Anisotropic Infaltion
and
Statistical Symmetry Breaking in the CMB

Jiro Soda
Kyoto University

Watanabe, Kanno, JS, arXiv:0902.2833; PRL 102, 191302, 2009.

Watanabe, Kanno, JS, arXiv:1003.0056; Prog. Theor. Phys. 123, 1041, 2010

Kanno, Watanabe, JS, arXiv:1010.5307; JCAP 1012:024, 2010.

Watanabe, Kanno, JS, arXiv:1011.3604; MNRAS 412:L83-L87, 2011.

Symmetry in Inflation

Let us recall basic features of inflation.

- **First of all, in order to have inflation, we need to assume initial homogeneity.**

- **In addition to this initial condition, we need to provide a sufficiently flat potential to realize the slow roll inflation. Hence, we have shift symmetry** $\phi \rightarrow \phi + c$

- **Once the slow roll inflation occurs, the cosmic no-hair conjecture suggests that the exponential expansion erases any classical anisotropy and leads to isotropic universe. This is nothing but the spatial de Sitter symmetry.**

- **de Sitter spacetime** $$ds^2 = -dt^2 + e^{2Ht}\left[dx^2 + dy^2 + dz^2\right]$$

has the temporal de Sitter symmetry $t \rightarrow t + c, \quad x^i \rightarrow e^{-Hc} x^i$

As is well known, inflation can generate primordial fluctuations.

The nature of primordial curvature fluctuations

Now, the symmetries determine the statistical nature of the fluctuations:

First of all, shift symmetry gives Gaussian statistics

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = P(\mathbf{k}_1,\mathbf{k}_2)$$

Moreover, initial homogeneity implies statistical homogeneity

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = \delta(\mathbf{k}_1+\mathbf{k}_2)P(\mathbf{k}_1)$$

And, spatial de Sitter symmetry accounts for statistical isotropy

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = \delta(\mathbf{k}_1+\mathbf{k}_2)P(k_1=|\mathbf{k}_1|)$$

Finally, temporal deSitter symmetry yields scale invariant spectrum

$$P(k)\approx const.$$

The above predictions are model independent and robust.

Why do we consider the statistical anisotropy?

Precision cosmology forces us to look at fine structures of fluctuations!

Violation of temporal de Sitter symmetry -> **spectral tilt**

There should be a slight tilt because the expansion is not exactly deSitter.
The deviation from deSitter can be characterized by the slow roll parameter.
Hence, the tilt should be of the order of the slow roll parameter.

Violation of shift symmetry -> **non-Gaussianity** **Maldacena 2003**

There should be small non-gaussianity of the order of the slow roll parameter because the shift symmetry is not exact.

Along the line of this thought,
it is natural to expect **Violation of spatial de Sitter symmetry.**

which must yield a deviation from the statistical isotropy.

Until now, there has been a psychological barrier to consider this possibility due to the cosmic no-hair conjecture.

Gumrukcuoglu, Contaldi, Peloso 2006

Ackerman, Carroll, Wise 2007
Pitrou, Perira, Uzan 2008; Yokoyama, Soda 2008

Gauge Kinetic Function in Inflation

In spite of this worry, it turns out that
the statistical anisotropy is ubiquitous in the framework of supergravity.

Watanabe, Kanno, Soda, PRL, 2009.

$$S = \int d^4x \left[\sqrt{-g}R + g_{i\bar{j}}\partial^\mu\phi^i\partial_\mu\phi^{\bar{j}} - e^{\kappa^2 K} g^{i\bar{j}} \left(D_i W D_j \bar{W} - 3\kappa^2 |W|^2 \right) - \frac{1}{4} \mathrm{Re}\, f_{ab}(\phi^i) F^{a\mu\nu} F^b_{\mu\nu} + \cdots \right]$$

Cosmological roles of Kahler potential K and super potential W
in inflation has been well discussed so far.

While, the role of gauge kinetic function f in inflation has been overlooked.

To our best knowledge, the gauge kinetic function has been used only
in the discussion of generation of primordial magnetic fields during inflation,.

But, … Cf. Kallosh's talk.

However, no one has noticed its role in inflation itself!!

We show that anisotropic inflation is hidden in the supergravity
and provide a counter example to the cosmic no-hair theorem.

Cf. Ford 1989; Kaloper 1991; Barrow, Hervik 2006;
Golovnev, Mukhanov, Vanchurin 2008; Kanno, et al. 2008

Power-law Inflation and Primordial Magnetic Fields

$$S = \int d^4x \sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}\left(\partial_\mu \phi\right)^2 - V(\phi)\right] \qquad V = V_0 e^{\lambda \frac{\phi}{M_p}}$$

In this case, it is well known that there exists an isotropic power law inflation

$$\boxed{ds^2 = -dt^2 + t^{4/\lambda^2}\left(dx^2 + dy^2 + dz^2\right)}$$

In this background, one can discuss generation of primordial magnetic fields

$$S = \int d^4x \sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}\left(\partial_\mu \phi\right)^2 - V(\phi) - \frac{1}{4} f^2(\phi)\, F_{\mu\nu}F^{\mu\nu}\right] \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

gauge kinetic function

$$f = f_0 e^{\rho \frac{\phi}{M_p}}$$

Ratra, 1992;
Bamba, Yokoyama 2004
Martin, Yokoyama 2008

They argued primordial magnetic fields can be generated with this model.
Here, we take into account backreaction of gauge fields.

Anisotropic Power-law inflation

Watanabe, Kanno, Soda, PRL, 2009.
Kanno, Watanabe, Soda, JCAP, 2010.

If we consider the backreaction, the same action leads to new solutions.

For homogeneous background, the time component can be eliminated by gauge transformation.

Let the direction of the vector be $x-$ axis. Then, the metric should be Bianchi Type-I

$$ds^2 = -dt^2 + e^{2\alpha(t)}\left[e^{-4\sigma(t)}dx^2 + e^{2\sigma(t)}\left(dy^2 + dz^2\right)\right]$$

For the parameter region $\lambda^2 + 2\rho\lambda - 4 > 0$, we found the following new solution

$$\boxed{ds^2 = -dt^2 + t^{2\omega}\left[t^{-4\zeta}dx^2 + t^{2\zeta}\left(dy^2 + dz^2\right)\right]}$$

$$\omega = \frac{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8}{6\lambda(\lambda + 2\rho)} \qquad \zeta = \frac{\lambda^2 + 2\rho\lambda - 4}{3\lambda(\lambda + 2\rho)}$$

Apparently, the expansion is anisotropic and its degree of anisotropy is given by

$$\boxed{\frac{\Sigma}{H} = \frac{\dot{\sigma}}{\dot{\alpha}} = \frac{1}{3}I\varepsilon_H}$$

$$I = \frac{\lambda^2 + 2\rho\lambda - 4}{\lambda^2 + 2\rho\lambda}$$

$$0 \le I < 1$$

$$\varepsilon_H = -\frac{\dot{H}}{H^2} = \frac{6\lambda(\lambda + 2\rho)}{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8}$$

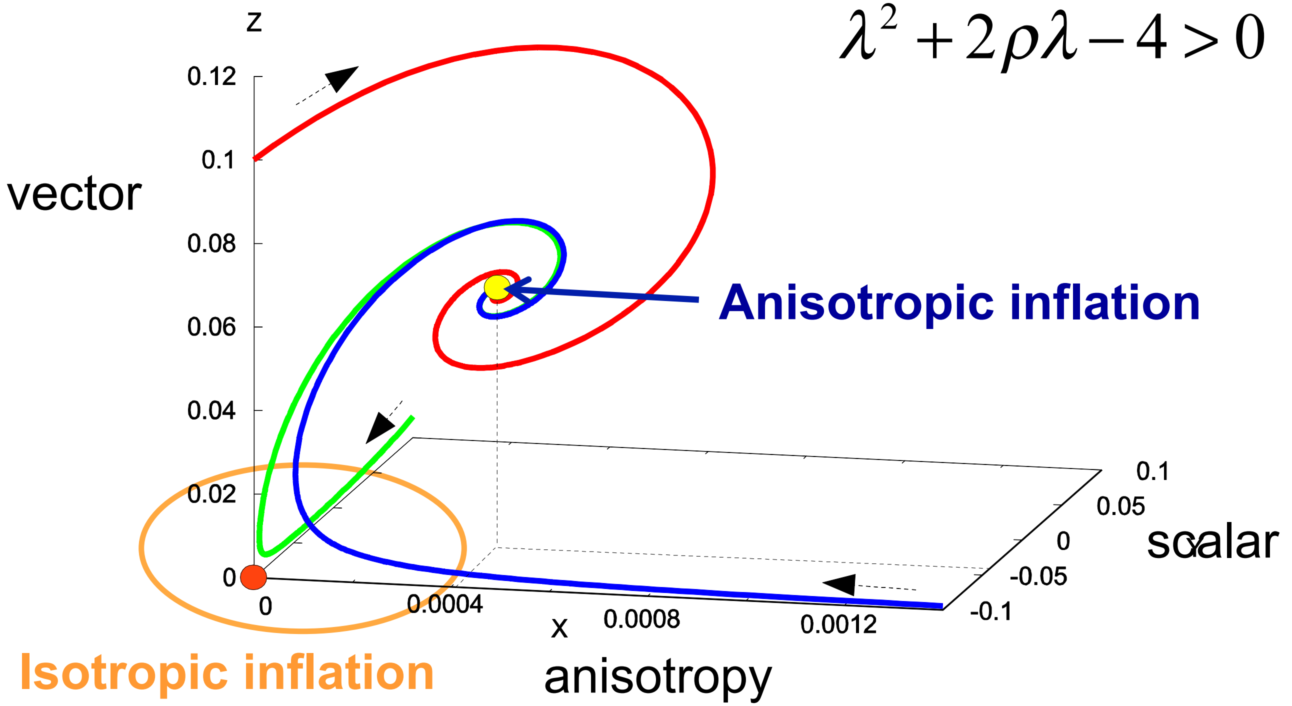
slow roll parameter

The phase flow

Kanno, Watanabe, Soda, JCAP, 2010.

Quantum fluctuations generate seeds of coherent vector fields.

$$\lambda^2 + 2\rho\lambda - 4 > 0$$



After a transient isotropic inflationary phase,
the universe enter into an anisotropic inflationary phase.

The result universally holds for other set of potential and gauge kinetic functions.

Generality of anisotropic inflation

Consider the slow roll phase $\varepsilon_H \ll 1$

In order for the vector contribution to increase, we need the condition $\dfrac{f_\phi}{\kappa f}\dfrac{V_\phi}{\kappa V} > 2$

As ϕ decreases, f^{-2} Increases. However,

$$3\dot{\alpha}\dot{\phi} = -V_\phi(\phi) + E^2 f^{-3}(\phi) f_\phi(\phi) e^{-4\alpha-4\sigma}$$

opposite to the potential force

Once the vector contributes the dynamics of the inflaton field,
the energy density of the vector field does not increase any more

The vector energy density saturates at $\dfrac{E^2 f^{-2}(\phi) e^{-4\alpha-4\sigma}}{2V} = \dfrac{1}{2}\dfrac{V_\phi}{V}\dfrac{f}{f_\phi} < 1$

$$\Rightarrow \quad \frac{\Sigma}{H} = \frac{\dot{\sigma}}{\dot{\alpha}} \approx \frac{E^2 f^{-2} e^{-4\alpha-4\sigma}}{V} \approx \frac{V_\phi}{V}\frac{f}{f_\phi} < \frac{1}{\kappa^2}\left(\frac{V_\phi}{V}\right)^2 \approx \varepsilon_H$$

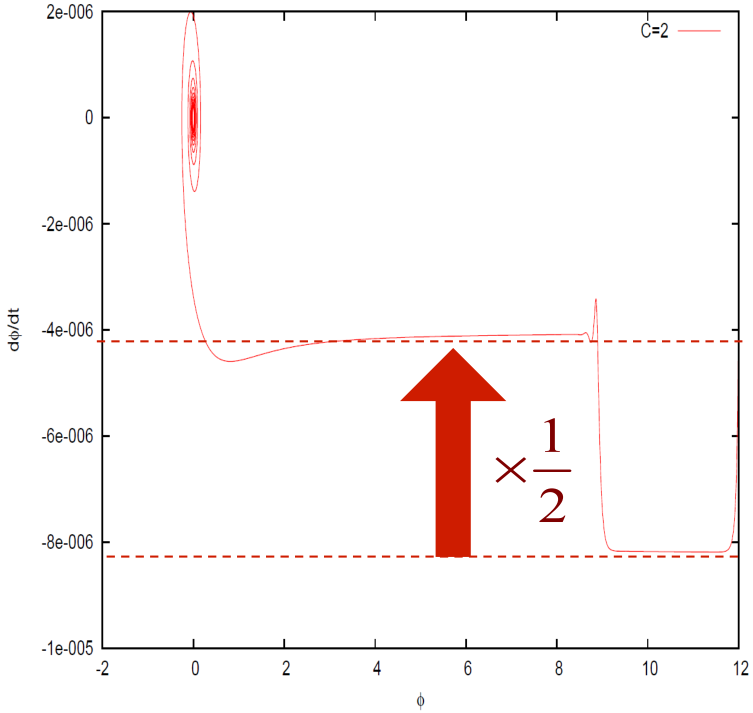
Example : chaotic inflation $V = \frac{1}{2} m^2 \phi^2$

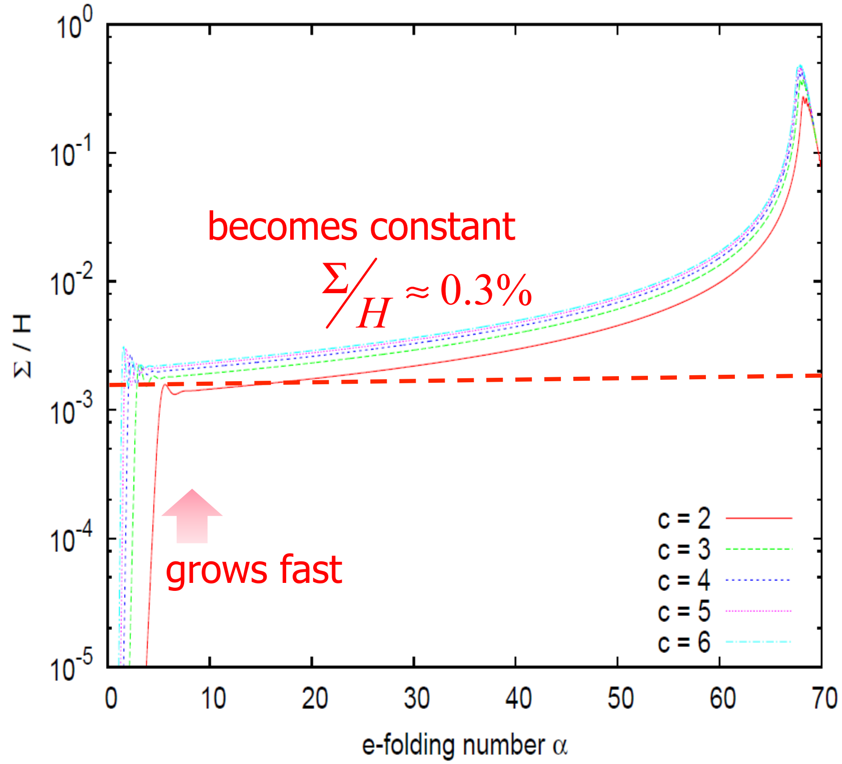
A simple choice is $f(\phi) = e^{c\kappa^2 \phi^2 / 2}$ $c > 1$ **Watanabe, Kanno, Soda, PRL, 2009.**

We find that the degree of anisotropy is written by the slow-roll parameter.

$$\boxed{\frac{\Sigma}{H} = \frac{1}{3} I \varepsilon_H}$$

: A universal relation $I = \frac{c-1}{c}$





Anisotropic Inflation is an attractor

$$ds^2 = -dt^2 + e^{2Ht}\left[e^{-4\Sigma t}dx^2 + e^{2\Sigma t}\left(dy^2 + dz^2\right)\right]$$

$$\boxed{\frac{\Sigma}{H} = \frac{1}{3} I \varepsilon_H} \qquad 0 \le I < 1$$

$$\Downarrow$$

Statistical Symmetry Breaking in the CMB

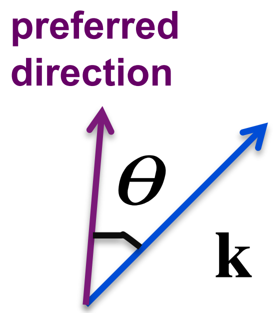
Predictions of anisotropic inflation

Thus, we found the following nature of primodordial fluctuations in anisotropic inflation.

Watanabe, Kanno, Soda, PTP, 2010.
Dulaney, Gresham, PRD, 2010.
Gumrukcuoglu,, Himmetoglu., Peloso PRD, 2010.

statistical anisotropy in curvature perturbations

$$P_s(\mathbf{k}) = P_s(k)\left[1 + g_s \sin^2\theta\right] \qquad g_s = 24\, I\, N^2(k)$$



statistical anisotropy in primordial GWs

$$P_t(\mathbf{k}) = P_t(k)\left[1 + g_t \sin^2\theta\right] \qquad g_t = 6\, I\, \varepsilon_H\, N^2(k)$$

cross correlation between curvature perturbations and primordial GWs

$$r_c = \frac{\langle \zeta G \rangle}{\langle \zeta\zeta \rangle} = -24\, I\, \varepsilon_H N^2(k)$$

TB correlation in CMB

These results give consistency relations between observables.

$$\boxed{4g_t = \varepsilon_H g_s \qquad r_c = -4g_t}$$

How to test the anisotropic inflation?

The current observational constraint is given by

WMAP constraint Pullen & Kamionkowski 2007 $g_s = 24\, I\, N^2(k) \leq 0.3$

Now, suppose we detected $g_s = 24\, I\, N^2(k) = 0.3 \quad \varepsilon_H = 0.02$

Then we could expect

- **statistical anisotropy in GWs** $g_t = 1.5 \times 10^{-3}$
- **cross correlation between curvature perturbations and GWs** $\dfrac{\langle \zeta G \rangle}{\langle \zeta\zeta \rangle} = -6 \times 10^{-3}$

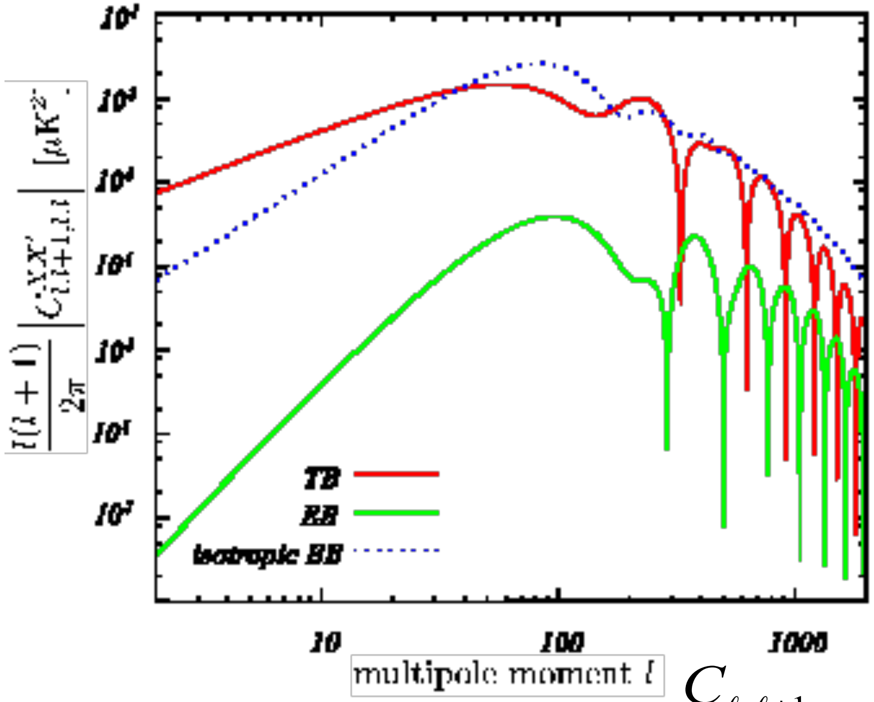
If these predictions are proved, it must be an evidence of anisotropic inflation!

We should look for the following signals in PLANCK data!

When we assume the tensor to the scalar ratio $r = 0.3$

and scalar anisotropy $g_s = 0.3$

The off-diagonal spectrum becomes **Watanabe, Kanno, Soda, MNRAS Letters, 2011.**



$C_{\ell,\ell+1}$

The anisotropic inflation can be tested through the CMB observation!

Cf. Ma, Effstatiou, Challinor 2011
Bartolo et al., 2011.
Huterer's talk

Summary

- We have shown that anisotropic inflation with a gauge kinetic function induces the statistical symmetry breaking in the CMB.

More precisely, we have given the predictions:

✓the statistical anisotropy in scalar and tensor fluctuations
✓the cross correlation between scalar and tensor

- Off-diagonal angular power spectrum can be used to prove or disprove our scenario.
- Our analysis gives a first cosmological constraint on gauge kinetic functions.
- As a by-product, we found a counter example to the cosmic no-hair conjecture.