Chain Inflation: Bubble Bubble Toil and Trouble

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Two types of Models of Inflation

- ***1) Tunneling Models:***

  Why Old Inflation Failed

  New proposals for tunneling models:

  (i) double-field inflation (Adams and Freese; Linde)

  (ii) extended inflation (Steinhardt)

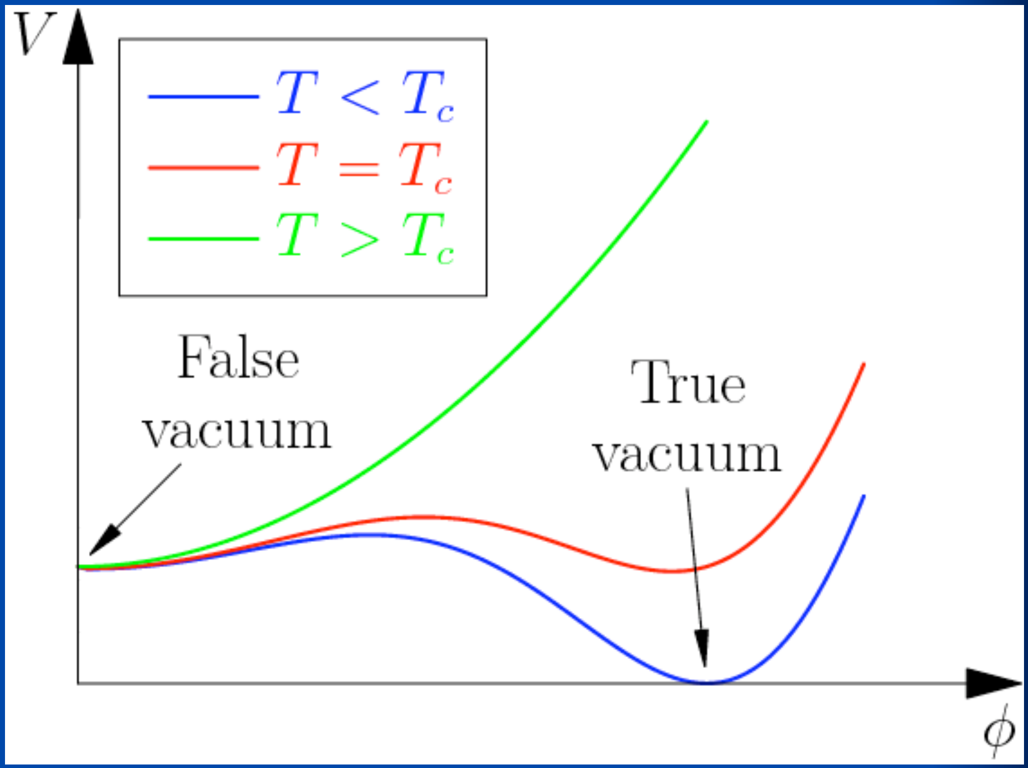
  (iii) Chain inflation (Freese and Spolyar)
- ***2) Rolling Models:***

  new inflation, chaotic inflation, hybrid inflation
- Natural inflation (Freese, Frieman, Olinto)

Old Inflation

Guth (1981)

- Temperature-dependent potential
- Initially at global minimum $\phi = 0$



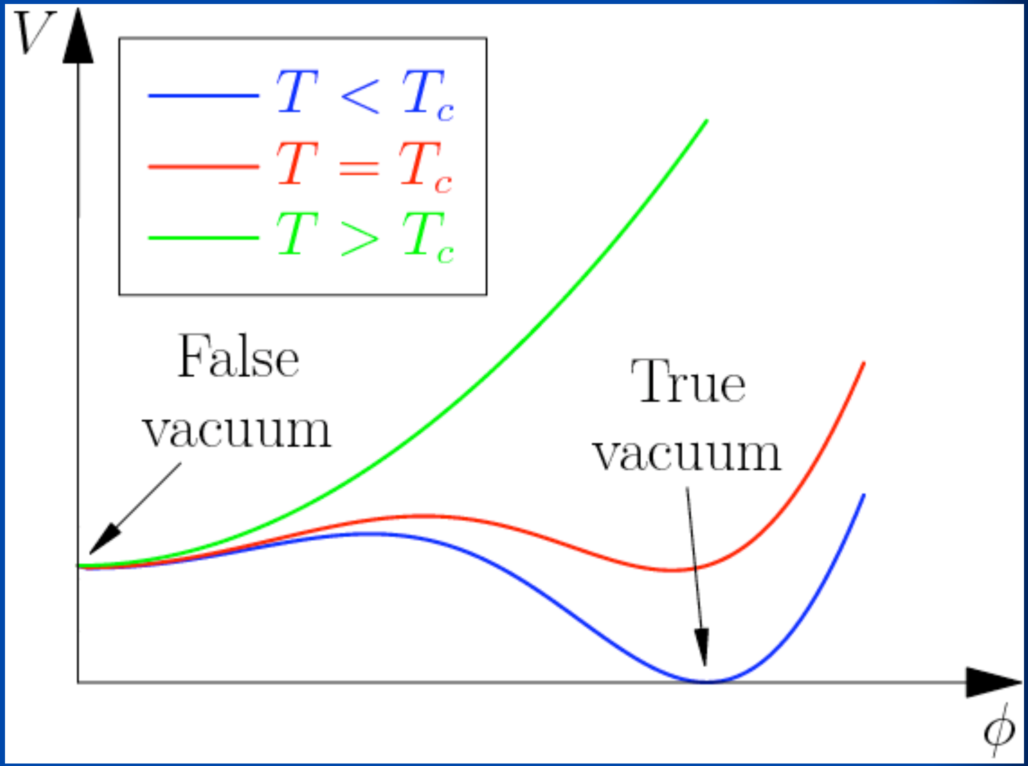
- When $T < T_c$, no longer global minimum
  $\Rightarrow$ false vacuum

Old Inflation

Guth (1981)

- Universe goes from false vacuum to true vacuum.

  How?

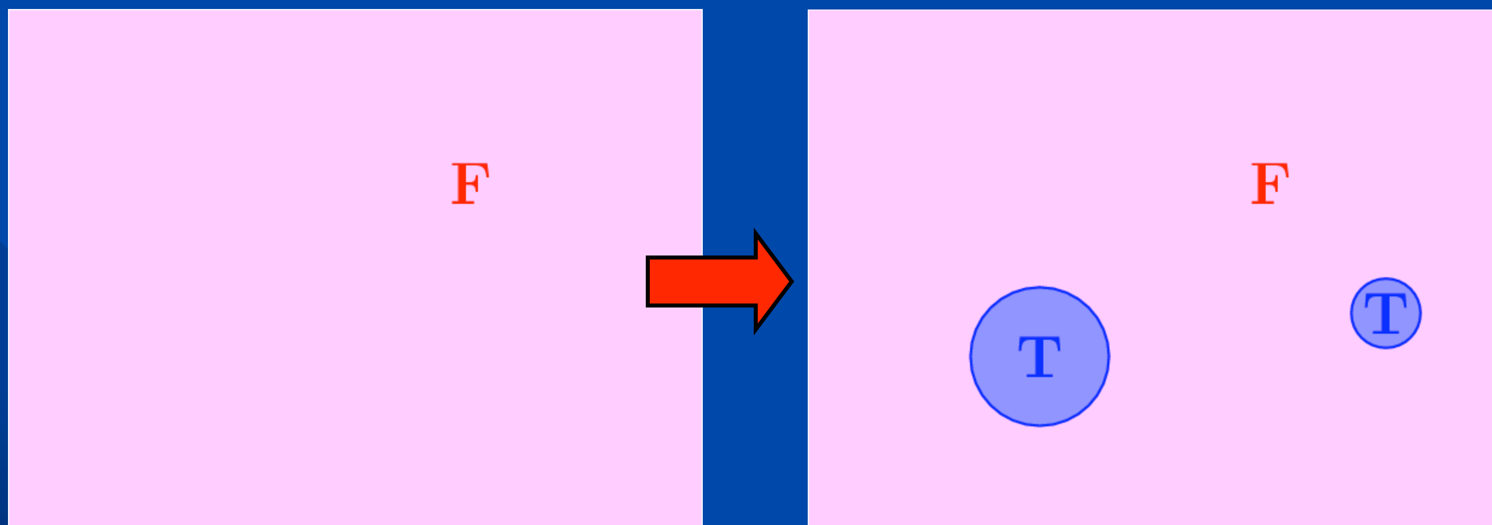


- Bubbles of true vacuum nucleate in a universe of false vacuum

  (first order phase transition)

Old Inflation

- Vacuum decay

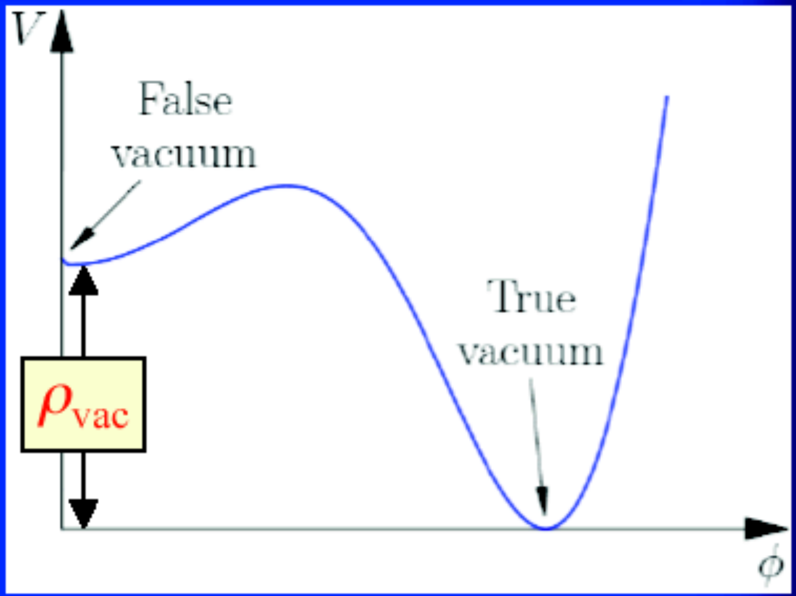


Entire universe is in false vacuum (**F**)

Nucleate bubbles of true vacuum (**T**)

Old Inflation

- With tunneling, the nucleation rate is slow, so the universe is trapped in the false vacuum for a long time



- The vacuum energy dominates over matter and radiation
  ⇒ de Sitter-like expanding universe

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{vac} = \text{constant}$$

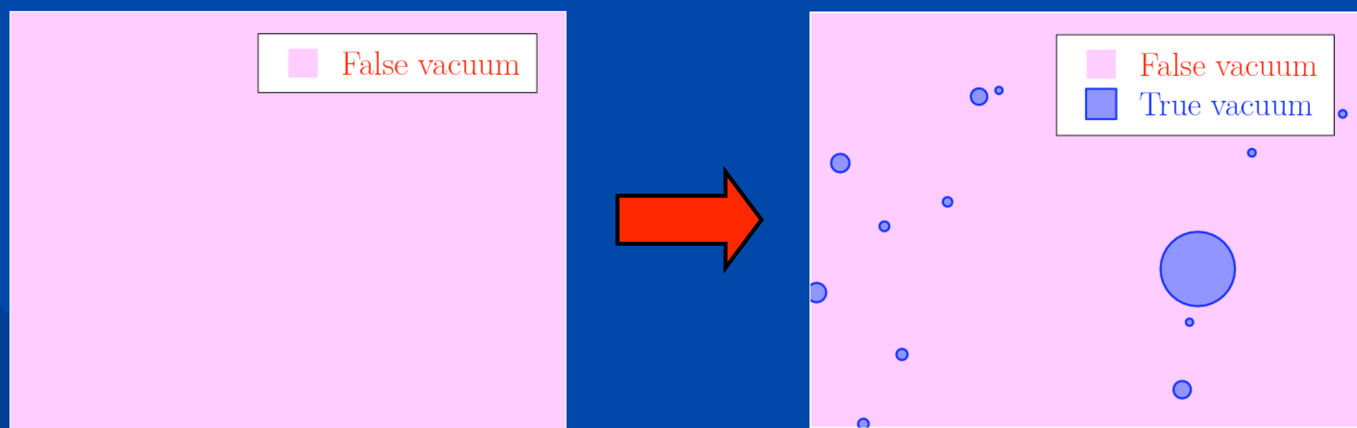
$$\text{solution:} \quad a \sim e^{Ht}, \quad H = \left[\frac{8\pi G}{3}\rho_{vac}\right]^{1/2}$$

- Enough inflation to solve problems:

$$a_{end} = a_{begin} \times 10^{27} = a_{begin} \times e^{65}$$

Old Inflation

- Vacuum decay: “swiss cheese problem”



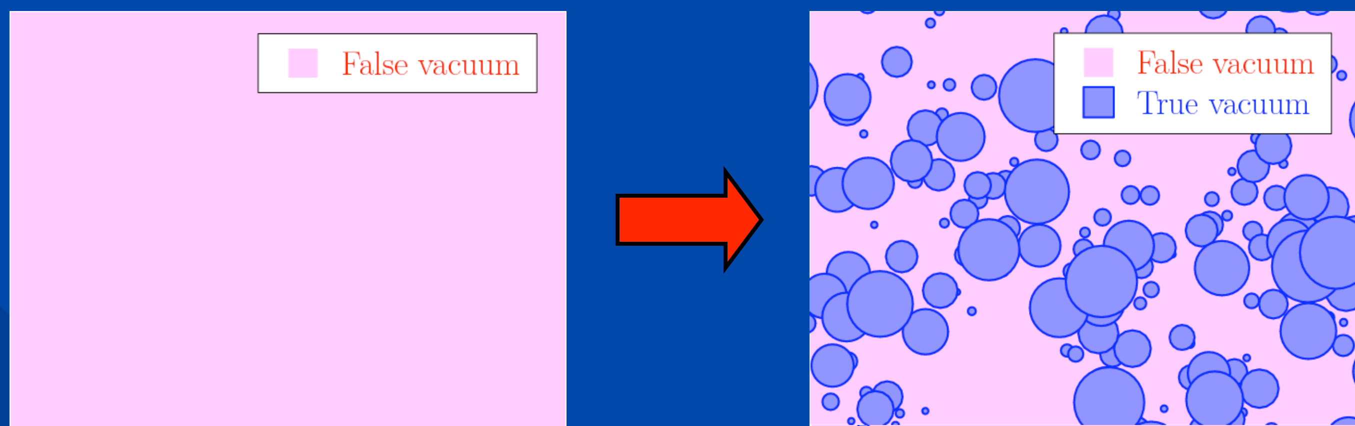
Entire universe is in false vacuum

Nucleate bubbles of true vacuum

Problem: bubbles never percolate & thermalize
$\Rightarrow$ NO REHEATING

Old Inflation

- Vacuum decay: “swiss cheese problem”



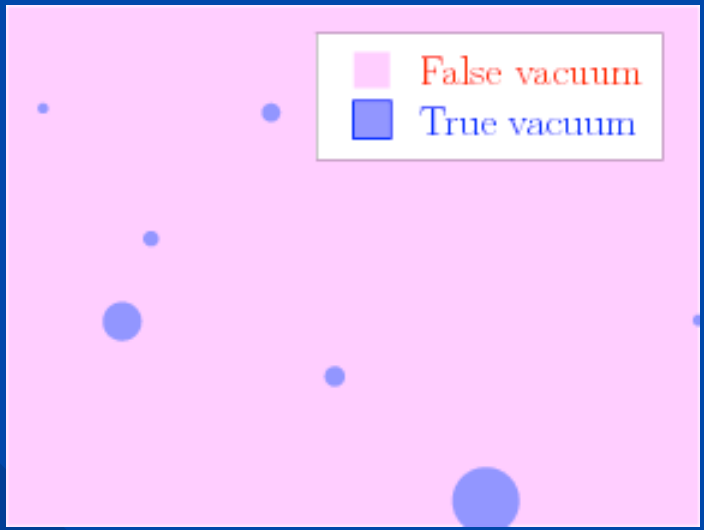
Entire universe is in false vacuum

Nucleate bubbles of true vacuum

Want rapid percolation

Old Inflation

- Bubbles inflate away faster than they form & grow
  $\Rightarrow$ no end to inflation & no reheating

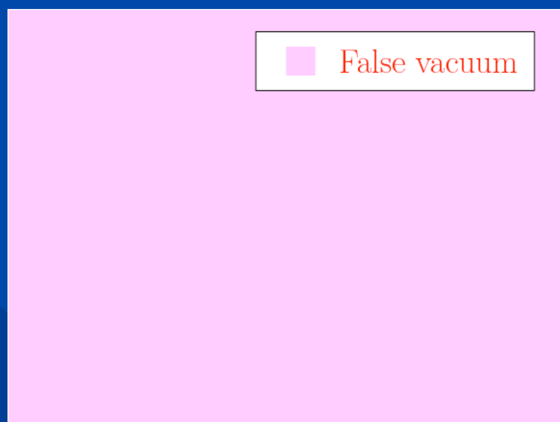


What is needed for tunneling inflation to work?

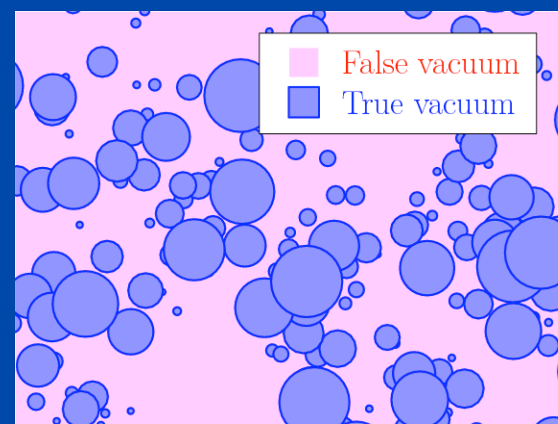
- Two requirements for inflation:
- 1) Sufficient Inflation: 60 e-foldings
- 2) The universe must thermalize and reheat; i.e. the entire universe must go through the phase transition at once. Then the phase transition completes.
- Can achieve both requirements with
- (i) time-dependent nucleation rate in <u>Double-field inflation</u> (Adams and Freese 1990; Linde 1990) with two coupled fields in a single tunneling event
- (ii) <u>Chain Inflation</u> (Freese and Spolyar 2005) with multiple tunneling events

Rapid phase transition leads to percolation (entire universe goes through phase transition at once)

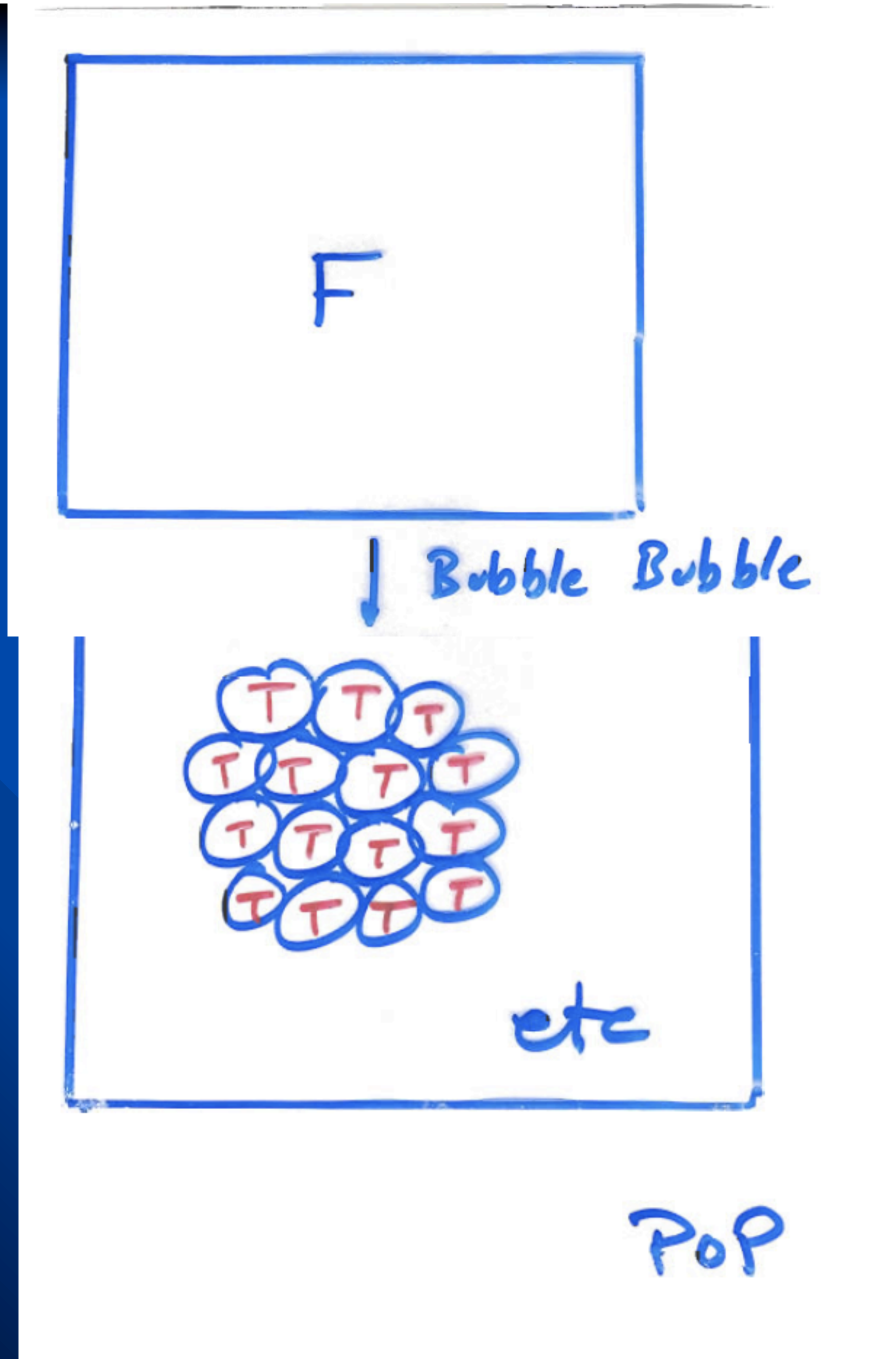
- Vacuum decay: “swiss cheese problem”



Entire universe is in false vacuum



Nucleate bubbles of true vacuum



This is what we want: Percolation

What is needed for tunneling inflation to work?

- Probability of a point remaining in false vacuum phase:

$$p(t) \sim e^{-\beta H t}, \beta = \Gamma / H^4$$

where Γ is the nucleation rate of bubbles and H is the expansion rate of the universe

- The number of e-foldings per tunneling event is

$$\chi = \int H dt \sim H\tau = \frac{3}{4\pi}\frac{H^4}{\Gamma}$$

- Graceful exit: Critical value of $\beta > \beta_{crit} = 9/4\pi$ is required to get percolation and reheating. In terms of number of efolds, this is $\chi < \chi_{crit} = 1/3$
- Sufficient Inflation requires $N_{tot} > 60$

β

Graceful Exit Achieved

Guth and Weinberg,1983

Turner, Weinberg, and Widrow, 1992

calculated that a critical value of

$$\beta = \frac{\Gamma}{H^4} \geq \beta_{crit} = 9/4\pi \qquad (8)$$

is required in order for percolation and thermalization to be achieved. In terms of e-foldings, this is

$$\chi \leq \chi_{crit} = 1/3. \qquad (9)$$

Two Requirements for Inflation

$$\beta = \Gamma / H^4$$

$$p(t) \sim \exp(-\frac{4\pi}{3}\beta H t) \sim \exp(-t/\tau)$$

- Lifetime of field
in metastable state:

$$\tau \sim \frac{3}{4\pi H \beta} = \frac{3}{4\pi}\frac{H^3}{\Gamma}$$

- Number of e-folds from
single tunneling event:

$$\chi = \int H dt \sim H\tau \sim 3/4\pi\beta$$

- Sufficient Inflation: $\chi_{\rm tot} > 60$
- Reheating:

$$\beta \geq \beta_{crit} = 9/4\pi \rightarrow \chi \leq \chi_{crit} = 1/3$$

How to achieve both criteria:

- Sufficient inflation: $\chi_{tot} \geq 60$
- Reheating: $\chi \leq \chi_{crit} = 1/3$
- With single tunneling event:
- "Double Field Inflation" (Adams + Freese 91; Linde 91) : time-dependent nucleation rate, couple two scalar fields
- With multiple tunneling events:

  CHAIN INFLATION

get a fraction of an e-fold at each stage, adds to more than 60 in the end

Double Field Inflation

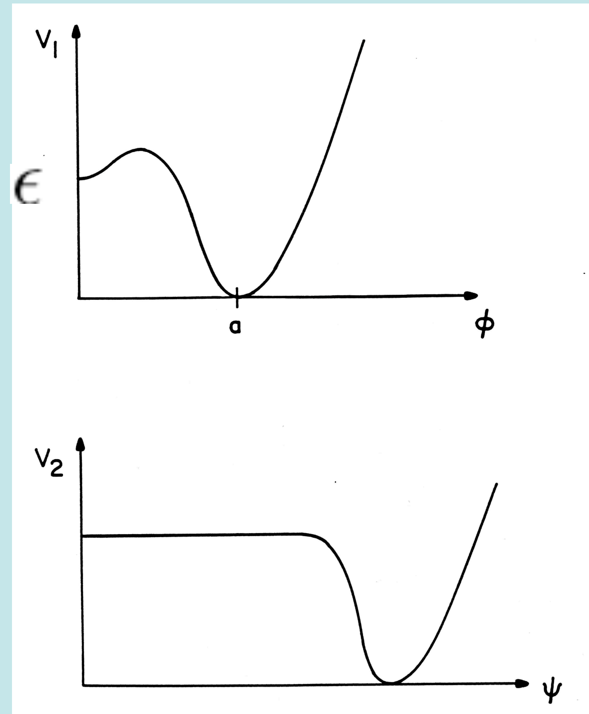
Adams and Freese; Linde (1990)

- Time dependent nucleation rate
- Couple 2 scalar fields

$$V_{tot} = V_1(\phi) + V_2(\psi) + V_{int}(\phi, \psi)$$

- Once the roller reaches its min,

$\epsilon_{eff} = \epsilon + f(t)a^4$ grows, tunneling rate increases. The tunneling rate is zero for ψ at top of potential, large as ψ approaches min (then, nucleation)

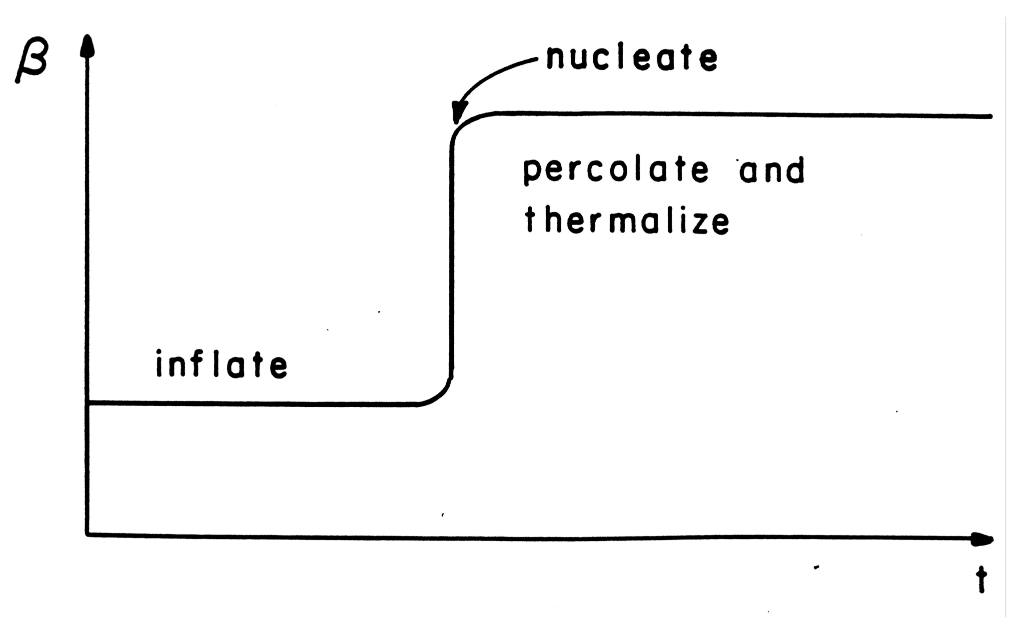


Required time dependence

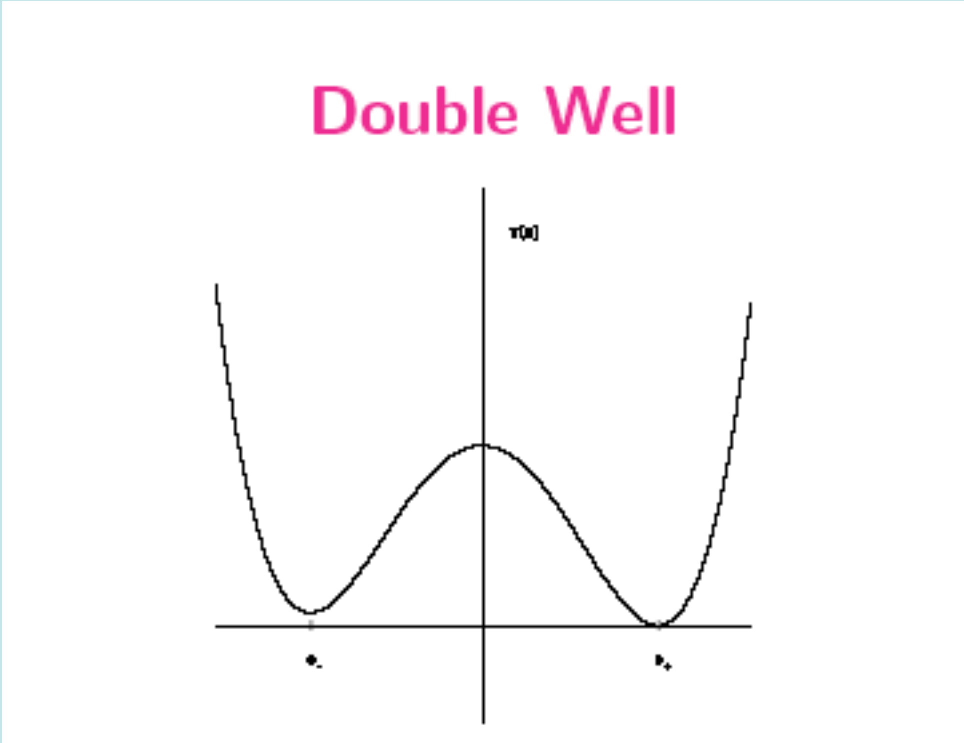
$$p(t) \sim E^{-\beta H t}, \beta = \Gamma / H^4$$

Need small β initially to inflate.

Then, suddenly, β gets larger so that all of universe goes from false to true vacuum at once. All bubbles of same size, get percolation and thermalization.



No Swiss Cheese!



ϵ is energy difference between vacua

Nucleation rate of true vacuum:

$$\Gamma \sim \epsilon \exp(-S_E), S_E = \frac{\pi^2}{6} \frac{\lambda^2 a^{12}}{\epsilon^3}$$ (thin wall)

(Callan and Coleman; Voloshin, Okun, and Obzarev))

Sensitivity of nucleation rate to parameters in the potential

- Sufficient inflation:

  number of e-folds= $\chi_{tot} = 3/4\pi\beta \geq 60$
- Followed by rapid nucleation:

  $$\beta > \beta_{crit} = 9/4\pi \ \ (\chi < 1/3)$$
- Both achieved by small change in ϵ/a^4

e.g. consider $\epsilon \sim$ TeV, 100 fields:

N=1000 for $a/\epsilon^{1/4} = 1.44891$

N=0.01 for $a/\epsilon^{1/4} = 1.43865$

To go from enough inflation to percolation, need this ratio to change by less than 2%

Toy model:

$$V_{tot}(\phi,\psi) = V_1(\phi) + V_2(\psi) + V_{int}(\phi,\psi)$$

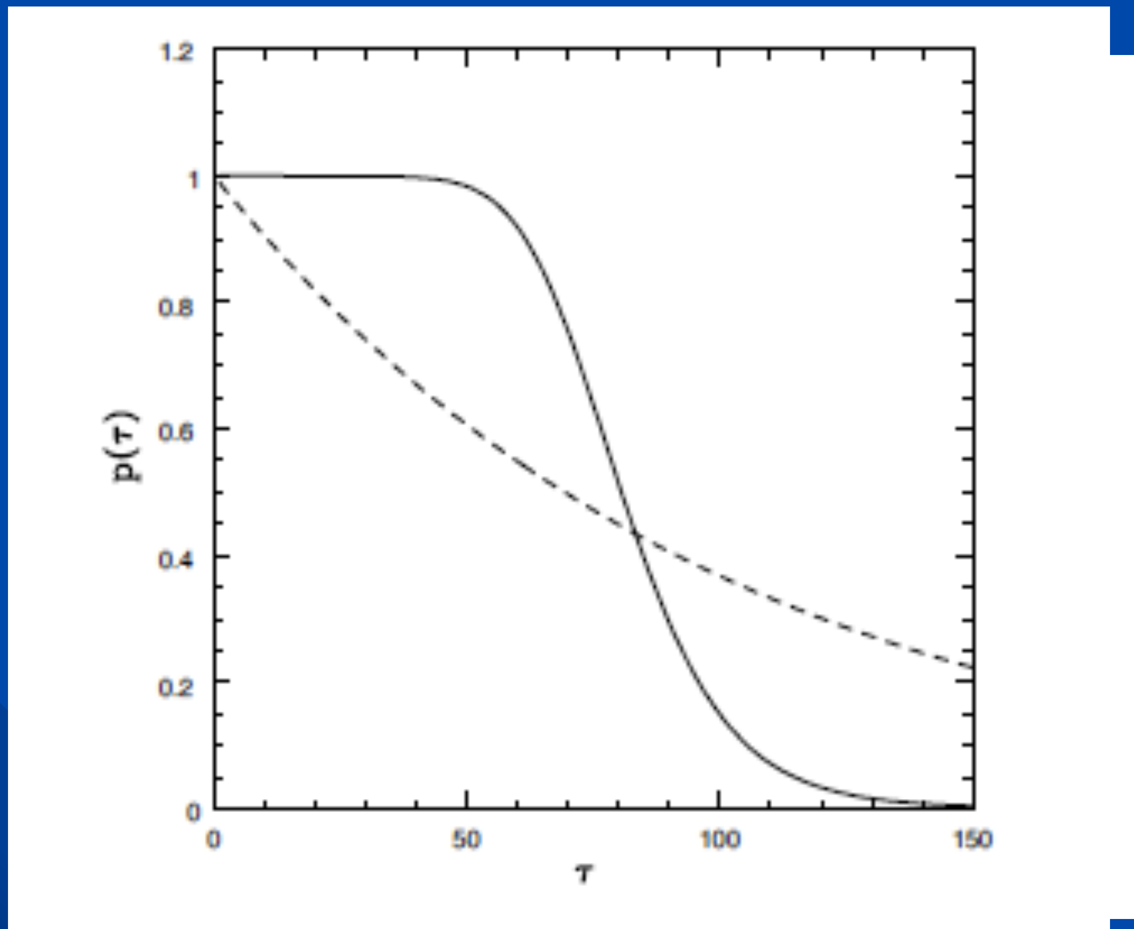
$$V_1(\phi) = \frac{1}{8}\lambda(\phi^2 - a^2)^2 - \frac{\epsilon}{2a}(\phi - a).$$

$$V_{int}(\phi,\psi) = -\gamma(\phi - a)\psi^3$$

$$\epsilon_{eff} = \epsilon + 2a\gamma\psi^3 .$$

$$\Gamma_N(t) = Ae^{-S_E}, \quad S_E = \frac{\pi^2}{6}\frac{\lambda^2 a^{12}}{\epsilon_{eff}^3}.$$

Psi is a roliing field. At first its value is small, epsilon is small, Phi does not tunnel. Then Psi rolls to its minimum, epsilon is large, Phi tunnels suddenly everywhere at once



Probability of remaining in the false vacuum as a function of time in old inflation (dotted line) vs. in double field inflation (solid line)

Double Field Inflation: Summary

- Time dependent nucleation rate allows the tunneling field to sit for a long time in a false vacuum while the tunneling rate is slow.
- Then, the tunneling rate suddenly speeds up e.g. due to increased energy difference between vacua or decreased barrier.
- The field tunnels immediately, allowing for percolation and reheating
- Other way to look at: field is rolling in one direction, suddenly it notices it can tunnel in a different direction because the barrier in that direction has decreased.
- The rolling field must take 60 efolds to get to the bottom: use axion to do it

Inflation Requires Two Basic Ingredients

- 1. Sufficient e-foldings of inflation
- 2. The universe must thermalize and reheat

- With single tunneling event:

"Double Field Inflation": time-dependent nucleation rate, couple two scalar fields (one tunneling and one rolling direction)

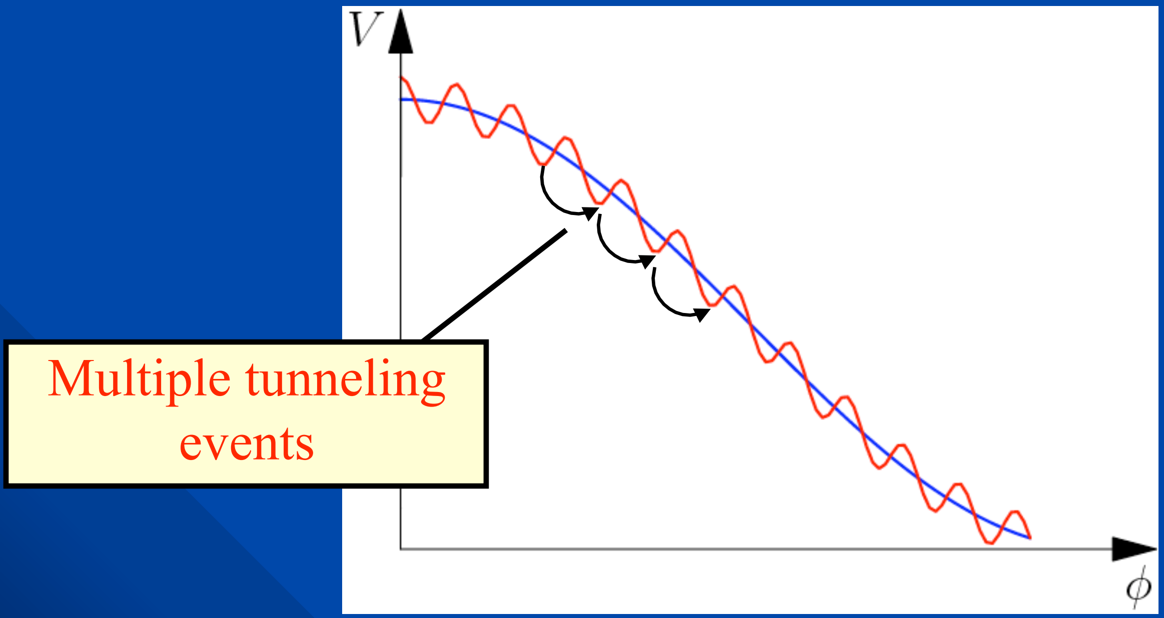
- With multiple tunneling events:

CHAIN INFLATION

get a fraction of an e-fold at each stage, adds to more than 60 in the end

Chain Inflation

Freese & Spolyar (2005)
Freese, Liu, & Spolyar (2005)



Relevant to:

- stringy landscape
- axionic particles

- Graceful exit:
  requires that the number of e-foldings per stage is $N < 1/3$
- Sufficient inflation:
  total number of e-foldings is $N_{tot} > 60$

Chain Inflation: Basic Setup

- The universe transitions from an initially high vacuum down towards zero, through a series of tunneling events.
- The picture to consider: tilted cosine
- Solves old inflation problem: Graceful Exit requires that the number of e-folds per stage < 1/3
- Sufficient Inflation requires a total number of e-folds > 60, hence there are many tunneling events

Rapid Tunneling

- Extreme sensitivity of tunneling rate to parameters in the potential is due to the exponential dependence on the bounce action
- Thus for natural parameter choices there will always be a direction of fast tunneling (e.g. in the landscape)

INFLATION AT ANY SCALE

- The tunneling rate depends on three parameters:
- energy difference between vacua
- barrier height
- width of potential

- When these numbers are comparable, the tunneling rate is just on the border between fast and slow tunneling. They can have any numbers you like. i.e. they can be GUT scale, GeV scale… as long as you reheat at the end.

NO fine-tuning

- The height and the width of the potential can be the same. In fact, it is when these two quantities are roughly comparable that the field is on the border of tunneling rapidly or never tunneling at all, so that any interaction is likely to cause the phase transition to proceed rapidly.

The String Landscape

- Long been recognized:

  vacua are not unique
- Degeneracy arises from:

  different Calabi Yau manifolds

  background fluxes
- Rough estimates can yield 10^200 to 10^500 vacua
- These vacua with different fluxes are disconnected in the mutidimensional landscape, with barriers in between them. Thus the only way to proceed from one to the other is via tunneling. Chain inflation is the result of rapid tunneling through a series of these vacua.

Analogy to Brown Teitelboim Mechanism

- 1+1 electrodynamics: in strong background E field, creation of e+e- pair
- Physically pulled apart; region between them has reduced value of field
- World lines of particles can be thought of as membranes, with E fluxes reduced by quantized amounts
- Series of nucleation of membranes in presence of background field, each time lowering the flux

Four Dimensional Generalization

- Four form field strength changes its value through bubble nucleation, $$\Delta F_4 = \pm q,$$

changing value of cosmological constant by quantized value $$\Lambda = \Lambda_{\text{bare}} + \frac{1}{2} F_4^2,$$

Four Dimensional Generalization

Tunneling in Bousso-Polchinski

- Extension of the Brown-Teitelboim mechanism
  Phys. Lett. B 19, 177 (1987); Nucl. Phys. B 297, 787 (1988).

- Make Λ dynamical: $\Lambda \to F_{(4)}$ (in four dimensions) where $F_{(4)}$ may arise from string/M-theory fluxes on cycles

$$F_{\mu\nu\rho\sigma} = nq\epsilon_{\mu\nu\rho\sigma}, \qquad n \in \mathbb{Z}$$

$$\Lambda = \Lambda_{\rm bare} + \tfrac{1}{2}n^2q^2$$

- This flux is reduced by the nucleation of membrane (wrapped p-brane) instantons

$n \to n-1$ gives an energy drop $\epsilon = -(n-\frac{1}{2})q^2$

- Four form field strength changes its value through bubble nucleation, $\Delta F_4 = \pm q,$

changing value of cosmological constant by quantized value

$$\Lambda = \Lambda_{\text{bare}} + \frac{1}{2} F_4^2,$$

- A simple estimate of the tunneling rate

$$\Gamma \sim e^{-S_E} \qquad S_E = \frac{27\pi^2}{2} \frac{\tau^4}{|\epsilon|^3}$$

ϵ = energy drop, τ = brane tension

(thin-wall instanton action ignoring gravity)

- Can S_E be of order 1?

Expectation: $\tau \sim$ large, $\epsilon \sim$ small $\Rightarrow$ slow tunneling

Another look at the bounce action

- For tunneling from $n \to n-1$, we estimate

$$S_E \approx \frac{27\pi^2}{2n^3}\frac{\tau^4}{q^6}$$

so we are interested in the ratio τ^4/q^6

- For string/M-theory branes, we may use the BPS condition $\tau = m_{\rm pl} q/\sqrt{2}$ to obtain

$$S_E \approx \frac{27\pi^2}{16n^3}\left(\frac{m_{\rm pl}^3}{\tau}\right)^2$$

(we expect $\tau/m_{\rm pl}^3 < 1$)

- So long as $\tau/m_{\rm pl}^3$ is not too small, we can compensate by taking n moderately large, and thus end up with reasonably fast tunneling

Example: wrapped M5-branes in BP

- Consider M-theory compactified on a 7-manifold with many 3-cycles
- Wrapped M5-branes have an effective 4-dimensional tension

$$\tau = 2\pi M_{11}^3 (V_3 M_{11}^3)$$

- Taking $M_{11} \sim 10^{-3}\, m_{\mathrm{pl}},\ V_3 M_{11}^3 \sim 10^3$ gives

$$\tau \sim 2\pi \times 10^{-6}\, m_{\mathrm{pl}}^3$$

- Then $n \sim (\text{few}) \times 10^3$ gives $S_E \sim \mathcal{O}(10\text{–}100)$
- Not all decay channels are fast — but you only need one

Back to the wrapped M5-brane example

- For

$$\tau \sim 2\pi \times 10^{-6}\, m_{\rm pl}^3 \qquad n \sim (\text{few}) \times 10^3$$

  each tunneling event drops the vacuum energy by

$$\epsilon \sim nq^2 = 2n\frac{\tau^2}{m_{\rm pl}^2} \sim 10^{-7}\, m_{\rm pl}^4$$

- We suppose inflation starts at $\Lambda_{\rm initial} \sim 10^{-4}\, m_{\rm pl}^4$ and ends at $\Lambda_{\rm final} \sim 10^{-8}\, m_{\rm pl}^4$

  We actually want $\Lambda_{\rm final} \approx 0$, however here we do not address the cosmological constant problem, so we assume $\Lambda_{\rm final}$ is of the order of ϵ

Back to the wrapped M5-brane example

- Using

$$\beta = \frac{\Gamma}{H^4} = 9\left(\frac{\Lambda}{m_{\rm pl}^4}\right)^{-2} e^{-S_E} \quad \text{where} \quad S_E \approx \frac{27\pi^2}{16n^3}\left(\frac{m_{\rm pl}^3}{\tau}\right)^2$$

the fast tunneling (percolation) requirement $\beta > 9/4\pi$ demands

$$n_{\rm initial} > 2735, \qquad n_{\rm final} > 2210$$

- Note that we need about 1000 tunneling events to reduce $\Lambda_{\rm initial}$

  These tunnelings can occur on different cycles

More to be done

- Better understand and characterize fast tunneling in the landscape
- What are the signatures of chain inflation?

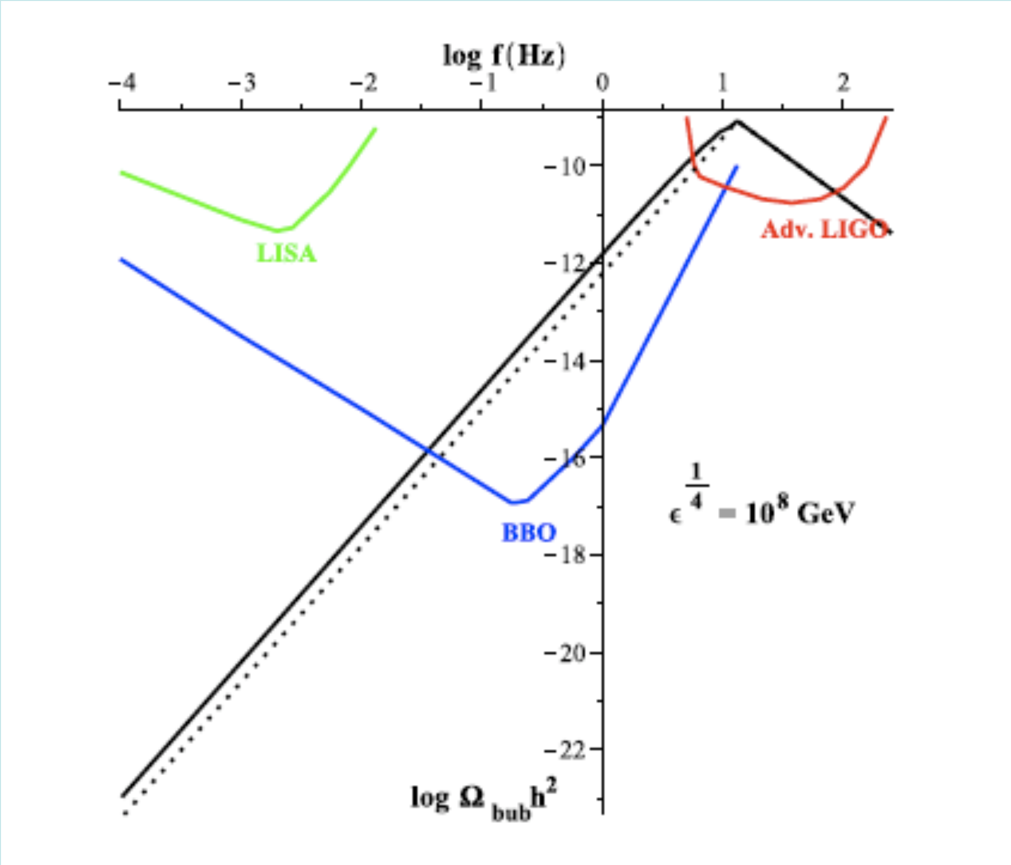
  density perturbations, tensor modes, etc.
- Go beyond the thin-wall approximation

Signatures of Chain Inflation

- Density Perturbations (Chialva and Danielsson)
- Gravity Waves: from bubble collisions as well as quantum fluctuations (A. Ashoorioon, J. Liu, A. Lopez, and K. Freese, in progress)

  Depending on parameters, detectable in Advanced LIGO

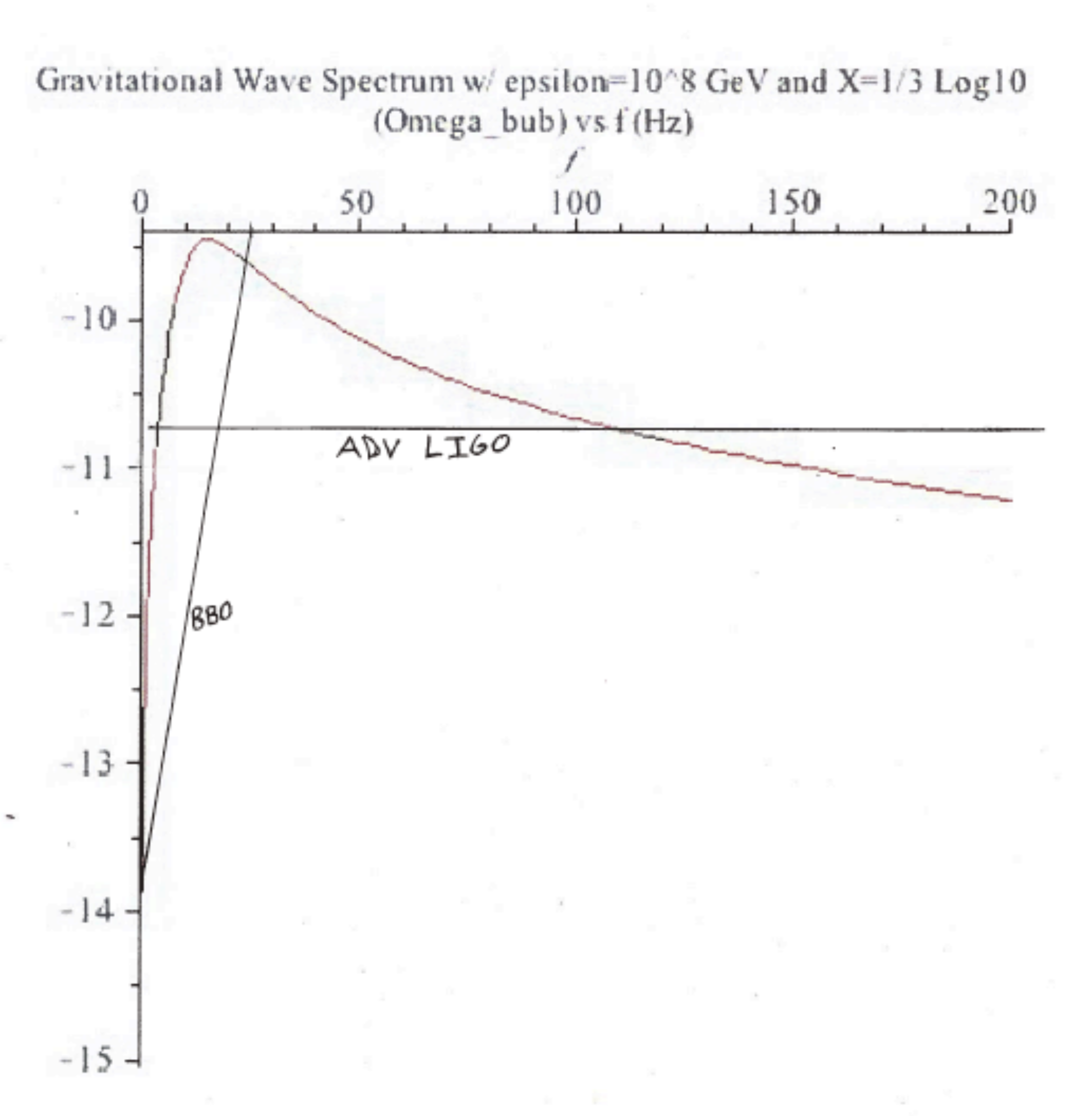
  n.b. prior work by Turner, Kosowsky, Watkins, Durrer



(Kosowsky, Turner, Watkins 1992)

Work with Alejandro Lopez and Jim Liu

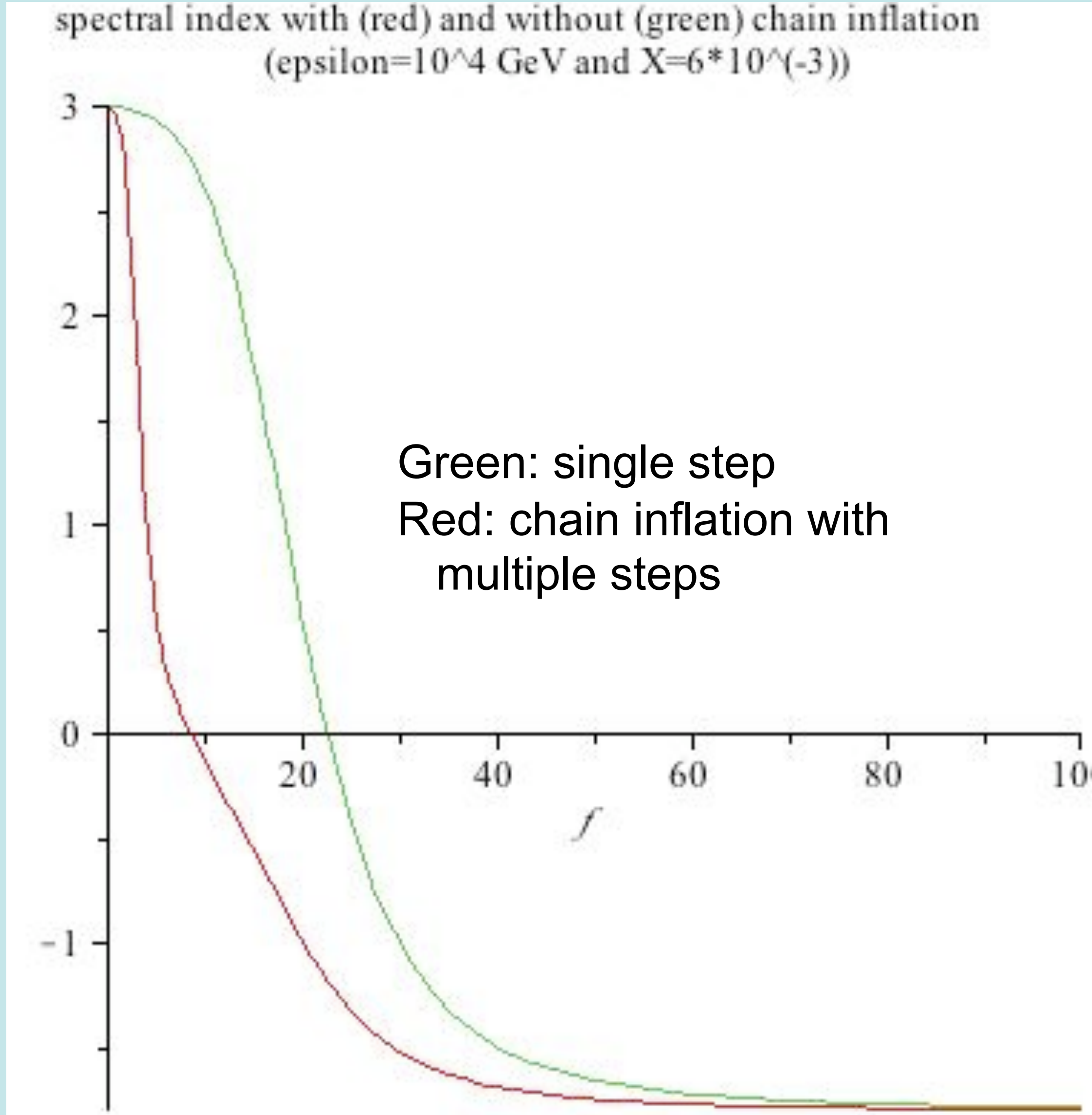
(using results of Caprini, Durrer, Servant GW due to bubble collsions for single step)



Tensor spectral
Index has
inflection point

(from adding
multiple steps
with f = H/chi.
As you go
down in the
potential H
is smaller but
less redshifting)

Work with Alejandro
Lopez



Chain Inflation with QCD Axion: would have been very economical

- Low scale inflation at 200 MeV: axion can simultaneously solve strong CP problem and provide inflation
- In addition to standard QCD axion, need (i) new heavy fermions to get many bumps in the theta field and (ii) tilt from soft breaking of underlying PQ symmetry

Inflating with the QCD Axion

K. Freese, J.T. Liu And D. Spolyar, hep-th/0502177

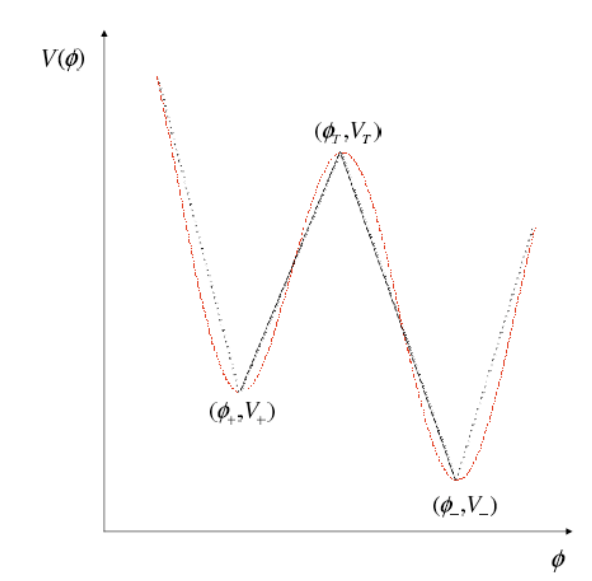
While the axion is $a\ priori$ a Goldstone boson of the spontaneously broken Peccei-Quinn symmetry $U(1)_{PQ}$, QCD instanton effects induce an axion potential with residual Z_N symmetry. The model we consider includes an additional explicit soft-breaking term, which tilts the instanton induced potential. While the complete form of the axion potential is dependent on non-perturbative effects, it is well modeled by a potential of the form

$$V(a) = V_0 \left[1 - \cos \frac{Na}{v} \right] - \eta \cos \left[\frac{a}{v} + \gamma \right]. \qquad (10)$$

Exact Tunneling Calculations (beyond thin wall)

- 1) General solution for quartic potentials (Adams 1993)

  2) Triangular Potentials (Duncan and Jensen 1992)



In the case of the QCD axion, tunneling is too slow (potential is too wide):

Can't chain inflate with it.

Amjad Ashoorioon, KF, and Jim Liu 2009

- Though chain inflation with QCD axion doesn’t seem to work, still it could work with axionic particle

TUNNELING INFLATION

- Need 60 efolds in total
- Need percolation for reheating:
- double field inflation
- chain inflation

  in string landscape

  with axionic particle

  can take place at low mass scales

  no fine tuning

Bubble Bubble Toil and Trouble

- Bubble bubble toil and trouble
- Fire burn and cauldron bubble
- Fillet of a fenny snake
- In the cauldron boil and bake
- Eye of newt and toe of frog
- Wool of bat and tongue of dog
- Adder’s fork and blind-worm’s sting
- Lizards’s leg and howlet’s wing
- For a charm of powerful trouble
- Like a hell-broth boil and bubble

Shakespeare (Macbeth)