

University of Minnesota
St. Anthony Falls Hydraulic Laboratory

Project Report No. 179

ICE FORMATION ON MINNESOTA LAKES
Use of LANDSAT Imagery and Weather Data
to Predict Freeze-Over Dates

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ABSTRACT

LANDSAT images taken in the fall of 1972, 1973, 1974 and 1975 were analyzed to estimate dates of ice formation on Minnesota lakes. Lakes located in the northern, central and southern parts of Minnesota were studied. Lake surface areas and depths were identified from a lake inventory. The observations derived from the satellite images were compared to theoretical prediction of freeze-over dates based on seasonal heat budget cycles and water temperature cycles. The theory was then applied to all of Minnesota to provide an estimate of average annual freeze-over dates of lakes located in different parts of the state and having different depths.

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LIST OF SYMBOLS AND NOTATIONS

A_1	Constant in regression equation for air temperature AT ($^{\circ}\text{F}$ or $^{\circ}\text{C}$)
A_2	Constant in regression equation for water temperature T ($^{\circ}\text{F}$ or $^{\circ}\text{C}$)
AT_i	Average air temperature on the i^{th} day or month ($^{\circ}\text{F}$ or $^{\circ}\text{C}$)
\overline{AT}_i	Normal daily air temperature on the i^{th} day or month ($^{\circ}\text{F}$ or $^{\circ}\text{C}$)
AT_i'	Air temperature departure from normal on the i^{th} day or month ($^{\circ}\text{F}$ or $^{\circ}\text{C}$)
B_1	Regression coefficient of cosine term for air temperature ($^{\circ}\text{F}$ or $^{\circ}\text{C}$)
B_2	Regression coefficient of cosine term for water temperature ($^{\circ}\text{F}$ or $^{\circ}\text{C}$)
c	Specific heat capacity of water (BTU lb $^{-1}$ $^{\circ}\text{F}^{-1}$ or calorie g $^{-1}$ $^{\circ}\text{C}^{-1}$)
C_1	Regression coefficient of sine term for air temperature ($^{\circ}\text{F}$ or $^{\circ}\text{C}$)
C_2	Regression coefficient of sine term for water temperature ($^{\circ}\text{F}$ or $^{\circ}\text{C}$)
d	Response coefficient of water temperature to air temperature departure
e_a	Actual air vapor pressure (lb ft $^{-1}$ or mb)
e_s	Standard air vapor pressure (lb ft $^{-1}$ or mb)
E	Equilibrium temperature ($^{\circ}\text{F}$ or $^{\circ}\text{C}$)
E_m	Mean equilibrium temperature ($^{\circ}\text{F}$ or $^{\circ}\text{C}$)
ΔE	Amplitude of the seasonal equilibrium temperature cycle ($^{\circ}\text{F}$ or $^{\circ}\text{C}$)
f	Relative humidity (%)
FW	Wind function (BTU ft $^{-2}$ day $^{-1}$ $^{\circ}\text{F}^{-1}$)
h	Mean (or median) depth of a lake (ft or m)
h_1	Mean (median or estimated) depth of the water body for which the coefficients A_1 , A_2 , B_1 , B_2 , C_1 , C_2 and d are determined from regression analysis (ft or m)
h_{lim}	Critical depth over which a lake will not freeze throughout a winter (ft or m)
I	$=H_i/(V\rho c)$ ($^{\circ}\text{F}$ day $^{-1}$ or $^{\circ}\text{C}$ day $^{-1}$)
i, j	index, subscripts

K	Bulk surface heat exchange coefficient (BTU ft ⁻² day ⁻¹ °F ⁻¹ or cal cm ⁻² day ⁻¹ °C ⁻¹)
m	= K / (ρch) (day ⁻¹)
p _a	Atmospheric pressure (lb ft ⁻² or mb)
t	Time (days)
T _i	Average water temperature on the i th day or month (°F or °C)
\bar{T}_i	Normal water temperature on the i th day or month (°F or °C)
T _i '	Water temperature departure from normal on the i th day or month (°F or °C)
T _d	Dew point (°F or °C)
T _s	Water surface temperature (°F or °C)
T _w	= (T+T _d)/2 (°F or °C)
$\Delta T_{h=h_1}$	The amplitude of the seasonal water temperature cycle generated by using coefficients determined from a water body having mean depth h ₁ (°F or °C)
W ₂	Wind velocity (mph) at 2 meters above ground surface
β	Proportionality constant between saturation vapor pressure difference and water temperature difference
δ	Phase lag of water temperature cycle (days)
δ	Seasonal phase lag of water temperature cycle (days)
w	equal 2π/365 for daily basis and (rad day ⁻¹) equal 2π/12 for monthly basis (rad month ⁻¹)
ρ	Density of water (lb ft ⁻³ or g cm ⁻³)
Δθ _v	Difference in virtual temperature between water surface and air (°F or °C)
σ	Standard deviation of K (BTU ft ⁻² day ⁻¹ °F ⁻¹ or cal cm ⁻² day ⁻¹ °C ⁻¹)

ICE FORMATION ON MINNESOTA LAKES

Use of LANDSAT Imagery and Weather Data to Predict Freeze-Over Dates

1. INTRODUCTION

In Minnesota, there are 15,290 lake basins larger than ten acres. Including Minnesota's portion of Lake Superior, lakes cover an area of 4,059 square miles or about 4.8 per cent of the State's area.

The lakes are not evenly distributed throughout the state; they are most numerous in the northeast and central parts of the state. The northwestern, extreme western, and southern parts of the state have a sparse distribution of lakes.

There exist about 2,400 detailed maps of Minnesota lakes showing depths. Lakes deeper than 100 feet (30 m) are exceptional in Minnesota, and many of the larger lakes are quite shallow. Mille Lacs Lake, for example, has a maximum depth of about 35 feet (10.5 m).

In Minnesota, almost all lakes are frozen over in winter. Ice coverage of Minnesota lakes is extensive and lasts from two to six months. Information on the period of ice coverage, that is, the dates of beginning and end of ice coverage under natural and unnatural conditions in lakes is scarce. Since lakes in Minnesota are an important resource, information on ice formation may prove helpful in lake and reservoir management. Ice coverage affects water quality of lakes e.g. through oxygen depletion, enhancement of sedimentation and through interaction with groundwater. Certain lake surveys require an ice cover, and rough fish is often removed just prior to ice formation.

An examination of the literature on freezing of lakes shows a growing interest in this phenomenon. Neumann (see e.g. Carlslaw and Jaeger, 1959) laid the foundation for the study of the freezing of an ice-water system by proposing an exact solution to the Stefan problem. Since then Portnov (1962), Jackson (1964), Langford (1966), and Boley (1968) presented series solutions to the melting and freezing problem. Numerical methods for the solution of one-dimensional heat conduction problems with melting or freezing have been proposed by Murray and Landis (1959). More recent treatment of the topic can

be found in papers by Westphal (1967); Fertuck, Spyker and Husband (1971); and Foss and Fan (1972 and 1974).

A method of prediction of water temperature in rivers and reservoirs was given by Raphael (1962). An energy budget was considered in the formulation. Weather records, inflow and outflow characteristics and the surface area and volume of the body of water are required as input. The method can be applied to shallow lakes, flowing streams, and detention reservoirs, in which a well-mixed condition persists. Several other methods have been published more recently.

Edinger et al (1968) proposed that the net rate of heat exchange at the water surface of a water body could be evaluated in terms of a thermal exchange coefficient and an equilibrium temperature, both of which depended on observable meteorological variables.

Vertical water temperature observations in lakes before ice formation were made by Yoshimura (1936), Scott (1964), and Bilello (1968). Observations of surface water temperature and ice regimes in portions of the Great Lakes system were made by a variety of investigators, e.g. by Webb (1972) in Georgian Bay.

Remote sensing techniques for the study of ice-covers have obvious advantages: (a) ease of data collection compared to ground/surveys in a hostile climate, (b) synoptic coverage of large areas, and (c) repeated sensing of identical areas. Satellite images for the study of ice-covers have been used for example by Lind (1973), Strong (1973), Tsang (1974), McGinnis and Schneider (1978).

Ice conditions in the vicinity of cooling water discharges from power plants in Minnesota were investigated from the air and on the ground by Stefan, Ford and Gulliver (1975).

In this report, a mathematical model which predicts the date of the first complete ice coverage of Minnesota lakes is presented. Ice covers visible on images taken by satellites (LANDSAT-1 and 2) are used to test the validity of the model.

2. INFORMATION FROM LANDSAT IMAGERY ON FREEZE-OVER

LANDSAT 1 and 2 images have been used to determine onset of ice coverage of lakes in northern, central and southern Minnesota. The scenes are identified in Fig. 1. They carry the identification numbers Path 29, Row 27 (North), Path 30, Row 28 (Central), and Path 29, Row 29 (South). Dates of available LANDSAT scenes and cloud coverage during the period of interest, i.e. from November 11, 1972 through January 17, 1976, are shown in Table 1.

Although the percentage cloud cover identified in Table 1 has been appreciable, much information could be extracted from LANDSAT scenes. LANDSAT imagery was acquired in the form of 9x9 black and white prints (Band No. 6) and for cloud covers up to 90 percent. Examination of the images under a magnifying glass shows ice covers quite clearly.

From 10 to 30 lakes were selected on each LANDSAT scene. The Minnesota Lake Inventory (1968) provided the name, the range, township and section, and an identification number for each selected lake. This information was subsequently used to find the median lake depth and the lake surface area from the Clean Lakes Inventory File (CLIF) of the Minnesota Pollution Control Agency.

Ice coverage was classified in 5 categories according to which portion of the lake was ice-covered.

- | | |
|-----------------------|---|
| (a) open water | ▼ |
| (b) nearly fully open | ● |
| (c) half open | ○ |
| (d) nearly closed | □ |
| (e) closed | △ |

Observations on ice coverage were tabulated.

Under clear sky, no difficulty was encountered in the interpretation. Water appeared black and ice greyish white or white on the images. In cases when light to heavy cloud cover prevailed, it was often difficult to distinguish ice and snow from overlying cloud cover on images. In such cases, no interpretation was made.

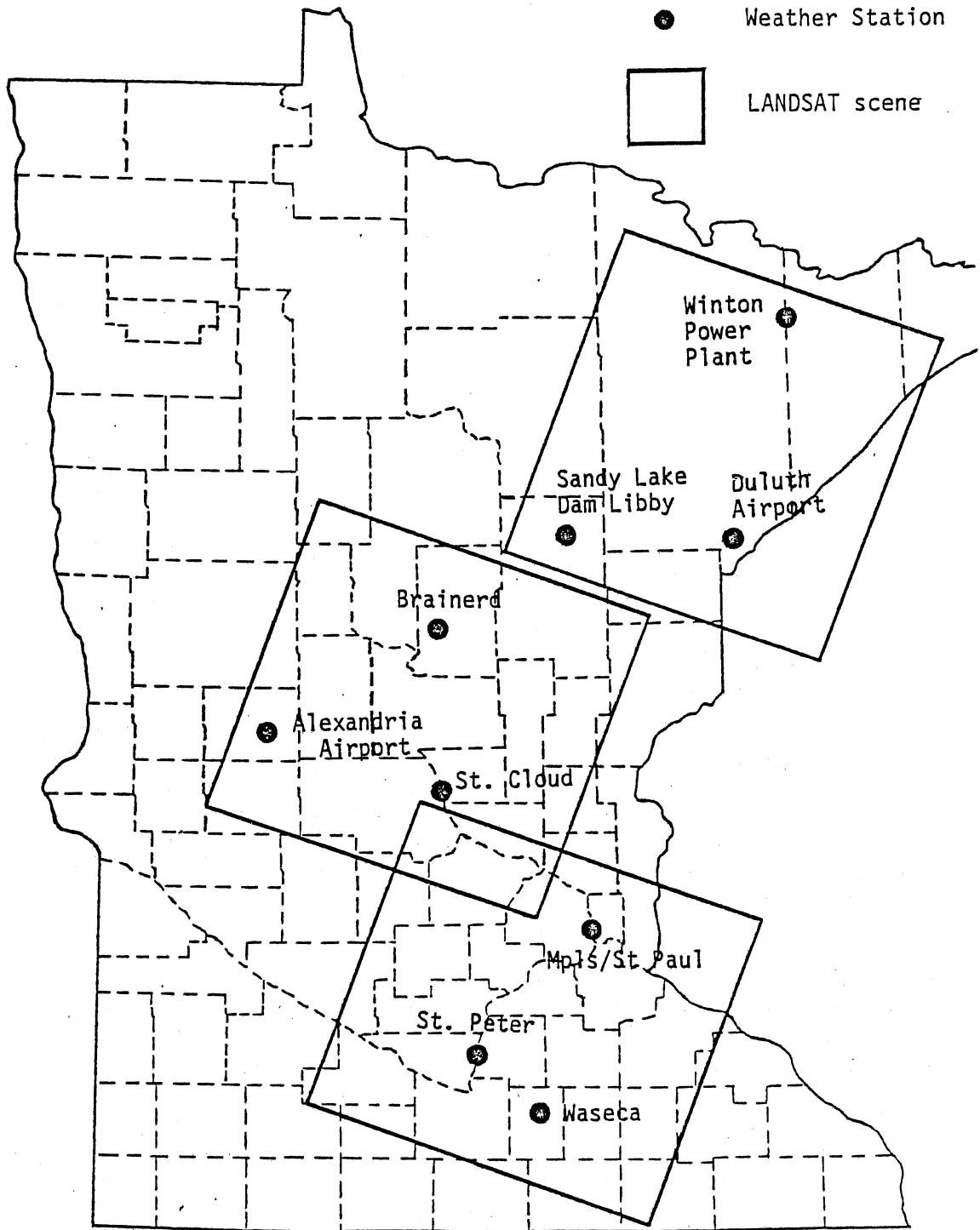


Fig. 1 - LANDSAT Scenes Selected for Study.

TABLE 1 (a). Available LANDSAT Scenes, Fall 1972

North

Date	Quality	Cloud Cover (%)
11/11/72	Excellent	90
11/29/72	Excellent	10
12/17/72	Excellent	90
1/04/73	Excellent	<u>30</u>
	Average	55

Central

Date	Quality	Cloud Cover (%)
11/30/72	Excellent	60
12/18/72	Excellent	50
1/05/73	Excellent	<u>0</u>
	Average	37

South

Date	Quality	Cloud Cover (%)
11/11/72	Excellent	90
11/29/72	Excellent	10
12/17/72	Excellent	20
1/04/73	Excellent	<u>20</u>
	Average	35

TABLE 1 (b). Available LANDSAT Scenes, Fall 1973

North

Date	Quality	Cloud Cover (%)
10/19/73	Excellent	50
11/06/73	Excellent	30
11/24/73	Unavailable	100
12/12/73	Excellent	90
12/30/73	Excellent	<u>10</u>
	Average	56

Central

Date	Quality	Cloud Cover (%)
10/20/73	Unsatisfactory	10
11/07/73	Unavailable	100
11/25/73	Unsatisfactory	90
12/13/73	Excellent	20
12/31/73	Unsatisfactory	20
1/18/74	Unsatisfactory	<u>70</u>
	Average	52

South

Date	Quality	Cloud Cover (%)
10/19/73	Excellent	10
11/06/73	Excellent	20
11/24/73	Unsatisfactory	90
12/12/73	Excellent	50
1/17/74	Excellent	<u>60</u>
	Average	46

TABLE 1 (c). Available LANDSAT Scenes, Fall 1974

North

Date	Quality	Cloud Cover (%)
11/19/74	Fair	90
12/07/74	Excellent	80
1/24/75	Fair	<u>90</u>
	Average	87

Central

Date	Quality	Cloud Cover (%)
11/02/74	Excellent	90
11/20/74	Fair	90
12/08/74	Unavailable	100
12/26/74	Unavailable	100
1/13/75	Unavailable	100
1/31/75	Excellent	<u>40</u>
	Average	87

South

Date	Quality	Cloud Cover (%)
11/01/74	Excellent	50
11/19/74	Fair	90
12/07/74	Excellent	30
1/24/75	Excellent	<u>80</u>
	Average	63

TABLE 1 (d). Available LANDSAT Scenes, Fall 1975

<u>North</u>		
Date	Quality	Cloud Cover (%)
10/27/75	Fair	90
11/05/75	Fair	90
11/14/75	Fair	10
11/23/75	Fair	60
12/02/75	Unavailable	100
12/11/75	Unsatisfactory	70
12/20/75	Unavailable	100
12/29/75	Fair	90
1/07/76	Fair	<u>20</u>
	Average	70

<u>Central</u>		
Date	Quality	Cloud Cover (%)
10/28/75	Excellent	80
11/06/75	Excellent	90
11/15/75	Fair	90
11/24/75	Fair	30
12/03/75	Unavailable	100
12/12/75	Excellent	20
12/21/75	Excellent	20
12/30/75	Excellent	40
1/08/76	Unavailable	100
1/17/76	Unavailable	<u>90</u>
	Average	66

<u>South</u>		
Date	Quality	Cloud Cover (%)
10/27/75	Excellent	0
11/05/75	Fair	10
11/14/75	Excellent	10
11/23/75	Fair	70
12/02/75	Unavailable	100
12/11/75	Unsatisfactory	90
12/20/75	Unavailable	100
12/29/75	Fair	80
1/07/76	Fair	20
1/16/76	Excellent	<u>10</u>
	Average	49

A. O. Lind (1973) in an earlier study already indicated the presence of various ice tones, patterns and arrangements of ice as well as open water in a LANDSAT image of Lake Champlain taken on January 8, 1973. In Lind's opinion, MSS band 5 imagery provided the most useful data. Lind states: "While it was not possible to differentiate open water from one or two-day old ice, it was possible to interpret the total signatures of the frozen portion in terms of freezing history or age. The dark gray tones of new smooth ice were found to contrast with the medium gray tones of older ice and the rough texture of wind-jammed bay ice."

During the study of the Minnesota lakes, the presence or absence of ice covers was generally well identified and few instances of ambiguity were encountered.

Occasionally, a lake remained open even after all the surrounding lakes were closed in winter and thawed in early spring before all the other lakes did. Such lakes were receiving artificial thermal input and must be treated separately in any analysis. A separate list of such water bodies is attached in Appendix A.

In addition, three weather stations in each LANDSAT frame were identified. They are also shown in Fig. 1. Since the objective was to relate weather to ice coverage, results on ice coverage were plotted separately for each group of lakes located in the vicinity of a particular weather station.

Figs. 2a, 2b, and 2c give examples of ice coverage of lakes at different locations and lakes of different depths on two or three different dates. Dark symbols identify open lakes and white symbols, ice covered lakes. The effect of lake depth on the delay in freeze-over is quite apparent. That delay was also determined theoretically. Not expected was the dependence on lake surface area. A sample graph showing the dependence of ice conditions on lake surface area is given in Fig. 3. The correlation is quite good. Sometimes lake depths and lake surface areas are related to each other in the sense that larger lakes are also deeper. This is, however, not generally true. It may be that wind effects also have an important role. Wind can prevent formation of a coherent ice cover and wind effects are stronger on lakes with larger fetch (surface area). Ice movement by wind is readily observed in large lakes and has been described, e.g., by Tsang (1974).

A graph in which the product of lake median depth and surface area has been used is given in Fig. 4. This may be the most appropriate representation.

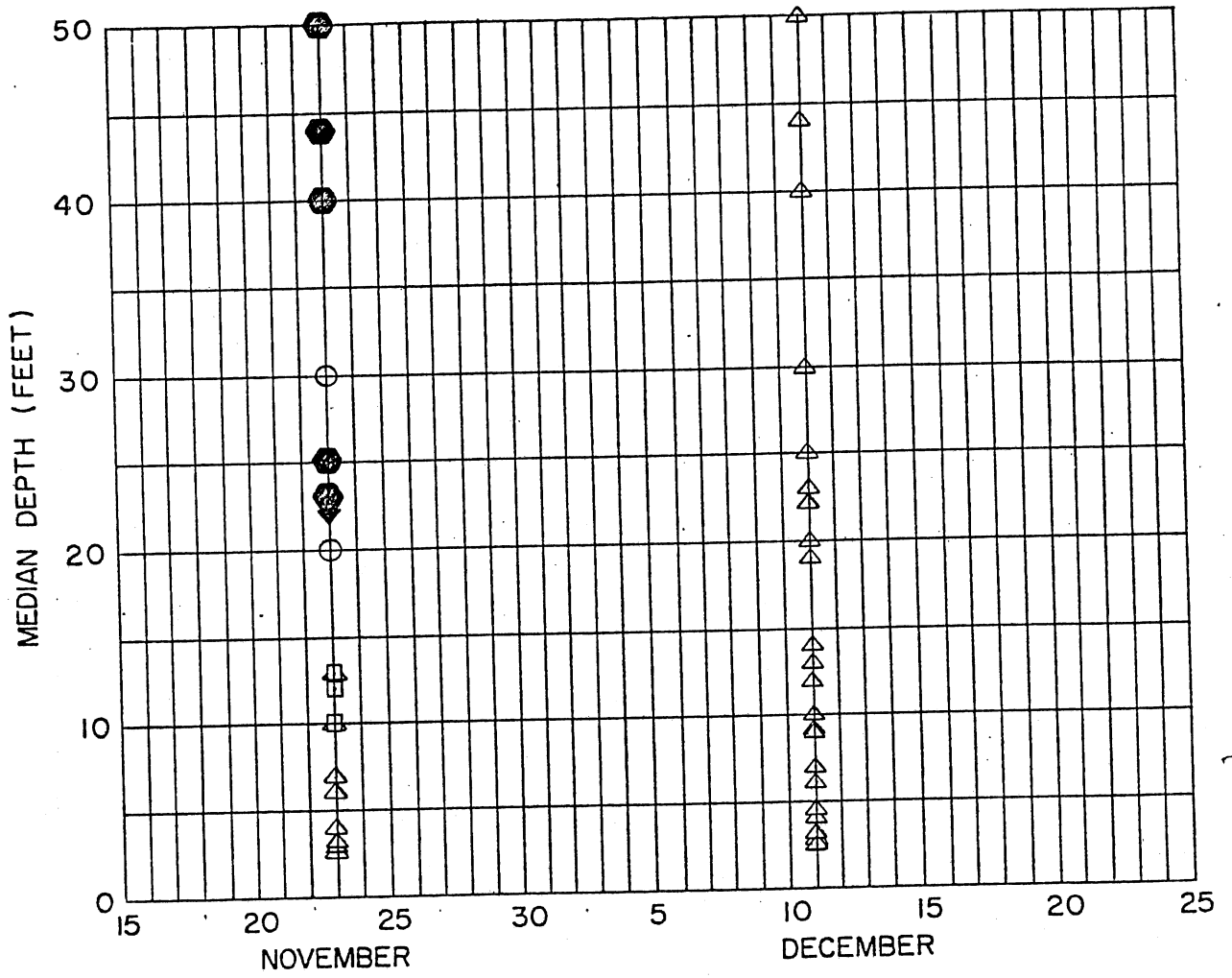


Fig. 2a - Lake ice coverage in fall 1975 as determined from LANDSAT images as a function of lake depth. Lakes near Winton Power Station.

(See legend on page 3.)

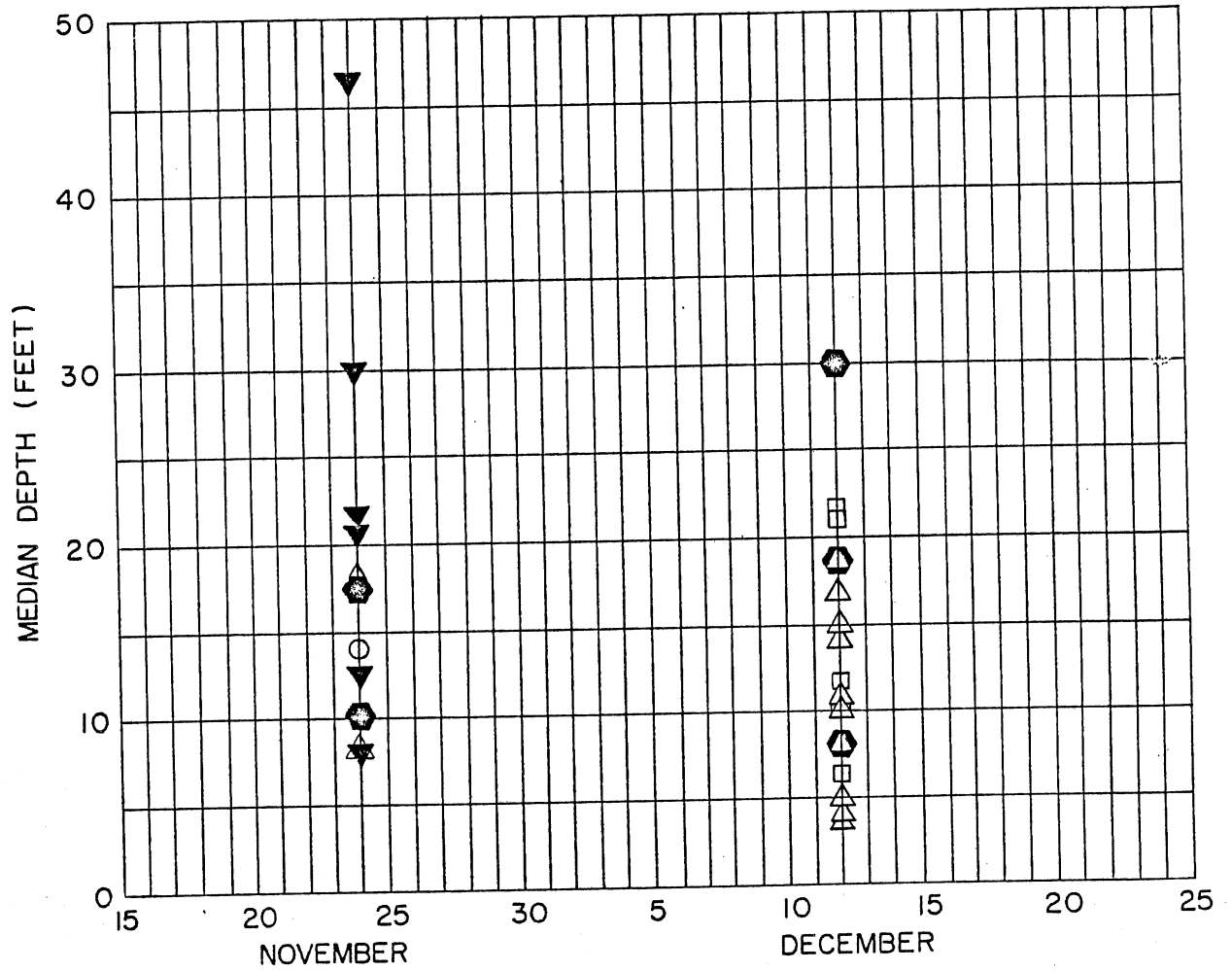


Fig. 2b - Lake ice coverage in Fall 1975 as determined from LANDSAT images as a function of lake depth. Lakes near Brainerd Weather Station.

(See legend on page 3.)

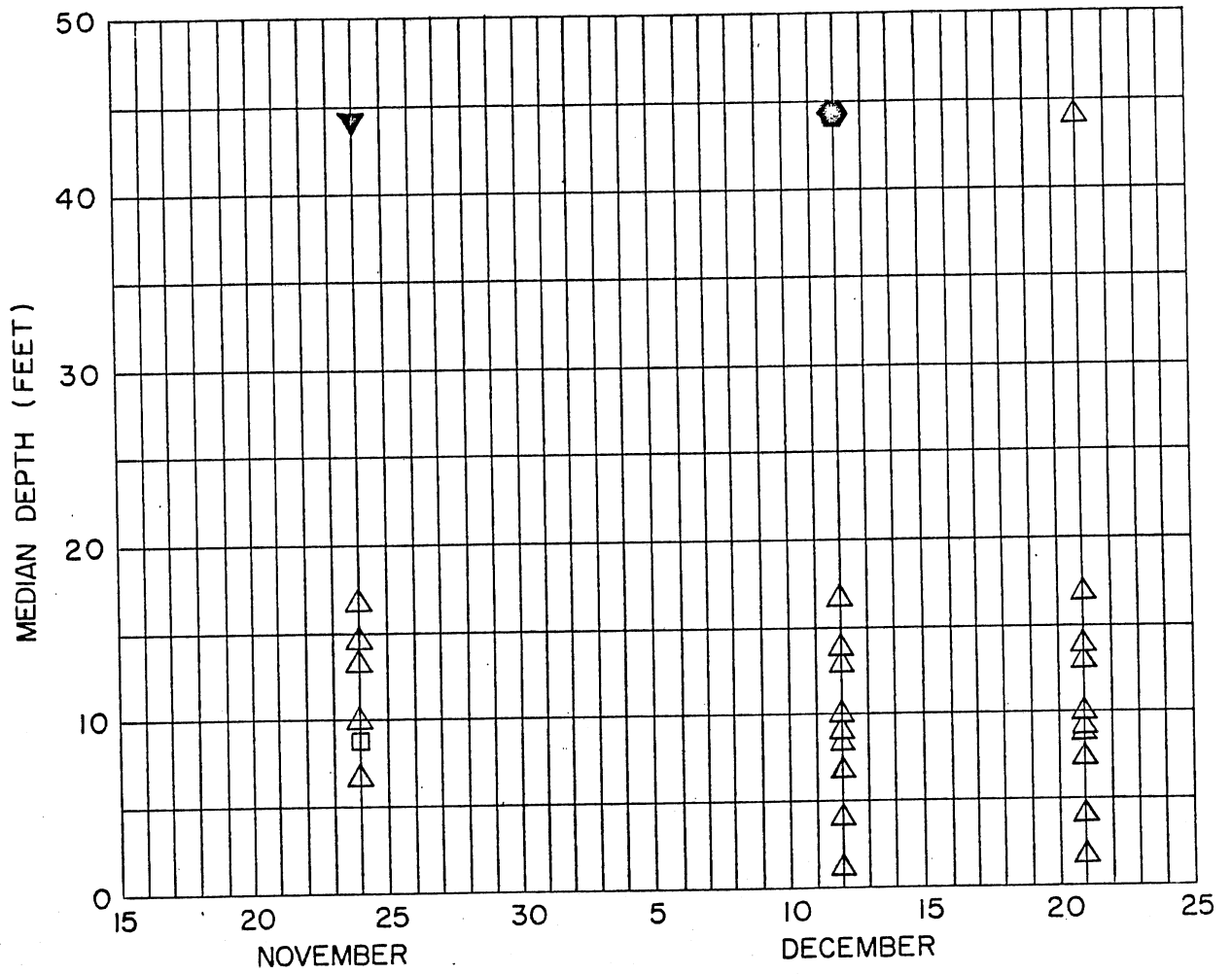


Fig. 2c - Lake ice coverage in Fall 1975 as determined from LANDSAT images as a function of lake depth. Lakes near St. Cloud Weather Station.

(See legend on page 3.)

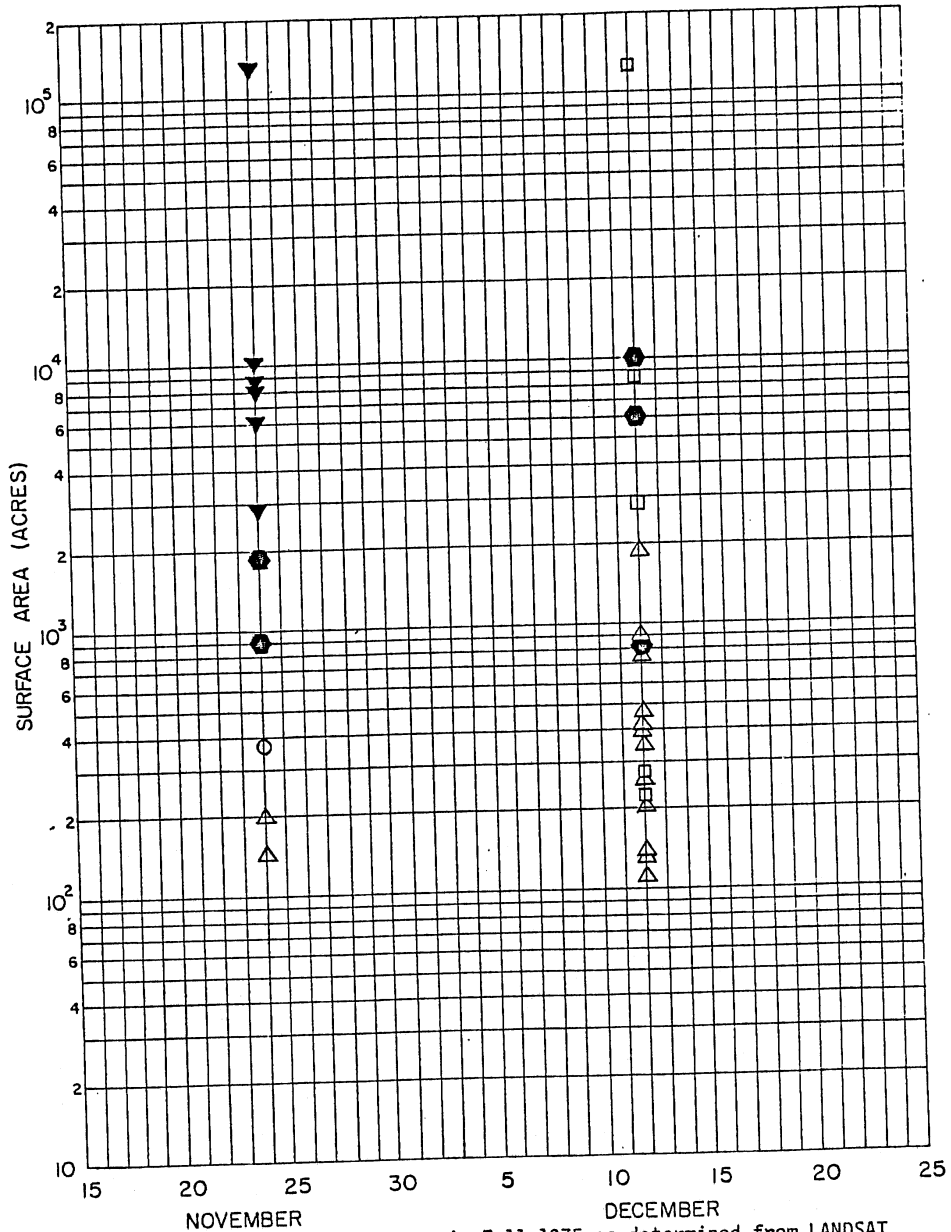


Fig. 3 - Lake ice coverage in Fall 1975 as determined from LANDSAT images as a function of lake surface area. Lakes near Brainerd Weather Station.

(See legend on page 3.)

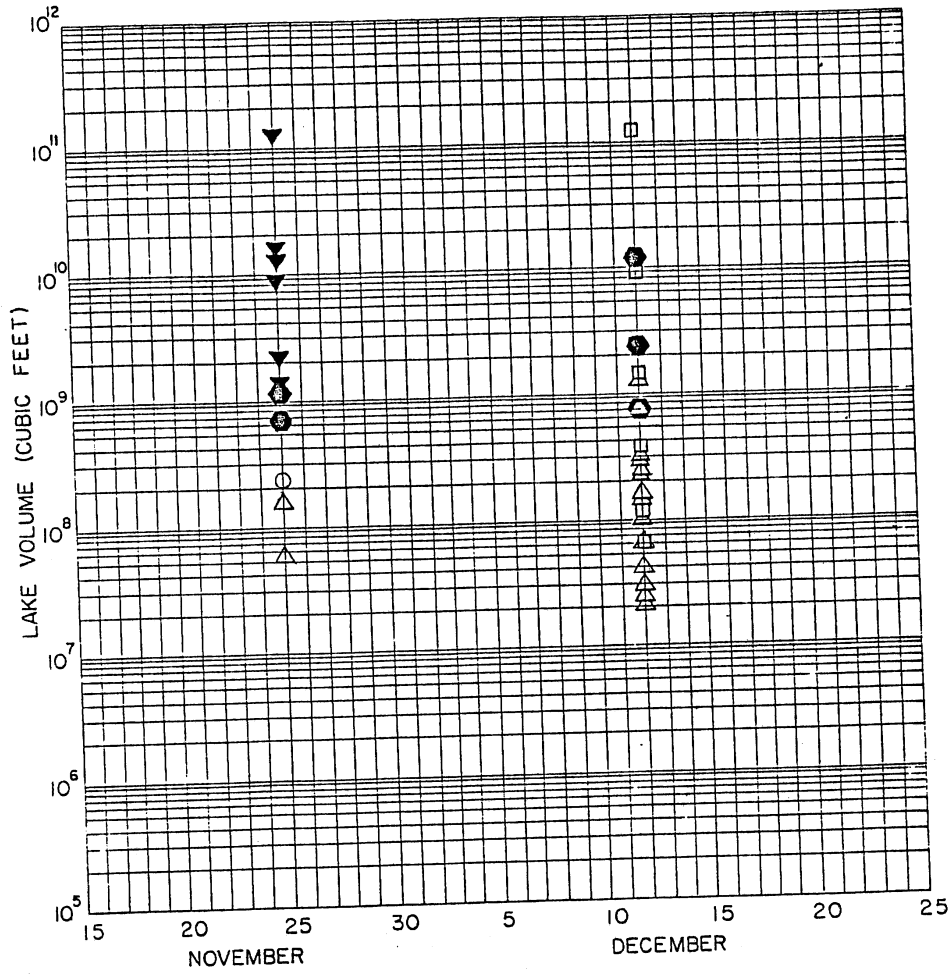


Fig. 4 - Lake ice coverage in Fall 1975 as determined from LANDSAT images as a function of lake volume. Lakes near Brainerd Weather Station.

(See legend on page 3.)

3. THEORY ON FREEZE-OVER

A mean seasonal water temperature cycle of a fully mixed (non-stratifying) lake can be approximated by the relationship

$$T = E_m + \frac{I}{m} + \Delta E \left[1 + \left(\frac{w}{m} \right)^2 \right]^{-1/2} \sin(wt - \delta) \quad (1)$$

Eq. (1) is valid only for water temperatures $T > 32^\circ\text{F}$ (0°C).

The foregoing equation is derived from the mean annual equilibrium temperature cycle as introduced by Edinger et al (1968).

$$E = E_m + \Delta E \sin(wt) \quad (2)$$

Eqs. (1) and (2) have been plotted schematically in Fig. 5. Equilibrium temperature E is by definition that water temperature at which the net heat transfer through the water surface is zero. Equilibrium temperature can be computed from weather data, specifically air temperature, solar radiation, dew point temperature, wind velocity and cloudiness ratio. Some actual data from a Vanderbilt University report (1971) are given in Fig. 6.

In Eq. (1) t = time in days, $w = 2\pi/365$, $m = K/\rho ch$, where K = an annual bulk surface conductance, ρc = specific heat per unit volume, h = mean depth of water body, I is the daily rate of artificial heat input, if any and $\delta = \arctan\left(\frac{w}{m}\right)$. Typical values for central Minnesota conditions are $E_m = 49^\circ\text{F}$, $\Delta E = 28^\circ\text{F}$, $K = 80 \text{ BTU ft}^{-2} \text{ day}^{-1} \text{ }^\circ\text{F}^{-1}$, $\rho c = 62.4 \text{ BTU ft}^{-3} \text{ }^\circ\text{F}^{-1}$ and $I = 0$ or non-zero, as the case may be.

A lake will freeze over when the freezing temperature T_f is reached during the cycle of decreasing water temperature (3rd quadrant of the sinusoidal temperature cycle). If Eq. (1) is solved for t at $T = T_f$, the result is

$$t = \frac{1}{w} \arcsin \left[\left\{ \frac{T_f - E_m - \frac{I}{m}}{\Delta E} \right\} \left\{ 1 + \left(\frac{w}{m} \right)^2 \right\}^{1/2} \right] + \frac{1}{w} \delta \quad (3)$$

The right hand side of Eq. (3) depends on lake depth h through the parameter m . A lake of zero or nearly zero depth has no lag δ and freezes over when equilibrium temperature reaches 32°F (0°C). This occurs at time

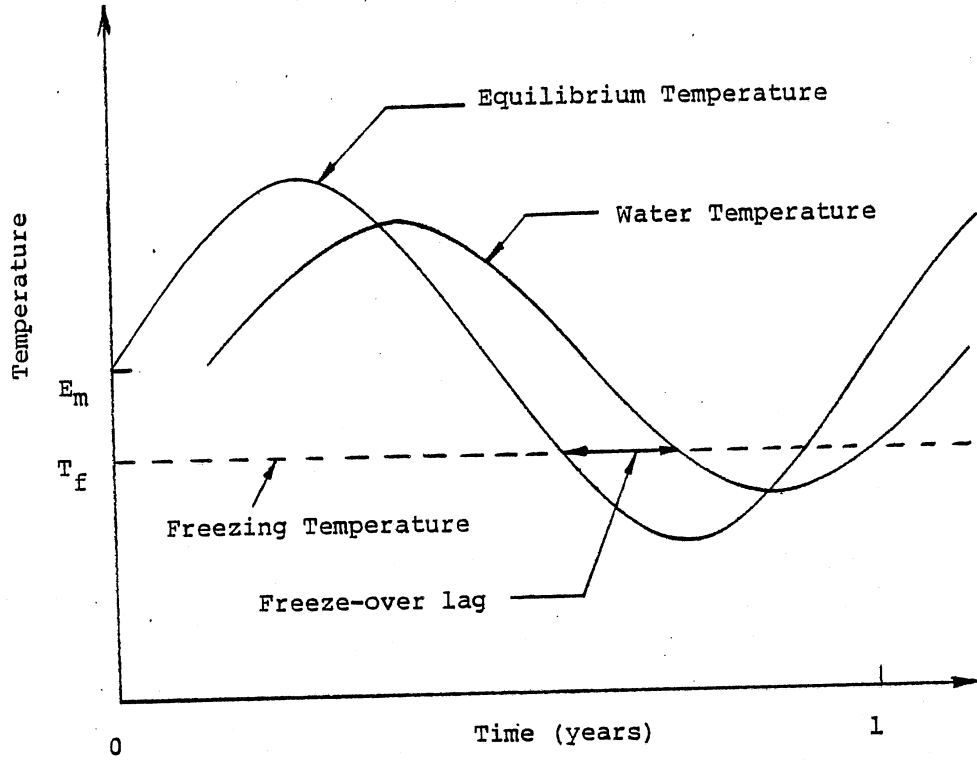
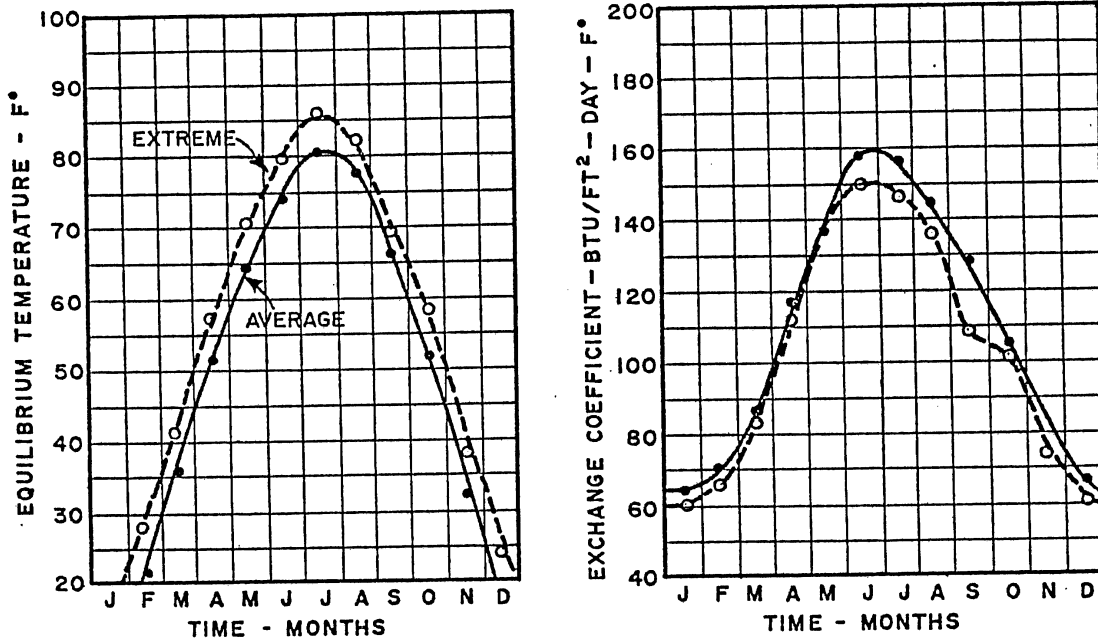
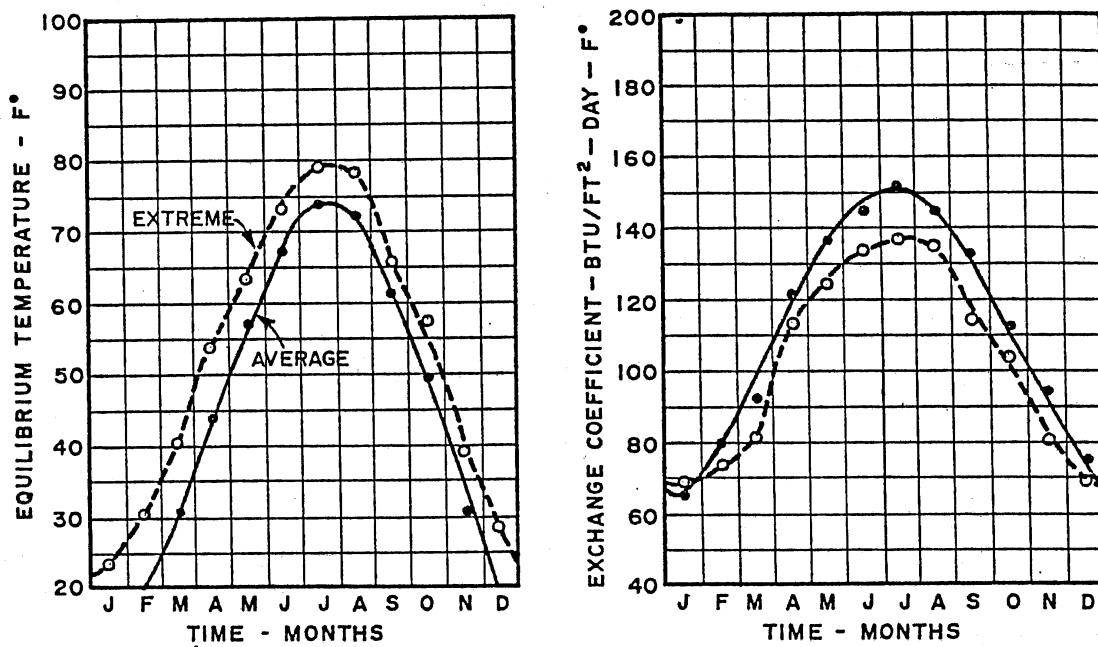


Fig. 5 - Schematic mean annual water temperature and equilibrium temperature cycles.



DULUTH, MINNESOTA



MINNEAPOLIS-ST. PAUL, MINNESOTA

Fig. 6 - Actual mean monthly values of equilibrium temperature and bulk surface heat exchange coefficients.

$$t = \frac{1}{w} \arcsin \frac{T_f - E_m}{\Delta E} \quad (4)$$

Lake depth produces a lag in the freeze-over date. The lag is

$$\text{lag} = \frac{1}{w} \arcsin \left(\frac{w}{m} \right) + \frac{1}{w} \left[\arcsin \left\{ \frac{T_f - E_m - \frac{I}{m}}{\Delta E} \right\} \left\{ 1 + \left(\frac{w}{m} \right)^2 \right\}^{1/2} \right] - \frac{1}{w} \left[\arcsin \left\{ \frac{T_f - E_m - \frac{I}{m}}{\Delta E} \right\} \right] \quad (5)$$

An example of function (5) has been plotted in Fig. 7. The relationship shown in Fig. 7 is for values of $E_m = 49^\circ\text{F}$, $\Delta E = 28^\circ\text{F}$, $K = 80 \text{ BTU ft}^{-2} \text{ day}^{-1} \text{ }^\circ\text{F}^{-1}$ and $I = 0$, which describe mean climatological conditions in central Minnesota. For other regions and specific years other numerical values would be obtained. Fig. 7 does specify an order of magnitude for the lag time. Since it reflects a nearly linear relationship it may be interpreted to mean that for every meter of median depth the freeze-over date will be delayed by approximately three days. Obviously, the total lag between lakes of different depths can add up to considerable lengths of time in terms of days and weeks. Information from satellite imagery given in Fig. 2 corroborates this information. To make the comparison easier, Fig. 8 has been prepared from Fig. 7. Similar theoretical curves were prepared for each weather station and the falls from 1972 through 1976. Details on the relationships used to compute values of the parameters E_m , ΔE and K from the individual weather station data can be found in Appendix A, Fu's M.S. thesis, University of Minnesota (1979), and in Appendix B.

The theory indicates that a lake will not freeze over at all if its depth exceeds a value which can be derived from the relationship

$$E_m + \frac{I}{m} - T_f = \Delta E \left[1 + \left(\frac{w}{m} \right)^2 \right]^{-1/2} \quad (6)$$

The limiting freeze-over depth is

$$h_{\text{lim}} = \frac{K}{w\rho c} \left[\left\{ \frac{\Delta E}{E_m + \frac{I}{m} - T_f} \right\}^2 - 1 \right]^{1/2} \quad (7)$$

For the before given numerical values $h_{\text{lim}} = 97.5 \text{ ft}$. A lake with a larger depth than 97.5 ft would, on an annual mean basis, not reach freezing temperature.

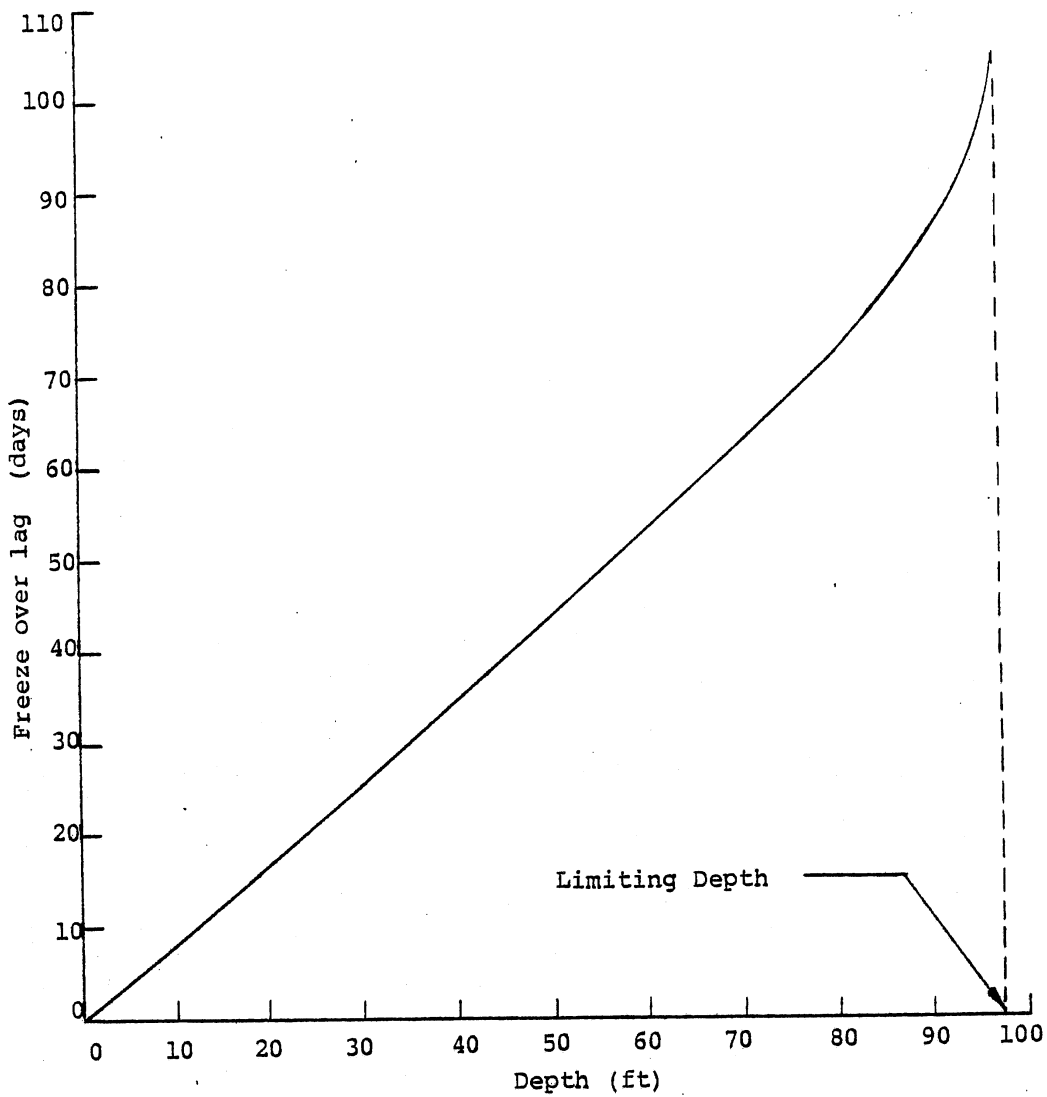


Fig. 7 - Theoretical time lag of freeze-over date as a function of mean lake depth for $E_m = 49^\circ\text{F}$, $\Delta E = 28^\circ\text{F}$, $K = 80 \text{ BTU ft}^{-2} \text{ day}^{-1} \text{ }^\circ\text{F}^{-1}$.

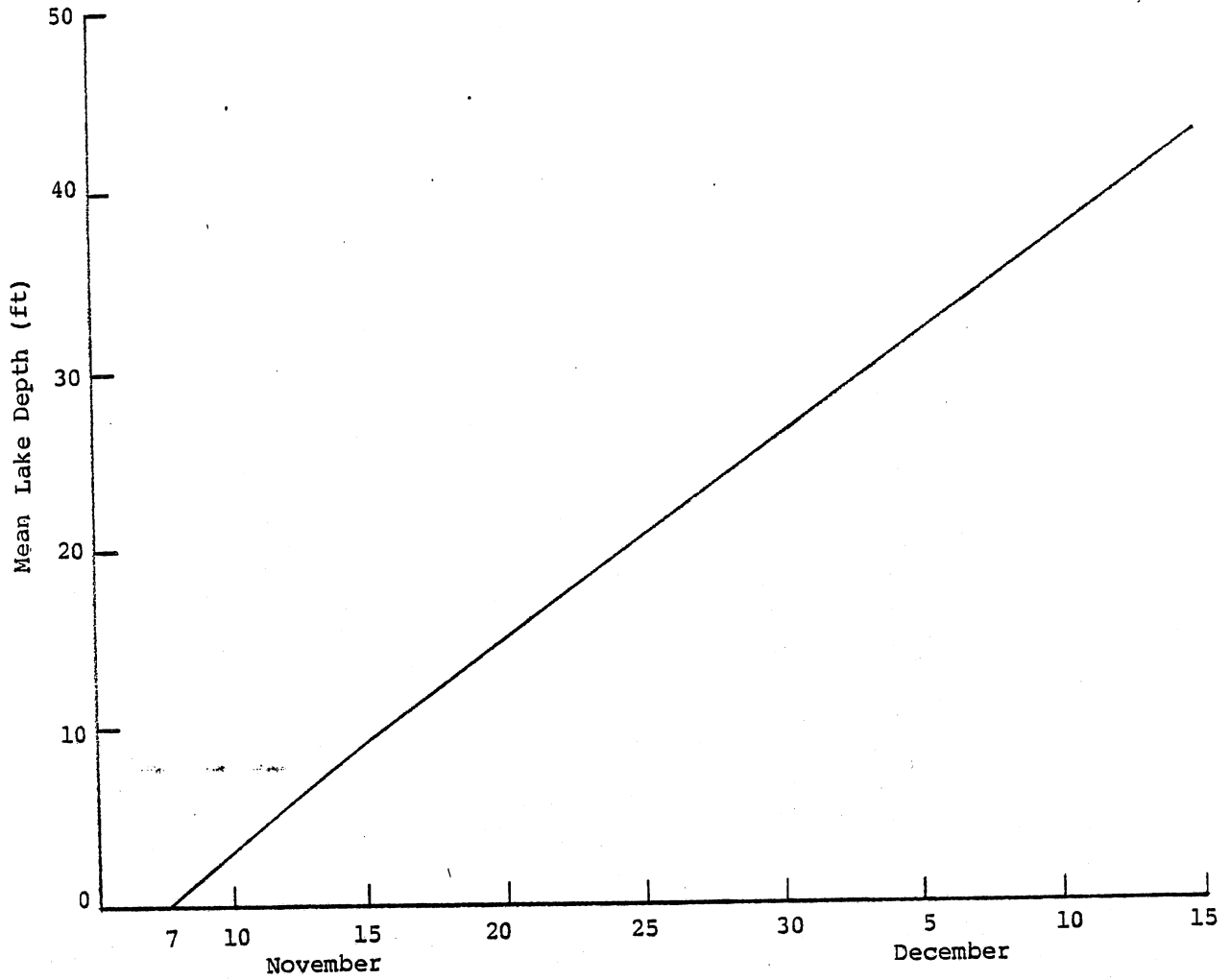


Fig. 8 - Mean annual theoretical freeze-over date for
 $E_m = 49^{\circ}\text{F}$, $\Delta E = 28^{\circ}\text{F}$ and $K = 80 \text{ BTU ft}^{-2} \text{ day}^{-1} \text{ }^{\circ}\text{F}^{-1}$.

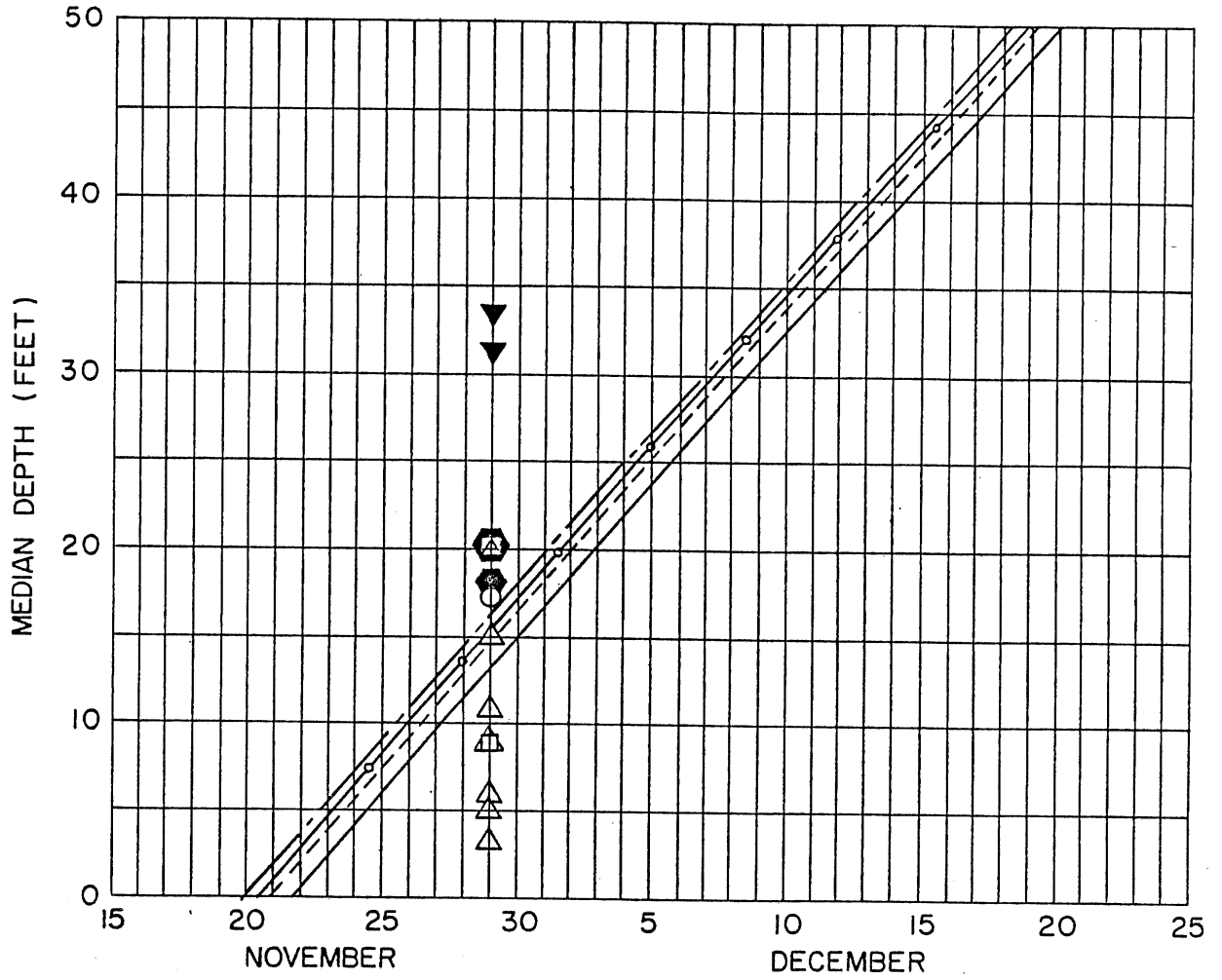
4. HINDCASTING OF FREEZE-OVER DATES AND VERIFICATION USING LANDSAT IMAGERY

LANDSAT imagery provided the only means to verify the relationships derived for forecasting freeze-over dates. Satellite imagery does not provide a continuous freeze-over history of a lake. It rather provides an occasional glimpse at ice conditions in an entire region. Typically, lakes of many different sizes and depths are found in a region, and those which have open water can be distinguished from those which are ice-covered. This information can be presented in the form already shown in Fig. 2 and can be superimposed and compared with the theory presented in the form of Fig. 8.

The forecasting of ice-covers would be made sometime in the fall, when the annual weather cycle is still incomplete. Estimates of the parameters E_m , ΔE , and K would be based on weather data available at the time of forecasting.

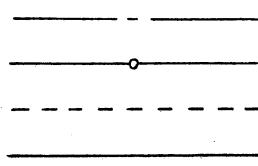
The effect of this limitation on the forecasting is shown in Fig. 9, where forecasts (hindcasts) at the end of four different months are compared with information from satellite imagery. It can be seen that the forecast freeze-over date shifts generally by one to three days as the season progresses.

The forecast lines in Fig. 9 take into consideration a winter stratification as shown in Appendix C. The method by which the air temperature data and K -values were extrapolated (in order to make predictions early in the fall) is shown in Appendix D.



Legend

Month at the end of which prediction was made



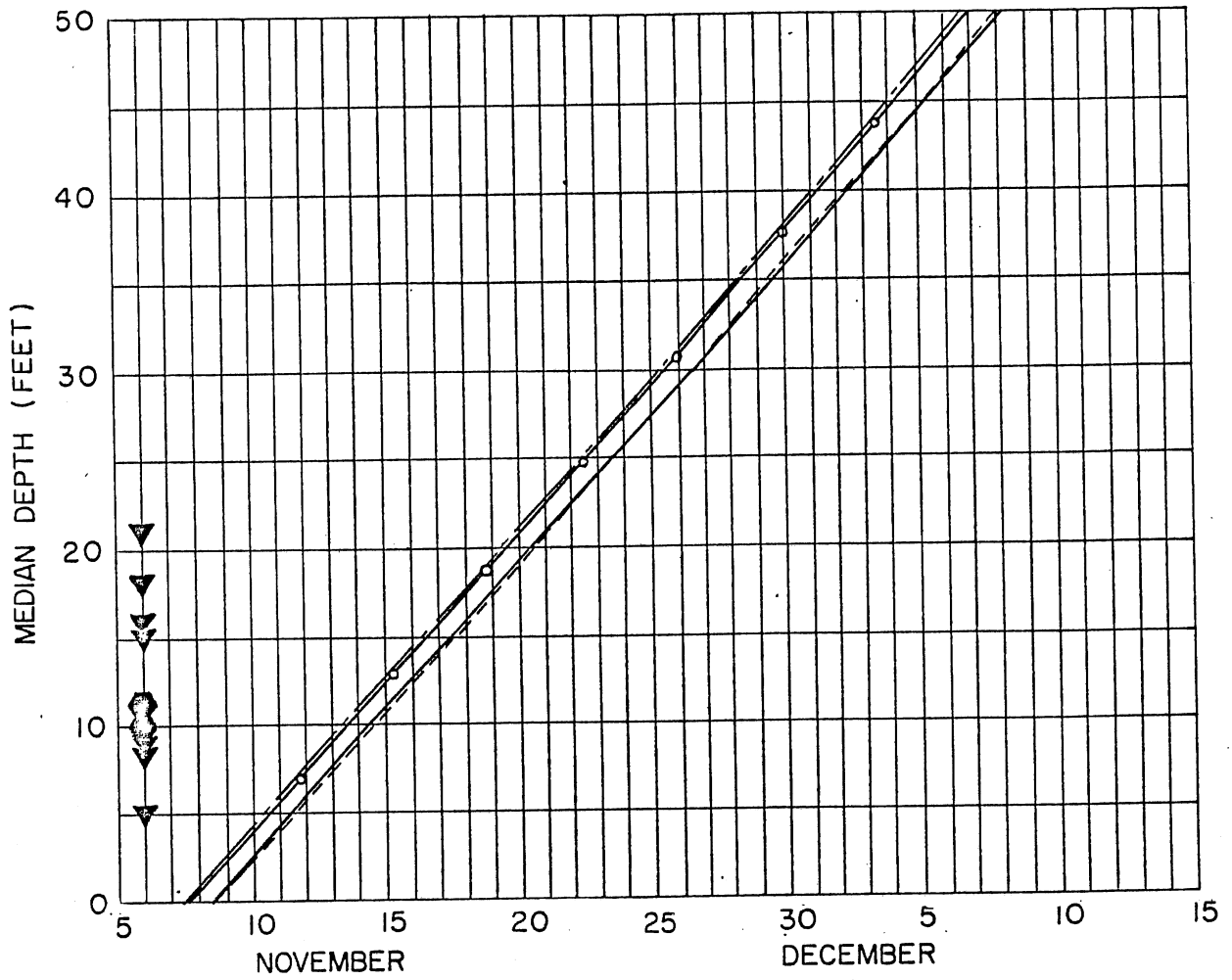
August

September

October

November and December

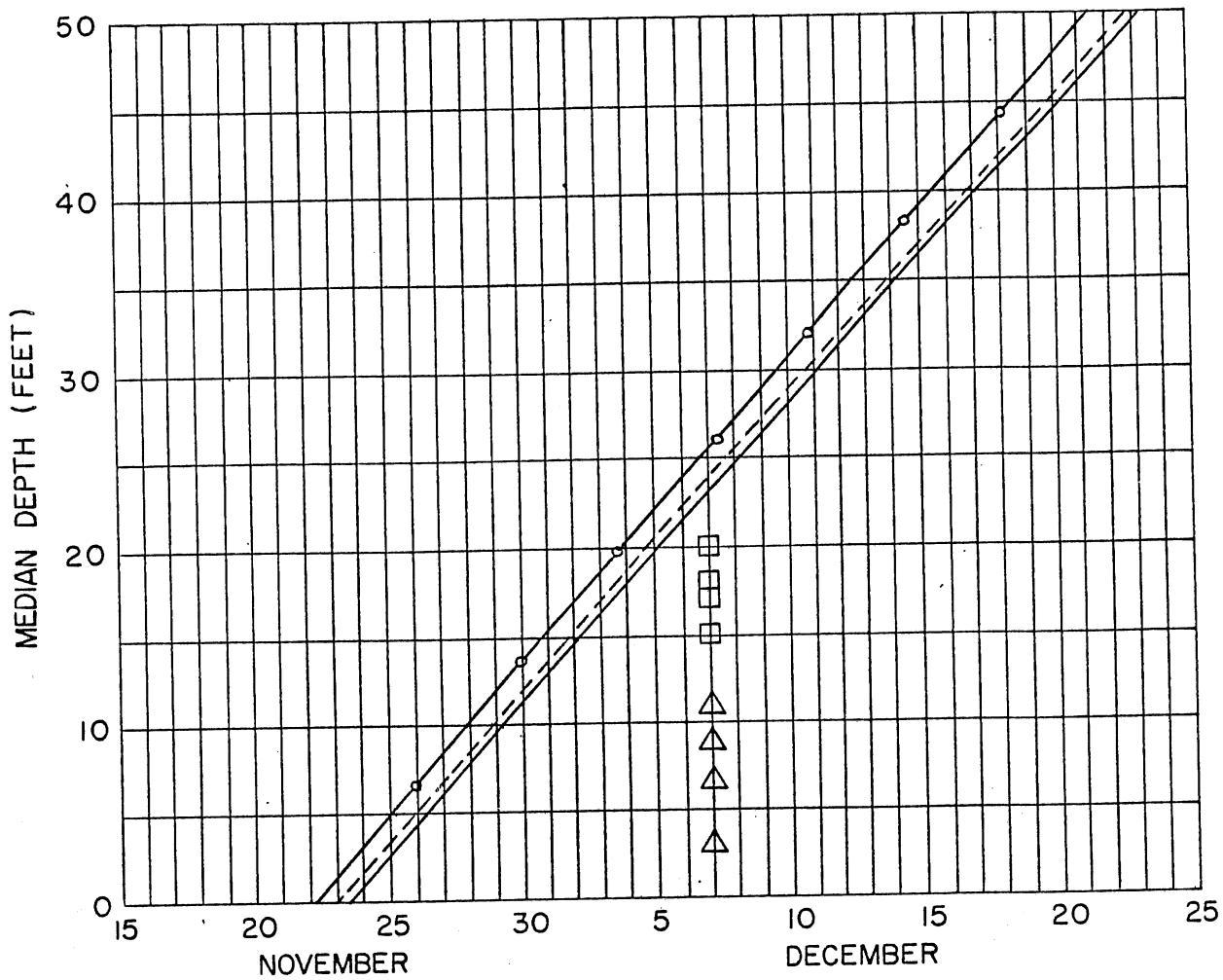
Fig. 9 (a) - Prediction of the beginning dates of freeze-over of lakes in the vicinity of Minneapolis/St. Paul in Fall 1972.



Legend

- | | |
|--|---------|
| Month at the end of which prediction was made. | |
| August | ----- |
| September | ———○——— |
| October | |
| November and December | ————— |

Fig. 9(b) Prediction of the beginning dates of freeze-over of lakes in the vicinity of Sandy Lake Dam Libby in Fall 1973.



Legend

Month at the end of which prediction was made.

—○—

August and September

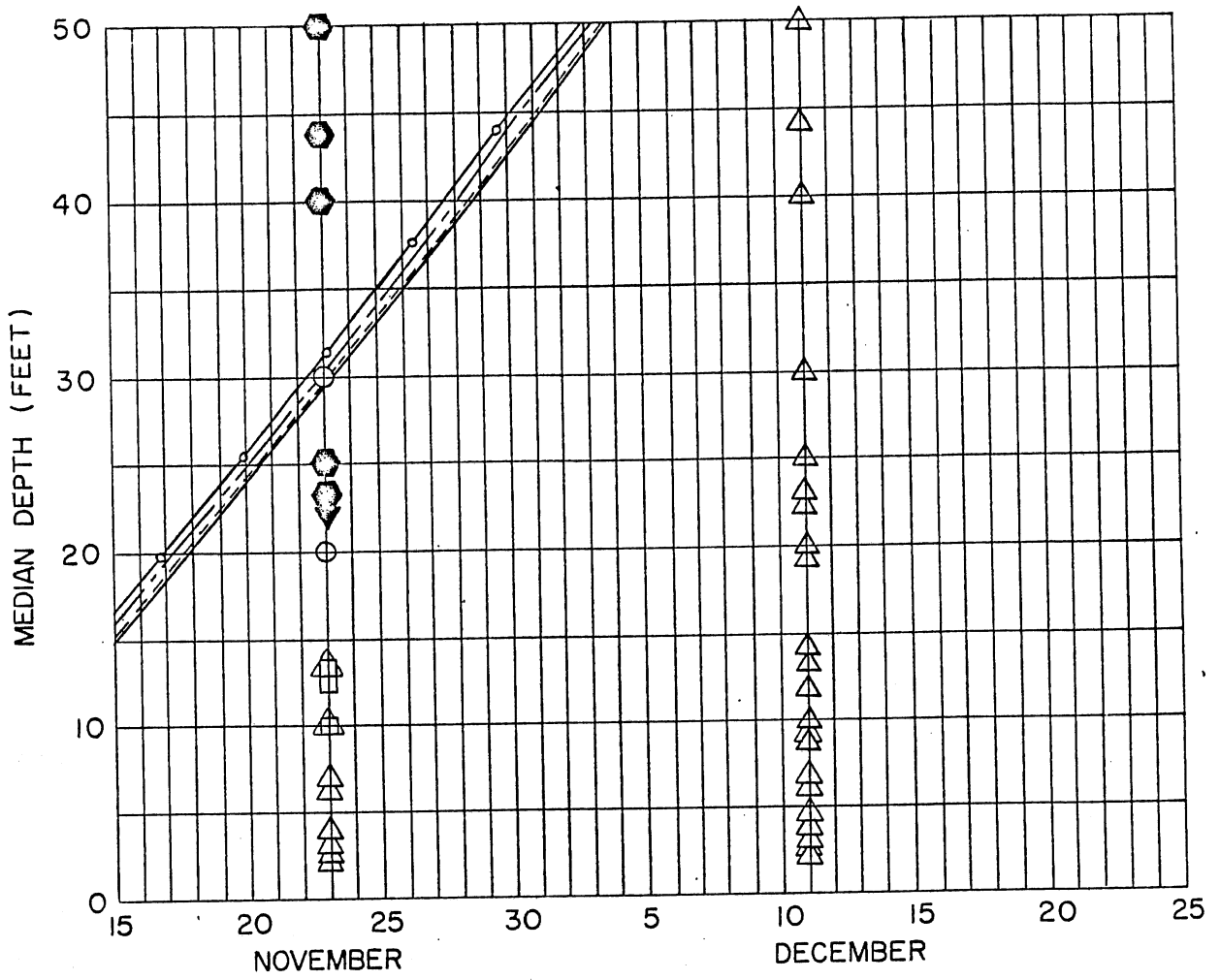
- - -

October

—

November and December

Fig. 9(c) - Prediction of the beginning dates of freeze-over of lakes in the vicinity of Minneapolis/St. Paul in Fall 1974.

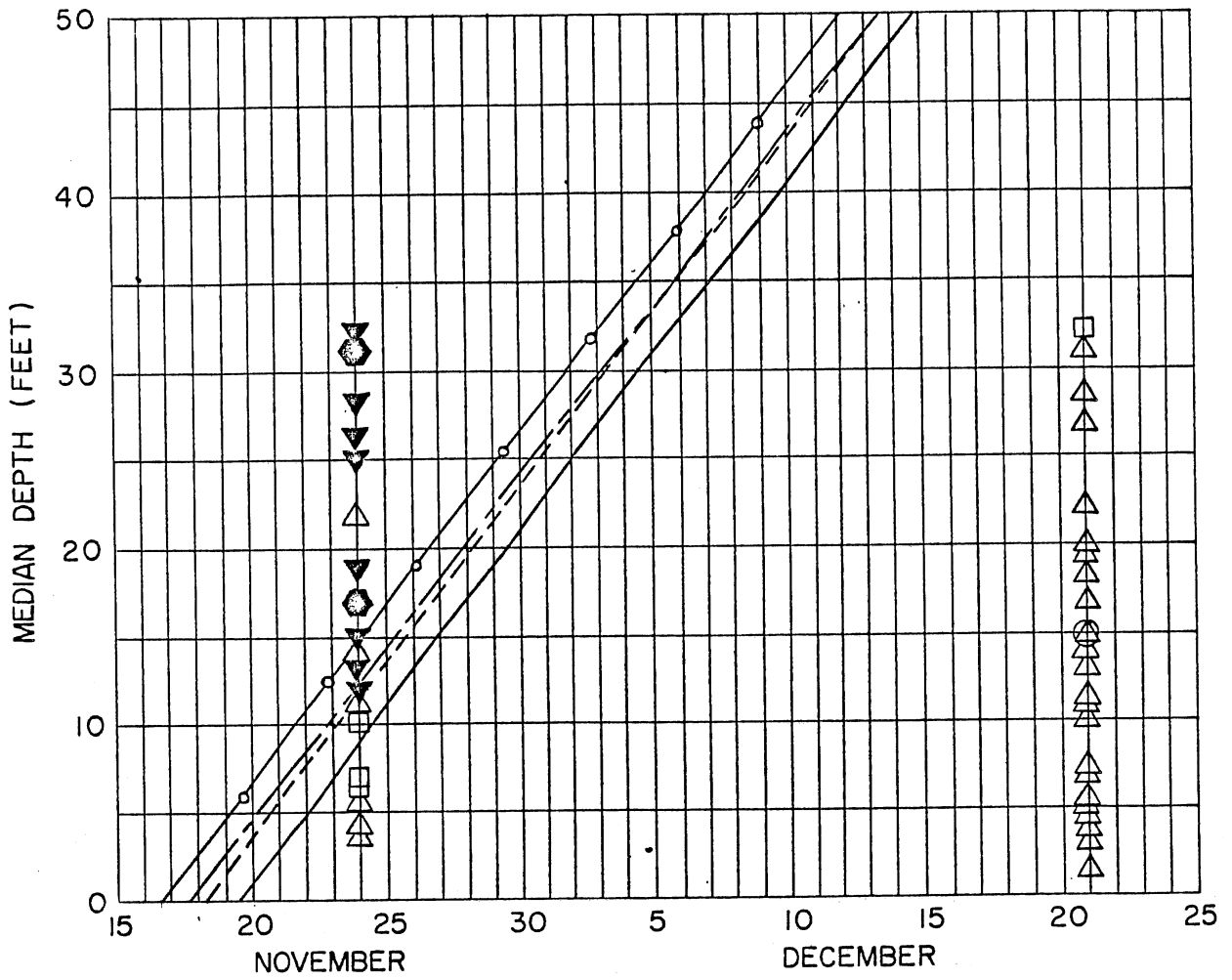


Legend

Month at the end of which prediction was made.

- August
- September
- October
- November and December

Fig. 9(d) Prediction of the beginning dates of freeze-over of lakes in the vicinity of Winton Power Plant in Fall 1975.



Legend

- Month at the end of which prediction was made.
- — — — — August
 - — — — — ○ — — — — — September
 - - - - - October
 - — — — — November and December

Fig. 9(e) Prediction of the beginning dates of freeze-over of lakes in the vicinity of Alexandria Airport in Fall 1975.

5. MEAN FREEZE-OVER DATES FOR MINNESOTA LAKES

The forecasting procedure derived by using information from three LANDSAT scenes (Fig. 1) was subsequently applied to all of Minnesota. The number of weather stations was expanded from nine to nineteen as shown in Fig. 10. The results of the weather data analysis applied to the freeze-over theory are shown in Figs. 11 through 12. Fig. 11 gives an approximate minimum depth, i.e. minimum mean depth of a lake which is unlikely to freeze over completely in an average. The value is an average of those found for the years from 1972 through 1977 and reflects the weather conditions during that time. Fig. 12 gives the average freeze-over date for lakes of different mean or median depths.

Additional information on the procedure used is given in Appendix E and F. A computer program carrying out the computations is listed in Appendix G.

In the early phase of this study, it was considered that the cumulative degree freezing days (Appendix H) might be related to freeze-over dates of lakes. This concept has not been further explored.

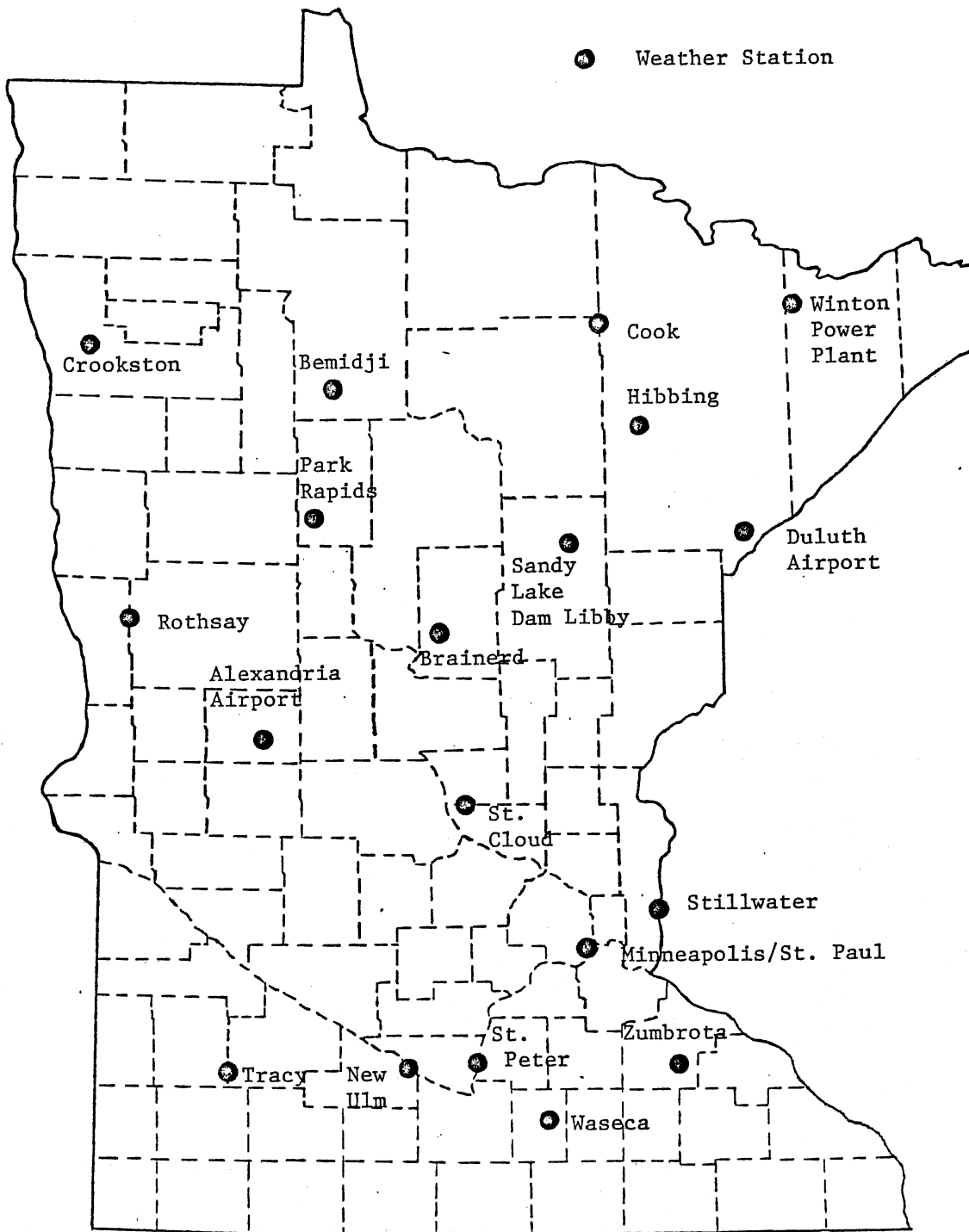


Fig. 10-Location of Weather Stations from which Weather Data are Used in Determining the Freeze-over Dates and Critical Lake Depths.

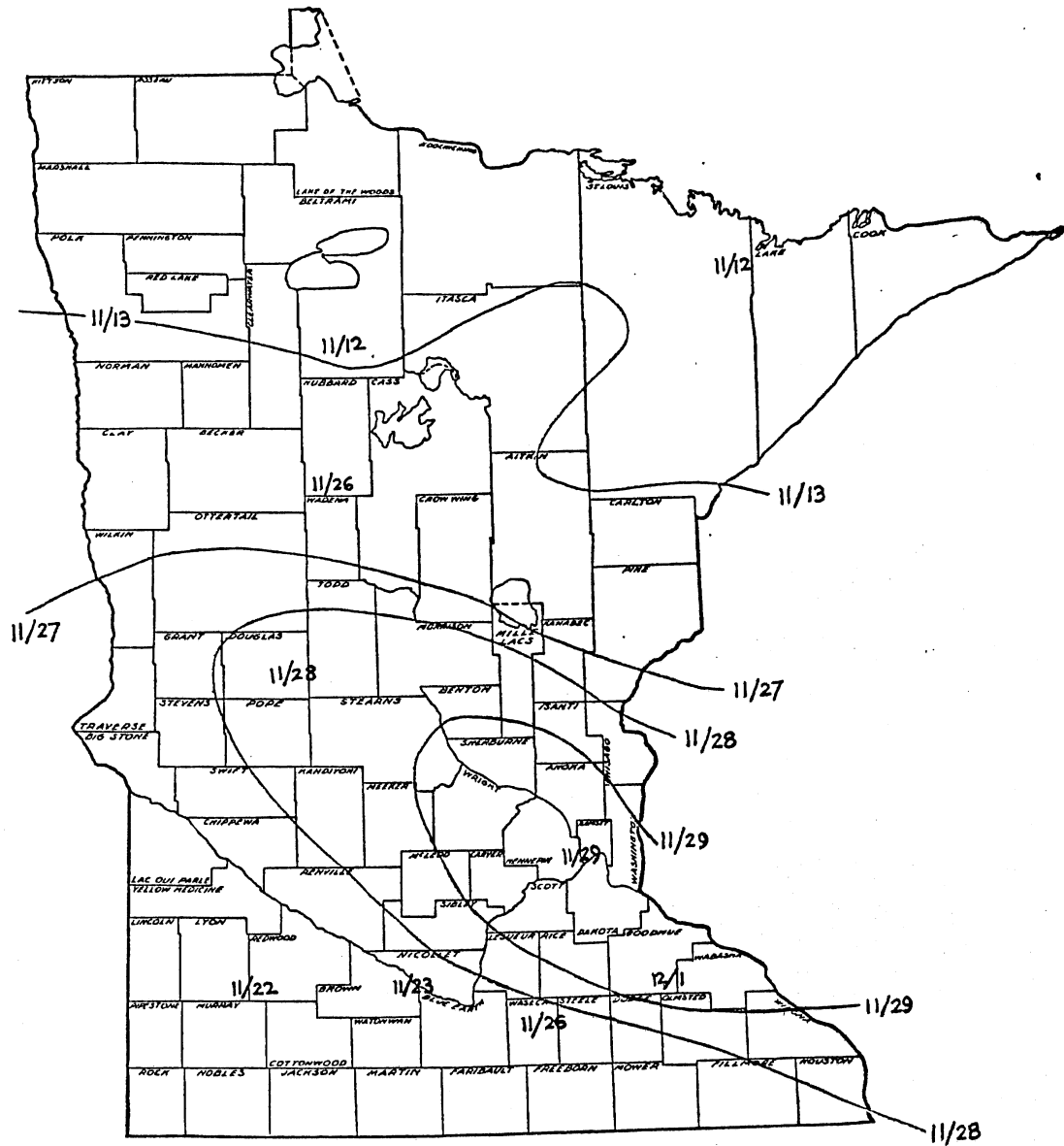


Fig. 12 (b)- Average Freeze-over Dates of Lakes Having Mean or Median Depths equal to 10 feet.

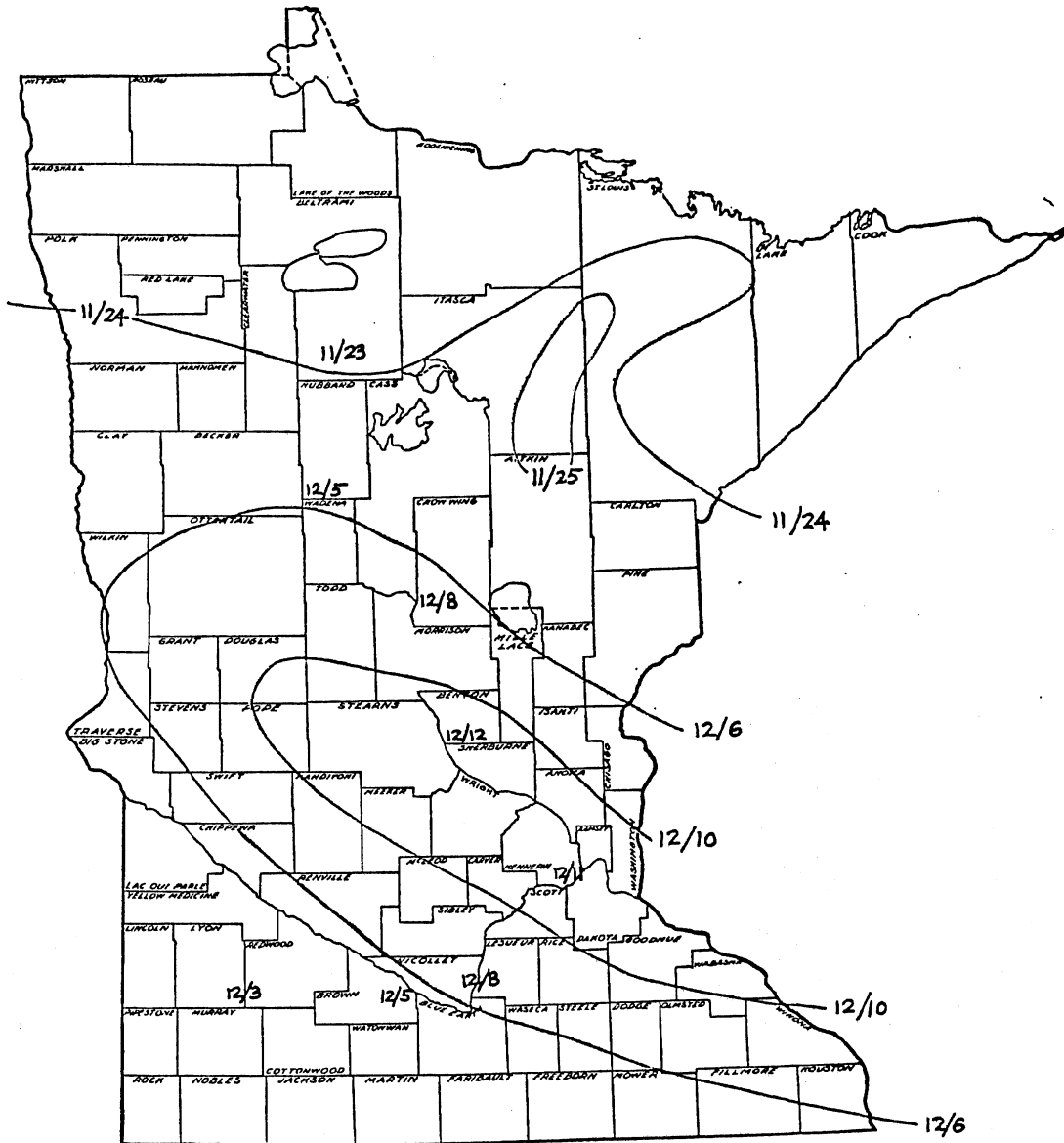


Fig. 12 (d) - Average Freeze-over Dates of Lakes Having Mean or Median Depths equal to 30 feet.

6. INFORMATION FROM LANDSAT IMAGERY ON SPRING MELTING OF LAKE
ICE-COVERS

The melting of ice covers on Minnesota lakes in the spring occurs typically between March and May. LANDSAT images for March, April and May 1976 have been used to extract information on ice conditions in spring. Average cloud cover in spring of 1976 was less than in fall of 1975 as shown in Table 2. This is typical also for other years. The procedure for LANDSAT image interpretation was similar to that described earlier for the fall of 1975. Results showing ice conditions as a function of lake depth and date similar to those given in Fig. 2 were plotted. They showed a very poor correlation of ice cover with depth. The reason, of course, is that the melting of the ice cover is only to a minor degree related to the heat budget of the lake water beneath the ice. The radiation balance on the ice as well as along the shorelines, and inflow of water from minor tributaries and overland flow are much more important. Radiation and overland flow, if at all related to lake morphology, would depend on lake surface area. Observations on ice coverage were therefore plotted versus lake surface area in Figs. 13a through 13f. It appears that the spring ice melting is very uniform in time and pretty much independent of lake size. A theory very different from that used for the freeze-over prediction is needed.

In the beginning of this study, it was conceived that the cumulative degree melting days (Appendix I) might be related to spring melting of lake ice-covers. This concept has not been further explored as the study was confined to the prediction of freeze-over dates of lakes.

TABLE 2. Available LANDSAT Scenes, Spring 1976

<u>North</u>		
Date	Quality	Cloud Cover (%)
3/01/76	Unavailable	100
3/10/76	Excellent	10
3/19/76	Fair	10
3/28/76	Fair	60
4/06/76	Excellent	20
4/15/76	Fair	70
4/24/76	Unavailable	100
5/03/76	Excellent	10
5/12/76	Excellent	10
5/21/76	Excellent	10
5/30/76	Excellent	60
Average		42

<u>Central</u>		
Date	Quality	Cloud Cover (%)
3/02/76	Unavailable	100
3/11/76	Fair	90
3/20/76	Unavailable	100
3/29/76	Excellent	90
4/07/76	Excellent	0
4/16/76	Fair	70
4/25/76	Excellent	10
5/04/76	Excellent	0
5/13/76	Unavailable	100
5/22/76	Unavailable	10
Average		57

<u>South</u>		
Date	Quality	Cloud Cover (%)
3/01/76	Unavailable	100
3/10/76	Excellent	10
3/19/76	Unavailable	100
3/28/76	Fair	20
4/06/76	Excellent	0
4/15/76	Fair	90
4/24/76	Unavailable	100
5/03/76	Excellent	30
5/12/76	Fair	80
5/21/76	Excellent	30
5/30/76	Excellent	90
Average		59

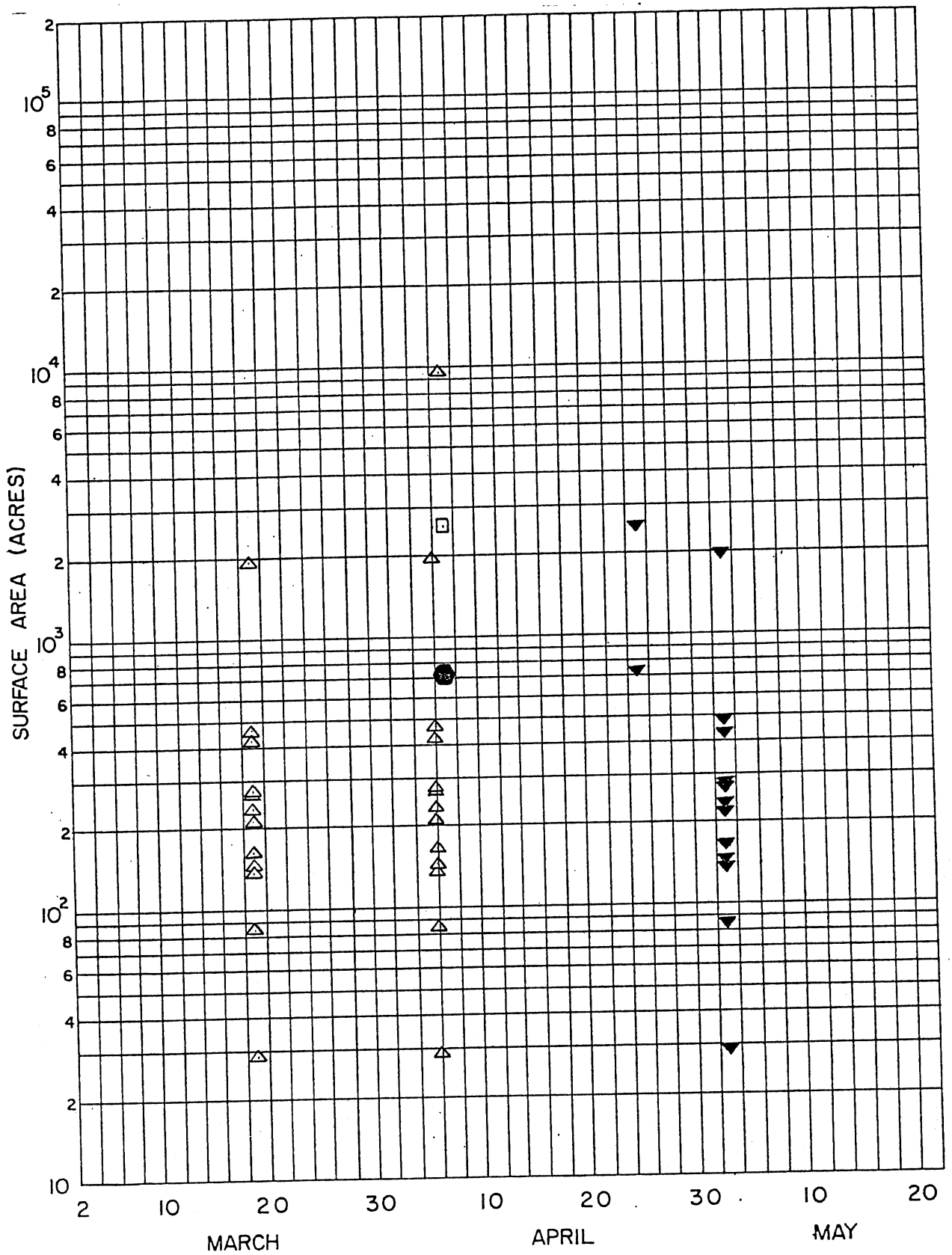


Fig. 13(b)-Lake ice coverage in spring 1976 as determined from LANDSAT images as a function of lake surface area. Lakes near Sandy Lake Dam Libby Weather Station.

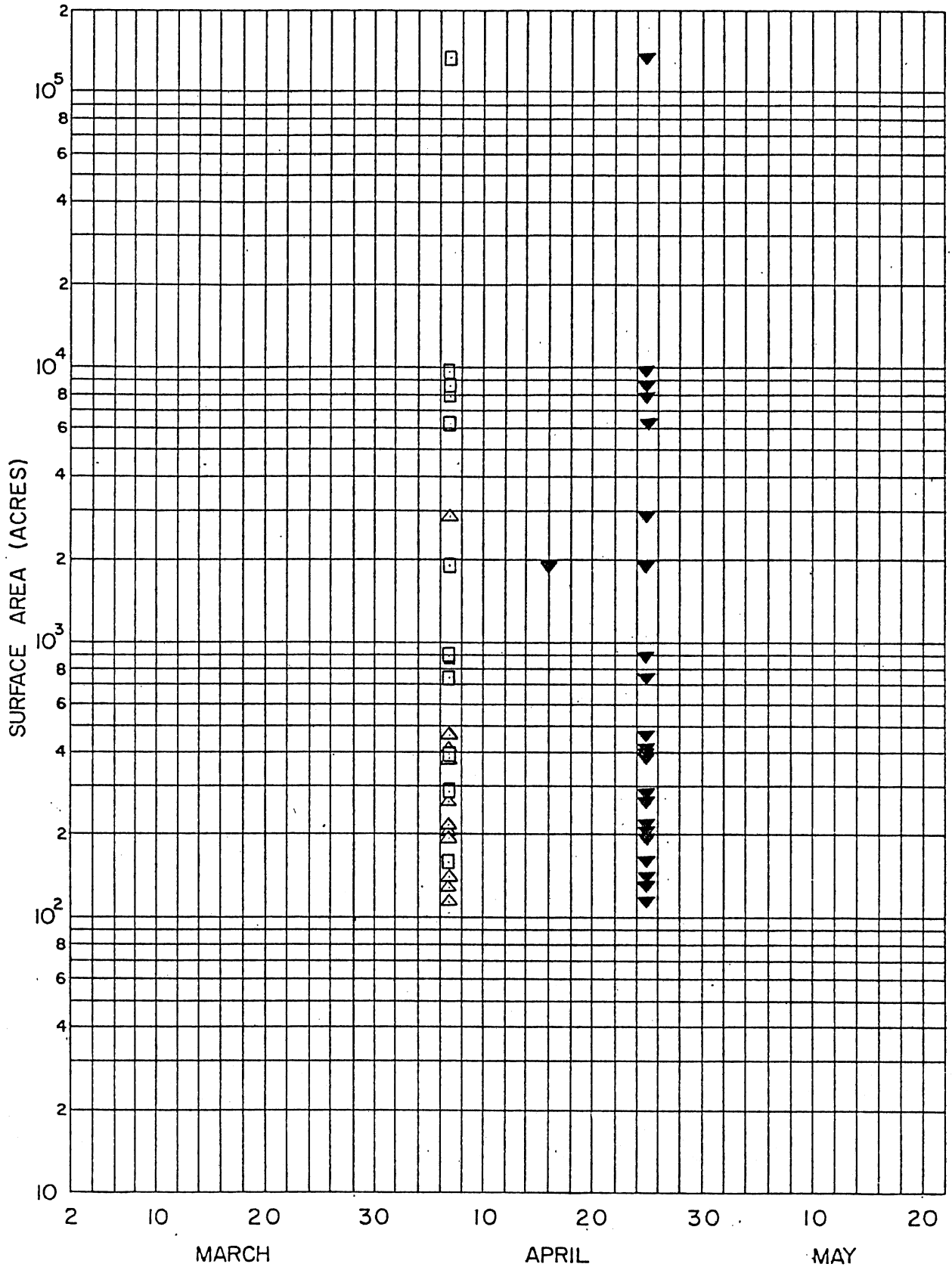


Fig. 13(c)-Lake ice coverage in spring 1976 as determined from LANDSAT images as a function of lake surface area. Lakes near Brainerd Weather Station.

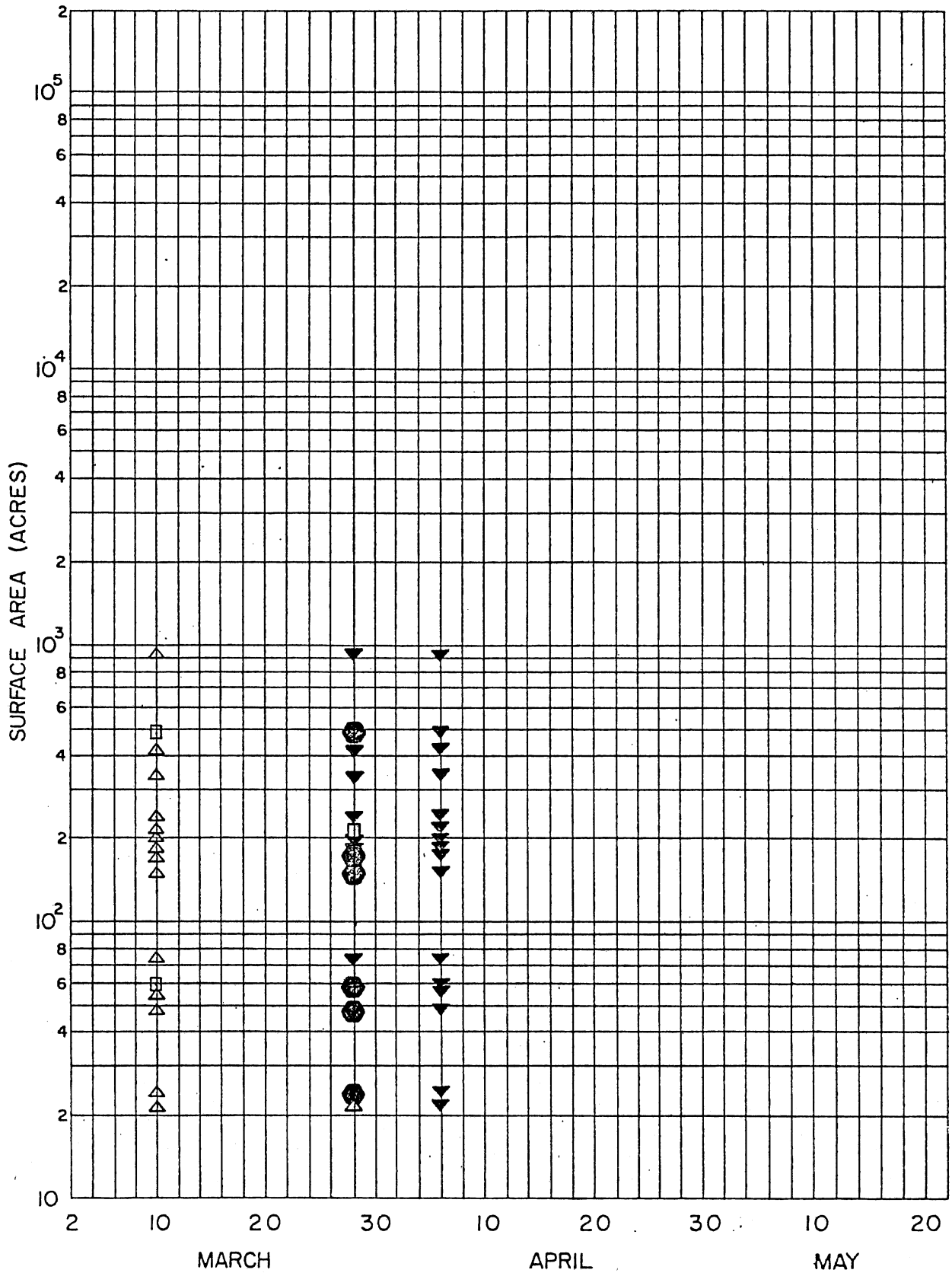


Fig. 13(e)-Lake ice coverage in spring 1976 as determined from LANDSAT images as a function of lake surface area. Lakes near Minneapolis/St. Paul Airport Weather Station.

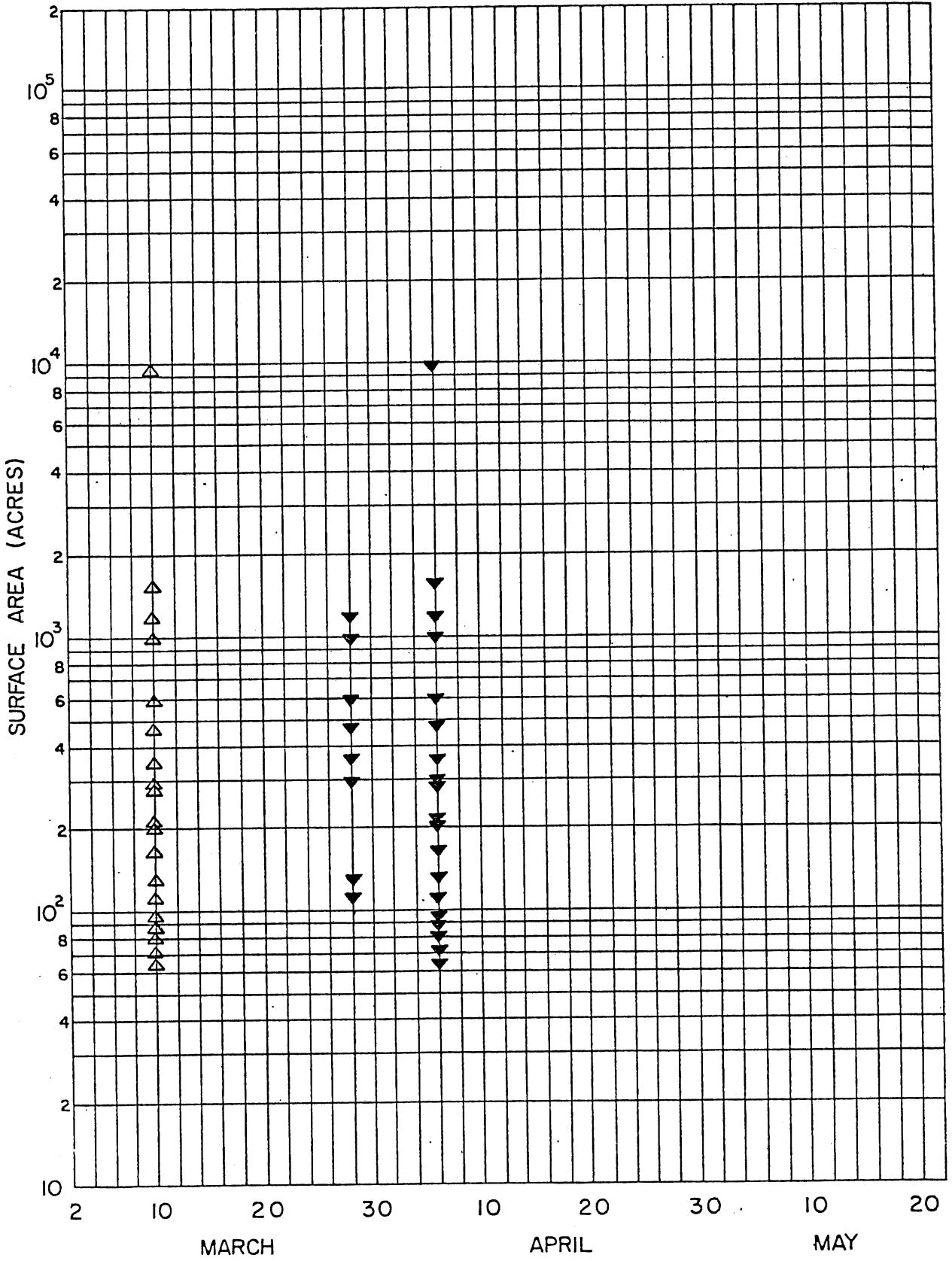


Fig. 13(f)-Lake ice coverage in spring 1976 as determined from LANDSAT images as a function of lake surface area. Lakes near Minneapolis/St. Paul Airport Weather Station.

7. CONCLUSIONS

- a) Satellite imagery is the only easily accessible and extensive source of information on ice coverage of Minnesota lakes.
- b) Ice coverage affects water quality of lakes e.g. through oxygen depletion, enhancement of sedimentation and interaction with groundwater. It is a potential factor in toxic material transport. Certain lake surveys require an ice cover. Removal of rough fish from lakes is often done just prior to ice formation.
- c) Cloud coverage hinders the use of LANDSAT imagery. Usefulness of satellite imagery in the fall of 1972, 1973, 1974, and 1975 is summarized in Table 1. Average cloud coverage during the period of lake ice formation in those four years was 59 percent. Nevertheless, enough information was available to verify a theoretical model for the prediction of freeze-over dates of Minnesota lakes. The model is based on a seasonal heat budget and takes into consideration lake depth. It was found that as a rule of thumb freeze-over date was delayed by three days for every additional meter of mean depth. A lake of 40 ft mean depth would therefore freeze about 27 days later than a lake of 10 ft depth.
- d) The predictive model was applied to different regions of Minnesota. The results are summarized in Fig. 12. Lakes in the northeastern part of Minnesota are predicted to freeze over first, those in the Metropolitan - St. Cloud area last. The lag for lakes of equal mean depth but located in those two extreme regions was predicted to be about 18 days.
- e) The lag-time due to depth and due to latitude are cumulative. A very shallow northeastern Minnesota lake would therefore freeze 26 days before a 20 ft (mean) lake in the Metropolitan area.
- f) The prediction of freeze-over dates at the end of the summer or in fall may miss the real date by as much as a week due to the uncertainty in weather between the date of the forecasting and the freeze-over date.
- g) Spring melting of ice covers observed on LANDSAT imagery bears little relationship to lake morphology. A separate theory needs to be considered.

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A P P E N D I C E S

APPENDIX A

MAJOR SOURCES OF HEATED DISCHARGES
INTO MINNESOTA SURFACE WATERS

<u>Plant</u>	<u>Owner</u>	<u>Receiving Water</u>
Fox Lake	INPD	Fox Lake near Fairmont
Aurora	MNPL	Colby Lake and Partridge River near Hoyt Lakes
Clay Boswell	MNPL	Mississippi River near Grand Rapids
Hibbard	MNPL	St. Louis Bay, Duluth
Blank Dog	NSP	Black Dog Lake and Minnesota River near Bloomington
High Bridge	NSP	Mississippi River in St. Paul
A. S. King	NSP	Lake St. Croix near Stillwater
Minnesota Valley	NSP	Minnesota River near Granite Falls
Monticello	NSP	Mississippi River near Monticello
Prairie Island	NSP	Mississippi River Pool No. 3 near Red Wing
Riverside	NSP	Mississippi River in Minneapolis
Hoot Lake	OTTP	Ottertail River and Reservoir near Fergus Falls
Ortonville	OTTP	Cooling Pond near Ortonville and Big Stone Lake

APPENDIX B

Seasonal Equilibrium Temperature
and Water Temperature Cycles of a Lake

The seasonal water temperature cycle expressed by Eq. (1) is in response to the equilibrium temperature cycle given by Eq. (2). Equation (2) is the forcing function and Eq. (1) is the response.

The question remains how, for a given site, the forcing function $E(t)$ and the bulk modules K of surface heat exchange can be established.

Two basically different approaches can be taken to evaluate E and K : one is based on weather data, the other on seasonal water temperature measurements. If the first route is chosen, records of four weather parameters are needed: solar radiation, windspeed, air temperature and dewpoint temperature (or relative humidity as a substitute). Only at first order weather stations are all four parameters continuously being measured. There are only a few of those in Minnesota. At some agricultural experiment stations the four parameters are also being measured, but generally the information source is very limited.

Air temperature records are usually available at many weather stations. A method to approximate water temperatures from air temperatures was therefore selected. The relationship between the air temperature and the water temperature is difficult to determine by a deterministic approach. A stochastic analysis such as proposed by Song et al (1973) among others is more practical and was used.

Song et al (1973) considered the water temperature (T) in a stream or a lake and the atmospheric air temperature (AT), in turn, to contain a deterministic part and a stochastic part. By correlating the deterministic part with the first harmonic of a Fourier series and by further assuming the existence of a correlation between the water temperature departure and the air temperature departure, the following relationship between air and water temperatures was derived:

$$T_i = (A_2 - dA_1) + (B_2 - dB_1) \sin \frac{2\pi i}{365} + (C_2 - dC_1) \cos \frac{2\pi i}{365} + dAT_i \quad (B-1)$$

Eq. (B-1) can be used to approximate water temperatures when air temperature records are available and when the coefficients $A_1, A_2, B_1, B_2, C_1, C_2$ and d have been determined by regression analysis from some actual water and air temperature measurements.

Such determinations have been made for various locations throughout the state of Minnesota by Song et al (1973) and further extended in a M.S. thesis presented to the University of Minnesota by Wong (1976). Coefficients are shown in Table B-1.

There will be a significant depth effect on the values of these coefficients, which is not represented by Eq. (B-1). How the coefficients will change as a function of depth is not entirely known. As a first approximation, the mean annual water temperature \bar{T} generated by Eq. (B-1) can be assumed to be the same for lakes of all depths in the vicinity of the water body from which the coefficients were determined. A shallow lake will however experience a bigger seasonal water temperature amplitude than a deep lake, given a set of climatic conditions. The amplitude of the seasonal water temperature cycle of a lake is inversely proportional to the damping factor $\left[1 + \left(\frac{w}{m}\right)^2\right]^{1/2}$ in Eq. (1). The following relationship thus can be developed:

$$\Delta T = \Delta T_{h=h_1} \left[\frac{1 + \left(\frac{w}{m_{h=h_1}}\right)^2}{1 + \left(\frac{w}{m}\right)^2} \right]^{1/2} \quad (B-2)$$

Furthermore

$$E_m + \frac{I}{m} = \bar{T} \quad (B-3)$$

and

$$\Delta E_m = \Delta T \left[1 + \left(\frac{w}{m}\right)^2 \right]^{1/2} \quad (B-4)$$

Substituting Eqs. (B-2), (B-3), and (B-4) into Eq. (B-5), we obtain

$$t = \frac{1}{w} \arcsin \left\{ \left(\frac{T - \bar{T} - \left(\frac{I}{m}\right)}{\Delta T_{h=h_1}} \right) \left[\frac{1 + \left(\frac{w \rho c}{K} h\right)^2}{1 + \left(\frac{w \rho c}{K} h_1\right)^2} \right]^{1/2} \right. \\ \left. + \frac{1}{w} \frac{3\pi}{2} + \frac{1}{w} \arctan \left(\frac{w \rho c}{K} h \right) \right\} \quad (B-5)$$

where $T_{h=h_1}$ is the amplitude of the water temperature cycle generated by using coefficients determined from a water body having mean depth h_1 . The mean depths h_1 used in the analysis are also shown in Table B-1.

Determination of K-values

Determination of the bulk surface heat transfer coefficient (K) follows the method proposed by Brady, Graves and Geyer (1969).

$$K = 15.7 + (\beta + 0.26) FW \quad (B-6)$$

in which

$$\beta = 0.255 - 0.0085 T_w + 0.000204 T_w^2 \quad (B-7)$$

$$T_w = \frac{T + T_d}{2} \quad (B-8)$$

where T is the water surface temperature and T_d is the dew point temperature, both in $^{\circ}F$. Since a well-mixed water body is assumed in the present case, the water surface temperature is equal to the water temperature of the entire water body. T_d can be obtained from weather stations or, if given relative humidity, calculated from the following relation suggested by Bosen as quoted by Linsley et al (1975).

$$T_d = (112 + 0.9AT)f^{0.125} + 0.1AT - 112 \quad (B-9)$$

Both T_d and air temperature (AT) are in $^{\circ}C$, and relative humidity (f) in per cent (%).

The wind function (FW) is adapted from a Shulyakovskiy formulation by Ryan and Harleman (1973)

$$FW = 14 W_2 + 22.4 (\Delta\theta_v)^{\frac{1}{3}} \quad (B-10)$$

in which FW is in $BTU \text{ ft}^{-2} \text{ day}^{-1} \text{ }^{\circ}F^{-1}$, W_2 is the wind velocity (mph) at 2m above ground, and

Weather Station	Original Location at which Coeff. were Determined	Estimated Depth (ft) of Water Body at Original Location	Air Temp (F)			Water Temp (F)			Respons. Coeff. d
			A ₁	B ₁	C ₁	A ₂	B ₂	C ₂	
Winton Power Plant	St Louis River at Forbes	5	33.97	-6.49	-29.27	37.26	-11.60	-33.22	0.316
Duluth Airport									
Sandy Lake Dam Libby									
Brainerd	Crow Wing River at Nimrod	5	37.49	-7.41	-32.52	47.15	-11.24	-28.24	0.537
Alexandria Airport									
St. Cloud									
Mpls/St Paul	Mississippi River at St. Paul	10	47.48	-8.29	-24.11	49.48	-11.75	-23.01	0.155
St. Peter	Minnesota River at Mankato	3	48.29	-7.88	-24.73	50.04	- 9.08	-24.72	0.462
Waseca									

Table B-1

Coefficients for Generation of Water Temperatures

$$\Delta\theta_v = T(1 + 0.378 \frac{e_s}{p_a}) - AT(1 + 0.378 \frac{e_a}{p_a}) \quad (B-11)$$

where T = water temperature ($^{\circ}R$)
 AT = air temperature ($^{\circ}R$)
 p_a = air pressure
 e_s = saturated air vapor pressure at temperature T
 e_a = actual air vapor pressure

The terms e_s/p_a and e_a/p_a are small. Introducing a standard atmospheric pressure will lead to only a trivial error, but save a lot of effort in collecting the required information. Therefore, instead of using Eq. (B-11) to determine the convective term ($\Delta\theta_v$), the following equation will be used:

$$\Delta\theta_v = T (1 + 0.378 \frac{e_s}{1013}) - AT (1 + 0.378 \frac{e_a}{1013}) \quad (B-12)$$

The determination of e_s and e_a can either be read from standard saturation vapor pressure table, or be calculated from the Goff-Gratch formula or its approximating formula (Ref. 28) in the following form

$$e_s \approx 3.8639 [(0.00738 T + 0.8072)^8 + 0.001316 - 0.000019 | 1.8 T + 48 |] \quad (B-13)$$

$$e_a \approx 33.8639 [(0.00738 AT + 0.8072)^8 + 0.001316 - 0.000019 | 1.8AT + 48 |] \quad (B-14)$$

where e_s and e_a are in millibars (mb), and T and AT in $^{\circ}C$.

If the height above ground at which the wind velocity is recorded is known, the data can be transformed to wind velocity at 2 m above ground by using a power law equation. If not, the acquired wind velocity (without any transformation) is used.

When the first term on the right-hand side of Eq. (B-11) is less than the second term, $\Delta\theta_v$ is set equal to zero.

APPENDIX C

Approximation of Water Temperature Profile
at Time of Freeze-over

Lakes in Minnesota are dimictic (Stefan, 1975). They have water surface temperatures above 4°C in summer and below 4°C in winter, and therefore turnover periods occur in spring and late autumn. Water temperatures in these lakes are transformed from an isothermal distribution in the summer and return to an isothermal condition in late fall (26) during which freeze-over of lakes often occurs. The water temperature profile at the time when freeze-over begins is not entirely isothermal from top to bottom, but often bears a form as sketched in Figure C-1. Measured from the water surface, the depth at which water of 4°C is first encountered may differ from one lake to another and from one year to another. Based on available measurements, a thickness of 6 feet for the transition layer has been chosen and its effect incorporated into the model. The formation of the transition layer will advance the freeze-over date as compared to fully isothermal conditions of 0°C.

For lakes having mean depths less than 6 feet, Eq. B-5 can be applied by setting $T_f = 32^\circ\text{F}$, that is, the lakes are isothermal at 32°F (0°C). The resulting equation is

$$t = \frac{1}{w} \arcsin \left\{ \left(\frac{32 - \bar{T} - \frac{I}{m}}{\Delta T_{h=h_1}} \right) \left[\frac{1 + \left(\frac{wQC}{K} h \right)^2}{1 + \left(\frac{wQC}{K} h_1 \right)^2} \right]^{1/2} \right. \\ \left. + \frac{1}{w} \frac{3\pi}{2} + \frac{1}{w} \arctan \left(\frac{wQC}{K} h \right) \right. \quad (C-1)$$

for $h < 6$ ft

At the time when freeze-over begins and for lakes having mean depths greater or equal to 6 feet, the water temperature profile as sketched in Fig. C-2 has been considered. The resulting equation corresponding to the water temperature profile in Fig. C-2 is

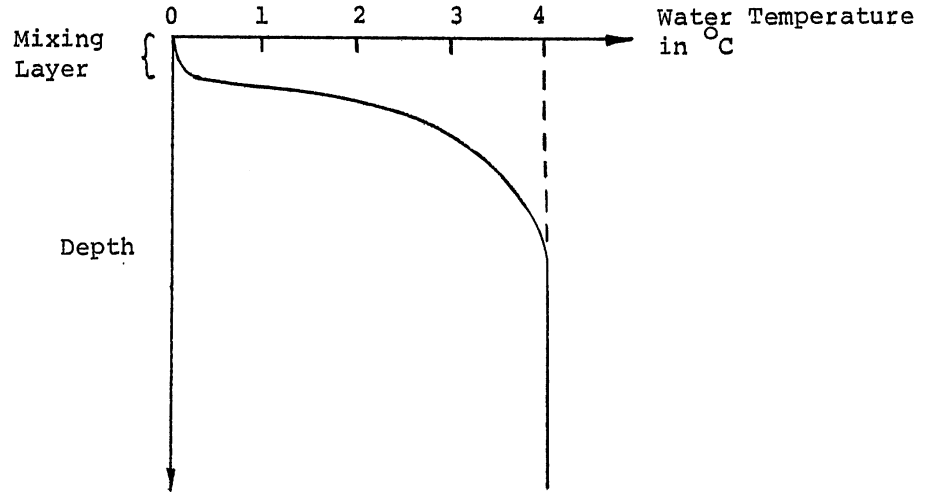


Fig. C-1. General Water Temperature Profile of Dimictic Lakes at Time of Freeze-over.

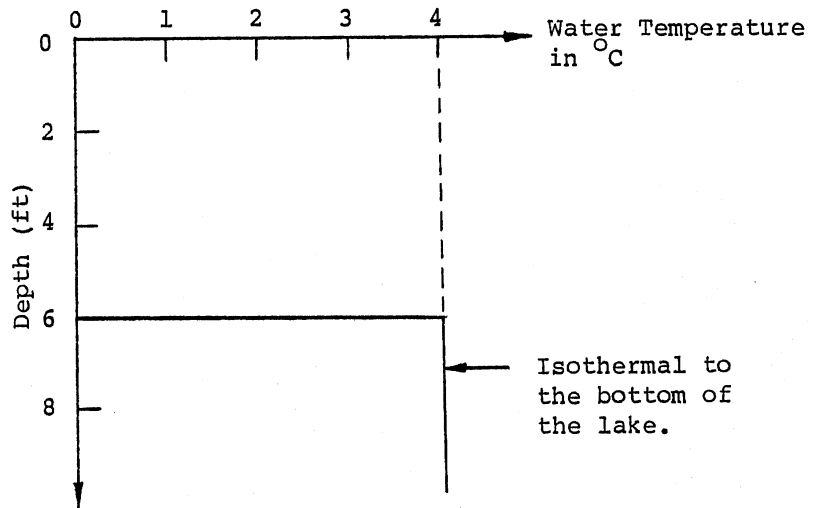


Fig. C-2. Simplified Water Temperature Profile of Dimictic Lakes of Depth ≥ 6 ft at Time of Freeze-over.

$$\begin{aligned}
 t = & \frac{1}{w} \frac{3\pi}{2} + \frac{1}{w} \arctan \left(\frac{w_0 c}{K} h \right) \\
 & + \frac{1}{w} \arcsin \left\{ \left(\frac{39.2 - \bar{T} - \frac{\bar{I}}{m}}{\Delta T_{h=h_1}} \right) \left[\frac{1 + \left(\frac{w_0 c}{K} h \right)^2}{1 + \left(\frac{w_0 c}{K} h_1 \right)^2} \right]^{\frac{1}{2}} \right\} \\
 & + \frac{1}{w} \arcsin \left\{ \left(\frac{32 - \bar{T} - \frac{\bar{I}}{m}}{\Delta T_{h=h_1}} \right) \left[\frac{1 + \left(\frac{w_0 c}{K} 6 \right)^2}{1 + \left(\frac{w_0 c}{K} h_1 \right)^2} \right]^{\frac{1}{2}} \right\} \\
 & - \frac{1}{w} \arcsin \left\{ \left(\frac{39.2 - \bar{T} - \frac{\bar{I}}{m}}{\Delta T_{h=h_1}} \right) \left[\frac{1 + \left(\frac{w_0 c}{K} 6 \right)^2}{1 + \left(\frac{w_0 c}{K} h_1 \right)^2} \right]^{\frac{1}{2}} \right\}
 \end{aligned} \tag{C-2}$$

for $h \geq 6$ ft

A derivation of Eq. (C-2) was given by Fu (1979).

In Eqs. C-1 and C-2, the lowest point of the water temperature cycle is forced to occur on January 15 of each year. In general, this is not necessarily true. Incorporation of a seasonal phase lag into the water temperature cycle was therefore explored. This modification and its resulting effect on prediction of freeze-over dates have been reported in Appendix J.

APPENDIX D

Application of Theory to Minnesota Lakes in Early Fall

If the theory presented is to be used for predictions in early fall, there will not be sufficient weather data available at the time when predictions are made. A method to estimate the unavailable (future) weather data (specifically, air temperatures, relative humidity and wind velocities) is necessary.

Using the least squares method to fit a sine function (with a phase lag) over the available monthly air temperature data, one can estimate the monthly air temperatures for the rest of the year.

It is found that the forecast results are relatively dependent on the air temperature of the month at the end of which predictions are made. In other words, a warmer month will lead to prediction of later freeze-over dates and a cooler month earlier freeze-over dates.

Careful review of the forecast results reveals that, for the northern part of the state, the results generally differ by only one day, whether the forecasts are made at the end of August, September, October, November or December. This is due to the fact that, in general, it is colder in the northern part than in the central and southern part of the state in late fall and winter. For northern Minnesota, the calculated water temperatures always reach 32°F in November and December and are close to 32°F in October. The predicted air temperatures in September are usually in reasonably good agreement with the observed records. Consequently, the forecast results made at the end of August, September, October, November or December are practically the same for northern Minnesota. The same conclusion does not hold true for central and southern Minnesota. The calculated water temperatures reach 32°F only in December for central and southern Minnesota. Thus, the results are relatively dependent upon the predicted air temperatures for the periods in which no meteorological data are available. Differences of up to one week between forecasts made at the end of August and November have been found. The results are practically the same if forecasts are made at the end of November and December.

Relative humidity and wind velocity for a specific location do not follow any easily describable cycle. It is therefore more practical to directly predict

K values, if feasible. K values do not follow a sinusoidal cycle. If average monthly K values determined from a long period of record can be used in place of the unknown K values and, at the same time, do not introduce appreciable deviations, the task will be simpler. This possibility has been investigated and the results follow.

The average monthly K values and the corresponding standard deviations for the months of September through December at five Minnesota weather stations have been calculated and are tabulated in Table D-1. These values were determined by using corresponding weather records ranging from 18 to 26 years. K values for other locations were obtained by interpolation between two of the five known stations. The results are tabulated in Table D-2.

Making predictions too early in the year will give results no better than mere guesses. The earliest time a prediction should be made is after August. The peak monthly air temperature will have been passed by the end of August and errors in the air temperature curve fitting processes will be reasonable. Only monthly average K values from September through December are listed in Tables D-1 and D-2.

Case studies conducted by Fu (1979) show that the use of mean monthly K values produces an error of less than one day in the prediction of the freeze-over date.

TABLE D-1

Statistics of Computed K Values

<u>Month</u>		Intl. Falls	Duluth Airport	St. Cloud	Mpls./ St.Paul	Roch- ester
September	No. of years	25	26	19	26	26
	Average K	95.4	104.9	125.1	124.2	142.6
	Std. Dev.	14.4	16.2	12.2	12.0	15.0
	Maximum	119.0	130.3	142.6	142.2	164.7
	Minimum	70.0	72.4	105.8	103.2	105.1
October	No. of years	24	25	18	25	25
	Average K	79.2	89.3	107.0	100.3	116.8
	Std. Dev.	11.4	14.8	12.5	11.4	14.1
	Maximum	107.5	120.2	127.7	122.5	137.2
	Minimum	58.8	57.4	87.4	75.1	91.0
November	No. of years	24	25	17	25	25
	Average K	92.3	98.0	90.6	96.1	108.7
	Std. Dev.	10.1	18.0	8.2	10.5	7.8
	Maximum	118.2	128.2	106.7	120.8	123.1
	Minimum	73.5	55.9	75.6	76.7	95.1
December	No. of years	25	25	17	25	25
	Average K	97.1	105.0	89.7	98.2	111.4
	Std. Dev.	7.8	12.7	8.1	8.1	11.4
	Maximum	113.7	126.9	103.3	112.5	126.4
	Minimum	86.2	76.7	75.7	75.1	84.6

*K is in BTU ft⁻² day⁻¹ °F⁻¹

TABLE D-2

Interpolated Average Monthly K

Month		Winton Power Plant	Duluth Airport	Sandy Lake Dam Libby	Brainerd	Alexandria Airport	St. Cloud Airport	Mpls./ St.Paul	St. Peter	Waseca
Sept.	Average K	100.2	104.9	104.9	115.0	120.1	125.1	124.2	133.4	133.4
	Std. Dev. (S.D.)	15.3	16.2	16.2	14.2	13.2	12.2	12.0	13.5	13.5
	Average K + S.D.	115.5	121.1	121.1	129.2	133.3	137.3	136.2	146.9	146.9
	Average K - S.D.	84.9	88.7	88.7	100.8	106.9	112.9	112.2	119.9	119.9
Oct.	Average K	84.3	89.3	89.3	98.2	102.6	107.0	100.3	108.6	108.6
	Std. Dev. (S.D.)	13.1	14.8	14.8	13.7	13.1	12.5	11.4	12.8	12.8
	Average K + S.D.	97.4	104.1	104.1	111.9	115.7	119.5	111.7	121.4	121.4
	Average K - S.D.	71.2	74.5	74.5	84.5	89.5	94.5	88.9	95.8	95.8
Nov.	Average K	95.2	98.0	98.0	94.3	92.5	90.6	96.1	102.4	102.4
	Std. Dev. (S.D.)	14.1	18.0	18.0	13.1	10.7	8.2	10.5	9.2	9.2
	Average K + S.D.	109.3	116.0	116.0	107.4	103.2	98.8	106.6	111.6	111.6
	Average K - S.D.	81.1	80.0	80.0	81.2	81.8	82.4	85.6	93.2	93.2
Dec.	Average K	101.1	105.0	105.0	97.4	93.6	89.7	98.2	104.8	104.8
	Std. Dev. (S.D.)	10.3	12.7	12.7	10.4	9.3	8.1	8.1	9.8	9.8
	Average K + S.D.	111.4	117.7	117.7	107.8	102.9	97.8	106.3	114.6	114.6
	Average K - S.D.	90.8	92.3	92.3	87.0	84.3	81.6	90.1	95.0	95.0

K is in $\text{BTU ft}^{-2} \text{ dat}^{-1} \text{ } ^\circ\text{F}^{-1}$

APPENDIX E

Hindcast of Past Freeze-over Dates Using Measured Annual Weather Cycles

Air temperature data from weather stations identified in Fig. 10 were obtained.

Necessary coefficients to generate water temperature were chosen from Table B-1. Water temperatures were calculated on a monthly basis using Eq. (B-1) in the form

$$T_i = (A_2 - dA_1) + (B_2 - dB_1) \sin \frac{2\pi i}{12} + (C_2 - dC_1) \cos \frac{2\pi i}{12} + dAT_i \quad (E-1)$$

where $i = 1, \dots, 12$

The main reason for calculating monthly water temperatures instead of daily water temperatures is to minimize computational time.

It is vital to point out that the coefficients listed in Table B-1 were determined from regression analysis by considering records of daily water temperature in periods of non-freezing water temperatures. It is therefore necessary to reset the generated water temperature to 32°F whenever it is less than 32°F. $\Delta \bar{T}$ and $\Delta T_{h=h_1}$ have to be determined from the synthetic monthly water temperatures before Eq. (C-1) or (C-2) can be used to predict onset of freeze-over. \bar{T} is the arithmetic mean determined from all generated monthly water temperatures. Then, by ignoring all freezing water temperatures (32°F) except the last one occurring in late spring and the first one occurring in late fall, a sine curve with a phase lag is fitted over the remaining water temperature values using the least squares method. $\Delta T_{h=h_1}$ is equal to the difference between the peak of the sine curve and \bar{T} . Ignoring all freezing water temperatures tends to over-estimate $\Delta T_{h=h_1}$, while including all freezing water temperatures tends to underestimate $\Delta T_{h=h_1}$.

A typical synthetic water temperature cycle is shown in Fig. E-1. Following the guideline set above, water temperatures from March through November are used in the curve fitting.

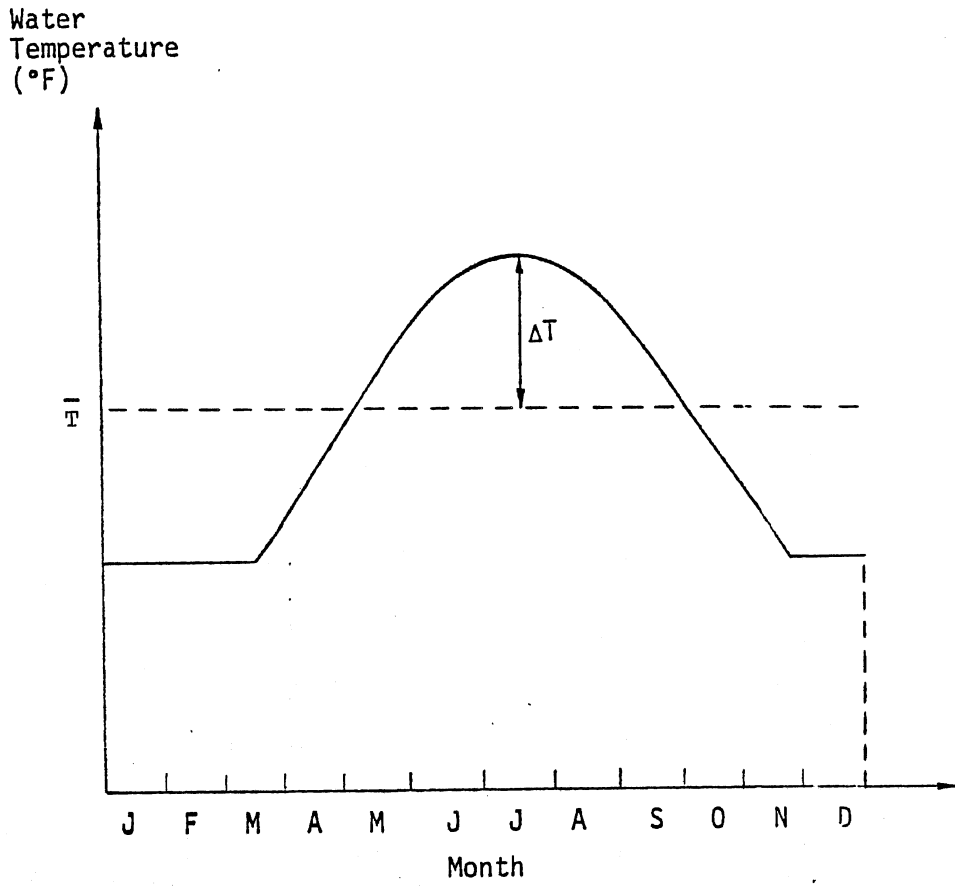


Figure E-1 - Determination of \bar{T} and ΔT .

The theory was then applied to hindcast the freeze-over dates of Minnesota lakes from fall, 1972 through fall, 1977. Whenever available, information extracted from images taken by LANDSAT-1 and 2 was used for comparison.

Little information on mean depths of lakes is available because mean depths cannot be accurately and easily measured. Whenever mean depth was not available, a median depth given in Ref. 3 has been used in the calculation.

Taken into account the accuracy in measuring the mean depth (or median depth) of a lake and the accuracy in interpreting data from imagery, the hindcast results on freeze-over agree with the satellite data reasonably well. In only two cases, the predicted dates were about 10 days late for lakes of 45 to 50 feet deep, while in most cases the deviation is between 1 and 5 days. The theory presented herein is well suited for shallow lakes. For a deep lake, the assumption of well-mixed condition does not hold very well.

It is also noteworthy that lake ice 1 to 2 days old may not be detected by MSS images because the ice is still very thin.

APPENDIX F

Numerical Example of Forecasting Freeze-over Date

As a numerical example, we want to predict the freeze-over date of a lake having a mean depth of 20 ft. at the end of September 1975. The lake has no artificial heat input ($I = 0$). The lake is located in Minneapolis/St. Paul.

- (i) The available meteorological data collected from Minneapolis/St. Paul Weather station are:

	J	F	M	A	M	J	J	A	S
	14.5	15.5	22.1	38.9	60.9	68.8	76.3	71.7	57.7
Air Temp. ($^{\circ}\text{F}$)	79	71	75	70	61	69	59	67	70
Wind (mph)	9.5	9.7	10.9	11.2	9.7	8.9	9.0	7.8	8.4

- (ii) Air temperatures for October, November and December are calculated to be 44.9°F , 28.9°F , 16.7°F , respectively.

- (iii) From Table 3,

$$A_1 = 47.48^{\circ}\text{F}$$

$$A_2 = 49.48^{\circ}\text{F}$$

$$B_1 = -8.29^{\circ}\text{F}$$

$$B_2 = -11.75^{\circ}\text{F}$$

$$C_1 = -24.11^{\circ}\text{F}$$

$$C_2 = -23.01^{\circ}\text{F}$$

$$d = 0.155$$

$$h_1 = 10 \text{ ft.}$$

Using equation (22), the calculated water temperatures are:

J	F	M	A	M	J	J	A	S	O	N	D
32.0	32.0	35.1	48.7	63.0	72.1	75.9	71.9	61.5	48.5	35.1	32.0

$$\begin{aligned} \text{(iv) } \bar{T} &= \frac{1}{12} (32.0 + 32.0 + 35.1 + 48.7 + 63.0 + 72.1 + 75.9 + \\ &\quad 71.9 + 61.5 + 48.5 + 35.1 + 32.0) \\ &= 50.7^{\circ}\text{F} \end{aligned}$$

- (v) Water Temperatures from February through December are used in least squares fitting.

$$\Delta T_1 = 12.0^\circ\text{F}$$

$$\Delta T_2 = 21.3^\circ\text{F}$$

$$\bar{T}_0 = 50.1^\circ\text{F}$$

(vi) $\Delta T_{h=h_1} = \Delta T_1 / [\cos(\arctan|\Delta T_2/\Delta T_1|)] + \bar{T} - \bar{T}_0 = 32.9^\circ\text{F}$

- (vii) Using Eqs. (11) through (15) and Eqs. (17 through (19), bulk surface heat transfer coefficients from January through September are

	J	F	M	A	M	J	J	A	S
K(BTU/ ft ² -day)	96.9	97.7	103.3	117.2	126.8	147.8	143.7	126.0	117.2

- (viii) From Table 4, the normal bulk surface heat transfer coefficients for October, November and December are 100.3, 96.1 and 98.2 BTU/ft²-day, respectively.

- (ix) Mean annual bulk surface heat transfer coefficient

$$= \frac{1}{12} (96.9 + 97.7 + 103.8 + 117.2 + 126.8 + 147.8 + 143.7 + 126.0 + 117.2 + 100.3 + 96.1 + 98.2)$$

$$= 114.3 \text{ BTU ft}^{-2} \text{ day}^{-1}$$

- (x) Since the lake is 20 ft deep, Eq. (10) should be used. Thus $t = 336.8$, which is December 2, 1975.

- (xi) $h_{\text{lim}} = 87 \text{ ft.}$

APPENDIX G

COMPUTER READOUT

```

1      C      .....THIS COMPUTER PROGRAM HINCASTS OR FORECASTS THE BEGINN-.....
2      C      .....-ING DATES OF FREEZE-OVER OF LAKES IN THE VICINITY OF A .....
3      C      .....WEATHER STATION FROM WHICH METEROLOGICAL DATA ARE ACQUI-.....
4      C      .....-RED. .....
5      C
6      C
7      PROGRAM ICE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
8      DIMENSION AT(12),WT(12),TD(12),WIND(12),RH(12),BULKK(12),TITLE(25)
9      +,SEASON(5),A(50,51),HMONTH(12)
10     DATA HINTRVL/5./
11     DATA (HMONTH(I),I=9,12)/3HSEP,3HOCT,3HNOV,3HDEC/
12     C
13     C      .....INPUT NAME OF WEATHER STATION.....
14     10     READ(5,1000,END=999) (TITLE(I),I=1,25)
15     1000   FORMAT(25A1)
16     C
17     C      .....INPUT SEASON AND YEAR.....
18     C      .....ODD=1. FOR 366 DAYS IN A YEAR, 0. OTHERWISE.....
19     READ(5,1001) (SEASON(I),I=1,5),IYEAR,ODD
20     1001   FORMAT(5A1,5X,I4,6X,F2.0)
21     C
22     C      .....INPUT COEFFICIENTS FOR WATER TEMPERATURE GENERATION.....
23     READ(5,1002) A1,A2,B1,B2,C1,C2,D
24     1002   FORMAT(7F10.3)
25     C
26     C      .....INPUT CORRESPONDING LAKE DEPTH AND NUMBER OF MONTHS OF .....
27     C      .....AVAILABLE METEROLOGICAL DATA. ....
28     READ(5,1006) H1, NUM
29     1006   FORMAT(F4.1,1X,I2)
30     C
31     C      .....INPUT AVAILABLE METEROLOGICAL DATA, .....
32     C      ..... AT = AIR TEMPERATURE IN DEGREE FARENHEITS .....

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33 C ..... WIND = WIND VELOCITY IN MPH .....
34 C ..... RH = RELATIVE HUMIDITY IN PERCENT .....
35 READ(5,1003) (AT(I),I=1,NUM)
36 1003 FORMAT(12F6.1)
37 READ(5,1003) (WIND(I),I=1,NUM)
38 READ(5,1003) (RH(I),I=1,NUM)
39 C
40 C .....PRINT OUT INPUT INFORMATION.....
41 WRITE(6,2000) (TITLE(I),I=1,25)
42 2000 FORMAT(1H1,5(/),10X,25A1)
43 IF(ODD.EQ.1.) GO TO 11
44 WRITE(6,2001) (SEASON(I),I=1,5),IYEAR
45 2001 FORMAT(10X,5A1,1X,I4,5X,*365 DAYS IN THIS YEAR*)
46 GO TO 12
47 11 WRITE(6,2002) (SEASON(I),I=1,5),IYEAR
48 2002 FORMAT(10X,5A1,1X,I4,5X,*366 DAYS IN THIS YEAR*)
49 12 WRITE(6,2003) A1,A2,B1,B2,C1,C2,D,H1
50 2003 FORMAT(3(/),10X,*A1 =*,F6.2,/,10X,*A2 =*,F6.2,/,10X,*B1 =*,F6.2,/,
51 /10X,*B2 =*,F6.2,/,10X,*C1 =*,F6.2,/,10X,*C2 =*,F6.2,/,11X,*D =*,F7
52 /.3,/,10X,*H1 =*,F6.2,3(/))
53 C
54 C .....INPUT MONTHLY AVERAGE BULK SURFACE HEAT TRANSFER COEFF- .....
55 C .....-ICIENTS( IN BTU PER SQ. FT. PER DAY PER DEGREE FARENHE-.....
56 C .....-ITS ) FOR FORECASTING PURPOSE. ....
57 IF(NUM.EQ.12) GO TO 65
58 READ(5,1004) (BULK(I),I=NUM+1,12)
59 1004 FORMAT(4F6.1)
60 LOOK1=1
61 LOOK2=NUM
62 C
63 C .....PREDICT MONTHLY AIR TEMPERATURES FOR THE REMAINING MONTHS....
64 CALL MATRIX(DIFF,SUMSIN,SUMCOS,SINSQ,COSSQ,PRODUCT,SUMAT,ATSIN,ATC

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```

65      +OS,NUM,AT,LOOK1,LOOK2)
66      CALL GAUSS(DIFF,SUMSIN,SUMCOS,SINSQ,COSSQ,PRODUCT,SUMAT,ATSIN,ATCO
67      +S,AVGAT,DAT1,DAT2)
68      DO 113 I=NUM+1,12
69      AT(I)=AVGAT-DAT1*SIN(0.5236*FLOAT(I))-DAT2*COS(0.5236*FLOAT(I))
70      113 CONTINUE
71      WRITE(6,2500)
72      2500 FORMAT(3(/))
73      C
74      C      .....START CALCULATION OF MONTHLY WATER TEMPERATURES.....
75      65  ACE=A2-D*A1
76          B=B2-D*B1
77          C=C2-D*C1
78          CALL WATER(ACE,B,C,D,AT,WT,TD,WIND,AWT,DWT,HK,RH,NUM,BULKK)
79          OMEG=2.*3.14159*62.4/(365.*HK)
80          WRITE(6,2004) IYEAR
81      2004  FDRMAT(5(/),35X,*MEDIAN DEPTH*,12X,* DAY OF YEAR *,15X,* DATE *
82          //35X,*OF LAKE (FT.)*,12X,* (JAN 1 BEING 1)*,15X,*(*,I4,*)*/35X,*---
83          /-----*,12X,*-----*,15X,*-----*)
84      C
85      C      .....START CALCULATION OF BEGINNING DATES OF FREEZE-OVER.....
86          DELWT=DWT*(1.+(OMEG*H1)**2)**0.5
87          TEMP3=ABS(ASIN((32.-AWT)*(1.+(OMEG*6.)**2)**0.5/DELWT)-ASIN((39.2
88          +-AWT)*(1.+(OMEG*6.)**2)**0.5/DELWT))
89          COUNT=0.
90      46  H=HINTRVL*COUNT
91          IF(H.GE.6.) GO TO 47
92          FREEZE=(1.5*3.14159-ASIN(((32.-AWT)*(1.+(OMEG*H)**2)**0.5)/DELWT)+
93          +ATAN(OMEG*H))*365./(2.*3.14159)
94          IF(FREEZE.GT.304.+ODD) GO TO 15
95          L=10
96          IDATE=FREEZE-273.-ODD

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97         IF(IDATE.NE.0) GO TO 29
98         IDATE=30
99         L=9
100        29  WRITE(6,2024) H,FREEZE,HMONTH(L),IDATE
101        2024 FORMAT(40X,F3.0,22X,F5.1,20X,A3,1X,I2)
102        COUNT=COUNT+1.
103        GO TO 46
104        49  H=HINTRVL*COUNT
105        47  TDAY=(1.5*3.14159-ASIN(((39.2-AWT)*(1.+(OMEG*H)**2)**0.5)/DELWT)+A
106        +TAN(OMEG*H)+TEMP3)*365./(2.*3.14159)
107        IF(TDAY.GT.304.+ODD) GO TO 64
108        L=10
109        IDATE=TDAY-273.-ODD
110        IF(IDATE.NE.0) GO TO 48
111        IDATE=30
112        L=9
113        48  WRITE(6,2024) H,TDAY, HMONTH(L),IDATE
114        IF(H.GE.50.) GO TO 24
115        COUNT=COUNT+1.
116        GO TO 49
117        78  H=HINTRVL*COUNT
118        IF(H.GE.6.) GO TO 79
119        FREEZE=(1.5*3.14159-ASIN(((32.-AWT)*(1.+(OMEG*H)**2)**0.5)/DELWT)+
120        +ATAN(OMEG*H))*365./(2.*3.14159)
121        15  IF(FREEZE.GT.334.+ODD) GO TO 16,
122        L=11
123        IDATE=FREEZE-304.-ODD
124        IF(IDATE.NE.0) GO TO 32
125        IDATE=31
126        L=10
127        32  WRITE(6,2024) H,FREEZE,HMONTH(L),IDATE
128        COUNT=COUNT+1.

```



```

129      GO TO 78
130      80  H=HINTRVL*COUNT
131      79  TDAY=(1.5*3.14159-ASIN(((39.2-AWT)*(1.+(OMEG*H)**2)**0.5)/DELWT)+A
132      +TAN(OMEG*H)+TEMP3)*365./(2.*3.14159)
133      64  IF(TDAY.GT.334.+ODD) GO TO 71
134      L=11
135      IDATE=TDAY-304.-ODD
136      IF(IDATE.NE.0) GO TO 77
137      IDATE=31
138      L=10
139      77  WRITE(6,2024) H,TDAY, HMONTH(L),IDATE
140      IF(H.GE.50.) GO TO 24
141      COUNT=COUNT+1.
142      GO TO 80
143      87  H=HINTRVL*COUNT
144      IF(H.GE.6.) GO TO 88
145      FREEZE=(1.5*3.14159-ASIN(((32.-AWT)*(1.+(OMEG*H)**2)**0.5)/DELWT)+
146      +ATAN(OMEG*H))*365./(2.*3.14159)
147      16  IF(FREEZE.GT.365.+ODD) GO TO 22
148      L=12
149      IDATE=FREEZE-334.-ODD
150      IF(IDATE.NE.0) GO TO 35
151      IDATE=30
152      L=11
153      35  WRITE(6,2024) H,FREEZE,HMONTH(L),IDATE
154      COUNT=COUNT+1.
155      GO TO 87
156      86  H=HINTRVL*COUNT
157      88  TDAY=(1.5*3.14159-ASIN(((39.2-AWT)*(1.+(OMEG*H)**2)**0.5)/DELWT)+A
158      +TAN(OMEG*H)+TEMP3)*365./(2.*3.14159)
159      71  IF(TDAY.GT.366.+ODD) GO TO 23
160      L=12

```

```

161          IDATE=TDAY-334.-000
162          IF(IDATE.NE.0) GO TO 75
163          IDATE=30
164          L=11
165          75  WRITE(6,2024) H,TDAY, HMONTH(L),IDATE
166          IF(H.GE.50.) GO TO 24
167          COUNT=COUNT+1.
168          GO TO 86
169          22  WRITE(6,2011) IYEAR
170          2011 FORMAT(3(/),10X,*ALL LAKES REMAINED OPEN AT THE END OF DEC. 31.*,I
171          /4)
172          GO TO 24
173          23  H=H-2.5
174          WRITE(6,2012) H,IYEAR
175          2012 FORMAT(3(/),10X,*LAKES HAVING MEDIAN DEPTH ABOVE APPROXIMATELY *,F
176          /3.0,* FEET REMAINED OPEN AT THE END OF DEC. 31.*,I4)
177          C
178          C      .....GO BACK FOR A NEW JOB, IF ANY.....
179          24  GO TO 10
180          999 STOP
181          END

```

```

1          SUBROUTINE MATRIX(DIFF,SUMSIN,SUMCOS,SINSQ,COSSQ,PRODUCT,SUMWT,WTS
2          +IN,WTCOS,N,WT,LOOK1,LOOK2)
3          C
4          C      .....THIS SUBROUTINE SETS UP THE ELEMENTS OF THE MATRIX FOR .....
5          C      .....THE CONSTANT TERM AND THE COEFFICIENTS OF THE FIRST .....

```

```

6      C      .....HARMONIC OF A FOURIER SERIES.
7      DIMENSION WT(12)
8      SUMSIN=0.
9      SUMCOS=0.
10     SINSQ=0.
11     COSSQ=0.
12     PRODUCT=0.
13     DIFF=FLOAT(N)
14     SUMWT=0.
15     WTSIN=0.
16     WTCOS=0.
17     OM=2.*3.14159/12.
18     DO 104 I=LOOK1,LOOK2
19     TIME=FLOAT(I)
20     TRIGSIN=SIN(OM*TIME)
21     TRIGCOS=COS(OM*TIME)
22     SUMSIN=SUMSIN+TRIGSIN
23     SUMCOS=SUMCOS+TRIGCOS
24     SINSQ=SINSQ+TRIGSIN**2
25     COSSQ=COSSQ+TRIGCOS**2
26     PRODUCT=PRODUCT+TRIGSIN*TRIGCOS
27     SUMWT=SUMWT+WT(I)
28     WTSIN=WTSIN+WT(I)*TRIGSIN
29     WTCOS=WTCOS+WT(I)*TRIGCOS
30     104 CONTINUE
31     RETURN
32     END

```

.....

```

1      SUBROUTINE WATER(ACE,B,C,D,AT,WT,TD,WIND,AWT,DWT,HK,RH,NUM,BULKK)
2      C
3      C      .....THIS SUBROUTINE CALCULATES WATER TEMPERATURES AND RETUR-.....
4      C      .....-NS THE VALUES OF THE AVERAGE ANNUAL WATER TEMPERATURE .....
5      C      .....AND THE AMPLITUDE OF THE WATER TEMPERATURE CYCLE OBTAIN-.....
6      C      .....-ED FROM LEAST SQUARES FITTING. ....
7      DIMENSION AT(12),WT(12),TD(12),WIND(12),BULKK(12),A(50,51),RH(12)
8      REAL MONTH
9      WRITE(6,2013)
10     2013  FORMAT(25X,*JAN.*,3X,*FEB.*,3X,*MAR.*,3X,*APR.*,3X,*MAY*,4X,*JUN.*
11           /,3X,*JUL.*,3X,*AUG.*,3X,*SEP.*,3X,*OCT.*,3X,*NOV.*,3X,*DEC.*,/)
12           WRITE(6,2014) (AT(I),I=1,12)
13     2014  FORMAT(10X,*AIR TEMP.*,5X,12(F5.1,2X))
14           MONTH=0.
15           SWT=0.
16           DO 100 I=1,12
17             MONTH=MONTH+1.
18             WT(I)=ACE+B*SIN(0.5236*MONTH)+C*COS(0.5236*MONTH)+D*AT(I)
19             IF(WT(I).LT.32.) WT(I)=32.
20             SWT=SWT+WT(I)
21     100   CONTINUE
22           WRITE(6,2015) (WT(I),I=1,12)
23     2015  FORMAT(8X,*WATER TEMP.*,5X,12(F5.1,2X))
24     C
25     C      .....CALCULATION OF BULK SURFACE HEAT TRANSFER COEFFICIENTS .....
26           CALL KCDEFF(WT,TD,WIND,HK,RH,AT,NUM,BULKK)
27     C
28     C      .....CALCULATE AVERAGE WATER TEMPERATURE, AWT.....
29           AWT=SWT/12.
30           DO 101 I=1,6
31             IF(WT(I).GT.32.) GO TO 27
32     101   CONTINUE

```

```

33      27      LOOK1=I-1
34          DO 103 I=9,12
35          IF(WT(I).EQ.32.) GO TO 28
36      103      CONTINUE
37      28      LOOK2=I
38          NO=LOOK2-LOOK1+1
39      C
40      C      .....SET UP EQUATION MATRIX.....
41          CALL MATRIX(DIFF,SUMSIN,SUMCOS,SINSQ,COSSQ,PRODUCT,SUMWT,WTSIN,WTC
42          +OS,NO,WT,LOOK1,LOOK2)
43      C
44      C      .....SOLVE EQUATION MATRIX.....
45          CALL GAUSS(DIFF,SUMSIN,SUMCOS,SINSQ,COSSQ,PRODUCT,SUMWT,WTSIN,WTCO
46          /S,AVGWT,DWT1,DWT2)
47      C
48      C      .....CALCULATE AMPLITUDE OF WATER TEMPERATURE CYCLE, DWT.....
49          DELTA=ATAN(DWT2/DWT1)
50          DWT3=DWT1/COS(DELTA)
51          DWT=AVGWT-AWT+DWT3
52          WRITE(6,2016) AWT,DWT
53      2016      FORMAT(3(/),20X,*AVERAGE WATER TEMP. FOR THIS YEAR =*,F5.1/23X,*AM
54          /PLITUDE OF WATER TEMP. CYCLE =*,F5.1)
55          WRITE(6,2999) HK
56      2999      FORMAT(27X,*AVERAGE BULK K COEFFICIENT =*,F5.1)
57          RETURN
58          END

```

```

1          SUBROUTINE KCOEFF(WT,TD,WIND,HK,RH,AT,NUM,BULKK)
2          C
3          C      .....THIS SUBROUTINE CALCULATES THE AVERAGE MONTHLY BULK      .....
4          C      .....SURFACE HEAT TRANSFER COEFFICIENTS.                      .....
5          DIMENSION WT(12),TD(12),WIND(12),RH(12),BULKK(12),AT(12)
6          DO 112 I=1,NUM
7          112  TD(I)=(172.8+0.9*AT(I))*(RH(I)/100.)*0.125+0.1*AT(I)-172.8
8              WRITE(6,2017) (TD(I),I=1,NUM)
9          2017  FORMAT(10X,*DEW POINT*,5X,12(F5.1,2X))
10             WRITE(6,2034) (RH(I),I=1,NUM)
11          2034  FORMAT(9X,*REL. HUMD.*,5X,12(F5.1,2X))
12             WRITE(6,2018) (WIND(I),I=1,NUM)
13          2018  FORMAT(10X,*WIND(MPH)*,5X,12(F5.1,2X))
14             SHK=0.
15             DO 102 I=1,NUM
16                 T=(WT(I)+TD(I))/2.
17                 BETA=0.255-0.0085*T+0.000204*T**2
18                 ES=33.8639*((0.00738*(5.*WT(I)/9.-32.)+0.8072)**8-0.000019*ABS(WT(
19 +I)-9.6)+0.001316)
20                 EA=RH(I)*0.338639*((0.00738*(5.*AT(I)/9.-32.)+0.8072)**8-0.000019*
21 +ABS(AT(I)-9.6)+0.001316)
22                 FTWS=(460.+WT(I))*(1.+0.378*ES/1013.)
23                 FTAS=(460.+AT(I))*(1.+0.378*EA/1013.)
24                 IF(FTWS.LE.FTAS) DCONV=0.
25                 IF(FTWS.GT.FTAS) DCONV=22.*(FTWS-FTAS)**0.3333
26                 FW=14.*WIND(I)+DCONV
27                 BULKK(I)=15.7+(BETA+0.26)*FW
28          102  CONTINUE
29             WRITE(6,2019) (BULKK(I),I=1,12)
30          2019  FORMAT(13X,*BULK K*,5X,12(F5.1,2X))
31             DO 114 I=1,12
32                 SHK=SHK+BULKK(I)

```

```

33      114 CONTINUE
34      HK=SHK/12.
35      RETURN
36      END

```

```

1          SUBROUTINE GAUSS(DIFF,SUMSIN,SUMCOS,SINSQ,COSSQ,PRODUCT,SUMWT,WTSI
2          /N,WTCOS,AVGWT,DWT1,DWT2)
3
4          C      .....THIS SUBROUTINE SOLVES A SET OF SIMULTANEOUS LINEAR      .....
5          C      .....EQUATIONS BY GAUSS-JORDAN ELIMINATION METHOD.          .....
6          DIMENSION A(50,51)
7          DATA N,M,EPS/3,4,1.0E-10/
8          NPLUSM=N+M
9          WRITE(6,2020), N,M,EPS,N,NPLUSM
10         2020 FORMAT(10(/),9H N      = , I8 / 9H M      = , I8 / 9H EPS      = ,
11         / 1PE13.1/22H0      A(1,1)...A( , I2, 1H, , I2, 1H) / 1H )
12         A(1,1)=DIFF
13         A(1,2)=-SUMSIN
14         A(1,3)=-SUMCOS
15         A(1,4)=SUMWT
16         A(2,1)=SUMSIN
17         A(2,2)=-SINSQ
18         A(2,3)=-PRODUCT
19         A(2,4)=WTSIN
20         A(3,1)=SUMCOS
21         A(3,2)=-PRODUCT
22         A(3,3)=-COSSQ

```

```

23      A(3,4)=WTCOS
24      DO 105 I=1,N
25      DO 106 J=N+2,NPLUSM
26      A(I,J)=0.
27      IF(I+N+1.EQ.J) A(I,J)=1.
28      106 CONTINUE
29      105 CONTINUE
30      DO 107 I=1,N
31      WRITE(6,2021) (A(I,J),J=1,NPLUSM)
32      2021 FORMAT(1H,7F13.7)
33      107 CONTINUE
34      C
35      C      .....BEGIN ELIMINATION PROCEDURE.....
36      DETER=1.
37      DO 108 K=1,N
38      C      .....UPDATE THE DETERMINANT VALUE.....
39      DETER=DETER*A(K,K)
40      C
41      C      .....CHECK FOR PIVOT ELEMENT TOO SMALL.....
42      IF(ABS(A(K,K)).GT.EPS) GO TO 25
43      WRITE(6,2022)
44      2022 FORMAT(37HOSMALL PIVOT - MATRIX MAY BE SINGULAR)
45      GO TO 26
46      C
47      C      .....NORMALIZE THE PIVOT ROW.....
48      25      KP1=K+1
49      DO 109 J=KP1,NPLUSM
50      109      A(K,J)=A(K,J)/A(K,K)
51      A(K,K)=1.
52      C
53      C      .....ELIMINATE K(TH) COLUMN ELEMENTS EXCEPT FOR PIVOT.....
54      DO 108 I=1,N

```



```

55      IF(I.EQ.K.OR.A(I,K).EQ.0.) GO TO 108
56      DO 110 J=KP1,NPLUSM
57      110  A(I,J)=A(I,J)-A(I,K)*A(K,J)
58          A(I,K)=0.
59      108  CONTINUE
60      C
61          WRITE(6,2023) DETER,N,NPLUSM
62      2023  FORMAT(9HODETER = ,E14.6/22H0
63          /  I2, 1H) / 1H )
64          DO 111 I=1,N
65      111  WRITE(6,2021) (A(I,J),J=1,NPLUSM)
66          AVGWT=A(1,N+1)
67          DWT1=A(2,N+1)
68          DWT2=A(3,N+1)
69      26   RETURN
70      END

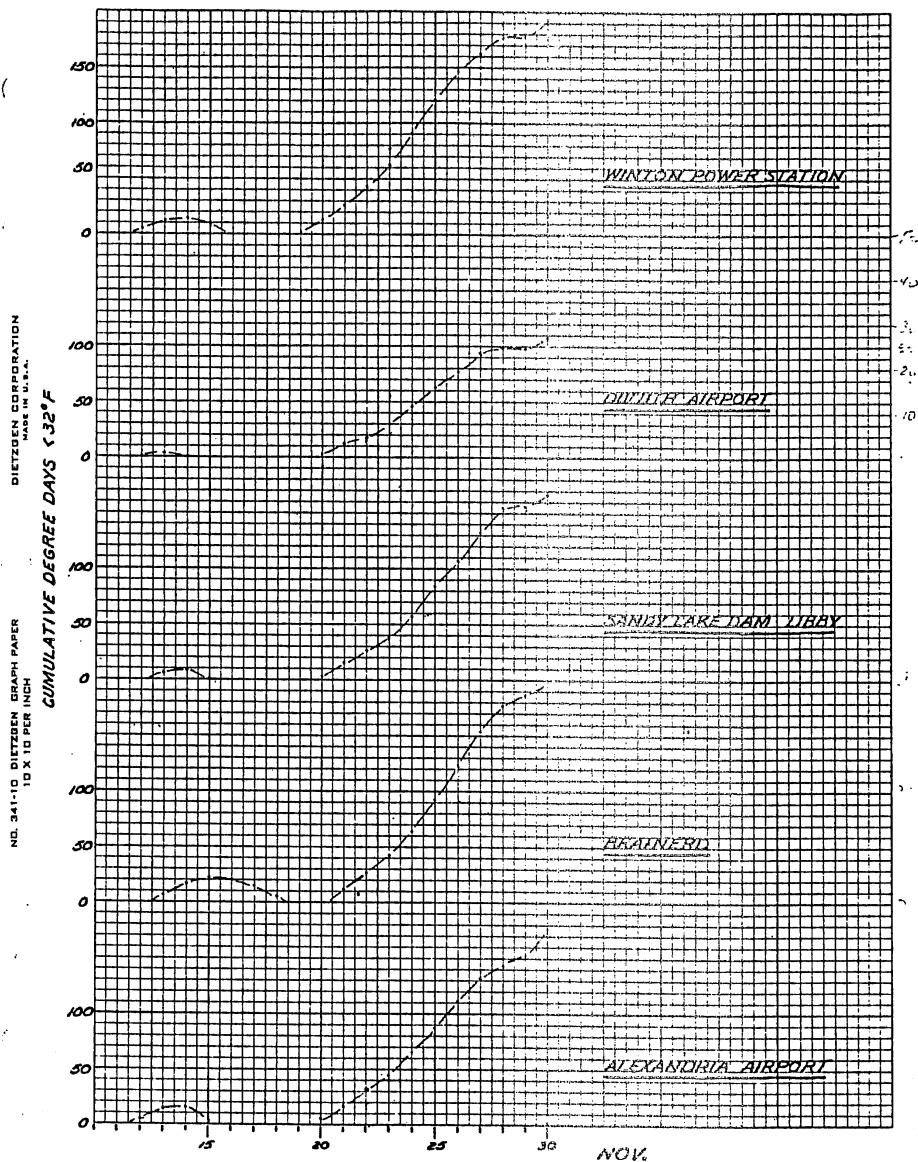
```

A(1,1)...A(, I2, 1H,,

APPENDIX H

CUMULATIVE DEGREE FREEZING DAYS

FALL 1975

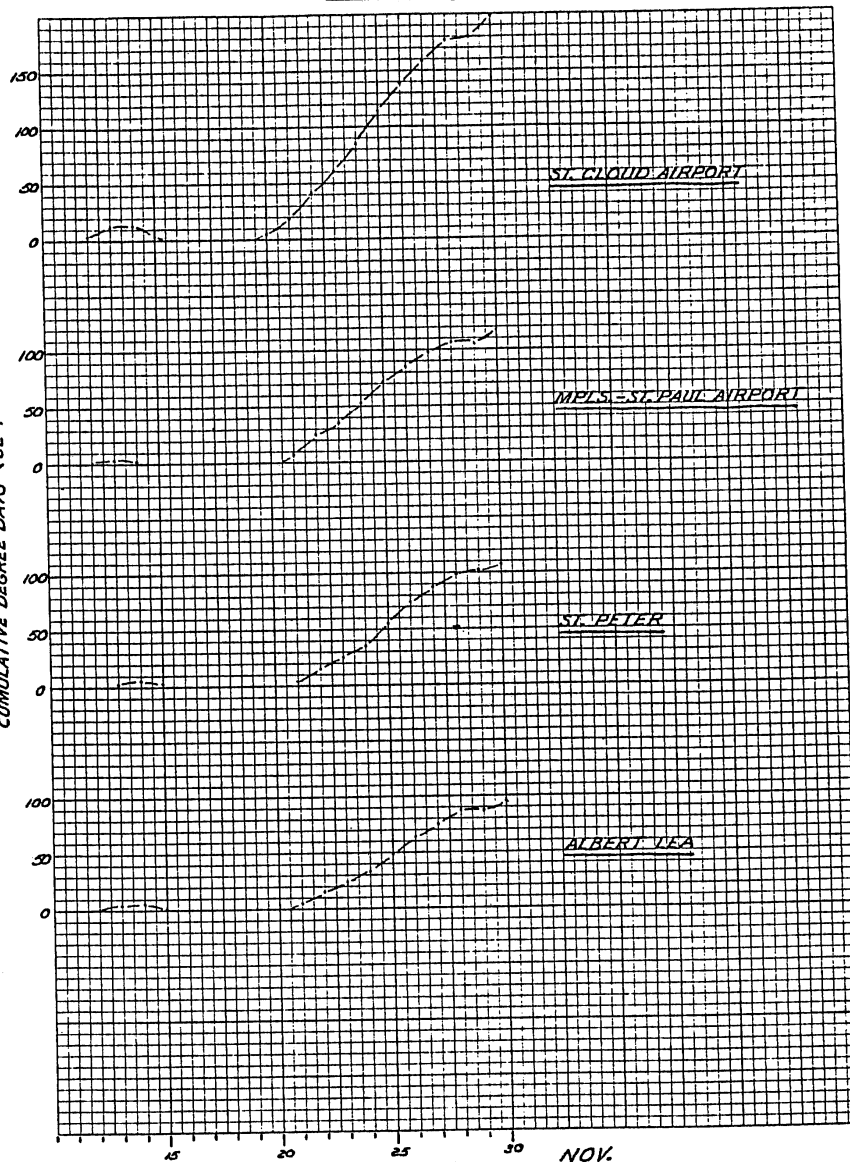


FALL 1975

DIETZGEN CORPORATION
MADE IN U.S.A.

NO. 341-10 DIETZGEN GRAPH PAPER
10 X 10 PER INCH

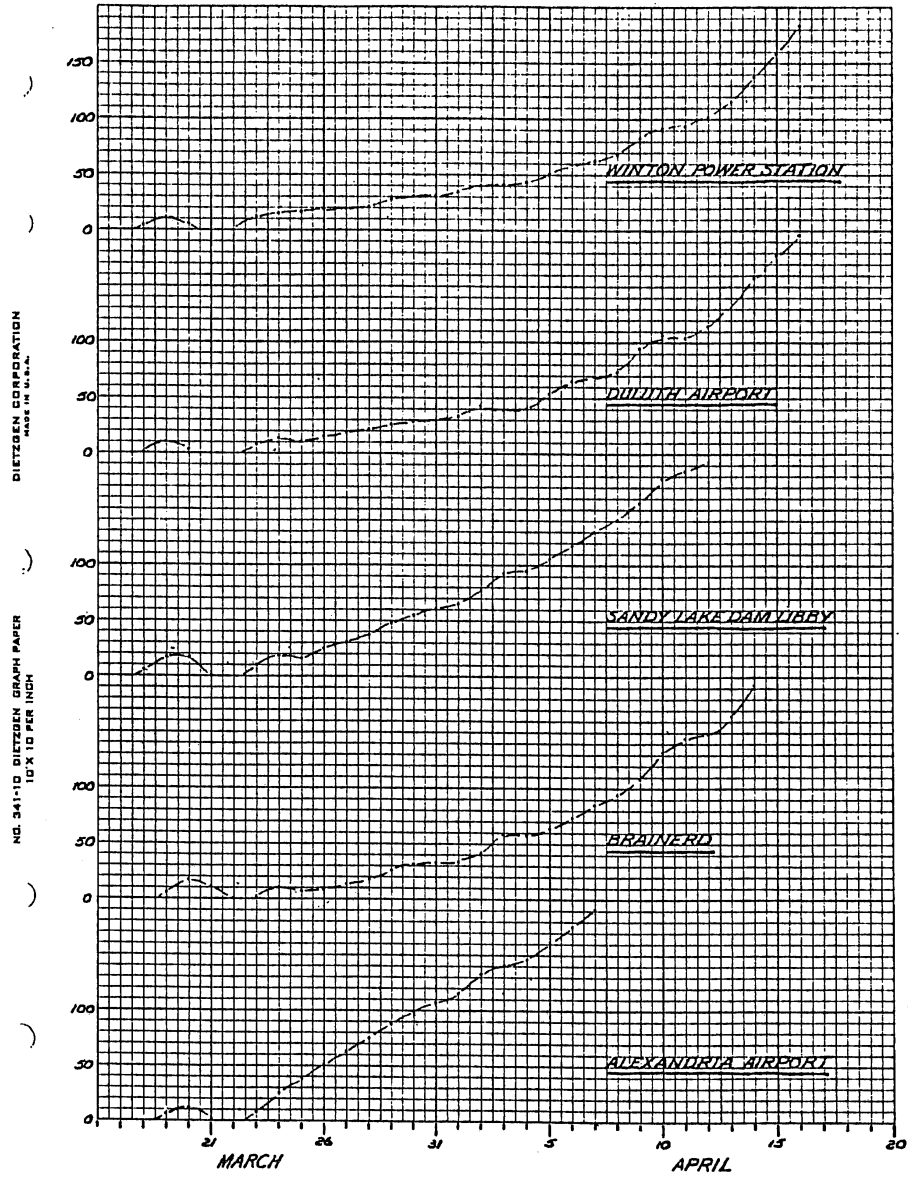
CUMULATIVE DEGREE DAYS $\leq 32^{\circ}\text{F}$



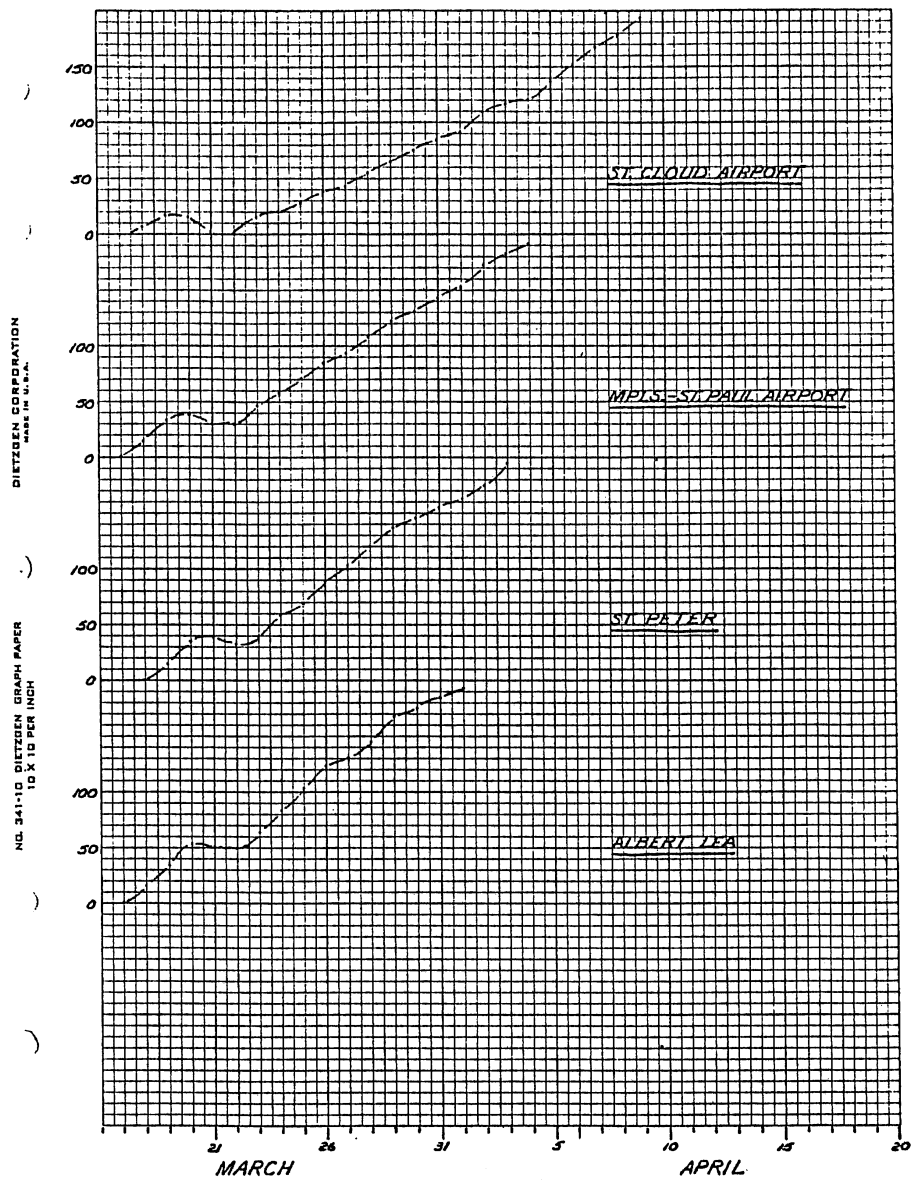
APPENDIX I

CUMULATIVE DEGREE MELTING DAYS

SPRING 1976



SPRING 1976



APPENDIX J

Incorporation of a Seasonal Phase Lag into the Water
Temperature Cycle and the Resulting Effect on
Prediction of Freeze-over Dates

In Eqs. C-1 and C-2 in Appendix C, the lowest point of the water temperature cycle is forced to occur on January 15 of each year, and therefore the term $3\pi/2\omega$ appears in the equations. However, the lowest monthly water temperature does not necessarily occur on January 15. In addition to the term $3\pi/2\omega$, a seasonal phase lag (δ_s) can be added to Eqs. C-1 and C-2 in Appendix C to account for the difference.

Mean monthly water temperatures were fitted to a Fourier series.

$$T_i = \bar{T}_O + \Delta T_1 \sin \frac{2\pi}{12} i + \Delta T_2 \cos \frac{2\pi}{12} i, \quad i = 1, \dots, 12 \quad (J-1)$$

The above mentioned phase lag (δ_s) in days can then be determined from

$$\delta_s = \tan^{-1} \left(\frac{\Delta T_2}{\Delta T_1} \right) * \frac{365}{360} \quad (J-2)$$

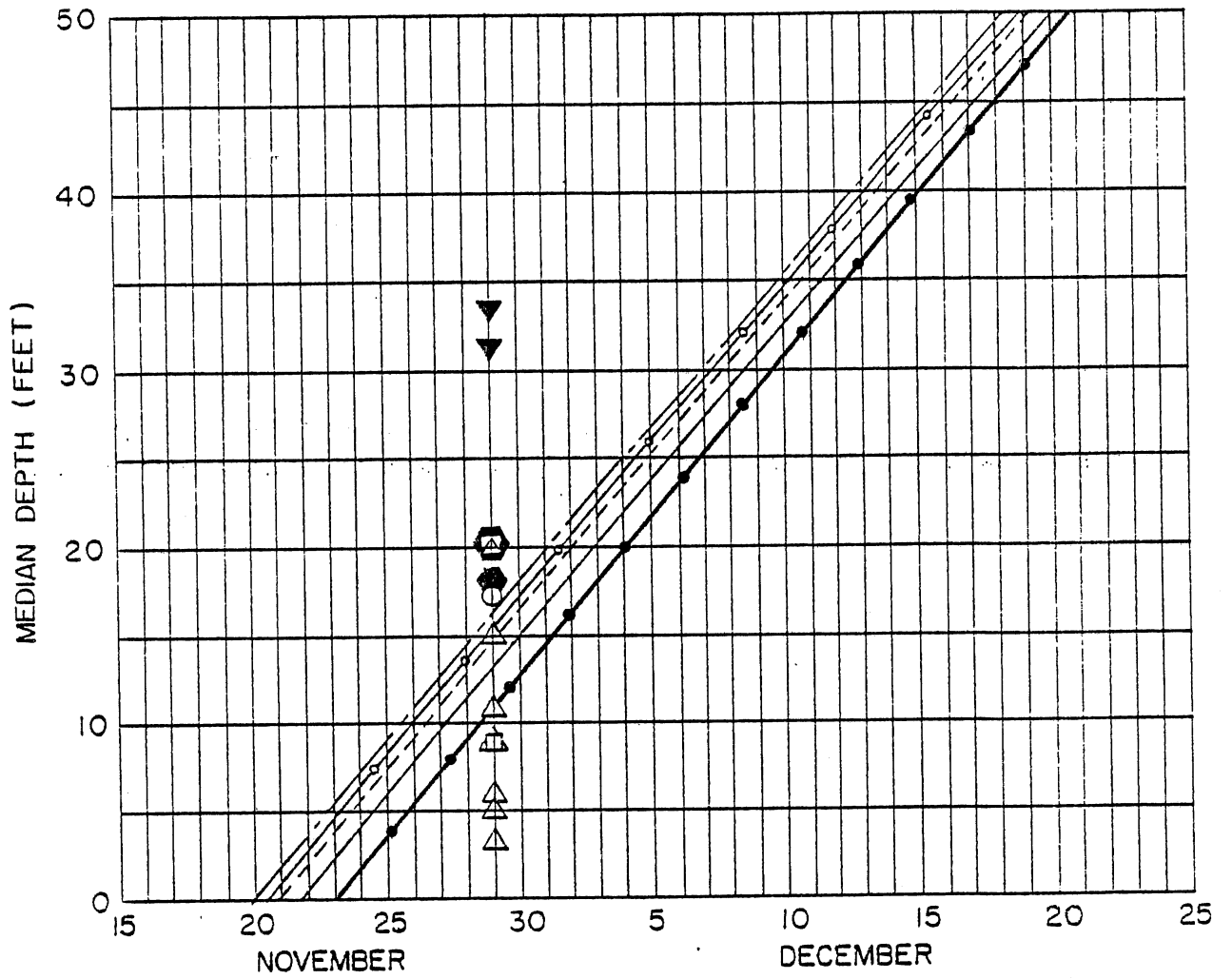
In Eqs. C-1 and C-2 in Appendix C, the monthly water temperature is assumed to occur on the 15th of each month. If the difference ($\delta_s - 15.5$) days is added to the right-hand side of Eqs. C-1 and C-2 in Appendix C, the freeze-over date will be obtained in Julian days (days after January 1). Only non-freezing water temperatures are used in the least squares fitting to determine the average water temperature \bar{T}_O and amplitudes, ΔT_1 and ΔT_2 . The modified procedure outlined above has been used to make prediction of freeze-over dates for lakes in Minnesota. Sample results are shown in Figs. J-1 to J-10. A comparison of those results with data from LANDSAT is shown in the figures.

In summary, if the modified procedure described here is used, the predicted freeze-over dates in northern Minnesota tend to be, on the average, 15 days earlier than the LANDSAT data would indicate and 14 days earlier than the results obtained in using Eqs. C-1 and C-2 in Appendix C.

In central Minnesota, the predicted freeze-over dates tend to be, on the average, 12 days later than the LANDSAT data would indicate and 8 days later than the results obtained in using Eqs. C-1 and C-2 in Appendix C.

In southern Minnesota, the predicted freeze-over dates agree closely (within a day on the average) with the LANDSAT data and the results obtained in using Eqs. C-1 and C-2 in Appendix C.

As a whole the procedure outlined in this appendix does not give results as good as those obtained by the procedures in appendix J.

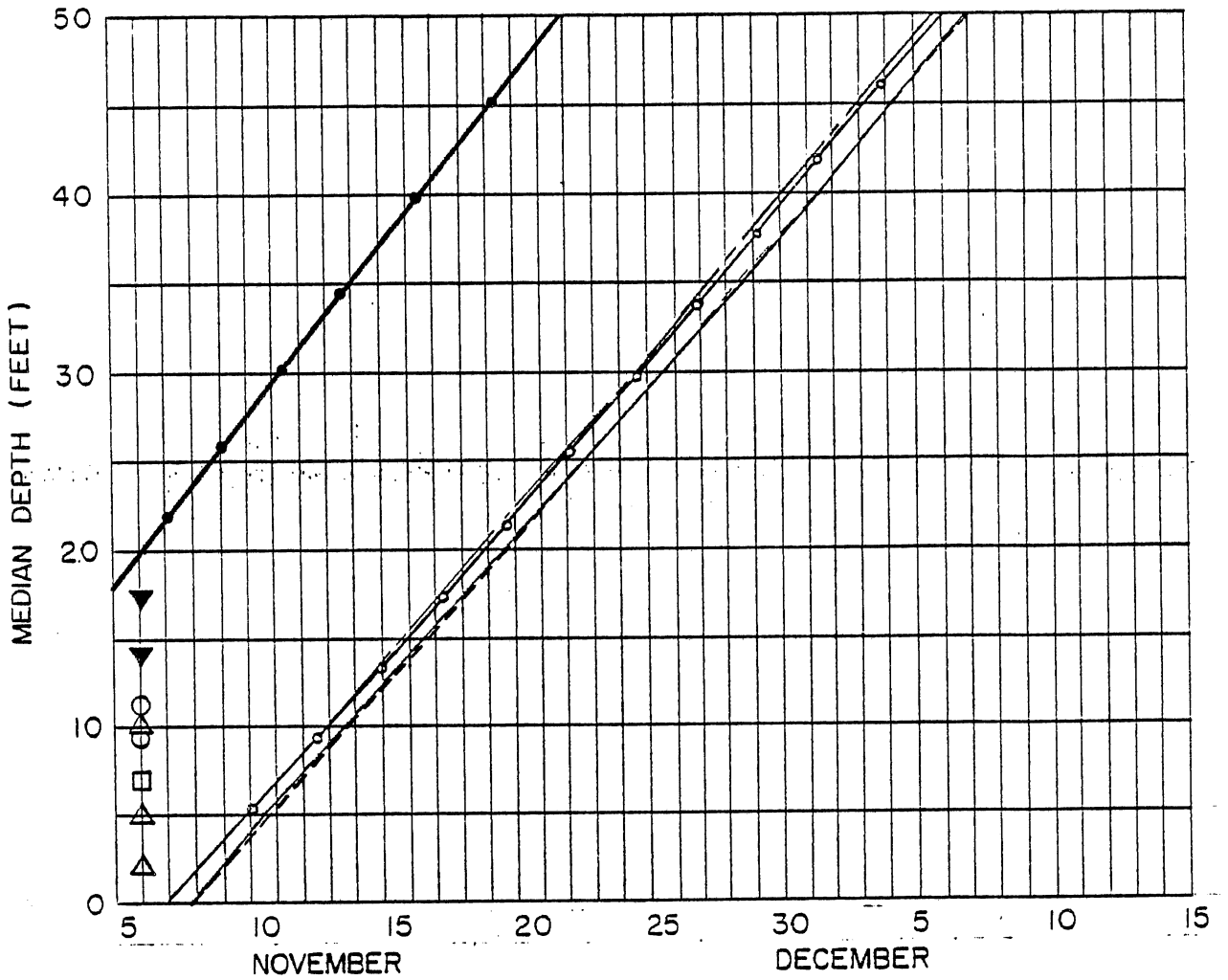


Legend

Month at the end of which prediction was made

- August
- September
- October
- November and December
- December (time lag incorporated)

Fig. J-1 - Prediction of the beginning dates of freeze-over of lakes in the vicinity of Minneapolis/St. Paul in Fall 1972.

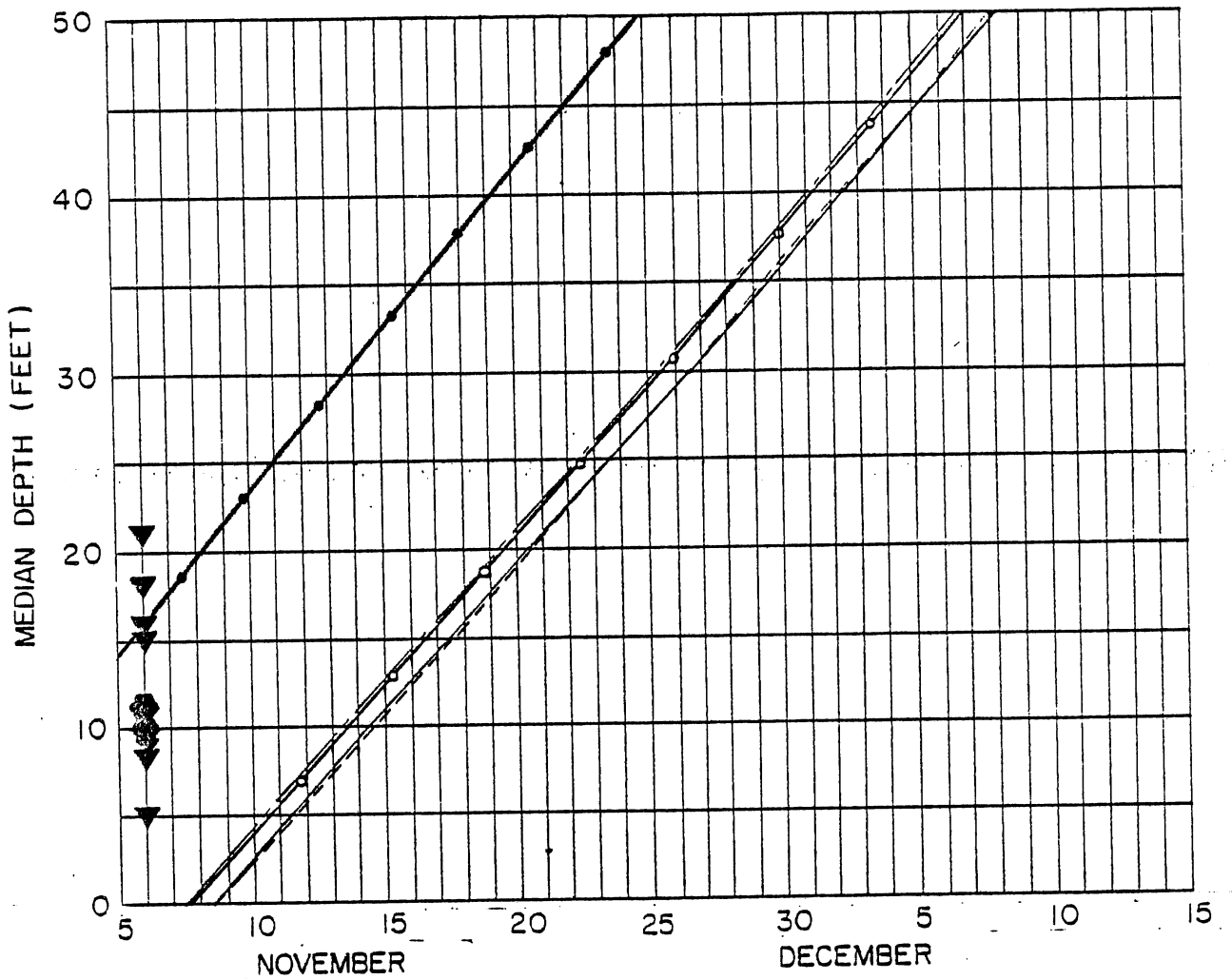


Legend

Month at the end of which prediction was made.

- August
- September
- October
- November and December
- December (time lag incorporated)

Fig. J-2 - Prediction of the beginning dates of freeze-over of lakes in the vicinity of Duluth Airport in Fall 1973.

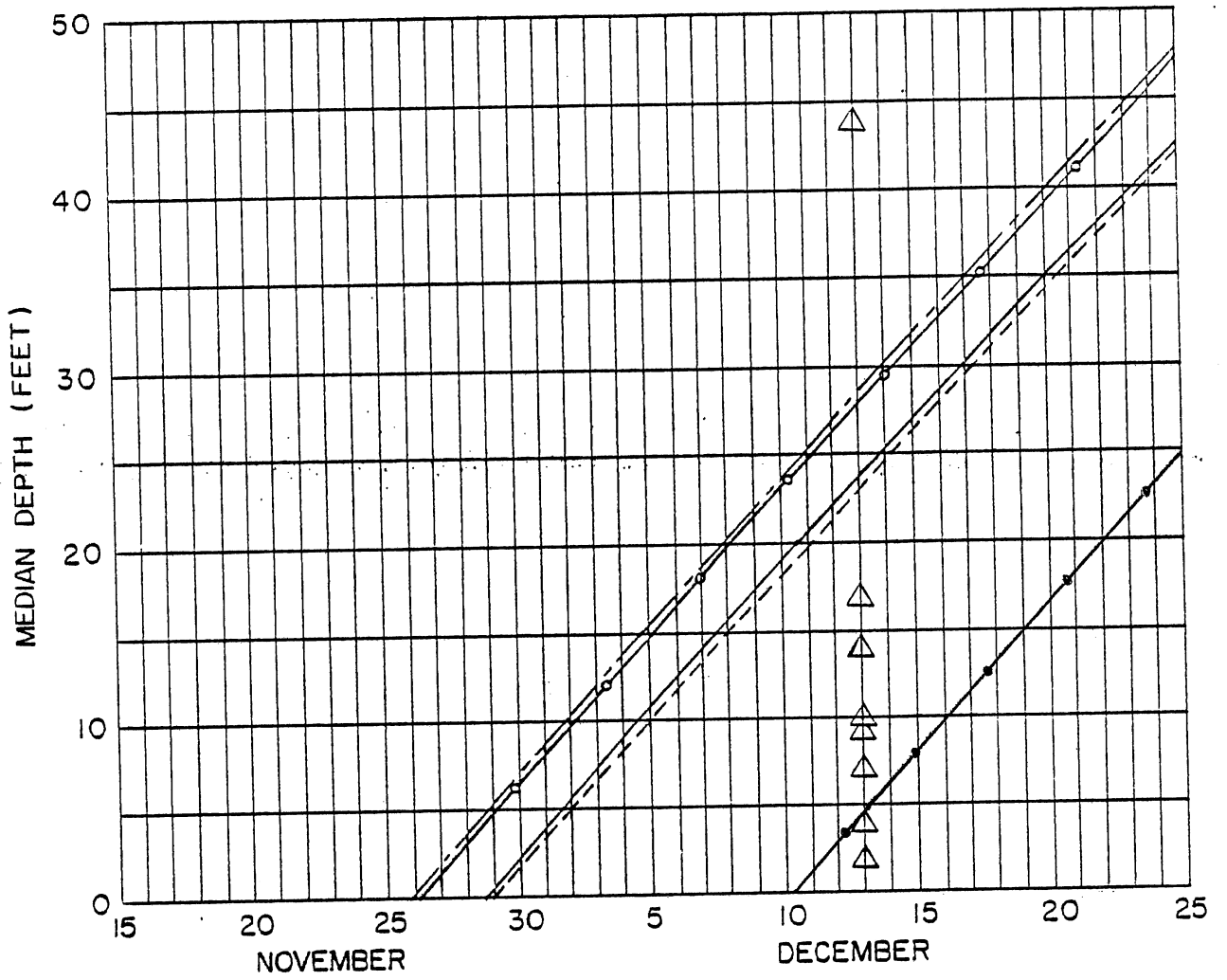


Legend

Month at the end of which prediction was made.

- August
- September
- October
- November and December
- December (time lag incorporated)

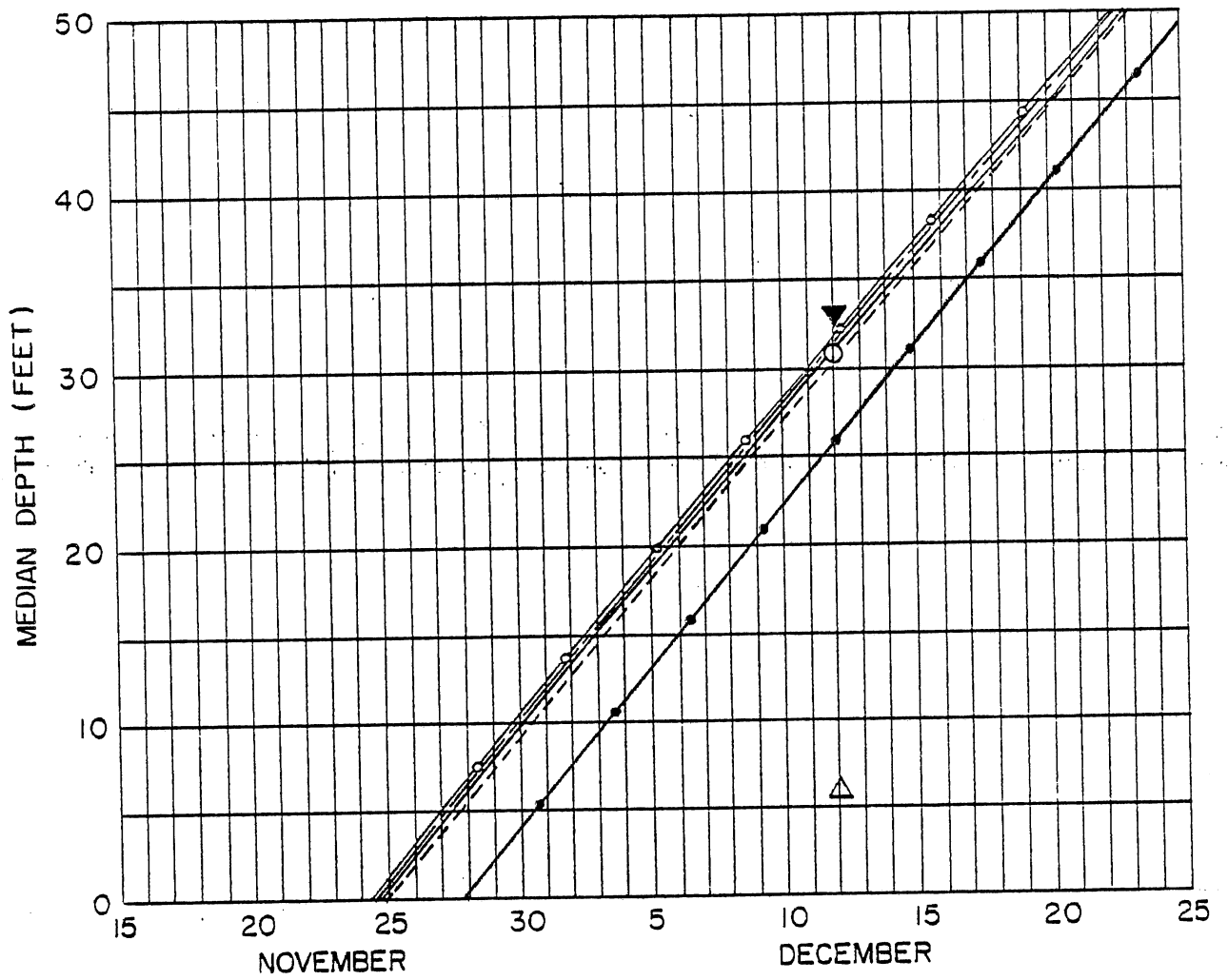
Fig. J-3 - Prediction of the beginning dates of freeze-over of lakes in the vicinity of Sandy Lake Dam Libby in Fall 1973.



Legend

- Month at the end of which prediction was made.
- August
 - September
 - October
 - November and December
 - December (time lag incorporated)

Fig. J-4 - Prediction of the beginning dates of freeze-over of lakes in the vicinity of St. Cloud in Fall 1973.

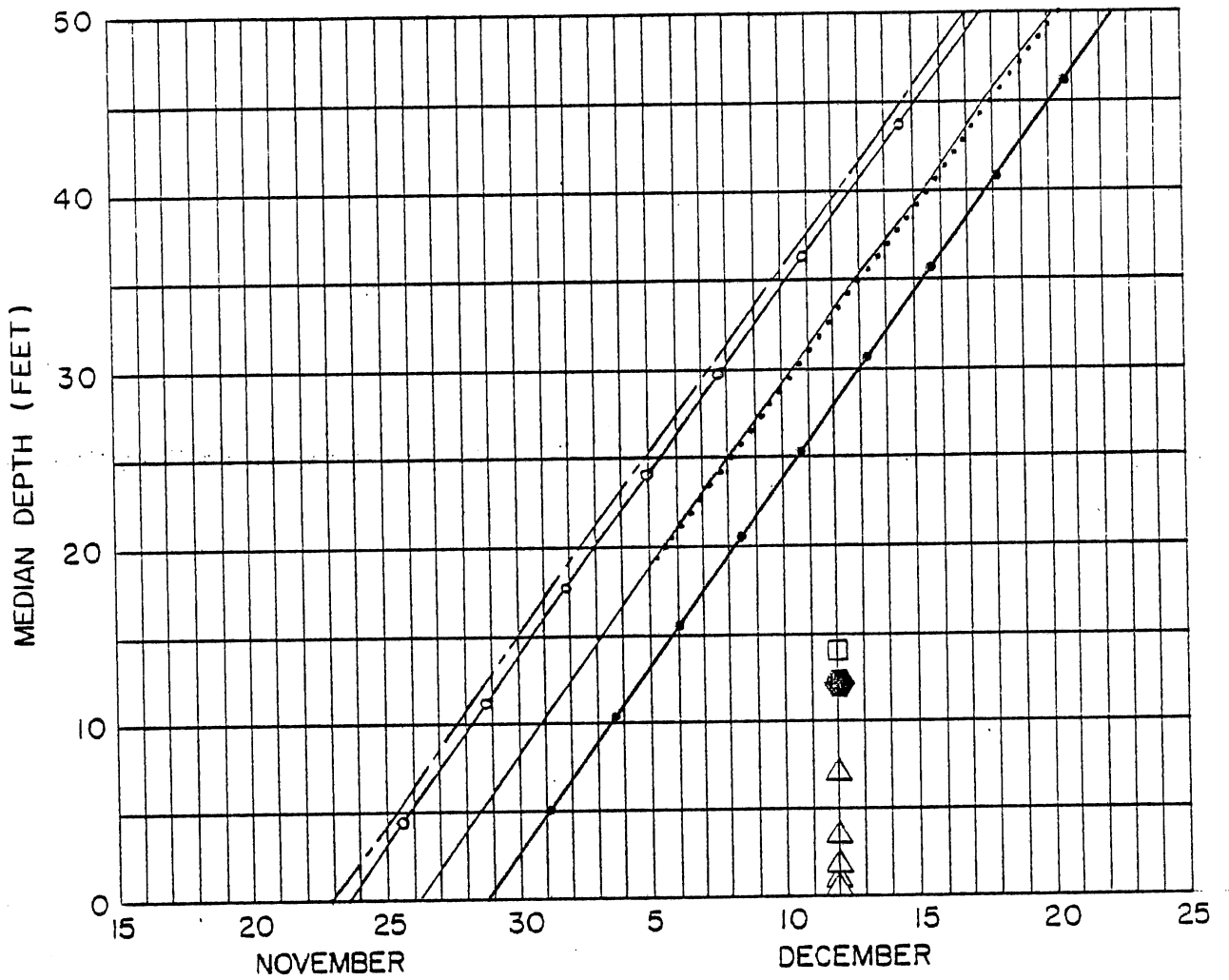


Legend

Month at the end of which prediction was made.

- August
- September
- October
- November and December
- December (time lag incorporated)

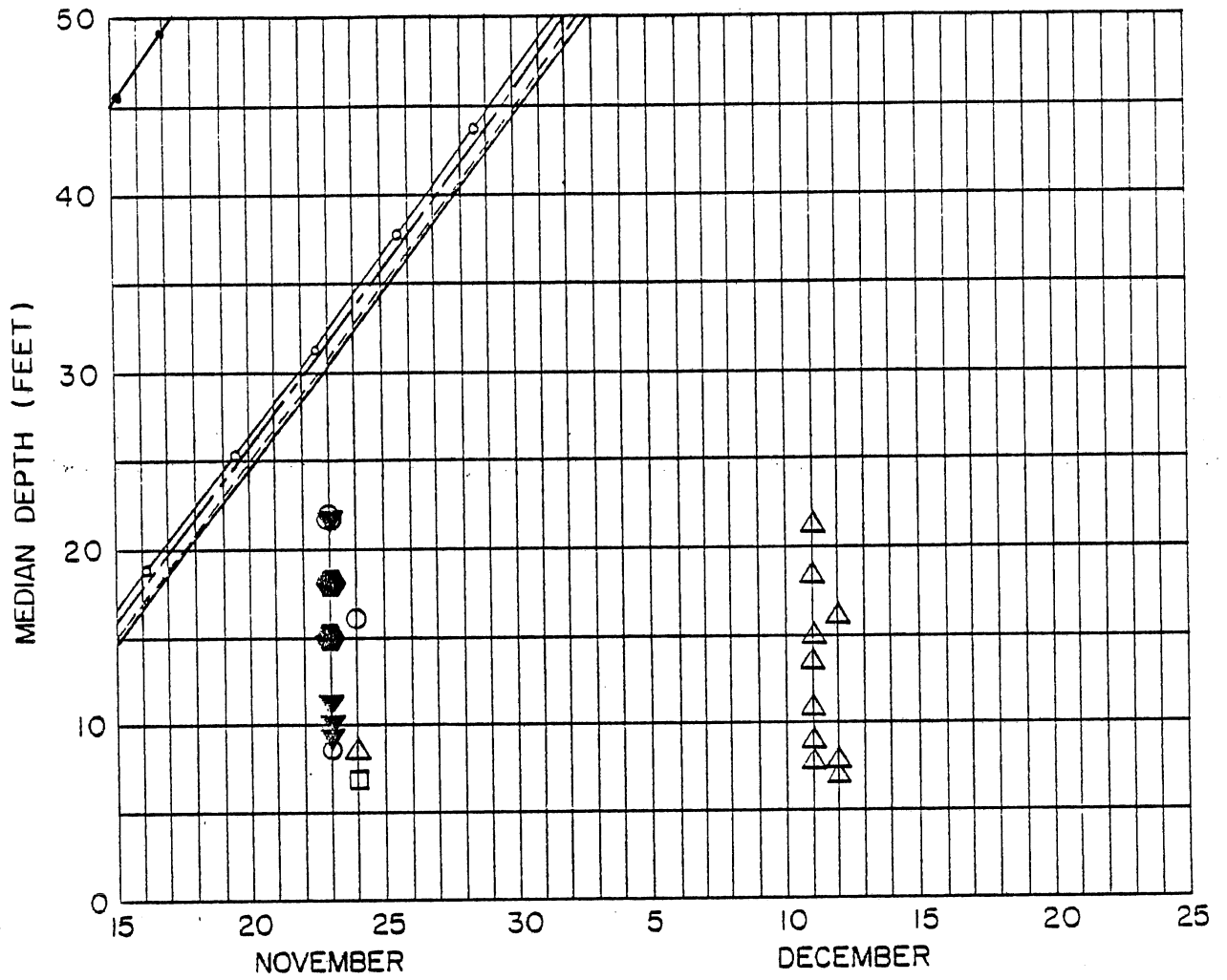
Fig. J-5 - Prediction of the beginning dates of freeze-over of lakes in the vicinity of Minneapolis/St. Paul in Fall 1973.



Legend

- Month at the end of which prediction was made.
- August
 - September
 - October and November
 - December
 - December (time lag incorporated)

Fig. J-6 - Prediction of the beginning dates of freeze-over of lakes in the vicinity of Waseca in Fall 1973.

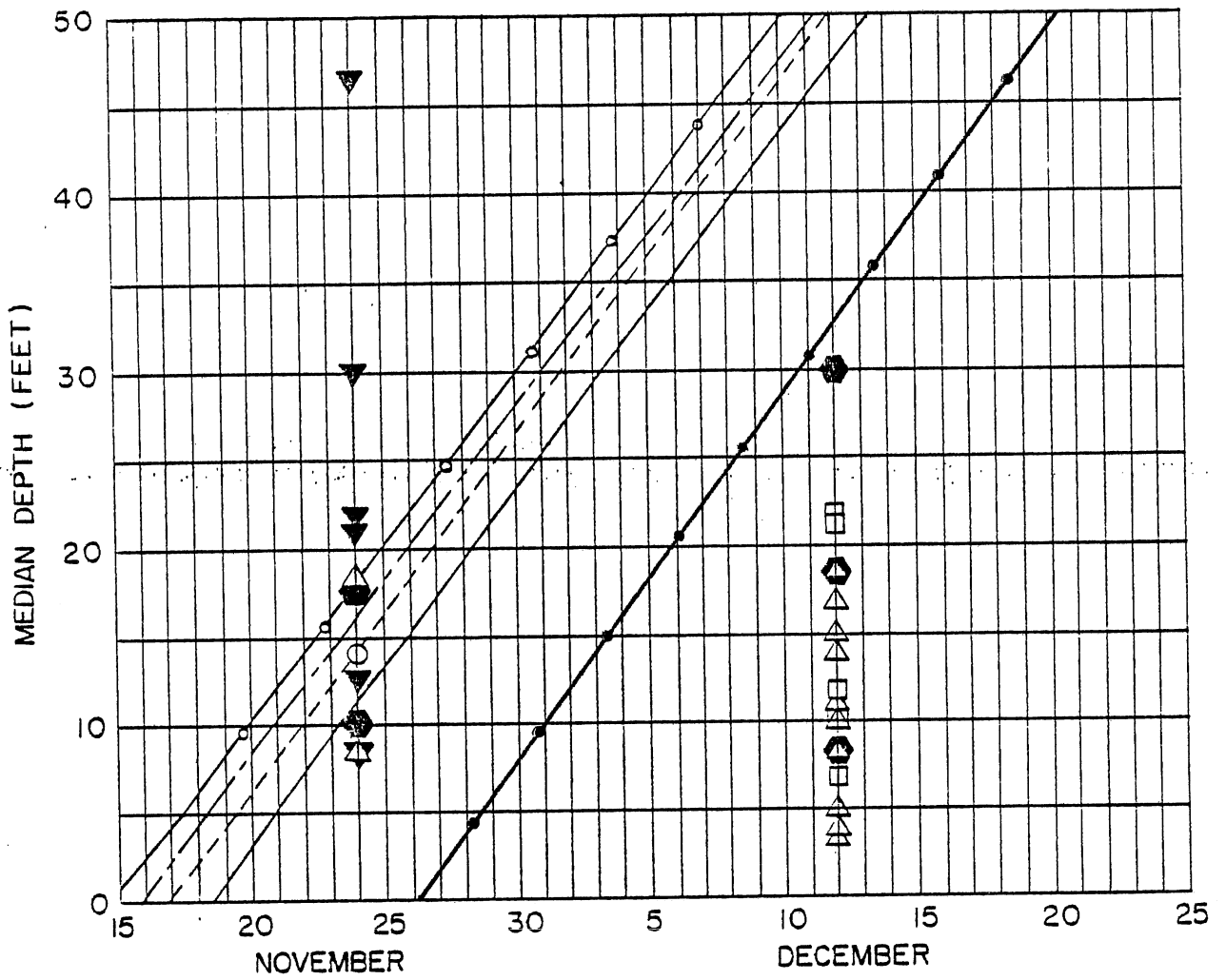


Legend

Month at the end of which prediction was made.

- August
- September
- October
- November and December
- December (time lag incorporated)

Fig. J-7 - Prediction of the beginning dates of freeze-over of lakes in the vicinity of Sandy Lake Dam Libby in Fall 1975.



Legend

Month at the end of which prediction was made.

----- August

August

—○— September

September

----- October

October

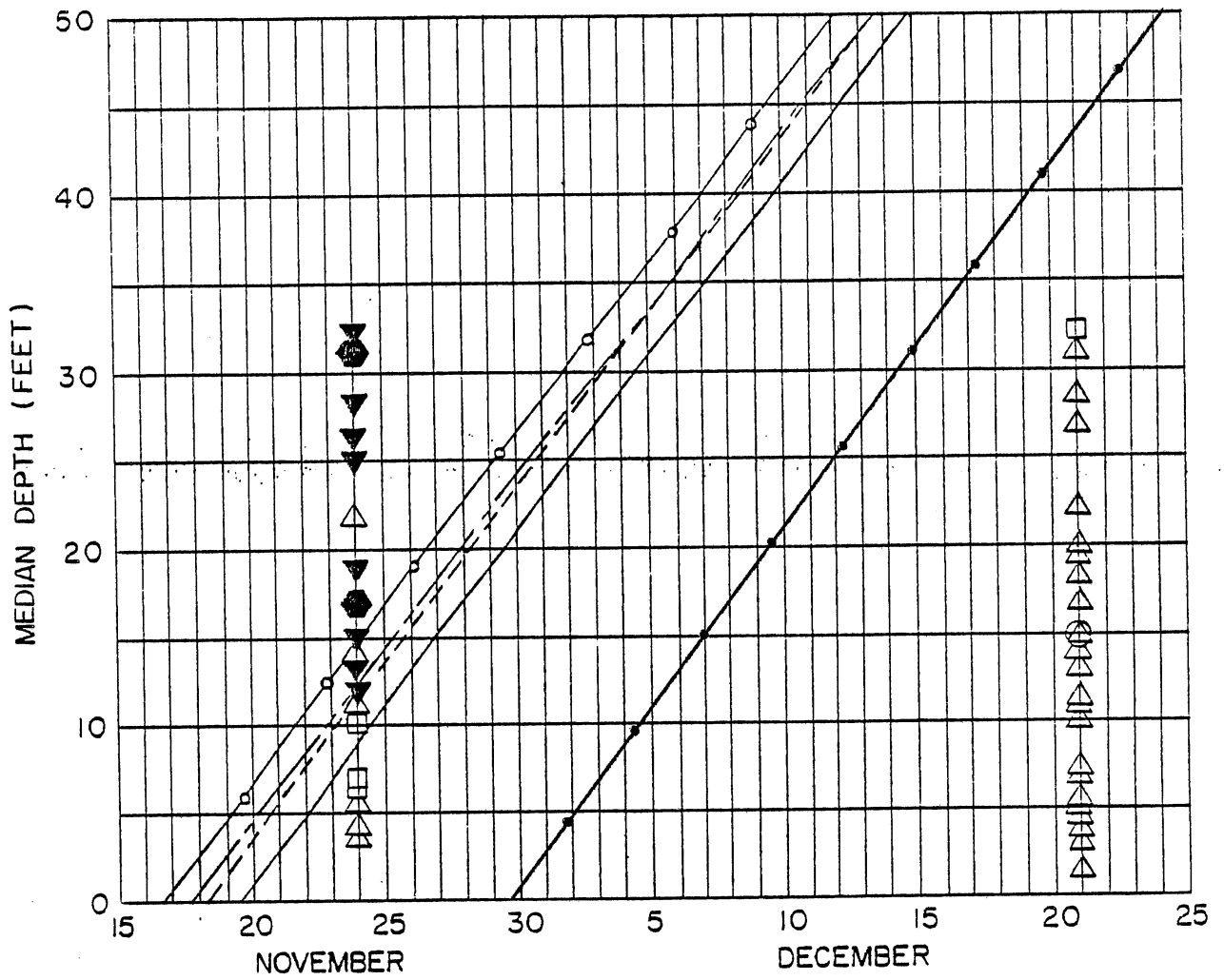
———— November and December

November and December

—●— December (time lag incorporated)

December (time lag incorporated)

Fig. J-8 - Prediction of the beginning dates of freeze-over of lakes in the vicinity of Brainerd in Fall 1975.

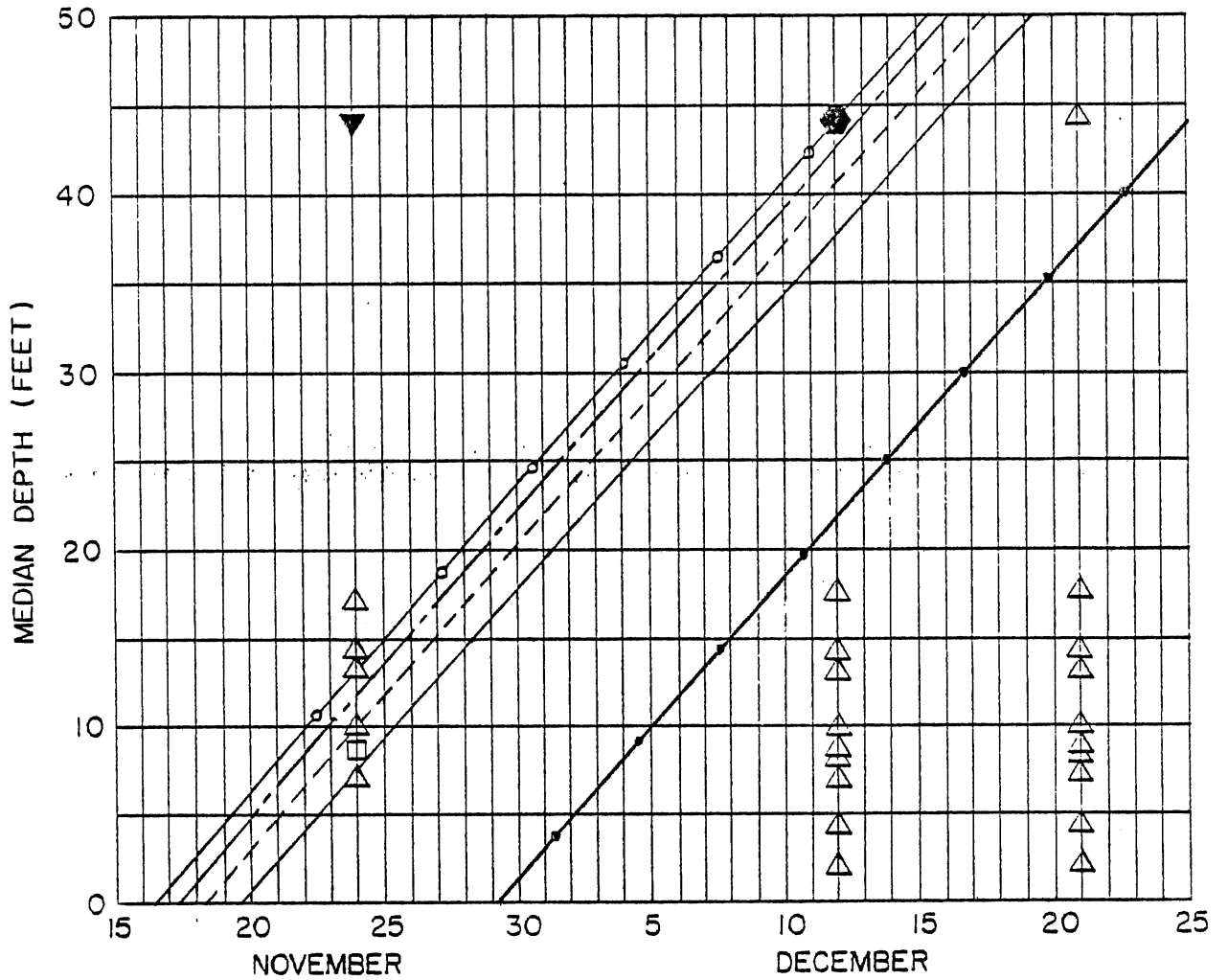


Legend

Month at the end of which prediction was made.

- August
- September
- October
- November and December
- December (time lag incorporated)

Fig. J-9 - Prediction of the beginning dates of freeze-over of lakes in the vicinity of Alexandria Airport in Fall 1975.



Legend

Month at the end of which prediction was made.

- August
- September
- October
- November and December
- December (time lag incorporated)

Fig. J-10 - Prediction of the beginning dates of freeze-over of lakes in the vicinity of St. Cloud in Fall 1975.