Explanations for Accuracy of the General Multivariate Formulas in Correcting for Range Restriction

Janet D. Held and Paul P. Foley

Navy Personnel Research and Development Center

Univariate and multivariate corrections for range restriction were compared using Navy applicant scores on the Armed Services Vocational Aptitude Battery (ASVAB). Two Navy school ASVAB selector composites were used separately and together to simulate three selection situations for nine selection ratios (SRs). The selectors then were used as predictors. Composite validities (the ASVAB Mechanical Comprehension test was the criterion) were corrected using the univariate and the general multivariate formulas. In general, multivariate corrections were more accurate than univariate corrections, notably when the univariate explicit selection variable was negatively skewed and correction violations (linearity and homoscedasticity) did not offset each other. Multivariate correction accuracy was attributed to the inclusion of variables with adequate distributional properties, the compensatory effects of regression weights, and the related psychometric principle that differentially weighting a large number of correlated predictor variables has little impact on a multiple correlation. *Index terms: correction formulas, explicit selection, incidental selection, multivariate correction formulas, restriction in range, validation studies.*

Since Pearson (1903) developed the correction formulas for restriction in range and Thorndike (1947, 1949) defined them for practical application, educational and personnel psychologists have conducted extensive research on the problem of restriction in range (Alexander, Carson, Alliger, & Barrett, 1984; Booth-Kewley, 1985; Greener & Osburn, 1979; Gross & Fleischman, 1983; Levin, 1972; Linn, 1968; Linn, Harnisch, & Dunbar, 1981; Mendoza, Hart, & Powell, 1991; Olson & Becker, 1983; Sands, Alf, & Abrahams, 1978). However, most of the research pertains to the univariate, not the multivariate, correction formulas (Lord & Novick, 1968; Novick & Thayer, 1969). This study investigated the use of multivariate corrections in some univariate as well as multivariate selection situations.

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The restriction in range problem in personnel selection research is to find the value of the predictor/criterion correlation, or validity, for an unrestricted applicant group that has only predictor information available. Given predictor/criterion information for a subset of the unrestricted group, the Pearson formulas estimate the unrestricted group validity. Restriction in range is classified as either explicit (direct) or incidental (indirect): *explicit* for scores on a predictor variable that is used to select individuals into the subset, and *incidental* for scores on another predictor variable that is correlated with the variable used for selection. The basis for the accuracy of the estimated unrestricted validity (corrected validity) is bivariate normality for the univariate selection situation with selection solely on the explicit selector variable, and multivariate normality for the multivariate selection situation with selection solely on the explicit selector variables.

Lawley (1943–44), however, relaxed the distributional assumptions, or the formal properties of normality, required for correction accuracy. For the univariate explicit case, only two properties of bivariate normality are required: (1) linearity of regression of the criterion, y, on the predictor, x; and (2) homoscedasticity of y error variance for all values of x. If selection occurs solely on x and the above properties hold, then the following slope and error identities are true, respectively,

$$B = S_{XY}/S_X^2 = b = s_{xy}/s_x^2 \,, \tag{1}$$

and

$$S_E^2 = S_Y^2\left(1 - R_{XY}^2\right) = s_e^2 = s_y^2\left(1 - r_{xy}^2\right), \tag{2}$$

where

B is the unrestricted slope (weight),
S_{XY} is the unrestricted covariance,
S_X^2 is the unrestricted predictor variance,
S_Y^2 is the unrestricted criterion variance,
S_E^2 is the unrestricted error in prediction variance,
R_{XY}^2 is the unrestricted squared validity, and
b, s_{xy}, s_x^2, s_y^2, s_e^2, and r_{xy}^2, are the respective restricted terms.

Algebraic manipulations with term substitutions of these identities result in solutions for the unknown unrestricted predictor/criterion covariance, S_{XY}, and the unknown unrestricted criterion standard deviation (SD), S_Y. These values and the known unrestricted predictor SD, S_X, are used to solve for the unrestricted validity from the formula,

$$R_{XY} = \frac{S_{XY}}{S_X S_Y} \,, \tag{3}$$

where

$$S_{XY} = bS_X^2 \,. \tag{4}$$

S_Y is the square root of S_Y^2, and

$$S_Y^2 = s_y^2 + b\left(S_{XY} - s_{xy}\right). \tag{5}$$

Equation 3, substituting the previous derivations for the two unknowns, is equivalent to the usual correction formula for the explicit case,

$$R_{XY} = \frac{r_{xy}\left(S_x/s_x\right)}{\left[1 - r_{xy}^2 + r_{xy}^2\left(S_x/s_x\right)^2\right]^{1/2}} \tag{6}$$

(Case 1 from Guilford, 1965, p. 343; Case A from Thorndike, 1982, p. 210). The validity equation used here will be Equation 3 because its use of covariance rather than correlation coefficients simplifies univariate and multivariate comparisons.

The general multivariate formulas are derived directly from the two bivariate identities as matrix algebra extensions (see Gulliksen, 1950, chap. 13 for derivations); more correctly, the general multivariate formulas include the bivariate formulas as a singular case. The univariate three-variable correction formula correcting for incidental selection, is

$$R_{1C} = \frac{r_{1C} + r_{12} + r_{2C}\left(\dfrac{S_2^2}{s_2^2} - 1\right)}{\left[1 + r_{12}^2\left(\dfrac{S_2^2}{s_2^2} - 1\right)\right]^{1/2}\left[1 + r_{2C}^2\left(\dfrac{S_2^2}{s_2^2} - 1\right)\right]^{1/2}} \,, \tag{7}$$

where

the subscript 1 designates the incidental selector variable (usually a candidate selection instrument),
the subscript 2 designates the explicit selector variable, and
the subscript c designates the criterion.

Equation 7 (Case 3 from Guilford, 1965, p. 344; Case C from Thorndike, 1982, p. 213) is just a special multivariate case of one explicit selector variable and two incidental selector variables.

Three formal properties of multivariate normality are required for correction accuracy: (1) linearity of regression of the criterion, y, on the predictors, x_i, where $i = 1, \ldots, p$ and p is the number of explicit selector variables; (2) homoscedasticity of y error variance for all values of x_i; and (3) y covariances unconditional on x_i. If explicit selection occurs solely on x_i and the above properties hold, then the following slope and error identities are true, respectively,

$$B_i = b_i \quad (\text{for all } i = 1, \ldots, p), \tag{8}$$

and

$$S_Y^2\left(1 - R_{Y.X_1,\ldots,X_p}^2\right) = s_y^2\left(1 - r_{y.x_1,\ldots,x_p}^2\right). \tag{9}$$

It follows that the solution and use of these identities parallel the univariate case to determine the unrestricted validity (Equation 3).

For the multivariate case,

$$\mathbb{W}_{XY} = \mathbb{C}_{XX}^{-1}\mathbb{C}_{XY} = \mathbb{w}_{xy} = \mathbb{c}_{xx}^{-1}\mathbb{c}_{xy} \,, \tag{10}$$

and

$$\mathbb{C}_{EE} = \mathbb{C}_{YY} - \mathbb{C}_{YX}'\mathbb{W}_{XY} = \mathbb{c}_{ee} = \mathbb{c}_{yy} - \mathbb{c}_{yx}'\mathbb{w}_{xy} \,, \tag{11}$$

where, for multiple explicit (predictor) and incidental (criterion) selector variables,

$\mathbb{W}_{XY}$ is the unrestricted slope (weight) variance-covariance matrix,

$\mathbb{C}_{XY}$ is the unrestricted predictor/criterion variance-covariance matrix,

$\mathbb{C}_{XX}$ is the unrestricted predictor variance-covariance matrix,

$\mathbb{C}_{YY}$ is the unrestricted criterion variance-covariance matrix,

$\mathbb{C}_{EE}$ is the error in prediction variance-covariance matrix, and

$\mathbb{w}_{xy}$, $\mathbb{c}_{xy}$, $\mathbb{c}_{xx}$, $\mathbb{c}_{yy}$, and $\mathbb{c}_{ee}$ are the respective restricted matrices.

In the multivariate case, the equivalent of Equation 3 could be formulated as

$$R_{XY} = \frac{C_{XY}}{\left(C_{XX}\right)^{1/2}\left(C_{YY}\right)^{1/2}} . \tag{12}$$

C_{XY} is the relevant covariance term in $\mathbb{C}_{XY}$ where

$$\mathbb{C}_{XY} = \mathbb{C}_{XX}\mathbb{w}_{xy} . \tag{13}$$

$\left(C_{YY}\right)^{1/2}$ is the criterion SD taken from $\mathbb{C}_{YY}$ where

$$\mathbb{C}_{YY} = \mathbb{c}_{yy} + \mathbb{w}'_{yx}\left(\mathbb{C}_{XY} - \mathbb{c}_{xy}\right). \tag{14}$$

$\left(C_{XX}\right)^{1/2}$ is the predictor SD taken from $\mathbb{C}_{XX}$.

In an attempt to deal with the common personnel selection research problem of isolating the explicit selector variable, some psychologists apply the general multivariate formulas using a wide range of incidental as well as explicit selector variables (Novick & Thayer, 1969). A major objective of this study was to explain the relative accuracy of this approach when univariate formulas give inaccurate corrections due to violations of correction assumptions. The violation of particular interest is linearity, because univariate corrections have been found to be sensitive to violations of linearity but robust to violations of homoscedasticity (Greener & Osburn, 1979). Further, although Lawley (1943–44) relaxed distributional assumptions for use of the univariate and multivariate correction formulas, a skewed distribution, which is a common data condition, can affect linearity (Brewer & Hills, 1969).

To understand the differences between univariate and multivariate formula accuracy, it is helpful to examine Gross' (1982) quotient (Q) formula for the bivariate case, which will be generalized to the multivariate case in this study of one criterion variable. Q for the bivariate case (explicit selector variable) is

$$Q_{\mathrm{U}} = \frac{\left(S_E^2/s_e^2\right)^{1/2}}{B/b} . \tag{15}$$

Q_{M} for the multivariate case can be formulated as

$$Q_{\mathrm{M}} = \frac{\left(C_{EE}^2/c_{ee}^2\right)^{1/2}}{\sum W_{XY}/\sum w_{xy}} , \tag{16}$$

where C_{EE}^2 and c_{ee}^2 represent the respective unrestricted and restricted variance errors of estimate taken after the last variable entry in the full regression equation for p predictors and one criterion. W_{XY} and w_{xy} represent the respective unrestricted and restricted slope (weight) sums of the p vector entries. Weights are raw score, or unstandardized, and derived from a forward accretion multiple regression procedure. (Unstandardized weights, not standardized weights, are derived in the correction formulas because the slopes for the restricted and unrestricted groups are assumed to be equal.)

Gross indicated that when the univariate correction assumptions of linearity and homoscedasticity are met, $S_E^2/s_e^2 = 1$ and $B/b = 1$, and, therefore, $Q = 1$. In this case, the conditions of linearity and homoscedasticity have been met and the univariate correction formulas are accurate. However, the linearity and homoscedasticity assumptions are not necessary conditions for the univariate formulas (assumed here for the multivariate formulas) to produce accurate results. For example, bivariate data often violate the homoscedasticity assumption—a common violation is that the conditional y variances are lower for extremes of x than they are in the middle range of x (Lord & Novick, 1968, p. 148). Given that selection is stringent and the slope flattens for extremes of x, the homoscedasticity and linearity violations can offset each other. In this situation, Q still can equal 1 and the univariate formula (Equation 3) is accurate. However, if the slope rises rather than flattens, Q is greater than 1 and the univariate formula overcorrects. If the slope decreases substantially, Q is less than 1 and the univariate formula undercorrects.

The functional relationships between the offsetting linearity and homoscedasticity violations can be assessed from a reduced form of the unrestricted validity formula (Equation 3),

$$R_{XY} = \frac{bS_X}{S_Y}. \tag{17}$$

The predictor SD, S_X, is the only known value in the unrestricted group. The remaining ratio term, b/S_Y, provides the same slope to error ratio relationship as the Q formula. In fact, Held & Foley (1993) found that the graph of Q values across selection ratios (SRs) paralleled the graph of univariate corrected validities under several violations of the bivariate correction assumptions.

Finally, the impact of skew on correction accuracy and Q is obvious from the slope formula,

$$b = s_{xy} / s_x^2. \tag{18}$$

High negative skew for an explicit selector variable results in dense scores in the right tail of the distribution. If selection is moderate to stringent, the reduced predictor score variance $\left(s_x^2\right)$ for high values of x raises b and the univariate formula overcorrects. Reduced covariance (s_{xy}) from multivariate selection, on the other hand, lowers b and the univariate formula undercorrects.

Background

The data used for this study, which compared univariate and multivariate corrections for restriction in range, were Navy applicant scores on the Armed Services Vocational Aptitude Battery (ASVAB). The ASVAB, used by all United States military services for selection and classification, measures cognitive abilities and academic and technical knowledge (Foley & Rucker, 1989; Kass, Mitchell, Grafton, & Wing, 1983; Prestwood, Vale, Massey, & Welsh, 1985). The Navy has 11 operational ASVAB selector composites that are comprised of various combinations of two to four tests. These composites are used to select recruits for Navy entry level technical schools. Validation studies are conducted periodically to ensure that each school is using the most valid selection (predictor) composite. The usual criterion is final school grade.

For each Navy school, the current operational ASVAB selection composite (explicit selector variable) and one or more candidate replacement composites (incidental selector variable) are evaluated after correcting validities for restriction in range. [The candidate replacement composite is a Navy operational selection composite(s) that most closely resembles an experimental composite that is derived from a multiple regression. The multiple regression is performed on a test development sample randomly determined in a cross-validation design. The validities are compared in a holdout sample.]

The bivariate formulas (Equation 6 or other versions) apply for the operational selection composite, whereas the three-variable formulas (Equation 7 or other versions) apply for the candidate replacement composite. The general multivariate formulas apply when explicit selection is multivariate (two or more operational selector composites or test score requirements).

The Navy currently performs multivariate corrections for all selection situations, treating all ASVAB tests as explicit selector variables. One of the objectives of this study was to assess this approach. Equation 12 applies for both the operational and candidate replacement composites. The differences are (1) the C_{XY} term for the operational composite is obtained from $\mathbf{C}_{XY}$, whereas for the candidate composite it is obtained from $\mathbf{C}_{YY}$ (the composite covariance is the sum of the individual composite test covariances) and (2) S_X for the operational composite is known, whereas for the candidate composite it is obtained from $\mathbf{C}_{YY}$, as is S_Y for both cases (Ghiselli, Campbell, & Zedeck, 1981, chap. 7 treats combining variances and covariances of the components of a composite to determine composite validities). Corrected composite validities and corrected composite validity differences are used with the Taylor-Russell tables (Taylor & Russell, 1939) to determine the utility of replacing the operational selection composite in terms of improved school success rates (see Held, 1992, for an example of the Navy's use of the Taylor-Russell tables).

The major objectives of this study were to (1) determine the relative accuracy of univariate and multivariate corrections for range restriction for several univariate and multivariate selection situations, (2) assess the efficacy of including incidental selector variables as explicit selector variables in multivariate corrections, and (3) determine if the multivariate formulas should be applied in other personnel selection situations.

Method

Unrestricted Group

The unrestricted group consisted of 147,288 Navy applicants who took the ASVAB in fiscal year 1988. A high school diploma requirement for female but not male enlistment may have resulted in a restriction of range for the females because of higher academic ability. Self-selection and unknown bias at the recruitment level also may have influenced the composition of the applicant group.

Variables

The variables were the 10 tests of the ASVAB (Foley & Rucker, 1989): General Science (GS), Arithmetic Reasoning (AR), Word Knowledge (WK), Paragraph Comprehension (PC), Numerical Operations (NO), Coding Speed (CS), Auto and Shop Information (AS), Mathematics Knowledge (MK), Mechanical Comprehension (MC), and Electronics Information (EI). Nine tests were used operationally; PC and WK were combined to form the Verbal composite (VE). The operational scores of record were used. These were standardized (means of 50 and SD of 10) with norms obtained from the American Youth Population (U.S. Department of Defense, 1982).

Selectors

The selectors were two substantially correlated (low .70s) Navy operational ASVAB composites: (1) General Technical (VEAR), comprised of VE and AR; and (2) Engineering (MKAS), comprised of MK and AS. These composites have been used jointly to select recruits for entry into Navy mechanical schools. Composite scores were integer-weighted sums of standardized scores (which are used operationally for recruit classification).

Criterion

The criterion was the ASVAB MC test. MC was substituted for final school grade because only a small fraction of Navy applicants would have attended the school in this study and received final school grades (the objective of the study was to compare corrected validities to observed unrestricted validities). MC, which is multidimensional, is not used in the two mechanical school selection composites and was considered an appropriate criterion for mechanical school prediction equations.

Statistical Analyses

Three selection situations were simulated for nine SRs (.10 to .90; the unrestricted SR was 1.00): (1) explicit selection on VEAR, incidental selection on MKAS (Selection Situation 1—SS1); (2) explicit selection on MKAS, incidental selection on VEAR (Selection Situation 2—SS2); and (3) explicit selection on both VEAR and MKAS (Selection Situation 3—SS3). Minimum qualifying scores to determine SRs were determined from non-normal cumulative frequency distributions.

Uncorrected validities for the explicit and incidental selector variables were corrected. In the univariate situation (U), the bivariate and univariate three-variable formulas were used as appropriate. U corrections for SS1 and SS2 used the bivariate formulas for the explicit selector and the univariate three-variable formulas for the incidental selector. U corrections for SS3 used the bivariate formulas for both explicit selectors. In the multivariate situations, two different multivariate corrections were used. The first used both VEAR and MKAS as explicit selector variables, referred to as M2; the second used the ASVAB GS, EI, NO, and CS tests as well as VEAR and MKAS as explicit selector variables, referred to as M6.

All corrections were calculated by MULTICOR (Sympson & Candell, 1983). The accuracy of the composite validities and the composite validity differences resulting from the three correction methods were compared graphically across SRs.

Q values for the U, M2, and M6 data were computed and graphed, with the expectation that the forms of the graphs would parallel the forms of the validity graphs. Because Q applies only to the explicit selector variable, there were only four Q graphs: (1) VEAR for SS1; (2) MKAS for SS2; (3) VEAR for SS3; and (4) MKAS for SS3.

Results

Table 1 gives ASVAB descriptive statistics for the unrestricted applicant group. The VEAR distribution skew was negative (–.28) whereas the MKAS distribution skew was positive but trivial (.05). Table 2 provides the predictor and criterion means and SDs for all SRs. Consistent with the negative VEAR skew, extreme selection on VEAR resulted in lower SDs for VEAR than were found for MKAS when selection was extreme on MKAS. For example, at SR = .10 for SS1 the VEAR SD was 1.77, whereas at SR = .10 for

Table 1
Descriptive Statistics for VEAR and MKAS, and Five ASVAB Tests

ASVAB	Mean	SD	Skew	Kurtosis
VEAR	101.03	14.49	–.28	–.62
MKAS	101.63	14.31	.05	–.68
GS	50.68	8.69	–.28	–.61
EI	50.23	9.03	–.01	–.64
NO	52.93	7.67	–.75	.19
CS	52.33	7.83	–.13	.31
MC	51.39	9.52	–.15	–.73

Table 2
Means and SDs for the Predictors (VEAR and MKAS) and the Criterion (MC)

		VEAR		MKAS		MC	
SS and SR	N	Mean	SD	Mean	SD	Mean	SD
SS1: Selection on VEAR							
1.00	147,288	101.03	14.49	101.63	14.31	51.39	9.52
.90	132,510	103.95	12.05	103.55	13.51	52.64	8.96
.80	119,222	106.09	10.73	105.10	13.02	53.62	8.63
.70	104,282	108.39	9.45	106.89	12.49	54.67	8.29
.60	87,537	110.94	8.09	108.97	11.89	55.85	7.93
.50	73,784	113.07	6.97	110.82	11.31	56.85	7.61
.40	60,226	115.22	5.84	112.77	10.71	57.88	7.28
.30	44,332	117.82	4.48	115.23	9.97	59.16	6.84
.20	28,533	120.51	3.11	117.89	9.14	60.53	6.37
.10	13,685	123.28	1.77	120.71	8.26	62.06	5.79
SS2: Selection on MKAS							
1.00	147,288	101.03	14.49	101.63	14.31	51.39	9.52
.90	133,241	103.00	13.40	104.15	12.56	52.62	8.95
.80	117,297	104.92	12.62	106.66	11.23	53.91	8.44
.70	104,719	106.43	12.02	108.60	10.30	54.93	8.03
.60	87,720	108.49	11.22	111.24	9.13	56.30	7.50
.50	73,003	110.37	10.45	113.61	8.15	57.52	7.03
.40	59,057	112.28	9.53	115.98	7.23	58.70	6.59
.30	45,681	114.14	8.61	118.48	6.31	59.88	6.14
.20	30,434	116.43	7.38	121.75	5.17	61.37	5.57
.10	15,737	118.86	5.99	125.84	3.76	63.06	4.92
SS3: Selection on VEAR and MKAS							
1.00	147,288	101.03	14.49	101.63	14.31	51.39	9.52
.90	124,982	104.78	11.80	105.06	12.35	53.32	8.64
.80	105,875	107.43	10.44	107.83	11.06	54.85	8.04
.70	89,901	109.78	9.20	110.09	10.16	56.11	7.58
.60	71,926	112.40	7.86	112.87	9.03	57.57	7.04
.50	58,020	114.49	6.77	115.18	8.11	58.75	6.59
.40	45,310	116.49	5.69	117.48	7.25	59.88	6.18
.30	32,184	118.72	4.43	120.01	6.38	61.13	5.71
.20	18,901	121.11	3.11	123.13	5.31	62.63	5.11
.10	7,436	123.60	1.80	126.85	3.95	64.32	4.39

SS2 the MKAS SD was 3.76.

Validities

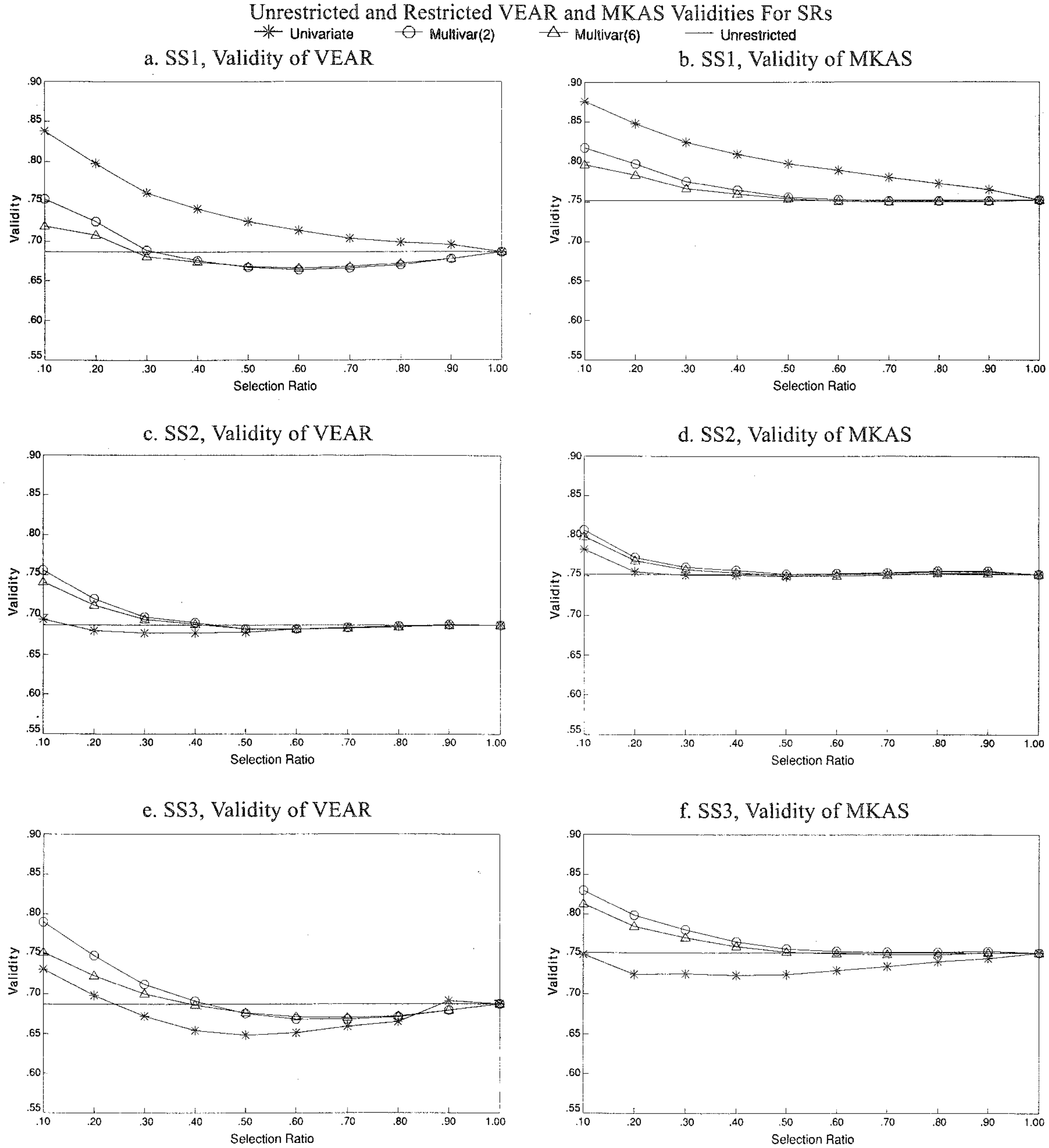
Table 3 presents the uncorrected and corrected validities for the three selection situations for all SRs. All corrected validities were more accurate than the respective uncorrected validities. Figure 1 displays the corrected validities from Table 3 along with the unrestricted validities (i.e., for SR = 1.00).

For selection on VEAR (SS1), univariate corrections overestimated the unrestricted validities, with errors increasing substantially for lower SRs (all correction methods exhibited upward validity drift at low SRs). In contrast, both multivariate correction methods were relatively accurate for the SR range, with M6 slightly more accurate than M2 for low SRs. For selection on MKAS (SS2), univariate and multivariate corrections were approximately equally accurate for SRs .30 and above. For selection on both VEAR and MKAS (SS3), univariate corrections substantially underestimated the unrestricted validities for most of the SR range, whereas multivariate corrections overestimated the unrestricted validity in the low SR range.

Table 3
Uncorrected and Corrected Validities for VEAR and MKAS for SS1, SS2, and SS3 for SRs from 1.00 to .10

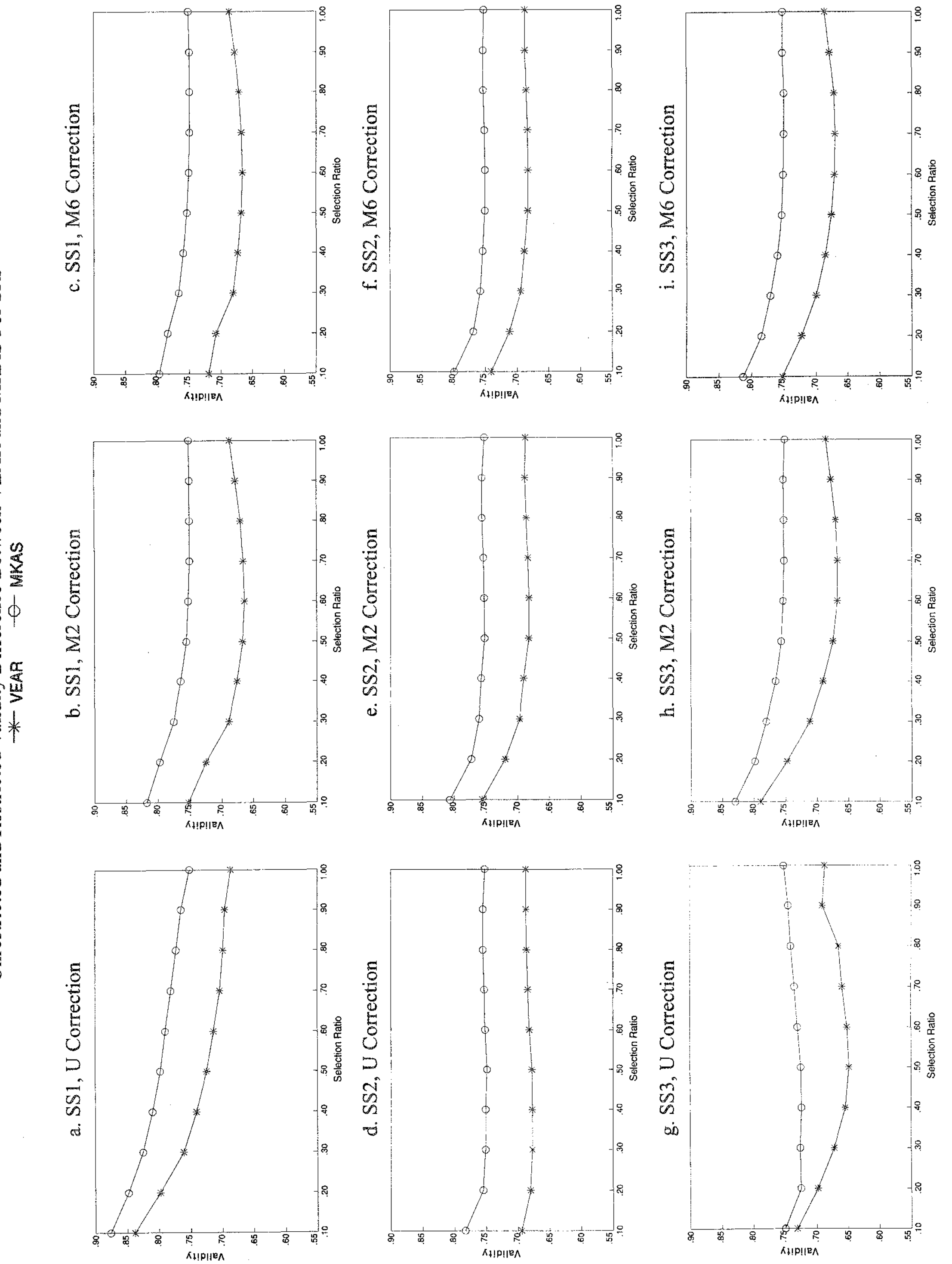
	VEAR Validities				MKAS Validities			
	Uncor-	Corrected			Uncor-	Corrected		
SS and SR	rected	U	M2	M6	rected	U	M2	M6
SS1: Selection on VEAR								
1.00	.687	.687	.687	.687	.752	.752	.752	.752
.90	.627	.696	.678	.678	.719	.765	.751	.750
.80	.586	.699	.670	.672	.700	.773	.751	.750
.70	.543	.704	.666	.668	.679	.781	.751	.750
.60	.495	.714	.664	.666	.655	.790	.753	.751
.50	.451	.725	.667	.668	.634	.798	.756	.754
.40	.406	.741	.676	.674	.614	.810	.765	.760
.30	.341	.761	.689	.681	.589	.825	.776	.767
.20	.273	.798	.725	.708	.564	.848	.798	.784
.10	.184	.838	.753	.719	.538	.876	.818	.797
SS2: Selection on MKAS								
1.00	.687	.687	.687	.687	.752	.752	.752	.752
.90	.640	.687	.688	.687	.711	.755	.756	.753
.80	.597	.686	.686	.685	.671	.755	.756	.753
.70	.562	.684	.684	.683	.636	.753	.754	.751
.60	.515	.682	.682	.682	.589	.752	.753	.750
.50	.472	.678	.682	.682	.542	.749	.752	.750
.40	.433	.677	.690	.688	.498	.751	.757	.754
.30	.394	.677	.697	.694	.449	.751	.761	.758
.20	.354	.680	.720	.712	.384	.755	.773	.769
.10	.309	.694	.756	.741	.314	.783	.807	.799
SS3: Selection on VEAR and MKAS								
1.00	.687	.687	.687	.687	.752	.752	.752	.752
.90	.602	.691	.679	.679	.694	.745	.754	.752
.80	.540	.665	.671	.672	.649	.741	.753	.750
.70	.486	.659	.668	.670	.610	.735	.753	.750
.60	.422	.651	.668	.671	.559	.730	.754	.751
.50	.369	.648	.675	.676	.513	.725	.757	.753
.40	.321	.654	.691	.686	.470	.724	.766	.760
.30	.267	.672	.712	.700	.426	.726	.781	.771
.20	.205	.698	.748	.723	.364	.725	.799	.785
.10	.132	.731	.790	.752	.299	.750	.830	.813

Figure 1
Unrestricted and Restricted VEAR and MKAS Validities For SRs



Figures 2a–2i compare the validity difference between VEAR and MKAS for the three selection situations and three correction methods for all SRs. The validity difference is depicted as the area bounded by the validity lines, with the unrestricted validity difference at SR = 1.00 (.752 − .687 = .065). In general, the nine graphs show that, for each SS, estimates of the unrestricted validity difference from the three

Figure 2
Unrestricted and Restricted Validity Difference Between VEAR and MKAS For SRs



correction methods were generally accurate for most of the SR ranges. However, when VEAR was an explicit selector variable and the correction was U (Figures 2a and 2g) and M2 (Figure 2h), the validity difference was substantially underestimated for low SRs. M6 results appeared most accurate when considering all three selection situations.

Q Values

Table 4 lists (for all SRs and SSs) standard errors of estimate from the least squares linear regression procedure that was used to calculate *Q*. Consistent with the heteroscedastic condition of interest in this study, errors decreased with lower SRs for all selection situations.

Table 4
Standard Errors of Estimate for the Three SSs

	U			
SS and SR	VEAR	MKAS	M2	M6
SS1: Selection on VEAR				
1.00	6.92	—	6.04	5.83
.90	6.98	—	6.07	5.84
.80	6.99	—	6.05	5.81
.70	6.97	—	6.00	5.76
.60	6.89	—	5.92	5.68
.50	6.79	—	5.83	5.59
.40	6.65	—	5.70	5.46
.30	6.43	—	5.50	5.27
.20	6.13	—	5.23	5.03
.10	5.69	—	4.87	4.69
SS2: Selection on MKAS				
1.00	—	6.28	6.04	5.83
.90	—	6.29	6.07	5.84
.80	—	6.25	6.05	5.81
.70	—	6.20	6.00	5.76
.60	—	6.07	5.89	5.65
.50	—	5.91	5.74	5.51
.40	—	5.72	5.56	5.33
.30	—	5.49	5.34	5.13
.20	—	5.15	5.00	4.82
.10	—	4.67	4.54	4.41
SS3: Selection on VEAR and MKAS				
1.00	6.92	6.28	6.04	5.83
.90	6.90	6.22	6.06	5.83
.80	6.77	6.12	6.01	5.77
.70	6.62	6.01	5.91	5.67
.60	6.38	5.84	5.76	5.52
.50	6.13	5.66	5.60	5.36
.40	5.85	5.45	5.40	5.18
.30	5.50	5.17	5.12	4.92
.20	5.00	4.76	4.72	4.55
.10	4.36	4.19	4.18	4.06

Univariate weights. Table 5 shows (also for all SRs and SSs) the unstandardized regression weights (slopes) for the explicit selector composites and the ASVAB tests treated as explicit selector variables. The sum of all explicit selector variable weights for each SR is listed for M2 and M6 (used to calculate *Q*). Values of *Q*, based on the standard errors of estimate in Table 4 and the weight sums in Table 5, are shown in Table 6. For explicit selection on VEAR (SS1), VEAR weights systematically increased for lowering SRs (attributed to VEAR negative skew). This increase, in conjunction with decreasing errors of estimate (see Table 4), accounted for univariate formula overcorrections (*Q* greater than 1). In contrast, for explicit selection on MKAS (SS2), MKAS weights decreased for lowering SRs (flattening slope). This decrease, in conjunction with decreasing errors of estimate (again, see Table 4), accounted for the accuracy of the univariate formula (*Q* equal to or approximating 1). For explicit selection on VEAR and MKAS (SS3), VEAR and MKAS weights systematically and substantially decreased from those obtained from univariate selection for lowering SRs (attributed to reduced covariance from multivariate selection). This decrease, in conjunction with less severe decreases in errors of estimate, accounted for univariate formula undercorrections (*Q* less than 1).

Plots of *Q* for U, M2, and M6 for the explicit selector variables used as explicit selection composites (VEAR for SS1, MKAS for SS2, and both VEAR and MKAS for SS3) paralleled those of the corrected validities in Figure 1 (1a, 1d, 1e, and 1f, respectively). Gross's *Q* equation for the bivariate correction case (Equation 15) and the multivariate version (Equation 16) provided indicators of correction accuracy.

Multivariate weights. The multivariate weights, or slopes, also are reported in Table 5. For all three selection situations, M2 and M6 for lowering SRs departed only slightly from the unrestricted weights. Further, weights for some variables exhibited compensatory, or canceling effects. That is, some test weights increased for lower SRs, whereas others decreased. Even negligible weights behaved systematically. The outcome for all selection situations was, generally, a systematic decrease of ASVAB weight sums for lowering SRs. This decrease, in conjunction with

Table 5
Unstandardized Regression Weights for Variables Treated as Explicit Selectors and Regression Weight Sums Across Tests for M2 and M6

	U		M2			M6						
SS and SR	VEAR	MKAS	VEAR	MKAS	Sum	VEAR	MKAS	GS	EI	NO	CS	Sum
SS1: Selection on VEAR												
1.00	.451	—	.180	.362	.542	.133	.276	.102	.194	−.063	.019	.661
.90	.466	—	.166	.370	.536	.125	.282	.099	.196	−.073	.016	.645
.80	.471	—	.154	.376	.530	.115	.285	.099	.197	−.078	.014	.632
.70	.475	—	.145	.378	.523	.109	.285	.101	.199	−.081	.011	.624
.60	.485	—	.139	.378	.517	.105	.283	.098	.202	−.083	.008	.613
.50	.493	—	.137	.377	.514	.104	.282	.096	.204	−.076	.001	.611
.40	.506	—	.141	.377	.518	.102	.279	.103	.204	−.069	−.003	.616
.30	.520	—	.147	.374	.521	.099	.275	.105	.206	−.057	−.010	.618
.20	.560	—	.177	.372	.549	.112	.269	.116	.204	−.039	−.013	.649
.10	.601	—	.194	.367	.561	.098	.263	.151	.190	−.029	−.012	.661
SS2: Selection on MKAS												
1.00	—	.500	.180	.362	.542	.133	.276	.102	.194	−.063	.019	.661
.90	—	.507	.179	.370	.549	.134	.279	.102	.195	−.068	.015	.657
.80	—	.504	.175	.371	.546	.133	.278	.101	.197	−.073	.016	.652
.70	—	.496	.171	.366	.537	.132	.272	.097	.198	−.074	.013	.638
.60	—	.484	.166	.360	.526	.129	.266	.093	.202	−.074	.012	.628
.50	—	.468	.163	.349	.512	.126	.256	.092	.201	−.073	.010	.612
.40	—	.454	.165	.340	.505	.129	.250	.089	.199	−.074	.008	.601
.30	—	.437	.167	.329	.496	.127	.241	.089	.197	−.068	.008	.594
.20	—	.414	.181	.311	.492	.133	.233	.093	.183	−.064	.009	.587
.10	—	.410	.193	.319	.512	.136	.242	.093	.172	−.036	−.002	.605
SS3: Selection on VEAR and MKAS												
1.00	.451	.500	.180	.362	.542	.133	.276	.102	.194	−.063	.019	.661
.90	.444	.486	.165	.375	.540	.125	.283	.099	.197	−.075	.014	.643
.80	.416	.472	.152	.378	.530	.116	.283	.098	.197	−.081	.013	.626
.70	.400	.455	.144	.374	.518	.112	.278	.097	.199	−.080	.008	.614
.60	.378	.436	.140	.367	.507	.109	.270	.094	.204	−.078	.004	.603
.50	.359	.417	.143	.357	.500	.111	.260	.090	.205	−.069	−.001	.591
.40	.349	.400	.154	.347	.501	.115	.252	.092	.203	−.064	−.003	.595
.30	.344	.381	.167	.339	.506	.115	.245	.100	.200	−.052	−.003	.605
.20	.336	.350	.195	.321	.516	.118	.232	.116	.190	−.044	.004	.616
.10	.321	.332	.216	.317	.533	.121	.235	.117	.172	−.023	−.003	.619

decreasing errors of estimate (see Table 4), accounted for the accuracy of the multivariate formula Q equal to or approximating 1. The multivariate overcorrections in the low SR range were accounted for by rising weights.

Discussion

Multivariate correction accuracy can be attributed to the inclusion of incidental selector variables with adequate distributional properties, the compensatory effects of regression weights, and the related psychometric principle that differentially weighting a large number of correlated predictor variables has little impact on a multiple correlation. The general multivariate correction formulas (as formulated in Equation 12) including incidental selector variables available for both restricted and unrestricted groups may be more accurate than the univariate correction formulas for the following univariate selection situations in which: (1) homoscedasticity and linearity violations for bivariate data can be assessed as nonoffsetting, (2) the incidental selector variables have more adequate distributional properties than the explicit selector variable, (3) multivariate selection is suspected to have occurred on variables related to the explicit selector, (4) small sample sizes could contribute

Table 6
Values of Q for U, M2, and M6 at SRs From 1.00 to .10 for SS1, SS2, and SS3

SS and	U			
SR	VEAR	MKAS	M2	M6
SS1: Selection on VEAR				
1.00	1.000	—	1.000	1.000
.90	1.024	—	.984	.974
.80	1.034	—	.976	.959
.70	1.046	—	.971	.955
.60	1.080	—	.973	.952
.50	1.114	—	.982	.964
.40	1.168	—	1.013	.995
.30	1.241	—	1.056	1.034
.20	1.402	—	1.170	1.138
.10	1.621	—	1.284	1.243
SS2: Selection on MKAS				
1.00	—	1.000	1.000	1.000
.90	—	1.012	1.008	.992
.80	—	1.013	1.006	.990
.70	—	1.005	.997	.977
.60	—	1.001	.995	.980
.50	—	.995	.994	.980
.40	—	.997	1.012	.995
.30	—	1.000	1.035	1.021
.20	—	1.010	1.097	1.074
.10	—	1.103	1.257	1.210
SS3: Selection on VEAR and MKAS				
1.00	1.000	1.000	1.000	1.000
.90	.987	.981	.993	.973
.80	.943	.969	.983	.957
.70	.927	.951	.977	.955
.60	.909	.938	.981	.963
.50	.899	.925	.995	.973
.40	.915	.922	1.034	1.013
.30	.960	.926	1.101	1.085
.20	1.031	.924	1.218	1.194
.10	1.130	.995	1.421	1.345

to weight instability, and (5) the validity of competing selection instruments are to be compared for moderate to stringent selection situations.

More studies of the general multivariate correction formulas treating a wide range of incidental selector variables as explicit selector variables should be conducted before the merits of the procedure can be generalized to the typical personnel or educational selection setting. Simulations could be conducted to vary distributional properties, selection situations, predictor intercorrelations, and validity magnitudes. Varying validity magnitudes would be especially useful because corrected validities have been found to be less accurate than unrestricted validities of low range (Greener & Osburn, 1979).

References

Alexander, R. A., Carson, K. P., Alliger, G. M., & Barrett, G. V. (1984). Correction for restriction of range when both X and Y are truncated. *Applied Psychological Measurement, 8*, 231–240.

Booth-Kewley, S. (1985). *An empirical comparison of the accuracy of univariate and multivariate corrections for range restriction* (NPRDC-TR-85-19). San Diego: Navy Personnel Research and Development Center.

Brewer, J. K., & Hills, J. R. (1969). Univariate selection: The effects of size of correlation, degree of skew, and degree of restriction. *Psychometrika, 34*, 347–361.

Foley, P. P., & Rucker, L. S. (1989). An overview of the Armed Services Vocational Aptitude Battery (ASVAB). In R. F. Dillon & J. W. Pellegrino (Eds.), *Testing: Theoretical and applied perspectives* (pp. 16–35). New York: Praeger Publishers.

Ghiselli, E. E., Campbell, J. P., & Zedeck, S. (1981). *Measurement theory for the behavioral sciences*. San Francisco: W. H. Freeman and Company.

Greener, J. M., & Osburn, H. G. (1979). An empirical study of the accuracy of corrections for restriction of range due to explicit selection. *Applied Psychological Measurement, 3*, 31–41.

Gross, A. L. (1982). Relaxing the assumptions underlying corrections for restriction in range. *Educational and Psychological Measurement, 42*, 795–801.

Gross, A. L., & Fleischman, L. (1983). Restriction of range corrections when both distribution and selection assumptions are violated. *Applied Psychological Measurement, 7*, 227–237.

Guilford, J. P. (1965). *Fundamental statistics in psychology and education*. New York: McGraw-Hill.

Gulliksen, H. (1950). *Theory of mental tests*. New York: Wiley.

Held, J. D. (1992). *Validation of Armed Services Vocational Aptitude Battery (ASVAB) selector composites: Radioman (RM) class "A" school* (NPRDC Tech. Rep. No. 92-16). San Diego: Navy Personnel Research and Development Center.

Held, J. D., & Foley, P. P. (1993). The multivariate correction formulas: Efficacy for univariate selection situations. *Proceedings of the 35th Annual Conference of the Military Testing Association* (pp. 686–691). Williamsburg VA: U.S. Coast Guard.

Kass, R. A., Mitchell, K. J., Grafton, F. C., & Wing, H. (1983). Factorial validity of the Armed Services Vocational Aptitude Battery (ASVAB), Forms 8, 9, and 10: 1981 Army applicant sample. *Educational and Psychological Measurement, 43*, 1077–1087.

Lawley, D. (1943-44). A note on Karl Pearson's selection formula. *Proceedings of the Royal Society of Edinburgh, 62*, (Section A), 28–30.

Levin, J. (1972). The occurrence of an increase in correlation by restriction in range. *Psychometrika, 37*, 93–97.

Linn, R. L. (1968). Range restriction problems in the use of self-selected groups for test validation. *Psychological Bulletin, 69*, 69–73.

Linn, R. L., Harnisch, D. L., & Dunbar, S. B. (1981). Correction for range restriction: An empirical investigation of conditions resulting in conservative corrections. *Journal of Applied Psychology,* 6, 655–663.

Lord, F. M., & Novick, M. R. (1968). *Statistical theories of mental test scores*. Reading MA: Addison-Wesley.

Mendoza, J. L., Hart, D. E., & Powell, A. (1991). A bootstrap confidence interval based on a correlation corrected for range restriction. *Multivariate Behavioral Research, 26*, 255–269.

Novick, M. R., & Thayer, D. T. (1969). *An investigation of the accuracy of the Pearson selection formulas* (ONR-RM-69-22). Princeton NJ: Educational Testing Service.

Olson, C. A., & Becker, B. E. (1983). A proposed technique for the treatment of restriction of range in selection validation. *Psychological Bulletin, 93*, 137–148.

Pearson, K. (1903). On the influence of natural selection on the variability and correlations of organs. *Philosophical Transactions of the Royal Society, London, 200*, (Series A), 1–66.

Prestwood, J. S., Vale, C. D., Massey, R. H., & Welsh, J. R. (1985). *Armed Services Vocational Aptitude Battery: Development of Forms 11, 12, and 13* (AFHRL Tech. Rep. No. 85-16). Brooks AFB TX: Air Force Human Resource Laboratory, Manpower and Personnel Division.

Sands, W. A., Alf, E. F., & Abrahams, N. M. (1978). Correction of validity coefficients for direct restriction in range occasioned by univariate selection. *Journal of Applied Psychology, 63*, 747–750.

Sympson, J. B., & Candell, G. (1983). *MULTICOR: A computer program for correcting means, standard deviations, and correlations for multivariate restriction in range* [Computer program]. San Diego: Navy Personnel Research and Development Center.

Taylor, H. C., & Russell, J. T. (1939). The relationship of validity coefficients to the practical effectiveness of tests in selection: Discussion and tables. *Journal of Applied Psychology, 23*, 565–578.

Thorndike, R. L. (1947). *Research problems and techniques* (Report No. 3). Washington DC: AAF Aviation Psychology Program Research Reports, U.S. Government Printing Office.

Thorndike, R. L. (1949). *Personnel selection: Test and measurement techniques*. New York: Wiley.

Thorndike, R. L. (1982). *Applied psychometrics*. Boston: Houghton-Mifflin.

U.S. Department of Defense. (1982). *Profile of American youth: Nationwide administration of the Armed Services Vocational Aptitude Battery*. Washington DC: Office of the Assistant Secretary of Defense (Manpower, Reserve Affairs, and Logistics).

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Author's Address

Send requests for reprints or further information to Janet Held, Personnel and Organizational Assessment Department (Code 12), Navy Personnel Research and Development Center, 53335 Ryne Road, San Diego CA 92152-7250, U.S.A. Internet: held@nprdc.navy.mil.