Conformal symmetry of inflationary perturbations

P.C., arXiv:1108.0874

with G. D'Amico, M. Musso and J. Noreña, arXiv:1106.1462
+ work in progress with M. Simonovic and J. Noreña
(+ K. Hinterbichler, L. Hui, J. Khoury and A. Nicolis)

Outline

Test scalars in de Sitter have conformal-invariant correlation functions

The consistency relation for the 3-point function does not receive linear correcti

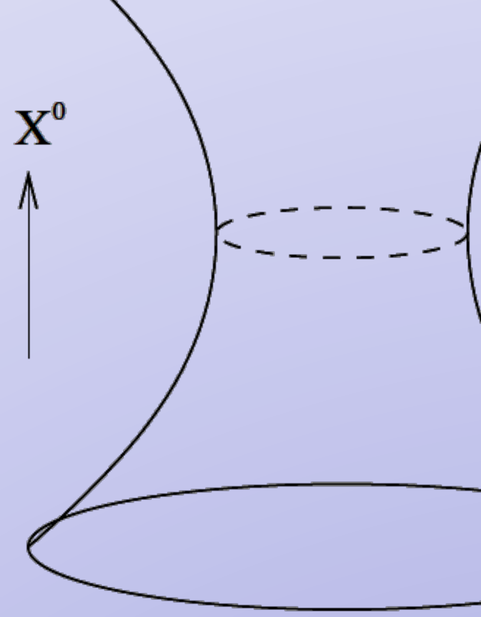
$$\langle\zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle \simeq -(2\pi)^3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)P(k_1)P(k_S)\left[\frac{\mathrm{d}\ln(k_S^3 P(k_S))}{\mathrm{d}\ln k_S}+\mathcal{O}\left(\frac{k_1^2}{k_S^2}\right)\right]$$

$$k_1 \to 0$$

Non-linear realization of conformal invariance constrains the squeezed limit of correlation functions

New consistency relations

Inflation takes place in ~ dS



$$ds^2 = \frac{1}{H^2\eta^2}(-d\eta^2 + d\vec{x}^2)$$

Translations, rotations: ok

Dilations (if slow-roll) $\quad \eta \to \lambda\eta, \ \vec{x} \to \lambda\vec{x}$

→ scale-invariance

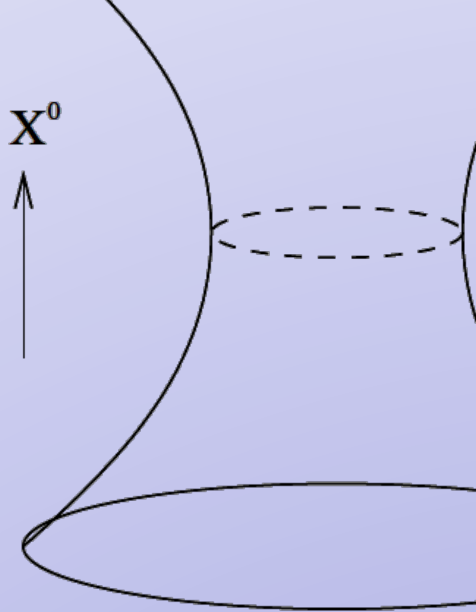
$$\varphi_{\vec{k}} \to \lambda^3 \varphi_{\lambda\vec{k}} \qquad \langle \varphi_{\vec{k}_1} \varphi_{\vec{k}_2} \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \frac{1}{k_1^3} F(k_1\eta)$$

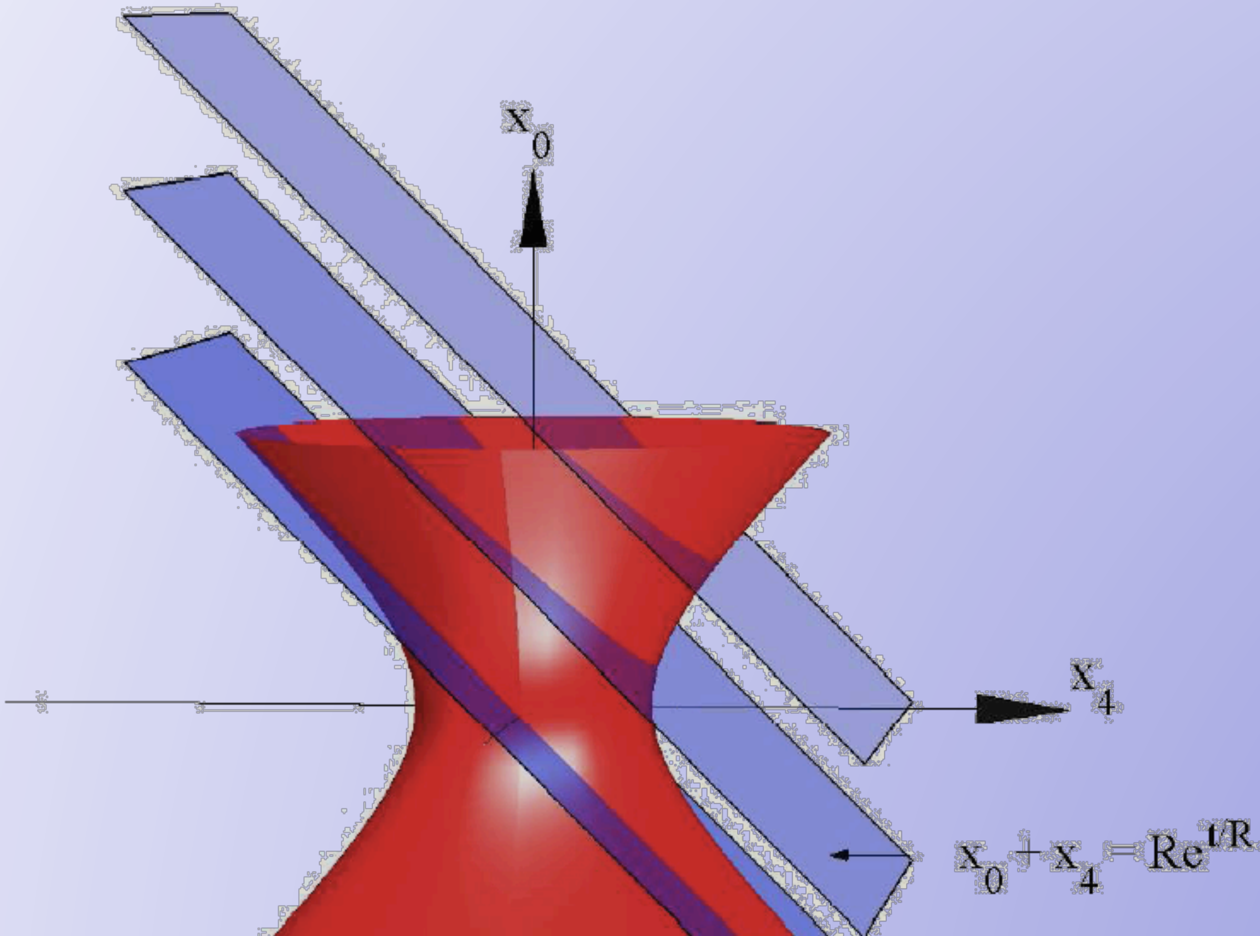
Special conformal

$$\eta \to \eta - 2\eta(\vec{b}\cdot\vec{x}) \,, \quad x^i \to x^i + b^i(-\eta^2 + \vec{x}^2) - 2x^i(\vec{b}\cdot\vec{x})$$

Inflation takes place in ~ dS

$$ds^2 = \frac{1}{H^2\eta^2}(-d\eta^2 + d\vec{x}^2)$$





Antoniadis, Mazur and Mottola, '11   Maldacena and Pi

perturbations are created by a sector with negligible interactions with the inflaton
correlation functions have the full SO(4,1) symmetry

They are conformal invariant

Independently of any details about this sector, even at strong coupling

We are interested in correlators at late times

$$x^i \to x^i + b^i \vec{x}^2 - 2x^i(\vec{b}\cdot\vec{x}) \qquad \eta \to \eta - 2\eta(\vec{b}\cdot\vec{x})$$

$$\varphi \sim \eta^\Delta \,, \quad \Delta = \frac{3}{2}\left(1 - \sqrt{1 - \frac{4m^2}{9H^2}}\right) \ll 1$$

This is the transformation of the a primary of conformal dim Δ

ample: $m = \sqrt{2}H \qquad \Delta = 1$

$$d^4x\sqrt{-g}\frac{M}{6}\varphi^3 \longrightarrow \langle\varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\varphi_{\vec{k}_3}\rangle = (2\pi)^3\delta(\sum_i \vec{k}_i)\frac{\pi}{16}MH^2\eta_*^3 \cdot \frac{1}{k_1k_2k}$$

$$\langle\varphi(\vec{x}_1)\varphi(\vec{x}_2)\varphi(\vec{x}_3)\rangle = \frac{MH^2\eta_*^3}{128\pi^2}\cdot\frac{1}{|\vec{x}_1 - \vec{x}_2||\vec{x}_1 - \vec{x}_3||\vec{x}_2 - \vec{x}_3|}$$

$$\varphi_{\vec{k}_3}\rangle = (2\pi)^3\delta(\sum_i \vec{k}_i)\frac{H^2}{\prod_i 2k_i^3}\frac{2M}{3}\left[\sum_i k_i^3(-1+\gamma+\log(-k_t\eta_*)) + k_1k_2k_3 - \sum_{i}\right.$$

$$k_t \equiv \sum_i k_i$$

Zaldarriaga '0
Seery, Malik,Lytl

$$\langle\varphi_1(\vec{x}_1)\varphi_2(\vec{x}_2)\varphi_3(\vec{x}_3)\rangle = \frac{MH^2}{48\pi^2}\log\frac{|\vec{x}_1-\vec{x}_2|}{A\eta_*}\log\frac{|\vec{x}_1-\vec{x}_3|}{A\eta_*}\log\frac{|\vec{x}_2-\vec{x}_3|}{A\eta_*}$$

Everything determined up to two constants

Independently of the interactions!

$$d^4x\sqrt{-g}\nabla_\mu\varphi_1\nabla^\mu\varphi_2\varphi_3 \longrightarrow \frac{1}{M}\int d^4x\sqrt{-g}\frac{1}{2}(\Box\varphi_3\varphi_1\varphi_2 - \Box\varphi_1\varphi_2\varphi_3 - \Box\varphi$$

$$\int d^4x \frac{1}{8M^4}(\partial_\mu\varphi)^2(\partial_\nu\varphi)^2$$

$$\langle \varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\varphi_{\vec{k}_3}\varphi_{\vec{k}_4}\rangle = (2\pi)^3\delta(\sum_i \vec{k}_i)\frac{1}{M^4}\frac{H^8}{\prod_i 2k_i^3}\left[-\frac{144k_1^2k_2^2k_3^2k_4^2}{k_t^5} - 4\left(\frac{12k_1k_2k_3k_4}{k_t^5} + \frac{3\prod_{i<j<l}k_ik_jk_l}{k_t^4} + \frac{\prod_{i<j}k_ik_j}{k_t^3}\right.\right.$$

$$(\vec{k}_3\cdot\vec{k}_4)+(\vec{k}_1\cdot\vec{k}_3)(\vec{k}_2\cdot\vec{k}_4)+(\vec{k}_1\cdot\vec{k}_4)(\vec{k}_2\cdot\vec{k}_3))+(\vec{k}_1\cdot\vec{k}_2)\left(\frac{4k_3^2k_4^2}{k_t^3}+\frac{12(k_1+k_2)k_3^2k_4^2}{k_t^4}+\frac{48k_1k_2k_3^2k_4^2}{k_t^5}\right)+6\text{perm}$$

Not so obvious it is conformal invariant…

I can check it in **Fourier space** Maldacena and Pim

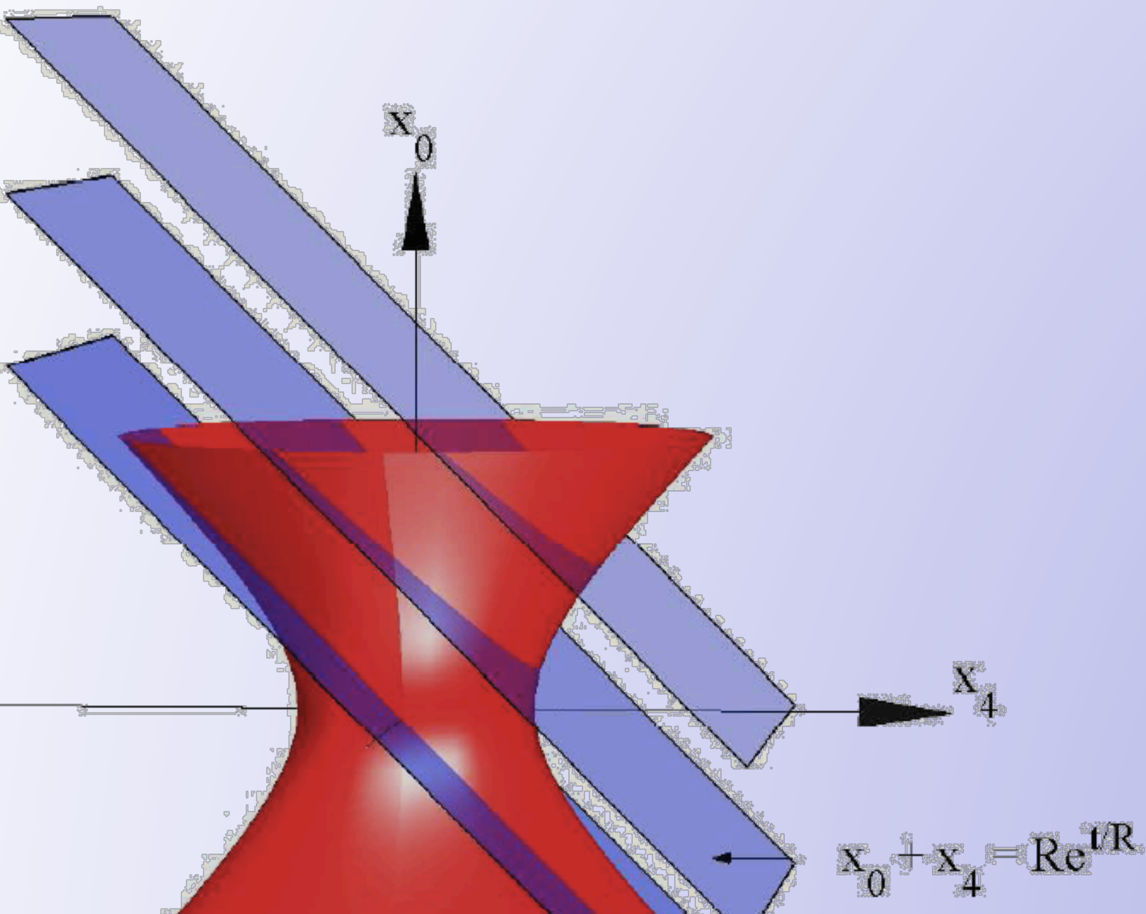
$$\sum_{a=1,2,3,4}\left[6\vec{b}\cdot\vec{\partial}_{k_a} - \vec{b}\cdot\vec{k}_a\vec{\partial}_{k_a}^2 + 2\vec{k}_i\cdot\vec{\partial}_{k_a}(\vec{b}\cdot\vec{\partial}_{k_a})\right]\langle\varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\varphi_{\vec{k}_3}\varphi_{\vec{k}_4}\rangle' = 0$$

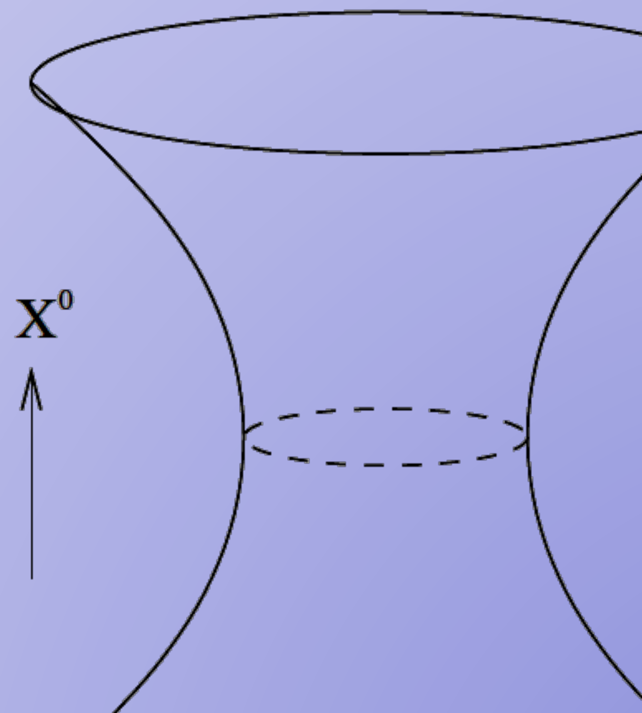
general: $F\left(\frac{r_{13}r_{24}}{r_{12}r_{34}}, \frac{r_{23}r_{41}}{r_{12}r_{34}}\right)\prod_{i<j} r_{ij}^{-2\Delta/3}$ 2 parameters instead of 5

If we see something beyond the spectrum

- Something not conformal would be a probe of a "sliced" de Sitter

- Something conformal would be a probe of pure de Sitter





Squeezed limit in single-field models: one of the modes is a classical bkg when the other two exit the H^{-1}

$$\langle \zeta_B(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle \simeq \langle \zeta_B(\vec{k}_1)\langle\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle_B\rangle \qquad k_1 \ll k_S$$

The long mode acts just as a rescaling of the coordinates

$$\langle\zeta(\vec{x}_2)\zeta(\vec{x}_3)\rangle_B = \xi(\tilde{\vec{x}}_2 - \tilde{\vec{x}}_3) \simeq \xi(\vec{x}_3 - \vec{x}_2) + \zeta_B(\vec{x}_+)(\vec{x}_3 - \vec{x}_2)\cdot\nabla\xi(\vec{x}_3 - \vec{x}_2$$

Going back to Fourier space we get the consistency relation

$$\langle\zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle \simeq -(2\pi)^3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)P(k_1)P(k_S)\frac{\mathrm{d}\ln(k_S^3P(k_S))}{\mathrm{d}\ln k_S}$$

$$\vec{k} = (\vec{k} \quad \vec{k})/2$$

owest order in derivatives

$$S_2 + S_3 = M_{\rm Pl}^2 \int d^4x \, \epsilon a^3 \left[(1+3\zeta_B)\dot{\zeta}^2 - (1+\zeta_B)\frac{(\partial_i \zeta)^2}{a^2} \right]$$

g mode reabsorbed by coordinate rescaling $\vec{x} \to (1+\zeta_B)\vec{x}$

rections:

me evolution of ζ is of order k^2

patial derivatives will be symmetrized with the short modes, giving k^2

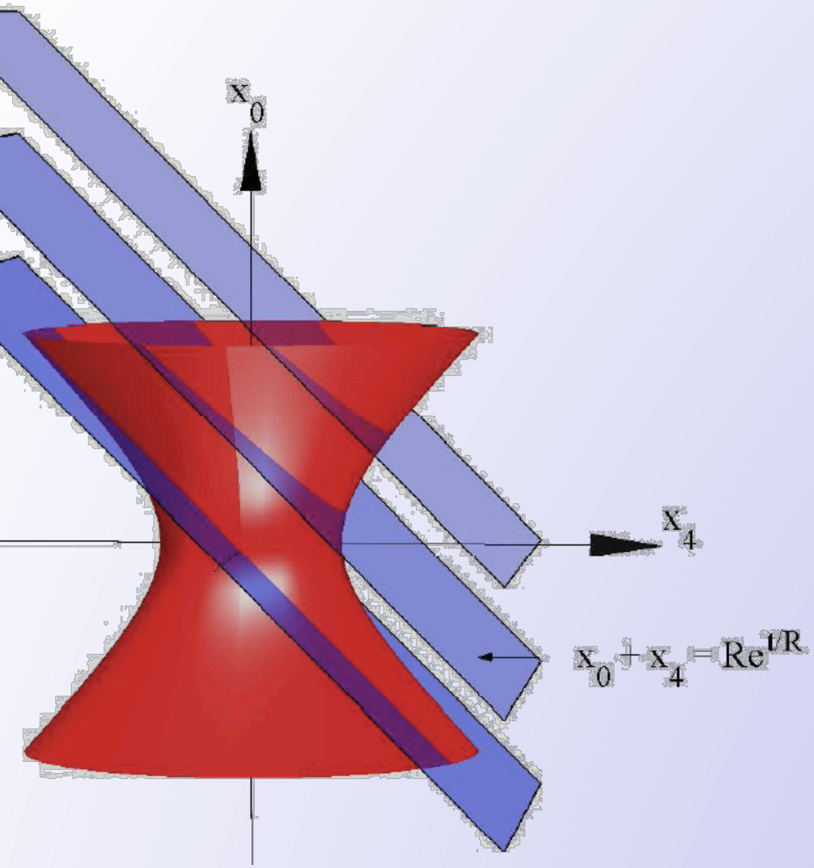
onstraint equations give order k^2 corrections

Final result: in the not-so-squeezed limit we have

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle \simeq -(2\pi)^3 \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3) P(k_1) P(k_S) \left[\frac{d \ln(k_S^3 P(k_S))}{\ldots} + \mathcal{O}\left(\frac{k_1^2}{\ldots}\right) \right.$$

in progress with Simonovic an
+ Hinterbichler, Hui, Khoury ar

The inflaton does not break SO(4,1) explicitely: it is still non-linearly realized



$$\phi(\vec{x},t) = t + \pi(\vec{x},t)$$

$$\eta \to \eta - 2\eta(\vec{b}\cdot\vec{x})$$

$$x^i \to x^i + b^i(-\eta^2 + \vec{x}^2) - 2x^i(\vec{b}$$

$$\boxed{\pi \to \pi + 2H^{-1}(\vec{b}\cdot\vec{x})}$$

A gradient of a background mode can be traded for a conformal transformation

(Assuming zero tilt for simplicity)

$$\langle \zeta_{\vec{q}}\, \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \ldots \rangle' = -\frac{1}{2} P(q) q^i D_i \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \ldots \rangle', \qquad |\vec{q}| \to 0$$

$$q^i D_i = \sum_{a=1}^{n} \left[6\vec{q} \cdot \vec{\partial}_{k_a} - \vec{q} \cdot \vec{k}_a \vec{\partial}^2_{k_a} + 2\vec{k}_a \cdot \vec{\partial}_{k_a} (\vec{q} \cdot \vec{\partial}_{k_a}) \right]$$

• The variation of the 2-point function is zero: no linear term in the 3 pf

Linear terms in the 4pf are connected with the conf variation of the 3-point functi

• No tilt suppression

It encodes at the level of observables the relation among operators

E.g.

$$(g^{00}+1)^2 \supset -2\dot{\pi}(\partial_\mu\pi)^2 + (\partial_\mu\pi)^4$$

✓ $\dot{\pi}^4$ has free coefficient, indeed it has no k^{-2} squeezed limit

✓ $(\nabla\pi)^4$ and $\dot{\pi}^2(\nabla\pi)^2$ are related to $\dot{\pi}(\nabla\pi)^2$ and $\dot{\pi}^3$

Also including gravity

So far we have neglected metric perturbations + taken exact dS but everything still holds

$$x^i \to x^i + b^i\vec{x}^2 - 2x^i(\vec{b}\cdot\vec{x})$$

Maldacena

$$\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle = (2\pi)^3\delta^3(\sum \vec{k}_i)\frac{\dot{\rho}_*^4}{\dot{\phi}_*^4}\frac{H_*^4}{M_{pl}^4}\frac{1}{\prod_i(2k_i^3)}\mathcal{A}_*$$

$$\mathcal{A} = 2\frac{\ddot{\phi}_*}{\dot{\phi}_*\dot{\rho}_*}\sum_i k_i^3 + \frac{\dot{\phi}_*^2}{\dot{\rho}_*^2}\left[\frac{1}{2}\sum_i k_i^3 + \frac{1}{2}\sum_{i\neq j} k_i k_j^2 + 4\frac{\sum_{i>}}{}\right.$$

$$\langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle^{\mathrm{CI}} = (2\pi)^3\delta(\sum_a \mathbf{k}_a)\frac{H_*^6}{4\epsilon^2\prod_a(2k_a^3)}\sum_{\mathrm{perms}}\mathcal{M}_4(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)$$

$$\mathcal{M}_4(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = -2\frac{k_1^2k_3^2}{k_{12}^2k_{34}^2}\frac{W_{24}}{k_t}\left(\frac{\mathbf{Z}_{12}\cdot\mathbf{Z}_{34}}{k_{34}^2} + 2\mathbf{k}_2\cdot\mathbf{Z}_{34} + \frac{3}{4}\sigma_{12}\sigma_{34}\right)$$
$$- \frac{1}{2}\frac{k_3^2}{k_{34}^2}\sigma_{34}\left(\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_t}W_{124} + 2\frac{k_1^2k_2^2}{k_t^3} + 6\frac{k_1^2k_2^2k_4}{k_t^4}\right),$$

$$\begin{aligned}\sigma_{ab} &= \mathbf{k}_a\cdot\mathbf{k}_b + k_b^2\,,\\ \mathbf{Z}_{ab} &= \sigma_{ab}\mathbf{k}_a - \sigma_{ba}\mathbf{k}_b\,,\\ W_{ab} &= 1 + \frac{k_a+k_b}{k_t} + \frac{2k_ak_b}{k_t^2}\,,\end{aligned}$$

Lidsey, Seery, Slo
Seery, Sloth, Vern

- Linear and non-linear realization of conformal invariance

- Future directions:

  - Relation with Ward identities for spontaneously broken symmetrie

  - Extension to models with SO(4,2)

  - Extension to correlators involving gravitons