Topological density wave states of non-zero angular momentum

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- Classification of density waves by angular momentum
- A perspective on d-density wave
- Mixed singlet-triplet order parameter
- Topological invariants
- Bulk-edge correspondence
- Phenemenology

- Pseudogap may be susceptible to a host of competing orders

- Interesting to explore interesting order parameters of unconventional symmetry

- Shall consider an order parameter closely related to singlet DDW that retains some of its primary signatures--broken translational symmetry implying a gap and a particle-hole condensate of non-zero angular momentum--but has important topological properties similar to recently discovered topological insulators

- May be even useful for URu_2Si_2

Nayak 2000

Spin singlet: $\langle \psi^\dagger_\sigma(k+Q)\psi_{\sigma'}(k)\rangle = \Phi_Q f(k)\delta_{\sigma,\sigma'}$

$f(k)$ an element of some representation of the space group of Q

Spin triplet: $\langle \psi^\dagger_\sigma(k+Q)\psi_{\sigma'}(k)\rangle = \vec{\Phi}_Q(k)\cdot\vec{\tau}_{\sigma,\sigma'}$

p-wave $\sin k_x,\ \sin k_y,\ \sin k_x \pm i \sin k_y$

d-wave

$\cos k_x - \cos k_y,\ \sin k_x \sin k_y,\ \cos k_x - \cos k_y \pm i \sin k_x \sin k_y$

$Q \equiv Q_0 = (\pi,\pi),$ **two-fold commensurate**

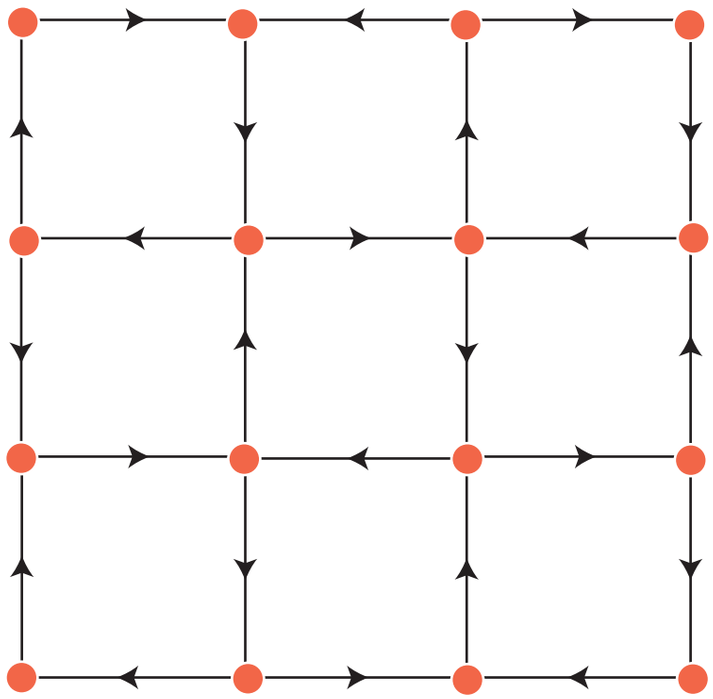
$Q \equiv Q_0 - \pi(2\times\frac{1}{8},0)$ **4-fold commensurate**

$Q \equiv Q_0 - \pi(2\eta,0)$ η **irrational, incommensurate**

Broken Symmetries

Singlet d-density wave order parameter--particle-hole condensate corresponding to circulating charge currents

Breaks time-reversal, parity, translation by a lattice spacing, and rotation by $\pi/2$



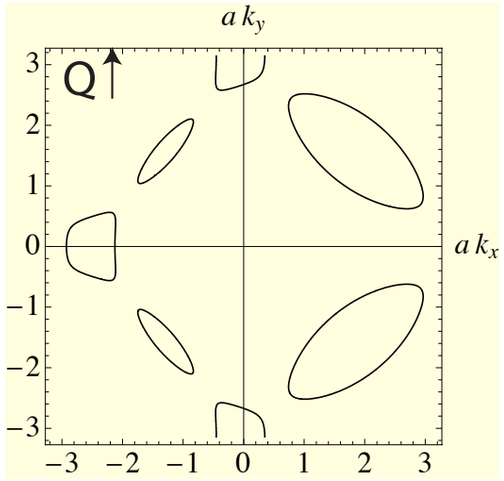
Estimated magnetic field is about 10 G based on the magnitude of the pseudogap

Triplet d-density wave does not break time reversal but breaks spin rotational symmetry corresponding to staggered circulating spin currents

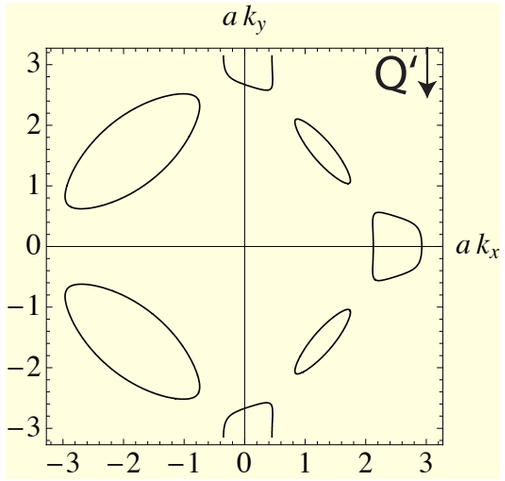
Strict incommensurate order

$$\eta = 0.09$$

$\mathbf{Q} = (\pi/a, \pi/a) - \pi(2\eta, 0)/a$



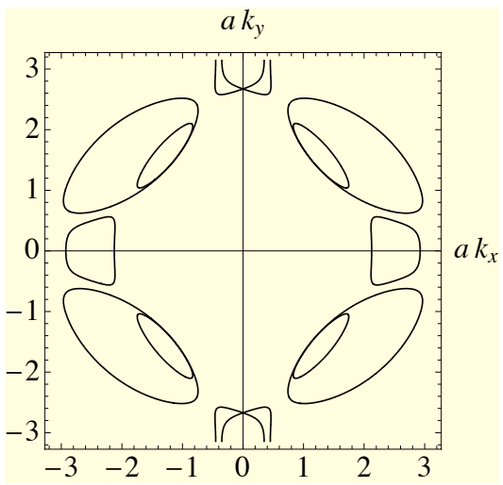
(a)



(b)

$\mathbf{Q}' = (\pi/a, \pi/a) + \pi(2\eta, 0)/a$

I. Dimov,
P. Goswami
X. Jia,
S. C.
Phys. Rev. B 78, 134529 (2008)



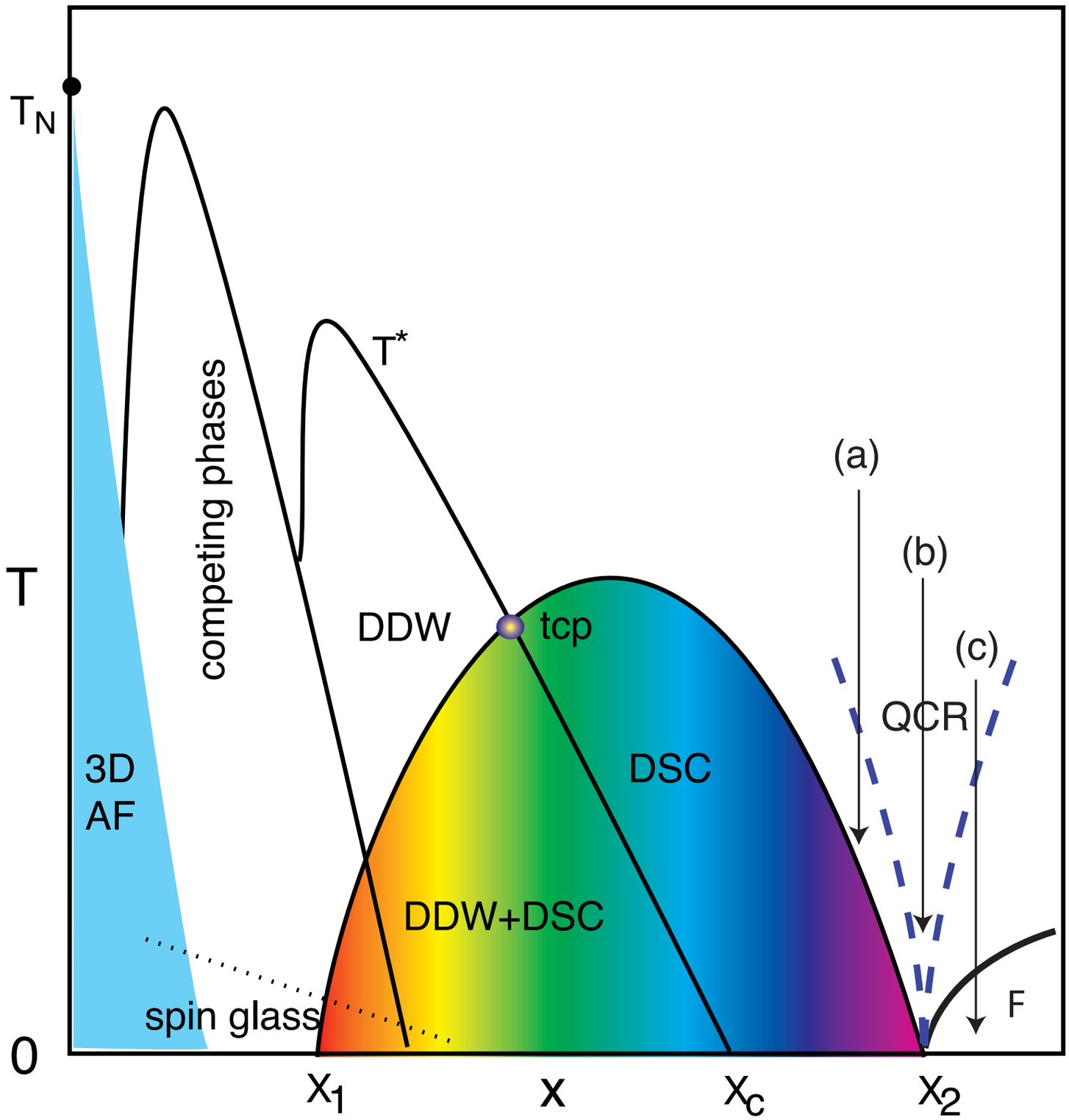
(c)

Note that both inversion and time reversal are broken separately but the product is preserved

The results are essentially identical to spiral SDW

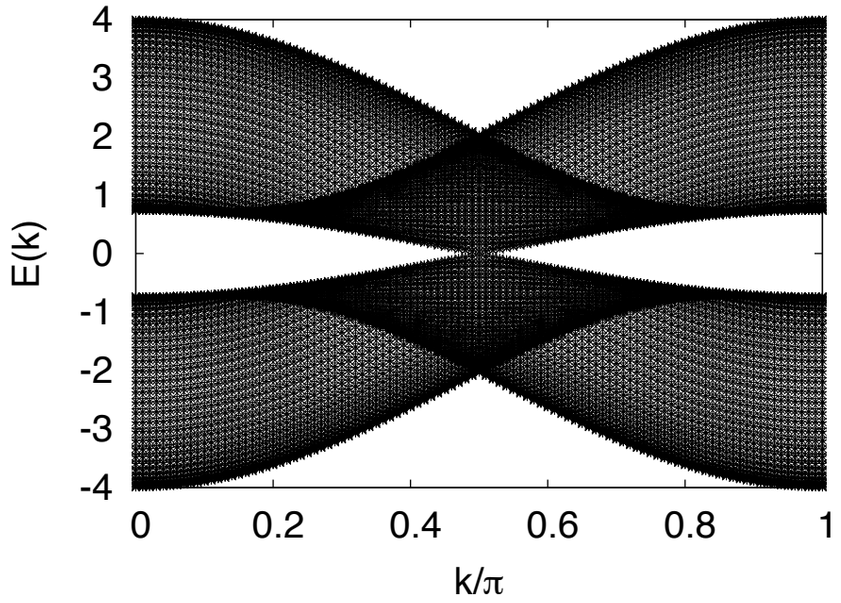
Could possibly be detected in spin resolved ARPES.

DDW exists in an intermediate range of doping



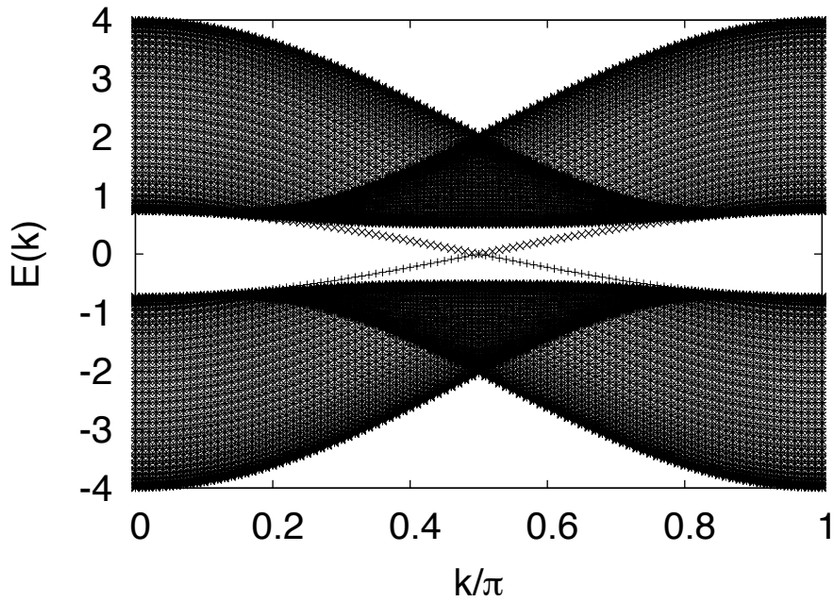
Trivial versus non-trivial topologies density waves

Singlet $id_{x^2-y^2}$ Trivial topology



Singlet $id_{x^2-y^2} + d_{xy}$ Non-trivial topology

S. Tewari, C. Zhang, V. Yakovenko, S. Das Sarma
PRL 100, 217004 (2008)
C. Zhang, S. Tewari, V. Yakovenko, S. Das Sarma PRB 78, 174508 (2008)

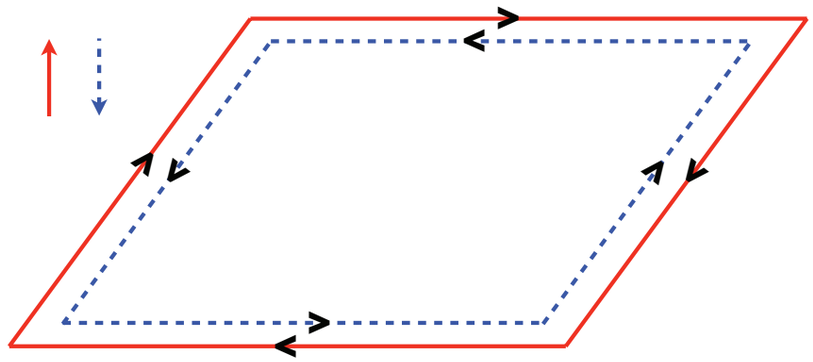


Mixed triplet-singlet order parameter $i\sigma d_{x^2-y^2} + d_{xy}$

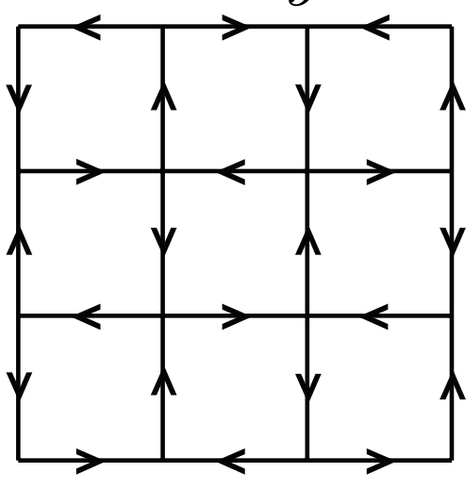
$\langle c^\dagger_{k+Q,\sigma} c_{k,\sigma'} \rangle = (\Phi^\mu(k)\tau^\mu)_{\sigma\sigma'}$ $\tau_0 = I$ $\tau_1,\ \tau_2,\ \tau_3$ **Pauli matrices**

$\Phi^3(k) \propto i\frac{W_0}{2}(\cos k_x - \cos k_y) \equiv iW_k$

$\Phi^0(k) \propto \Delta_0 \sin k_x \sin k_y \equiv \Delta_k$



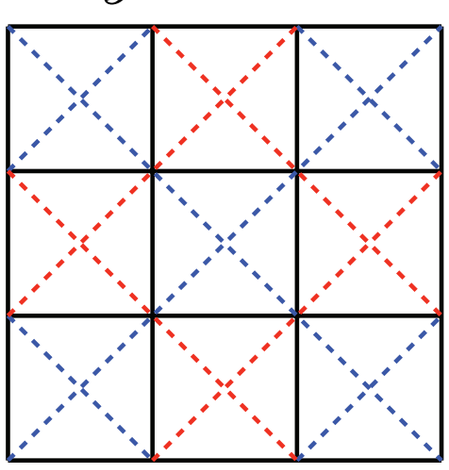
$i\sigma d_{x^2-y^2}$



(a)

circulating spin current

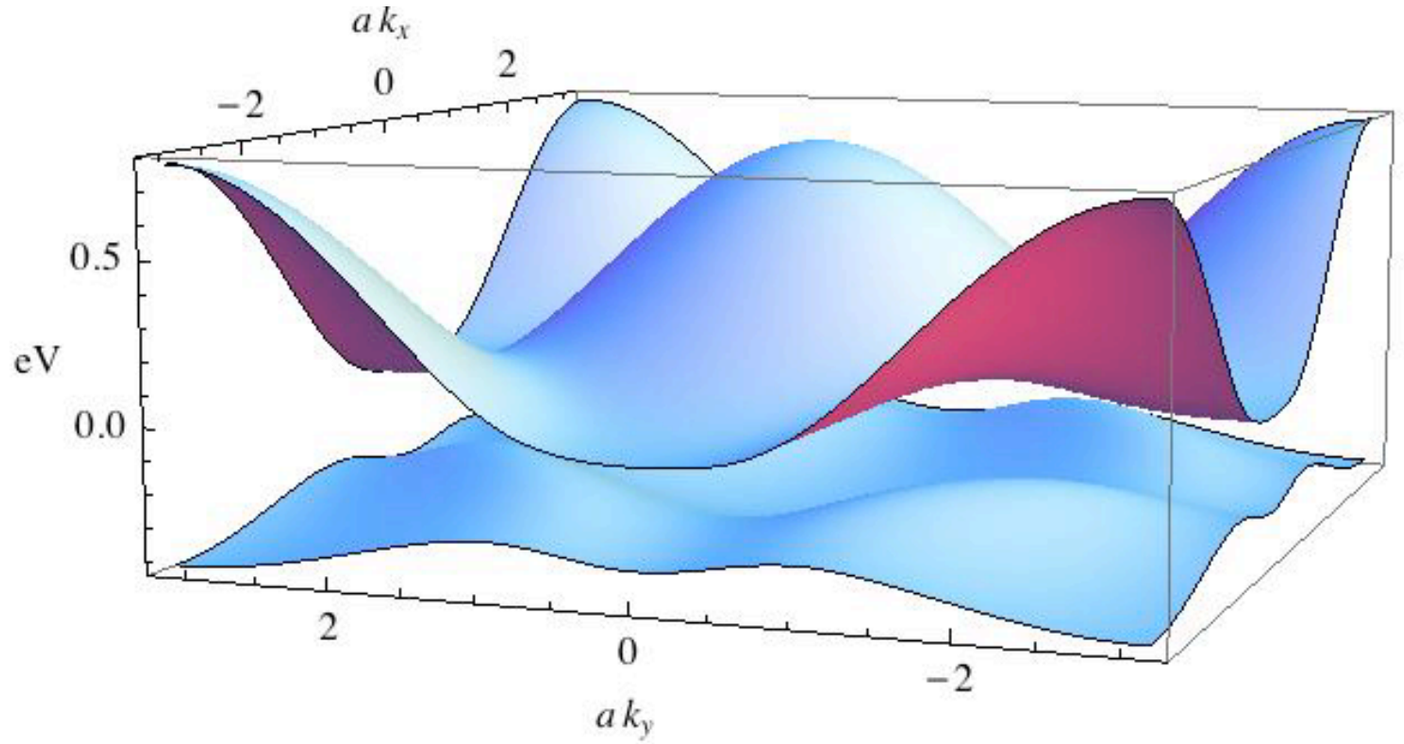
d_{xy}



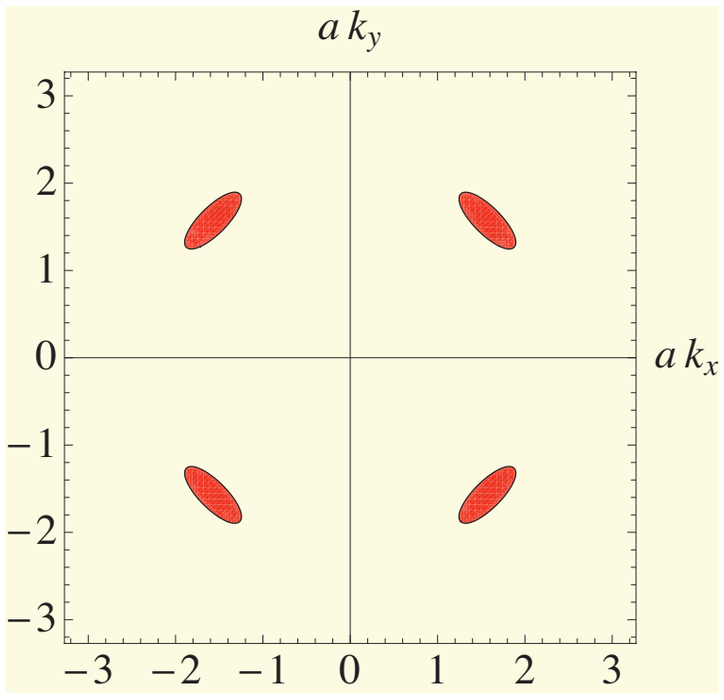
(b)

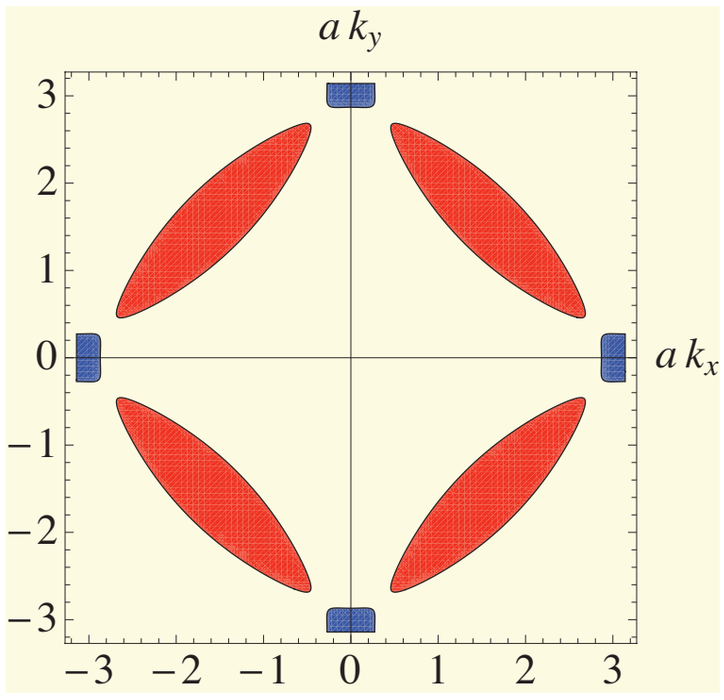
modulation of nn hopping

Band structure



Lifshitz transitions





Berry curvature

$$N_{\sigma,\pm} = \int_{RBZ} \frac{d^2k}{2\pi} \Omega_{\sigma,\pm}$$

$$= \pm\sigma \int_{RBZ} \frac{d^2k}{2\pi} \frac{tW_0\Delta_0}{E_k^3} (\sin^2 k_y + \sin^2 k_x \cos^2 k_y)$$

$$= \pm\sigma.$$

$+$: upper band

$-$: lower band

$$E_k = \sqrt{\epsilon_{1k}^2 + W_k^2 + \Delta_k^2}$$

Vanishes unless both components of the order parameter are non-zero

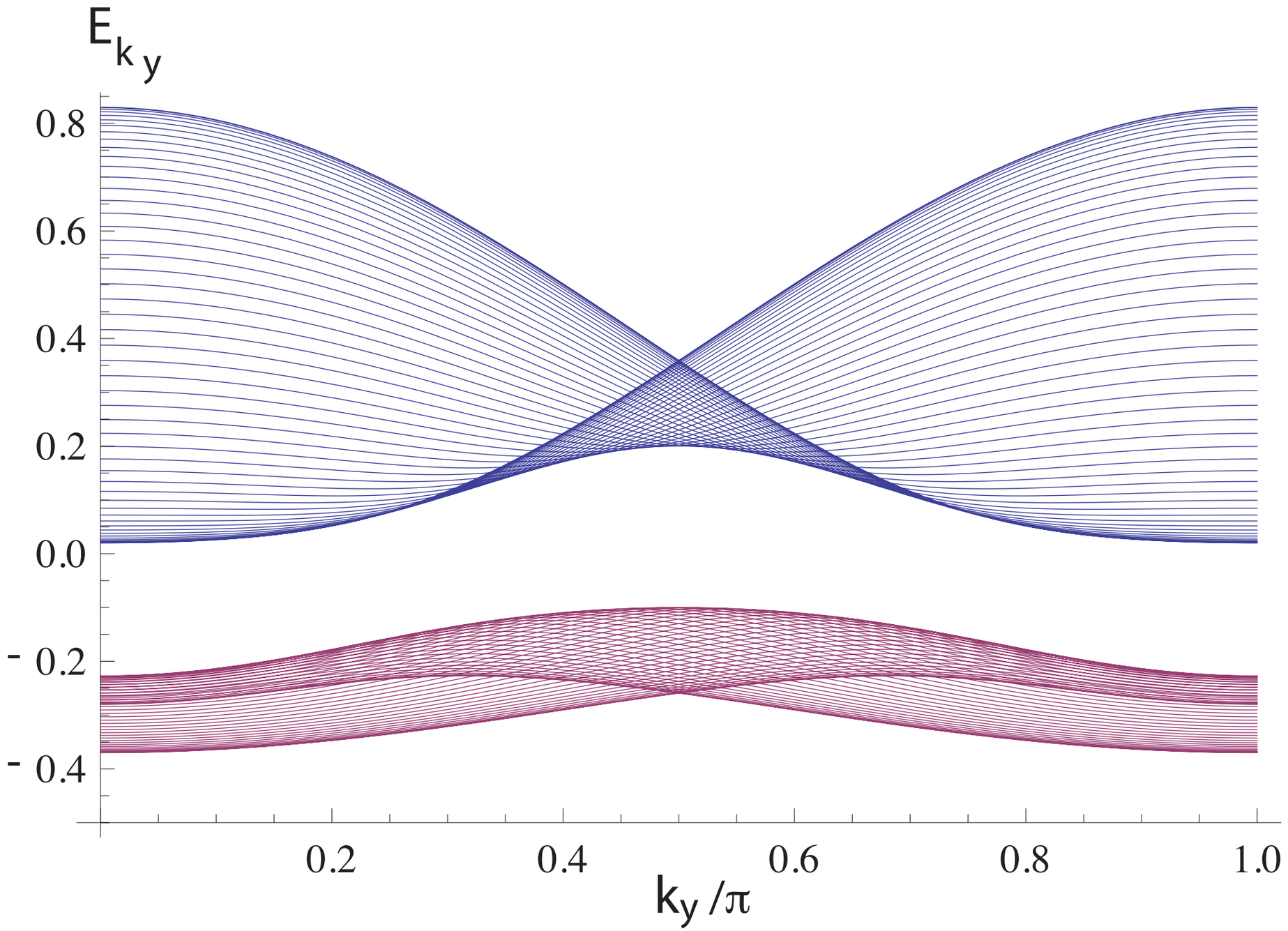
$$N = N_{\uparrow,-} + N_{\downarrow,-} = 0$$

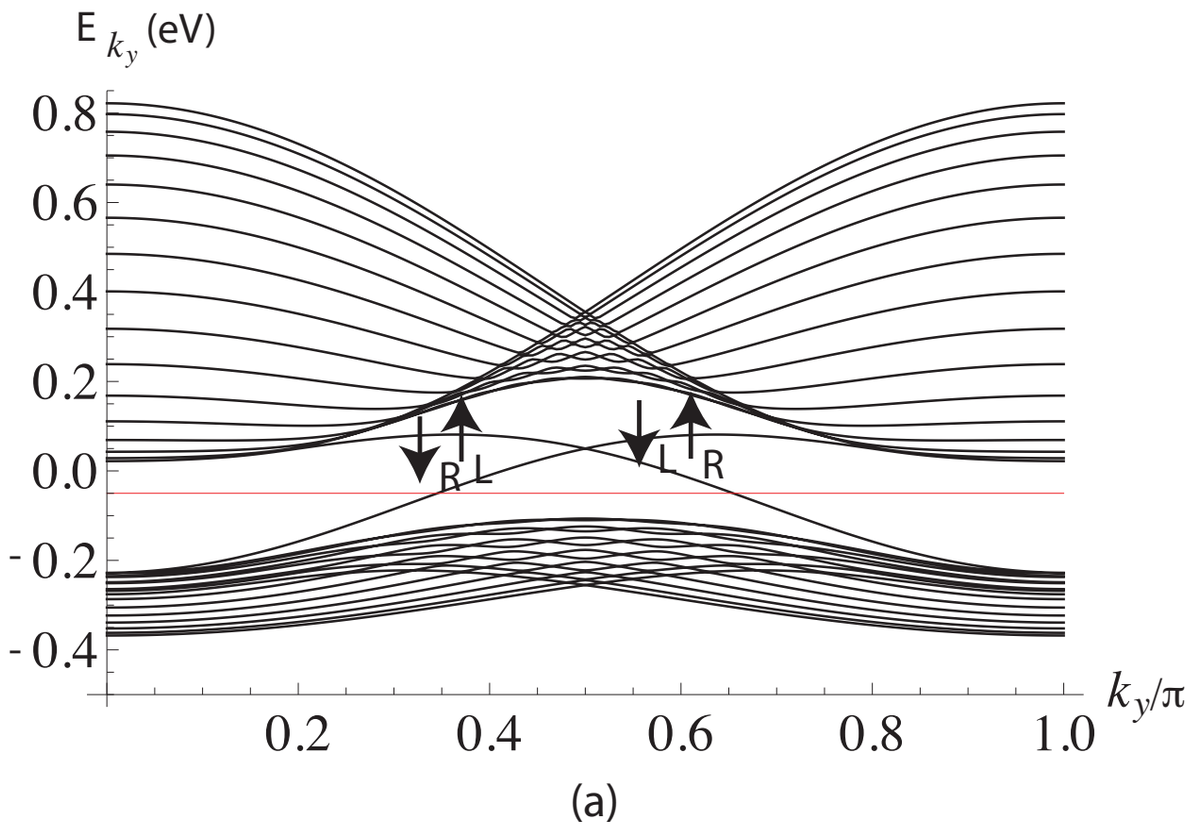
$$N_{\rm spin} = N_{\uparrow,-} - N_{\downarrow,-} = (-1) - 1 = -2$$

$$\sigma_{xy}^{\rm spin} = -\frac{e^2}{h}\frac{\hbar}{2e} N_{\rm spin} = \frac{e}{2\pi}$$

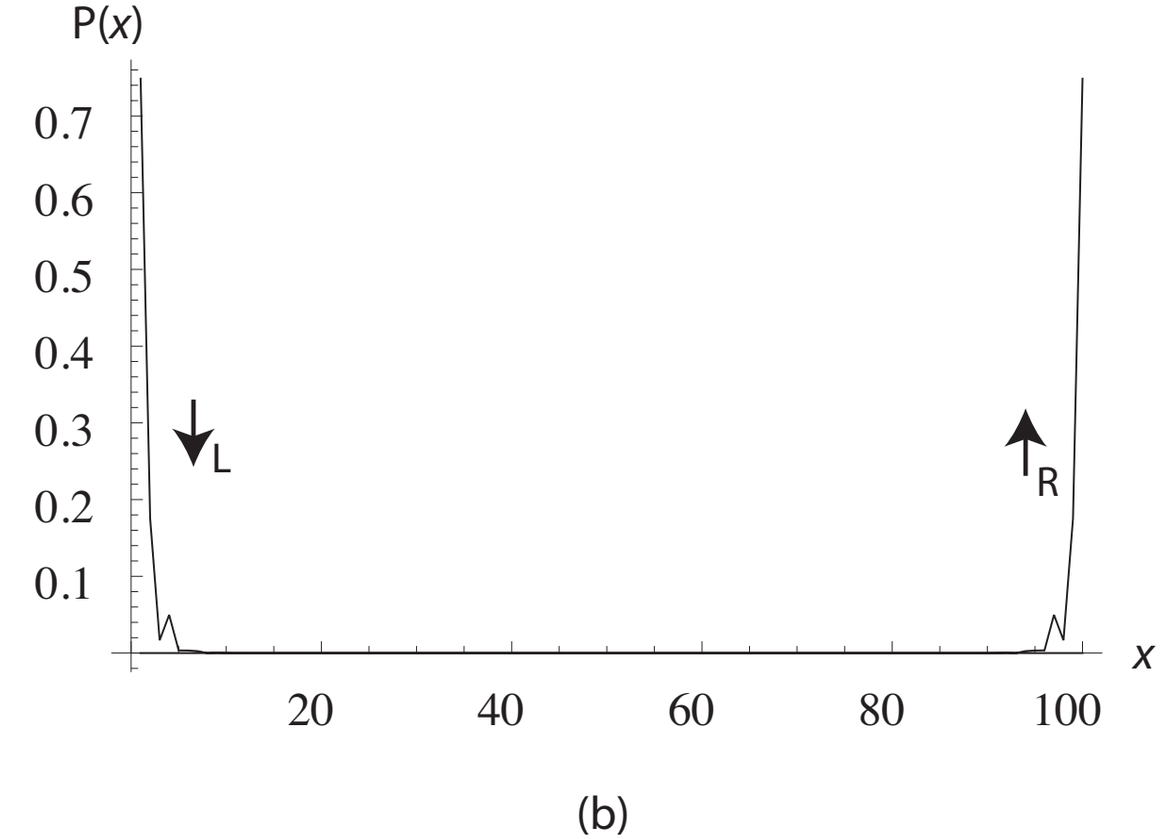
In the metallic state, there is no quantization but still spin Hall effect

Bulk spectrum with P. B. C. in both x and y-direction





(a)



(b)

Torus cut to a cylinder: open boundary condition along x, periodic along y

$$H_{1D} = \sum_{k_y,i,j} \Psi^{\dagger}_{i,k_y} A_{ij}(k_y) \Psi_{j,k_y}$$

$$A_{ij} : \ 4N \times 4N \text{ matrix}$$

$$\Psi_{i,k_y} = \left(c_{i,k_y,\uparrow}, c_{i,k_y+\pi,\uparrow}, c_{i,k_y,\downarrow}, c_{i,k_y+\pi,\downarrow}\right)^T$$

Possible experimental signatures

- Neutron: no magnetic elastic peak (unlike singlet DDW), inelastic signature possible
- NQR shift, in the presence of spin-orbit coupling
- Quantum oscillation properties are similar to the singlet case
- Two-magnon Raman
- Signature in ultrasound propagation because of the d_xy component

$$H_{probe} = \int d^2x [2(\vec{F} \cdot \vec{\Phi}_Q)(\vec{F} \cdot \vec{\Phi}_Q^*) - |\vec{\Phi}_Q|^2 F^2]$$

F: photons, neutrons, or nuclear spins

- Microscopic models? Correlated hopping?

- $i\sigma d_{x^2-y^2}$ and d_{xy} belong to two distinct irreducible representations on a square lattice, and should have distinct transitions. d_{xy} transition should be at higher temperature, as it only breaks discrete symmetry, whereas $i\sigma d_{x^2-y^2}$ breaks SU(2) and requires interlayer coupling, typically very small.