Gravity Waves and Anisotropic Stress in Inflation

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Outline

- Anisotropic stress τ_{ij} in inflation
- Classical source – particle production
- $<\tau_{ij}>=0$, quantum fluctuations of stress tensor
- Effects on gravity waves or tensor modes

Gravitational Waves

Einstein–Hilbert action $$I_G = \frac{1}{16\pi G}\int d^4x\sqrt{g}\,R$$

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2$$

$$= a^2(\eta)\left(d\eta^2 - d\mathbf{x}^2\right)$$

$$g_{\mu\nu} = a^2(\eta)(\eta_{\mu\nu} + h_{\mu\nu}), \quad h_{\mu\nu} \ll 1,$$

Synchronous
Transverse
Traceless gauge

$$h_{00} = h_{0i} = 0, \quad h^k_k = \partial_i h^{ij} = 0,$$

$$I_{\text{graviton}} = \frac{1}{16\pi G}\int d^4x\; a^2\;\frac{1}{4}\partial_\mu h_{ij}\partial^\mu h^{ij}$$

$$h_{ij}(x) = h(x;\mathbf{k},\lambda)\,\epsilon_{ij}(\mathbf{k};\lambda)\,,$$

polarization tensor and $\lambda = +, \times$.

$$\epsilon_{ij}(\mathbf{k};\lambda)\epsilon^{ij}(\mathbf{k};\lambda') = 2\delta_{\lambda\lambda'}$$

$$I_{\text{graviton}} = \frac{1}{16\pi G}\int d^4x\; a^2(\eta)\;\frac{1}{2}\left[(\partial_\mu h(x;\mathbf{k},+))^2 + (\partial_\mu h(x;\mathbf{k},\times))^2\right]$$

$$h(x;\mathbf{k},\lambda) = (2\pi)^{-\frac{3}{2}} h_\lambda(\eta;\mathbf{k})\, e^{i\mathbf{k}\cdot\mathbf{x}} + \text{H.c.}$$

equation of motion for the wave amplitude

$$\ddot{h}_\lambda + 2\frac{\dot{a}}{a}\dot{h}_\lambda + k^2 h_\lambda = 0$$

Inflation $\longrightarrow$ Radiation-dominated $\longrightarrow$ matter-dominated

choose
Bunch-Davis vacuum
or positive-energy solution

$$|h_\lambda(\eta;k)|^2 k^3 \simeq 8\pi G H^2 \left[\frac{3j_1(k\eta)}{k\eta}\right]^2$$

scale-invariant
power spectrum

$$\frac{3j_1(k\eta)}{k\eta} \simeq 1 \quad \text{as} \quad k\eta \ll 1\,.$$

Anisotropy Stress

energy-momentum tensor $\delta T_{ij} = \bar{p}\, h_{ij} + a^2 \delta_{ij} \delta p + a^2 \pi_{ij}$

$\downarrow$ perfect fluid

$\downarrow$ anisotropic stress tensor

$$\ddot{h}_{ij} + 2\frac{\dot{a}}{a}\dot{h}_{ij} + k^2 h_{ij} = 16\pi G a^2 \pi_{ij}.$$

traceless transverse anisotropic stress tensor $\pi_{ii} = 0, \quad \partial_i \pi_{ij} = 0$

Free-streaming of relativistic particles χ (classical) with energy density ρ_χ

$$\pi_{ij} = -4\rho_\chi \int_{\eta_i}^{\eta} K(k\eta - k\eta')\dot{h}_{ij}(\eta')\, d\eta'$$ a back reaction

Kernel $$K(u) = \frac{j_2(u)}{u^2} = -\frac{\sin u}{u^3} - \frac{3\cos u}{u^4} + \frac{3\sin u}{u^5} \qquad u = k\eta$$

Inflation $$a = -\frac{1}{H\eta} \qquad H^2 = \frac{8\pi G}{3}\rho_\phi$$

$$\boxed{\frac{d^2\tilde{h}}{du^2} - \frac{2}{u}\frac{d\tilde{h}}{du} + \tilde{h} = -\frac{24\epsilon}{u^2}\int_{u_i}^{u} K(u-u')\frac{d\tilde{h}}{du'}\, du'}$$

$$\epsilon \equiv \frac{\rho_\chi}{\rho_\phi}$$

$$h = (8\pi G)^{\frac{1}{2}} H k^{-\frac{3}{2}} \tilde{h}$$

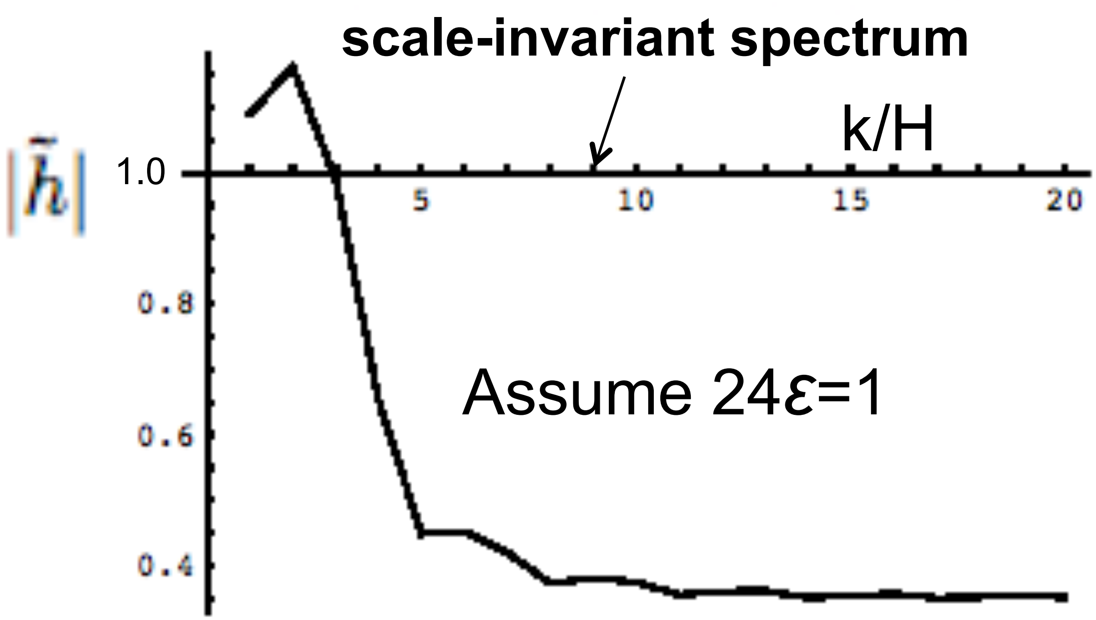
Damping of Tensor Modes

In standard slow-roll inflation

$$\rho_\chi \simeq H^4 \,, \quad \epsilon \simeq \frac{H^4}{\rho_\phi} \simeq \frac{H^2}{M_{pl}^2} \lesssim 10^{-10}$$

In trapped inflation

$$\rho_\chi \simeq k_*^4 \quad \text{where } k_* \simeq \sqrt{g\dot{\phi}} \,, \quad \epsilon \simeq 10^{-2}$$



k/H=1 corresponds to the present horizon if inflation lasts 60 e-folds

Quantum Anisotropic Stress

Generation of gravity waves by a fluctuating source:

Integrate $\Box_S h_\mu{}^\nu = -16\pi G\, S_\mu{}^\nu$

using a retarded Green's function

$$\Box_S G_R(x,x') = -\frac{\delta(x-x')}{\sqrt{-\gamma}}$$

$g_{\mu\nu} = \gamma_{\mu\nu} + h_{\mu\nu}$

and form the metric correlation function

$$K_\mu{}^\nu{}_\rho{}^\sigma(x,x') = \langle h_\mu{}^\nu(x)\, h_\rho{}^\sigma(x')\rangle - \langle h_\mu{}^\nu(x)\rangle\langle h_\rho{}^\sigma(x')\rangle$$

in terms of a stress tensor correlation function

$$C_\mu{}^\nu{}_\rho{}^\sigma(x,x') = \langle S_\mu{}^\nu(x)\, S_\rho{}^\sigma(x')\rangle - \langle S_\mu{}^\nu(x)\rangle\langle S_\rho{}^\sigma(x')\rangle$$

Conformally invariant fields:

$$C^{RW}_{\mu\nu\alpha\beta}(x,x') = a^{-4}(\eta)\, a^{-4}(\eta')\, C^{flat}_{\mu\nu\alpha\beta}(x,x')$$

Take spatial Fourier transforms:

$$\hat{A}(\eta,\mathbf{k}) \equiv \frac{1}{(2\pi)^3}\int d^3x\, e^{i\mathbf{k}\cdot\mathbf{x}} A(\eta,\mathbf{x})$$

and take $\mathbf{k}$ to be in the z-direction.

Need only x & y components of $\hat{C}_\mu{}^\nu{}_\rho{}^\sigma(\eta_1,\eta_2,k)$

Note: "power spectrum" in cosmology usually refers to

$$\mathcal{P}(k) = 4\pi k^3\, P(k)$$

Here $P(k)$ is a spatial component of $\hat{K}_\mu{}^\nu{}_\rho{}^\sigma(\eta,\eta,k)$ ***equal time correlation***

First model: assume that the gravity wave fluctuations vanish at some initial time $\eta = \eta_0$ (the beginning of inflation) and then integrate forward in time to the end of inflation at $\eta = \eta_r$

Power spectrum:

$$P_s(k) = 64(2\pi)^8 \int_{\eta_0}^{\eta_r} d\eta_1 \int_{\eta_0}^{\eta_r} d\eta_2 \, \hat{G}(\eta, \eta_1, k) \, \hat{G}(\eta, \eta_2, k) \, \hat{C}_{flat}(\eta_1 - \eta_2, k)$$

Result:

$$P_s(k) = -\frac{H^2 \, S^2}{3\pi^2 \, k}(1 + k^2 \, H^{-2})$$

S = expansion factor during inflation

Three remarkable features:

1) Negative power

2) Grows as the duration of inflation increases

$\propto S^2$

3) Highly blue tilted

$$P(k) \propto k$$

Second model: assume that the coupling to the fluctuating stress tensor is switch on gradually with a switching function $e^{p\eta}$

Now 1/p is the approximate conformal time at which the interaction begins.

Explore the exponentially switched model:

$$P_e(k) = -\frac{H^3(1 + k^2/H^2)\,S}{8\pi^2\,k^2}$$

Corresponding spatial correlation function:

$$C_e(r) = -\frac{H^3\,S}{4r}$$

(A delta function term has been dropped.)

Consider modes of the order of the horizon size today:

If $h > 10^{-5}$ $(C_e > 10^{-10})$

the tensor perturbations would have been detected.

This leads to the constraint:

$$S < 10^{40} \left(\frac{10^{16} GeV}{E_R}\right)^7$$

Allows enough inflation to solve the horizon and flatness problems

Can we take the power spectrum seriously for much smaller wavelengths?

At scales of the order of 100km, LIGO has set limits of $h < 10^{-24}$

This implies

$$S < 10^{25} \left(\frac{10^{13} GeV}{E_R}\right)^7$$

and

$$E_R < 10^{13} GeV$$

assuming efficient reheating to an energy of E_R

Can we take the power spectrum seriously for much smaller wavelengths?

Use of transplanckian modes.

Dominant contribution comes from modes wavelengths far less than the Planck length at the beginning of inflation.

Is this a problem?

This effect might offer a probe of transplanckian physics in the form of a non-Gaussian, non-scale invariant spectrum of gravity waves.

Summary

- Classical anisotropic stress (model dependent) damps tensor modes
- Quantum anisotropic stress (intrinsic) generates tensor modes

  - non-Gaussian and a blue spectrum
  - this spectrum may imply a limit on inflation e-folds
  - may also offer a test of trans-planckian physics