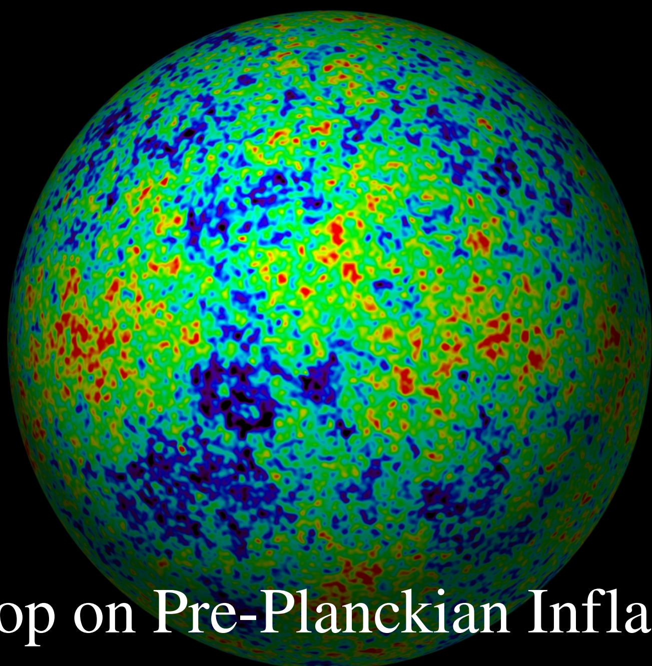
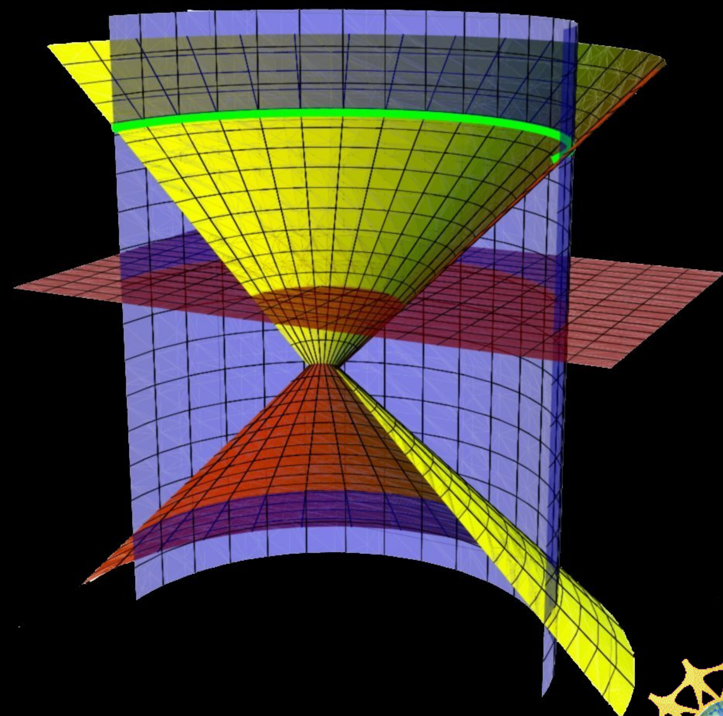
Scale-Invariant Perturbations: Is Inflation the Only Way?

Will Kinney

University at Buffalo The State University of New York

KICP





Workshop on Pre-Planckian Inflation
University of Minnesota
8 October 2011

NSF

CMB: Basic Properties

- Adiabatic density perturbations
- Superhorizon correlations
- Gaussian statistics
- Scale Invariance

Q: What does this really tell us?

Generating Superhorizon Perturbations

Punch line:

In an *expanding universe*, to generate perturbations consistent with observation, must have one of:

(1) Accelerated Expansion

(2) Superluminal Sound Speed

(3) Super-Planckian Energy Density

(Geshnizjani, WHK, Moradinezhad Dizgah, arXiv:1107.1241)

The Canonical Case

Mukhanov-Sasaki variable for curvature perturbations with *constant sound speed*

$$v \equiv z\zeta \qquad z = a\sqrt{2\epsilon}$$

$$\boxed{v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0}$$

The Canonical Case

Mukhanov-Sasaki variable for curvature perturbations with *constant sound speed*

$$v \equiv z\zeta \qquad z = a\sqrt{2\epsilon}$$

$$\boxed{v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0}$$

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{3}{2}(\rho + p) \qquad \text{Inflation:} \ \ \epsilon < 1$$

The Freezeout Horizon

Scale invariance:

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0 \qquad \frac{z''}{z} = \frac{2}{\tau^2}$$

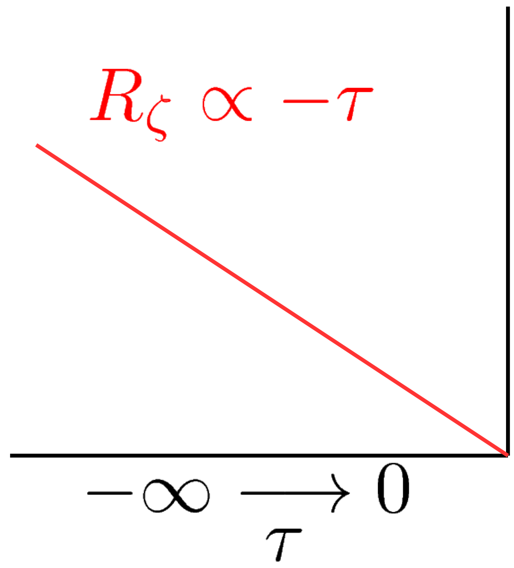
The Freezeout Horizon

Scale invariance:

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0 \qquad \frac{z''}{z} = \frac{2}{\tau^2} \equiv R_\zeta^{-2}$$

Freezeout Horizon

Perturbations are generated when *Freezeout Horizon* shrinks.



Two Horizons

Hubble Length: $$R_H = \frac{1}{aH} = \frac{a}{a'}$$

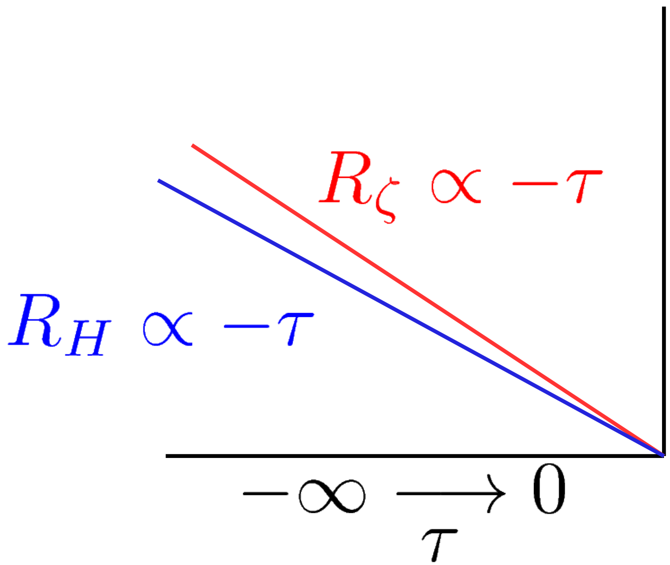
Freezeout Horizon: $$R_\zeta = \sqrt{\frac{a\sqrt{\epsilon}}{(a\sqrt{\epsilon})''}}$$

Two Horizons

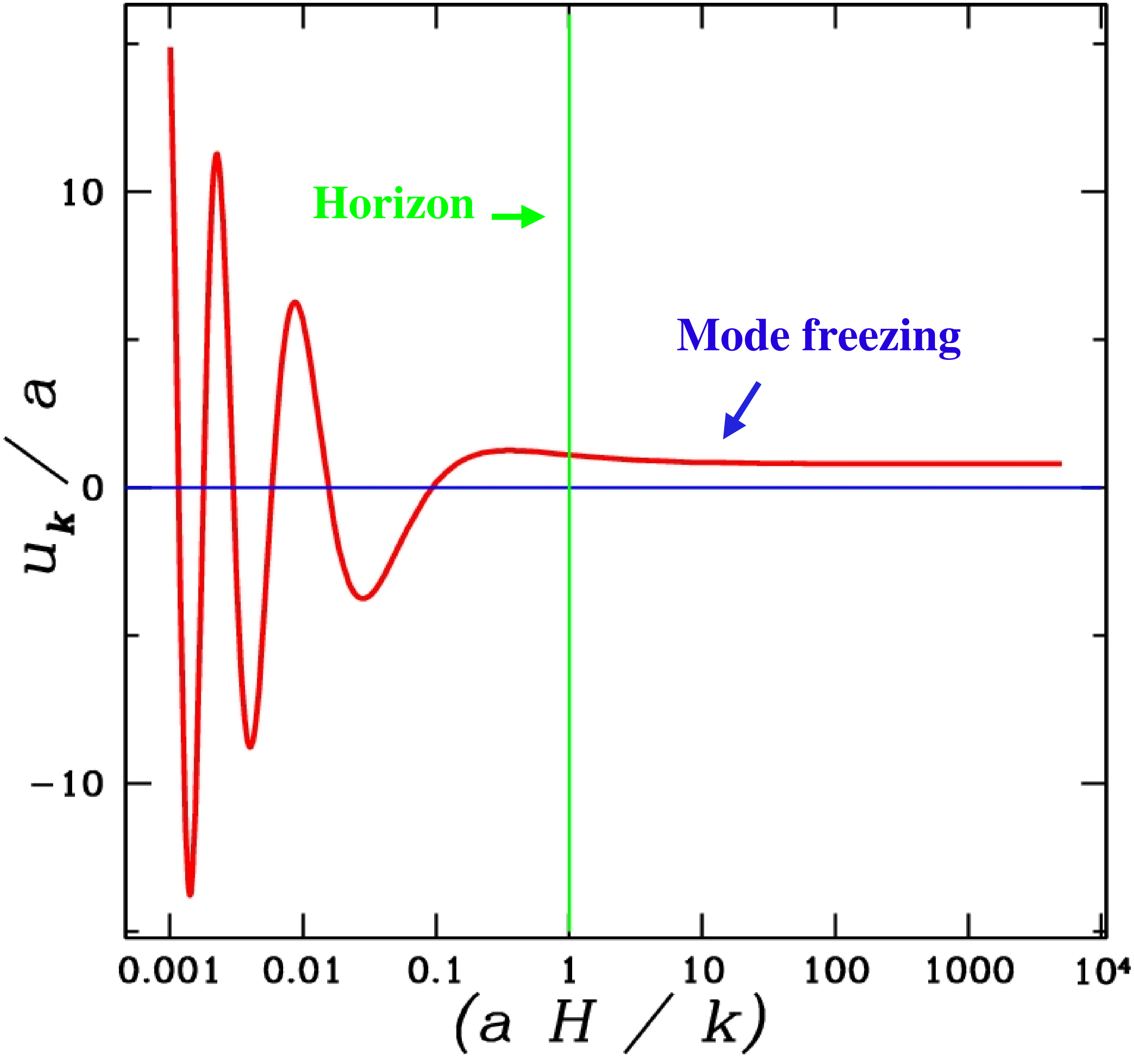
Hubble Length: $R_H = \dfrac{1}{aH} = \dfrac{a}{a'} \propto -\tau$

Freezeout Horizon: $R_\zeta = \sqrt{\dfrac{a\sqrt{\epsilon}}{(a\sqrt{\epsilon})''}} \propto -\tau$

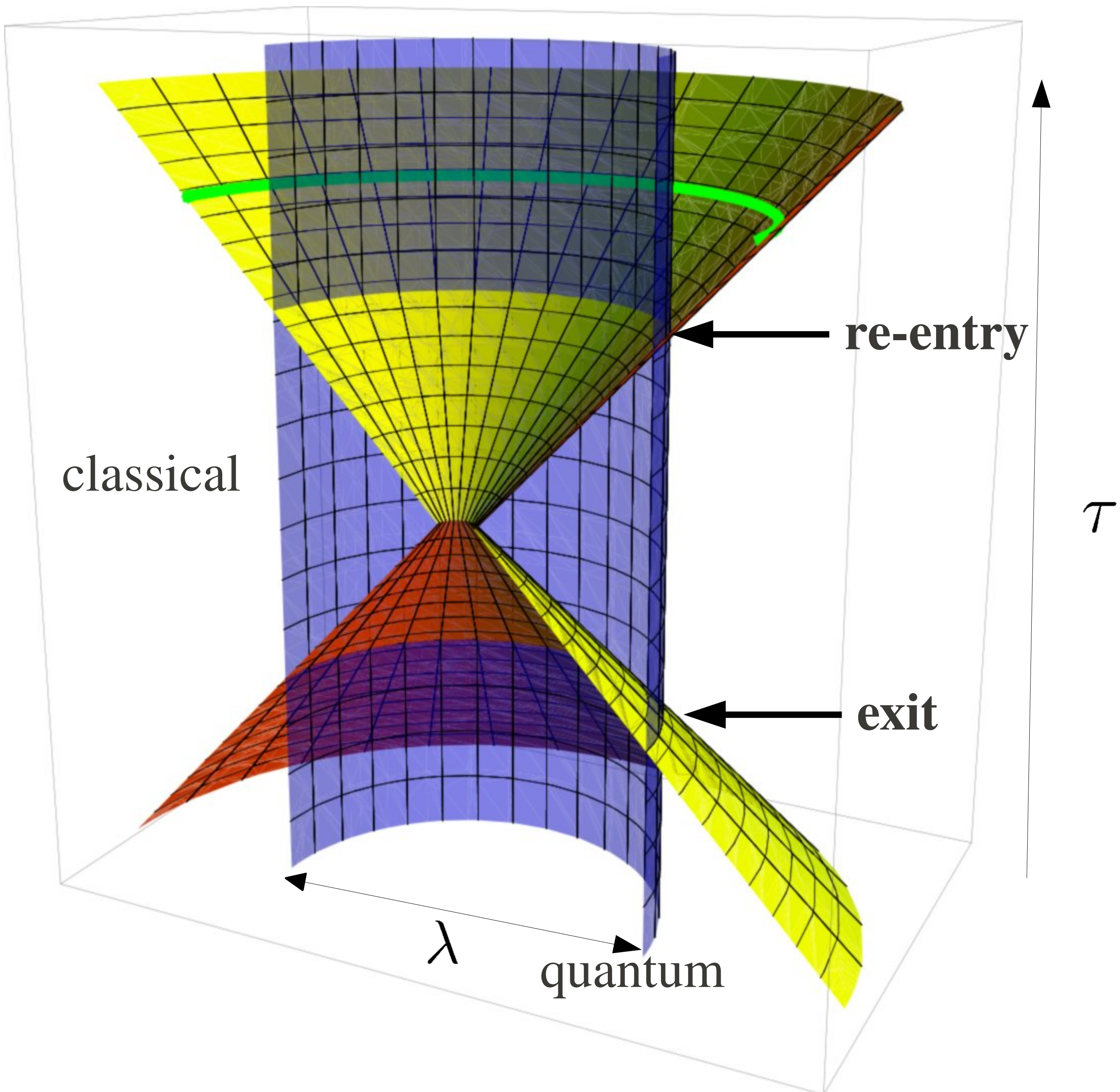
Slow Roll Inflation: $\epsilon \simeq \text{const.} \ll 1$



Generation of Perturbations



Mode Exit and Reentry



Two Horizons

Hubble Length: $R_H = \frac{1}{aH} = \frac{a}{a'}$

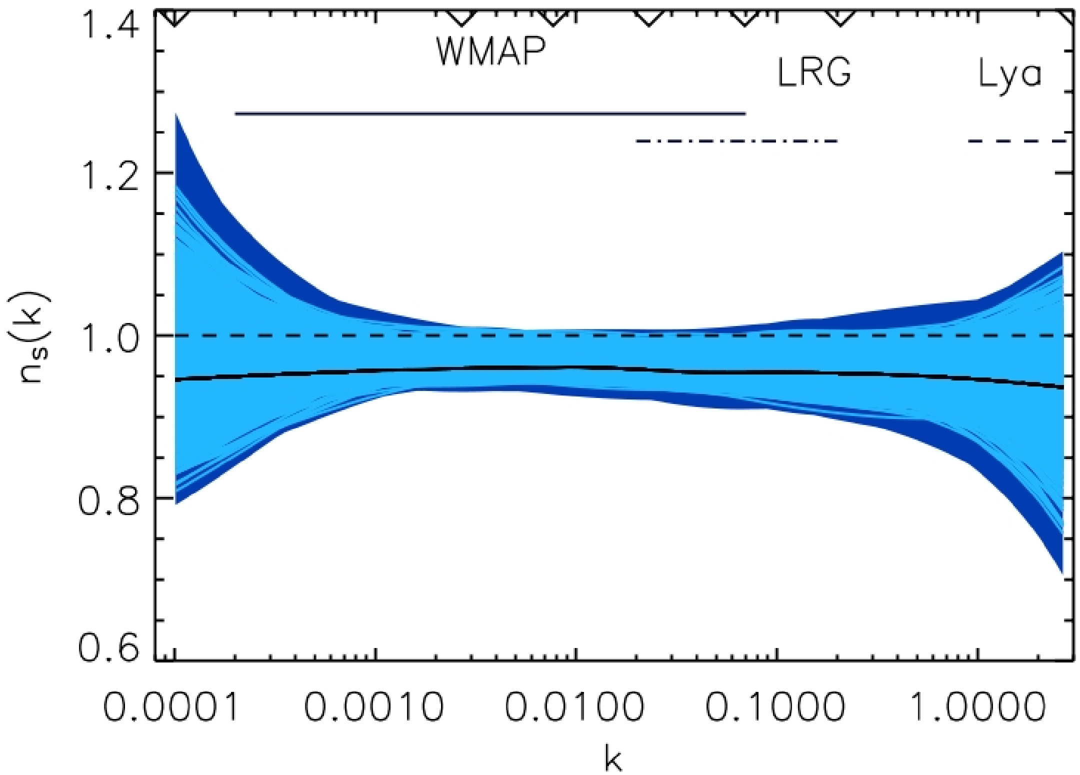
Freezeout Horizon: $R_\zeta = \sqrt{\frac{a\sqrt{\epsilon}}{(a\sqrt{\epsilon})''}}$

General Case: $R_\zeta \neq R_H$

What conditions allow us to have a shrinking freezeout horizon without inflation?

(Khoury & Miller, arXiv:1012.0846)

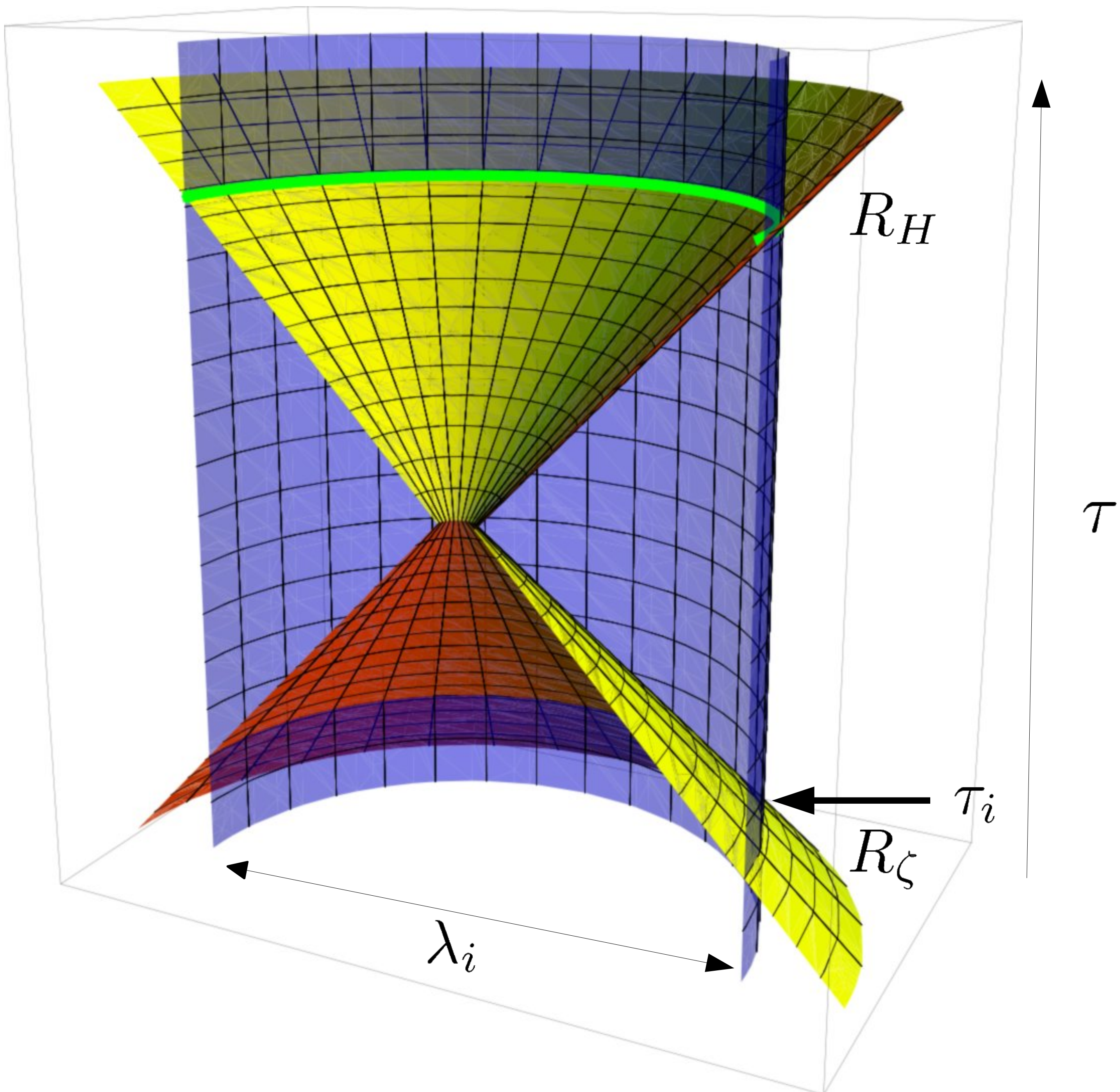
What We Know



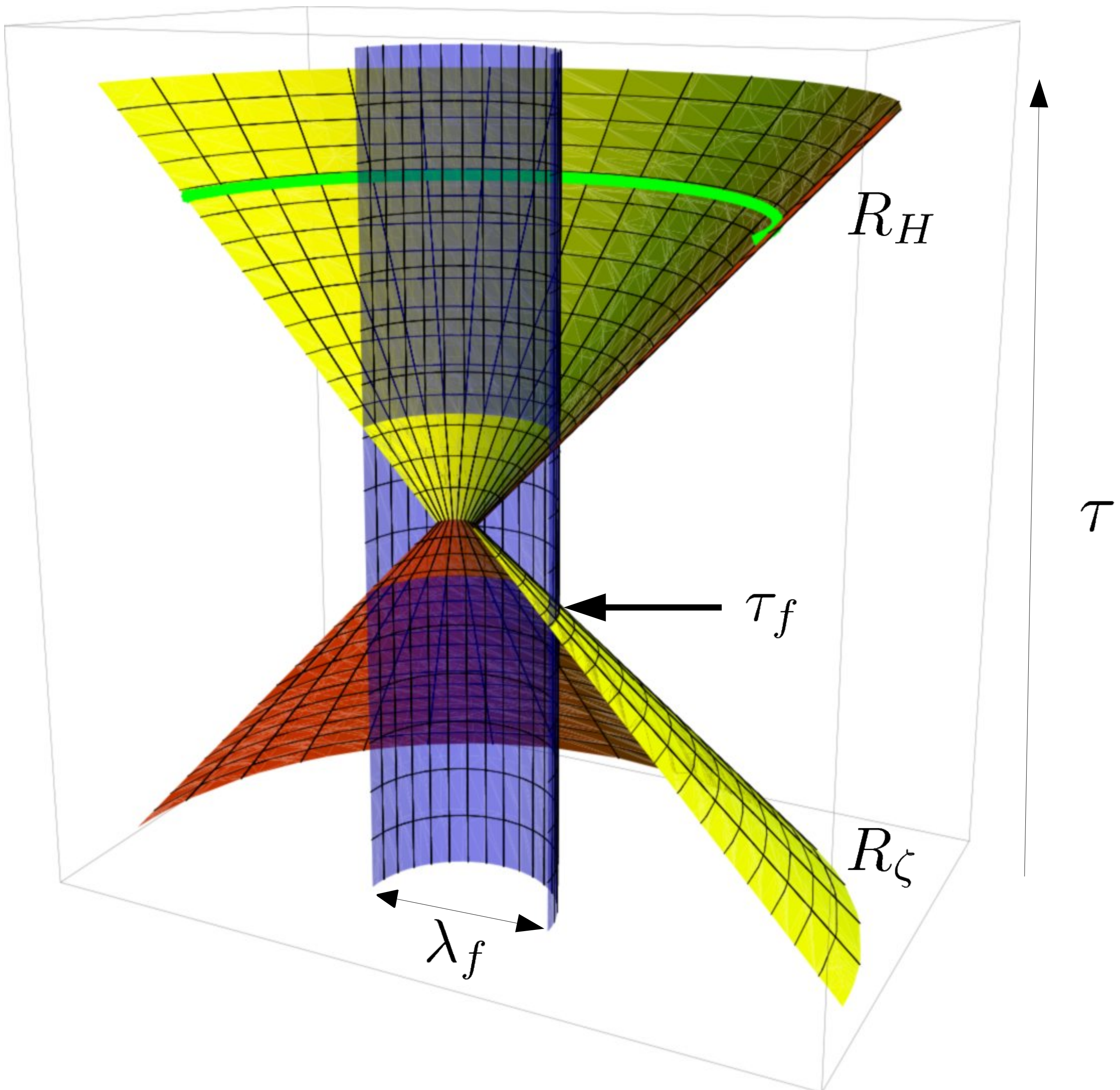
Peiris & Verde, arXiv:0912.0268

Power spectrum is approximately scale-invariant over *at least* a factor of 1000 in wavelength.

Longer Wavelength Modes Exit Earlier



Shorter Wavelength Modes Exit Later



Horizon Crossing and Scale

Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

Horizon Crossing: $\lambda_i(\tau_i) = R_\zeta(\tau_i) = |\tau_i|$

$$\lambda_f(\tau_f) = R_\zeta(\tau_f) = |\tau_f| > R_H(\tau_f)$$

Horizon Crossing and Scale

Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

Horizon Crossing: $\lambda_i(\tau_i) = R_\zeta(\tau_i) = |\tau_i|$

$$\lambda_f(\tau_f) = R_\zeta(\tau_f) = |\tau_f| > R_H(\tau_f)$$

CMB / LSS: $\lambda_i \geq 1000\lambda_f$

$$\boxed{\frac{\tau_f - \tau_i}{R_H(\tau_f)} > 1000} \qquad (\tau < 0)$$

Continuity

Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

Continuity: $$\frac{\dot{\rho}}{\rho} = -2\epsilon H$$

$$\ln \frac{\rho_i}{\rho_f} = 2\int_{t_i}^{t_f} \epsilon H dt = 2\int_{\tau_i}^{\tau_f} \epsilon R_H^{-1} d\tau$$

Continuity

Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

Continuity: $$\frac{\dot{\rho}}{\rho} = -2\epsilon H$$

$$\ln \frac{\rho_i}{\rho_f} = 2 \int_{t_i}^{t_f} \epsilon H dt = 2 \int_{\tau_i}^{\tau_f} \epsilon R_H^{-1} d\tau$$

$$> 2R_H^{-1}(\tau_f) \int_{\tau_i}^{\tau_f} \epsilon d\tau \quad (\dot{R}_H > 0)$$

Continuity

Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

Continuity: $\dfrac{\dot{\rho}}{\rho} = -2\epsilon H$

$$\ln \frac{\rho_i}{\rho_f} = 2\int_{t_i}^{t_f} \epsilon H dt = 2\int_{\tau_i}^{\tau_f} \epsilon R_H^{-1} d\tau$$

$$> 2R_H^{-1}(\tau_f)\int_{\tau_i}^{\tau_f} \epsilon d\tau \qquad (\dot{R}_H > 0)$$

$$> 2R_H^{-1}(\tau_f)\,(\tau_f - \tau_i) \qquad (\epsilon > 1)$$

Density

Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

CMB/LSS: $\dfrac{\tau_f - \tau_i}{R_H(\tau_f)} > 1000$

Continuity: $\ln \dfrac{\rho_i}{\rho_f} > 2\dfrac{\tau_f - \tau_i}{R_H(\tau_f)}$

Density

Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

CMB/LSS: $$\frac{\tau_f - \tau_i}{R_H(\tau_f)} > 1000$$

Continuity: $$\ln \frac{\rho_i}{\rho_f} > 2\frac{\tau_f - \tau_i}{R_H(\tau_f)} > 2000$$

$$\boxed{\rho_i > 10^{868} \rho_f!}$$

$$\rho_f \geq (100 \text{ MeV})^4 \Rightarrow \rho_i \gg M_P^4$$

The Non-Canonical Case

Mukhanov-Sasaki variable for curvature perturbations with *variable sound speed*

$$v \equiv q\zeta \qquad q = \frac{a\sqrt{2\epsilon}}{\sqrt{c_S}}$$

$$\boxed{v_k'' + \left(k^2 - \frac{q''}{q}\right) v_k = 0}$$

Time variable: $dy \equiv c_S d\tau$

(Khoury & Piazza, arXiv:0811.3633)

The Non-Canonical Case

Mukhanov-Sasaki variable for curvature perturbations with *variable sound speed*

$$v \equiv q\zeta \qquad q = \frac{a\sqrt{2\epsilon}}{\sqrt{c_S}}$$

$$v_k'' + \left(k^2 - \frac{q''}{q}\right) v_k = 0$$

Time variable: $dy \equiv c_S d\tau$

Scale Invariance: $R_\zeta = \sqrt{\frac{q}{q''}} \propto -y$

Horizon Crossing and Scale

Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

Horizon Crossing: $\lambda_i(\tau_i) = R_\zeta(\tau_i) = |y_i|$

$$\lambda_f(\tau_f) = R_\zeta(\tau_f) = |y_f| > R_H(\tau_f)$$

Horizon Crossing and Scale

Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

Horizon Crossing: $\lambda_i(\tau_i) = R_\zeta(\tau_i) = |y_i|$

$$\lambda_f(\tau_f) = R_\zeta(\tau_f) = |y_f| > R_H(\tau_f)$$

$$y_f - y_i = \int_{\tau_i}^{\tau_f} c_S d\tau = \bar{c}_S \left(\tau_f - \tau_i\right)$$

$$\boxed{\frac{\bar{c}_S(\tau_f - \tau_i)}{R_H(\tau_f)} > 1000}$$

Density

Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

CMB/LSS: $$\frac{\bar{c}_S(\tau_f - \tau_i)}{R_H(\tau_f)} > 1000$$

Continuity: $$\ln \frac{\rho_i}{\rho_f} > 2\frac{\tau_f - \tau_i}{R_H(\tau_f)} > \frac{2000}{\bar{c}_S}$$

Density

Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

CMB/LSS: $$\frac{\bar{c}_S(\tau_f - \tau_i)}{R_H(\tau_f)} > 1000$$

Continuity: $$\ln\frac{\rho_i}{\rho_f} > 2\frac{\tau_f - \tau_i}{R_H(\tau_f)} > \frac{2000}{\bar{c}_S}$$

$$\begin{array}{l} \rho_i \leq M_P^4 \\ \rho_f \geq (100MeV)^4 \end{array} \Longrightarrow \bar{c}_s > 10$$

Broken Scale Invariance

What if scale invariance is broken?

$$P(k) \propto k^{n_S - 1},\ n_S \neq 1$$

$$\boxed{R_\zeta^{-2} = \frac{q''}{q} = \frac{2 + (3/2)(1 - n_S)}{y^2}}$$

$$q = \frac{a\sqrt{2\epsilon}}{\sqrt{c_S}} \qquad dy \equiv c_S d\tau$$

Limit still holds!

Summary

In an *expanding universe*, to generate perturbations consistent with observation, must have one of:

(1) Accelerated Expansion

(2) Superluminal Sound Speed

(3) Super-Planckian Energy Density

(Geshnizjani, WHK, Moradinezhad Dizgah, arXiv:1107.1241)