上州人

上州無智亦無才
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至誠依神期勝利

鑑三

Generalized G-inflation

Galileon

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Tsutomu Kobayashi, Masahide Yamaguchi, & JY

“G-inflation: Inflation driven by the Galileon Field”
1008.0603[hep-th] Phys. Rev. Lett. 135(2010)230302.

“Primordial non-Gaussianity from G-inflation”
1103.1740[hep-th], Phys. Rev. D 83(2011)103524.

“Generalized G-inflation: Inflation with the most general second-order field equations” 1105.5723 Prog. Theor. Phys. 126(2011)511

K. Kamada, T. Kobayashi, M. Yamaguchi, & JY

“Higgs G-inflation” 1012.4238[hep-th], Phys.Rev.D83(2011)083515

X. Gao, T. Kobayashi, M. Yamaguchi, & JY

“Primordial non-Gaussianities of gravitational waves in the most general single-field inflation model” 1108.3513[astro-ph.CO]

The Galileon (Nicolis, Rattazzi, & Trincherini 2009)

Higher derivative theory with a Galilean shift symmetry $\partial_\mu \phi \to \partial_\mu \phi + \mathrm{const}_\mu$ in the flat spacetime.

$$\mathcal{L}_1 = \phi$$

$$\mathcal{L}_2 = \left(\nabla\phi\right)^2$$

motivated by brane bending mode in the DGP model at the decoupling limit.

$$\mathcal{L}_3 = \left(\nabla\phi\right)^2 \Box\phi$$

$$\mathcal{L}_4 = \left(\nabla\phi\right)^2 \left[2\left(\Box\phi\right)^2 - 2\left(\nabla_\mu\nabla_\nu\phi\right)^2 \right]$$

$$\mathcal{L}_5 = \left(\nabla\phi\right)^2 \left[\left(\Box\phi\right)^3 - 3\left(\Box\phi\right)\left(\nabla_\mu\nabla_\nu\phi\right)^2 + 2\left(\nabla_\mu\nabla_\nu\phi\right)^3 \right]$$

$$\left(\nabla_\mu\nabla_\nu\phi\right)^2 = \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi$$

$$\left(\nabla_\mu\nabla_\nu\phi\right)^3 = \nabla_\mu\nabla_\nu\phi\nabla^\nu\nabla^\lambda\phi\nabla_\lambda\nabla^\mu\phi$$

⇒ Field equation contains derivatives up to second-order at most.

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nonreleativistic limit of 4D probe brane action in 5D theory (de Rham & Tolley 2010)

$$\mathcal{L}_4 = (\nabla\phi)^2 \left[2(\Box\phi)^2 - 2(\nabla_\mu\nabla_\nu\phi)^2 \right]$$

$$\mathcal{L}_5 = (\nabla\phi)^2 \left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3 \right]$$

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Field equation contains derivatives up to second-order at most.

Galilean symmetry exists only in flat spacetime.

The Galileon (Nicolis, Rattazzi, & Trincherini 2009)

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$$(\nabla_\mu\nabla_\nu\phi)^3 = \nabla_\mu\nabla_\nu\phi\nabla^\nu\nabla^\lambda\phi\nabla_\lambda\nabla^\mu\phi$$

$\Rightarrow$ Field equation contains derivatives up to second-order at most.

Extension of the symmetry to curved space in brane world (Burrage, de Rham & Heisenberg 2011, Goon, Hinterbichler, & Trodden 2011,...)

Covariantization: Generalized Galileon

(Deffayet, Deswer, & Esposito-Farese, 2009)
(Deffayet, Esposito-Farese, & Vikman, 2009)

no longer Galilean invariant
but field equations remain of second-order.

Generic theory with second-order field eqs.

(Deffayet, Gao, Steer, Zahariade, 1103.3260[hep-th])

$$\mathcal{L}_2 = \boxed{K(\phi, X)}$$

$$\mathcal{L}_3 = -\boxed{G_3(\phi, X)}\Box\phi$$

4 arbitrary functions of ϕ and $X \equiv -\frac{1}{2}(\partial\phi)^2$

$$\mathcal{L}_4 = \boxed{G_4(\phi, X)}R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \qquad G_{iX} \equiv \frac{\partial G_i}{\partial X}$$

$$\mathcal{L}_5 = \boxed{G_5(\phi, X)}G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

This theory includes the Einstein action with $G_4 \supset M_{Pl}^2/2$
as well as a nonminimal coupling with $G_4 \supset -\xi\phi^2/2$.

Gravity is naturally included by construction.

In fact the most general scalar+gravity theory that yields second-order field eqs. was discovered by Horndeski already in 1974. (recently revisited by Charmousis et al. 1106.2000)

$$\mathcal{L}_H = \delta^{\alpha\beta\gamma}_{\mu\nu\sigma}\left[\kappa_1 \nabla^\mu\nabla_\alpha\phi R_{\beta\gamma}{}^{\nu\sigma} + \left(\tfrac{2}{3}\kappa_{1X}\nabla^\mu\nabla_\alpha\phi + 2\kappa_{3X}\nabla_\alpha\phi\nabla^\mu\phi\right)\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi + \kappa_3\nabla_\alpha\phi\nabla^\mu\phi R_{\beta\gamma}{}^{\nu\sigma}\right]$$

$$+\delta^{\alpha\beta}_{\mu\nu}\left[F R_{\alpha\beta}{}^{\mu\nu} + 2F_X\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi + 2\kappa_8\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\right] - 6\left(F_\phi - X\kappa_8\right)\Box\phi + \kappa_9$$

with $\delta^{\alpha\beta\gamma}_{\mu\nu\sigma} = 3!\,\delta^{[\alpha}_{\mu}\delta^{\beta}_{\nu}\delta^{\gamma]}_{\sigma}$ $\quad F_X = 2\left(\kappa_3 + 2X\kappa_{3X} - \kappa_{1\phi}\right)$ $\quad \kappa_1, \kappa_3, \kappa_8, \kappa_9(\phi, X)$

We have found that the Generalized Galileon is equivalent to Horndeski theory by the following identification.

$$K = \kappa_9 + 4X\int^X \mathrm{d}X'\left(\kappa_{8\phi} - 2\kappa_{3\phi\phi}\right),$$

$$G_3 = 6F_\phi - 2X\kappa_8 - 8X\kappa_{3\phi} + 2\int^X \mathrm{d}X'(\kappa_8 - 2\kappa_{3\phi}),$$

$$G_4 = 2F - 4X\kappa_3,$$

$$G_5 = -4\kappa_1,$$

Generalized G-inflation

Inflation from a gravity + scalar system $S=\sum_{i=2}^{5}\int \mathcal{L}_i\sqrt{-g}\,d^4x$

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$G_4 \supset M_{Pl}^2/2$ gives the Einstein action

$$\mathcal{L}_4 = \boxed{G_4(\phi, X)}R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

This theory includes

Generalized G-inflation is a framework to study the most general single-field inflation model with second-order field equations.

G-inflation model

$K(\phi, X) - G(\phi, X)\Box\phi$

Kinetically driven G-inflation: A simple example

- $K(\phi, X) \equiv K(X), \;\; G_3(\phi, X) \equiv gX \equiv X/M^3, \; G_4 = M_{Pl}^2/2,$ and $G_5 = 0.$

  seek for a solution with $H = \text{const.}$ and $\dot{\phi} = \text{const.}$

$$3M_{Pl}^2 H^2 = \rho, \quad M_{Pl}^2\left(3H^2 + \cancel{2\dot{H}}\right) = -p$$

with $\rho = 2K_X X - K + 3G_{3X} H\dot{\phi}^3 \cancel{- 2G_{3\phi} X}$

$$p = K - 2\left(\cancel{G_{3\phi}} + \cancel{G_{3X}\ddot{\phi}}\right) X$$

For $\rho = -p = -K = \text{const.} > 0$ we set $\boxed{D \equiv K_X + 3gH\dot{\phi} = 0}$

- The simplest solution

$$\mathcal{L} = -X + \frac{X^2}{2M^3\mu} - \frac{X}{M^3}\Box\phi \;\Longrightarrow\; X \simeq M^3\mu, \;\; H^2 \simeq \frac{M^3\mu}{6M_{Pl}^2}.$$

$\mu =$ const. during de Sitter inflation

Similar to k-inflation model but crucial difference arises in the perturbation calculations.

Fluctuations in Generalized G-inflation

$$ds^2 = -N^2 dt^2 + \gamma_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

$$N = 1 + \alpha, \quad N_i = \partial_i \beta,$$

$$\gamma_{ij} = a^2(t) e^{2\mathcal{R}}\left(\delta_{ij} + h_{ij} + \frac{1}{2} h_{ik} h_{kj}\right)$$

$$h_{ii} = 0 = h_{ij,j}$$

work in the unitary gauge where the scalar field is homogeneous $\phi = \phi(t)$.

Tensor Perturbations

★ The quadratic action $\alpha = \beta = \mathcal{R} = 0$

$$S_T^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x\, a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\vec{\nabla} h_{ij})^2\right]$$

$$\mathcal{F}_T := 2\left[G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)\right], \quad > 0$$

$$\mathcal{G}_T := 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right] \quad > 0$$

for stability

The "sound" velocity $c_T^2 \equiv \mathcal{F}_T / \mathcal{G}_T$ deviates from unity if $G_{4X} \neq 0$, $G_{5X} \neq 0$ or $G_{5\phi} \neq 0$.

★ Tensor spectral index and amplitude

$$n_T = 3 - 2\nu_T = -\frac{4\varepsilon + 3f_T - g_T}{2(1-\varepsilon - s_T)}$$

Blue spectrum if $4\varepsilon + 3f_T - g_T < 0$.

$$\varepsilon \equiv -\frac{\dot{H}}{H^2}, \quad f_T \equiv \frac{\dot{\mathcal{F}}_T}{H\mathcal{F}_T}, \quad g_T \equiv \frac{\dot{\mathcal{G}}_T}{H\mathcal{G}_T}, \quad s_T \equiv \frac{\dot{c}_T}{Hc_T}.$$

$$\mathcal{P}_T(k) \simeq \frac{1}{4\pi^2}\frac{H^2}{\mathcal{F}_T c_T}\bigg|_{SHC}$$

Curvature Perturbations

We adopt the unitary gauge in which ϕ is homogeneous, $\delta\phi = 0$.

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\partial_i \beta dt dx^i + a^2(1+2\mathcal{R})d\mathbf{x}^2$$

Curvature Perturbation

As usual,

① Expand the action to the second order.

② Eliminate α and β using constraint equations.

③ Obtain a quadratic action for $\mathcal{R}$.

$$S_S{}^{(2)} = \frac{1}{2}\int dt d^3x a^3 \left[\mathcal{G}_S \dot{\mathcal{R}}^2 - \frac{\mathcal{F}_S}{a^2}(\nabla\mathcal{R})^2 \right],$$

$$\mathcal{G}_T \dot{\mathcal{R}} = \Theta\alpha,$$

$$\frac{\nabla^2}{a^2}\left(\mathcal{G}_T\mathcal{R} + a^2\Theta\beta\right) = \Sigma\alpha + 3\Theta\dot{\mathcal{R}}$$

where

$$\mathcal{F}_S := \frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{a}{\Theta}\mathcal{G}_T^2\right) - \mathcal{F}_T,$$

$$\mathcal{G}_S := \frac{\Sigma}{\Theta^2}\mathcal{G}_T^2 + 3\mathcal{G}_T.$$

with

$$\begin{aligned}
\Sigma := \; & XK_X + 2X^2K_{XX} + 12H\dot{\phi}XG_{3X} \\
& +6H\dot{\phi}X^2G_{3XX} - 2XG_{3\phi} - 2X^2G_{3\phi X} - 6H^2G_4 \\
& +6\Big[H^2\left(7XG_{4X} + 16X^2G_{4XX} + 4X^3G_{4XXX}\right) \\
& -H\dot{\phi}\left(G_{4\phi} + 5XG_{4\phi X} + 2X^2G_{4\phi XX}\right)\Big] \\
& +30H^3\dot{\phi}XG_{5X} + 26H^3\dot{\phi}X^2G_{5XX} \\
& +4H^3\dot{\phi}X^3G_{5XXX} - 6H^2X\left(6G_{5\phi}\right. \\
& \left.+9XG_{5\phi X} + 2X^2G_{5\phi XX}\right), \\
\Theta := \; & -\dot{\phi}XG_{3X} + 2HG_4 - 8HXG_{4X} \\
& -8HX^2G_{4XX} + \dot{\phi}G_{4\phi} + 2X\dot{\phi}G_{4\phi X} \\
& -H^2\dot{\phi}\left(5XG_{5X} + 2X^2G_{5XX}\right) \\
& +2HX\left(3G_{5\phi} + 2XG_{5\phi X}\right).
\end{aligned}$$

$$S_S^{(2)} = \frac{1}{2}\int dt d^3x a^3 \left[\mathcal{G}_S \dot{\mathcal{R}}^2 - \frac{\mathcal{F}_S}{a^2}(\nabla \mathcal{R})^2 \right],$$

No ghosts, No gradient instability if $\mathcal{G}_S > 0, \quad c_S^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S} > 0.$

In k inflation where $G_3 = G_5 = 0,\ G_4 = M_{Pl}^2/2$ hold, we find $\mathcal{F}_S = M_{Pl}^2 \varepsilon = -M_{Pl}^2 \dot{H}/H^2$, which means that $\dot{H} > 0$ is prohibited by the stability condition.

But in G-inflation $\dot{H} > 0$ is possible.

★ Scalar spectral index and amplitude

$$n_S - 1 = 3 - 2\nu_S = -\frac{4\varepsilon + 3f_S - g_S}{2(1 - \varepsilon - s_S)} \qquad \mathcal{P}_S(k) \cong \frac{1}{4\pi^2}\frac{H^2}{\mathcal{F}_S c_S}\bigg|_{SHC}$$

★ The tensor-to-scalar ratio $$r = \frac{\mathcal{P}_T(k)}{\mathcal{P}_S(k)} = 16\frac{\mathcal{F}_S}{\mathcal{F}_T}\frac{c_S}{c_T}$$

Standard inflation $\mathcal{L} = X - V[\phi]$ $r = 16\varepsilon$

k-inflation $\mathcal{L} = K(\phi, X)$ $r = 16\varepsilon c_S$

G-inflation $\mathcal{L} = K(\phi, X) - G(\phi, X)\Box\phi$ $r = 16\mathcal{F}_S c_S$

Generalized G-inflation $r = 16\frac{\mathcal{F}_S}{\mathcal{F}_T}\frac{c_S}{c_T}$

NOT slow-roll suppressed

Small sound speed and large tensor-to-scalar ratio are compatible.

Specific Models

Example I: More on Kineticallly Driven G-Inflation

A model with full shift symmetry

- $K(\phi, X) \equiv K(X), \;\; G_3(\phi, X) \equiv gX \equiv X/M^3, \;\; G_4 = M_{Pl}^2/2, \;\; \text{and } G_5 = 0.$
- The simplest solution

$$K(X) \equiv -X + \frac{X^2}{2M^3\mu} \;\Longrightarrow\; X \simeq M^3\mu, \;\; H^2 \simeq \frac{M^3\mu}{6M_{Pl}^2}.$$

$\mu =$ const. during de Sitter inflation

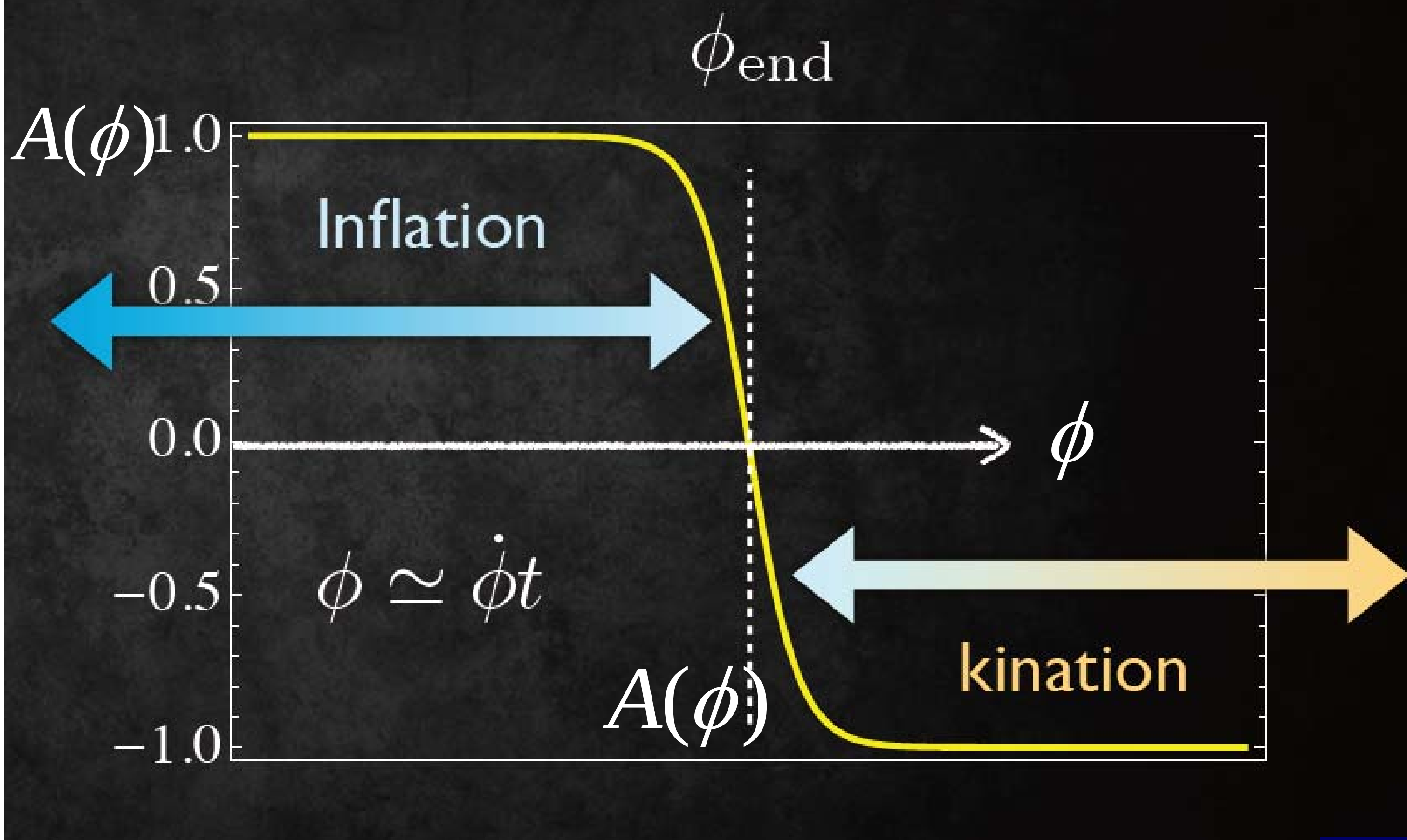
Inflation does not end.

- $$K(X) \equiv -X + \frac{X^2}{2M^3\mu} \;\Longrightarrow\; K(\phi, X) \equiv -A(\phi)X + \frac{X^2}{2M^3\mu}$$

Inflation can be terminated by flipping the sign here.

A simple choice: $A(\phi) \equiv \tanh\left[\lambda\left(\phi_{end} - \phi\right)/M_{Pl}\right]$ with $\lambda = O(1)$.

Numerical solutions indicate ϕ stalls within one e-fold after crossing ϕ_{end} and all higher derivative terms become negligible. This function breaks the shift symmetry (severely) only in the vicinity of ϕ_{end}.



$K(X) \simeq X$

$$\boxed{\rho = \frac{\dot{\phi}^2}{2} \propto a^{-6}(t).}$$

$w = 1$

- The Universe is reheated through gravitational particle production. (Ford 87)
- The Universe will eventually be dominated by radiation.

$$\boxed{T_R \approx 0.01 \frac{H_{\rm inf}^2}{M_{Pl}} = 2 \times 10^7 \left(\frac{r}{0.1} \right) \mathrm{GeV}}$$

r : tensor-to-scalar ratio

Perturbative stability of kinetic G-inflation

$$S_S^{(2)} = \frac{1}{2}\int dt d^3x a^3 \left[\mathcal{G}_S \dot{\mathcal{R}}^2 - \frac{\mathcal{F}_S}{a^2}(\nabla \mathcal{R})^2 \right],$$

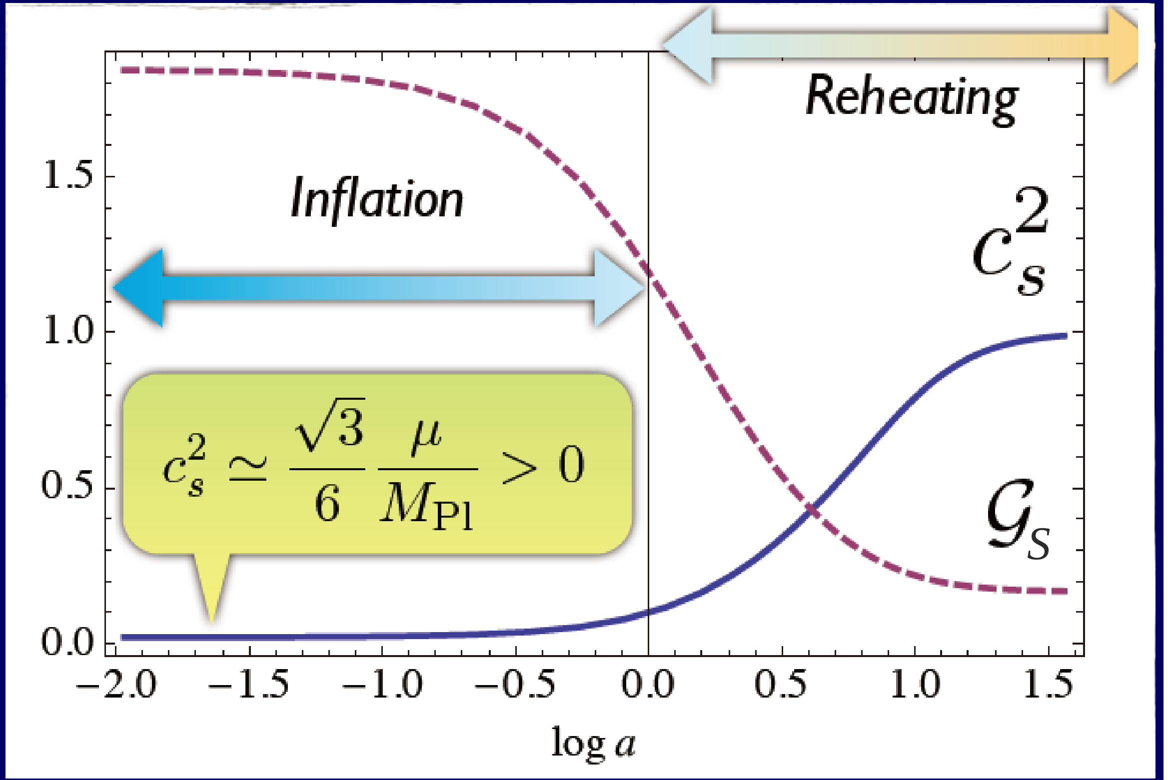
No ghosts, No gradient instability if $\mathcal{G}_S > 0, \quad c_S^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S} > 0.$

Our simple model satisfies these requirements.

$$K(\phi, X) \equiv -A(\phi)X + \frac{X^2}{2M^3\mu}$$

$$G_3(\phi, X) \equiv \frac{X}{M^3}$$

$r = 16\mathcal{F}_S c_S$ can be as large as $r = 0.17$, saturating the observational bound.



Tensor mode can be large and blue!

In G-inflation, the null energy condition may be violated,

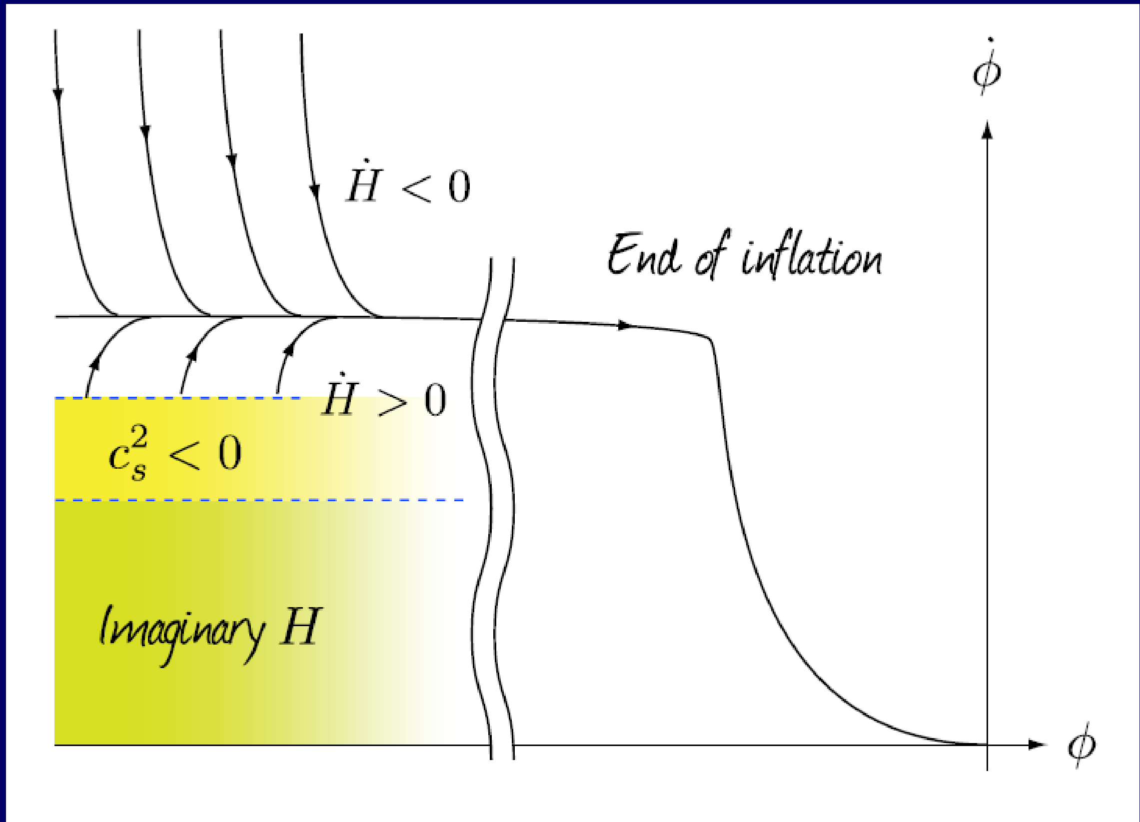
$$2M_{Pl}^2\dot{H} = -(\rho + p) = -(2K_X X + 3G_{3X} H\dot{\phi}^3 - 4G_{3\phi} X - 2G_{3X}\ddot{\phi} X) > 0.$$

It can be violated without instabilities, keeping $c_s^2 > 0$.

The tensor spectral index can be positive,

$$n_T = -2\varepsilon > 0.$$

Short wave tensor fluctuations may have a larger amplitude at formation.

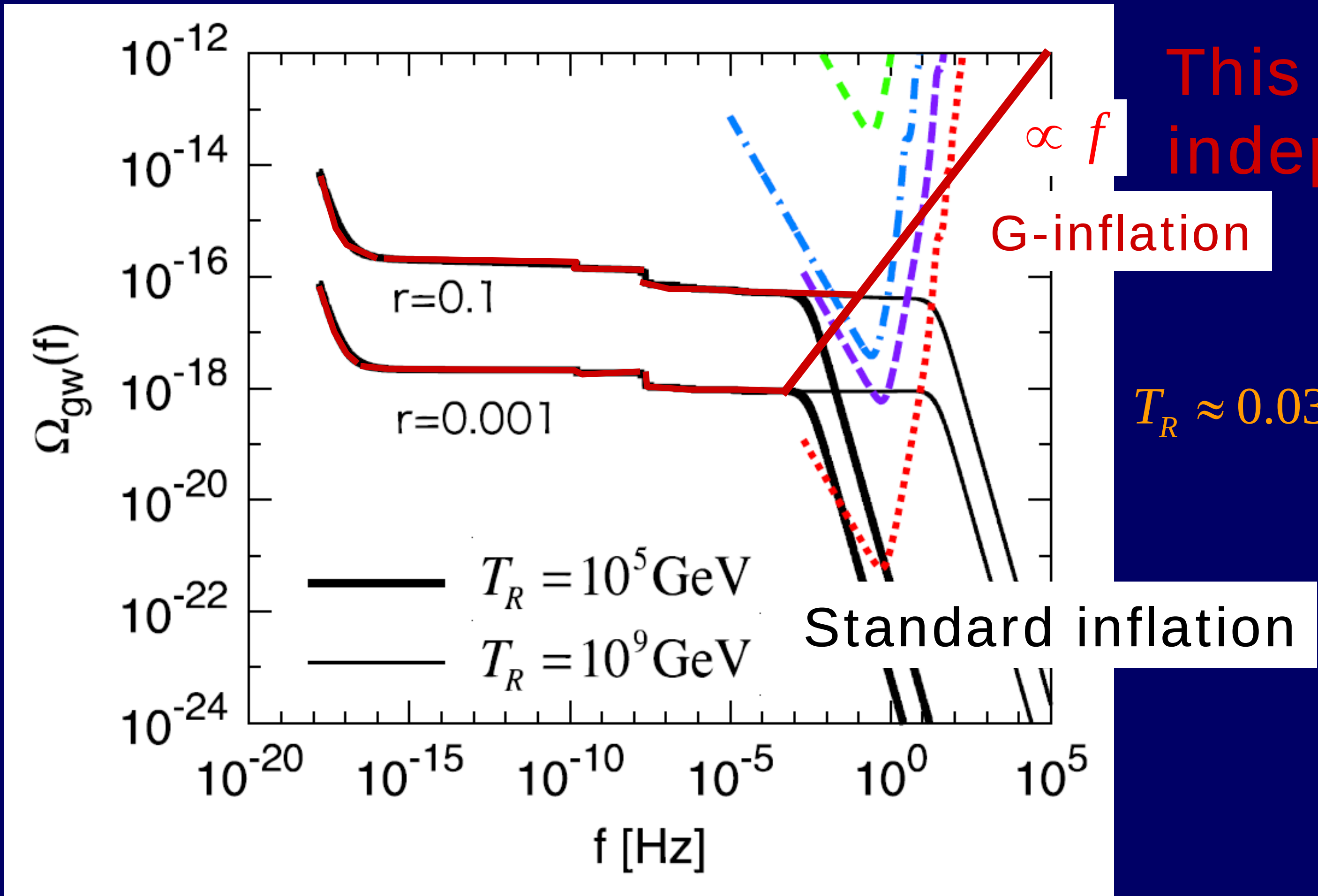


Furthermore, after inflation...

The Universe is dominated by the kinetic energy of the inflaton with equation of state $w = 1$.

The fractional energy density of short-wave gravitational waves will be enhanced.

(Chiba Tashiro Sasaki 2004)

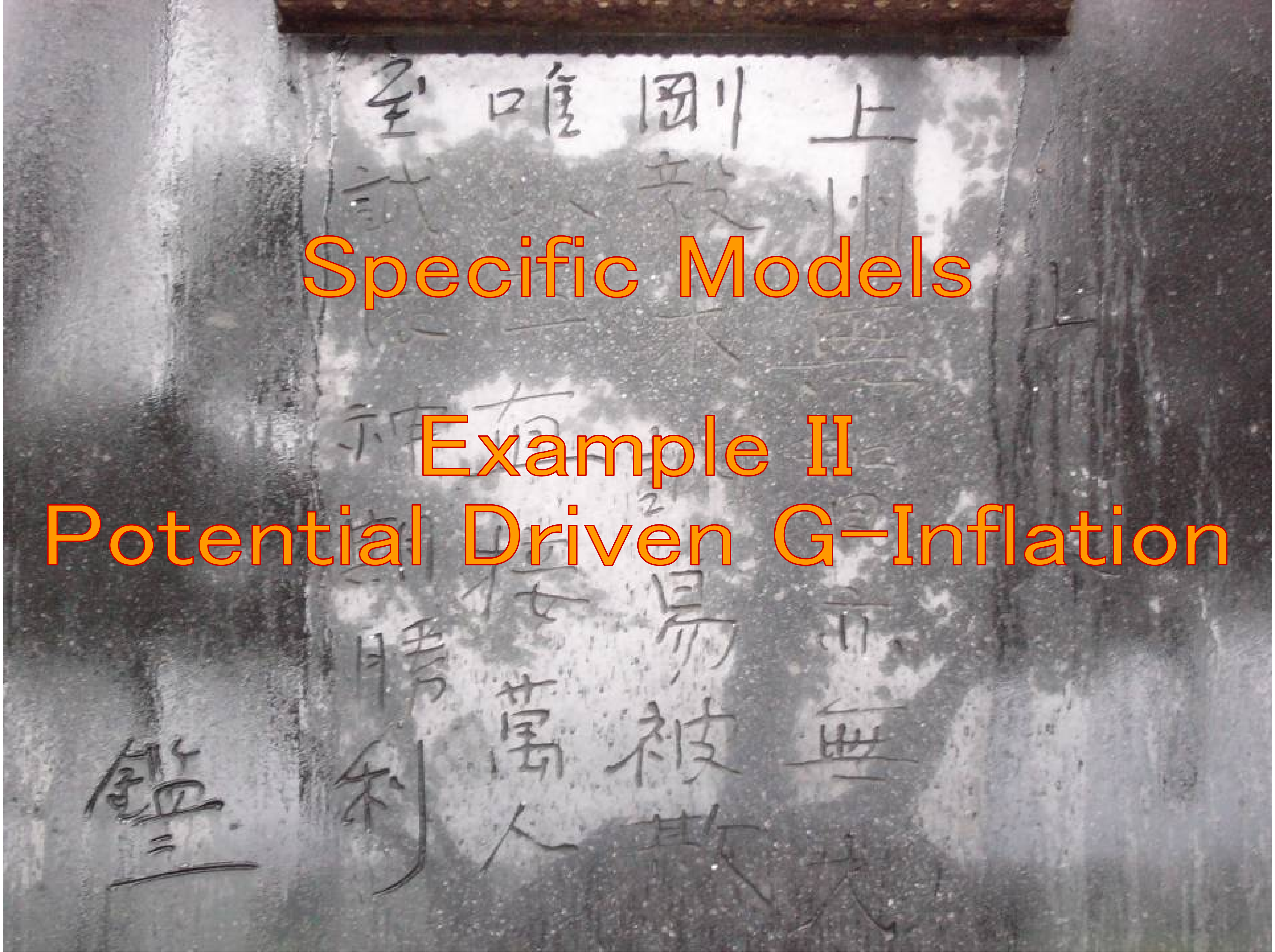


This part is independent of r.

$$T_R \approx 0.03 \frac{H_{\inf}^2}{M_{Pl}} = 6 \times 10^7 \left(\frac{r}{0.1} \right) \mathrm{GeV}$$

Specific Models

Example II
Potential Driven G-Inflation



Potential driven G-inflation

$$\mathcal{L}_\phi = X - V(\phi) - g(\phi) X \Box \phi,$$

Inflation is driven by the Potential.

Background equations of motion

$$3M_{Pl}^2 H^2 = \left[1 - gH\dot{\phi}(6-\alpha)\right] X + V(\phi)$$

$$M_{Pl}^2 \dot{H} = -\left[1 - gH\dot{\phi}(3+\eta-\alpha)\right] X$$

$$\left[3 - \eta - gH\dot{\phi}(9 - 3\varepsilon - 6\eta + 2\eta\alpha)\right] H\dot{\phi} + (1+2\beta)V'(\phi) = 0$$

Slow-roll parameters $\varepsilon = -\dfrac{\dot{H}}{H^2}, \quad \eta = -\dfrac{\ddot{\phi}}{H\dot{\phi}}, \quad \alpha = \dfrac{g'\dot{\phi}}{gH}, \quad \beta = \dfrac{g''X^2}{V'(\phi)}.$

For $|\varepsilon|, |\mu|, |\alpha|, |\beta| \ll 1$ we find slow-roll EOMs

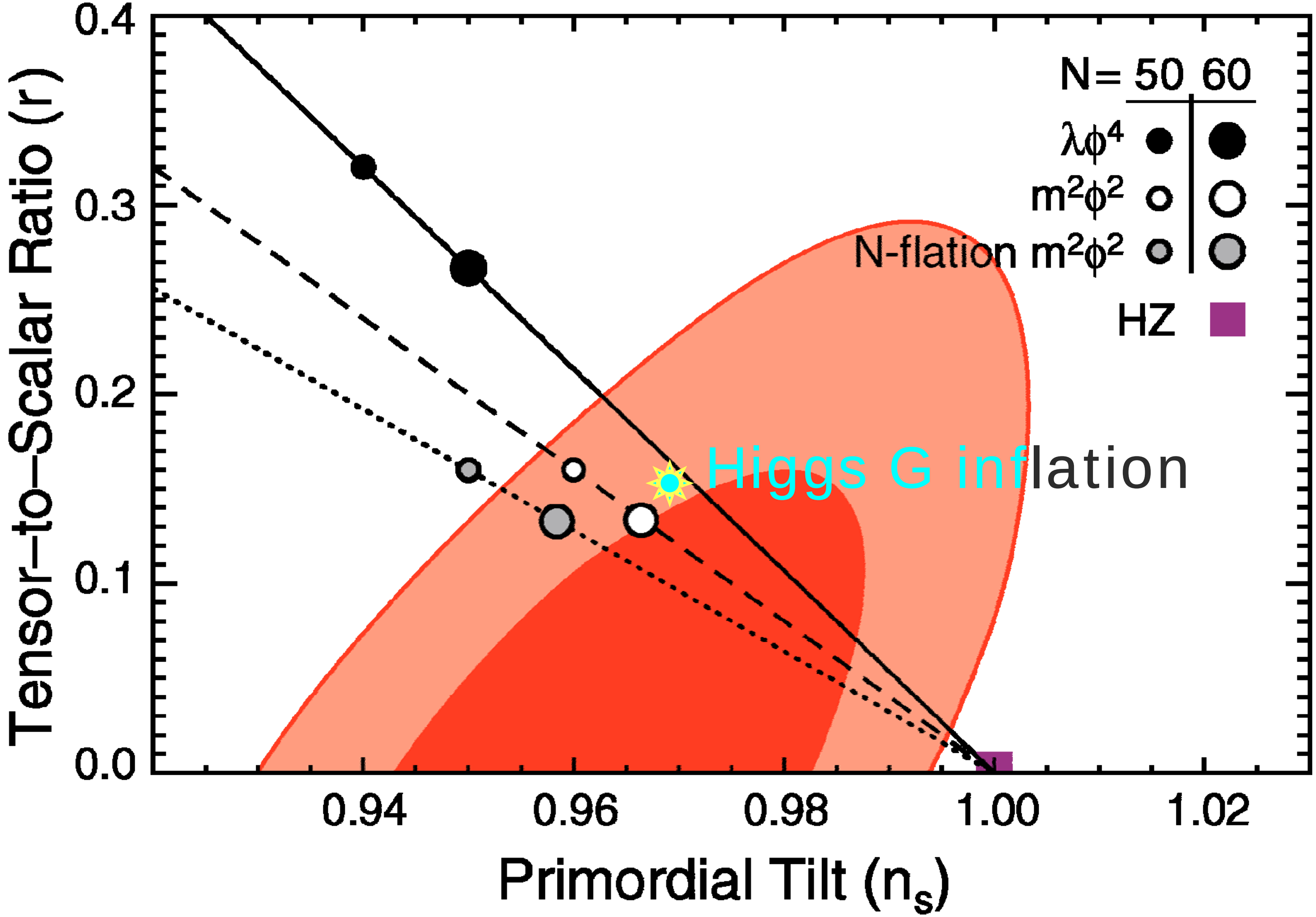
$$3H\dot{\phi}\left(1 - 3gH\dot{\phi}\right) + V'(\phi) \simeq 0 \qquad 3M_{Pl}^2 H^2 \simeq V(\phi)$$

This extra friction term enhances inflation and makes it possible to drive inflation by the standard Higgs field.

$$\mathcal{O}_\phi = X - \frac{\lambda}{4}\phi^4 - \frac{X\phi\Box\phi}{M^4}$$

$$\mathcal{P}_{\mathcal{R}}(k) = 2.4\times 10^{-9} \,@\, k = 0.002\mathrm{Mpc}^{-1} (\mathcal{N}_{\mathrm{COBE}} = 60)$$

$$\boxed{M \simeq 4.7\times 10^{-6}\lambda^{-\frac{1}{4}} M_{Pl} \simeq 10^{13}\mathrm{GeV}.}$$



Nongaussianity

$$\mathcal{L}_\phi = K(\phi, X) - G(\phi, X)\Box\phi$$

see also
Mizuno & Koyama (2010)
Burrage, de Rham, Seery & Tolley (2011)
Creminelli et al (2011)
DeFelice & Tsujikawa (2011)
Gao and Steer (2011)
Renaux-Petel (2011)

$G_4(\phi, X)$ and $G_5(\phi, X)$ do not generate new terms

We adopt the unitary gauge in which ϕ is homogeneous, $\delta\phi = 0$.

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\partial_i\beta dt dx^i + a^2(1+2\mathcal{R})d\mathbf{x}^2$$

Curvature Perturbation

As usual,

① Expand the action to the second order.

② Eliminate α and β using constraint equations.

③ Obtain a cubic action for $\mathcal{R}$.

$$S_3 = \int dt d^3x\, a^3 \left[\frac{\mathcal{C}_1}{H}\dot{\mathcal{R}}^3 + \mathcal{C}_2\mathcal{R}\dot{\mathcal{R}}^2 + \frac{\mathcal{C}_3}{a^4H^2}\partial^2\mathcal{R}(\partial\mathcal{R})^2 + \frac{\mathcal{C}_4}{a^2H^2}\dot{\mathcal{R}}^2\partial^2\mathcal{R} + \mathcal{C}_5 H\mathcal{R}^2\dot{\mathcal{R}} \right.$$
$$\left. + \frac{\mathcal{C}_6}{a^4H}\partial^2\mathcal{R}(\partial\mathcal{R}\cdot\partial\chi) + \frac{\mathcal{C}_7}{a^4}\partial^2\mathcal{R}(\partial\chi)^2 + \frac{\mathcal{C}_8}{a^2}\mathcal{R}(\partial\mathcal{R})^2 + \frac{\mathcal{C}_9}{a^2}\dot{\mathcal{R}}(\partial\mathcal{R}\cdot\partial\chi) + \frac{2}{a^3}f(\mathcal{R})\left.\frac{\delta L}{\delta\mathcal{R}}\right|_1\right],$$

$$\chi := \partial^{-2}\Lambda, \quad \Lambda := \frac{a^2}{\Theta^2}X\mathcal{G}\dot{\mathcal{R}} = \frac{a^2\sigma}{c_s^2}\dot{\mathcal{R}}.$$

$$\mathcal{C}_1 = -\frac{H}{\Theta}\frac{\sigma}{c_s^2}\left(1+2\frac{\mathcal{I}}{\mathcal{G}}\right) - 2\dot{\phi}X\left(G_X + XG_{XX}\right)\frac{H\sigma}{c_s^2\Theta^2} + \frac{H^2\sigma}{c_s^4\Theta^2},$$

$$\mathcal{C}_2 = \frac{\sigma}{c_s^2}\left[3 - \frac{H^2}{c_s^2\Theta^2}\left(3+\epsilon+\frac{2\dot{\Theta}}{H\Theta}\right)\right],$$

$$\mathcal{C}_3 = -\frac{H^2\dot{\phi}XG_X}{\Theta^3},$$

$$\mathcal{C}_4 = \frac{2H^2\dot{\phi}X\left(G_X+XG_{XX}\right)}{\Theta^3},$$

$$\Theta = H - \dot{\phi}XG_X$$

$$M_{Pl} = 1$$

$$\mathcal{C}_5 = \frac{\sigma}{2c_s^2H}\frac{d}{dt}\left(\frac{H^2\delta}{c_s^2\Theta^2}\right),$$

$$\mathcal{C}_6 = \frac{2H\dot{\phi}XG_X}{\Theta^2},$$

$$\mathcal{C}_7 = \frac{\sigma}{4} - \frac{\dot{\phi}XG_X}{\Theta},$$

$$\mathcal{C}_8 = -\sigma + \frac{H^2}{\Theta^2}\frac{\sigma}{c_s^2}\left(1-\epsilon-2s-\frac{2\dot{\Theta}}{H\Theta}\right),$$

$$\mathcal{C}_9 = \frac{\sigma}{c_s^2}\left(-\frac{2H}{\Theta}+\frac{\sigma}{2}\right),$$

Generic result

$$f_{\rm NL} = \mathcal{O}\left(\frac{\tilde{\sigma}^2}{c_s^2}\right) + \mathcal{O}\left(\tilde{\sigma}^2 \frac{XG_{XX}}{G_X}\right) + \mathcal{O}\left(\tilde{\sigma}\frac{\mathcal{I}}{\mathcal{G}}\right), \quad \tilde{\sigma} := \max\{1, \sigma\}.$$

$$\mathcal{I} := XK_{XX} + \frac{2X^2}{3}K_{XXX} + H\dot{\phi}G_X + 6X^2G_X^2 + 5H\dot{\phi}XG_{XX} + 6X^3G_XG_{XX} + 2H\dot{\phi}X^2G_{XXX} - \frac{2X}{3}\left(2G_{\phi X} + XG_{\phi XX}\right).$$

$$\mathcal{G} := K_X + 2XK_{XX} + 6G_XH\dot{\phi} + 6G_X^2X^2 - 2\left(G_\phi + XG_{\phi X}\right) + 6G_{XX}HX\dot{\phi}$$

$$\sigma \equiv \mathcal{F}_S = -\frac{\dot{\Theta}}{\Theta^2} - \frac{\Theta - H}{\Theta}, \left(\Theta = H - \frac{G_X\dot{\phi}^3}{2M_{Pl}^2}\right) \quad \left(\sigma \xrightarrow{G_X \to 0} \varepsilon\right)$$

is not necessarily small.

Large $r = 16\mathcal{F}_S c_s$ and large f_{NL} are compatible.

★ Kinetically driven G-inflation

Nonlinearity parameter is determined by $\mathcal{F}_S$, c_s etc..

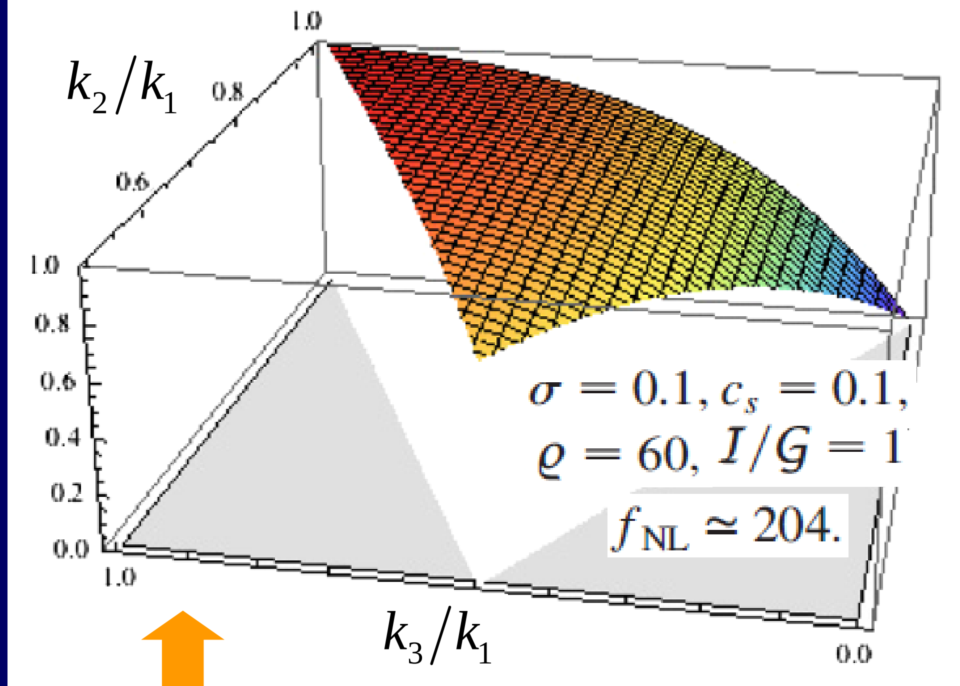
★ Potential driven G-inflation

NonGaussianity is small: $f_{NL} = \dfrac{235}{3888} \simeq 0.06.$

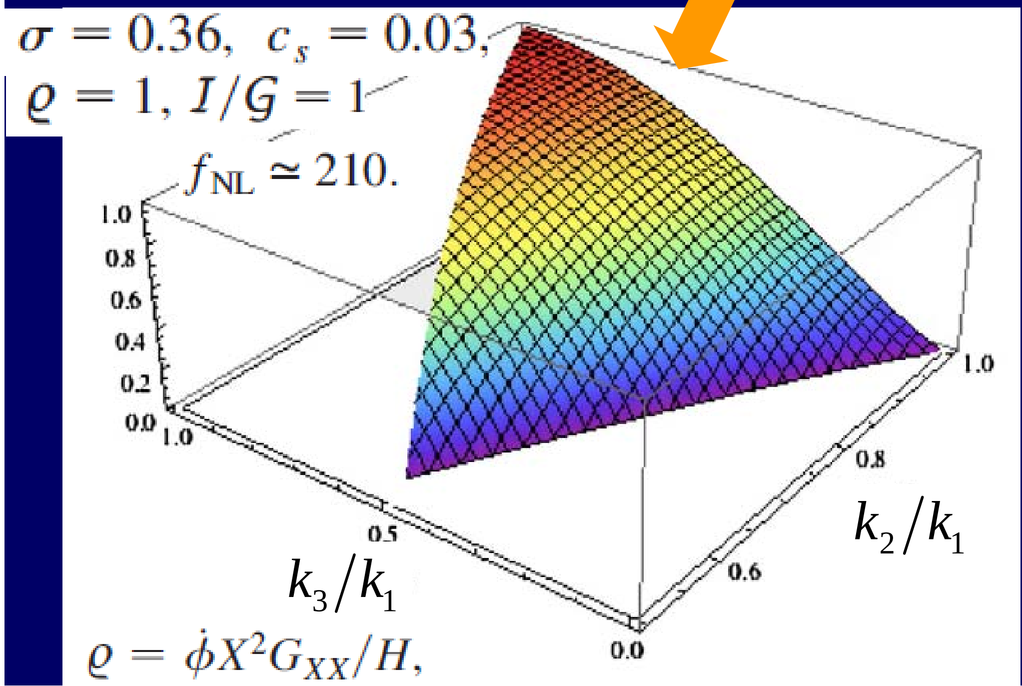
Contours of

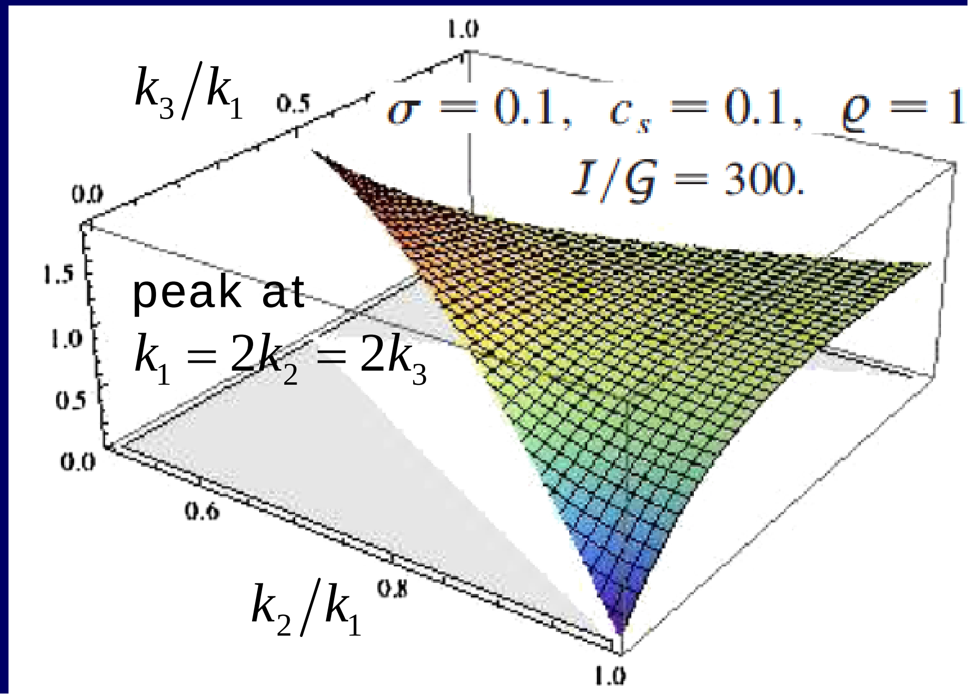
$\mathcal{A}(1, k_2/k_1, k_3/k_1)(k_2/k_1)^{-1}(k_3/k_1)^{-1}$

normalized to unity at the equilateral configuration.



$$f_{\rm NL} = \mathcal{O}\left(\frac{\tilde{\sigma}^2}{c_{\rm s}^2}\right) + \mathcal{O}\left(\tilde{\sigma}^2 \frac{X G_{XX}}{G_X}\right) + \mathcal{O}\left(\tilde{\sigma}\frac{\mathcal{I}}{\mathcal{G}}\right)$$





Conclusion

Generalized G-inflation is a single-field inflation model based on the most general Lagrangian that yields second-order field equations.

Generalized Galileon = Horndeski Theory

- ★ Large tensor-to-scalar ratio and large nongaussianity are compatible.
- ★ Parameter space for the standard inflation is also extended (Standard Higgs can be the inflaton.)
- ★ Stable violation of null energy condition $\left(\dot{H} > 0\right)$ is possible.