Geometric Origin of Coincidences and Hierarchies in the Landscape

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Joint work with

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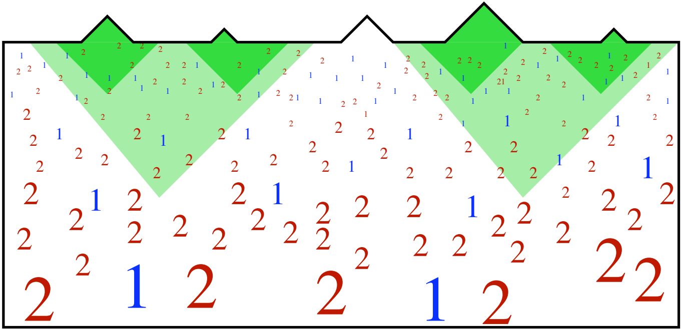
Eternal Inflation and the Measure Problem

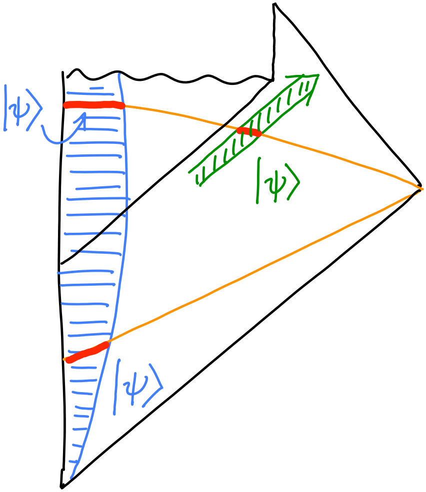


- Infinitely many pockets of each vacuum
- Each pocket contains infinitely many observers (if any)
- Relative probabilities are ill-defined:

$$\frac{p_1}{p_2} = \frac{\langle N_1 \rangle}{\langle N_2 \rangle} = \frac{\infty}{\infty}$$

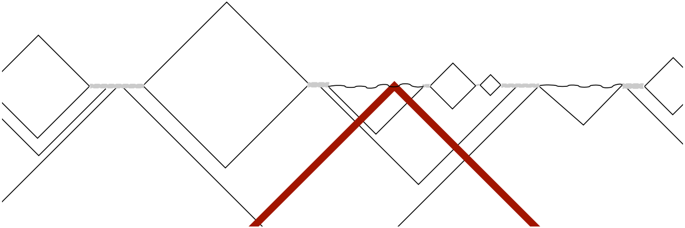
- Need a cutoff to render $\langle N_I \rangle$'s finite

Restricting to the Causal Patch



- If black hole evaporation is unitary, then globally it would lead to quantum xeroxing, which conflicts with the linearity of quantum mechanics
- But no observer can see both copies
- Physics need only describe experiments that can actually be performed, so we lose nothing by restricting to a causal patch

Causal Patch Cut-off



- Restrict to the causal past of the future endpoint of a geodesic RB 2006
- First example of a "local" measure: keep neighborhood of worldline. (Turns out to be dual to global "light-cone time" measure. RB 2009, RB & Yang 2009
- In metastable vacua with $\Lambda > 0$, this means counting events inside the cosmological horizon.
- In this talk I will consider the $\Lambda > 0$ half of the landscape

Claim:

The causal patch cutoff predicts

$$\log t_{\rm c} \sim \log t_\Lambda \approx \log t_{\rm obs} \approx \frac{1}{2}\log \mathcal{N}$$

independently of specific anthropic assumptions.

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This result follows chiefly from the geometric properties of the causal patch. I will show this in two steps.

First step

Given t_{obs} (arbitrary but fixed),
compute the probability distribution

$$\frac{d^2 p}{d \log t_{\text{c}} \; d \log t_{\Lambda}}$$

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$$= \frac{d^2 \tilde{p}}{d \log t_{\text{c}} \, d \log t_{\Lambda}} \, n_{\text{obs}}(t_{\text{c}}, t_{\Lambda}; t_{\text{obs}})$$

(The second step will be to vary t_{obs}.)

Prior probability

Cosmological constant:

$$t_\Lambda \sim \Lambda^{-1/2}$$

Taylor-expand around $\Lambda = 0$:

$$\frac{d\tilde{p}}{d\Lambda} = \text{const} \to \frac{d\tilde{p}}{d\log t_\Lambda} \sim t_\Lambda^{-2}$$

[Weinberg 1987]

Prior probability

Curvature:

$$\log t_\mathrm{c} \sim N_\mathrm{inf}\ .$$

Assume mild preference for short inflation, e.g.

$$\frac{d\tilde{p}}{d\log t_\mathrm{c}} \equiv g(\log t_\mathrm{c}) \sim (\log t_\mathrm{c})^{-4}$$

[Freivogel et al. 2005; McAllister's talk]

Details will be irrelevant. In fact, let's set $g = \mathrm{const}$ for now.

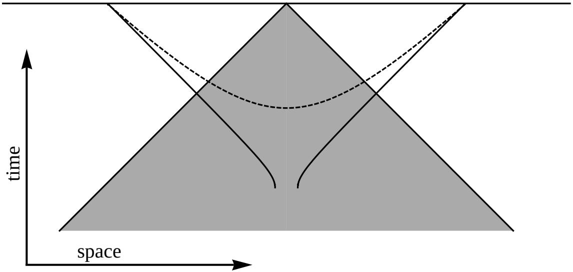
First step

Given $t_{\rm obs}$ (arbitrary but fixed),
compute the probability distribution

$$\frac{d^2 p}{d \log t_{\rm c} \; d \log t_\Lambda}$$

$$= \frac{d^2 \tilde{p}}{d \log t_{\rm c} \; d \log t_\Lambda} \; n_{\rm obs}(t_{\rm c}, t_\Lambda; t_{\rm obs})$$

Number of Observers



$$n_{\rm obs} = \alpha\, M_{CP}(t_{\rm c}, t_\Lambda; t_{\rm obs})$$

where M_{CP} is the total matter mass inside the causal patch, and α is the number of observers per unit mass.

Set $\alpha = \text{const}$ for now. $\rightarrow$ Need to compute M_{CP}, which is easy.

A Bit of Geometry

The metric in a bubble universe with (t_c, t_Λ) is

$$ds^2 = -dt^2 + a^2(t)[d\chi^2 + \sinh^2\chi \, d\Omega^2]$$

where

$$a(t) \sim \begin{cases} t_c^{1/3} t^{2/3} \,, & t < t_c \\ t \,, & t_c < t < t_\Lambda \\ t_\Lambda e^{t/t_\Lambda - 1} \,, & t_\Lambda < t \,. \end{cases}$$

(For $t_c > t_\Lambda$, there is no period of curvature domination; set $t_c \to t_\Lambda$ in that regime, in all formulas displayed.)

A Bit of Geometry

Mass inside the causal patch:

$$M_{CP} = \rho_\mathrm{m}\, a^3\, V_\mathrm{com} = t_\mathrm{c}\; V_\mathrm{com}[\chi_{CP}(t_\mathrm{obs})]$$

Comoving volume inside a sphere in hyperbolic space:

$$V_\mathrm{com} \sim \begin{cases} \chi^3 & \text{for } \chi \lesssim 1 \;\; (t_\Lambda < t_\mathrm{obs}) \\ e^{2\chi} & \text{for } \chi \gtrsim 1 \;\; (t_\Lambda > t_\mathrm{obs}). \end{cases}$$

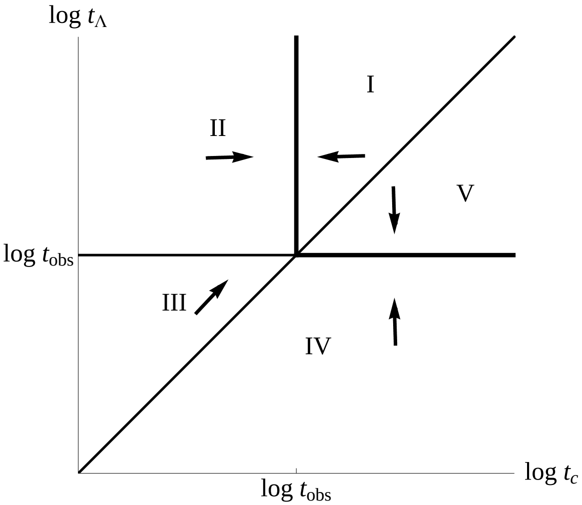
Comoving radius of the causal patch:

$$\chi_{CP}(t) \sim \begin{cases} 1 + \log(t_\Lambda/t_\mathrm{c}) + 3[1 - (\frac{t}{t_\mathrm{c}})^{1/3}], & t < t_\mathrm{c} \\ 1 + \log(t_\Lambda/t)\;, & t_\mathrm{c} < t < t_\Lambda \\ e^{-t/t_\Lambda}\;, & t_\Lambda < t \end{cases}$$

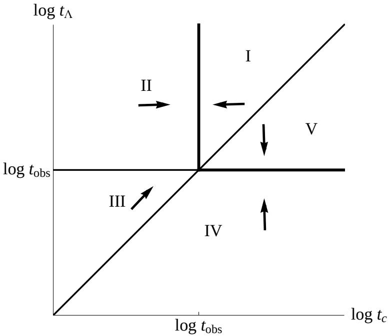
Putting it all together

$$\frac{dp}{d\log t_c \; d\log t_\Lambda} \sim \begin{cases} 1/t_c \,, & t_{obs} < t_c < t_\Lambda \qquad I \\ t_c/t_{obs}^2 \,, & t_c < t_{obs} < t_\Lambda \qquad II \\ (t_c/t_\Lambda^2) e^{-3t_{obs}/t_\Lambda} \,, & t_c < t_\Lambda < t_{obs} \qquad III \\ 1/t_\Lambda \,, & t_{obs} < t_\Lambda < t_c \qquad V \\ t_\Lambda^{-1} e^{-3t_{obs}/t_\Lambda} \,, & t_\Lambda < t_{obs},\; t_\Lambda < t_c \qquad IV \end{cases}$$

Ok this is what this looks like

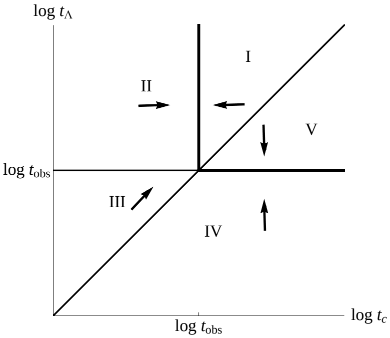


2 degenerate lines of max probability



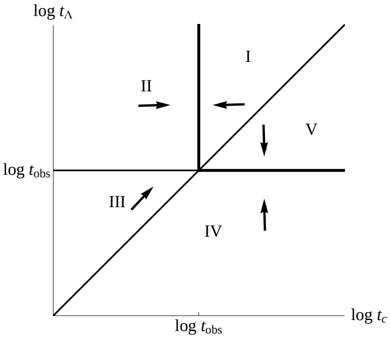
- power law in $t_{\rm c}$, $t_\Lambda \to$ exponential in log ...
- probability distribution dominated by maxima
- which are two degenerate half-lines

2 degenerate lines of max probability



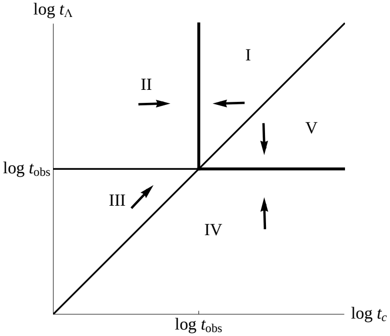
- The degeneracy in the t_c direction is lifted by g with any mild suppression of large $\log t_c$.
- But what about (anthropic) α? Does it modify anything?

2 degenerate lines of max probability



- The degeneracy in the $t_{\rm c}$ direction is lifted by g with any mild suppression of large log $t_{\rm c}$.
- But what about (anthropic) α? Does it modify anything?
- NO.

1 degenerate line of max probability



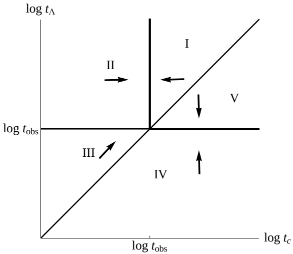
Problem #1: not integrable

Solution: the landscape is finite:

$$\Lambda_{\rm min} \sim \frac{1}{\mathcal{N}}$$

$$t_{\Lambda,{\rm max}} \sim \mathcal{N}^{1/2}$$

0 degenerate lines of max probability



Problem #2: the prediction is now:

$$\log t_{obs} \sim \log t_c$$

$$\log t_\Lambda \in (\log t_{obs}, \log t_{\Lambda,max})$$

Unless $t_{obs} \sim t_{\Lambda,max}$, this conflicts with observation.
(Λ should be unobservably small.)

Second step

Let t_{obs} scan.

Probability distribution over t_{obs}

Integrate out t_c, t_Λ :

$$\frac{dp}{d\log t_{\text{obs}}} \sim \frac{1}{t_{\text{obs}}} \log \frac{t_{\Lambda,\text{max}}}{t_{\text{obs}}}$$

Probability distribution over t_{obs}

Integrate out t_c, t_Λ :

$$\frac{dp}{d\log t_{\text{obs}}} \sim \frac{1}{t_{\text{obs}}} \log \frac{t_{\Lambda,\text{max}}}{t_{\text{obs}}} \ \alpha(t_{\text{obs}}) f(t_{\text{obs}}) \ ,$$

where f is the fraction of vacua that have observers at t_{obs}.

Probability distribution over $t_{\rm obs}$

Integrate out $t_{\rm c}, t_\Lambda$:

$$\frac{dp}{d\log t_{\rm obs}} \sim \frac{1}{t_{\rm obs}} \log\frac{t_{\Lambda,\rm max}}{t_{\rm obs}} \;\; \alpha(t_{\rm obs}) f(t_{\rm obs}) \; ,$$

where f is the fraction of vacua that have observers at $t_{\rm obs}$.

One expects that both α and f grow with $t_{\rm obs}$. Let us make the stronger but plausible assumption that

$$\alpha f \sim t_{\rm obs}^{1+\epsilon} \; , \;\; \epsilon > 0 \; .$$

Probability distribution over t_{obs}

Then

$$\frac{dp}{d\log t_{\text{obs}}} \sim t_{\text{obs}}^{\epsilon} \log \frac{t_{\Lambda,\text{max}}}{t_{\text{obs}}} \ ,$$

which predicts

$$t_{\text{obs}} \sim t_{\Lambda,\text{max}} \sim \mathcal{N}^{1/2} \ .$$

Combined with the previous result, we find the claimed relation

$$\log t_c \sim \log t_\Lambda \approx \log t_{\text{obs}} \approx \frac{1}{2} \log \mathcal{N} \ .$$