals of two good analysts are perfectly correlated. Similarly, $\Pr(s_A, s_B|x, g_A, b_B)$, $\Pr(s_A, s_B|x, b_A, g_B)$ and $\Pr(s_A, s_B|x, b_A, b_B)$ are calculated using the assumption that if at least one of the analysts is bad, their signals are conditionally independent. Taken together (detailed calculations are in Appendix A.3)

$$\Pr(g_A|\hat{s}_A^h, \hat{s}_B^h, x^H) = \lambda \left[\frac{\{\gamma_g \sigma_h + (1-\gamma_g)\sigma_l\}\{\gamma\sigma_h + (1-\gamma)\sigma_l\} + \frac{c}{\lambda}(\sigma_h - \sigma_l)^2}{\{\gamma\sigma_h + (1-\gamma)\sigma_l\}^2 + c(\sigma_h - \sigma_l)^2} \right] \quad (3.9)$$

where $c \equiv \lambda^2 \gamma_g(1-\gamma_g)$ is defined as the "correlation effect[3] ." The term $\lambda^2\gamma_g(1-\gamma_g)$ captures the effect of the correlation between the private signals of the analysts. Note that if the correlation between the signals vanishes (in the limit)

$$\begin{aligned} \lim_{c \to 0} \Pr(g_A|\hat{s}_A^h, \hat{s}_B^h, x^H) &= \lambda \left[\frac{\gamma_g \sigma_h + (1-\gamma_g)\sigma_l}{\gamma\sigma_h + (1-\gamma)\sigma_l} \right] \\ &= \Pr(g_A|\hat{s}_A^h, x^H) \end{aligned} \quad (3.10)$$

which has the same expression found in Property 1 in Chapter 2, in which an analyst's reputation depends only on his own forecast. The intuition is that if analyst signals are conditionally independent (in the limit), one analyst's signal (and thus, his type) does not depend on the other analyst's signal (or forecast), and therefore, an analyst's reputation will not depend on the other analyst's forecast (signal). Property 3 below generalizes the analyst reputation result in (3.9) to other combinations of analyst forecasts and reported earnings.

Property 2 *(Reputation with Multiple Analysts)*
*For $i \in \{A, B\}$, analyst i's reputation, given high reported earnings (*i.e., *x^H), is*

$$\begin{aligned} \Pr(g_i|\hat{s}_i^h, \hat{s}_{-i}^h, x^H) &= \lambda \left[\frac{\{\gamma_g \sigma_h + (1-\gamma_g)\sigma_l\}\Sigma_1 + \frac{c}{\lambda}(\sigma_h - \sigma_l)^2}{\{\Sigma_1\}^2 + c(\sigma_h - \sigma_l)^2} \right] \\ \Pr(g_i|\hat{s}_i^h, \hat{s}_{-i}^l, x^H) &= \lambda \left[\frac{\{\gamma_g \sigma_h + (1-\gamma_g)\sigma_l\}\Sigma_2 - \frac{c}{\lambda}(\sigma_h - \sigma_l)^2}{\Sigma_1\Sigma_2 - c(\sigma_h - \sigma_l)^2} \right] \\ \Pr(g_i|\hat{s}_i^l, \hat{s}_{-i}^h, x^H) &= \lambda \left[\frac{\{\gamma_g (1-\sigma_h) + (1-\gamma_g)(1-\sigma_l)\}\Sigma_1 - \frac{c}{\lambda}(\sigma_h - \sigma_l)^2}{\Sigma_1\Sigma_2 - c(\sigma_h - \sigma_l)^2} \right] \\ \Pr(g_A|\hat{s}_i^l, \hat{s}_{-i}^l, x^H) &= \lambda \left[\frac{\{\gamma_g (1-\sigma_h) + (1-\gamma_g)(1-\sigma_l)\}\Sigma_2 + \frac{c}{\lambda}(\sigma_h - \sigma_l)^2}{\{\Sigma_2\}^2 + c(\sigma_h - \sigma_l)^2} \right] \end{aligned}$$

[3] For a more general correlation level $\rho \in (0, 1]$ (see footnote 2), the correlation effect will be $c = \rho\lambda^2\gamma_g(1-\gamma_g)$, strictly increasing in ρ. Also, if there is perfect correlation between two "bad" analysts, and if one of the analysts is "good", then analyst signals are conditionally independent, the correlation effect term would be, $c = (1-\lambda)^2\gamma_b(1-\gamma_b)$.

Similarly, analyst i's reputation, given x^L, is

$$\begin{aligned}
\Pr(g_i|\hat{s}_i^h, \hat{s}_{-i}^h, x^L) &= \lambda\left[\frac{\{(1-\gamma_g)\sigma_h + \gamma_g\sigma_l\}\Sigma_3 + \frac{c}{\lambda}(\sigma_h-\sigma_l)^2}{\{\Sigma_3\}^2 + c(\sigma_h-\sigma_l)^2}\right] \\
\Pr(g_i|\hat{s}_i^h, \hat{s}_{-i}^l, x^L) &= \lambda\left[\frac{\{(1-\gamma_g)\sigma_h + \gamma_g\sigma_l\}\Sigma_4 - \frac{c}{\lambda}(\sigma_h-\sigma_l)^2}{\Sigma_3\Sigma_4 - c(\sigma_h-\sigma_l)^2}\right] \\
\Pr(g_i|\hat{s}_i^l, \hat{s}_{-i}^h, x^L) &= \lambda\left[\frac{\{(1-\gamma_g)(1-\sigma_h) + \gamma_g(1-\sigma_l)\}\Sigma_3 - \frac{c}{\lambda}(\sigma_h-\sigma_l)^2}{\Sigma_3\Sigma_4 - c(\sigma_h-\sigma_l)^2}\right] \\
\Pr(g_A|\hat{s}_i^l, \hat{s}_{-i}^l, x^L) &= \lambda\left[\frac{\{(1-\gamma_g)(1-\sigma_h) + \gamma_g(1-\sigma_l)\}\Sigma_4 + \frac{c}{\lambda}(\sigma_h-\sigma_l)^2}{\{\Sigma_4\}^2 + c(\sigma_h-\sigma_l)^2}\right].
\end{aligned}$$

where $c = \lambda^2\gamma_g(1-\gamma_g)$, $\Sigma_1 = \gamma\sigma_h + (1-\gamma)\sigma_l$, $\Sigma_2 = \gamma(1-\sigma_h) + (1-\gamma)(1-\sigma_l)$, $\Sigma_3 = (1-\gamma)\sigma_h + \gamma\sigma_l$ and $\Sigma_4 = (1-\gamma)(1-\sigma_h) + \gamma(1-\sigma_l)$.

Two other important properties of an analyst's reputation in the presence of a second analyst are detailed below. These properties are not proved here because the proofs are very straightforward.

Property 3 *(Symmetry)*
For analyst $i \in \{A, B\}$ and $j \in \{h, l\}$, $k \in \{h, l\}$

$$\Pr(g_i|\hat{s}_i^j, \hat{s}_{-i}^k, x) = \Pr(g_{-i}|\hat{s}_i^k, \hat{s}_{-i}^j, x).$$

For example

$$\begin{aligned}
\Pr(g_A|\hat{s}_A^h, \hat{s}_B^h, x^H) &= \Pr(g_B|\hat{s}_A^h, \hat{s}_B^h, x^H) \\
\Pr(g_A|\hat{s}_A^l, \hat{s}_B^l, x^H) &= \Pr(g_B|\hat{s}_A^l, \hat{s}_B^l, x^H) \\
\Pr(g_A|\hat{s}_A^h, \hat{s}_B^l, x^H) &= \Pr(g_B|\hat{s}_A^l, \hat{s}_B^h, x^H) \\
\Pr(g_A|\hat{s}_A^l, \hat{s}_B^h, x^H) &= \Pr(g_B|\hat{s}_A^h, \hat{s}_B^l, x^H).
\end{aligned}$$

This property holds because the analysts are identical in every respect, except for their private signals, and are using symmetric strategies.

Property 4 *(Ordering)*
For analyst $i \in \{A, B\}$

$$\Pr(g_i|\hat{s}_i = x, \hat{s}_{-i} \neq x, x) \geqslant \Pr(g_i|\hat{s}_i \neq x, \hat{s}_{-i} = x, x).$$

For example, for analyst A

$$\begin{aligned}\Pr(g_A|\hat{s}_A^h, \hat{s}_B^l, x^H) &\geqslant \Pr(g_A|\hat{s}_A^l, \hat{s}_B^h, x^H)\\ \Pr(g_A|\hat{s}_A^l, \hat{s}_B^h, x^L) &\geqslant \Pr(g_A|\hat{s}_A^h, \hat{s}_B^l, x^L).\end{aligned}$$

This property implies that an analyst's reputation is higher, when his forecast matches with the reported earnings but the other analyst's does not, than when his forecast does not match the earnings but the other analyst's does.

3.5 Reputational Ranking Payoff Function

I consider two types of reputational ranking payoff function – one linear, and the other non-linear – which satisfy the properties, $\frac{\partial U_i}{\partial r_i} > 0$ and $\frac{\partial U_i}{\partial r_{-i}} < 0$ for all i, as described earlier in (3.1). The functional forms are discussed below.

$$\textbf{Linear :} \quad U_i(r_i, r_{-i}) = \beta_0 + \beta_1 r_i - \beta_2 r_{-i}, \text{ where } \beta_0, \beta_1, \beta_2 > 0. \tag{3.11}$$

An example of a linear payoff function is $U_i(r_i, r_{-i}) = r_i - \frac{r_i + r_{-i}}{2}$. Here, each analyst's reputational payoff depends on how his own reputation compares with a summary statistic (the mean) of both the analysts' reputations. As a special case of linear functional form, in this example, $\beta_0 = 0$, $\beta_1 = \beta_2 = \frac{1}{2}$.

$$\textbf{Non-linear:} \quad U_i(r_i, r_{-i}) = \frac{r_i}{r_i + \upsilon r_{-i}}, \text{ where } \upsilon > 0. \tag{3.12}$$

This simple non-linear reputational ranking payoff function has the natural property that for $i \in \{A, B\}$

$$U_i(\hat{s}_i^h, \hat{s}_{-i}^l, x^H) \geqslant U_i(\hat{s}_i^h, \hat{s}_{-i}^h, x^H) \geqslant U_i(\hat{s}_i^l, \hat{s}_{-i}^h, x^H) \tag{3.13}$$

$$U_i(\hat{s}_i^l, \hat{s}_{-i}^h, x^L) \geqslant U_i(\hat{s}_i^l, \hat{s}_{-i}^l, x^L) \geqslant U_i(\hat{s}_i^h, \hat{s}_{-i}^l, x^L) \tag{3.14}$$

which implies that the reputational ranking payoff of an analyst is higher, when his forecast matches the reported earnings and the other analyst's forecast does not, than when both analysts' forecasts match or when his forecast does not match but the other analyst's does. This property is a direct consequence of Property 4. Furthermore

$$U_i(\hat{s}_i^h, \hat{s}_{-i}^h, x^H) = U_i(\hat{s}_i^l, \hat{s}_{-i}^l, x^H) = \frac{1}{1+\upsilon} = U_i(\hat{s}_i^l, \hat{s}_{-i}^l, x^L) = U_i(\hat{s}_i^h, \hat{s}_{-i}^h, x^L) \tag{3.15}$$

which implies that if both analysts make the same forecast, regardless of whether it does or does not match the reported earnings, the analysts' reputational ranking payoffs are the same.

I now define the convexity of a non-linear reputational ranking payoff function, and show, in Lemma 5 below, how the parameter υ in U is related to this convexity.

Definition 7 *(Convexity of Reputational Ranking Payoff Function)*

$$\textit{Convexity Ratio } (\phi) \equiv \frac{\textit{marginal gain of being ranked higher}}{\textit{marginal loss of being ranked lower}} > 0$$

where

$$\begin{aligned}\textit{marginal gain of being ranked higher} &\equiv U_i(r_i > r_{-i}) - U_i(r_i = r_{-i}) \\ \textit{marginal loss of being ranked lower} &\equiv U_i(r_i = r_{-i}) - U_i(r_i < r_{-i}).\end{aligned}$$

Figure 4, Panel A shows a convex reputational ranking payoff function. Figure 4, Panel B illustrates how the reputational ranking payoff function (U) changes with the convexity ratio (ϕ). As shown, the reputational ranking function becomes more convex with an increase in ϕ. Lemma 5 establishes a relationship between the convexity ratio and the parameter υ of the reputational ranking payoff function U.

Lemma 5 *Suppose the reputational ranking payoff function is* $U_i(r_i, r_{-i}) = \frac{r_i}{r_i + \upsilon r_{-i}}$. *Then the convexity ratio* (ϕ) *is increasing in* υ.

3.6 Equilibrium Analysis

In this section I define and characterize the equilibrium of the model when each analyst is concerned with maximizing his expected reputational ranking payoff in the market. Throughout, the term equilibrium refers to a perfect Bayesian equilibrium with symmetric strategies. Given that the analysts do not know their types and are identical in every aspect except in their private signals, the most plausible equilibrium is an equilibrium with symmetric strategies.

Definition 8 *An equilibrium consists of analysts' forecasting strategies* (σ_i, σ_{-i}) *such that*
(i) *for each* $s_i \in S_i$*, analyst* i *solves*

$$\max_{\hat{s}_i} \mathbb{E}_{x,\hat{s}_{-i}}[U_i(\hat{s}_i, \hat{s}_{-i}, x) \mid s_i, \sigma_{-i}] \tag{3.16}$$

(ii) *given* $(\hat{s}_i, \hat{s}_{-i})$ *and the realization of* x*, the market's belief about analyst* i*'s type,* $Pr(g_i \mid \hat{s}_i, \hat{s}_{-i}, x)$*, is consistent with Bayes' rule.*

Condition (i) states that each analyst maximizes his expected reputational ranking payoff for each of his signal types, taking the other analyst's strategy as given. Condition (ii) states that the market's beliefs about the analysts' types are consistent with Bayes' rule.

3.6.1 Analyst Signals are Conditionally Independent

I start with a case in which analysts' private signals are conditionally independent, and the reputational ranking payoff function is linear in analysts' reputations.

Proposition 4 *If analyst signals are conditionally independent and the reputational ranking payoff function is linear, then each analyst's equilibrium forecasting behavior will be the same as that in the baseline model with a single analyst.*

The intuition of this result is that the conditional (on state variable x) independence of analysts' private signals ensures that each analyst's posterior reputation depends only on his own forecast (and not on the other analyst's forecast) and the reported earnings. Therefore, in effect, each analyst is concerned with maximizing his own expected reputational value in the market as in the baseline model with a single analyst. Also, the linearity of the reputational ranking payoff function is useful for the linearity of the expectation operator.

The result is interesting because even though each analyst is competing against the other analyst to get a higher reputational ranking, the equilibrium behavior does not change from the baseline model. A similar result has been obtained by Ottaviano and Sorensen (2006a) with a somewhat different modeling choice of relative reputation. In their model, an expert's payoff is a function of his type relative to other experts' types.

In my model, an analyst's payoff is a function of his reputation – the market's belief about his type – relative to the other analyst's reputations.

What happens when the reputational ranking payoff function is non-linear? It is difficult to obtain a result similar to that in Proposition 4 with a non-linear payoff function due to the fact that the expectation operator is not transferable to a non-linear function. However, I can derive a restricted result that if the correlation between analyst signals vanishes (in the limit), then there exists an equilibrium in which each analyst's equilibrium forecasting behavior will be the same as that in the baseline model with a single analyst. This result is discussed in more depth in Corollary 2 in the next section.

3.6.2 Analyst Signals are Conditionally Correlated

In this section, I characterize the equilibrium for the case in which analyst signals are conditionally correlated, and the reputational ranking payoff function is non-linear as defined earlier in (3.12). By Definition 8, analyst i's objective is

$$\max_{\hat{s}_i} \mathbb{E}_{x,\hat{s}_{-i}}[U_i(\hat{s}_i, \hat{s}_{-i}, x) \mid s_i, \sigma_{-i}].$$

For ease of illustration, I define a few terms. Let for every i

$$\Delta R_{i,h} \equiv R_i(\hat{s}_i^h | s_i^h) - R_i(\hat{s}_i^l | s_i^h) \tag{3.17}$$

$$\Delta R_{i,l} \equiv R_i(\hat{s}_i^h | s_i^l) - R_i(\hat{s}_i^l | s_i^l). \tag{3.18}$$

$\Delta R_{i,h}$ is the marginal gain of analyst i for reporting a high forecast ($\hat{s}^h$) versus a low forecast ($\hat{s}^l$) when he receives a *high* signal (s^h). Similarly, $\Delta R_{i,l}$ is the marginal gain of analyst i for reporting a high forecast versus a low forecast when he receives a *low* signal. Note that the subscripts h and l in ΔR denote an analyst's private signal, high and low respectively. To mention analysts' strategies explicitly, and to show that $\Delta R_{i,h}$ is a function of prior q, I will often write $\Delta R_{i,h}(\sigma_h, \sigma_l, q)$. Similarly, I will write $\Delta R_{i,l}(\sigma_h, \sigma_l, q)$ for $\Delta R_{i,l}$.

For example, to calculate analyst A's expected reputational ranking payoff when he receives a high signal and makes a high forecast

$$
\begin{aligned}
& R_A(\hat{s}_A^h|s_A^h) \equiv \mathbb{E}_{x,\hat{s}_B}[U_A(\hat{s}_A^h, \hat{s}_B, x)|s_A^h, \sigma_B] \\
&= \Pr(\hat{s}_B^h, x^H|s_A^h)U_A(\hat{s}_A^h, \hat{s}_B^h, x^H) + \Pr(\hat{s}_B^l, x^H|s_A^h)U_A(\hat{s}_A^h, \hat{s}_B^l, x^H) + \\
&\quad \Pr(\hat{s}_B^h, x^L|s_A^h)U_A(\hat{s}_A^h, \hat{s}_B^h, x^L) + \Pr(\hat{s}_B^l, x^L|s_A^h)U_A(\hat{s}_A^h, \hat{s}_B^l, x^L) \\
&= \Pr(x^H|s_A^h)\{\Pr(\hat{s}_B^h|x^H, s_A^h)U_A(\hat{s}_A^h, \hat{s}_B^h, x^H) + \Pr(\hat{s}_B^l|x^H, s_A^h)U_A(\hat{s}_A^h, \hat{s}_B^l, x^H)\} + \\
&\quad \Pr(x^L|s_A^h)\{\Pr(\hat{s}_B^h|x^L, s_A^h)U_A(\hat{s}_A^h, \hat{s}_B^h, x^L) + \Pr(\hat{s}_B^l|x^L, s_A^h)U_A(\hat{s}_A^h, \hat{s}_B^l, x^L)\}.
\end{aligned}
$$

The values of $U_A(\hat{s}_A, \hat{s}_B, x)$ (or $U_A(r_A(\hat{s}_A, \hat{s}_B, x), r_B(\hat{s}_A, \hat{s}_B, x))$) are calculated using the reputational ranking payoff function defined in (3.12), and $r_A(\hat{s}_A, \hat{s}_B, x)$ and $r_B(\hat{s}_A, \hat{s}_B, x)$ are calculated by Property 2. The terms $\Pr(\hat{s}_B|x, s_A)$ are calculated from the joint distribution of the private signals of two analysts and analyst B's strategy. For example, $\Pr(\hat{s}_B^h|x^H, s_A^l)$ is calculated as follows

$$
\begin{aligned}
\Pr(\hat{s}_B^h|x^H, s_A^h) &= \Pr(s_B^h|x^H, s_A^h)\Pr(\hat{s}_B^h|s_B^h) + \Pr(s_B^l|x^H, s_A^h)\Pr(\hat{s}_B^h|s_B^l) \\
&= \Pr(s_B^h|x^H, s_A^h)\sigma_h + \Pr(s_B^l|x^H, s_A^h)\sigma_l
\end{aligned}
$$

where

$$
\Pr(s_B^h|x^H, s_A^h) = \frac{\Pr(s_A^h, s_B^h|x^H)}{\Pr(s_A^h|x^H)} \text{ and } \Pr(s_B^l|x^H, s_A^h) = \frac{\Pr(s_A^h, s_B^l|x^H)}{\Pr(s_A^h|x^H)}.
$$

The joint distribution of analyst signals, for example $\Pr(s_A^h, s_B^l|x^H)$, is calculated as follows

$$
\begin{aligned}
\Pr(s_A^h, s_B^l|x^H) &= \sum_{\theta_A}\sum_{\theta_B}\Pr(\theta_A, \theta_B)\Pr(s_A^h, s_B^l|x^H, \theta_A, \theta_B) \\
&= \Pr(g_A, g_B)\Pr(s_A^h, s_B^l|x^H, g_A, g_B) + \Pr(g_A, b_B)\Pr(s_A^h, s_B^l|x^H, g_A, b_B) \\
&\quad + \Pr(b_A, g_B)\Pr(s_A^h, s_B^l|x^H, b_A, g_B) \\
&\quad + \Pr(b_A, b_B)\Pr(s_A^h, s_B^l|x^H, b_A, b_B)
\end{aligned}
$$

which can be simplified (detailed calculations are shown in the proof of Property 5 in Appendix A.3) to

$$
\begin{aligned}
\Pr(s_A^h, s_B^l|x^H) &= \gamma(1-\gamma) - \lambda^2\gamma_g(1-\gamma_g) \\
&= \gamma(1-\gamma) - c. \qquad (3.19)
\end{aligned}
$$

Note that the joint distribution of analyst signals is comprised of two terms. The first term represents the joint probability of the signals as if the signals were conditionally independent. The second term introduces the correlation between the signals. Furthermore, the correlation effect is negative (the second term is negative) in this case because the analysts have different signals. When the analysts have the same signal, the correlation effect is positive. The following property summarizes the joint distribution of the analyst signals.

Property 5 *(Joint Distribution of Signals)*
$(i)\, \Pr(s_i^h, s_{-i}^h|x^H) = \Pr(s_i^l, s_{-i}^l|x^L) = \gamma^2 + c$
$(ii)\, \Pr(s_i^h, s_{-i}^l|x^H) = \Pr(s_i^l, s_{-i}^h|x^H) = \Pr(s_i^l, s_{-i}^h|x^L) = \Pr(s_i^h, s_{-i}^l|x^L) = \gamma(1-\gamma) - c$
$(iii)\, \Pr(s_i^l, s_{-i}^l|x^H) = \Pr(s_i^h, s_{-i}^h|x^L) = (1-\gamma)^2 + c$

where $c \equiv \lambda^2 \gamma_g (1 - \gamma_g)$

I will now characterize the equilibrium of the model with reputational ranking. For a fully informative equilibrium to exist, the following two conditions are required to be satisfied. For every i

$$\Delta R_{i,h}(\sigma_h = 1, \sigma_l = 0, q) \geqslant 0 \tag{3.20}$$

$$\Delta R_{i,l}(\sigma_h = 1, \sigma_l = 0, q) \leqslant 0. \tag{3.21}$$

The following proposition characterizes the equilibrium. Figure 5 shows the equilibrium regions.

Proposition 5 *(Characterization of Equilibrium)*
If analyst signals are conditionally correlated and the reputational ranking payoff function is non-linear as defined in (3.12), then there exists an equilibrium which can be expressed as follows:
(i) if $q \in [1 - q_{rc}, q_{rc}]$*, then there is a fully informative equilibrium,*
(ii) if either $q < 1 - q_{rc}$ *or* $q > q_{rc}$*, then the equilibrium is noninformative*

where

$$
\begin{aligned}
q_{rc} &= \underbrace{\gamma}_{\textit{Absolute Reputation Effect}} - \underbrace{\underbrace{c}_{\textit{Correlation Effect}} \underbrace{[\frac{\phi-1}{\phi(1-\gamma)+\gamma}]}_{\textit{Convexity Effect}}}_{\textit{Reputational Ranking Effect}} \in (\frac{1}{2},1) \\
\phi &= \frac{1-\gamma_g+\upsilon\gamma_g}{\gamma_g+\upsilon(1-\gamma_g)} \in (\frac{1-\gamma_g}{\gamma_g}, \frac{\gamma_g}{1-\gamma_g}) \\
c &= \lambda^2\gamma_g(1-\gamma_g).
\end{aligned}
$$

Two aspects of the equilibrium are noteworthy. First, like the equilibrium in the baseline model with a single analyst, this equilibrium *always* has two regions : fully informative and noninformative. However, the intervals of priors for which those equilibrium regions exist can be different from those in the baseline model. Also, while the equilibrium regions crucially depend on the convexity parameter υ of the reputational ranking payoff function (more on this in Proposition 6), it is interesting to note that since $q_{rc} > \frac{1}{2}$, and so, $q \in [1-q_{rc}, q_{rc}]$ is nonempty, there is always an interval of priors for which a fully informative exists. Similarly, since $q_{rc} < 1$, and so, $q \in (q_{rc}, 1)$ and $q \in (0, 1-q_{rc})$ are nonempty, there is always a noninformative equilibrium as well. Note that the subscript of q_{rc}, r means "ranking", and c means "correlation."

Second, q_{rc} – the maximum value of the market prior for which there is a fully informative equilibrium – has two components. The first component is γ, which captures the effect of each analyst's concern for maximizing his own reputation, without any concern about his reputational ranking relative to the other analyst. I call this the "absolute reputation effect". This was the only effect found in the baseline model with a single analyst. The second component captures the effect of each analyst's concern for maximizing his reputational ranking payoff. I will call this the "reputational ranking effect". This component, in turn, comprises two elements: the conditional correlation effect between analyst signals ("correlation effect"), and the convexity of the reputational ranking payoff function ("convexity effect"). If the marginal gain of being ranked higher is greater than the marginal loss of being ranked lower (*i.e.*, $\phi > 1$), then the convexity effect is positive, and $q_{rc} < \gamma$. On the other hand, if the marginal gain of being ranked higher is smaller than the marginal loss of being ranked lower (*i.e.*, $\phi < 1$), then the convexity effect is negative, and $q_{rc} > \gamma$. The correlation effect complements the convexity effect in the sense that higher values of c make the convexity effect greater, which in turn, make the overall reputational ranking effect bigger.

One important thing to note in the expression of q_{rc} is that the effect of conditional correlation between analyst signals does not depend on my assumption of perfect correlation between two "good" analysts rather than between two "bad" analysts. For the latter case, as mentioned earlier in footnote 3, the value of the correlation effect will be, $c = (1-\lambda)^2\gamma_b(1-\gamma_b)$. Since analysts do not know their talents, what is crucial here is that there is some ex-ante positive correlation between analyst signals.

Finally, there is one additional point to note : if either $\phi = 1$ or $c \to 0$ or both, then $q_{rc} = \gamma$, which implies that there exists an equilibrium in which each analyst will behave exactly the same way as the single analyst in the baseline reputation model. Note that this result is somewhat restricted compared to Proposition 4, in the sense that it cannot be claimed here that each analyst will *always* behave the same way as in the baseline model. The following corollaries formalize this intuition.

Corollary 1 *If the reputational ranking payoff function is non-linear as in (3.12), and the marginal gain of being ranked higher is the same as the marginal loss of being ranked lower (i.e., $\phi = 1$), then there exists an equilibrium in which each analyst's equilibrium forecasting behavior will be the same as that in the baseline model with a single analyst.*

Corollary 2 *If the reputational ranking payoff function is non-linear as in (3.12), and the conditional correlation between the analyst signals vanishes (in the limit), then there exists an equilibrium in which each analyst's equilibrium forecasting behavior will be the same as that in the baseline model with a single analyst.*

The implication of Corollary 1 is that even if there is a conditional correlation between analyst signals, the reputational ranking will have no effect, at least in one equilibrium, when the reputational reward for being ranked higher is exactly the same as the reputational penalty for being ranked lower. Corollary 2 can be interpreted as an extension of Proposition 4 for a simple non-linear reputational ranking payoff function.

Next, I discuss how the convexity parameter of the reputational ranking payoff function and the correlation effect influence the information content of analyst forecasts.

Proposition 6 *(Informativeness of Equilibrium)*
(i) The region of the market priors for which there is a fully informative equilibrium decreases, and the region of priors for which there is a noninformative equilibrium increases, with the convexity parameter of the reputational ranking payoff function. Formally, q_{rc} is decreasing in υ
(ii) The effect of convexity of the reputational ranking payoff function is accentuated with the correlation effect between the analyst signals. Formally, if $\upsilon > 1$, then q_{rc} is decreasing in c; however, if $\upsilon < 1$, then q_{rc} is increasing in c.

Proposition 6 is the main result of this chapter. It states that the greater the marginal gain of being ranked higher compared to the marginal loss of being ranked lower (*i.e.*, greater values of ϕ, which, in turn, imply higher values of υ, by Lemma 5), the lower the informativeness of analyst forecasts. Furthermore, this convexity effect of the reputational payoff function has greater impact on the information content of analyst forecasts with higher values of the correlation effect between the analyst signals. Recall that the marginal gain of being ranked higher is the difference in analyst payoffs between being ranked higher than and being ranked the same as the other analyst. Similarly, the marginal loss of being ranked lower is the payoff difference between being ranked the same as and being ranked lower than the other analyst.

To understand the implication of this result, suppose that the analysts are compensated in a way such that if an analyst is ranked higher than his peer, he is rewarded by a certain amount; however if the same analyst is ranked lower than the other analyst, he is *not* penalized as much as he was rewarded. The popular press often suggests that while the sell-side analysts are paid huge bonuses when they are ranked higher ("All Star" status), they are not sufficiently penalized when they fall down the ranks (unless they are fired). Proposition 6 implies that such a lopsided compensation structure can potentially incentivize sell-side analysts to reveal less of their private information to the market. This result also states that if the marginal reward for being ranked higher is smaller than the marginal penalty for being ranked lower, then analyst forecasts can have greater information content.

To understand the intuition of this result, note that each analyst – with the objective of maximizing his expected reputational ranking payoff – wants to be ranked higher by differentiating himself from the other analyst (*i.e.*, by issuing a different forecast).

However, this differentiation helps only when his forecast matches the reported earnings, and the other analyst's forecast does not. If, to the contrary, the analyst's forecast does not match the reported earnings and the other analyst's forecast does, then the first analyst is ranked lower, and the differentiation does not help. As such, each analyst wants to maximize the likelihood of being ranked higher by differentiating himself from the other analyst, and minimize the likelihood of being ranked lower by moving with (*i.e.*, not differentiating from) the other analyst.

Note also that the analysts are making simultaneous forecasts, without the knowledge of the other analyst's signal and forecast. Assuming symmetric strategy, and knowing that analyst signals are *ex-ante* (conditionally) positively correlated, the best strategy for each analyst – to differentiate himself from the other analyst – is to forecast against his own signal. As such, to increase the likelihood of being ranked higher, each analyst will forecast against his own signal. However, to decrease the likelihood of being ranked lower, each analyst will tend to move with the other analyst by honestly forecasting his own signal.

On balance, the optimal strategy of each analyst – forecasting against or in the direction of his signal – depends on the relative payoff of being ranked higher or lower than the other analyst. If the marginal gain of being ranked higher (than being ranked the same as the other analyst) is greater than the marginal loss of being ranked lower (than being ranked the same), then the optimal strategy of each analyst will be to issue a forecast different from his private signal, thereby reducing the informativeness of his forecast in equilibrium. On the other hand, if the reward for being ranked higher is smaller than the penalty for being ranked lower, then the optimal strategy of each analyst will be to honestly forecast his own signal, thereby leading to a fully informative equilibrium.

3.7 Summary of Results

In this chapter I considered a model in which two analysts are making simultaneous forecasts about a company's earnings, each trying to be ranked higher than the other in terms of their reputations in the market. In order to focus primarily on the competition between the analysts for higher reputational ranking payoffs, I considered only the reputational concerns of the analysts. There is no short-term price impact motive in this model.

I considered cases in which analyst signals are either conditionally independent, or conditionally correlated. In the first case, I found that if analyst signals are conditionally independent, and the reputational ranking payoff function is linear, then reputational ranking has no effect, in the sense that each analyst will behave exactly the same way as the analyst in the baseline reputation model, in which a single analyst is maximizing his expected reputation in the market.

In the second case of a simple non-linear reputational ranking payoff function, I found that if analyst signals are conditionally correlated, the reputational ranking does have an impact. The main result of this chapter is that the higher the convexity of the reputational ranking payoff function is, the lower the information content of an analyst forecast. The convexity of the payoff function is represented by the ratio of the marginal gain of being ranked higher (than being ranked the same as the other analyst) to the marginal loss of being ranked lower (than being ranked the same).

In a related result, I showed that even if analyst signals are conditionally correlated, if the marginal gain of being ranked higher is the same as the marginal loss of being ranked lower, then there is at least one equilibrium in which each analyst will behave exactly the same way as the analyst in the baseline model – there is no reputational ranking effect.

Chapter 4

Conclusions and Future Research

In the empirical accounting and finance literature, analyst forecasts are commonly used as proxies for investors' earnings expectations. The implicit assumption is that analyst forecasts truthfully reveal analysts' private information. In this paper, I demonstrated that given an analyst's incentives to maximize his short-term price-impact profits and his long-term reputational value in the labor market, his forecasts often fall short of the truthful revelation of his private information. Indeed, a fully informative equilibrium exists only at the intermediate values of the market's prior expectation of a company's earnings. At extreme values of the prior, reputational concerns create a "conformist" bias – an analyst will tend to bias his forecast towards the market's prior – leading to no information revelation in equilibrium.

The perverse effect of reputational concerns worsens if analysts compete for a better reputational ranking in the market, and the reputational payoff structure is convex. In particular, an analyst compensation structure in which the reward for being ranked higher is greater than the penalty for being ranked lower can potentially impair the information content of analyst forecasts. It is often suggested in the popular press that on Wall Street, "All Star" analysts are paid substantially higher than their average counterparts, yet analysts are not penalized accordingly if they rank lower (conditional on the analyst not being fired). My result suggests that such convexity in a reputational payoff structure can influence analysts to reveal less information in equilibrium.

A short-term price-impact motive alone can also create a perverse incentive. For a single analyst solely concerned with maximizing his price-impact profits, honest forecasting is impossible. However, interestingly, the consideration of price-impact profits and reputational concerns in conjunction provides a stronger inducement to report honestly than either incentive in isolation. This result underscores the role of the short-term price-impact incentive in improving the information content of analyst forecasts.

There are several directions in which the stylized models in this paper can be extended. One useful extension would be an analysis of a continuous signal and message space for the analysts. When an expert's only objective is to maximize his reputational value in the labor market, it has been shown, for a continuous signal space, that a fully informative equilibrium cannot exist (Ottaviani and Sorensen 2006a). However, what the equilibrium would be remains an open question, except for a very restrictive case with a multiplicative linear signal structure (Ottaviani and Sorensen 2006b).

Another useful extension would be the analysis of an endogenous reputation in a dynamic setting in which the market's posterior belief of an analyst's talent would be used as the prior reputation in the next period. In his seminal paper on managerial reputation, Holmstrom (1999) considers a case in which a manager's productive ability – initially unknown to the manager and the market – is revealed over time through the manager's choice of effort in equilibrium. In cases closer to my model, in which there is no effort choice by an expert, typically one type of expert is assumed to be "honest", *i.e.*, a 'crazy' type who always reveals his private information honestly (*e.g.*, Kreps and Wilson 1982; Benabou and Laroque 1992). In my model, the honesty of an analyst is endogenous and, hence, cannot be assumed. Moreover, integrating the comparison of one expert's reputation relative to others adds further complexity to my model in a dynamic setting.

References

[1] Benabou, R., and G. Laroque. 1992. Using privileged information to manipulate markets: Insiders, gurus, and credibility. *Quarterly Journal of Economics* August:921-958.

[2] Bernhardt, B., M. Campello, and E. Kutsaoti. 2004. Analyst compensation and forecast bias. *Working Paper*, University of Illinois.

[3] Beyer, A., and I. Guttman. 2011. The effect of trading volume on analysts' forecast bias. *Accounting Review*, 86(2):451-481.

[4] Crawford, V., and J. Sobel. 1982. Strategic information transmission. *Econometrica* 50:1431-1452.

[5] Dasgupta, A., and A. Prat. 2008. Information aggregation in financial markets with career concerns. *Journal of Economic Theory* 143:83-113.

[6] Effinger, M., and M. Polborn. 2001. Herding and anti-herding: A model of reputational differentiation. *European Economic Review* 45:385-403.

[7] Graham, J. 1999. Herding among investment newsletters: Theory and evidence. *Journal of Finance* February:237-268.

[8] Holmstrom, B. 1982. Managerial incentive problems: A dynamic perspective. Reprinted in *Review of Economic Studies(1999)* 66:169-182.

[9] Hong, H., and J. Kubik. 2003. Analyzing the analysts: Career concerns and biased earnings forecasts. *Journal of Finance* 58:313-351.

[10] Jackson, A. 2005. Trade generation, reputation, and sell-side analysts. *Journal of Finance* April:673-717.

[11] Kreps, D., and R. Wilson. 1982. Reputation and imperfect information. *Journal of Economic Theory* 27:253-279.

[12] Leone, A., and J. Wu. 2007. What does it take to become a superstar? Evidence from institutional investor rankings of financial analysts. *Working Paper*, University of Rochester.

[13] Lim, T. 2001. Rationality and analysts' forecasting bias. *Journal of Finance*, 56:369-385.

[14] Mikhail, M., B. Walther, and R. Willis. 1999. Does forecast accuracy matter to security analysts? *Accounting Review* 74(2): 185-200.

[15] Morgan, J., and P. C. Stocken. 2003. An analysis of stock recommendations. *RAND Journal of Economics* 34(1):183-203.

[16] Ottaviani, M., and P. Sorensen. 2001. Information aggregation in debate : Who should speak first? *Journal of Public Economics* 81 : 393-421.

[17] Ottaviani, M., and P. Sorensen. 2006a. Reputational cheap talk. *RAND Journal of Economics* 37(1):155-175.

[18] Ottaviani, M., and P. Sorensen. 2006b. Professional advice. *Journal of Economic Theory* 126:120-142.

[19] Park, C., and E. Stice. 2000. Analyst forecasting ability and the stock price reaction to forecast revisions. *Review of Accounting Studies* 5:259-272

[20] Prendergast, C., and L. Stole. 1996. Impetuous youngsters and jaded old-timers: Acquiring a reputation of learning. *Journal of Political Economy* 104:1105-1134.

[21] Richardson, S.,S. Teoh and P. Wysocki. 2004. The walkdown to beatable analysts' forecasts: the role of equity issuance and insider trading incentives. *Contemporary Accounting Research* 21:885-924.

[22] Scharfstein, D. S., and J. C. Stein. 1990. Herd behavior and investment. *American Economic Review* June:465-479.

[23] Stickel, S. 1992. Reputation and performance among security analysts. *Journal of Finance* 47(5):1811-1836.

[24] Trueman, B. 1994. Analyst forecasts and herding behavior. *Review of Financial Studies* 7(1):97-124.

Appendix A

Calculation of Prices and Analyst Reputation

A.1 Calculation of Prices

The price of the company's stock after the analyst makes a high forecast is

$$P(\hat{s}^h) \equiv E[x|\hat{s}^h] = x^H \Pr(x^H|\hat{s}^h) + x^L \Pr(x^L|\hat{s}^h)$$

By Bayes' rule, $\Pr(x^H|\hat{s}^h) = \left[\frac{\Pr(x^H)\Pr(\hat{s}^h|x^H)}{\Pr(x^H)\Pr(\hat{s}^h|x^H)+\Pr(x^L)\Pr(\hat{s}^h|x^L)}\right]$.
Also, $\Pr(\hat{s}^h|x^H) = \Pr(\hat{s}^h|s^h)\Pr(s^h|x^H) + \Pr(\hat{s}^h|s^l)\Pr(s^l|x^H) = \sigma_h\gamma + \sigma_l(1-\gamma)$.
Similarly, $\Pr(\hat{s}^h|x^L) = \sigma_h(1-\gamma) + \sigma_l\gamma$.
Thus

$$\Pr(x^H|\hat{s}^h) = \frac{[\sigma_h\gamma + \sigma_l(1-\gamma)]q}{[\sigma_h\gamma + \sigma_l(1-\gamma)]q + [\sigma_h(1-\gamma) + \sigma_l\gamma](1-q)}.$$

Similarly

$$\Pr(x^L|\hat{s}^h) = \frac{[\sigma_h(1-\gamma) + \sigma_l\gamma](1-q)}{[\sigma_h\gamma + \sigma_l(1-\gamma)]q + [\sigma_h(1-\gamma) + \sigma_l\gamma](h-q)}.$$

Taken together

$$P(\hat{s}^h) = \frac{x^H[\sigma_h\gamma + \sigma_l(1-\gamma)]q + x^L[\sigma_h(1-\gamma) + \sigma_l\gamma](1-q)}{[\sigma_h\gamma + \sigma_l(1-\gamma)]q + [\sigma_h(1-\gamma) + \sigma_l\gamma](1-q)}.$$

The price for a low forecast, $P(\hat{s}^l)$, has been calculated in a similar way.

A.2 Single Analyst Reputation

Proof of Property 1. I will prove part (i). Part (ii) can be proved by an analogous argument. Using the expressions of reputation

$$\begin{aligned}
\Pr(g|\hat{s}^h, x^H) - \Pr(g|\hat{s}^l, x^H) &= \lambda \frac{\gamma_g \sigma_h + (1-\gamma_g)\sigma_l}{\gamma\sigma_h + (1-\gamma)\sigma_l} \\
&\quad - \frac{\gamma_g(1-\sigma_h) + (1-\gamma_g)(1-\sigma_l)}{\gamma(1-\sigma_h) + (1-\gamma)(1-\sigma_l)} \\
&= \lambda \left[\frac{N}{\{\gamma\sigma_h + (1-\gamma)\sigma_l\}\{\gamma(1-\sigma_h) + (1-\gamma)(1-\sigma_l)\}} \right]
\end{aligned}$$

where, analyst's average signal precision, $\gamma = \lambda\gamma_g + (1-\lambda)\gamma_b$ and the numerator

$$\begin{aligned}
N &\equiv \{\gamma_g\sigma_h + (1-\gamma_g)\sigma_l\}\{\gamma(1-\sigma_h) + (1-\gamma)(1-\sigma_l)\} \\
&\quad -\{\gamma_g(1-\sigma_h) + (1-\gamma_g)(1-\sigma_l)\}\{\gamma\sigma_h + (1-\gamma)\sigma_l\}.
\end{aligned}$$

After some algebraic manipulations, $N = (1-\lambda)(\gamma_g - \gamma_b)(\sigma_h - \sigma_l) \geqslant 0$ because $\sigma_h \geqslant \sigma_l$. ■

A.3 Reputation with Multiple Analysts

Proof of Property 2. I will prove only the first case. The rest can be proved by an analogous argument. Without loss of generality, fix $i = A$.

$$\begin{aligned}
&\Pr(g_A|\hat{s}_A^h, \hat{s}_B^h, x^H) \\
&= \frac{\Pr(g_A, g_B|\hat{s}_A^h, \hat{s}_B^h, x^H) + \Pr(g_A, b_B|\hat{s}_A^h, \hat{s}_B^h, x^H)}{\sum_{\theta_A}\sum_{\theta_B} \Pr(\theta_A, \theta_B|\hat{s}_A^h, \hat{s}_B^h, x^H)} \\
&= \frac{\frac{\Pr(g_A,g_B)\Pr(\hat{s}_A^h,\hat{s}_B^h|x^H,g_A,g_B)}{\Pr(\hat{s}_A^h,\hat{s}_B^h|x^H)} +}{\frac{\Pr(g_A,g_B)\Pr(\hat{s}_A^h,\hat{s}_B^h|x^H,g_A,g_B)}{\Pr(\hat{s}_A^h,\hat{s}_B^h|x^H)} + \frac{\Pr(g_A,b_B)\Pr(\hat{s}_A^h,\hat{s}_B^h|x^H,g_A,b_B)}{\Pr(\hat{s}_A^h,\hat{s}_B^h|x^H)} +} \\
&\quad \frac{\frac{\Pr(g_A,b_B)\Pr(\hat{s}_A^h,\hat{s}_B^h|x^H,g_A,b_B)}{\Pr(\hat{s}_A^h,\hat{s}_B^h|x^H)}}{\frac{\Pr(b_A,g_B)\Pr(\hat{s}_A^h,\hat{s}_B^h|x^H,b_A,g_B)}{\Pr(\hat{s}_A^h,\hat{s}_B^h|x^H)} + \frac{\Pr(b_A,b_B)\Pr(\hat{s}_A^h,\hat{s}_B^h|x^H,b_A,b_B)}{\Pr(\hat{s}_A^h,\hat{s}_B^h|x^H)}}
\end{aligned}$$

Continuing from the previous page,

$$
\begin{aligned}
&\Pr(g_A|\hat{s}_A^h, \hat{s}_B^h, x^H) \\
&= \frac{\Pr(g_A, g_B)\Pr(\hat{s}_A^h, \hat{s}_B^h|x^H, g_A, g_B) +}{\Pr(g_A, g_B)\Pr(\hat{s}_A^h, \hat{s}_B^h|x^H, g_A, g_B) + \Pr(g_A, b_B)\Pr(\hat{s}_A^h, \hat{s}_B^h|x^H, g_A, b_B) +} \\
&\quad \frac{\Pr(g_A, b_B)\Pr(\hat{s}_A^h, \hat{s}_B^h|x^H, g_A, b_B)}{\Pr(b_A, g_B)\Pr(\hat{s}_A^h, \hat{s}_B^h|x^H, b_A, g_B) + \Pr(b_A, b_B)\Pr(\hat{s}_A^h, \hat{s}_B^h|x^H, b_A, b_B)} \\
&= \frac{\text{Numerator}}{\text{Denominator}}
\end{aligned}
$$

Also, since analysts are playing symmetric strategies (*i.e.*, $\sigma_{A,h} = \sigma_{B,h} \equiv \sigma_h$ and $\sigma_{A,l} = \sigma_{B,l} \equiv \sigma_l$), for every $\theta \in \{g, b\}$

$$
\begin{aligned}
&\Pr(\hat{s}_A^h, \hat{s}_B^h|x^H, \theta_A, \theta_B) = \Pr(s_A^h, s_B^h|x^H, \theta_A, \theta_B)\sigma_h^2 + \Pr(s_A^h, s_B^l|x^H, \theta_A, \theta_B)\sigma_h\sigma_l + \\
&\Pr(s_A^l, s_B^h|x^H, \theta_A, \theta_B)\sigma_l\sigma_h + \Pr(s_A^l, s_B^l|x^H, \theta_A, \theta_B)\sigma_l^2
\end{aligned}
$$

Thus

$$
\begin{aligned}
&\text{Denominator} \\
&= \Pr(g_A, g_B)\Pr(\hat{s}_A^h, \hat{s}_B^h|x^H, g_A, g_B) + \Pr(g_A, b_B)\Pr(\hat{s}_A^h, \hat{s}_B^h|x^H, g_A, b_B) + \\
&\quad \Pr(b_A, g_B)\Pr(\hat{s}_A^h, \hat{s}_B^h|x^H, b_A, g_B) + \Pr(b_A, b_B)\Pr(\hat{s}_A^h, \hat{s}_B^h|x^H, b_A, b_B) \\
&= \lambda^2[\Pr(s_A^h, s_B^h|x^H, g_A, g_B)\sigma_h^2 + \Pr(s_A^h, s_B^l|x^H, g_A, g_B)\sigma_h\sigma_l + \\
&\quad \Pr(s_A^l, s_B^h|x^H, g_A, g_B)\sigma_l\sigma_h + \Pr(s_A^l, s_B^l|x^H, g_A, g_B)\sigma_l^2] \\
&\quad +\lambda(1-\lambda)[\Pr(s_A^h, s_B^h|x^H, g_A, b_B)\sigma_h^2 + \Pr(s_A^h, s_B^l|x^H, g_A, b_B)\sigma_h\sigma_l + \\
&\quad \Pr(s_A^l, s_B^h|x^H, g_A, b_B)\sigma_l\sigma_h + \Pr(s_A^l, s_B^l|x^H, g_A, b_B)\sigma_l^2] \\
&\quad +(1-\lambda)\lambda[\Pr(s_A^h, s_B^h|x^H, b_A, g_B)\sigma_h^2 + \Pr(s_A^h, s_B^l|x^H, b_A, g_B)\sigma_h\sigma_l + \\
&\quad \Pr(s_A^l, s_B^h|x^H, b_A, g_B)\sigma_l\sigma_h + \Pr(s_A^l, s_B^l|x^H, b_A, g_B)\sigma_l^2] \\
&\quad +(1-\lambda)^2[\Pr(s_A^h, s_B^h|x^H, b_A, b_B)\sigma_h^2 + \Pr(s_A^h, s_B^l|x^H, b_A, b_B)\sigma_h\sigma_l + \\
&\quad \Pr(s_A^l, s_B^h|x^H, b_A, b_B)\sigma_l\sigma_h + \Pr(s_A^l, s_B^l|x^H, b_A, b_B)\sigma_l^2] \qquad \text{(A.1)}
\end{aligned}
$$

By the assumption that signals of two good analysts are perfectly positively correlated

$$
\begin{aligned}
\Pr(s_A^h, s_B^h|x^H, g_A, g_B) &= \gamma_g \\
\Pr(s_A^l, s_B^l|x^H, g_A, g_B) &= 1 - \gamma_g \\
\Pr(s_A^h, s_B^l|x^H, g_A, g_B) &= \Pr(s_A^l, s_B^h|x^H, g_A, g_B) = 0
\end{aligned}
$$

If at least one of the analysts is bad, their signals are conditionally independent

$$\begin{aligned}
\Pr(s_A^h, s_B^h | x^H, g_A, b_B) &= \gamma_g \gamma_b = \frac{\gamma_g}{2} \\
\Pr(s_A^l, s_B^l | x^H, g_A, b_B) &= (1-\gamma_g)(1-\gamma_b) = \frac{1-\gamma_g}{2} \\
\Pr(s_A^h, s_B^h | x^H, b_A, b_B) &= \gamma_b \gamma_b = \frac{1}{4} \\
\Pr(s_A^l, s_B^l | x^H, b_A, b_B) &= (1-\gamma_b)(1-\gamma_b) = \frac{1}{4}
\end{aligned}$$

Replacing these values in (A.1), and after some algebra and collecting terms

$$\begin{aligned}
& \text{Denominator} \\
= \; & \sigma_h^2\gamma^2 + 2\sigma_h\sigma_l\gamma(1-\gamma) + \sigma_l^2(1-\gamma)^2 + \lambda^2\gamma_g(1-\gamma_g)\{\sigma_h^2 - 2\sigma_h\sigma_l + \sigma_l^2\} \\
= \; & \{\gamma\sigma_h + (1-\gamma)\sigma_l\}^2 + c(\sigma_h - \sigma_l)^2 \qquad \text{(A.2)}
\end{aligned}$$

Similarly, after some algebra

$$\begin{aligned}
& \text{Numerator} \\
= \; & \Pr(g_A, g_B)\Pr(\hat{s}_A^h, \hat{s}_B^h | x^H, g_A, g_B) + \Pr(g_A, b_B)\Pr(\hat{s}_A^h, \hat{s}_B^h | x^H, g_A, b_B) \\
= \; & \lambda^2[\Pr(s_A^h, s_B^h | x^H, g_A, g_B)\sigma_h^2 + \Pr(s_A^h, s_B^l | x^H, g_A, g_B)\sigma_h\sigma_l + \\
& \Pr(s_A^l, s_B^h | x^H, g_A, g_B)\sigma_l\sigma_h + \Pr(s_A^l, s_B^l | x^H, g_A, g_B)\sigma_l^2] + \\
& \lambda(1-\lambda)[\Pr(s_A^h, s_B^h | x^H, g_A, b_B)\sigma_h^2 + \Pr(s_A^h, s_B^l | x^H, g_A, b_B)\sigma_h\sigma_l + \\
& \Pr(s_A^l, s_B^h | x^H, g_A, b_B)\sigma_l\sigma_h + \Pr(s_A^l, s_B^l | x^H, g_A, b_B)\sigma_l^2] \\
= \; & \lambda\left[\sigma_h\gamma(\{\gamma_g\sigma_h + (1-\gamma_g)\sigma_l\} + (1-\gamma)\sigma_l\{\gamma_g\sigma_h + (1-\gamma_g)\sigma_l\}\right] \\
& +\lambda\gamma_g(1-\gamma_g)(\sigma_h - \sigma_l)^2 \\
= \; & \lambda\left[\{\gamma_g\sigma_h + (1-\gamma_g)\sigma_l\}\{\gamma\sigma_h + (1-\gamma)\sigma_l\} + \frac{c}{\lambda}(\sigma_h - \sigma_l)^2\right] \qquad \text{(A.3)}
\end{aligned}$$

Thus, by (A.2) and (A.3)

$$\begin{aligned}
\Pr(g_A | \hat{s}_A^h, \hat{s}_B^h, x^H) &= \frac{\text{Numerator}}{\text{Denominator}} \\
&= \lambda\left[\frac{\{\gamma_g\sigma_h + (1-\gamma_g)\sigma_l\}\{\gamma\sigma_h + (1-\gamma)\sigma_l\} + \frac{c}{\lambda}(\sigma_h - \sigma_l)^2}{\{\gamma\sigma_h + (1-\gamma)\sigma_l\}^2 + c(\sigma_h - \sigma_l)^2}\right] \\
&= \lambda\left[\frac{\{\gamma_g\sigma_h + (1-\gamma_g)\sigma_l\}\Sigma_1 + \frac{c}{\lambda}(\sigma_h - \sigma_l)^2}{\{\Sigma_1\}^2 + c(\sigma_h - \sigma_l)^2}\right]
\end{aligned}$$

where $\Sigma_1 = \gamma\sigma_h + (1-\gamma)\sigma_l$. ■

Proof of Property 5. By definition

$$\begin{aligned}
&\Pr(s_A, s_B|x) \qquad\qquad \text{(A.4)}\\
&= \sum_{\theta_A}\sum_{\theta_B} \Pr(\theta_A,\theta_B)\Pr(s_A,s_B|x,\theta_A,\theta_B)\\
&= \Pr(g_A,g_B)\Pr(s_A,s_B|x,g_A,g_B) + \Pr(g_A,b_B)\Pr(s_A,s_B|x,g_A,b_B) +\\
&\quad \Pr(b_A,g_B)\Pr(s_A,s_B|x,b_A,g_B) + \Pr(b_A,b_B)\Pr(s_A,s_B|x,b_A,b_B)\\
&= \lambda^2\Pr(s_A,s_B|x,g_A,g_B) + \lambda(1-\lambda)\Pr(s_A|x,g_A)\Pr(s_B|x,b_B) +\\
&\quad (1-\lambda)\lambda\Pr(s_A|x,b_A)\Pr(s_B|x,g_B) + (1-\lambda)^2\Pr(s_A|x,b_A)\Pr(s_B|x,b_B)
\end{aligned}$$

The last three terms of the third equality follow from the assumption that the signals of analysts are conditionally independent if at least one of the analysts is bad. The first term involves the correlation of signals between two good analysts. By assumption

$$\Pr(s_A,s_B|x,g_A,g_B) = \gamma_g \text{ if } s_A = s_B = x \qquad \text{(A.5)}$$

$$\Pr(s_A,s_B|x,g_A,g_B) = 1-\gamma_g \text{ if } s_A = s_B \neq x \qquad \text{(A.6)}$$

$$\Pr(s_A,s_B|x,g_A,g_B) = 0 \text{ if } s_A \neq s_B \qquad \text{(A.7)}$$

I will first prove part (ii) of Property 5. From (A.4)

$$\begin{aligned}
&\Pr(s_A^h, s_B^l|x^H)\\
&= \lambda^2\Pr(s_A^h,s_B^l|x^H,g_A,g_B) + \lambda(1-\lambda)\Pr(s_A^h|x^H,g_A)\Pr(s_B^l|x^H,b_B) +\\
&\quad (1-\lambda)\lambda\Pr(s_A^h|x^H,b_A)\Pr(s_B^l|x^H,g_B) + (1-\lambda)^2\Pr(s_A^h|x^H,b_A)\Pr(s_B^l|x^H,b_B)\\
&= \lambda(1-\lambda)\gamma_g(1-\gamma_b) + (1-\lambda)\lambda\gamma_b(1-\gamma_g) + (1-\lambda)^2\gamma_b(1-\gamma_b)\\
&= \lambda^2\gamma_g(1-\gamma_g) + \lambda(1-\lambda)\gamma_g(1-\gamma_b) + (1-\lambda)\lambda\gamma_b(1-\gamma_g) +\\
&\quad (1-\lambda)^2\gamma_b(1-\gamma_b) - \lambda^2\gamma_g(1-\gamma_g)\\
&= \gamma(1-\gamma) - c
\end{aligned}$$

where $c \equiv \lambda^2\gamma_g(1-\gamma_g)$. In the second equality above, $\Pr(s_A^h, s_B^l|x^H, g_A, g_B) = 0$ using (A.7). In general, it can be shown that

$$\Pr(s_A = x, s_B \neq x \mid x) = \gamma(1-\gamma) - c.$$

This proves part (ii).

Again from (A.4), and using (A.5), part (i) can be derived more generally as

$$\Pr(s_A = x, s_B = x \mid x) = \gamma^2 + c$$

Similarly, by (A.4) and (A.6), part (iii) can be derived more generally as

$$\Pr(s_A \neq x, s_B \neq x \mid x) = (1-\gamma)^2 + c$$

Note also that $\Pr(s_A|s_B)$ can be calculated as follows

$$\Pr(s_A|s_B) = \Pr(x^H|s_B)\Pr(s_A|s_B, x^H) + \Pr(x^L|s_B)\Pr(s_A|s_B, x^L)$$

where $\Pr(x|s_B)$ and $\Pr(s_A|s_B, x)$ can be calculated as follows

$$\Pr(x|s_B) = \frac{\Pr(x)\Pr(s_B|x)}{\Pr(s_B)}$$

$$\Pr(s_A|s_B, x) = \frac{\Pr(s_A, s_B|x)}{\Pr(s_B|x)}.$$

■

Appendix B

Proofs

B.1 Single Analyst Case

Proof of Lemma 2. Since by definition, $\gamma = \lambda\gamma_g + (1-\lambda)\gamma_b = \frac{1+\lambda(2\gamma_g-1)}{2}$, which is increasing in λ, it is enough to show that $\frac{\partial\pi(\hat{s})}{\partial\gamma} \geqslant 0$. I will show here the case for $\frac{\partial\pi(\hat{s}^h)}{\partial\gamma} \geqslant 0$. The case for $\pi(\hat{s}^l)$ can be shown analogously.

For every $\sigma_h \geqslant \sigma_l$

$$\begin{aligned} \pi(\hat{s}^h) &= \left|\frac{(2\gamma-1)q(1-q)(\sigma_h-\sigma_l)}{\sigma_h[\gamma q+(1-\gamma)(1-q)]+\sigma_l[(1-\gamma)q+\gamma(1-q)]}\right| \\ &= \frac{(2\gamma-1)q(1-q)(\sigma_h-\sigma_l)}{\sigma_h[\gamma q+(1-\gamma)(1-q)]+\sigma_l[(1-\gamma)q+\gamma(1-q)]}. \end{aligned}$$

Differentiating $\pi(\hat{s}^h)$ with respect to γ

$$\begin{aligned} \frac{\partial\pi(\hat{s}^h)}{\partial\gamma} &= q(1-q)(\sigma_h-\sigma_l)\frac{\partial}{\partial\gamma}\left[\frac{(2\gamma-1)}{\sigma_h[\gamma q+(1-\gamma)(1-q)]+\sigma_l[(1-\gamma)q+\gamma(1-q)]}\right] \\ &= q(1-q)(\sigma_h-\sigma_l)\left[\frac{N}{\{\sigma_h[\gamma q+(1-\gamma)(1-q)]+\sigma_l[(1-\gamma)q+\gamma(1-q)]\}^2}\right] \end{aligned}$$

where

$$\begin{aligned} N \equiv\ & 2\sigma_h[\gamma q+(1-\gamma)(1-q)]+2\sigma_l[(1-\gamma)q+\gamma(1-q)]- \\ & (2\gamma-1)[\sigma_h(q-(1-q))+\sigma_l(-q+(1-q))] \end{aligned}$$

Thus

$$sgn(\frac{\partial\pi(\hat{s}^h)}{\partial\gamma}) = sgn(N)$$

Collecting terms and simplifying N (numerator)

$$\begin{aligned} N = \sigma_h[2\{\gamma q + (1-\gamma)(1-q)\} - (2\gamma-1)\{q-(1-q)\}] \\ +\sigma_l[2\{(1-\gamma)q + \gamma(1-q)\} - (2\gamma-1)\{-q+(1-q)\}] \\ = \quad \sigma_h[2\gamma q + 2(1-\gamma)(1-q) - (2\gamma-1)(2q-1)] \\ +\sigma_l[2(1-\gamma)q + 2\gamma(1-q) - (2\gamma-1)(1-2q)] \\ = \quad \sigma_h + \sigma_l \geqslant 0 \end{aligned}$$

Thus $\frac{\partial \pi(\hat{s}^h)}{\partial \gamma} \geqslant 0$. ■

Proof of Lemma 3. I will prove this lemma by contradiction. Suppose that there exists an interval $q \in (0,1)$ for which a fully informative equilibrium exists. For the putative equilibrium to exist, the following inequalities have to be satisfied

$$\begin{aligned} \pi(\hat{s}^h) &\geqslant \pi(\hat{s}^l) \\ \pi(\hat{s}^h) &\leqslant \pi(\hat{s}^l) \end{aligned}$$

However, both the inequalities can hold simultaneously only if

$$\pi(\hat{s}^h) = \pi(\hat{s}^l)$$

Now, given the market's conjecture of a fully informative equilibrium (*i.e.*, $\sigma_h = 1,\ \sigma_l = 0$), equations (2.12) lead to

$$\begin{aligned} \pi(\hat{s}^h;\ \sigma_h = 1, \sigma_l = 0) = \frac{(x^H - x^L)q(1-q)(2\gamma-1)}{[\gamma q + (1-\gamma)(1-q)]} \\ \pi(\hat{s}^l;\ \sigma_h = 1, \sigma_l = 0) = \frac{(x^H - x^L)q(1-q)(2\gamma-1)}{[(1-\gamma)q + \gamma(1-q)]} \end{aligned}$$

Now

$$\begin{aligned} \pi(\hat{s}^h;\ \sigma_h &= 1, \sigma_l = 0) = \pi(\hat{s}^l;\ \sigma_h = 1, \sigma_l = 0) \\ &\Rightarrow q = \frac{1}{2} \end{aligned} \tag{B.1}$$

Thus, for a fully informative equilibrium to exist, it has to be true that $q = \frac{1}{2}$, which is not an interval, as assumed. This leads to a contradiction. Therefore, there does not exist any fully informative equilibrium at any interval of prior of earnings.

Furthermore, $\pi(\hat{s}^h;\ \sigma_h = 1, \sigma_l = 0) \geqslant \pi(\hat{s}^l;\ \sigma_h = 1, \sigma_l = 0)$ if $q \leqslant \frac{1}{2}$, which means that given the market's naïve conjecture of a fully informative equilibrium, the analyst has an incentive to issue a high forecast if the prior is pessimistic. Similarly, $\pi(\hat{s}^l;\ \sigma_h = 1, \sigma_l = 0) > \pi(\hat{s}^h;\ \sigma_h = 1, \sigma_l = 0)$ if $q > \frac{1}{2}$. ■

Proof of Proposition 1. Consider the case for $0 < q < \frac{1}{2}$. Suppose that the market conjectures that the analyst's strategy is $(\sigma_h = 1,\ \sigma_l \in (0, 1))$. I will show that the conjectured pair (σ_h, σ_l) is consistent in equilibrium. Two inequalities that need to hold for this equilibrium to exist are (i) $\pi(\hat{s}^h) \geqslant \pi(\hat{s}^l)$ and (ii) $\pi(\hat{s}^h) = \pi(\hat{s}^l)$. For both of these inequalities to be satisfied simultaneously, I need

$$\begin{aligned}
& \pi(\hat{s}^h) = \pi(\hat{s}^l) \\
\Rightarrow\ & |\frac{(x^H - x^L)q(1-q)(2\gamma-1)(1-\sigma_l)}{[\gamma q + (1-\gamma)(1-q)] + \sigma_l[(1-\gamma)q + \gamma(1-q)]}| \\
=\ & |\frac{(x^H - x^L)q(1-q)(2\gamma-1)(1-\sigma_l)}{(1-\sigma_l)[(1-\gamma)q + \gamma(1-q)]} \\
\Rightarrow\ & [\gamma q + (1-\gamma)(1-q)] + \sigma_l[(1-\gamma)q + \gamma(1-q)] = (1-\sigma_l)[(1-\gamma)q + \gamma(1-q)] \\
\Rightarrow\ & q[\sigma_h\gamma + \sigma_l(1-\gamma)] + (1-q)[\sigma_h(1-\gamma) + \sigma_l\gamma] = 1/2 \\
\Rightarrow\ & \sigma_l = \frac{(2\gamma-1)(1-2q)}{2\{q(1-\gamma) + (1-q)\gamma\}}.
\end{aligned} \tag{B.2}$$

Note that since $0 < q < \frac{1}{2}$, $\sigma_l > 0$. Also, $q(1-\gamma) + (1-q)\gamma > \frac{1}{2}$ because $q < \frac{1}{2}$. So, $\sigma_l < (2\gamma - 1)(1 - 2q)$. Furthermore, $(2\gamma - 1) < 1$ because $\gamma < 1$, and $(1 - 2q) < 1$ because $q > 0$. Thus, $\sigma_l < (2\gamma - 1)(1 - 2q) < 1$. Taken together $\sigma_l \in (0, 1)$. Thus, if $0 < q < \frac{1}{2}$, then the strategy pair $(\sigma_h = 1,\ \sigma_l \in (0, 1))$ is consistent in equilibrium. This proves the existence of the equilibrium.

Now differentiating σ_l in (B.2) with respect to q

$$\begin{aligned}
\frac{\partial\sigma_l}{\partial q} &= \frac{\partial}{\partial q}[\frac{(2\gamma-1)(1-2q)}{2\{q(1-\gamma) + (1-q)\gamma\}}] \\
&= \frac{(2\gamma-1)}{2}[\frac{-2\{q(1-\gamma) + (1-q)\gamma\} - (1-2q)\{(1-\gamma) - \gamma\}}{\{q(1-\gamma) + (1-q)\gamma\}^2}] \\
&= \frac{(2\gamma-1)}{2}[\frac{1}{\{q(1-\gamma) + (1-q)\gamma\}^2}][-2\{q(1-2\gamma) + \gamma\} - (1-2q)(1-2\gamma)] \\
&= \frac{(2\gamma-1)}{2}[\frac{1}{\{q(1-\gamma) + (1-q)\gamma\}^2}][2q(2\gamma-1) - 2\gamma + (1-2q)(2\gamma-1)] \\
&= -\frac{(2\gamma-1)}{2}[\frac{1}{\{q(1-\gamma) + (1-q)\gamma\}^2}] < 0
\end{aligned}$$

This implies that σ_l (*i.e.*, $\Pr(\hat{s}^h|s^l)$) increases – or the informativeness of the equilibrium decreases – as q decreases. This proves the last part of (i) which states that the farther q is from $\frac{1}{2}$ (*i.e.*, q is more towards zero), the less information is revealed.

Part (ii) – the case for $\frac{1}{2} < q < 1$ – can be proved in an analogous manner.

I have already shown in (B.1) in the proof of Lemma 3 that the analyst can fully reveal his private signals in equilibrium only if $q = \frac{1}{2}$. This proves part (iii) of the proposition. ■

Proof of Lemma 4. Consider the case $0 < q < \frac{1}{2}$. Take price-impact profits, $\pi(\hat{s}^h) = \pi(\hat{s}^l) = \frac{(x^H - x^L)q(1-q)(2\gamma-1)(1-\sigma_l)}{[\gamma q + (1-\gamma)(1-q)] + \sigma_l[(1-\gamma)q + \gamma(1-q)]}$. By replacing the equilibrium $\sigma_l = \frac{(2\gamma-1)(1-2q)}{2\{q(1-\gamma)+(1-q)\gamma\}}$ by (B.2) from the proof of Proposition 1, I get an analyst's equilibrium price-impact profits

$$\begin{aligned}\pi(\hat{s}) &= \left|\frac{(x^H - x^L)q(1-q)(2\gamma-1)(1-\sigma_l)}{(1-\sigma_l)[(1-\gamma)q + \gamma(1-q)]}\right| \\ &= \frac{(x^H - x^L)q(1-q)(2\gamma-1)}{(1-\gamma)q + \gamma(1-q)} \qquad \text{(B.3)}\end{aligned}$$

Differentiating $\pi(\hat{s})$ with respect to q

$$\begin{aligned}\frac{\partial \pi(\hat{s})}{\partial q} &= (x^H - x^L)(2\gamma - 1)\frac{\partial}{\partial q}\left[\frac{q(1-q)}{(1-\gamma)q + \gamma(1-q)}\right] \\ &= (x^H - x^L)(2\gamma-1)\left[\frac{(1-2q)\{\gamma - q(2\gamma-1)\} + q(1-q)(2\gamma-1)}{\{(1-\gamma)q + \gamma(1-q)\}^2}\right]\end{aligned}$$

Note that for $0 < q < \frac{1}{2}$, $\gamma - q(2\gamma - 1) > 0$. Also, since $(x^H - x^L)(2\gamma - 1) > 0$, $(1-2q) > 0$ for $0 < q < \frac{1}{2}$ and $q(1-q)(2\gamma-1) > 0$

$$\frac{\partial \pi(\hat{s})}{\partial q} > 0 \text{ if } 0 < q < \frac{1}{2}$$

This means that equilibrium price-impact profits is increasing in q if $q \in (0, \frac{1}{2})$. Similarly

$$\frac{\partial \pi(\hat{s})}{\partial q} < 0 \text{ if } \frac{1}{2} < q < 1$$

Taken together, it is easy to see that $\pi(\hat{s})$ decreases as q moves farther from $q = \frac{1}{2}$. It is also straightforward to see from (B.3) that $\pi(\hat{s}) \to 0$ as either $q \to 0$ or $q \to 1$.

Since $\frac{\partial \pi(\hat{s})}{\partial q} > 0$ in $q \in (0, \frac{1}{2})$ and $\frac{\partial \pi(\hat{s})}{\partial q} < 0$ in $q \in (\frac{1}{2}, 1)$, it is straightforward to see that equilibrium price-impact profits achieves the global maximum at $q = \frac{1}{2}$. ■

Proof of Proposition 2. I will prove this proposition by using Lemma 6 below. In part (a) of the lemma, I first show that if the market priors are optimistic, then an analyst with a high signal will always issue a high forecast. However, if he receives a low signal, then to what extent he can credibly communicate his private signal depends on the interval in which the prior lies. Specifically, if the prior is in the intermediate range, then he forecasts his low signal; if the prior is sufficiently high, then the analyst randomizes between high and low forecasts; if the prior is at the extreme, *i.e.*, close to 1, then to appear a good type, he will report high ignoring his own low signal, leading to no information transmission in equilibrium. Part (b) of Lemma 6 describes the analyst's forecasting behavior for $q \leqslant \frac{1}{2}$.

To identify the intervals of priors in which different types of equilibria lie, I define the end points (see Figure 3, Panel C; note that to avoid notational clutter, α in the parenthesis has been dropped in the figure) as follows

$$\overline{q}(\alpha) \equiv \max\{q : (\sigma_h = 1, \sigma_l = 0) \text{ is an equilibrium}\} \tag{B.4}$$

$$\underline{q}(\alpha) \equiv \min\{q : (\sigma_h = 1, \sigma_l = 0) \text{ is an equilibrium}\} \tag{B.5}$$

$$q_{\max}(\alpha) \equiv \max\{q : (\sigma_h = 1, \sigma_l \in (0,1)) \text{ is an equilibrium}\} \tag{B.6}$$

$$q_{\min}(\alpha) \equiv \min\{q : (\sigma_h \in (0,1), \sigma_l = 0) \text{ is an equilibrium}\} \tag{B.7}$$

For any fixed α, the end points $\underline{q}(\alpha)$ and $\overline{q}(\alpha)$ identify the lower and upper bounds of the prior for which there is a fully informative equilibrium. Thus, the strategy pair $(\sigma_h = 1, \sigma_l = 0)$ will be an equilibrium if $q \in [\underline{q}(\alpha), \overline{q}(\alpha)]$. Again, for any given α, $q_{\max}(\alpha)$ is the upper bound of prior for which there is a partially informative equilibrium, with strategy pair $(\sigma_h = 1, \sigma_l \in (0,1))$. If $q > q_{\max}(\alpha)$, the equilibrium is noninformative with strategy pair $(\sigma_h = 1, \sigma_l = 1)$. Similarly, $q_{\min}(\alpha)$ is the lower bound of prior for which there is a partially informative equilibrium, with strategy pair $(\sigma_h \in (0,1), \sigma_l = 0)$. If $q < q_{\min}(\alpha)$, the equilibrium is noninformative with strategy pair $(\sigma_h = 0, \sigma_l = 0)$.

Finally, since I focus only on natural equilibria (see my discussion subsequent to Definition 2), I consider $\sigma_h \geqslant \sigma_l$.

Lemma 6 *(a) If $q \geqslant \frac{1}{2}$, then there exists an $\alpha_{\max}$ such that for all $\alpha \leq \alpha_{\max} \in (0,1)$, the following pair (σ_h, σ_l) forms an equilibrium in the given interval*
(i) $\sigma_h = 1$, $\sigma_l = 0$ if $\frac{1}{2} \leqslant q \leqslant \overline{q}(\alpha)$

(ii) $\sigma_h = 1$, $\sigma_l \in (0,1)$ *if* $\overline{q}(\alpha) < q \leqslant q_{\max}(\alpha)$

(iii) $\sigma_h = 1 = \sigma_l$ *if* $q_{\max}(\alpha) < q < 1$

(b) If $q \leqslant \frac{1}{2}$, *then there exists an* $\hat{\alpha}_{\max}$ *such that for all* $\alpha \leq \hat{\alpha}_{\max} \in (0,1)$, *the following pair* (σ_h, σ_l) *forms an equilibrium in the given interval*

(i) $\sigma_h = 1$, $\sigma_l = 0$ *if* $\underline{q}(\alpha) \leqslant q \leqslant \frac{1}{2}$

(ii) $\sigma_h \in (0,1)$, $\sigma_l = 0$ *if* $q_{\min}(\alpha) \leqslant q < \underline{q}(\alpha)$

(iii) $\sigma_h = 0 = \sigma_l$ *if* $0 < q < q_{\min}(\alpha)$

Proof. I will prove part (a) of Lemma 6. Part (b) can be proved by an analogous argument. To prove part (a), I will use Claims 1-6 below.

Claim 1 $\triangle R_h(\sigma_h, \sigma_l, q) \geqslant \triangle R_l(\sigma_h, \sigma_l, q)$ *for every* $q \in (0,1)$

Claim 2 $\triangle R_h(\sigma_h = 1, \sigma_l, q) > 0$ *for every* $q \geqslant \frac{1}{2}$

Claim 3 *(i)* $\triangle\, \pi(\sigma_h = 1, \sigma_l, q) < 0$ *if either* $\{q > \frac{1}{2}$ *and* $\sigma_l \in [0,1]\}$ *or* $\{q \geqslant \frac{1}{2}$*and* $\sigma_l \in (0,1]\}$

(ii) $\triangle\, \pi(\sigma_h = 1, \sigma_l, q) = 0$ *only if* $q = \frac{1}{2}$ *and* $\sigma_l = 0$.

Claim 4 *If* $q \geqslant \frac{1}{2}$, *then there exists an* $\alpha'_{\max} \equiv \frac{1}{1+\left[\frac{x^H - x^L}{\left(\frac{2\lambda(\gamma g - \gamma)}{\gamma(1-\gamma)}\right)}\right]} \in (0,1)$ *such that for all* $\alpha < \alpha'_{\max}$, $\triangle V_h(\sigma_h = 1, \sigma_l, q, \alpha) > 0$.

Claim 4 shows that if $q \geqslant \frac{1}{2}$, and α not very high, then the analyst will always be better off forecasting high when he receives a high signal.

Claim 5 *There exists an* $\alpha''_{\max} \in (0,1)$ *such that for all* $\alpha < \alpha''_{\max}$, $\frac{\partial \triangle V_l(\sigma_h = 1, \sigma_l, q, \alpha)}{\partial q} > 0$ *for every* $q \geqslant \gamma$.

Claim 6 *Suppose* $\alpha < \alpha''_{\max}$. *There exists a* $\overline{q}(\alpha) \in (\gamma, 1)$ *such that if* $\frac{1}{2} \leqslant q \leqslant \overline{q}(\alpha)$, *then* $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, \alpha) \leqslant 0$.

Claim 6 shows that if α is not very high, then given the market conjecture that the analyst is honestly forecasting his private signals, an analyst with a low signal will be better of forecasting low if $q \leqslant \overline{q}(\alpha)$.

Claim 7 *There exists a* $q_{\max}(\alpha) \in (\frac{1}{2}, 1)$ *such that if* $q > q_{\max}(\alpha)$, *then* $\triangle V_l(\sigma_h = 1, \sigma_l, q, \alpha) > 0$, *and if* $q \leqslant q_{\max}(\alpha)$, *then there exists a* $\sigma_l^*(q)$ *such that* $\triangle V_l(\sigma_h = 1, \sigma_l^*(q) \in (0,1), q, \alpha) = 0$.

Claim 7 shows that if α is not very high, there is a $q_{\max}(\alpha)$, beyond which an analyst will always be better off by forecasting high even if his signal is low. However, if $q \leqslant q_{\max}(\alpha)$, and the analyst gets a low signal, he will strictly randomize between high and low forecasts.

Proof of Claim 1. As defined earlier in (2.15)

$$\begin{aligned}
\triangle R_h &\equiv R(\hat{s}^h|s^h) - R(\hat{s}^l|s^h) && \text{(B.8)}\\
&= \Pr(x^H|s^h)[\Pr(g|\hat{s}^h, x^H) - \Pr(g|\hat{s}^l, x^H)] - \Pr(x^L|s^h)[\Pr(g|\hat{s}^h, x^L) - \Pr(g|\hat{s}^l, x^L)]\\
\triangle R_l &\equiv R(\hat{s}^h|s^l) - R(\hat{s}^l|s^l) && \text{(B.9)}\\
&= \Pr(x^H|s^l)[\Pr(g|\hat{s}^h, x^H) - \Pr(g|\hat{s}^l, x^H)] - \Pr(x^L|s^l)[\Pr(g|\hat{s}^h, x^L) - \Pr(g|\hat{s}^l, x^L)].
\end{aligned}$$

Subtracting

$$\begin{aligned}
\triangle R_h(\sigma_h, \sigma_l, q) - \triangle R_l(\sigma_h, \sigma_l, q) &= [\Pr(x^H|s^h) - \Pr(x^H|s^l)][\Pr(g|\hat{s}^h, x^H) - \Pr(g|\hat{s}^l, x^H)]\\
&\quad + [\Pr(x^L|s^h) - \Pr(x^L|s^l)][\Pr(g|\hat{s}^h, x^L) - \Pr(g|\hat{s}^l, x^L)]\\
&= [\Pr(x^H|s^h) - \Pr(x^H|s^l)][\Pr(g|\hat{s}^h, x^H) - \Pr(g|\hat{s}^l, x^H)]\\
&\quad + [\Pr(x^L|s^l) - \Pr(x^L|s^h)][\Pr(g|\hat{s}^l, x^L) - \Pr(g|\hat{s}^h, x^L)].
\end{aligned}$$

However, $\Pr(x^H|s^h) \geqslant \Pr(x^H|s^l)$ and $\Pr(x^L|s^l) \geqslant \Pr(x^L|s^h)$ for every $q \in (0,1)$. Also, by Property 1, $\Pr(g|\hat{s}^h, x^H) \geqslant \Pr(g|\hat{s}^l, x^H)$, and $\Pr(g|\hat{s}^l, x^L) \geqslant \Pr(g|\hat{s}^h, x^L)$. Taken together

$$\triangle R_h(\sigma_h, \sigma_l, q) - \triangle R_l(\sigma_h, \sigma_l, q) \geqslant 0.$$

■

Proof of Claim 2. By (B.8)

$$\begin{aligned}
\triangle R_h(\sigma_h &= 1, \sigma_l, q) = \Pr(x^H|s^h)[\Pr(g|\hat{s}^h, x^H, \sigma_h = 1)\\
-\Pr(g|\hat{s}^l, x^H, \sigma_h &= 1)] + \Pr(x^L|s^h)[\Pr(g|\hat{s}^h, x^L, \sigma_h = 1) - \Pr(g|\hat{s}^l, x^L, \sigma_h = 1)].
\end{aligned}$$

Using the expressions of analyst's reputation from Property 1, and after some algebraic manipulations

$$\begin{aligned}\Pr(g|\hat{s}^h, x^H, \sigma_h &= 1) - \Pr(g|\hat{s}^l, x^H, \sigma_h = 1) \\ &= \lambda\left[\frac{\gamma_g + (1-\gamma_g)\sigma_l}{\gamma + (1-\gamma)\sigma_l}\right] - \lambda\left[\frac{1-\gamma_g}{1-\gamma}\right] = \lambda\left[\frac{\gamma_g - \gamma}{(1-\gamma)(\gamma + (1-\gamma)\sigma_l)}\right]\end{aligned}$$

$$\begin{aligned}\Pr(g|\hat{s}^h, x^L, \sigma_h &= 1) - \Pr(g|\hat{s}^l, x^L, \sigma_h = 1) \\ &= \lambda\left[\frac{(1-\gamma_g) + \gamma_g\sigma_l}{(1-\gamma) + \gamma\sigma_l}\right] - \lambda\left[\frac{\gamma_g}{\gamma}\right] = -\lambda\left[\frac{\gamma_g - \gamma}{\gamma(1-\gamma+\gamma\sigma_l)}\right].\end{aligned}$$

Thus

$$\begin{aligned}\triangle R_h(\sigma_h &= 1, \sigma_l, q) \\ &= \Pr(x^H|s^h)\left[\lambda\frac{\gamma_g-\gamma}{(1-\gamma)(\gamma+(1-\gamma)\sigma_l)}\right] + \Pr(x^L|s^h)\left[-\lambda\frac{\gamma_g-\gamma}{\gamma(1-\gamma+\gamma\sigma_l)}\right] \\ &= \Pr(x^H|s^h)\left[\lambda\frac{\gamma_g-\gamma}{(1-\gamma)(\gamma+(1-\gamma)\sigma_l)}\right] - \\ &\quad \{1-\Pr(x^H|s^h)\}\left[\lambda\frac{\gamma_g-\gamma}{\gamma(1-\gamma+\gamma\sigma_l)}\right]\end{aligned}$$

But $\Pr(x^H|s^h) = \frac{q\gamma}{q\gamma+(1-q)(1-q\gamma)}$ is increasing in q, which implies that

$$\triangle R_h(\sigma_h = 1, \sigma_l, q) \text{ is increasing in } q.$$

Now, calculating $\triangle R_h$ at $q = \frac{1}{2}$

$$\begin{aligned}\triangle R_h(\sigma_h &= 1, \sigma_l, q = \frac{1}{2}) \\ &= \Pr(x^H|s^h)\left[\lambda\frac{\gamma_g-\gamma}{(1-\gamma)(\gamma+(1-\gamma)\sigma_l)}\right] - \Pr(x^L|s^h)\left[\lambda\frac{\gamma_g-\gamma}{\gamma(1-\gamma+\gamma\sigma_l)}\right] \\ &= \gamma\left[\lambda\frac{\gamma_g-\gamma}{(1-\gamma)(\gamma+(1-\gamma)\sigma_l)}\right] - (1-\gamma)\left[\lambda\frac{\gamma_g-\gamma}{\gamma(1-\gamma+\gamma\sigma_l)}\right] \\ &= \lambda\left[\frac{(\gamma_g-\gamma)(2\gamma-1)\{\gamma(1-\gamma)+\sigma_l\}}{\{(1-\gamma)(\gamma+(1-\gamma)\sigma_l)\}\{\gamma(1-\gamma+\gamma\sigma_l)\}}\right] > 0.\end{aligned}$$

Thus, $\triangle R_h(\sigma_h = 1, \sigma_l, q) > 0$ for every $q \geqslant \frac{1}{2}$. ■

Proof of Claim 3. Using the expressions of price-impact profits from (2.12)

$$\begin{aligned}\triangle\pi(\sigma_h &= 1, \sigma_l, q) \equiv \pi(\hat{s}^h; \sigma_h = 1, \sigma_l) - \pi(\hat{s}^l; \sigma_h = 1, \sigma_l) \\ &= \frac{(x^H - x^L)q(1-q)(2\gamma-1)(1-\sigma_l)}{\{\gamma q + (1-\gamma)(1-q)\} + \sigma_l\{(1-\gamma)q + \gamma(1-q)\}} \\ &\quad - \frac{(x^H - x^L)q(1-q)(2\gamma-1)}{(1-\gamma)q + \gamma(1-q)}\end{aligned}$$

After some algebra

$$\begin{aligned}
\triangle\pi(\sigma_h &= 1, \sigma_l, q) \\
&= \left[\frac{(x^H - x^L)q(1-q)(2\gamma-1)}{\gamma q + (1-\gamma)(1-q) + \sigma_l\{(1-\gamma)q + \gamma(1-q)\}}\right] * \\
&\quad \left[\frac{-\{\gamma q + (1-\gamma)(1-q)\} + (1-2\sigma_l)\{(1-\gamma)q + \gamma(1-q)\}}{(1-\gamma)q + \gamma(1-q)}\right] \\
&= \left[\frac{(x^H - x^L)q(1-q)(2\gamma-1)}{\gamma q + (1-\gamma)(1-q) + \sigma_l\{(1-\gamma)q + \gamma(1-q)\}}\right] * \\
&\quad \left[\frac{-1 + \{(1-\gamma)q + \gamma(1-q)\} + (1-2\sigma_l)\{(1-\gamma)q + \gamma(1-q)\}}{(1-\gamma)q + \gamma(1-q)}\right] \\
&= \left[\frac{(x^H - x^L)q(1-q)(2\gamma-1)}{\gamma q + (1-\gamma)(1-q) + \sigma_l\{(1-\gamma)q + \gamma(1-q)\}}\right] * \\
&\quad \left[\frac{2(1-\sigma_l)\{(1-\gamma)q + \gamma(1-q)\} - 1}{(1-\gamma)q + \gamma(1-q)}\right]
\end{aligned}$$

The second equality follows from the fact that $\{\gamma q + (1-\gamma)(1-q)\} = 1 - \{(1-\gamma)q + \gamma(1-q)\}$. Now

$$\begin{aligned}
sgn(\triangle\pi(\sigma_h &= 1, \sigma_l, q)) \\
&= sgn(2(1-\sigma_l)\{(1-\gamma)q + \gamma(1-q)\} - 1)
\end{aligned}$$

Also, since $\{(1-\gamma)q+\gamma(1-q)\}$ is decreasing in q and $\{(1-\gamma)q+\gamma(1-q)\} = \frac{1}{2}$ at $q = \frac{1}{2}$, it follows that if $q \leqslant \frac{1}{2}$, then $\{(1-\gamma)q+\gamma(1-q)\} \leqslant \frac{1}{2}$ and $2(1-\sigma_l)\{(1-\gamma)q+\gamma(1-q)\} \leqslant 1 - \sigma_l$. Taken together

$$2(1-\sigma_l)\{(1-\gamma)q + \gamma(1-q)\} - 1 \leqslant -\sigma_l \text{ if } q \geqslant \frac{1}{2} \text{ and } \sigma_l \in [0,1]$$

which further implies that

$$\begin{aligned}
\text{(i) } 2(1-\sigma_l)\{(1-\gamma)q + \gamma(1-q)\} - 1 &< 0 \text{ if either } \{q > \frac{1}{2} \text{ and} \\
\sigma_l &\in [0,1]\} \text{ or } \{q \geqslant \frac{1}{2} \text{and } \sigma_l \in (0,1]\} \\
\text{(ii) } 2(1-\sigma_l)\{(1-\gamma)q + \gamma(1-q)\} - 1 &= 0 \text{ only if} \\
q &= \frac{1}{2} \text{ and } \sigma_l = 0.
\end{aligned}$$

Therefore

$$\begin{aligned}\triangle\pi(\sigma_h &= 1,\sigma_l,q)<0 \text{ if either}\\ \{q &> \frac{1}{2} \text{ and } \sigma_l\in[0,1]\} \text{ or } \{q\geqslant\frac{1}{2}\text{and } \sigma_l\in(0,1]\}\\ \triangle\pi(\sigma_h &= 1,\sigma_l,q)=0 \text{ only if } q=\frac{1}{2} \text{ and } \sigma_l=0.\end{aligned}$$

■

Proof of Claim 4. By (2.17)

$$\triangle V_h(\sigma_h=1,\sigma_l,q,\alpha)=\alpha\triangle\pi(\sigma_h=1,\sigma_l,q)+(1-\alpha)\triangle R_h(\sigma_h=1,\sigma_l,q).$$

Case 1 : If $q>\frac{1}{2}$ and $\sigma_l\in(0,1]$, then

$$\triangle\pi(\sigma_h=1,\sigma_l,q)<0 \quad \text{by Claim 3}$$

$$\triangle R_h(\sigma_h=1,\sigma_l,q)>0 \quad \text{by Claim 2.}$$

Now, since $\triangle V_h(\sigma_h=1,\sigma_l,q,\alpha)$ is continuous in α, there exists an $\overline{\alpha}(\sigma_l,q)\in(0,1)$ such that if $\alpha\leq\overline{\alpha}(\sigma_l,q)$, then

$$\begin{aligned}V_h(\sigma_h &= 1,\sigma_l,q,\alpha)\geqslant 0\\ \text{or, } \alpha\triangle\pi(\sigma_h &= 1,\sigma_l,q)+(1-\alpha)\triangle R_h(\sigma_h=1,\sigma_l,q)\geqslant 0\end{aligned}\tag{B.10}$$

and $\overline{\alpha}(\sigma_l,q)$ satisfies the equation

$$\triangle V_h(\sigma_h=1,\sigma_l,q,\overline{\alpha}(\sigma_l,q))=0$$

which further implies

$$\begin{aligned}\overline{\alpha}(\sigma_l,q)\triangle\pi(\sigma_h &= 1,\sigma_l,q)+(1-\overline{\alpha}(\sigma_l,q))\triangle R_h(\sigma_h=1,\sigma_l,q)=0\\ &\Rightarrow \overline{\alpha}(\sigma_l,q)=\frac{1}{1+[\frac{-\triangle\pi(\sigma_h=1,\sigma_l,q)}{\triangle R_h(\sigma_h=1,\sigma_l,q)}]}\end{aligned}\tag{B.11}$$

Now, define

$$\alpha'_{\max}\equiv\min_{\sigma_l,q}\{\overline{\alpha}(\sigma_l,q)\}\in(0,1).\tag{B.12}$$

The term $\alpha'_{\max}$ is the minimum value of $\overline{\alpha}(\sigma_l,q)$, for any value of $(\sigma_l\in(0,1], q\in(\frac{1}{2},1))$, which satisfies (B.10). Thus, for all $\alpha\leq\alpha'_{\max}$

$$\triangle V_h(\sigma_h=1,\sigma_l,q,\alpha)\geqslant 0 \text{ if } q>\frac{1}{2}.$$

What is the value of $\alpha^{'}_{\max}$? Since $\triangle R_h(\sigma_h = 1, \sigma_l, q) > 0$ and is strictly increasing in q for every $q > \frac{1}{2}$, $\triangle R_h$ will achieve its minimum, for any given value of σ_l, at $q \to \frac{1}{2}$. Also, since $\triangle\pi(\sigma_h = 1, \sigma_l, q) < 0$ for every $q > \frac{1}{2}$, it is obvious that the minimum $\overline{\alpha}(\sigma_l, q)$ will be an α which will satisfy (B.10) at $q \to \frac{1}{2}$ for every value of σ_l.

Now, from equations (2.12)

$$\begin{aligned}\triangle\,\pi(\sigma_h &= 1, \sigma_l, q) \equiv \pi(\hat{s}^h; \sigma_h = 1, \sigma_l) - \pi(\hat{s}^l; \sigma_h = 1, \sigma_l)\\ &= \frac{(x^H - x^L)(2\gamma - 1)q(1-q)(1-\sigma_l)}{\gamma q + (1-\gamma)(1-q) + \sigma_l[(1-\gamma)q + \gamma(1-q)]}\\ &\quad -\frac{(x^H - x^L)(2\gamma - 1)q(1-q)}{(1-\gamma)q + \gamma(1-q)}\end{aligned} \tag{B.13}$$

is decreasing in σ_l and thus, $(-\triangle\,\pi(\sigma_h = 1, \sigma_l, q))$ is increasing in σ_l for any q. Also

$$\lim_{q\to\frac{1}{2}} \triangle\, R_h(\sigma_h = 1, \sigma_l, q) = \left(\frac{\lambda(\gamma_g - \gamma)(2\gamma - 1)}{\gamma(1-\gamma)}\right)\left(\frac{\gamma(1-\gamma) + (\gamma^2 + 1 - \gamma)\sigma_l}{(\gamma + (1-\gamma)\sigma_l)(1 - \gamma + \gamma\sigma_l)}\right)$$

is first increasing and then decreasing in σ_l, and achieves the minimum at both $\sigma_l = 0$ and 1. Thus the minimum value is $\lim_{q\to\frac{1}{2}} \triangle\, R_h(\sigma_h = 1, \sigma_l = 0, q) = \lim_{q\to\frac{1}{2}} \triangle\, R_h(\sigma_h = 1, \sigma_l = 1, q) = \frac{\lambda(\gamma_g - \gamma)(2\gamma - 1)}{\gamma(1-\gamma)}$. Also, the maximum value of $\lim_{q\to\frac{1}{2}}(-\triangle\,\pi(\sigma_h = 1, \sigma_l, q))$ for any given σ_l is $\lim_{q\to\frac{1}{2}}(-\triangle\,\pi(\sigma_h = 1, \sigma_l = 1, q)) = \frac{(x^H - x^L)(2\gamma - 1)}{2}$. Taken together

$$\alpha^{'}_{\max} \equiv \min_{\sigma_l, q\in(\frac{1}{2},1)} \{\overline{\alpha}(\sigma_l, q)\} = \frac{1}{1 + [\frac{\frac{(x^H - x^L)}{2\lambda(\gamma_g - \gamma)}}{\gamma(1-\gamma)}]}. \tag{B.14}$$

Case 2: If $q = \frac{1}{2}$ and $\sigma_l = 0$, then

$$\triangle\pi(\sigma_h = 1, \sigma_l, q) = 0 \quad \text{by Claim 3}$$

$$\triangle R_h(\sigma_h = 1, \sigma_l, q) > 0 \quad \text{by Claim 2.}$$

Thus, $\triangle V_h(\sigma_h = 1, \sigma_l, q, \alpha) \geqslant 0$ for every α.

Taken together both cases 1 and 2, there exists an $\alpha'_{\max} \equiv \frac{1}{1+[\frac{\frac{(x^H-x^L)}{2\lambda(\gamma_g-\gamma)}}{\gamma(1-\gamma)}]}$ such that

$$\text{for every } \alpha \leq \alpha'_{\max}, \text{ if } q \geqslant \frac{1}{2}, \text{ then } \triangle V_h(\sigma_h = 1, \sigma_l, q, \alpha) \geqslant 0.$$

■

Proof of Claim 5. By (2.17)

$$\triangle V_l(\sigma_h = 1, \sigma_l, q, \alpha) = \alpha \triangle \pi(\sigma_h = 1, \sigma_l, q) + (1-\alpha) \triangle R_l(\sigma_h = 1, \sigma_l, q).$$

By (B.13)

$$\begin{aligned} \triangle\pi(\sigma_h &= 1, \sigma_l, q) \\ &= \frac{(x^H - x^L)(2\gamma-1)q(1-q)(1-\sigma_l)}{\gamma q + (1-\gamma)(1-q) + \sigma_l[(1-\gamma)q + \gamma(1-q)]} - \\ &\quad \frac{(x^H - x^L)(2\gamma-1)q(1-q)}{(1-\gamma)q + \gamma(1-q)}. \end{aligned}$$

Now, again by definition

$$\begin{aligned} \triangle R_l(\sigma_h &= 1, \sigma_l, q) \equiv R(\hat{s}^h|s^l; \sigma_h = 1, \sigma_l) - R(\hat{s}^l|s^l; \sigma_h = 1, \sigma_l) \\ &= \Pr(x^H|s^l)\left[\Pr(g|\hat{s}^h, x^H, \sigma_h = 1) - \Pr(g|\hat{s}^l, x^H, \sigma_h = 1)\right] \\ &\quad + \Pr(x^L|s^l)\left[\Pr(g|\hat{s}^h, x^L, \sigma_h = 1) - \Pr(g|\hat{s}^l, x^L, \sigma_h = 1)\right] \\ &= \Pr(x^H|s^l)\left[\lambda\frac{\gamma_g - \gamma}{(1-\gamma)(\gamma + (1-\gamma)\sigma_l)}\right] + \\ &\quad + \Pr(x^L|s^l)\left[-\lambda\frac{\gamma_g - \gamma}{\gamma(1-\gamma+\gamma\sigma_l)}\right]. \end{aligned} \tag{B.15}$$

Now

$$\frac{\partial \triangle V_l(\sigma_h = 1, \sigma_l, q, \alpha)}{\partial q} = \alpha\frac{\partial \triangle \pi(\sigma_h = 1, \sigma_l, q)}{\partial q} + (1-\alpha)\frac{\partial \triangle R_l(\sigma_h = 1, \sigma_l, q)}{\partial q}$$

Note that $\frac{\partial \triangle R_l(\sigma_h=1,\sigma_l,q)}{\partial q}$ is strictly positive. If $\frac{\partial\triangle\pi(\sigma_h=1,\sigma_l,q)}{\partial q} \geqslant 0$, then $\frac{\partial\triangle V_l(\sigma_h=1,\sigma_l,q,\alpha)}{\partial q} > 0$ for all α. However, if $\frac{\partial\triangle\pi(\sigma_h=1,\sigma_l,q)}{\partial q} < 0$ for $q > \gamma$, then since $\frac{\partial\triangle V_l(\sigma_h=1,\sigma_l,q,\alpha)}{\partial q}$ is continuous in α, there exists an $\underline{\alpha}(\sigma_l, q) \in (0,1)$ such that if $\alpha < \underline{\alpha}(\sigma_l, q)$, then

$$\frac{\partial \triangle V_l(\sigma_h = 1, \sigma_l, q, \alpha)}{\partial q} > 0 \tag{B.16}$$

and $\underline{\alpha}(\sigma_l, q)$ satisfies the equation

$$\frac{\partial \triangle V_l(\sigma_h = 1, \sigma_l, q, \underline{\alpha}(\sigma_l, q))}{\partial q} = 0$$

As before, define

$$\alpha''_{\max} \equiv \min_{\sigma_l, q \in (\gamma,1)} \{\underline{\alpha}(\sigma_l, q)\} \in (0,1). \tag{B.17}$$

Thus, there exists an $\alpha''_{\max} \in (0,1)$ such that for every $\alpha < \alpha''_{\max}$, $\frac{\partial \triangle V_l(\sigma_h=1,\sigma_l,q,\alpha)}{\partial q}$ > 0 for every $q \in (\gamma, 1)$. ■

Proof of Claim 6. Using (B.15)

$$\begin{aligned}
\triangle R_l(\sigma_h &= 1, \sigma_l = 0, q) \\
&= \lambda \frac{(\gamma_g - \gamma)}{\gamma(1-\gamma)} [\Pr(x^H | s^l) - \Pr(x^L | s^l)] \\
&= [\lambda \frac{(\gamma_g - \gamma)}{\gamma(1-\gamma)}][\frac{q-\gamma}{q(1-\gamma) + (1-q)\gamma}].
\end{aligned} \tag{B.18}$$

Also, by (B.13)

$$\begin{aligned}
\triangle \pi(\sigma_h &= 1, \sigma_l = 0, q) \\
&= \frac{(x^H - x^L)(2\gamma - 1)^2 q(1-q)(1-2q)}{\{q\gamma + (1-q)(1-\gamma)\}\{q(1-\gamma) + (1-q)\gamma\}}
\end{aligned} \tag{B.19}$$

Thus, using (B.18) and (B.19)

$$\begin{aligned}
\triangle V_l(\sigma_h &= 1, \sigma_l = 0, q, \alpha) \\
&= \alpha \triangle \pi(\sigma_h = 1, \sigma_l = 0, q) + (1-\alpha) \triangle R_l(\sigma_h = 1, \sigma_l = 0, q) \\
&= \alpha \left[\frac{(x^H - x^L)(2\gamma - 1)^2 q(1-q)(1-2q)}{\{q\gamma + (1-q)(1-\gamma)\}\{q(1-\gamma) + (1-q)\gamma\}}\right] + \\
&\quad (1-\alpha)\left[\{\lambda \frac{(\gamma_g - \gamma)}{\gamma(1-\gamma)}\}\{\frac{q-\gamma}{q(1-\gamma) + (1-q)\gamma}\}\right]
\end{aligned} \tag{B.20}$$

From (B.20), I can see that $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, \alpha)$ has the following properties:

(i) $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, \alpha) < 0$ if $\frac{1}{2} \leqslant q \leqslant \gamma$ (since both $\triangle R_l(\sigma_h = 1, \sigma_l = 0, q) < 0$ and $\triangle \pi(\sigma_h = 1, \sigma_l = 0, q) \leqslant 0$ if $q \in [\frac{1}{2}, \gamma]$)

(ii) $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, \alpha) > 0$ if $q \to 1$ (since $\triangle \pi(\sigma_h = 1, \sigma_l = 0, q) = 0$ and $\triangle R_l(\sigma_h = 1, \sigma_l = 0, q) > 0$ at $q \to 1$)

However, for every $\alpha < \alpha''_{\max}$, since $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, \alpha)$ is strictly increasing in q for every $q > \gamma$ (by Claim 5), by the Intermediate Value Theorem, I can say that there exists a unique $\overline{q}(\alpha) \in (\gamma, 1)$ such that $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, \alpha)$ crosses zero at $q = \overline{q}(\alpha)$. Thus, for every $\alpha < \alpha''_{\max}$, there exits a unique $\overline{q}(\alpha) \in (\gamma, 1)$ such that if $\frac{1}{2} \leqslant q \leqslant \overline{q}(\alpha)$, then $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, \alpha) \leqslant 0$. ■

Proof of Claim 7. By (2.14)

$$\triangle V_l(\sigma_h = 1, \sigma_l, q, \alpha) = \alpha \triangle \pi(\sigma_h = 1, \sigma_l, q) + (1 - \alpha) \triangle R_l(\sigma_h = 1, \sigma_l, q).$$

Now, from equations (B.13)

$$\begin{aligned} \triangle\pi(\sigma_h &= 1, \sigma_l, q) \\ &= \frac{(x^H - x^L)(2\gamma - 1)q(1-q)(1-\sigma_l)}{\gamma q + (1-\gamma)(1-q) + \sigma_l[(1-\gamma)q + \gamma(1-q)]} - \frac{(x^H - x^L)(2\gamma - 1)q(1-q)}{(1-\gamma)q + \gamma(1-q)}. \end{aligned}$$

Together with the facts that $\frac{(x^H - x^L)q(1-q)(2\gamma-1)(1-\sigma_l)}{\gamma q + (1-\gamma)(1-q) + \sigma_l[(1-\gamma)q + \gamma(1-q)]}$ is decreasing in σ_l and

$$\min_{\sigma_l} \left[\frac{(x^H - x^L)q(1-q)(2\gamma-1)(1-\sigma_l)}{\gamma q + (1-\gamma)(1-q) + \sigma_l[(1-\gamma)q + \gamma(1-q)]} \right] = 0,$$

I have $\frac{(x^H - x^L)q(1-q)(2\gamma-1)(1-\sigma_l)}{\gamma q + (1-\gamma)(1-q) + \sigma_l[(1-\gamma)q + \gamma(1-q)]} \geqslant 0$. Therefore

$$\triangle \pi(\sigma_h = 1, \sigma_l, q) \geqslant -\frac{(x^H - x^L)q(1-q)(2\gamma - 1)}{(1-\gamma)q + \gamma(1-q)}. \tag{B.21}$$

By (B.15)

$$\begin{aligned} \triangle R_l(\sigma_h &= 1, \sigma_l, q) \\ &= \Pr(x^H|s^l)\left[\lambda \frac{\gamma_g - \gamma}{(1-\gamma)(\gamma + (1-\gamma)\sigma_l)}\right] + \\ &\quad + \Pr(x^L|s^l)\left[-\lambda \frac{\gamma_g - \gamma}{\gamma(1 - \gamma + \gamma\sigma_l)}\right]. \end{aligned} \tag{B.22}$$

By (B.21) and (B.22)

$$\begin{aligned} \triangle V_l(\sigma_h &= 1, \sigma_l, q, \alpha) \\ &\geqslant \alpha\left[-\frac{(x^H - x^L)(2\gamma - 1)q(1-q)}{q(1-\gamma)q + (1-q)\gamma}\right] + \\ &\quad (1-\alpha)[\Pr(x^H|s^l)\left(\lambda \frac{\gamma_g - \gamma}{(1-\gamma)(\gamma + (1-\gamma)\sigma_l)}\right) \\ &\quad + \Pr(x^L|s^l)\left(-\lambda \frac{\gamma_g - \gamma}{\gamma(1 - \gamma + \gamma\sigma_l)}\right)]. \end{aligned}$$

Now, define

$$\begin{aligned} f(q,\sigma_l) \quad &\equiv \quad \alpha[-\frac{(x^H-x^L)q(1-q)(2\gamma-1)}{q(1-\gamma)q+(1-q)\gamma}] \\ &+(1-\alpha)[\Pr(x^H|s^l)\left(\lambda\frac{\gamma_g-\gamma}{(1-\gamma)(\gamma+(1-\gamma)\sigma_l)}\right) \\ &+\Pr(x^L|s^l)\left(-\lambda\frac{\gamma_g-\gamma}{\gamma(1-\gamma+\gamma\sigma_l)}\right)]. \end{aligned}$$

such that

$$\triangle\, V_l(\sigma_h=1,\sigma_l,q,\alpha)\geqslant f(q,\sigma_l). \tag{B.23}$$

Since $\left[\Pr(x^H|s^l)\left(\lambda\frac{\gamma_g-\gamma}{(1-\gamma)(\gamma+(1-\gamma)\sigma_l)}\right)+\Pr(x^L|s^l)\left(-\lambda\frac{\gamma_g-\gamma}{\gamma(1-\gamma+\gamma\sigma_l)}\right)\right]$ is increasing in q and is negative at $q=\frac{1}{2}$ for any σ_l, I have

$$\Pr(x^H|s^l)\left(\lambda\frac{\gamma_g-\gamma}{(1-\gamma)(\gamma+(1-\gamma)\sigma_l)}\right)+\Pr(x^L|s^l)\left(-\lambda\frac{\gamma_g-\gamma}{\gamma(1-\gamma+\gamma\sigma_l)}\right)<0 \text{ if } q\leqslant\frac{1}{2}$$

Thus

$$f(q,\sigma_l)<0 \text{ if } q\leqslant\frac{1}{2}.$$

However

$$f(q,\sigma_l)>0 \text{ if } q\to 1.$$

Since $f(q,\sigma_l)$ is a quadratic function of q for any given σ_l, by the Intermediate Value Theorem, there exists a *unique* $q_{\max}(\alpha)\in(\frac{1}{2},1)$ such that

$$f(q_{\max}(\alpha),\sigma_l)=0 \text{ and } f(q,\sigma_l)>0 \text{ if } q\in(q_{\max}(\alpha),1).$$

Therefore, there exists a unique $q_{\max}(\alpha)\in(\frac{1}{2},1)$ such that

$$\begin{aligned} \triangle V_l(\sigma_h \quad &= \quad 1,\sigma_l,q_{\max}(\alpha),\alpha)\geqslant f(q)=0 \text{ and} \\ \triangle V_l(\sigma_h \quad &= \quad 1,\sigma_l,q,\alpha)\geqslant f(q)>0 \text{ if } q\in(q_{\max}(\alpha),1). \end{aligned}$$

Now for any given $q\leqslant q_{\max}(\alpha)$, there exists a $\sigma_l^*(q)$ such that

$$\triangle\, V_l(\sigma_h=1,\sigma_l^*(q)\in(0,1),q,\alpha)=0 \tag{B.24}$$

Intuitively, $\sigma_l^*(q)$ is the equilibrium randomization strategy of the analyst, when he receives a low signal, for any given q. ∎

Now, I return to proving part (a) of Lemma 6. Define

$$\alpha_{\max} \equiv \min\{\alpha'_{\max}, \alpha''_{\max}\} \in (0,1).$$

Part (i) follows from Claims 4 and 6, which taken together, imply that for every $\alpha \leq \alpha_{\max}$, if $\frac{1}{2} \leqslant q \leqslant \overline{q}(\alpha)$

$$\triangle V_h(\sigma_h = 1, \sigma_l, q, \alpha) \geqslant 0 \text{ and } \triangle V_l(\sigma_h = 1, \sigma_l = 0, q, \alpha) \leqslant 0.$$

Part (iii) follows from Claims 4 and 7, which taken together, imply that for every $\alpha \leq \alpha_{\max}$, if $q_{\max}(\alpha) < q < 1$

$$\triangle V_h(\sigma_h = 1, \sigma_l, q, \alpha) > 0 \text{ and } \triangle V_l(\sigma_h = 1, \sigma_l, q, \alpha) > 0.$$

Part (ii). Suppose $\alpha \leq \alpha_{\max}$. For $\overline{q}(\alpha) < q \leqslant q_{\max}(\alpha)$

$$\triangle V_h(\sigma_h = 1, \sigma_l, q, \alpha) \geqslant 0 \text{ and } \triangle V_l(\sigma_h = 1, \sigma_l, q, \alpha) = 0$$

which implies that there exists a partially informative equilibrium for every $q \in (\overline{q}(\alpha), q_{\max}(\alpha)]$. The equilibrium randomization strategy $\sigma_l^*(q)$ varies with q, and satisfies

$$\triangle V_l(\sigma_h = 1, \sigma_l^*(q) \in (0,1), q, \alpha) = 0$$

as in (B.24). This completes the proof of part (a). ■

Part (b) can be proved in an analogous argument. By symmetry, it can be shown that $\underline{q}(\alpha) = 1 - \overline{q}(\alpha)$, $q_{\min}(\alpha) = 1 - q_{\max}(\alpha)$, and $\hat{\alpha}_{\max} = \alpha_{\max}$. ■

Finally, I return to completing the proof of Proposition 2. Now, using the results of Lemma 6, the results of Proposition 2 follow directly.

Proof of Proposition 3. Suppose $\alpha \leq \alpha_{\max}$. I will first show that if $\alpha_2 > \alpha_1$, then $\overline{q}(\alpha_2) \geqslant \overline{q}(\alpha_1)$. By definition (B.4)

$$\overline{q}(\alpha) \equiv \max\{q : (\sigma_h = 1, \sigma_l = 0) \text{ is an equilibrium}\}$$

Proposition 2 and definition (B.4), together, imply

$$\begin{aligned} \triangle V_l(\sigma_h &= 1, \sigma_l = 0, \overline{q}(\alpha), \alpha) = 0 \\ \triangle V_l(\sigma_h &= 1, \sigma_l = 0, q, \alpha) < 0 \text{ if } q < \overline{q}(\alpha) \end{aligned}$$

At $\alpha = \alpha_1$, $\triangle V_l(\sigma_h = 1, \sigma_l = 0, \overline{q}(\alpha_1), \alpha_1) = 0$. However, by Claim 3, $\triangle\pi(\sigma_h = 1, \sigma_l = 0, \overline{q}(\alpha_1)) < 0$. Therefore $\triangle R_l(\sigma_h = 1, \sigma_l = 0, \overline{q}(\alpha_1)) > 0$.

At $\alpha_2 > \alpha_1$

$$\begin{aligned}\triangle V_l(\sigma_h &= 1, \sigma_l = 0, \overline{q}(\alpha_1), \alpha_2) \\ &= \alpha_2 \triangle \pi(\sigma_h = 1, \sigma_l = 0, \overline{q}(\alpha_1)) + (1-\alpha_2) \triangle R_l(\sigma_h = 1, \sigma_l = 0, \\ &\quad \overline{q}(\alpha_1)) \\ &< 0\end{aligned}$$

Since $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, \alpha)$ is strictly increasing in q for every $q > \gamma$ (by Claim 5), and $\overline{q}(\alpha) > \gamma$, there exists $\overline{q}(\alpha_2) \geqslant \overline{q}(\alpha_1)$ such that

$$\triangle V_l(\sigma_h = 1, \sigma_l = 0, \overline{q}(\alpha_2), \alpha_2) = 0.$$

This shows that if $\alpha_2 > \alpha_1$, then $\overline{q}(\alpha_2) \geqslant \overline{q}(\alpha_1)$.

Next, I will show that if $\alpha_2 > \alpha_1$, then $q_{\max}(\alpha_2) \geqslant q_{\max}(\alpha_1)$. By definition (B.6)

$$q_{\max}(\alpha) \equiv \max\{q : (\sigma_h = 1, \sigma_l \in (0,1)) \text{ is an equilibrium }\}$$

By Proposition 2 and the definition of $q_{\max}$ in (B.6), I have

$$\triangle V_l(\sigma_h = 1, \sigma_l^*(q_{\max}(\alpha)), q_{\max}(\alpha), \alpha) = 0.$$

where $\sigma_l^*(q)$ is the equilibrium value of σ_l for any given q, and $\sigma_h = 1$. At $\alpha = \alpha_1$, I have $\triangle V_l(\sigma_h = 1, \sigma_l^*(q_{\max}(\alpha_1)), q_{\max}(\alpha_1), \alpha_1) = 0$. However, by Claim 3, $\triangle\pi(\sigma_h = 1, \sigma_l^*(q_{\max}(\alpha_1)), q_{\max}(\alpha_1)) < 0$. Thus, $\triangle R_l(\sigma_h = 1, \sigma_l^*(q_{\max}(\alpha_1)), q_{\max}(\alpha_1)) > 0$.

Now for $\alpha_2 > \alpha_1$

$$\begin{aligned}\triangle V_l(\sigma_h &= 1, \sigma_l^*(q_{\max}(\alpha_1)), q_{\max}(\alpha_1), \alpha_2) \\ &= \alpha_2 \triangle \pi(\sigma_h = 1, \sigma_l^*(q_{\max}(\alpha_1)), q_{\max}(\alpha_1)) + (1-\alpha_2) \triangle R_l(\sigma_h = 1, \\ &\quad \sigma_l^*(q_{\max}(\alpha_1)), q_{\max}(\alpha_1)) \\ &< 0.\end{aligned}$$

Again, since by definition (B.6), $\triangle V_l(\sigma_h = 1, \sigma_l^*(q_{\max}(\alpha)), q_{\max}(\alpha), \alpha) = 0$, and $\triangle V_l(\sigma_h = 1, \sigma_l, q, \alpha)$ is strictly increasing in q for every $q > \gamma$ (by Claim 5), there exists $q_{\max}(\alpha_2) \geqslant q_{\max}(\alpha_1)$ such that

$$\triangle V_l(\sigma_h = 1, \sigma_l^*(q_{\max}(\alpha_2)), q_{\max}(\alpha_2), \alpha_2) = 0.$$

This shows that if $\alpha_2 > \alpha_1$, then $q_{\max}(\alpha_2) \geqslant q_{\max}(\alpha_1)$. ∎

B.2 Two Analysts Case

Proof of Lemma 5. Fix $i = A$. Analyst A is ranked higher than analyst B if $r_A(\hat{s}_A, \hat{s}_B, x) > r_B(\hat{s}_A, \hat{s}_B, x)$. By Property 3 and 4, $r_A(\hat{s}_A, \hat{s}_B, x) > r_B(\hat{s}_A, \hat{s}_B, x)$ if analyst A's forecast matches with the reported earnings, but analyst B's does not. Similarly, $r_A(\hat{s}_A, \hat{s}_B, x) < r_B(\hat{s}_A, \hat{s}_B, x)$ if A's forecast does not match the reported earnings, but B's does.

Fix $x = x^H$. With $U_A(r_A, r_B) = \frac{r_A}{r_A + \upsilon r_B}$

$$
\begin{aligned}
U_A(r_A &> r_B) = \frac{r_A(\hat{s}_A^h, \hat{s}_B^l, x^H)}{r_A(\hat{s}_A^h, \hat{s}_B^l, x^H) + \upsilon r_B(\hat{s}_A^h, \hat{s}_B^l, x^H)} \\
U_A(r_A &< r_B) = \frac{r_A(\hat{s}_A^l, \hat{s}_B^h, x^H)}{r_A(\hat{s}_A^l, \hat{s}_B^h, x^H) + \upsilon r_B(\hat{s}_A^l, \hat{s}_B^h, x^H)} \\
U_A(r_A &= r_B) = \frac{1}{1+\upsilon}
\end{aligned}
$$

Therefore, the marginal gain (or loss) of analyst A being ranked higher (or lower) than analyst B, compared to both being ranked the same, are

$$
\begin{aligned}
\text{Gain} &\equiv U_A(r_A > r_B) - U_A(r_A = r_B) \\
&= \left(\frac{\upsilon}{1+\upsilon}\right)\left[\frac{r_A(\hat{s}_A^h, \hat{s}_B^l, x^H) - r_B(\hat{s}_A^h, \hat{s}_B^l, x^H)}{r_A(\hat{s}_A^h, \hat{s}_B^l, x^H) + \upsilon r_B(\hat{s}_A^h, \hat{s}_B^l, x^H)}\right] > 0 \\
\text{Loss} &\equiv U_A(r_A = r_B) - U_A(r_A < r_B) \\
&= \left(\frac{\upsilon}{1+\upsilon}\right)\left[\frac{r_B(\hat{s}_A^l, \hat{s}_B^h, x^H) - r_A(\hat{s}_A^l, \hat{s}_B^h, x^H)}{r_A(\hat{s}_A^l, \hat{s}_B^h, x^H) + \upsilon r_B(\hat{s}_A^l, \hat{s}_B^h, x^H)}\right] > 0
\end{aligned}
$$

However, in the numerator of loss above, $r_B(\hat{s}_A^l, \hat{s}_B^h, x^H) = r_A(\hat{s}_A^h, \hat{s}_B^l, x^H)$ and $r_A(\hat{s}_A^l, \hat{s}_B^h, x^H) = r_B(\hat{s}_A^h, \hat{s}_B^l, x^H)$ by Property 3 (symmetry). Thus

$$
\text{Loss} = \left(\frac{\upsilon}{1+\upsilon}\right)\left[\frac{r_A(\hat{s}_A^h, \hat{s}_B^l, x^H) - r_B(\hat{s}_A^h, \hat{s}_B^l, x^H)}{r_A(\hat{s}_A^l, \hat{s}_B^h, x^H) + \upsilon r_B(\hat{s}_A^l, \hat{s}_B^h, x^H)}\right].
$$

Now, by the definition of ϕ

$$
\begin{aligned}
\phi &\equiv \frac{\text{Gain}}{\text{Loss}} \\
&= \frac{r_A(\hat{s}_A^l, \hat{s}_B^h, x^H) + \upsilon r_B(\hat{s}_A^l, \hat{s}_B^h, x^H)}{r_A(\hat{s}_A^h, \hat{s}_B^l, x^H) + \upsilon r_B(\hat{s}_A^h, \hat{s}_B^l, x^H)} \\
&= \frac{r_A(\hat{s}_A^l, \hat{s}_B^h, x^H) + \upsilon r_B(\hat{s}_A^l, \hat{s}_B^h, x^H)}{r_B(\hat{s}_A^l, \hat{s}_B^h, x^H) + \upsilon r_A(\hat{s}_A^l, \hat{s}_B^h, x^H)}.
\end{aligned}
$$

In the second equality, $r_A(\hat{s}_A^h, \hat{s}_B^l, x^H) = r_B(\hat{s}_A^l, \hat{s}_B^h, x^H)$ and $r_B(\hat{s}_A^h, \hat{s}_B^l, x^H) = r_A(\hat{s}_A^l, \hat{s}_B^h, x^H)$ by Property 3. Now, differentiating ϕ with respect to υ, and using the fact that $r_B(\hat{s}_A^l, \hat{s}_B^h, x^H) > r_A(\hat{s}_A^l, \hat{s}_B^h, x^H)$

$$
\begin{aligned}
& \frac{\partial \phi}{\partial \upsilon} \\
= & \frac{r_B(\hat{s}_A^l, \hat{s}_B^h, x^H)\{r_B(\hat{s}_A^l, \hat{s}_B^h, x^H) + \upsilon r_A(\hat{s}_A^l, \hat{s}_B^h, x^H)\}}{\{r_B(\hat{s}_A^l, \hat{s}_B^h, x^H) + \upsilon r_A(\hat{s}_A^l, \hat{s}_B^h, x^H)\}^2} \\
& - \frac{r_A(\hat{s}_A^l, \hat{s}_B^h, x^H)\{r_A(\hat{s}_A^l, \hat{s}_B^h, x^H) + \upsilon r_B(\hat{s}_A^l, \hat{s}_B^h, x^H)\}}{\{r_B(\hat{s}_A^l, \hat{s}_B^h, x^H) + \upsilon r_A(\hat{s}_A^l, \hat{s}_B^h, x^H)\}^2} \\
> & \ 0
\end{aligned}
$$

This completes the proof of Lemma 5. ∎

Proof of Proposition 4. Consider analyst A. The hypothesis that analyst signals are conditionally independent implies that

$$\Pr(g_A|\hat{s}_A, \hat{s}_B, x) = \Pr(g_A|\hat{s}_A, x)$$

and

$$\Pr(g_B|\hat{s}_A, \hat{s}_B, x) = \Pr(g_B|\hat{s}_B, x)$$

The intuition is that since each analyst's type (good or bad) is determined by his private signal, his posterior of type (*i.e.*, reputation) is stochastically independent of the other analyst's signal and forecast.

Analyst A's objective is $\max_{\hat{s}_A} R_A(\hat{s}_A|s_A)$, where $R_A(\hat{s}_A|s_A) \equiv \mathbb{E}_{x,\hat{s}_B}[U_A(\hat{s}_A, \hat{s}_B, x)|s_A, \sigma_B]$, as defined earlier. Using the linear reputational ranking payoff function, as defined in (3.11), $R_A(\hat{s}_A|s_A)$ can be expressed as

$$
\begin{aligned}
R_A(\hat{s}_A|s_A) &\equiv \mathbb{E}_{x,\hat{s}_B}[U_A(\hat{s}_A, \hat{s}_B, x)|s_A, \sigma_B] \\
&= \mathbb{E}_{x,\hat{s}_B}[\beta_0 + \beta_1 \Pr(g_A|\hat{s}_A, \hat{s}_B, x) - \beta_2 \Pr(g_B|\hat{s}_A, \hat{s}_B, x)|s_A, \sigma_B] \\
&= \beta_0 + \beta_1 \mathbb{E}_{x,\hat{s}_B}[\Pr(g_A|\hat{s}_A, \hat{s}_B, x)|s_A, \sigma_B] - \beta_2 \mathbb{E}_{x,\hat{s}_B}[\Pr(g_B|\hat{s}_A, \hat{s}_B, x)|s_A, \sigma_B] \\
&= \beta_0 + \beta_1 \mathbb{E}_x[\Pr(g_A|\hat{s}_A, x)|s_A] - \beta_2 \mathbb{E}_{x,\hat{s}_B}[\Pr(g_B|\hat{s}_B, x)|s_A, \sigma_B] \\
&= \beta_0 + \beta_1 \mathbb{E}_x[\Pr(g_A|\hat{s}_A, x)|s_A] - \beta_2 \mathbb{E}_x[\Pr(g_B|x)|s_A] \\
&= \beta_0 + \beta_1 \mathbb{E}_x[\Pr(g_A|\hat{s}_A, x)|s_A] - \beta_2 \Pr(g_B) \\
&= \beta_0 + \beta_1 \mathbb{E}_x[\Pr(g_A|\hat{s}_A, x)|s_A] - \beta_2 \lambda
\end{aligned}
$$

The third equality follows from the conditional independence of analyst signals. In fourth equality, iterated expectation on $\hat{s}_B$ leads to $\mathbb{E}_{x,\hat{s}_B}[\Pr(g_B|\hat{s}_B, x)|s_A, \sigma_B] = \mathbb{E}_x[\Pr(g_B|x)|s_A]$. Further, iterated expectation on x leads to $\mathbb{E}_x[\Pr(g_B|x)|s_A] = \Pr(g_B) = \lambda$.

Analyst A's objective is now reduced to

$$\max_{\hat{s}_A}\{\beta_0 + \beta_1\mathbb{E}_x[\Pr(g_A|\hat{s}_A, x)|s_A] - \beta_2\lambda\}$$

which does not depend on analyst B's forecast. Thus, analyst A's equilibrium behavior will be the same as the analyst in the baseline model, in which an analyst is maximizing his own expected reputation in the market. ■

Proof of Proposition 5. I will prove this proposition by the following claims.

Claim 8 *When the market and each analyst believes that each analyst is honestly reporting his private signal (i.e., $\hat{s} = s$),then an analyst with a high signal will forecast high if $q \geqslant 1 - q_{rc}$, and with a low signal will forecast low if $q \leqslant q_{rc}$.*

Claim 9 $q_{rc} = \gamma - \lambda^2\gamma_g(1 - \gamma_g)[\frac{\phi-1}{\phi(1-\gamma)+\gamma}] \in (\frac{1}{2}, 1)$ *where* $\phi = \frac{1-\gamma_g+\upsilon\gamma_g}{\gamma_g+\upsilon(1-\gamma_g)} \in (\frac{1-\gamma_g}{\gamma_g}, \frac{\gamma_g}{1-\gamma_g})$.

Claim 10 *If either $q > q_{rc}$ or $q < 1 - q_{rc}$, then the equilibrium is noninformative.*

Fix $i = A$. Before I start proving the claims, I will derive expressions for $\Delta R_{A,h}$ and $\Delta R_{A,l}$. By definition

$$\begin{aligned}
R_A(\hat{s}_A^h|s_A^l) &\equiv \mathbb{E}_{x,\hat{s}_B}[U_A(\hat{s}_A^h, \hat{s}_B, x)|s_A^l, \sigma_B] \\
&= \Pr(\hat{s}_B^h, x^H|s_A^l)U_A(\hat{s}_A^h, \hat{s}_B^h, x^H) + \Pr(\hat{s}_B^l, x^H|s_A^l)U_A(\hat{s}_A^h, \hat{s}_B^l, x^H) + \\
&\quad \Pr(\hat{s}_B^h, x^L|s_A^l)U_A(\hat{s}_A^h, \hat{s}_B^h, x^L) + \Pr(\hat{s}_B^l, x^L|s_A^l)U_A(\hat{s}_A^h, \hat{s}_B^l, x^L) \\
&= \Pr(x^H|s_A^l)\{\Pr(\hat{s}_B^h|x^H, s_A^l)U_A(\hat{s}_A^h, \hat{s}_B^h, x^H) + \Pr(\hat{s}_B^l|x^H, s_A^l)U_A(\hat{s}_A^h, \hat{s}_B^l, x^H)\} + \\
&\quad \Pr(x^L|s_A^l)\{\Pr(\hat{s}_B^h|x^L, s_A^l)U_A(\hat{s}_A^h, \hat{s}_B^h, x^L) + \Pr(\hat{s}_B^l|x^L, s_A^l)U_A(\hat{s}_A^h, \hat{s}_B^l, x^L)\}
\end{aligned}$$

Similarly

$$\begin{aligned}
R_A(\hat{s}_A^l|s_A^l) &\equiv \mathbb{E}_{x,\hat{s}_B}[U_A(\hat{s}_A^l, \hat{s}_B, x)|s_A^l, \sigma_B] \\
&= \Pr(x^H|s_A^l)\{\Pr(\hat{s}_B^h|x^H, s_A^l)U_A(\hat{s}_A^l, \hat{s}_B^h, x^H) + \Pr(\hat{s}_B^l|x^H, s_A^l)U_A(\hat{s}^l, \hat{s}_B^l, x^H)\} + \\
&\quad \Pr(x^L|s_A^l)\{\Pr(\hat{s}_B^h|x^L, s_A^l)U_A(\hat{s}_A^l, \hat{s}_B^h, x^L) + \Pr(\hat{s}_B^l|x^L, s_A^l)U_A(\hat{s}_A^l, \hat{s}_B^l, x^L)\}
\end{aligned}$$

Thus, using definition (3.18), for $i = A$

$$\begin{aligned}
\Delta R_{A,l}(\sigma_h, \sigma_l, q) &\equiv R_A(\hat{s}_A^h|s_A^l) - R_A(\hat{s}_A^l|s_A^l) \\
&= \Pr(x^H|s_A^l)[\Pr(\hat{s}_B^h|x^H, s_A^l)\{U_A(\hat{s}_A^h, \hat{s}_B^h, x^H) - U_A(\hat{s}_A^l, \hat{s}_B^h, x^H)\} + \\
&\quad \Pr(\hat{s}_B^l|x^H, s_A^l)\{U_A(\hat{s}_A^h, \hat{s}_B^l, x^H) - U_A(\hat{s}^l, \hat{s}_B^l, x^H)\}] + \\
&\quad + \Pr(x^L|s_A^l)[\Pr(\hat{s}_B^h|x^L, s_A^l)\{U_A(\hat{s}_A^h, \hat{s}_B^h, x^L) - U_A(\hat{s}_A^l, \hat{s}_B^h, x^L)\} + \\
&\quad \Pr(\hat{s}_B^l|x^L, s_A^l)\{U_A(\hat{s}_A^h, \hat{s}_B^l, x^L) - U_A(\hat{s}_A^l, \hat{s}_B^l, x^L)\}] \\
&= \Pr(x^H|s_A^l)[\Pr(\hat{s}_B^h|x^H, s_A^l)\{U_A(\hat{s}_A^l, \hat{s}_B^l, x^H) - U_A(\hat{s}_A^l, \hat{s}_B^h, x^H)\} + \\
&\quad \Pr(\hat{s}_B^l|x^H, s_A^l)\{U_A(\hat{s}_A^h, \hat{s}_B^l, x^H) - U_A(\hat{s}^h, \hat{s}_B^h, x^H)\}] + \\
&\quad + \Pr(x^L|s_A^l)[-\Pr(\hat{s}_B^h|x^L, s_A^l)\{U_A(\hat{s}_A^l, \hat{s}_B^h, x^L) - U_A(\hat{s}_A^l, \hat{s}_B^l, x^L)\} - \\
&\quad \Pr(\hat{s}_B^l|x^L, s_A^l)\{U_A(\hat{s}_A^h, \hat{s}_B^h, x^L) - U_A(\hat{s}_A^h, \hat{s}_B^l, x^L)\}] \\
&= \Pr(x^H|s_A^l)[\Pr(\hat{s}_B^h|x^H, s_A^l)Loss_H + \Pr(\hat{s}_B^l|x^H, s_A^l)Gain_H] \\
&\quad + \Pr(x^L|s_A^l)[\Pr(\hat{s}_B^h|x^L, s_A^l)(-Gain_L) + \Pr(\hat{s}_B^l|x^L, s_A^l)(-Loss_L)] \qquad \text{(B.25)}
\end{aligned}$$

where

$$\begin{aligned}
Gain_H &\equiv U_A(\hat{s}_A^h, \hat{s}_B^l, x^H) - U_A(\hat{s}_A^h, \hat{s}_B^h, x^H) > 0 \\
Loss_H &\equiv U_A(\hat{s}_A^l, \hat{s}_B^l, x^H) - U_A(\hat{s}_A^l, \hat{s}_B^h, x^H) > 0 \\
Gain_L &\equiv U_A(\hat{s}_A^l, \hat{s}_B^h, x^L) - U_A(\hat{s}_A^l, \hat{s}_B^l, x^L) > 0 \\
Loss_L &\equiv U_A(\hat{s}_A^h, \hat{s}_B^h, x^L) - U_A(\hat{s}_A^h, \hat{s}_B^l, x^L) > 0 \qquad \text{(B.26)}
\end{aligned}$$

Note that in the second equality of (B.25)

$$U_A(\hat{s}_A^l, \hat{s}_B^l, x^H) = U_A(\hat{s}_A^h, \hat{s}_B^h, x^H) = U_A(\hat{s}_A^h, \hat{s}_B^h, x^L) = U_A(\hat{s}_A^l, \hat{s}_B^l, x^H \quad \left(= \frac{1}{1+v}\right)$$

by (3.12), because an analyst's reputational ranking payoff remains the same if both analysts make the same forecast regardless of whether their forecasts match or do not match the reported earnings. By Definition 7, "gain" represents an analyst's marginal gain in payoff for being ranked higher than the other analyst, and "loss" represents an analyst's marginal loss in payoff for being ranked lower. Subscripts H and L in gain and loss imply that the corresponding reported earnings are high and low.

The equation (B.25) represents analyst A's (similar expression for analyst B) expected net payoff of reporting a high versus low forecast, when he receives a low signal.

The intuition is that when the reported earning is high ($i.e.$, x^H), reporting high provides analyst A with a net positive payoff ($i.e.$, $\Pr(\hat{s}_B^h|x^H, s_A^l)Loss_H + \Pr(\hat{s}_B^l|x^H, s_A^l)Gain_H > 0$). However, when the reported earning is low ($i.e.$, x^L), reporting high provides him a net negative payoff ($i.e.$, $\Pr(\hat{s}_B^h|x^L, s_A^l)(-Gain_L) + \Pr(\hat{s}_B^l|x^L, s_A^l)(-Loss_L) < 0$).

Furthermore, by (B.25)

$$\begin{aligned}
&\Delta R_{A,l}(\sigma_h, \sigma_l, q) \\
&= \Pr(x^H|s_A^l)[\Pr(\hat{s}_B^h|x^H, s_A^l)Loss_H(\sigma_h, \sigma_l) + \Pr(\hat{s}_B^l|x^H, s_A^l)Gain_H(\sigma_h, \sigma_l)] \\
&\quad + \Pr(x^L|s_A^l)[\Pr(\hat{s}_B^h|x^L, s_A^l)(-Gain_L(\sigma_h, \sigma_l)) + \Pr(\hat{s}_B^l|x^L, s_A^l)(-Loss_L(\sigma_h, \sigma_l))] \\
&= \beta_l(\sigma_h, \sigma_l)[\frac{\Pr(x^H|s_A^l)}{\Pr(x^L|s_A^l)} - \\
&\quad \frac{\Pr(\hat{s}_B^h|x^L, s_A^l)Gain_L(\sigma_h, \sigma_l) + \Pr(\hat{s}_B^l|x^L, s_A^l)Loss_L(\sigma_h, \sigma_l)}{\Pr(\hat{s}_B^h|x^H, s_A^l)Loss_H(\sigma_h, \sigma_l) + \Pr(\hat{s}_B^l|x^H, s_A^l)Gain_H(\sigma_h, \sigma_l)}] \\
&= \beta_l(\sigma_h, \sigma_l)\left[\frac{\Pr(x^H|s_A^l)}{\Pr(x^L|s_A^l)} - \psi_l(\sigma_h, \sigma_l)\right] \qquad \text{(B.27)}
\end{aligned}$$

where

$$\begin{aligned}
\beta_l(\sigma_h, \sigma_l) &\equiv \Pr(x^L|s_A^l)[\Pr(\hat{s}_B^h|x^H, s_A^l)Loss_H(\sigma_h, \sigma_l) + \Pr(\hat{s}_B^l|x^H, s_A^l)Gain_H(\sigma_h, \sigma_l)] \\
&> 0 \qquad \text{(B.28)}
\end{aligned}$$

$$\psi_l(\sigma_h, \sigma_l) \equiv \frac{\Pr(\hat{s}_B^h|x^L, s_A^l)Gain_L(\sigma_h, \sigma_l) + \Pr(\hat{s}_B^l|x^L, s_A^l)Loss_L(\sigma_h, \sigma_l)}{\Pr(\hat{s}_B^h|x^H, s_A^l)Loss_H(\sigma_h, \sigma_l) + \Pr(\hat{s}_B^l|x^H, s_A^l)Gain_H(\sigma_h, \sigma_l)} \qquad \text{(B.29)}$$

The intuition of ψ_l is that it is the ratio of the net expected payoff of analyst A when the reported earnings is low and he forecasts high (versus low) to the net expected payoff when the earnings is high, and he forecasts high (versus low). The subscript l in β_l and ψ_l refers to the analyst A's private signal, which is low in this case.

Similarly, when analyst A receives a high signal, his net expected payoff of reporting a high versus low forecast is

$$\begin{aligned}
&\Delta R_{A,h}(\sigma_h,\sigma_l,q) \equiv R_A(\hat{s}_A^h|s_A^h) - R_A(\hat{s}_A^l|s_A^h) \\
&= \Pr(x^H|s_A^h)[\Pr(\hat{s}_B^h|x^H,s_A^h)Loss_H(\sigma_h,\sigma_l) + \Pr(\hat{s}_B^l|x^H,s_A^h)Gain_H(\sigma_h,\sigma_l)] \\
&\quad + \Pr(x^L|s_A^h)[\Pr(\hat{s}_B^h|x^L,s_A^h)Gain_L(\sigma_h,\sigma_l) + \Pr(\hat{s}_B^l|x^L,s_A^h)Loss_L(\sigma_h,\sigma_l)] \\
&= \beta_h(\sigma_h,\sigma_l)[\frac{\Pr(x^H|s_A^h)}{\Pr(x^L|s_A^h)} - \\
&\quad \frac{\Pr(\hat{s}_B^h|x^L,s_A^h)Gain_L(\sigma_h,\sigma_l) + \Pr(\hat{s}_B^l|x^L,s_A^h)Loss_L(\sigma_h,\sigma_l)}{\Pr(\hat{s}_B^h|x^H,s_A^h)Loss_H(\sigma_h,\sigma_l) + \Pr(\hat{s}_B^l|x^H,s_A^h)Gain_H(\sigma_h,\sigma_l)}] \\
&= \beta_h(\sigma_h,\sigma_l)\left[\frac{\Pr(x^H|s_A^h)}{\Pr(x^L|s_A^h)} - \psi_h(\sigma_h,\sigma_l)\right] \qquad \text{(B.30)}
\end{aligned}$$

where

$$\begin{aligned}
\beta_h(\sigma_h,\sigma_l) &\equiv \Pr(x^L|s_A^h)[\Pr(\hat{s}_B^h|x^H,s_A^h)Loss_H(\sigma_h,\sigma_l) + \Pr(\hat{s}_B^l|x^H,s_A^h)Gain_H(\sigma_h,\sigma_l)] \\
&> 0 \qquad \text{(B.31)}
\end{aligned}$$

$$\psi_h(\sigma_h,\sigma_l) \equiv \frac{\Pr(\hat{s}_B^h|x^L,s_A^h)Gain_L(\sigma_h,\sigma_l) + \Pr(\hat{s}_B^l|x^L,s_A^h)Loss_L(\sigma_h,\sigma_l)}{\Pr(\hat{s}_B^h|x^H,s_A^h)Loss_H(\sigma_h,\sigma_l) + \Pr(\hat{s}_B^l|x^H,s_A^h)Gain_H(\sigma_h,\sigma_l)} \qquad \text{(B.32)}$$

Similar to ψ_l, the term ψ_h is the ratio of the net (loss) expected payoff of analyst A when the reported earnings is low and he forecasts high, to his net (gain) expected payoff when the earnings is low and he forecasts high. The subscript h represents the analyst A's private signal, which is high in this case.

Now, I will start proving Claims 8-10.

Proof of Claim 8. When the market and each analyst believes that each analyst is honestly reporting his private signal ($i.e.$, $\hat{s} = s$ or $\sigma_h = 1, \sigma_l = 0$), then an analyst (consider analyst A) with a low signal will forecast low if and only if

$$\begin{aligned}
&\Delta R_{A,l}(\sigma_h = 1, \sigma_l = 0, q) \leqslant 0 \\
&\Rightarrow \beta_l(\sigma_h = 1, \sigma_l = 0)\left[\frac{\Pr(x^H|s_A^l)}{\Pr(x^L|s_A^l)} - \psi_l(\sigma_h = 1, \sigma_l = 0)\right] \leqslant 0 \\
&\Rightarrow \frac{\Pr(x^H|s_A^l)}{\Pr(x^L|s_A^l)} \leqslant \psi_l(\sigma_h = 1, \sigma_l = 0) \text{ since } \beta_l > 0 \text{ by (B.28)} \\
&\Rightarrow q \leqslant \frac{1}{1 + \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1}{\psi_l(\sigma_h=1,\sigma_l=0)}\right)} \equiv q_{rc} \qquad \text{(B.33)}
\end{aligned}$$

Note that since $\psi_l(\sigma_h = 1, \sigma_l = 0) > 0$, $q_{rc} < 1$.

Similarly, given the market's and each analyst's conjecture of honest forecasting, analyst A with a high signal will forecast high if and only if

$$\begin{aligned}
&\Delta R_{A,h}(\sigma_h = 1, \sigma_l = 0, q) \geqslant 0 \\
&\quad\Rightarrow \quad \beta_h(\sigma_h = 1, \sigma_l = 0)\left[\frac{\Pr(x^H|s_A^h)}{\Pr(x^L|s_A^h)} - \psi_h(\sigma_h = 1, \sigma_l = 0)\right] \geqslant 0 \\
&\quad\Rightarrow \quad \frac{\Pr(x^H|s_A^h)}{\Pr(x^L|s_A^h)} \geqslant \psi_h(\sigma_h = 1, \sigma_l = 0) \;\; \text{since } \beta_h > 0 \text{ by (B.31)} \\
&\quad\Rightarrow \quad q \geqslant \frac{1}{1 + \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1}{\psi_h(\sigma_h=1,\sigma_l=0)}\right)} \qquad \text{(B.34)}
\end{aligned}$$

By symmetry, it can be shown that $\frac{1}{1+\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1}{\psi_h(\sigma_h=1,\sigma_l=0)}\right)} = 1 - q_{rc}$. Thus, by (B.33) and (B.34)

$$\begin{aligned}
\Delta R_{A,l}(\sigma_h &= 1, \sigma_l = 0, q) \equiv R_A(\hat{s}_A^h|s_A^l) - R_A(\hat{s}_A^l|s_A^l) \leqslant 0 \text{ if } q \leqslant q_{rc} \\
\Delta R_{A,h}(\sigma_h &= 1, \sigma_l = 0, q) \equiv R_A(\hat{s}_A^h|s_A^h) - R_A(\hat{s}_A^l|s_A^h) \geqslant 0 \text{ if } q \geqslant 1 - q_{rc}.
\end{aligned}$$

This shows that when the market and each analyst believes that each analyst is honestly reporting his private signal (*i.e.*,$\sigma_h = 1, \sigma_l = 0$),then an analyst with a low signal will forecast low if $q \leqslant q_{rc}$, and with a high signal will forecast high if $q \geqslant 1 - q_{rc}$. ■

Proof of Claim 9. As defined in (B.33) in the proof of Claim 8

$$q_{rc} = \frac{1}{1 + (\frac{1-\gamma}{\gamma})\left(\frac{1}{\psi_l(\sigma_h=1,\sigma_l=0)}\right)}$$

Also, by (B.29)

$$\psi_l(\sigma_h = 1, \sigma_l = 0) = \qquad \text{(B.35)}$$
$$\frac{\Pr(s_B^h|x^L, s_A^l)Gain_L(\sigma_h = 1, \sigma_l = 0) + \Pr(s_B^l|x^L, s_A^l)Loss_L(\sigma_h = 1, \sigma_l = 0)}{\Pr(s_B^h|x^H, s_A^l)Loss_H(\sigma_h = 1, \sigma_l = 0) + \Pr(s_B^l|x^H, s_A^l)Gain_H(\sigma_h = 1, \sigma_l = 0)}$$

Now, by equations (B.26), and Property 2 and 5, and after some algebra

$$\begin{aligned} Gain_H(\sigma_h &= 1, \sigma_l = 0) = (\frac{2\upsilon}{\upsilon+1})\left[\frac{\gamma_g - \gamma}{(1-\lambda)\{\gamma_g + \upsilon(1-\gamma_g)\}}\right] \\ &= Gain_L(\sigma_h = 1, \sigma_l = 0) \equiv Gain(\sigma_h = 1, \sigma_l = 0) \end{aligned} \tag{B.36}$$

$$\begin{aligned} Loss_H(\sigma_h &= 1, \sigma_l = 0) = (\frac{2\upsilon}{\upsilon+1})\left[\frac{\gamma_g - \gamma}{(1-\lambda)\{1-\gamma_g + \upsilon\gamma_g\}}\right] \\ &= Loss_L(\sigma_h = 1, \sigma_l = 0) \equiv Loss(\sigma_h = 1, \sigma_l = 0) \end{aligned} \tag{B.37}$$

which leads to

$$\begin{aligned} &\psi_l(\sigma_h = 1, \sigma_l = 0) \\ &= \frac{\Pr(s_B^h|x^L, s_A^l)Gain(\sigma_h = 1, \sigma_l = 0) + \Pr(s_B^l|x^L, s_A^l)Loss(\sigma_h = 1, \sigma_l = 0)}{\Pr(s_B^h|x^H, s_A^l)Loss(\sigma_h = 1, \sigma_l = 0) + \Pr(s_B^l|x^H, s_A^l)Gain(\sigma_h = 1, \sigma_l = 0)} \\ &= \frac{\Pr(s_B^h|x^L, s_A^l)\left(\frac{Gain(\sigma_h=1,\sigma_l=0)}{Loss(\sigma_h=1,\sigma_l=0)}\right) + \Pr(s_B^l|x^L, s_A^l)}{\Pr(s_B^l|x^H, s_A^l)\left(\frac{Gain(\sigma_h=1,\sigma_l=0)}{Loss(\sigma_h=1,\sigma_l=0)}\right) + \Pr(s_B^h|x^H, s_A^l)} \\ &= \frac{\Pr(s_B^h|x^L, s_A^l)\phi(\sigma_h = 1, \sigma_l = 0) + \Pr(s_B^l|x^L, s_A^l)}{\Pr(s_B^l|x^H, s_A^l)\phi(\sigma_h = 1, \sigma_l = 0) + \Pr(s_B^h|x^H, s_A^l)} \end{aligned} \tag{B.38}$$

Now, using Property 5 in the expression of $\psi_l(\sigma_h = 1, \sigma_l = 0)$

$$\begin{aligned} &1 + (\frac{1-\gamma}{\gamma})\left(\frac{1}{\psi_l(\sigma_h = 1, \sigma_l = 0)}\right) \\ &= 1 + (\frac{1-\gamma}{\gamma})\left[\frac{\Pr(s_B^l|x^H, s_A^l)\phi(\sigma_h = 1, \sigma_l = 0) + \Pr(s_B^h|x^H, s_A^l)}{\Pr(s_B^h|x^L, s_A^l)\phi(\sigma_h = 1, \sigma_l = 0) + \Pr(s_B^l|x^L, s_A^l)}\right] \\ &= 1 + (\frac{1-\gamma}{\gamma})\left[\frac{\{1-\gamma+\frac{c}{1-\gamma}\}\phi(\sigma_h = 1, \sigma_l = 0) + \{\gamma - \frac{c}{1-\gamma}\}}{\{1-\gamma-\frac{c}{\gamma}\}\phi(\sigma_h = 1, \sigma_l = 0) + \{\gamma + \frac{c}{\gamma}\}}\right] \\ &= \frac{\gamma + (1-\gamma)\phi(\sigma_h = 1, \sigma_l = 0)}{\gamma^2 + c + \{\gamma(1-\gamma) + c\}\phi(\sigma_h = 1, \sigma_l = 0)} \end{aligned}$$

Thus

$$
\begin{aligned}
q_{rc} \equiv & \frac{1}{1+(\frac{1-\gamma}{\gamma})\left(\frac{1}{\psi_l(\sigma_h=1,\sigma_l=0)}\right)} \\
= & \frac{1}{\frac{\gamma+(1-\gamma)\phi(\sigma_h=1,\sigma_l=0)}{\gamma^2+c+\{\gamma(1-\gamma)+c\}\phi(\sigma_h=1,\sigma_l=0)}} \\
= & \frac{\gamma^2+c+\{\gamma(1-\gamma)+c\}\phi(\sigma_h=1,\sigma_l=0)}{\gamma+(1-\gamma)\phi(\sigma_h=1,\sigma_l=0)} \\
= & \gamma - c\left[\frac{\phi-1}{\phi(1-\gamma)+\gamma}\right] \qquad \text{(B.39)}
\end{aligned}
$$

where, by definition, $c = \lambda^2\gamma_g(1-\gamma_g)$. For notational simplicity, $(\sigma_h = 1, \sigma_l = 0)$ has been dropped from the expression of ϕ.

Now, using the expressions for Gain and Loss from (B.36) and (B.37) and simplifying

$$
\begin{aligned}
\phi \equiv & \frac{Gain(\sigma_h=1,\sigma_l=0)}{Loss(\sigma_h=1,\sigma_l=0)} \\
= & \frac{(\frac{2\upsilon}{\upsilon+1})[\frac{\gamma_g-\gamma}{\gamma_g(1-\gamma)+\upsilon\gamma(1-\gamma_g)-(\upsilon+1)\frac{c}{\lambda}}]}{(\frac{2\upsilon}{\upsilon+1})[\frac{\gamma_g-\gamma}{(1-\gamma_g)\gamma+\upsilon\gamma_g(1-\gamma)-(\upsilon+1)\frac{c}{\lambda}}]} \\
= & \frac{(1-\gamma_g)\gamma+\upsilon\gamma_g(1-\gamma)-(\upsilon+1)\frac{c}{\lambda}}{\gamma_g(1-\gamma)+\upsilon\gamma(1-\gamma_g)-(\upsilon+1)\frac{c}{\lambda}} \\
= & \frac{(1-\gamma_g)\gamma+\upsilon\gamma_g(1-\gamma)-(\upsilon+1)\lambda\gamma_g(1-\gamma_g)}{\gamma_g(1-\gamma)+\upsilon\gamma(1-\gamma_g)-(\upsilon+1)\lambda\gamma_g(1-\gamma_g)} \\
= & \frac{1-\gamma_g+\upsilon\gamma_g}{\gamma_g+\upsilon(1-\gamma_g)} \qquad \text{(B.40)}
\end{aligned}
$$

Thus, by (B.39) and (B.40), I have $q_{rc} = \gamma - \lambda^2\gamma_g(1-\gamma_g)[\frac{\phi-1}{\phi(1-\gamma)+\gamma}]$ where $\phi = \frac{1-\gamma_g+\upsilon\gamma_g}{\gamma_g+\upsilon(1-\gamma_g)}$.

Also, since ϕ is increasing in υ, and $\lim_{\upsilon\to 0}\phi = \frac{1-\gamma_g}{\gamma_g}$ and $\lim_{\upsilon\to\infty}\phi = \frac{\gamma_g}{1-\gamma_g}$

$$
\phi \in (\frac{1-\gamma_g}{\gamma_g}, \frac{\gamma_g}{1-\gamma_g}). \qquad \text{(B.41)}
$$

Now, is $q_{rc} > \frac{1}{2}$ so that $q \in [1 - q_{rc}, q_{rc}]$ is nonempty?

$$
\begin{aligned}
& q_{rc} - \frac{1}{2} \\
= \;& \gamma - c[\frac{\phi - 1}{\phi(1-\gamma)+\gamma}] - \frac{1}{2} \\
= \;& \left(\gamma - \frac{1}{2}\right) - \lambda^2 \gamma_g (1-\gamma_g)[\frac{\phi - 1}{\phi(1-\gamma)+\gamma}] \\
> \;& \left(\gamma - \frac{1}{2}\right) - \lambda^2 \gamma_g (1-\gamma_g)[\frac{\frac{\gamma_g}{1-\gamma_g} - 1}{\frac{\gamma_g}{1-\gamma_g}(1-\gamma)+\gamma}] \qquad \text{(B.42)}
\end{aligned}
$$

since $[\frac{\phi-1}{\phi(1-\gamma)+\gamma}]$ is increasing in $\phi \in (\frac{1-\gamma_g}{\gamma_g}, \frac{\gamma_g}{1-\gamma_g})$ and thus, $[\frac{\phi-1}{\phi(1-\gamma)+\gamma}] < [\frac{\frac{\gamma_g}{1-\gamma_g}-1}{\frac{\gamma_g}{1-\gamma_g}(1-\gamma)+\gamma}]$. Using $\gamma = \frac{1+\lambda(2\gamma_g-1)}{2}$ and after some algebra $\frac{\frac{\gamma_g}{1-\gamma_g}-1}{\frac{\gamma_g}{1-\gamma_g}(1-\gamma)+\gamma} = \frac{2(2\gamma_g-1)}{1-\lambda(2\gamma_g-1)^2}$. Thus

$$
\begin{aligned}
& \left(\gamma - \frac{1}{2}\right) - \lambda^2 \gamma_g (1-\gamma_g)[\frac{\frac{\gamma_g}{1-\gamma_g} - 1}{\frac{\gamma_g}{1-\gamma_g}(1-\gamma)+\gamma}] \\
= \;& \frac{2\gamma - 1}{2} - \frac{2\lambda^2\gamma_g(1-\gamma_g)(2\gamma_g-1)}{1-\lambda(2\gamma_g-1)^2} \\
= \;& \frac{\lambda(2\gamma_g - 1)}{2} - \frac{2\lambda^2\gamma_g(1-\gamma_g)(2\gamma_g-1)}{1-\lambda(2\gamma_g-1)^2} \\
= \;& \frac{\lambda(2\gamma_g - 1)}{2}\left[1 - \frac{4\lambda\gamma_g(1-\gamma_g)}{1-\lambda(2\gamma_g-1)^2}\right] \\
= \;& \frac{\lambda(2\gamma_g - 1)}{2}\left[\frac{1-\lambda}{1-\lambda(2\gamma_g-1)^2}\right] > 0 \qquad \text{(B.43)}
\end{aligned}
$$

which implies $q_{rc} - \frac{1}{2} > 0$ and $q_{rc} > \frac{1}{2}$.

Also, since $q_{rc} \equiv \frac{1}{1+(\frac{1-\gamma}{\gamma})\left(\frac{1}{\psi_l(\sigma_h=1,\sigma_l=0)}\right)}$ and by (B.38) $\psi_l(\sigma_h = 1, \sigma_l = 0) > 0$, $q_{rc} < 1$.Taken together

$$
q_{rc} \in (\frac{1}{2}, 1). \qquad \text{(B.44)}
$$

■

Proof of Claim 10. I will show that if $q > q_{rc}$, then an analyst's optimal reporting strategy will always be to forecast high regardless of his private signal. Knowing this strategy, the market will completely ignore the analyst's forecasts, leading to no information transmission in equilibrium. By an analogous argument, it can be shown that if

$q < 1 - q_{rc}$, then an analyst's optimal reporting strategy will always be to forecast low regardless of his private signals, again, leading to a noninformative equilibrium.

Using (B.27)

$$\begin{aligned} &\Delta R_{A,l}(\sigma_h = 1, \sigma_l, q) \\ &= \beta_l(\sigma_h = 1, \sigma_l)\left[\frac{\Pr(x^H|s_A^l)}{\Pr(x^L|s_A^l)} - \psi_l(\sigma_h = 1, \sigma_l)\right] \end{aligned} \tag{B.45}$$

where

$$\begin{aligned} &\psi_l(\sigma_h = 1, \sigma_l) \equiv \\ &\left[\frac{\Pr(\hat{s}_B^h|x^L, s_A^l)Gain_L(\sigma_h = 1, \sigma_l) + \Pr(\hat{s}_B^l|x^L, s_A^l)Loss_L(\sigma_h = 1, \sigma_l)}{\Pr(\hat{s}_B^h|x^H, s_A^l)Loss_H(\sigma_h = 1, \sigma_l) + \Pr(\hat{s}_B^l|x^H, s_A^l)Gain_H(\sigma_h = 1, \sigma_l)}\right] \end{aligned} \tag{B.46}$$

Now, by equations (B.26), and Property 2, and after some algebra

$$\begin{aligned} Gain_H(\sigma_h &= 1, \sigma_l) = (\frac{2\upsilon}{\upsilon+1})\left[\frac{\gamma_g - \gamma}{(1-\lambda)\{\gamma_g + \upsilon(1-\gamma_g)\} + (1+\lambda)(1+\upsilon)(1-\gamma_g)\sigma_l}\right] \\ Loss_H(\sigma_h &= 1, \sigma_l) = (\frac{2\upsilon}{\upsilon+1})\left[\frac{\gamma_g - \gamma}{(1-\lambda)\{1-\gamma_g + \upsilon\gamma_g\} + (1+\lambda)(1+\upsilon)(1-\gamma_g)\sigma_l}\right] \\ Gain_L(\sigma_h &= 1, \sigma_l) = (\frac{2\upsilon}{\upsilon+1})\left[\frac{\gamma_g - \gamma}{(1-\lambda)\{\gamma_g + \upsilon(1-\gamma_g)\} + (1+\lambda)(1+\upsilon)\gamma_g\sigma_l}\right] \\ Loss_L(\sigma_h &= 1, \sigma_l) = (\frac{2\upsilon}{\upsilon+1})\left[\frac{\gamma_g - \gamma}{(1-\lambda)\{1-\gamma_g + \upsilon\gamma_g\} + (1+\lambda)(1+\upsilon)\gamma_g\sigma_l}\right] \end{aligned} \tag{B.47}$$

Also, for $(\sigma_h = 1, \sigma_l)$

$$\begin{aligned} \Pr(\hat{s}_B^h|x^H, s_A^l) &= \Pr(s_B^h|x^H, s_A^l) + \Pr(s_B^l|x^H, s_A^l)\sigma_l \\ \Pr(\hat{s}_B^l|x^H, s_A^l) &= \Pr(s_B^l|x^H, s_A^l)(1-\sigma_l) \\ \Pr(\hat{s}_B^h|x^L, s_A^l) &= \Pr(s_B^h|x^L, s_A^l) + \Pr(s_B^l|x^L, s_A^l)\sigma_l \\ \Pr(\hat{s}_B^l|x^L, s_A^l) &= \Pr(s_B^l|x^L, s_A^l)(1-\sigma_l) \end{aligned} \tag{B.48}$$

Using Property 5

$$\begin{aligned} \Pr(s_B^h|x^H, s_A^l) &= \gamma - \frac{c}{1-\gamma};\ \Pr(s_B^l|x^H, s_A^l) = 1 - \gamma + \frac{c}{1-\gamma} \\ \Pr(s_B^h|x^L, s_A^l) &= 1 - \gamma - \frac{c}{\gamma};\ \Pr(s_B^l|x^L, s_A^l) = \gamma + \frac{c}{\gamma} \end{aligned} \tag{B.49}$$

Now, using (B.46)-(B.49), and after some algebra, it can be shown (proved in Claim 10.1 below) that $\psi_l(\sigma_h = 1, \sigma_l)$ reaches maximum at $\sigma_l = 0$.

Claim 10.1: $\psi_l(\sigma_h = 1, \sigma_l)$ reaches its global maximum at $\sigma_l = 0$.

Sketch of Proof. By definition (to avoid notational clutter, I omit $(\sigma_h = 1, \sigma_l)$ in the parenthesis)

$$\begin{aligned}
\psi_l \equiv & \left[\frac{\Pr(\hat{s}_B^h|x^L, s_A^l)Gain_L + \Pr(\hat{s}_B^l|x^L, s_A^l)Loss_L}{\Pr(\hat{s}_B^h|x^H, s_A^l)Loss_H + \Pr(\hat{s}_B^l|x^H, s_A^l)Gain_H}\right] \\
= & \left[\frac{Loss_L}{Loss_H}\right]\left[\frac{\Pr(\hat{s}_B^h|x^L, s_A^l)\left(\frac{Gain_L}{Loss_L}\right) + \Pr(\hat{s}_B^l|x^L, s_A^l)}{\Pr(\hat{s}_B^h|x^H, s_A^l) + \Pr(\hat{s}_B^l|x^H, s_A^l)\left(\frac{Gain_H}{Loss_H}\right)}\right] \\
= & \left[\frac{Loss_L}{Loss_H}\right]\left[\frac{\Pr(\hat{s}_B^h|x^L, s_A^l)\phi_L + \Pr(\hat{s}_B^l|x^L, s_A^l)}{\Pr(\hat{s}_B^h|x^H, s_A^l) + \Pr(\hat{s}_B^l|x^H, s_A^l)\phi_H}\right] \\
= & [\text{Part}_1]\,[\text{Part}_2]
\end{aligned} \tag{B.50}$$

where

$$\begin{aligned}
\text{Part}_1 \equiv & \frac{Loss_L}{Loss_H} \\
= & \frac{(1-\lambda)\{1-\gamma_g+\upsilon\gamma_g\} + (1+\lambda)(1+\upsilon)(1-\gamma_g)\sigma_l}{(1-\lambda)\{1-\gamma_g+\upsilon\gamma_g\} + (1+\lambda)(1+\upsilon)\gamma_g\sigma_l}
\end{aligned} \tag{B.51}$$

$$\begin{aligned}
\text{Part}_2 \equiv & \frac{\Pr(\hat{s}_B^h|x^L, s_A^l)\phi_L + \Pr(\hat{s}_B^l|x^L, s_A^l)}{\Pr(\hat{s}_B^h|x^H, s_A^l) + \Pr(\hat{s}_B^l|x^H, s_A^l)\phi_H} \\
= & \left[\frac{\Pr(\hat{s}_B^l|x^L, s_A^l)}{\Pr(\hat{s}_B^l|x^H, s_A^l)}\right]\left[\frac{\frac{\Pr(\hat{s}_B^h|x^L,s_A^l)}{\Pr(\hat{s}_B^l|x^L,s_A^l)}\phi_L + 1}{\frac{\Pr(\hat{s}_B^h|x^H,s_A^l)}{\Pr(\hat{s}_B^l|x^H,s_A^l)} + \phi_H}\right] \\
= & \left[\frac{\Pr(s_B^l|x^L, s_A^l)(1-\sigma_l)}{\Pr(s_B^l|x^H, s_A^l)(1-\sigma_l)}\right]\left[\frac{\left(\frac{1-\Pr(\hat{s}_B^l|x^L,s_A^l)}{\Pr(\hat{s}_B^l|x^L,s_A^l)}\right)\phi_L + 1}{\left(\frac{1-\Pr(\hat{s}_B^l|x^H,s_A^l)}{\Pr(\hat{s}_B^l|x^H,s_A^l)}\right) + \phi_H}\right] \\
= & \left[\frac{\Pr(s_B^l|x^L, s_A^l)}{\Pr(s_B^l|x^H, s_A^l)}\right]\left[\frac{\left(\frac{1}{\Pr(\hat{s}_B^l|x^L,s_A^l)} - 1\right)\phi_L + 1}{\left(\frac{1}{\Pr(\hat{s}_B^l|x^H,s_A^l)} - 1\right) + \phi_H}\right] \\
= & \left[\frac{\gamma + \frac{c}{\gamma}}{1-\gamma+\frac{c}{1-\gamma}}\right]\left[\frac{\left(\frac{1}{\left(\gamma+\frac{c}{\gamma}\right)(1-\sigma_l)} - 1\right)\phi_L + 1}{\left(\frac{1}{\left(1-\gamma+\frac{c}{1-\gamma}\right)(1-\sigma_l)} - 1\right) + \phi_H}\right]
\end{aligned} \tag{B.52}$$

and

$$\phi_H = \frac{Gain_H}{Loss_H} = \frac{(1-\lambda)\{1-\gamma_g+\upsilon\gamma_g\}+(1+\lambda)(1+\upsilon)(1-\gamma_g)\sigma_l}{(1-\lambda)\{\gamma_g+\upsilon(1-\gamma_g)\}+(1+\lambda)(1+\upsilon)(1-\gamma_g)\sigma_l} \quad \text{(B.53)}$$

$$\phi_L = \frac{Gain_L}{Loss_L} = \frac{(1-\lambda)\{1-\gamma_g+\upsilon\gamma_g\}+(1+\lambda)(1+\upsilon)\gamma_g\sigma_l}{(1-\lambda)\{\gamma_g+\upsilon(1-\gamma_g)\}+(1+\lambda)(1+\upsilon)\gamma_g\sigma_l} \quad \text{(B.54)}$$

Also by (B.50)

$$\frac{\partial\psi_l}{\partial\sigma_l} = [\text{Part}_2]\left[\frac{\partial \text{Part}_1}{\partial\sigma_l}\right] + [\text{Part}_1]\left[\frac{\partial \text{Part}_2}{\partial\sigma_l}\right]$$

It is easy to see that both Part_1 and Part_2 are positive and $\frac{\partial \text{Part}_1}{\partial\sigma_l} < 0$. Thus, if $\frac{\partial \text{Part}_2}{\partial\sigma_l} \leqslant 0$, then $\frac{\partial\psi_l}{\partial\sigma_l} < 0$. However, if $\frac{\partial \text{Part}_2}{\partial\sigma_l} > 0$, then $sgn\left(\frac{\partial\psi_l}{\partial\sigma_l}\right)$ will depend on the relative values of $[\text{Part}_2]\left[\frac{\partial \text{Part}_1}{\partial\sigma_l}\right]$ and $[\text{Part}_1]\left[\frac{\partial \text{Part}_2}{\partial\sigma_l}\right]$. Note also that $\frac{\partial\phi_H}{\partial\sigma_l}, \frac{\partial\phi_L}{\partial\sigma_l} > 0$ if $\upsilon < 1$ and $\frac{\partial\phi_H}{\partial\sigma_l}, \frac{\partial\phi_L}{\partial\sigma_l} < 0$ if $\upsilon > 1$. Thus, I will consider three cases.

Case 1: $\upsilon = 1$. It is easy to show that $\text{Part}_2 = 1$. Thus, $\frac{\partial\psi_l}{\partial\sigma_l} < 0$, and ψ_l achieves its global maximum at $\sigma_l = 0$.

Case 2: $\upsilon < 1$. Since the first component of Part_2 (see (B.52)) does not depend on σ_l, $sgn\left(\frac{\partial \text{Part}_2}{\partial\sigma_l}\right)$ will depend on $sgn\left(\frac{\partial}{\partial\sigma_l}\left[\frac{\left(\frac{1}{\left(\gamma+\frac{c}{\gamma}\right)(1-\sigma_l)}-1\right)\phi_L+1}{\left(\frac{1}{\left(1-\gamma+\frac{c}{1-\gamma}\right)(1-\sigma_l)}-1\right)+\phi_H}\right]\right)$. The expressions $\left(\frac{1}{\left(\gamma+\frac{c}{\gamma}\right)(1-\sigma_l)}-1\right)$ and $\left(\frac{1}{\left(1-\gamma+\frac{c}{1-\gamma}\right)(1-\sigma_l)}-1\right)$ are increasing and convex in σ_l. However, with $\upsilon < 1$, and thus, $\phi < 1$, the rate of increase of the expression $\left(\frac{1}{\left(\gamma+\frac{c}{\gamma}\right)(1-\sigma_l)}-1\right)\phi_L$ in the numerator will be somewhat damped compared to the similar expression in the denominator, leading to $\left[\frac{\left(\frac{1}{\left(\gamma+\frac{c}{\gamma}\right)(1-\sigma_l)}-1\right)\phi_L+1}{\left(\frac{1}{\left(1-\gamma+\frac{c}{1-\gamma}\right)(1-\sigma_l)}-1\right)+\phi_H}\right]$, as a whole, being decreasing in σ_l,which, in turn, makes $\frac{\partial \text{Part}_2}{\partial\sigma_l} < 0$. Thus, $\frac{\partial\psi_l}{\partial\sigma_l} < 0$, and ψ_l achieves its global maximum at $\sigma_l = 0$.

Case 3: $\upsilon > 1$. In this case, $\left[\frac{\left(\frac{1}{\left(\gamma+\frac{c}{\gamma}\right)(1-\sigma_l)}-1\right)\phi_L+1}{\left(\frac{1}{\left(1-\gamma+\frac{c}{1-\gamma}\right)(1-\sigma_l)}-1\right)+\phi_H}\right]$ will increase with σ_l , except when λ is very low, and υ is very high, in which case the expression first decreases and then increases with σ_l. Overall, ψ_l either strictly decreases with σ_l or first decreases and then increases with σ_l. If ψ_l strictly decreases with σ_l, then, like cases 1 and 2, ψ_l achieves its global maximum at $\sigma_l = 0$. Even in the case when ψ_l first decreases and then increases with σ_l, among two possible maxima – that is, $\sigma_l = 0$ or 1 – it can be shown that $\psi_l(\sigma_h = 1, \sigma_l = 0) > \psi_l(\sigma_h = 1, \sigma_l = 1)$, implying that ψ_l achieves its global maximum at $\sigma_l = 0$.

Thus, ψ_l achieves its global maximum at $\sigma_l = 0$ for all possible values of λ, γ_g and υ. ■

I am now back to proving Claim 10. By Claim 8

$$\begin{aligned} &\Delta R_{A,l}(\sigma_h = 1, \sigma_l = 0, q = q_{rc}) = 0 \\ &\Rightarrow \quad \beta_l(\sigma_h = 1, \sigma_l = 0)\left[\frac{\Pr(x^H|s_A^l; q = q_{rc})}{\Pr(x^L|s; q = q_{rc})} - \psi_l(\sigma_h = 1, \sigma_l = 0)\right] = 0 \end{aligned}$$

which implies

$$\frac{\Pr(x^H|s_A^l; q = q_{rc})}{\Pr(x^L|s_A^l; q = q_{rc})} - \psi_l(\sigma_h = 1, \sigma_l = 0) = 0$$

However, since $\psi_l(\sigma_h = 1, \sigma_l)$ achieves its maximum at $\sigma_l = 0$

$$\frac{\Pr(x^H|s_A^l; q = q_{rc})}{\Pr(x^L|s_A^l; q = q_{rc})} - \psi_l(\sigma_h = 1, \sigma_l \in [0,1]) \geqslant 0$$

Furthermore, since $\left[\frac{\Pr(x^H|s_A^l)}{\Pr(x^L|s_A^l)}\right]$ is strictly increasing in q

$$\frac{\Pr(x^H|s_A^l; q > q_{rc})}{\Pr(x^L|s_A^l; q > q_{rc})} - \psi_l(\sigma_h = 1, \sigma_l \in [0,1]) > 0$$

Also, since $\beta_l(\sigma_h = 1, \sigma_l) > 0$

$$\begin{aligned} \Delta R_{A,l}(\sigma_h &= 1, \sigma_l, q > q_{rc}) \\ &= \beta_l(\sigma_h = 1, \sigma_l)\left[\frac{\Pr(x^H|s_A^l; q > q_{rc})}{\Pr(x^L|s_A^l; q > q_{rc})} - \psi_l(\sigma_h = 1, \sigma_l)\right] > 0 \end{aligned} \tag{B.55}$$

Again, by Claim 8

$$\begin{aligned} &\Delta R_{A,h}(\sigma_h = 1, \sigma_l = 0, q = 1 - q_{rc}) = 0 \\ &\Rightarrow \quad \beta_h(\sigma_h = 1, \sigma_l = 0)\left[\frac{\Pr(x^H|s_A^h; q = 1 - q_{rc})}{\Pr(x^L|s_A^h; q = 1 - q_{rc})} - \psi_h(\sigma_h = 1, \sigma_l = 0)\right] = 0 \end{aligned}$$

Similar to the argument to get (B.55)

$$\begin{aligned}\Delta R_{A,h}(\sigma_h &= 1, \sigma_l, q) = \beta_h(\sigma_h = 1, \sigma_l)\left[\frac{\Pr(x^H|s_A^h)}{\Pr(x^L|s_A^h)} - \psi_h(\sigma_h = 1, \sigma_l)\right] > 0 \\ \text{if } q &> 1 - q_{rc}.\end{aligned}$$

Since $q_{rc} > \frac{1}{2}$, and thus, $q_{rc} > 1 - q_{rc}$, I have

$$\begin{aligned}\Delta R_{A,h}(\sigma_h &= 1, \sigma_l, q) = \beta_h(\sigma_h = 1, \sigma_l)\left[\frac{\Pr(x^H|s_A^h)}{\Pr(x^L|s_A^h)} - \psi_h(\sigma_h = 1, \sigma_l)\right] > 0 \\ \text{if } q &> q_{rc}\end{aligned} \tag{B.56}$$

By (B.55) and (B.56), if $q > q_{rc}$

$$\begin{aligned}\Delta R_{A,l}(\sigma_h &= 1, \sigma_l, q) \equiv R_A(\hat{s}_A^h|s_A^l) - R_A(\hat{s}_A^l|s_A^l) > 0 \\ \Delta R_{A,h}(\sigma_h &= 1, \sigma_l, q) \equiv R_A(\hat{s}_A^h|s_A^h) - R_A(\hat{s}_A^l|s_A^h) > 0\end{aligned}$$

and the strategy pair $(\sigma_h = 1, \sigma_l = 1)$ is an equilibrium. Thus, when the prior is sufficiently high (*i.e.*, $q > q_{rc}$), an analyst will always forecast high regardless of his private signal. By symmetry, it can be shown that if the prior is sufficiently low (*i.e.*, $q < 1 - q_{rc}$), then an analyst will always forecast low regardless of his private signal. In both the cases, the equilibrium is noninformative. ■

Finally, I return to completing the proof of Proposition 5. Part (i) follows directly from Claim 8 and 9. Part (ii) follows from Claim 10. ■

Proofs of Corollary 1 and 2. By Claim 10

$$q_{rc} = \gamma - \lambda^2\gamma_g(1-\gamma_g)[\frac{\phi - 1}{\phi(1-\gamma)+\gamma}]$$

It is easy to see that

$$q_{rc} = \gamma \text{ if either } \phi = 1,\ c \to 0 \text{ or both.}$$

However, if $q_{rc} = \gamma$, then each analyst's equilibrium behavior is the same as that in the baseline reputation model with only one analyst. ■

Proof of Proposition 6. By Proposition 5

$$q_{rc} = \gamma - c\left[\frac{\phi - 1}{\phi(1-\gamma)+\gamma}\right]$$

where ϕ is a function of υ. By Chain Rule

$$\frac{\partial}{\partial \upsilon} q_{rc} = \left[\frac{\partial q_{rc}}{\partial \phi}\right]\left[\frac{\partial \phi}{\partial \upsilon}\right]$$

Since $\frac{\partial}{\partial \phi}\left[\frac{\phi-1}{\phi(1-\gamma)+\gamma}\right] > 0$

$$\frac{\partial q_{rc}}{\partial \phi} = \frac{\partial}{\partial \phi}\left[\gamma - \lambda^2 \gamma_g (1-\gamma_g)\left(\frac{\phi-1}{\phi(1-\gamma)+\gamma}\right)\right] < 0$$

Also, by Lemma 5, and more specifically, in this case

$$\frac{\partial \phi}{\partial \upsilon} = \frac{\partial}{\partial \upsilon}\left[\frac{1-\gamma_g+\upsilon\gamma_g}{\gamma_g+\upsilon(1-\gamma_g)}\right] > 0.$$

Taken together

$$\frac{\partial}{\partial \upsilon} q_{rc} = \left[\frac{\partial q_{rc}}{\partial \phi}\right]\left[\frac{\partial \phi}{\partial \upsilon}\right] < 0. \tag{B.57}$$

To prove part (ii), note that

$$\begin{aligned}
\frac{\partial}{\partial c} q_{rc} &= -\frac{\partial}{\partial c} c\left[\frac{\phi-1}{\phi(1-\gamma)+\gamma}\right] \\
&= \frac{1-\phi}{\phi(1-\gamma)+\gamma} < 0 \text{ if } \phi > 1 \text{ and } > 0 \text{ if } \phi < 1 \qquad \text{(B.58)}
\end{aligned}$$

However, since $\phi > 1 \Leftrightarrow \upsilon > 1$ and $\phi < 1 \Leftrightarrow \upsilon < 1$, the result follows. ■

Appendix C

Figures

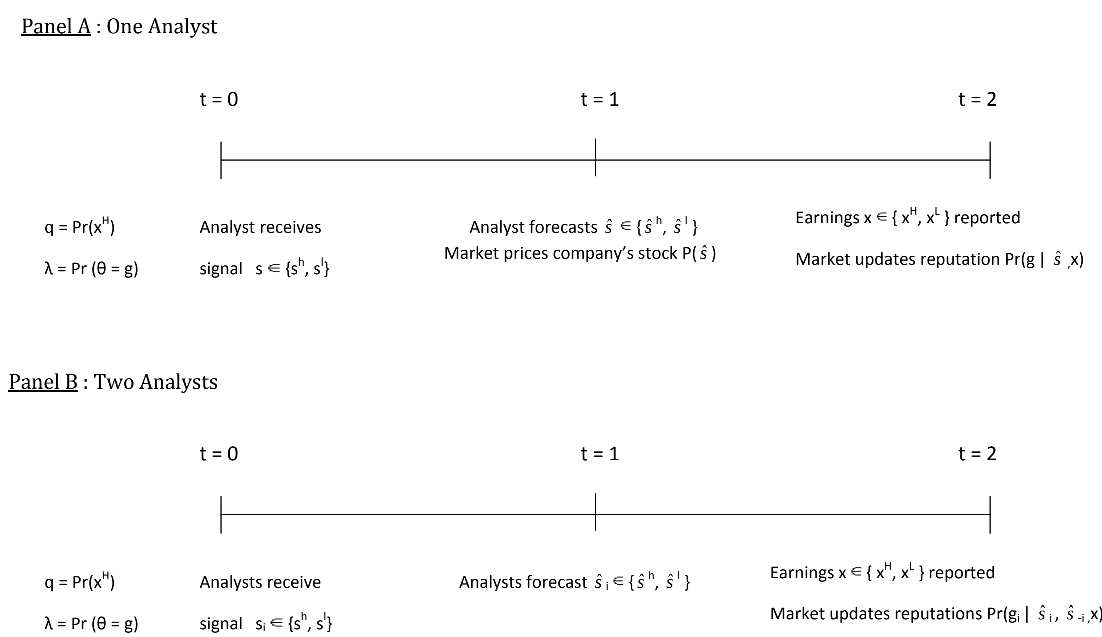


Figure C.1: Sequence of Events

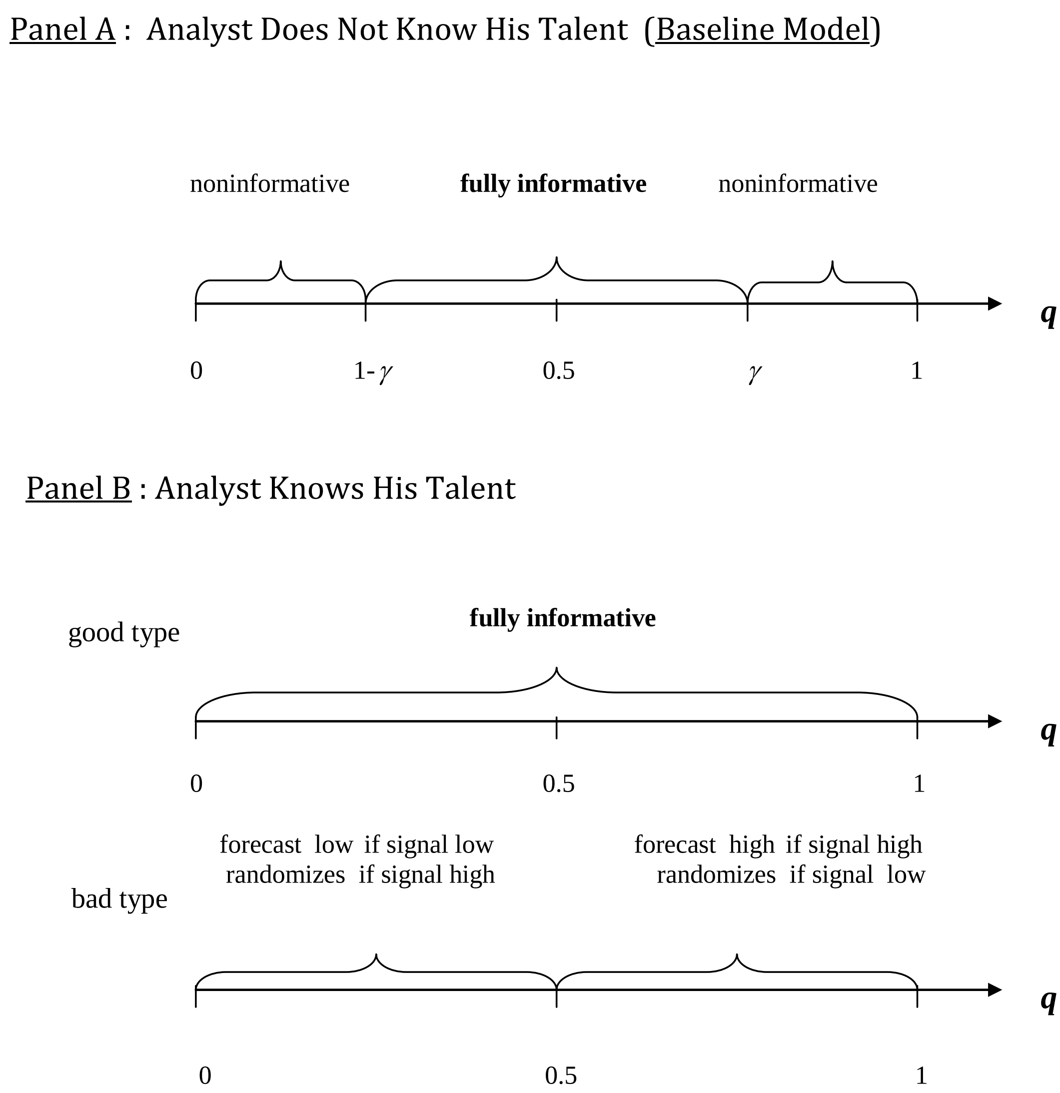
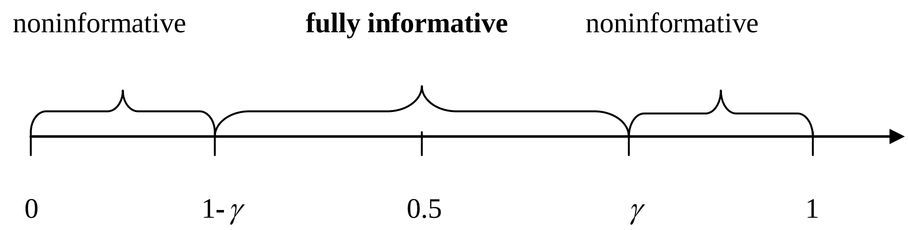
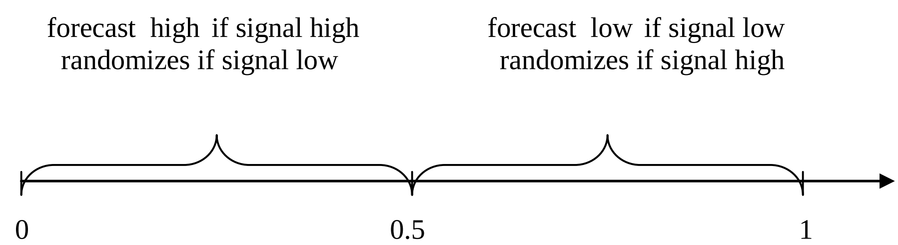


Figure C.2: Equilibrium with Only Reputation Motive

Panel A: Reputation Motive (Baseline Model)



Panel B: Price-Impact Motive



Panel C: Both Price-Impact & Reputation Motives

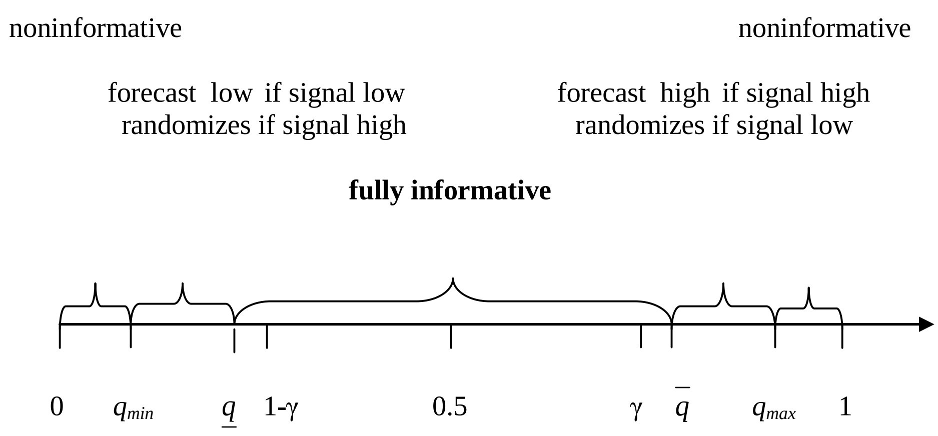


Figure C.3: Equilibrium Regions (Single Analyst)

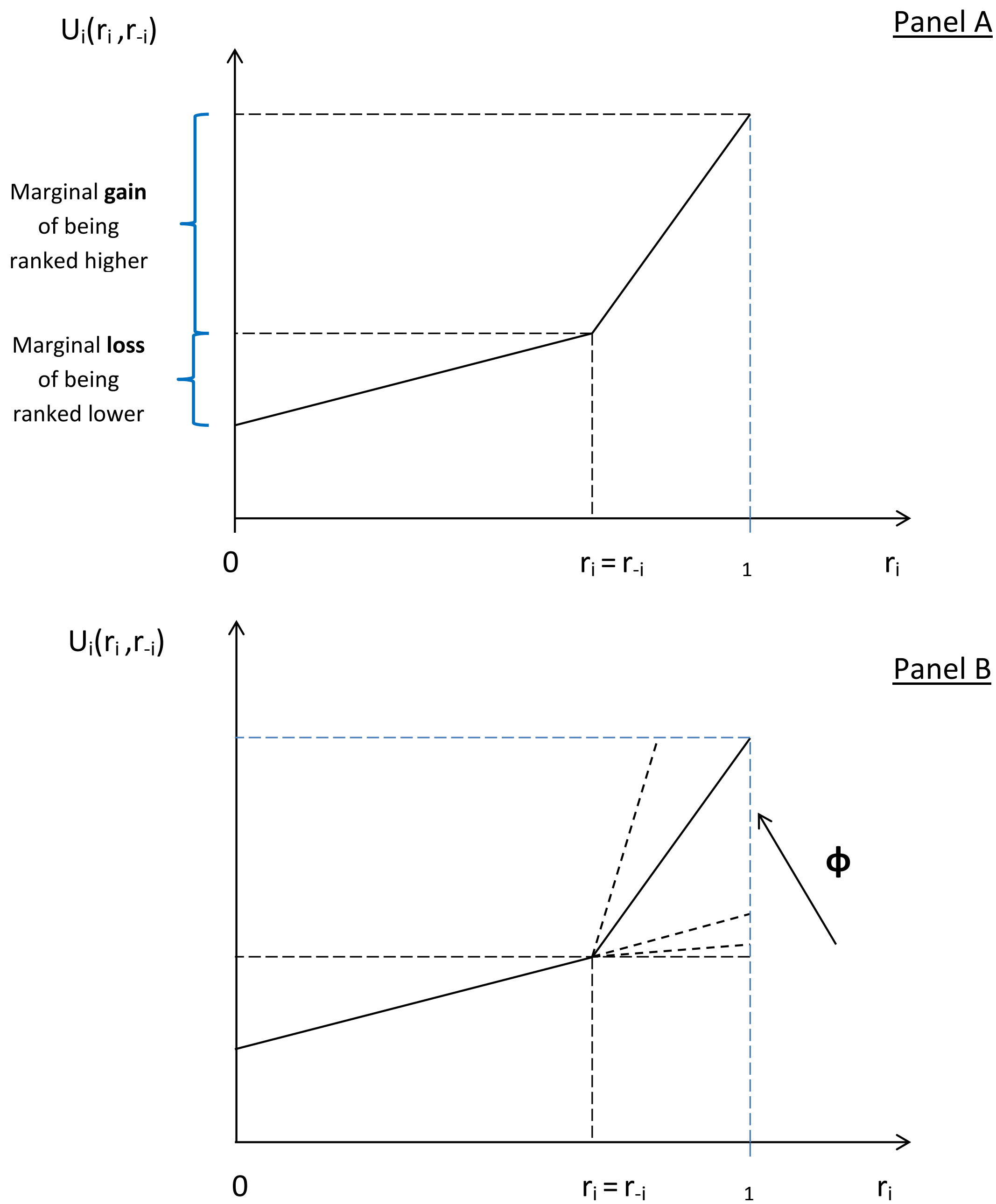


Figure C.4: Convexity of Reputational Ranking Payoff Function

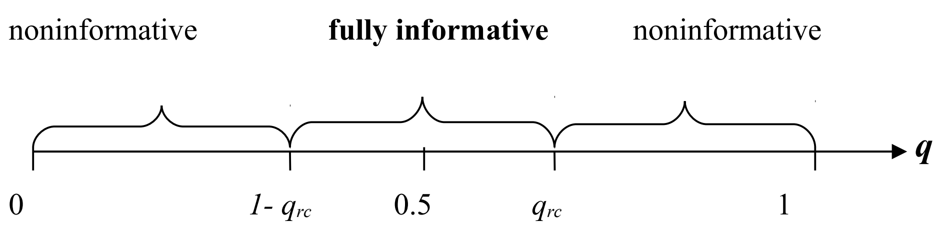


Figure C.5: Equilibrium with Reputational Ranking Motive