University of Minnesota
ST. ANTHONY FALLS HYDRAULIC LABORATORY

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# MIXING AND HEAT TRANSFER OF COOLING WATER DISCHARGES FROM THE MONTICELLO NUCLEAR POWER GENERATING PLANT INTO THE MISSISSIPPI RIVER

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# ABSTRACT

Water temperature data obtained in the Mississippi River downstream from the Monticello Nuclear Power Generating Plant during 32 field surveys have been analyzed and reduced to a form which permits the prediction of certain thermal plume characteristics using readily available plant operational, hydrologic and meteorological input data. To achieve this goal, dimensionless parameters are derived by analysis and applied in order to reduce the field data to simple graphical form and in some cases to semi-empirical equations. All results indicate that lateral turbulent mixing is the main mechanism by which cooling water effluents from the Monticello plant are diluted over the first three miles downstream from the outlet. There is no significant horizontal stratification and not much heat transfer to the atmosphere within the river reach studied.

# List of Symbols

$A$ = cross-sectional area

$A_o$ = cross-sectional area of outlet channel

$A_s$ = surface area enclosed by an isotherm

$A_T$ = total surface area of survey reach

$b$ = river width

$b_o$ = mean outlet width

$c_p$ = specific heat

$C$ = coefficient

$D_y$ = transverse turbulent diffusion coefficient

$f$ = Darcy - Weisbach friction factor

$F_o$ = outlet densimetric Froude number

$g$ = acceleration of gravity

$h$ = mean river depth

$h_o$ = mean outlet depth

$H_s$ = total solar radiation

$k$ = absolute channel roughness

$K_s$ = bulk surface heat exchange coefficient

$m$ = inverse of time constant in the surface heat exchange process

$n$ = coefficient, exponent

$p$ = coefficient

$Q_p$ = plant discharge rate of cooling water

$Q_R$ = river flow rate

$T(x,y)$ = local water temperature

## List of Symbols (Cont.)

$T_A$ = air temperature

$T_D$ = dew point temperature

$T_o$ = outlet water temperature

$T_R$ = river water temperature

$T_{st}$ = temperature standard

$T_m$ = mixed river temperature

$T_w$ = wet bulb temperature

$u_*$ = shear velocity

$V$ = mean river flow velocity

$V_o$ = mean discharge (outlet) velocity

$W$ = wind velocity

$x$ = distance from outlet channel along right shore

$y$ = distance from right shore transverse to river flow direction

$\left(\frac{y}{b}\right)_{max}$ = maximum lateral spreading ratio

$z$ = vertical coordinate (distance below water surface)

$\rho$ = mean water density

$\Delta\rho$ = density differential between river water and outlet

$\theta$ = excess temperature = $T-T_R$

$\theta_E$ = $T_E-T_R$ = excess equilibrium temperature

$\theta_o$ = $T_o-T_R$ = excess outlet temperature

$\frac{x_{max}}{b}$ = maximum longitudinal spreading ratio

## A. OBJECTIVE OF STUDY

The objective of the study described herein is to provide equations and graphs which will aid in predicting certain mixing and spreading characteristics of the cooling water discharge from the Monticello Power Generating Plant into the Mississippi River. Mixing and spreading will be described in terms of three parameters:

(a) maximum lateral spread of water of a given temperature,

(b) maximum longitudinal spread of water of a given temperature, and

(c) total surface area enclosed by a given isotherm.

The information provided will make it possible to select appropriate discharge temperatures and flow rates such that water temperature requirements specified by the permit are not violated.

## B. SITE DESCRIPTION AND FIELD SURVEYS

Numerous water temperature surveys have been conducted by NSP personnel in the immediate vicinity and downstream from the Monticello cooling water outfall. The dates of the surveys, the flow conditions through the plant and in the Mississippi River, and the weather conditions at the time of the survey are listed in Tables 1, 3 and 4 respectively. Each survey took about six to nine hours to complete. The data from these surveys were made available for this study.

All surveys, without exception, have shown that the thermal plume resulting from the cooling water discharge follows the right bank of the river for the first three miles. The lateral spread of the warm water into the Mississippi River as it moves downstream is very gradual. Typical temperature distributions in cross-sections downstream from the outlet and the configuration of the water surface isotherms are shown in Figs. 1 and 2; they were taken from an analysis of some 1971 data.[*] Some of the data used are listed in Northern States Power Company's Annual Reports of the Environmental Monitoring and Ecological Studies Program.

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[*] Stefan, H. and Skoglund, T., "Evaluation of Water Temperature Fields Resulting from Heated Water Discharges," Proceedings of the First World Congress on Water Resources, Vol. IV, pp. 32-47, Chicago, September 1973.

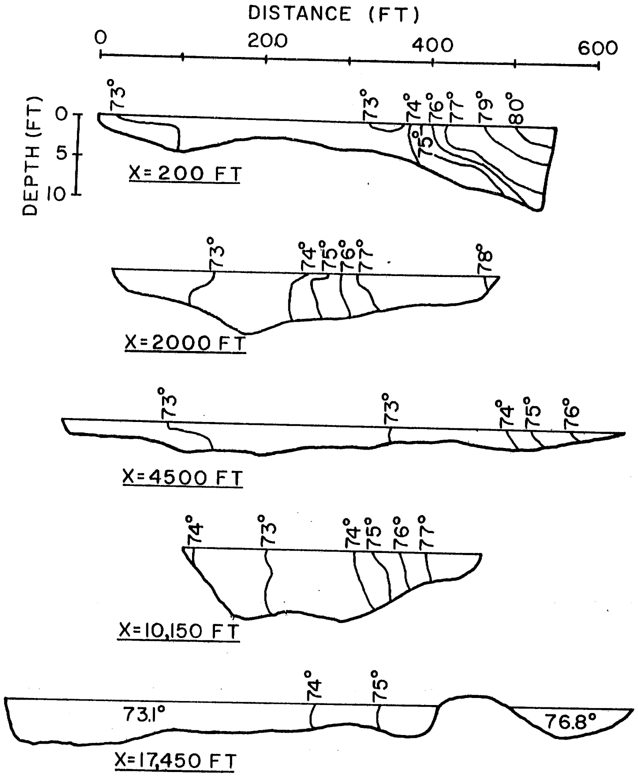

Figure 1. Water Temperature Distribution in Mississippi River downstream of Monticello on July 1, 1971. River cross sections at different distances X from the outfall.

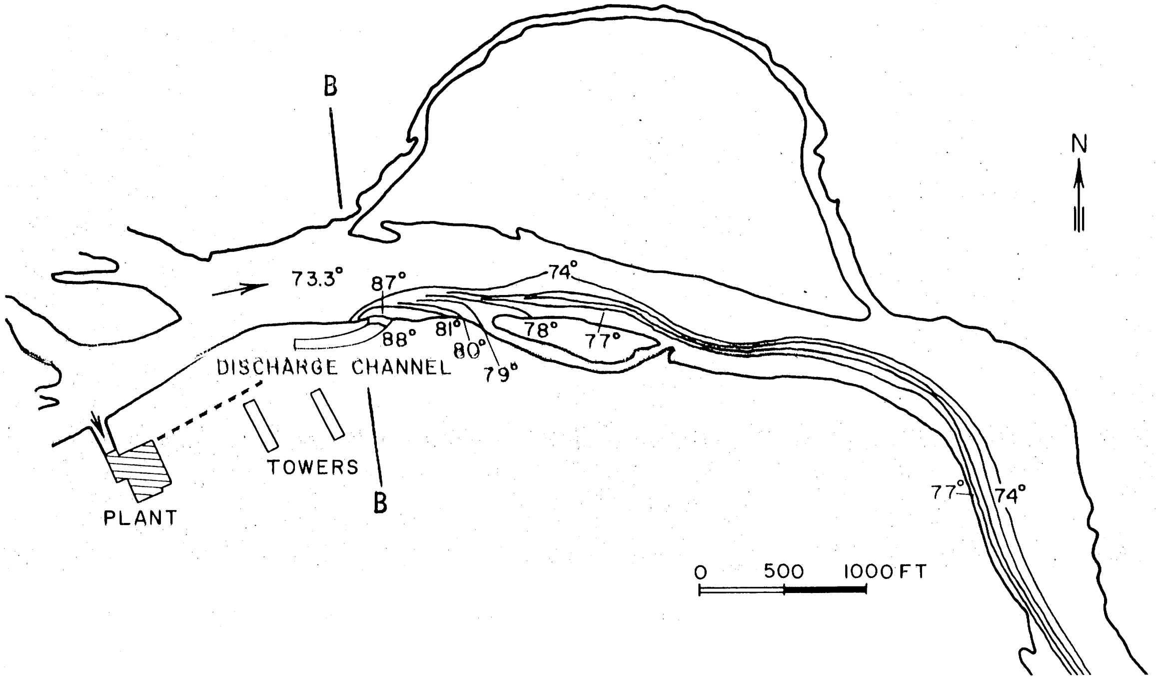

Figure 2. Isotherms at Surface of Mississippi River downstream of Monticello on July 1, 1971.

## C. MATHEMATICAL ANALYSIS OF MIXING AND HEAT TRANSFER CHARACTERISTICS

### 1. General Approach

The method of analysis **applied in this study uses a three-step approach:**

1. A mathematical description of the main features of the effluent flow field, with emphasis on mixing and surface heat transfer. Idealization of the channel geometry **is necessary.**
2. Derivation of dimensionless numbers containing all significant dependent and independent variables. This is accomplished by normalization and rearrangement of the mathematical results.
3. Reduction of all field temperature measurements in terms of the pertinent dimensionless parameters and graphical representation of the results. Subsequent establishment of analytical expressions to correlate these dimensionless numbers.

The above approach combines the best available knowledge and understanding of the flow and exchange processes at the cooling water outfall with numerous actual field observations. The tie is made by the principles of dynamic similarity. Expressing observed fluid flow phenomena in dimensionless form is a classical procedure (see e.g. fluid mechanics textbooks on pipe friction analysis, drag and lift on submerged bodies, or EPA's workbook on plume analysis).

The final product of the study are graphs or equations which can be used as deterministic design tools to interpolate and extrapolate (within reason) from observed flow and mixing conditions.

### 2. Cold Water Wedge

The flow from the cooling water outlet channel at the Monticello plant into the Mississippi River is at low velocity and at a small angle with respect to the shoreline. When the river flow is on the order of 4,000 cfs, the maximum depth of the outlet channel as measured at transect B (shown only in field survey drawings) is on the order of 2.5 ft and the channel cross-sectional area on the order of 220 $ft^2$. At a channel discharge rate of 650 cfs, which is close to the maximum flow used since the beginning of plant operation, the discharge velocity is therefore on the order of only 3.0 fps. Most of the time the discharge velocity is significantly less.

Because of its shallow depth, the outlet channel flow is always well mixed vertically, except if the cooling water discharge rate is unusually low. As an example, a temperature range from 66°F to 48°F from top to bottom was observed in the outlet channel at transect B on November 9, 1971, when the reported flow in the channel was 208 cfs. The upstream river temperature was 33.7°F, indicating that the wedge penetrating into the outlet channel was a mixed water wedge.

When a cold water wedge forms, a two-layered flow analysis indicates that an internally critical flow conditions forms at the outlet. The densimetric Froude number for the upper layer is then unity

$$F_o = \frac{V_o}{\sqrt{\frac{\Delta\rho_o}{\rho} g h_o}} = 1.0$$

where $V_o$ = average discharge velocity

$h_o$ = mean outlet depth

$g$ = acceleration of gravity

$\Delta\rho_o$ = density differential between river water and discharged cooling water

$\rho$ = mean water density

The above condition may be used to determine a range of $\Delta\rho$'s which will prevent cold water penetration and stratification in the outlet channel.

## 3. Lateral Turbulent Diffusion Process

Discharges from the outlet channel of the Monticello plant into the Mississippi River are at velocities generally smaller than the velocity with which the river flows past the outlet. This is shown subsequently by comparing Tables 1 and 3. The effluent flow and the river flow encounter at an angle somewhere between 0 and 60°. Experience indicates that secondary currents and increased turbulence are produced where streams of different velocity and flow direction meet.

Numerous temperature surveys made in the immediate vicinity of the outfall by NSP personnel between 1971 and 1973 indicate a very rapid dilution of the effluent near the outlet. Isotherms are stretched out in the direction of the river flow. Simultaneously with the advection, that is the carrying-away of the warm water with the river flow, some spread of the warm water in lateral direction, that is transverse to the general river flow occurs. The river generally is well-mixed in vertical direction as indicated by the vertical isotherms found in numerous surveys. The well-mixed condition in vertical direction and the very gradual lateral spread of the warm water can be attributed to river turbulence.

There is evidence of secondary effects contained in some of the field temperature data. Tilted isotherms near the outlet indicate lateral surface spreading caused by the buoyancy of the warm water effluent. Other irregularities in isotherm patterns probably reflect secondary motions caused by irregular river morphology.

No jet effect of significance at the outlet and no wind effect on the isotherms could be detected. Transport and mixing of the effluent are dominated by the gravity-induced river flow.

Heat transfer through the water surface is an insignificant factor in the temperature distribution within several thousand feet from the outlet, as shown by an order of magnitude estimate of the rate of heat transfer through the water surface compared to the total rate of heat discharged. The exact value is weather dependent.

For the reasons given above, the first and most appropriate idealization of the flow and mixing processes immediately downstream from the cooling water outlet channel of the Monticello plant is a two-dimensional turbulent diffusion model for discharge from a point or line source into a moving stream. The river flow is approximated by parallel streamlines. The equation describing the simultaneous longitudinal advection and lateral turbulent diffusion process downstream from the cooling water outfall under flow conditions which do not vary with time is the simplified, two dimensional, steady-state, depth-integrated diffusion equation.

$$\frac{\partial}{\partial x}\left(uz\theta\right) = \frac{\partial}{\partial y}\left(D_y z \frac{\partial \theta}{\partial y}\right)$$

Longitudinal turbulent diffusion and transverse advection have been ignored.

In this relationship

$\theta$ = excess temperature of the warm water = $T(x,y) - T_R$
$u$ = river flow velocity
$T, T_R$ = local and natural river water temperature, respectively
$x$ = coordinate in river flow direction
$y$ = coordinate transverse to river flow direction
$D_y$ = transverse (lateral) turbulent diffusion coefficient

Since it would be very cumbersome if not impossible to describe local velocities $u$ and local depths $z$ as functions of $x$ and $y$, these parameters are replaced by their cross-sectional mean values $V = Q/A$ and $h = A/b$, where $A$ is the river cross-sectional area and $b$ is the river width. The result is

$$V \frac{\partial \theta}{\partial x} = D_y \frac{\partial^2 \theta}{\partial y^2}$$

For the injection of heat at a continuous rate, $Q_p \theta_o \rho c_p$ at point $x = 0$, $y = 0$, the well-known solution of the diffusion equation is

$$\frac{\theta}{\theta_o} = \frac{Q_p}{hV(4\pi D_y \frac{x}{V})^{1/2}} \exp\left(\frac{-y^2}{4D_y \frac{x}{V}}\right)$$

This equation is already normalized and provides the temperature distribution in the entire idealized flow and mixing field. It can be used to provide more specific information such as: the maximum lateral spread of an isotherm, the maximum longitudinal spread of an isotherm and the surface area enclosed by an isotherm for point-source-like conditions which may approximate real conditions at some distance from the outlet.

### a. Maximum Lateral Spread $y_{max}$

The above equation is differentiated with respect to $x$ and the term $dy/dx$ set equal to zero. The resulting equation obtained after some simplification is:

$$\frac{y_{max}}{b} \frac{\theta}{\theta_o} = 0.43 \frac{Q_p}{Q_R}$$

where $y_{max}$ is the maximum spread of the isotherm with temperature difference $\theta$.

where $Q_R$ = river flow = Vhb.

## b. Maximum Longitudinal Spread $x_{max}$

The form of the general solution indicates that $x_{max}$ is obtained for $y = 0$. After some simplification, the solution is

$$\frac{x_{max}}{b}\left(\frac{\theta}{\theta_o}\right)^2 = \left(\frac{Q_p}{Q_R}\right)^2 \cdot \frac{bV}{4\pi D_y}$$

The turbulent eddy diffusivity in a stream is known to be related to river flow characteristics. A generally accepted relationship is of the form

$$D_y = \text{const } h\, u_*$$

where $u_*$ = the shear velocity of a stream

$$u_* = V\sqrt{\frac{f}{8}}$$

and therefore

$$D_y = c\ hV\sqrt{\frac{f}{8}}$$

Actually c is a function of the aspect ratio b/h of the channel as a data summary given by H. Fischer* indicates. Also the friction factor f is a function of relative roughness k/h. Combining all this information indicates that the term $\frac{bV}{4\pi D_y}$ depends on river aspect ratio, b/h, although perhaps not very strongly. The river aspect ratio in turn is a function of river flow $Q_R$. Hence

$$\frac{x_{max}}{b}\left(\frac{\theta}{\theta_o}\right)^2 = \left(\frac{Q_p}{Q_R}\right)^2 F(Q_R)$$

## c. Surface Areas Enclosed by Isotherms $A_s$

In a previous study (Stefan and Skoglund, 1973) it was shown that the 1971 temperature survey data for the Monticello cooling water mixing zone could be

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*H. Fischer, "Longitudinal Dispersion and Turbulent Mixing in Open Channel Flow," Annual Reviews of Fluid Mechanics, Vol. 5, 1973, pp. 59-78.

reduced to dimensionless form by using a dimensionless relationship of the form

$$\frac{A_s V}{Q_p} = F\left(\frac{\theta}{\theta_o}, \frac{Q_p}{Q_R}\right)$$

where $A_s$ is the surface area enclosed by an isotherm of excess temperature $\theta$. This relationship will be used again.

## 4. Surface Heat Transfer

In the very far field of the Monticello cooling water plume the lateral mixing process described above continues. In addition, heat transfer through the water surface becomes increasingly important. A term describing that process can be incorporated into the original heat transport equation. If the equilibrium temperature concept is used, the equation is

$$V\frac{\partial\theta}{\partial x} - D_y\frac{\partial^2\theta}{\partial y^2} = m\theta - m(T_R - T_E)$$

where

$m = \frac{K_s}{\rho c_p h}$ = time constant

$K_s$ = unit (bulk) rate of surface heat transfer

$\rho c_p$ = specific heat per unit volume

$T_E$ = equilibrium temperature

$T_R$ = river water temperature upstream from outfall

The solution of this equation for a point source, as described earlier is,

$$\frac{\theta}{\theta_o} - \frac{T_E - T_R}{T_o - T_R}\left(1 - e^{-mx/V}\right) = \frac{Q_p}{hV\left(4\pi D_y \frac{x}{V}\right)^{1/2}} \exp\left(-\frac{y^2}{4D_y\frac{x}{V}} - \frac{mx}{V}\right)$$

This equation reduces to the nearfield equation for $m = 0$. As $x$ becomes larger, the additional terms become increasingly important. It is readily apparent that two additional dimensionless parameters are introduced. They are $\frac{T_E - T_R}{T_o - T_R} = \frac{\theta_E}{\theta_o}$ and $\frac{mx}{V}$. The last parameter contains a variable distance $x$ which is known only if field data are available. For the purpose of prediction $x$ will be replaced by the total surface area of the river reach under investigation divided by the mean river width. The dimensionless parameter then has the form

$$\frac{mA_T}{bV}$$

or

$$\frac{K_s A_T}{\rho c_p Q_R}$$

## 5. Dimensionless Parameters Used For The Plotting of Field Data

The mathematical analysis of the idealized outfall given in the preceeding section has produced the dimensionless parameters which describe the essential dynamic features of the cooling water flow at Monticello. The equations between these dimensionless parameters are, however, based on a very simplified geometry and are not necessarily applicable to the Monticello site without a comparison with measured data. The actual river geometry at Monticello must be incorporated into the analytical solution.

To achieve this, a classical procedure of hydrodynamic analysis is followed: Field data are used to compute actual numerical values of the dimensionless parameters. These numbers are then graphed against each other and analytical relationships are derived. These relationships then contain the effects of the real river geometry. A significant feature of this real geometry is the limited width $b$ of the river (about 500 ft for average flows). Because $b$ is also variable with distance $y_{max}/b$ is replaced by $(y/b)_{max}$.

The following relationships are sought:

$$\left(\frac{y}{b}\right)_{max} = F_1\left(\frac{\theta}{\theta_o}, \frac{Q_p}{Q_R}, F_o, \frac{T_E-T_R}{T_o-T_R}, \frac{mA_T}{bV}\right)$$

$$\frac{x_{max}}{b} = F_2\left(\frac{\theta}{\theta_o}, \frac{Q_p}{Q_R}, F_o, \frac{T_E-T_R}{T_o-T_R}, \frac{mA_T}{bV}\right)$$

$$\frac{A_s V}{Q_p} = F_3\left(\frac{\theta}{\theta_o}, \frac{Q_p}{Q_R}, F_o, \frac{T_E-T_R}{T_o-T_R}, \frac{mA_T}{bV}\right)$$

The dimensionless numbers on the right side are listed in an anticipated decreasing order of significance. The flow field within 17,000 feet from the outlet will be analyzed. Most of the temperature drop occurs in that reach.

## D. DATA BASE AND COMPUTATION OF DIMENSIONLESS NUMBERS

### 1. Water Temperature T

The water temperature data used were from surveys conducted from 1971 through 1973 by NSP staff. Water temperatures were measured at 19 transects between the Monticello plant and the City of Monticello during 35 surveys. The data have been presented in graphical form in horizontal or vertical sections. Examples of these presentations can be found in the 1971 Annual Report of the NSP Environmental Monitoring and Environmental Studies Program.

The lateral spread of the cooling water into the Mississippi River was measured on the original blueprints of the transects. The distance of each isotherm from the right bank (on which the plant discharge channel is located) was measured at the water surface.

### 2. Effluent Temperature $T_o$

Discharge temperatures were obtained by computing the area-weighted average of the temperatures measured in transect B near the end of the outlet channel. With some exceptions, temperatures in the channel at that location were nearly uniform. On two occasions, November 9, 1971 and April 4, 1972, when channel discharges were only 205 cfs and 300 cfs respectively, a mixed water wedge penetrated the channel. The discharge temperature was then computed for the area above the thermocline.

### 3. Ambient River Temperature $T_R$

Values reported in the river portion of transect B a short distance upstream from the outlet were averaged. This section was selected to be representative of natural ambient river temperatures.

### 4. Cooling Water Discharge Rate $Q_p$

Canal discharges of cooling water $Q_p$ were obtained from NSP effluent reports. Hourly rates of discharge, as measured at the overflow weir, were averaged between the hours of 10:00 a.m. and 5:00 p.m., the usual duration of a water temperature survey.

There was strong fluctuation in $Q_p$ during each of the temperature surveys on June 22, 1971 and November 9, 1971. $Q_p$ for each survey was not well defined. Therefore, the data for these surveys were not considered in the final results of the study. In addition, $Q_p$ for the temperature survey on June 19, 1973 was very low. The data for this survey also was ignored. Plant discharges are shown in Table 1.

### 5. Outlet Depth $h_o$

To compute an average discharge velocity and a densimetric Froude number at the outlet, an average outlet depth was required. It could be expected that this depth would depend primarily on river stage or river flow. Average outlet channel depths measured on the blueprints of all the transects were plotted versus river flow. The function

$$h_o = 0.04\, Q_R^{0.5} \qquad \text{where } h_o \text{ is in feet, and } Q_R \text{ is in cfs}$$

provided a reasonable fit to the data which showed significant scatter, particularly for the low flows in the range from 2,000 to 4,000 cfs. A better data base for the depths of the outlet channel would be desirable.

## TABLE 1

## OUTLET CHANNEL DISCHARGE CHARACTERISTICS, TEMPERATURES AND DENSIMETRIC FROUDE NUMBERS

| Date of Temperature Survey | $Q_p$ (cfs) | $\frac{Q_p}{Q_R}$ | $h_o$ (ft) | $b_o$ (ft) | $A_o$ ($ft^2$) | $V_o$ (fps) | $T_o$ (°F) | $F_o$ |
|---|---|---|---|---|---|---|---|---|
| 7-1-71 | 558 | .070 | 3.58 | 97 | 347 | 1.61 | 87.9 | 4.04 |
| 7-6-71 | 558 | .083 | 3.29 | 100 | 329 | 1.70 | 99.5 | 2.63 |
| 8-2-71 | 519 | .152 | 2.34 | 95 | 222 | 2.34 | 80.5 | 6.34 |
| 8-3-71 | 594 | .188 | 2.25 | 98 | 221 | 2.69 | 92.1 | 5.24 |
| 8-13-71 | 524 | .262 | 1.79 | 147 | 263 | 1.99 | 86.2 | 6.10 |
| 8-19-71 | 585 | .269 | 1.86 | 100 | 186 | 3.15 | 103.9 | 5.98 |
| 9-2-71 | 633 | .223 | 2.13 | 100 | 213 | 2.97 | 87.0 | 8.06 |
| 9-20-71 | 625 | .284 | 1.88 | 95 | 179 | 3.49 | 74.2 | 11.6 |
| 2-22-72* | 527 | .074 | 2.64 | 80 | 211 | 2.50 | 82.8 | 2.72 |
| 2-23-72* | 526 | .121 | 2.63 | 80 | 210 | 2.50 | 62.4 | 8.40 |
| 3-1-72* | 526 | .125 | 2.59 | 90 | 233 | 2.26 | 60.2 | 8.52 |
| 4-4-72 | 300 | .021 | 4.79 | 85 | 407 | 0.74 | 83.6 | 0.97 |
| 5-23-72 | 594 | .041 | 4.80 | 85 | 408 | 1.46 | 84.5 | 2.62 |
| 6-22-72 | 543 | .079 | 3.32 | 87 | 289 | 1.88 | 99.1 | 2.58 |
| 7-5-72 | 559 | .131 | 2.61 | 85 | 222 | 2.52 | 91.3 | 4.81 |
| 8-3-72 | 598 | .036 | 5.16 | 90 | 464 | 1.29 | 80.4 | 2.98 |

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*Ice cover noted during temperature survey.

## TABLE 1 (Cont.)

## OUTLET CHANNEL DISCHARGE CHARACTERISTICS, TEMPERATURES AND DENSIMETRIC FROUDE NUMBERS

| Date of Temperature Survey | $Q_p$ (cfs) | $\frac{Q_p}{Q_R}$ | $h_o$ (ft) | $b_o$ (ft) | $A_o$ ($ft^2$) | $V_o$ (fps) | $T_o$ (°F) | $F_o$ |
|---|---|---|---|---|---|---|---|---|
| 8-14-72 | 651 | .043 | 4.95 | 90 | 446 | 1.46 | 86.6 | 2.45 |
| 8-18-72 | 633 | .046 | 4.67 | 90 | 420 | 1.51 | 89.5 | 2.73 |
| 8-23-72 | 626 | .048 | 4.57 | 90 | 411 | 1.52 | 82.5 | 3.14 |
| 8-30-72 | 627 | .043 | 4.60 | 92 | 423 | 1.48 | 86.6 | 2.55 |
| 9-7-72 | 597 | .054 | 4.21 | 87 | 366 | 1.63 | 94.0 | 2.12 |
| 9-15-72 | 571 | .077 | 3.44 | 89 | 306 | 1.87 | 94.2 | 2.66 |
| 6-27-73 | 558 | .150 | 2.44 | 85 | 207 | 2.70 | 93.5 | 5.21 |
| 7-2-73 | 573 | .135 | 2.61 | 83 | 217 | 2.64 | 86.8 | 6.50 |
| 7-5-73 | 572 | .149 | 2.48 | 90 | 223 | 2.57 | 87.4 | 6.57 |
| 7-12-73 | 573 | .166 | 2.35 | 90 | 212 | 2.70 | 89.9 | 7.12 |
| 7-16-73 | 569 | .153 | 2.44 | 86 | 210 | 2.71 | 85.8 | 7.36 |
| 7-19-73 | 557 | .141 | 2.51 | 90 | 226 | 2.46 | 86.7 | 7.03 |
| 7-25-73 | 573 | .239 | 1.96 | 90 | 176 | 3.26 | 89.6 | 9.37 |
| 7-30-73 | 571 | .176 | 2.28 | 85 | 194 | 2.94 | 81.7 | 8.39 |
| 8-13-73<br>a | 586 | .079 | 3.46 | 85 | 294 | 1.99 | 86.6 | 4.40 |
| 8-13-73<br>b | 573 | .077 | 3.46 | 89 | 308 | 1.86 | 105.2 | 2.39 |

## 6. River Flow $Q_R$

Mississippi River discharge records at the Monticello plant site were noted to be less than the reported discharge data from the U.S. Geological Survey (U.S.G.S.) gaging station near Royalton,[*] Minnesota during all of the temperature survey dates in 1971 and for 8 of 10 dates in 1973. The river discharges as reported by plant personnel during the temperature survey dates in 1972, however, were noted to be more representative of the flows expected. Since the contributing drainage area between the gaging station near Royalton and the plant site is about 2,200 square miles, the discharges reported for the temperature surveys in 1971 and 1973 are likely to be in error since there is no appreciable diversion and/or storage of continuous river discharge between the two locations. In addition, the staff gage at the plant site is reported to be near the location of the cooling water intake. Water levels in the immediate vicinity of the intake would likely be subject to some drawdown effect. The drawdown effect would be dependent upon the cooling water withdrawal rate. Therefore, an effort was made to determine discharge data for the plant site which would be more representative of actual river discharges. The discharge data would need to be compatible with the other input parameters needed for purposes of study.

Streamflow data from U.S.G.S. gaging stations on the Mississippi River and major tributaries between the gaged sites near Royalton to that near Anoka, Minnesota (both sites on the Mississippi River main stem) were used in establishing probable river discharges at the plant site. The gaging station on the Mississippi River near Anoka was used as an index station for which contributing streamflows below the plant site to that gage location were subtracted. River discharges were determined for the time period between 10:00 a.m. and 5:00 p.m. for those days during which the temperature surveys were made. Computed discharges per square mile as determined from recorded streamflow data were plotted against the representative drainage areas in order to establish discharge data for ungaged contributing drainage areas. Bi-hourly discharge data for the gaging stations were used for predicting runoff when available. Otherwise, daily average values of discharge were used. The U.S.G.S. gaging stations and pertinent data considered in the analysis are presented in Table 2. Times of travel for river flow were

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[*] above Monticello.

considered in the analysis, utilizing an assumed mean channel velocity of 3 feet per second (fps). This travel time would, in part, simulate the lag characteristics of discharge within the individual drainage basins. Contributing streamflows from major tributaries and local inflows to the Mississippi River above the gaging station near Anoka thus were subtracted from the recorded discharges of the index station to establish the probable river discharges at the plant site for all but four temperature survey dates in 1973 (June 19, June 27, July 2, and July 5). Discharges for these dates were estimated from daily flows reported at the gaging station near Royalton. Estimated discharges for the dates of the temperature surveys are presented in Table 3. It should be noted that the estimated discharge values for the 1972 temperature survey dates compared well with those reported by NSP.

The Mississippi River discharges, as estimated for the plant site, were plotted against those discharge values reported by NSP. For the present, this graph can be used by NSP for estimating a reliable discharge from its own staff gage measurements. However, it is recommended that an adequate staff gage location be established at a location near the plant site in the near future. Also, it is likely that current-meter measurements could be made at the Highway 25 bridge at Monticello. The correlation of stage with discharge for the two locations should be firmly established and checked periodically with consideration given to cooling water intake discharge and canal discharge flow from the plant to the river.

## 7. River Depth h

To compute an average river flow velocity, a mean river depth was required. Transect B, immediately upstream from the end of the outlet channel, was selected as a representative river section. Mean river depths in transect B were determined from the drawings provided and plotted versus river flow $Q_R$. The function

$$h = 0.038\, Q_R^{0.52} \qquad \text{where } h \text{ is in feet, and } Q_R \text{ is in cfs}$$

provided a reasonable fit to the data.

## TABLE 2

## U.S. GEOLOGICAL SURVEY GAGING STATIONS AND MISCELLANEOUS SITES WITH DRAINAGE AREAS

| | River | Gaging Station Location | Station Number | Drainage Area (sq. mi.) |
|---|---|---|---|---|
| Continuous-Record Stations | | | | |
| | Mississippi | near Royalton | 05267000 | 11,600 |
| | Elk | near Big Lake | 05275000 | 615 |
| | Crow | at Rockford | 05280000 | 2,520 |
| | Rum | near St. Francis | 05286000 | 1,360 |
| | Mississippi | near Anoka | 05288500 | 19,100 |
| Miscellaneous Sites | | | | |
| | Mississippi | at St. Cloud | --------- | 13,320 |
| | Mississippi | at Clearwater | --------- | 13,655 |
| | Mississippi | at Monticello | --------- | 13,800 |
| | Mississippi | at Elk River | --------- | 14,500 |
| | Mississippi | at Anoka | 05283500 | 17,100 |

## TABLE 3

## MISSISSIPPI RIVER DISCHARGE CHARACTERISTICS AND AMBIENT TEMPERATURES AT TRANSECT B**

| Date | $Q_R$ (cfs) | h (ft) | b (ft) | b/h | V (fps) | A (ft$^2$) | $T_R$ (°F) |
|---|---|---|---|---|---|---|---|
| 7-1-71 | 8,000 | 4.07 | 470 | 123 | 4.19 | 1,910 | 72.9 |
| 7-6 | 6,760 | 3.73 | 470 | 134 | 3.86 | 1,750 | 76.7 |
| 8-2 | 3,420 | 2.62 | 473 | 191 | 2.78 | 1,240 | 66.8 |
| 8-3 | 3,170 | 2.51 | 472 | 199 | 2.68 | 1,180 | 67.9 |
| 8-13 | 2,000 | 1.98 | 467(est.) | 253 | 2.15 | 925 | 74.0 |
| 8-19 | 2,170 | 2.07 | 467 | 242 | 2.24 | 965 | 78.2 |
| 9-2 | 2,840 | 2.38 | 470 | 210 | 2.55 | 1,120 | 74.0 |
| 9-20 | 2,200 | 2.08 | 477 | 241 | 2.25 | 990 | 61.0 |
| 2-22-72* | 4,350 | 2.96 | 468 | 169 | 3.12 | 1,390 | 32.0 |
| 2-23* | 4,340 | 2.96 | 468 | 169 | 3.12 | 1,390 | 32.0 |
| 3-1* | 4,190 | 2.91 | 472 | 172 | 3.07 | 1,370 | 32.0 |
| 4-4 | 14,300 | 5.51 | 469 | 91 | 5.53 | 2,580 | 32.0 |
| 5-23 | 14,400 | 5.53 | 470 | 90 | 5.55 | 2,600 | 70.5 |
| 6-22 | 6,910 | 3.77 | 467 | 133 | 3.90 | 1,760 | 68.2 |
| 7-5 | 4,270 | 2.93 | 474 | 170 | 3.09 | 1,390 | 69.7 |
| 8-3 | 16,600 | 5.95 | 471 | 84 | 5.95 | 2,800 | 72.4 |
| 8-14 | 15,300 | 5.70 | 466 | 88 | 5.71 | 2,660 | 71.6 |

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*Ice cover noted during temperature survey.

**Upstream from outlet channel.

## TABLE 3 (cont.)

## MISSISSIPPI RIVER DISCHARGE CHARACTERISTICS AND AMBIENT TEMPERATURES AT TRANSECT B

| Date | $Q_R$ (cfs) | h (ft) | b (ft) | b/h | V (fps) | A ($ft^2$) | $T_R$ (°F) |
|---|---|---|---|---|---|---|---|
| 8-18 | 13,700 | 5.37 | 468 | 93 | 5.41 | 2,510 | 76.8 |
| 8-23 | 13,100 | 5.26 | 471 | 95 | 5.30 | 2,480 | 71.3 |
| 8-30 | 14,600 | 5.57 | 470 | 90 | 5.59 | 2,620 | 71.2 |
| 9-7 | 11,100 | 4.83 | 475 | 103 | 4.91 | 2,290 | 64.6 |
| 9-15 | 7,430 | 3.92 | 470 | 128 | 4.04 | 1,840 | 64.0 |
| 6-27-73 | 3,720 | 2.73 | 464 | 183 | 2.90 | 1,270 | 70.4 |
| 7-2 | 4,260 | 2.93 | 467 | 171 | 3.09 | 1,370 | 73.9 |
| 7-5 | 3,840 | 2.78 | 464 | 180 | 2.94 | 1,290 | 75.0 |
| 7-12 | 3,450 | 2.63 | 468 | 190 | 2.79 | 1,230 | 78.2 |
| 7-16 | 3,730 | 2.73 | 468 | 183 | 2.90 | 1,280 | 74.4 |
| 7-19 | 3,950 | 2.82 | 469 | 177 | 2.98 | 1,320 | 77.0 |
| 7-25 | 2,400 | 2.18 | 469 | 230 | 2.35 | 1,020 | 77.7 |
| 7-30 | 3,250 | 2.55 | 469 | 196 | 2.72 | 1,200 | 69.6 |
| 8-13<br>a | 7,460 | 3.92 | 470 | 127 | 4.05 | 1,840 | 74.6 |
| 8-13<br>b | 7,460 | 3.92 | 470 | 127 | 4.05 | 1,840 | 74.6 |

## 8. River Width b

The mean river width in transect B was measured on the drawings provided for each survey. The dependence on river flow $Q_R$ was determined. The main channel width reported at transect B for the surveys remained fairly constant at about 470 feet, indicating that most flows were contained within steep-sided channel banks.

## 9. Mean River Flow Velocity V

The mean river flow velocity $V$ for each survey was computed for transect B only since the velocity at this section would be representative of the free-stream velocity at the downstream transect locations. The mean river flow velocity was determined by dividing the river flow $Q_R$ by the cross-sectional area of flow. The cross-sectional area of flow was determined by multiplying the surface width $b$ by the mean depth of flow $h$. The relationship

$$V = Q_R/bh = 0.056\ Q_R^{0.48}$$

where V is in fps,
$Q_R$ is in cfs, and
b and h are in feet

was used to determine the mean river flow velocity.

## 10. Spread of Warm Water y

The lateral spread of cooling water into the Mississippi River was measured along all of the transects as drawn on the original blueprints. The distance of each isotherm reported from the right bank (looking downstream) was measured at the water surface. A total of approximately 4,800 measurements (35 surveys * 17 transects * 8 isotherms) was recorded.

## 11. Surface Area $A_s$

The water surface area enclosed by each isotherm in every survey was computed from the lateral surface spread values y.

$$A_s(T) = \sum_{i=1}^{n} a_i y_i(T)$$

$a_i$ is a weighting factor equal to the sum of one half the distance to the adjacent transects

$$a_i = 1/2(x_{i+1} + x_{i-1} - 2x_i)$$

where i refers to a transect number and x is the distance from the outlet channel.

## 12. Maximum Longitudinal Spread of Warm Water $x_{max}$

The maximum longitudinal spread of each isotherm as noted downstream from the outlet channel at or near the right bank (looking downstream) was measured on the blueprint drawings. Theoretically, $x_{max}$ is always obtained near the river bank.

Interpretation of field temperature survey data showed this condition for most cases. However, some temperature isotherms achieved an indicated maximum downstream distance away from the right bank. For these cases, the $x_{max}$ distance was measured at the point of the indicated maximum distance shown on the blueprint drawings. Isotherms shown as continuing downstream of transect 17 were neglected in the theoretical analyses since lack of data downstream of transect 17 would preclude accurate evaluation of the longitudinal temperature profiles.

## 13. Weather Conditions

Field measurements were taken by NSP staff to note weather conditions during each of the temperature surveys. The weather data reported on the blueprint drawings is included in Table 4. The effect of weather conditions on the thermal discharges was considered in the study.

## 14. Equilibrium Temperature $T_E$ and Bulk Surface Heat Exchange Coefficient $K_s$

Values of $T_E$ and $K_s$ were computed by the method proposed by Brady, Graves and Geyer (1969). The required weather data were obtained in the following way:

Dew point temperatures $T_d$ were computed from wet bulb and dry bulb temperatures reported by NSP, using a table reported by Linsley, Kohler and Paulhus.[1]

Wind velocities W were those reported by NSP on the blueprints of the temperature surveys.

Solar radiation $H_s$ was that measured by Dr. R. Baker at St. Paul.[2] A total daily value was reported and computed equilibrium temperatures will therefore be representative of daytime hours.

A representative water temperature for the computation of $T_E$ and $K_s$ was considered to be the mixed river temperature

$$T_m = T_R + (T_o - T_R)\,\frac{Q_p}{Q_R}$$

This temperature was selected, although the river is not well mixed, because $T_o$ would have been too high and $T_R$ too low to be representative for surface heat exchange. Values of $T_E$ and $K_s$ are reported in Table 4.

## 15. Dimensionless Numbers

Computations of the dimensionless numbers listed in the theoretical analysis did not present any particular difficulties. The $Q_p/Q_R$ ratios are included in Table 1. Outlet densimetric Froude numbers are included in Table 1. Dimensionless surface heat transfer parameters are given in Table 4. The total surface area $A_T$ within the survey reach was measured to be $9.41 \times 10^6$ ft$^2$.

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[1] Linsley, R.K., Kohler, M.A. and Paulhus, J.L.H., Hydrology for Engineers, McGraw-Hill Book Co., 1958.

[2] Correlation between St. Paul and Monticello radiation data should be good because of the short distance between these locations (about 40 miles).

## TABLE 4

## SITE WEATHER CHARACTERISTICS AND HEAT EXCHANGE PARAMETERS

| Date of Temperature Survey | $T_W$ (°F) | $T_A$ (°F) | W (mph) | Percent Cloud Cover | $T_D$ (°F) | $H_S$ $\left(\frac{\text{BTU}}{\text{ft}^2\text{day}}\right)$ | $T_E$ (°F) | $K_S$ $\left(\frac{\text{BTU}}{\text{ft}^2\text{day}\,^\circ\text{F}}\right)$ | $\frac{\theta_E}{\theta_o}$ | $\frac{K_S A_T}{\rho c_p Q_R}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 7-1-71 | 63.6 | 73.6 | 8 | 50 | 57.3 | 2,238 | 77.3 | 112 | .293 | .024 |
| 7-6-71 | 71.8 | 84.4 | 8 | 75 | 66.2 | 2,077 | 82.7 | 126 | .263 | .033 |
| 8-2-71 | 60.0 | 66.0 | 8 | 85 | 56.2 | 1,468 | 70.3 | 104 | .256 | .053 |
| 8-3-71 | 63.5 | 70.5 | 6 | 52 | 59.5 | 1,750 | 78.5 | 92 | .438 | .051 |
| 8-13-71 | 72.5 | 81.3 | 12 | 23 | 68.1 | 1,839 | 78.6 | 175 | .377 | .153 |
| 8-19-71 | 72.4 | 79.4 | 6 | Clear | 69.4 | 794 | 76.6 | 110 | -.062 | .088 |
| 9-2-71 | 78.0 | 88.0 | 13.5 | Clear | 74.0 | 1,879 | 82.9 | 212 | .685 | .130 |
| 9-20-71 | 59.0 | 68.0 | 6.5 | 48 | 52.6 | 1,698 | 72.3 | 86 | .856 | .068 |
| 2-22-72 | N.R.* | 4.0 | 8.2 | 10 | --- | 1,928 | --- | --- | --- | --- |
| 2-23-72 | 20.0 | 21.6 | 16.0 | N.R.* | 15.5 | 1,419 | 27.1 | 122 | -.161 | .049 |
| 3-1-72 | N.R.* | 6.6 | 16.4 | 100 | --- | 786 | --- | --- | --- | --- |
| 4-4-72 | 26.3 | 33.0 | 18-20 | 70 | 9.2 | 1,839 | 21.1 | 153 | .211 | .019 |
| 5-23-72 | 67.5 | 75.6 | 0-5 | 100 | 63.4 | 1,907 | 87.3 | 80 | 1.20 | .010 |
| 6-22-72 | 65.6 | 73.6 | 5-8 | 4 | 61.3 | 2,774 | 89.8 | 97 | .699 | .025 |
| 7-5-72 | 60.6 | 72.0 | 3-5 | 24 | 52.6 | 2,161 | 80.6 | 77 | .505 | .032 |
| 8-3-72 | 59.4 | 67.0 | 5-8 | 62 | 54.2 | 2,399 | 79.5 | 95 | .888 | .010 |

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*Not recorded

## TABLE 4 (cont.)

## SITE WEATHER CHARACTERISTICS AND HEAT EXCHANGE PARAMETERS

| Date of Temperature Survey | $T_w$ (°F) | $T_A$ (°F) | W (mph) | Percent Cloud Cover | $T_D$ (°F) | $H_s$ $\left(\frac{BTU}{ft^2 day}\right)$ | $T_E$ (°F) | $K_s$ $\left(\frac{BTU}{ft^2 day\,^oF}\right)$ | $\frac{\theta_E}{\theta_o}$ | $\frac{K_s A_T}{\rho c_p Q_R}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 8-14-72 | 65.3 | 78.2 | 8-10 | 47 | 57.7 | 1,944 | 74.0 | 119 | .160 | .014 |
| 8-18-72 | 74.0 | 84.0 | 10-12 | 75 | 69.8 | 1,923 | 81.2 | 168 | .346 | .022 |
| 8-23-72 | 54.0 | 58.0 | 12-15 | 100 | 51.0 | 480 | 53.9 | 166 | -1.55 | .022 |
| 8-30-72 | 71.8 | 78.3 | 12-15 | 100 | 69.1 | 1,432 | 76.3 | 198 | .331 | .024 |
| 9-7-72 | 54.7 | 56.3 | 8-10 | 100 | 53.3 | 536 | 58.3 | 107 | -.214 | .017 |
| 9-15-72 | 57.7 | 62.7 | 12-15 | 23 | 54.2 | 1,698 | 64.9 | 159 | .030 | .037 |
| 6-27-73 | 62.0 | 66.0 | 15.0 | 100 | 60.0 | 1,132 | 65.5 | 204 | -.212 | .096 |
| 7-2-73 | 68.8 | 79.6 | 10.0 | Clear | 63.3 | 2,507 | 81.1 | 141 | .558 | .058 |
| 7-5-73 | 67.8 | 80.0 | 5.0 | Clear | 61.6 | 2,489 | 88.3 | 93 | 1.07 | .042 |
| 7-12-73 | 73.0 | 83.3 | 9.0 | 50 | 68.6 | 2,282 | 84.8 | 141 | .564 | .071 |
| 7-16-73 | 65.7 | 78.7 | 9.0 | Clear | 58.2 | 2,492 | 78.4 | 123 | .351 | .058 |
| 7-19-73 | 64.5 | 72.7 | 7.0 | Clear | 59.8 | 2,419 | 82.1 | 108 | .526 | .048 |
| 7-25-73 | 70.0 | 77.5 | 9.0 | 33 | 66.4 | 1,984 | 80.8 | 138 | .260 | .100 |
| 7-30-73 | 62.5 | 68.0 | 10.0 | 85 | 59.4 | 1,814 | 73.4 | 130 | .314 | .070 |
| 8-13-73 a | 67.5 | 72.9 | 4.0 | 62 | 66.3 | 1,353 | 81.1 | 91 | .542 | .021 |
| 8-13-73 b | 67.5 | 72.9 | 4.0 | 62 | 66.3 | 1,353 | 81.1 | 91 | .212 | .021 |

## E. DIMENSIONLESS GRAPHS AND EQUATIONS OF PLUME CHARACTERISTICS

### a. Maximum Lateral Spread of Cooling Water

The maximum lateral spread of the cooling water from the Monticello plant in the Mississippi River usually occurs at the water surface. Isotherms in the river are nearly vertical most of the time, indicating intensive vertical mixing.

The distance at the water surface from the right bank, called $y$, was measured for each isotherm reported in each transect and then divided by the local water surface width $b$ in that transect. The resulting dimensionless values $y/b$, called relative spread, were obtained. The highest value of $y/b$ in any transect for a given water temperature was called $(y/b)_{max}$ and was plotted against the dimensionless water temperature increment $\theta/\theta_o$ to which it belonged. The resulting graphs, given in Fig. 3a, b, c, and d, provided the following information:

1. The maximum relative spread $(y/b)_{max}$ is strongly dependent on the temperature ratio $\theta/\theta_o$. As $\theta/\theta_o$ decreases (going downstream), $(y/b)_{max}$ increases.
2. Despite the scatter in the data, it is evident that the relationship $(y/b)_{max} = f(\theta/\theta_o)$ also depends on the plant discharge to river flow ratio $Q_p/Q_R$. Smaller values of $Q_p/Q_R$ are associated with a lesser spread.
3. An effect of buoyancy, as measured by the outlet densimetric Froude number, on the maximum relative spread could not be detected. There were noted differences in maximum lateral spread $(y/b)_{max}$ at high values of $\theta/\theta_o$, particularly near the outlet, and for comparable values of $Q_p/Q_R$, but they did not seem related to densimetric Froude numbers. It is conceivable that not enough data in the immediate vicinity of the outlet were collected to assess buoyancy effects.

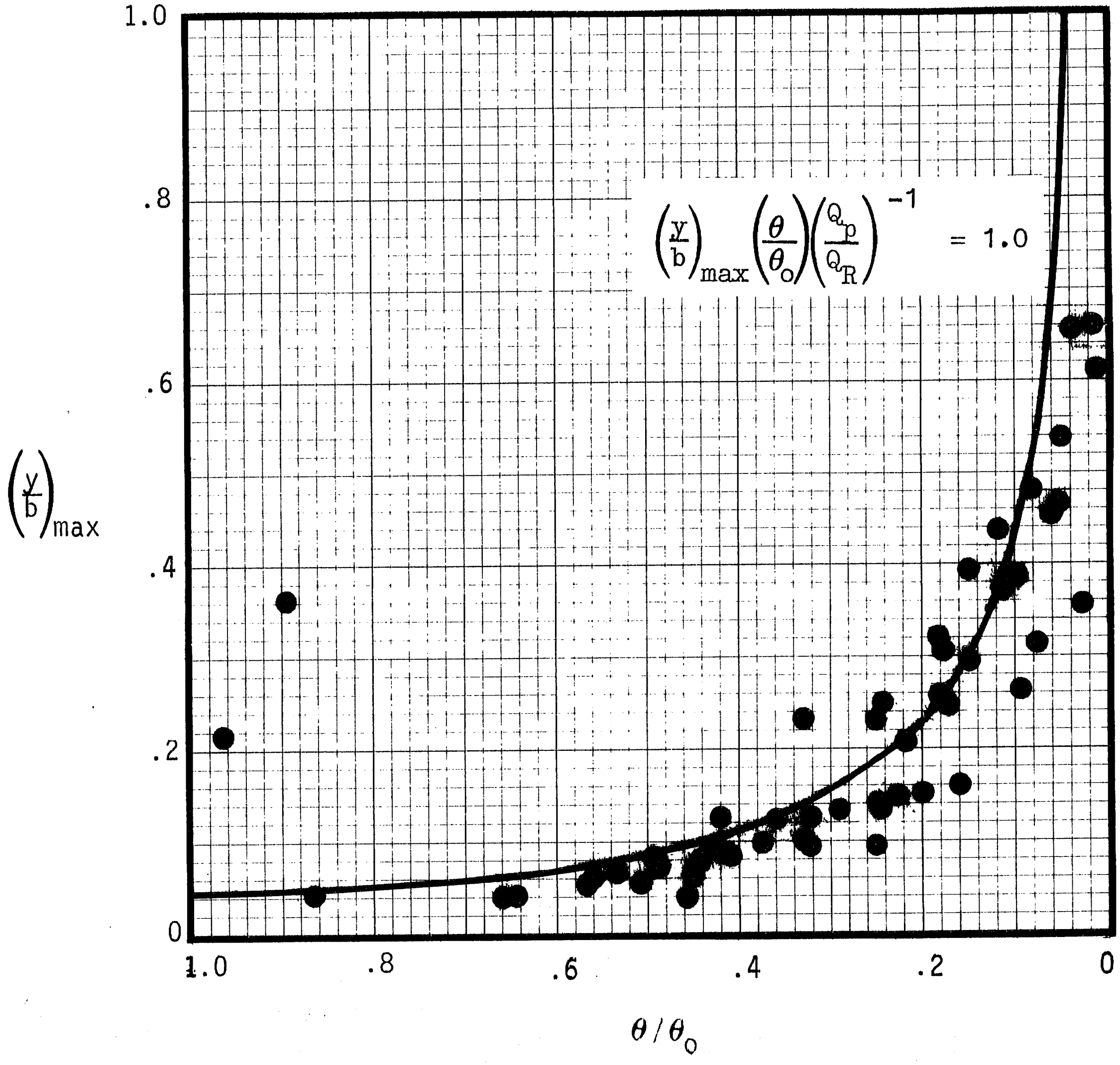


Fig. 3a - Maximum lateral spread as a function of excess temperature at $Q_p/Q_R$ **from** 0.036 to 0.054.

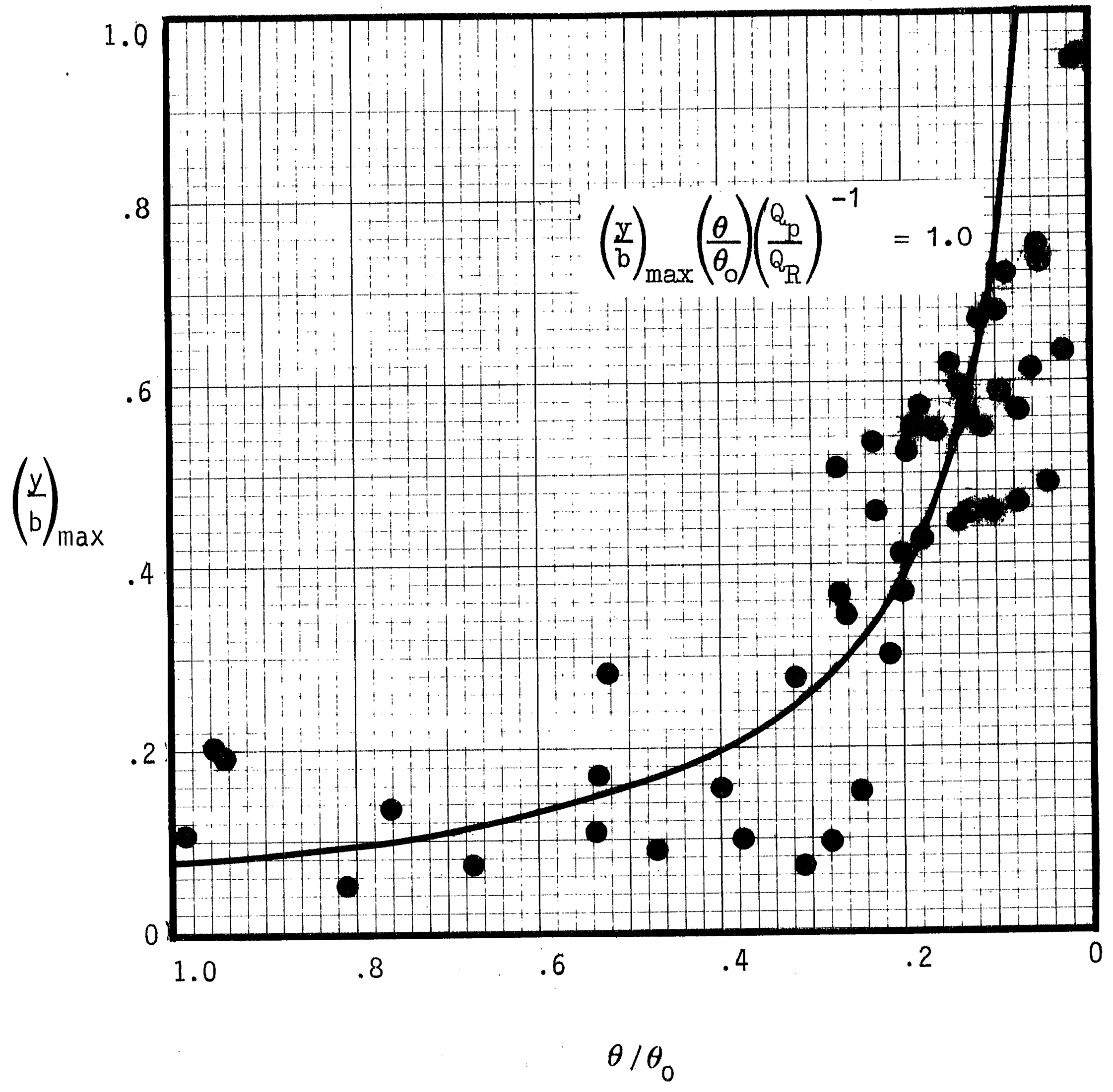
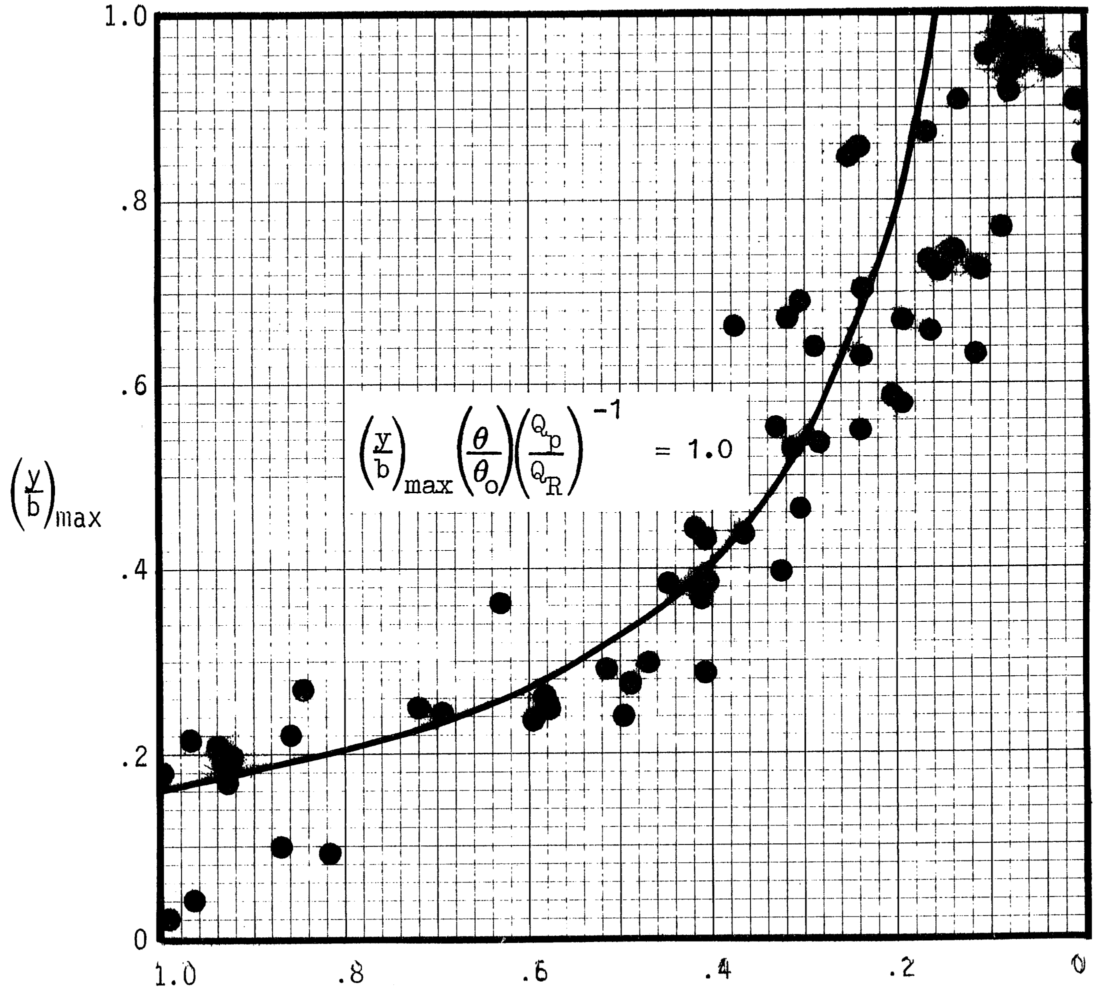


Fig 3b - Maximum lateral spread as a function of excess temperature at $Q_p/Q_R$ from 0.070 to 0.083.



$\theta/\theta_o$

Fig. 3c - Maximum lateral spread as a function of excess temperature at $Q_p/Q_R$ from 0.135 to 0.188.

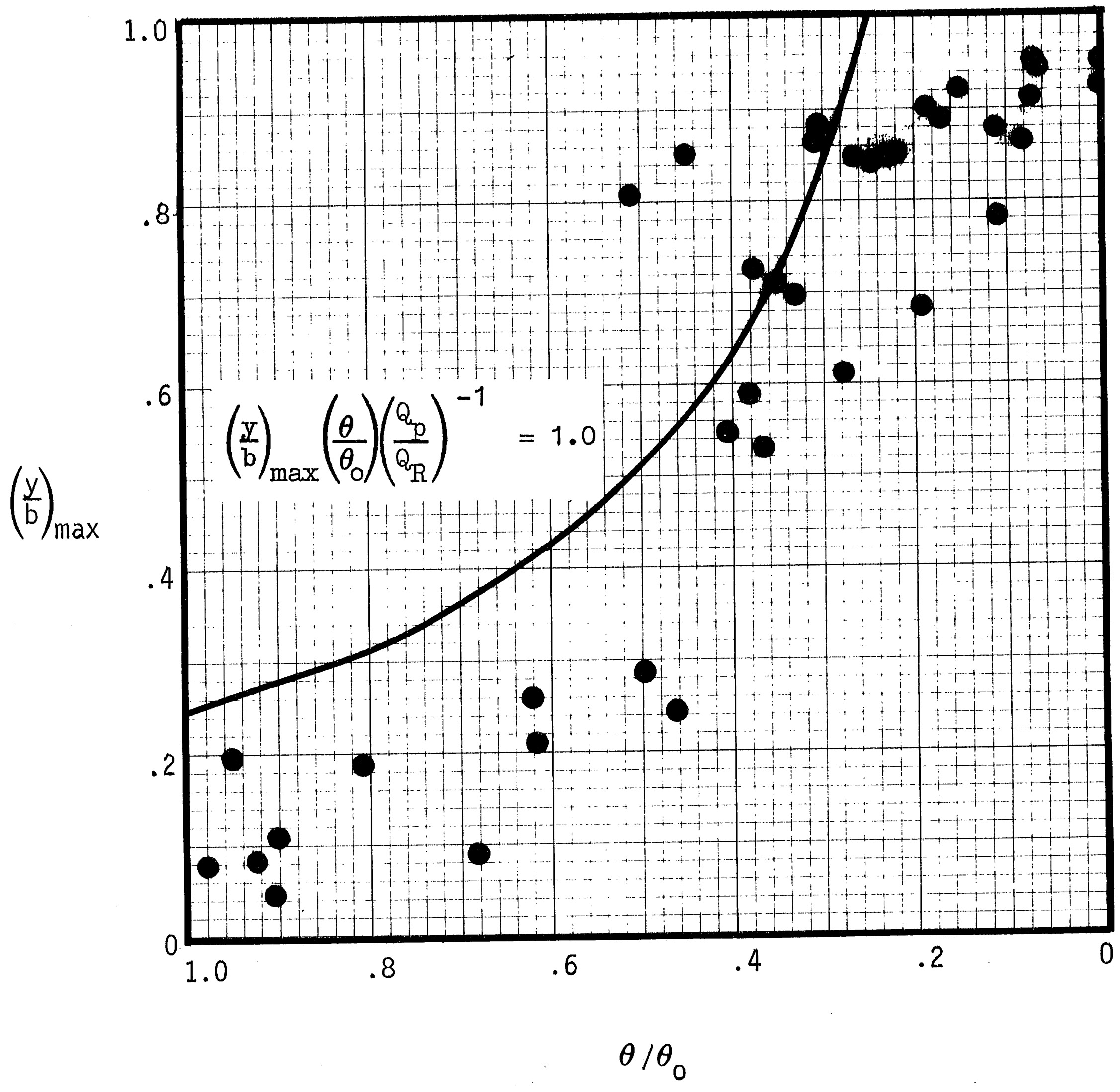


Fig. 3d - Maximum lateral spread as a function of excess temperature at $Q_p/Q_R$ from 0.223 to 0.284.

4. With few exceptions, all of the data were collected in summer. A few winter and spring data are given in Fig. 4. Initial spreading seems to be stronger than for summer data, but no definitive conclusions can be drawn from the data in Fig. 4

Fig. 3 can be used to interpolate for specified values of $(y/b)_{max}$. Results are given in Fig. 5 for $(y/b)_{max} = 0.3$, 0.5 and 0.7. The data, as well as the equations given on p. 7, indicate that for a fixed value of $(y/b)_{max}$ the relationship between $\theta/\theta_o$ and $Q_p/Q_R$ should be linear. It is actually found that the relationship

$$\left(\frac{y}{b}\right)_{max}\left(\frac{\theta}{\theta_o}\right)\left(\frac{Q_p}{Q_R}\right)^{-1} = 1.0$$

represents the mean trend of the data reasonably well.

The format of the foregoing equation agrees with the theory, but the value of the constant is much larger than the theoretical value because the theory ignores the momentum of the discharge and other effects.

Some of the actual data points lie above the curve given by the above equation. The equation for the envelope was therefore also determined. It is

$$\left(\frac{y}{b}\right)_{max}\left(\frac{\theta}{\theta_o}\right)\left(\frac{Q_p}{Q_R}\right)^{-0.75} = 1.0$$

This relationship may be used e.g. to estimate discharge water temperatures $T_o$ which will meet a midstream water temperature standard $T_{st}$. From the equation, one obtains for the maximum admissible plant temperature rise between the intake and the outlet channels

$$\frac{1}{2}\left(\frac{T_{st} - T_R}{T_o - T_R}\right)\left(\frac{Q_p}{Q_R}\right)^{-0.75} = 1.0 \qquad \text{or}$$

$$T_o = T_R + \frac{1}{2}\left(T_{st} - T_R\right)\left(\frac{Q_p}{Q_R}\right)^{-0.75}$$

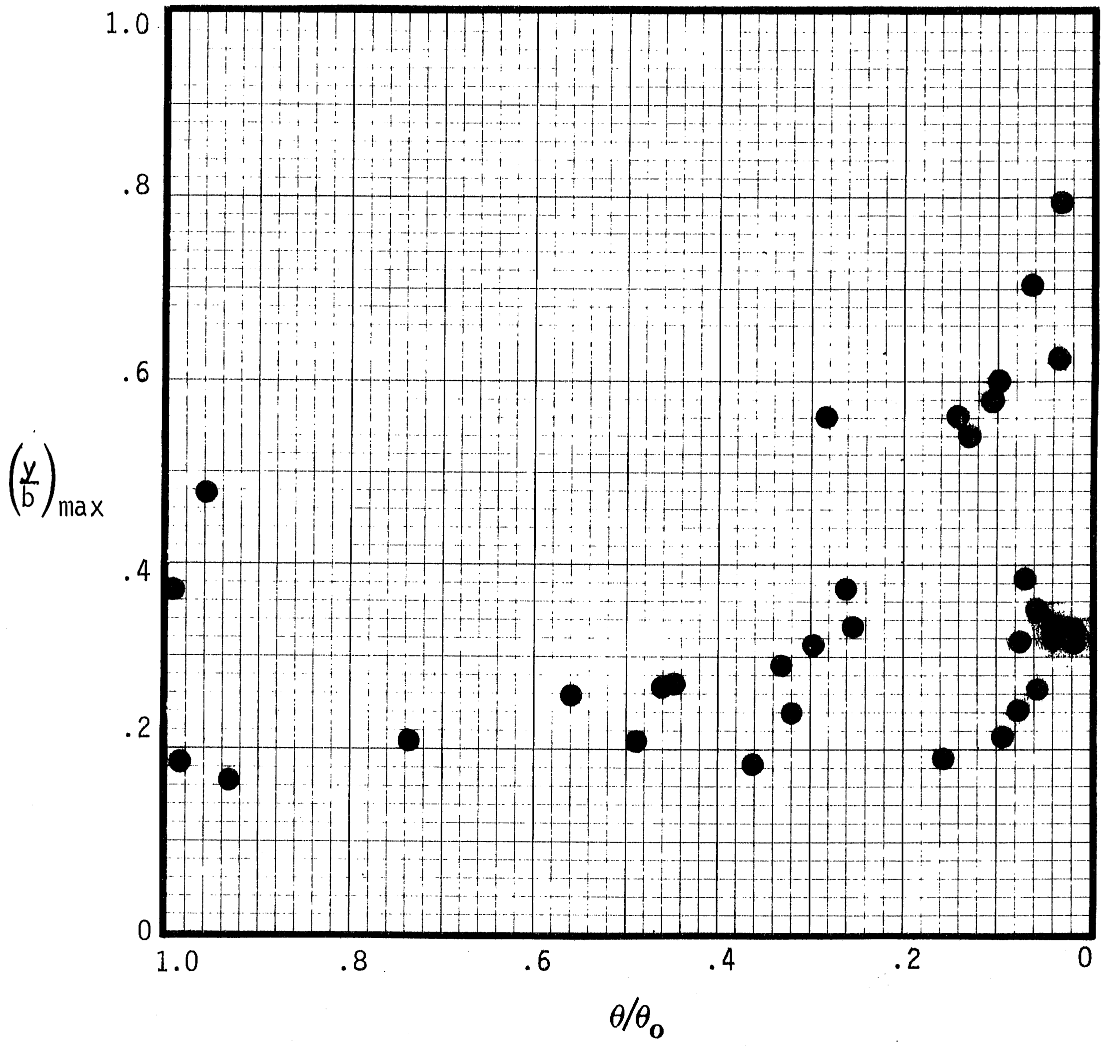


Fig. 4 - Maximum lateral spread as a function of excess temperature for <u>winter conditions</u> at $Q_p/Q_R$ from 0.021 to 0.125.

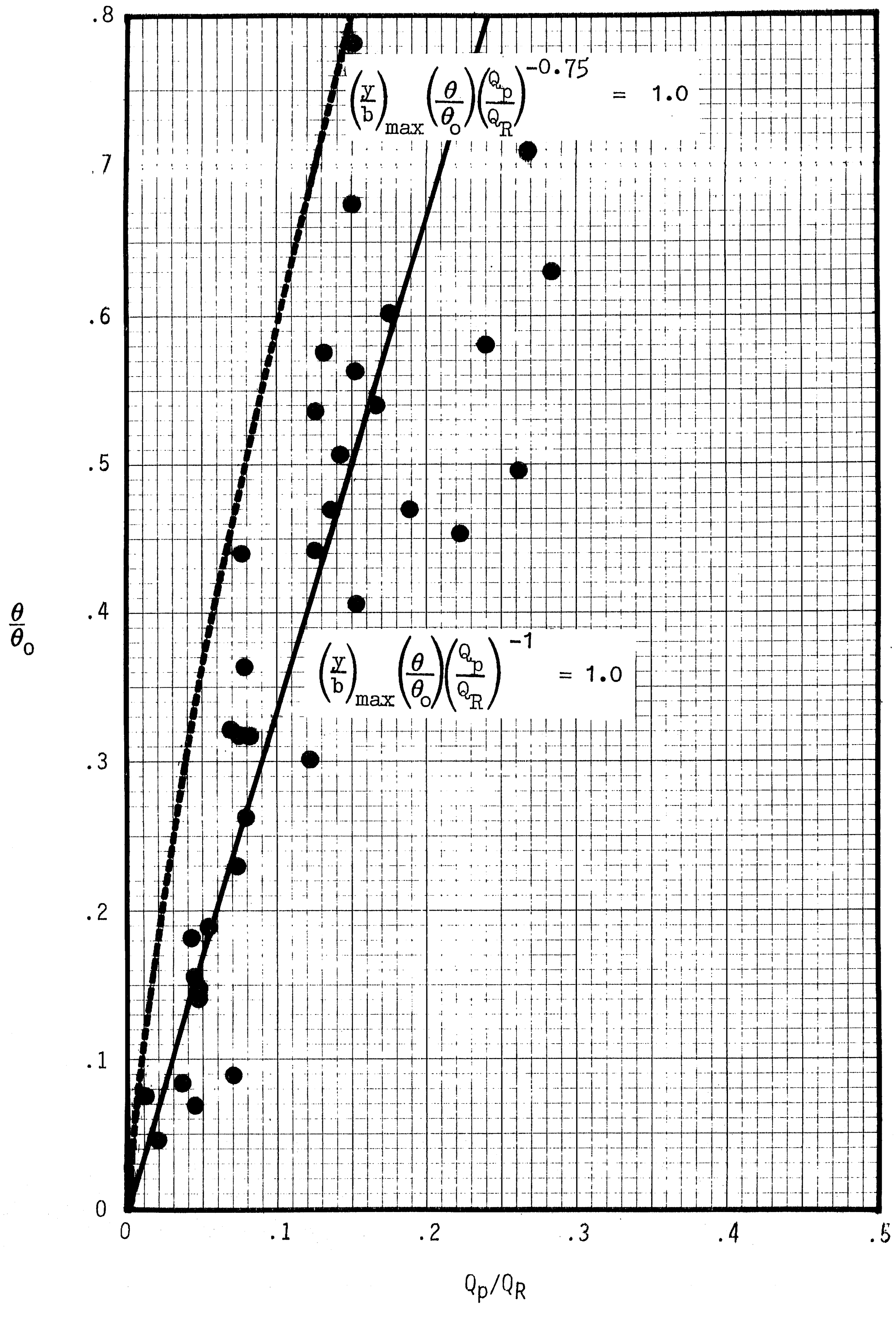


Fig. 5a - Excess temperature vs. plant to river flow ratio for maximum lateral spread $(y/b)_{max} = 0.3$.

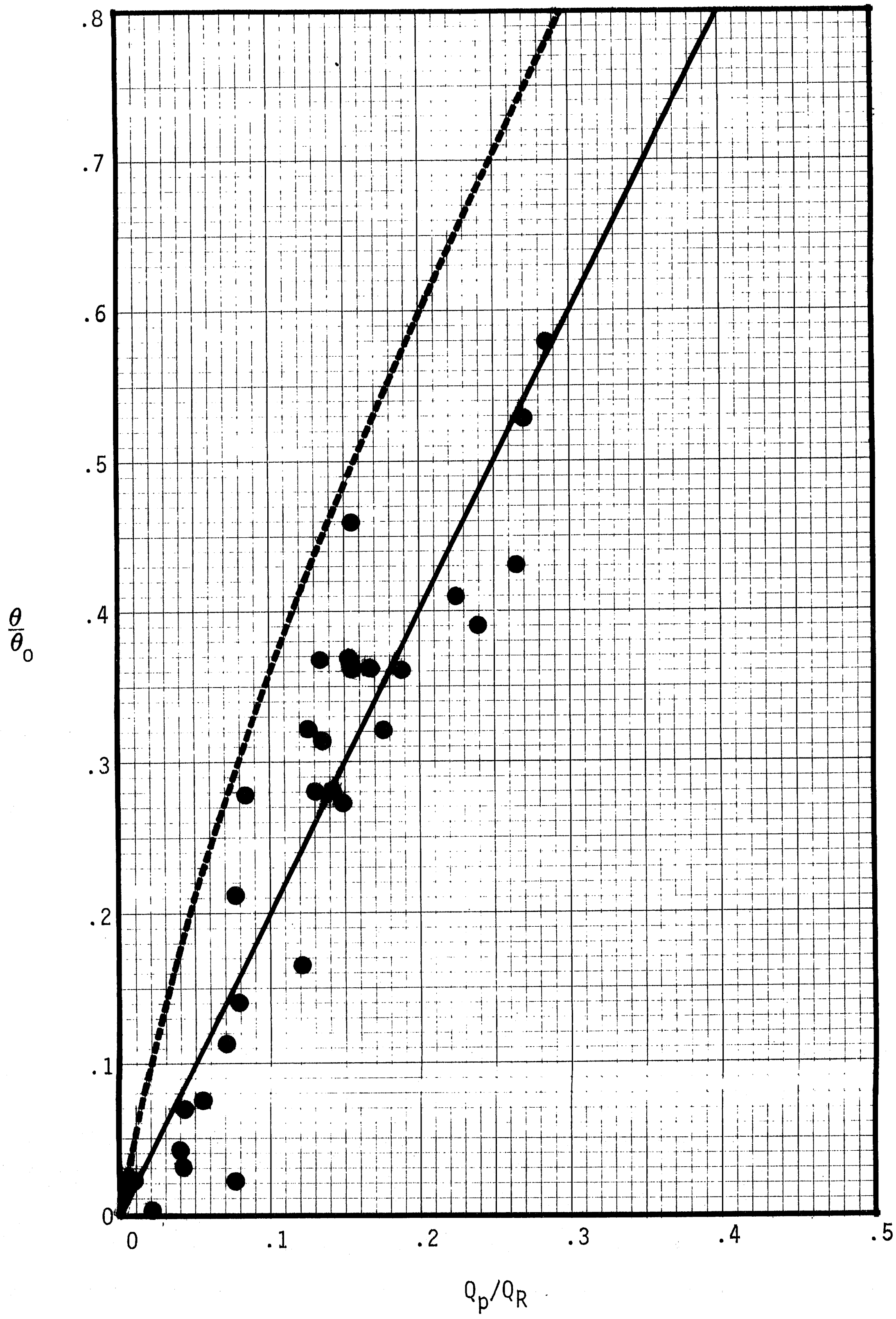


Fig. 5b - Excess temperature vs. plant to river flow ratio for maximum lateral spread $(y/b)_{max} = 0.5$.

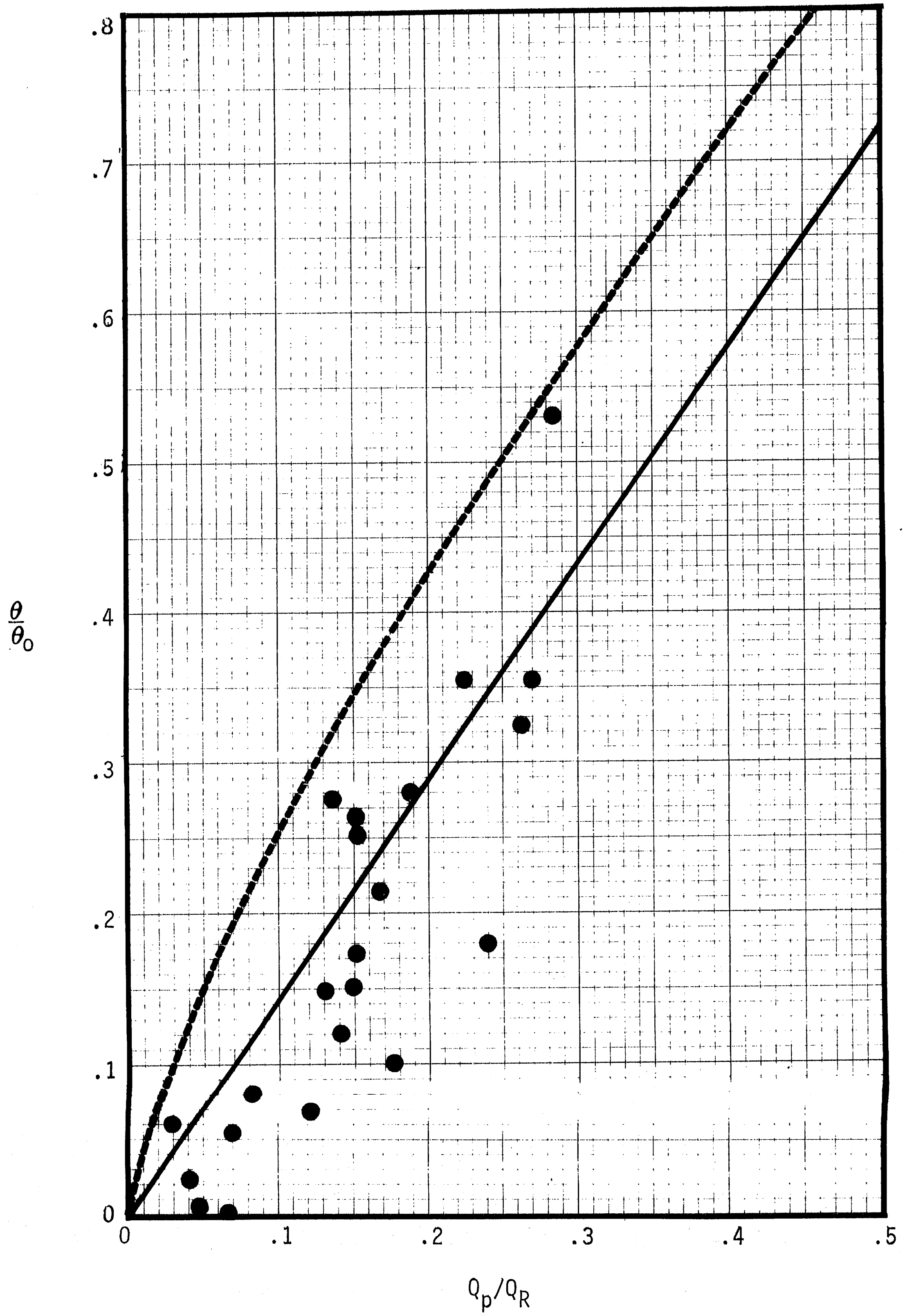


Fig. 5c - Excess temperature vs. plant to river flow ratio for maximum lateral spread $(y/b)_{max} = 0.7$.

$T_{st}$ is the water temperature standard in midstream; $T_R$ is the intake river temperature; $T_o$ is the outlet channel temperature.

Example: In order to maintain a water temperature of 90°F or less over half the stream width when the river temperature is 78°F, the river flow is 2,000 cfs and the plant discharge is 600 cfs, the discharge temperature must not exceed

$$T_o = 78 + (90 - 78)(0.5)\left(\frac{2000}{600}\right)^{0.75} = 92.8^{\circ}F$$

If the river flow is 5,000 cfs instead of 2,000 cfs

$$T_o = 78 + (90 - 78)(0.5)\left(\frac{5000}{600}\right)^{0.75} = 107.4^{\circ}F$$

The above shows that there is a large potential for dilution when the river flow is high, but that there is very little when the river flow is low compared to the cooling water demand. Extrapolation to low river flow conditions from the above data base may be questioned, but does not appear unreasonable. Extrapolation will show that $\theta/\theta_o = 1$ when $Q_p/Q_R = 0.4$. This means that dilution of the effluent cooling water in a mixing zone is not acceptable when the plant discharge is 40% of the river flow or more. In other words, at a maximum plant discharge of 650 cfs, river flows must be more than 1,625 cfs to take advantage of dilution and to meet the water temperature requirement for the midstream.

Some additional refinement and discussion of the above results is in order:

Because the discharged cooling water is warmer than the river water and therefore slightly lighter than the river water, one should expect some buoyancy induced effects, particularly near the outlet channel. Indeed, some of the field survey data show two such effects: (a) penetration of a cold water (actually mixed water) wedge into the outlet channel, and (b) buoyancy induced surface spread of discharged cooling water near the outlet.

(a) The cold water wedge formation is seen in the outlet channel isotherms for the surveys of November 9, 1971 and April 4, 1972. On both occasions the outlet densimetric Froude number $F_o$ was less than 1.0, mainly due to low cooling water discharge rates (below the design flow). The outlet densimetric Froude number $F_o$ is used as a measure of the buoyancy of the effluent relative to its inertia.

(b) The buoyant spread at the water surface is seen in the tilting of the isotherms at the outlet in the surveys of August 13, August 19, and September 20, in 1971. The resulting vertical temperature gradients are rapidly destroyed by vertical mixing as the cooling water is carried downstream. Isotherms are generally vertical or nearly vertical in transect 4 for these cases (about 1,000 ft from the outlet).

It was noted that the existence of an ice cover on the river served to enhance surface spread. More winter surveys are required to substantiate this preliminary finding. The probable reason is that an ice cover eliminates surface currents in the river; the effluent flow can subsequently develop a surface spreading pattern near the outlet more readily.

The occasional and very local observations described above did not seem to have any significant impact on the maximum temperature spread $(y/b)_{max}$. $F_o$ values were calculated for all $(y/b)_{max}$ values, but variations in $F_o$ values did not seem to correlate with $(y/b)_{max}$ - values in any detectable way. The conclusion at this time, therefore, is that no buoyancy effect on the maximum spread of the cooling water effluent from the Monticello generating plant in the Mississippi River could be detected.

A regression analysis of all data points and including densimetric outlet Froude numbers $F_o$ yielded the relationship

$$\left(\frac{y}{b}\right)_{max}\left(\frac{\theta}{\theta_o}\right)^{0.77}\left(\frac{Q_p}{Q_R}\right)^{-0.75}\left(F_o\right)^{-0.03} \doteq C$$

The low value of the exponent for $F_o$ bears out the low significance of buoyancy for the spreading of cooling water near the Monticello outlet.

Under low river flow conditions, cooling water withdrawal and subsequent discharge will represent a large portion of the total flow. A low-flow analysis for the Monticello site is still lacking, but flow records from gaging stations above and below the Monticello site (noted in Table 1) have been analyzed by the District Office of the U.S. Geological Survey in St. Paul with respect to flow frequencies. Such data could be utilized in extrapolating low flow conditions of the Mississippi River at the site.

As indicated in a previous section, dilution of the effluent in a mixing zone is not practically feasible when cooling water effluent rates exceed 40% of the river flow, because effluent temperatures may not experience significant dilution before reaching midstream.

Associated with low river flow conditions is the reduction in water depths in portions of the river channel. In some transects the river channel section will be very much reduced, not only in depth but also in width. In some portions the water depth may be only inches. It is for these reasons that extrapolation from the present data base to extreme low flows (less than 1,500 cfs) may produce significant error.

The effect of heat loss to the atmosphere on lateral spread of cooling water was also investigated. A regression analysis using all five independent variables listed on page 11 and an exponential form of the equation resulted in the relationship

$$\left(\frac{y}{b}\right)_{max}\left(\frac{\theta}{\theta_o}\right)^{0.776}\left(\frac{Q_p}{Q_R}\right)^{-0.665}\left(F_o\right)^{-0.094}\left(\frac{T_E-T_R}{T_o-T_R}\right)^{0.062}\left(\frac{K_s A_T}{Q_R \rho c_p}\right)^{-0.065} = 0.50$$

The low values of the exponents for the last three terms indicate the small significance of buoyancy and surface heat exchange on the lateral spread of the cooling water effluent at Monticello.

All of the before-given equations for $\left(\frac{y}{b}\right)_{max}$ give a finite value for $\left(\frac{y}{b}\right)_{max}$ when $\frac{\theta}{\theta_o} = 1$. This value does not always reflect the real behavior of the plume near the outlet, because the momentum effects and the real dimensions of the outlet are ignored. It is therefore suggested that the equations be limited to $0 < \frac{\theta}{\theta_o} < 0.8$ and $0 < \frac{Q_p}{Q_R} < 0.4$.

### b. Maximum Longitudinal Spread of Cooling Water

Dimensionless plots of the data are shown in Fig. 6a, b, c, and d for the summer and in Fig. 7 for the winter. There are two somewhat distinct regions in each plot. A relatively fast drop in $\theta/\theta_o$ values for $x_{max}/b$ of less than 1 or 2.[*] For the average river width of b = 500 ft, the rapid drop-off represents the first 500 to 1,000 ft beyond the outlet. Only two of the transects in which water temperatures were measured were within 500 ft from the outlet, all others were beyond. Most of the $x_{max}$ -values reported on the NSP drawings for the first 500 ft are the result of interpolations, rather than actual measurements. The data analysis therefore ignored all data points $x_{max}/b < 1.0$.

A linear regression on the maximum longitudinal spread data gave the relationship

$$\left(\frac{x_{max}}{b}\right)\left(\frac{\theta}{\theta_o}\right)^2\left(\frac{Q_p}{Q_R}\right)^{-2} = 59.0$$

This equation agrees in form with the one developed from the two-dimensional turbulent diffusion equation on p. 8.

A better fit of the equation with the data is obtained if the exponents are not imposed. The relationship thus obtained is

$$\left(\frac{x_{max}}{b}\right)\left(\frac{\theta}{\theta_o}\right)^{2.88}\left(\frac{Q_p}{Q_R}\right)^{-1.15} = 3.9$$

The multiple regression coefficient is 0.77 in that case.

The effects of buoyancy and surface heat transfer could not be found in a reliable way from the available data.

By further analysis it could be shown, however, that the width to depth ratio, b/h, of the river has some effect.

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[*] The higher value applies to the larger discharge rates.

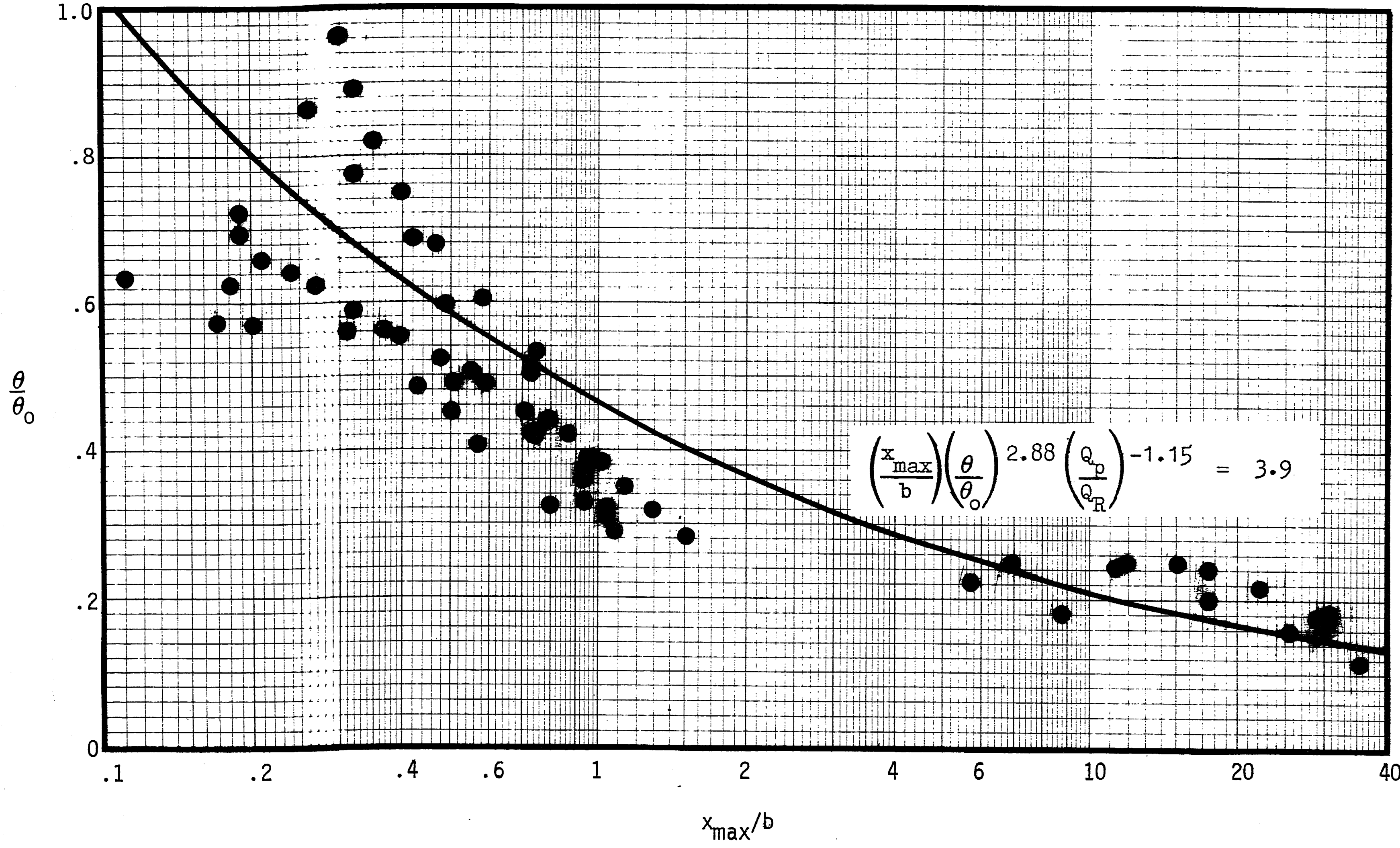

Fig. 6a - Maximum longitudinal spread as a function of excess temperature at $Q_p/Q_R$ from 0.036 to 0.054.

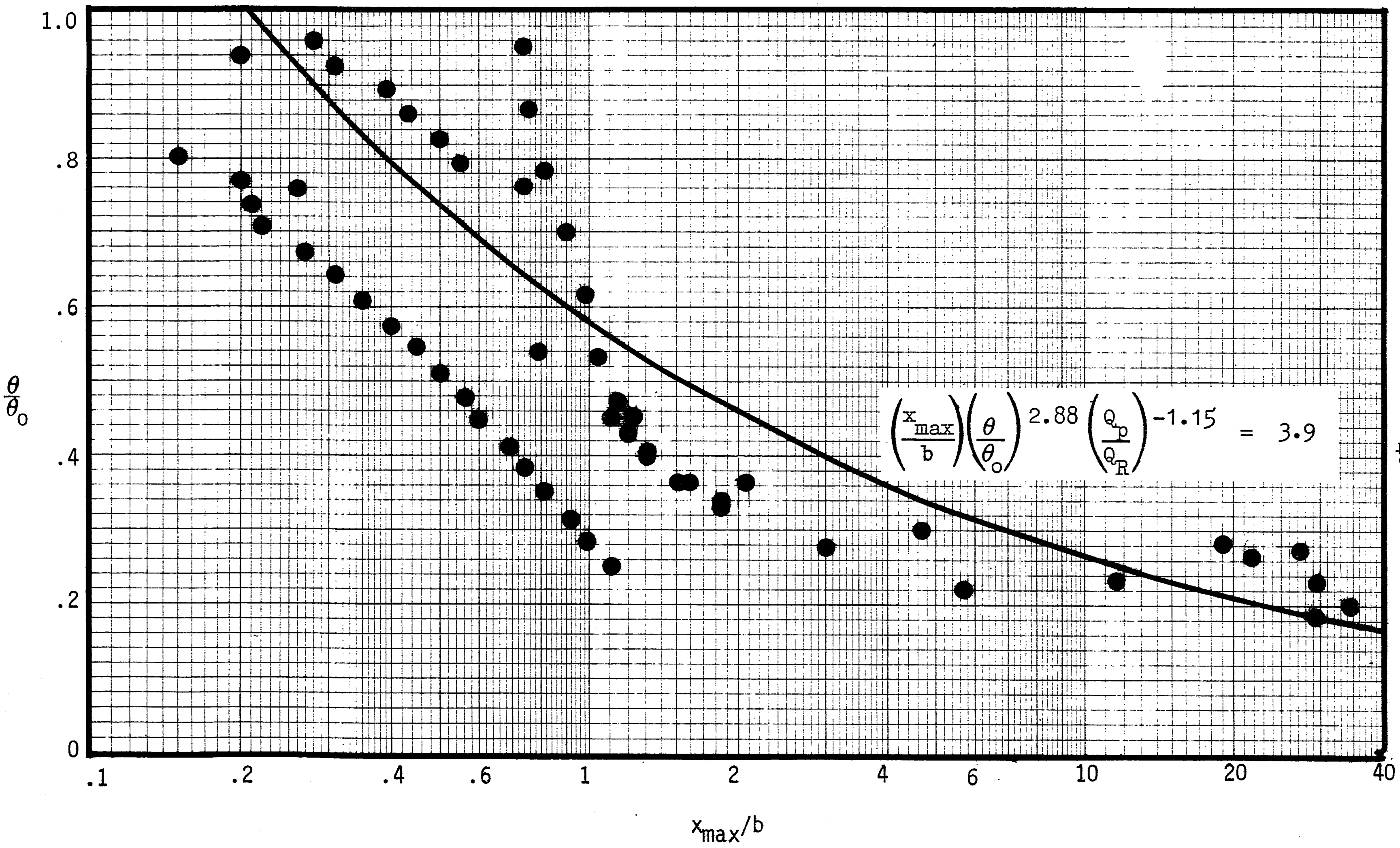

Fig. 6b - Maximum longitudinal spread as a function of excess temperature at $Q_p/Q_R$ from 0.070 to 0.083.

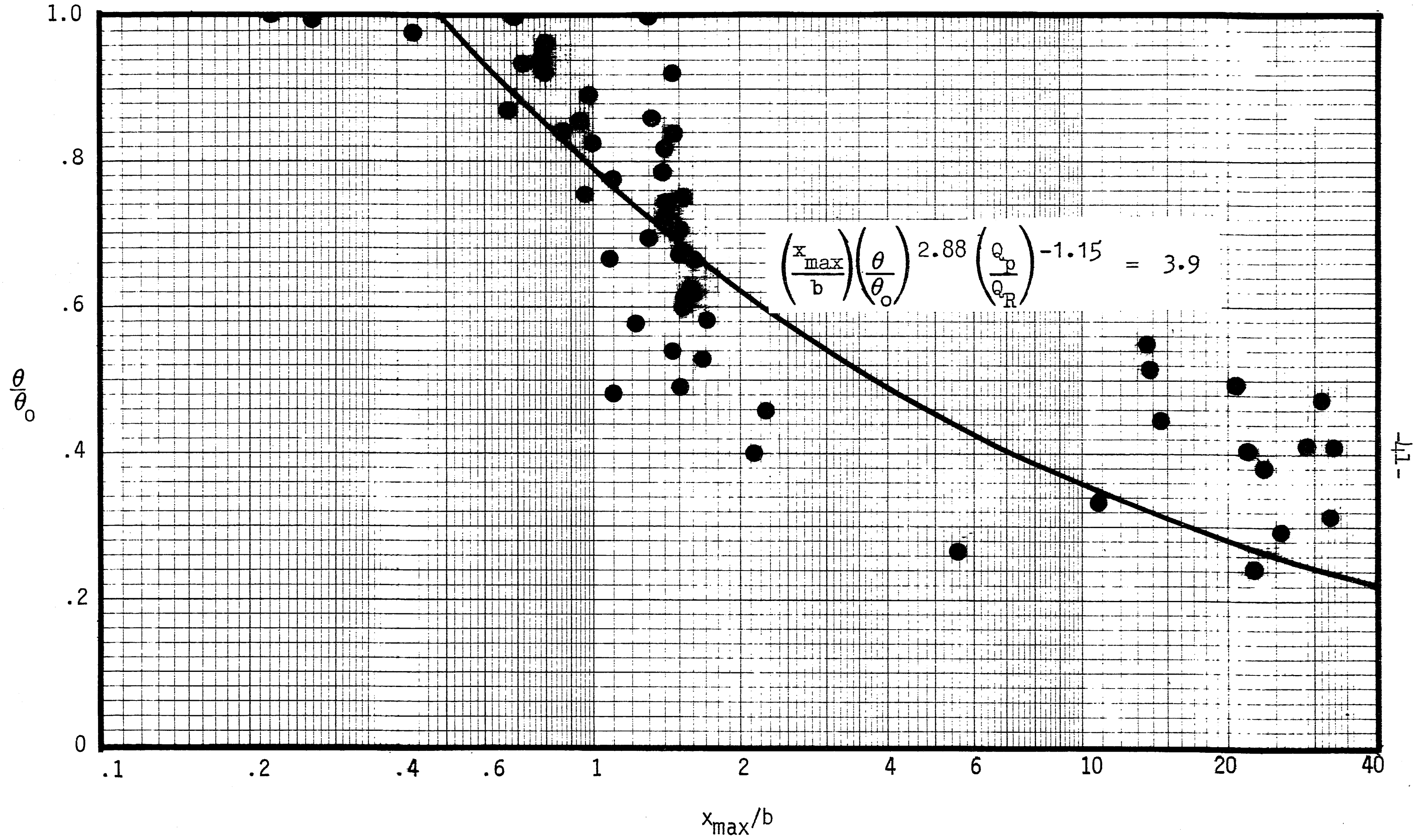

Fig. 6c - Maximum longitudinal spread as a function of excess temperature at $Q_p/Q_R$ from 0.135 to 0.188.

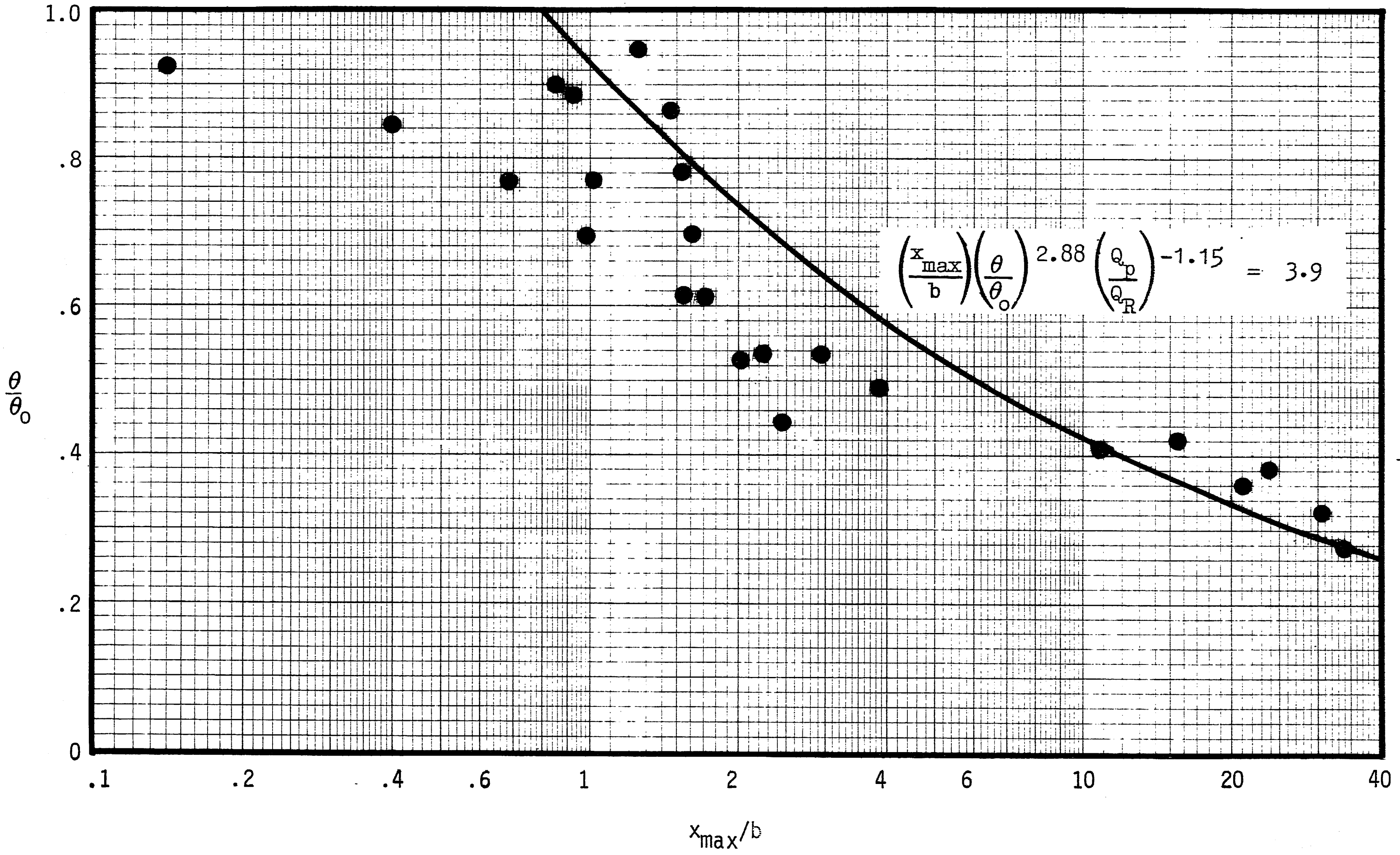

Fig. 6d - Maximum longitudinal spread as a function of excess temperature at $Q_p/Q_R$ from 0.239 to 0.284.

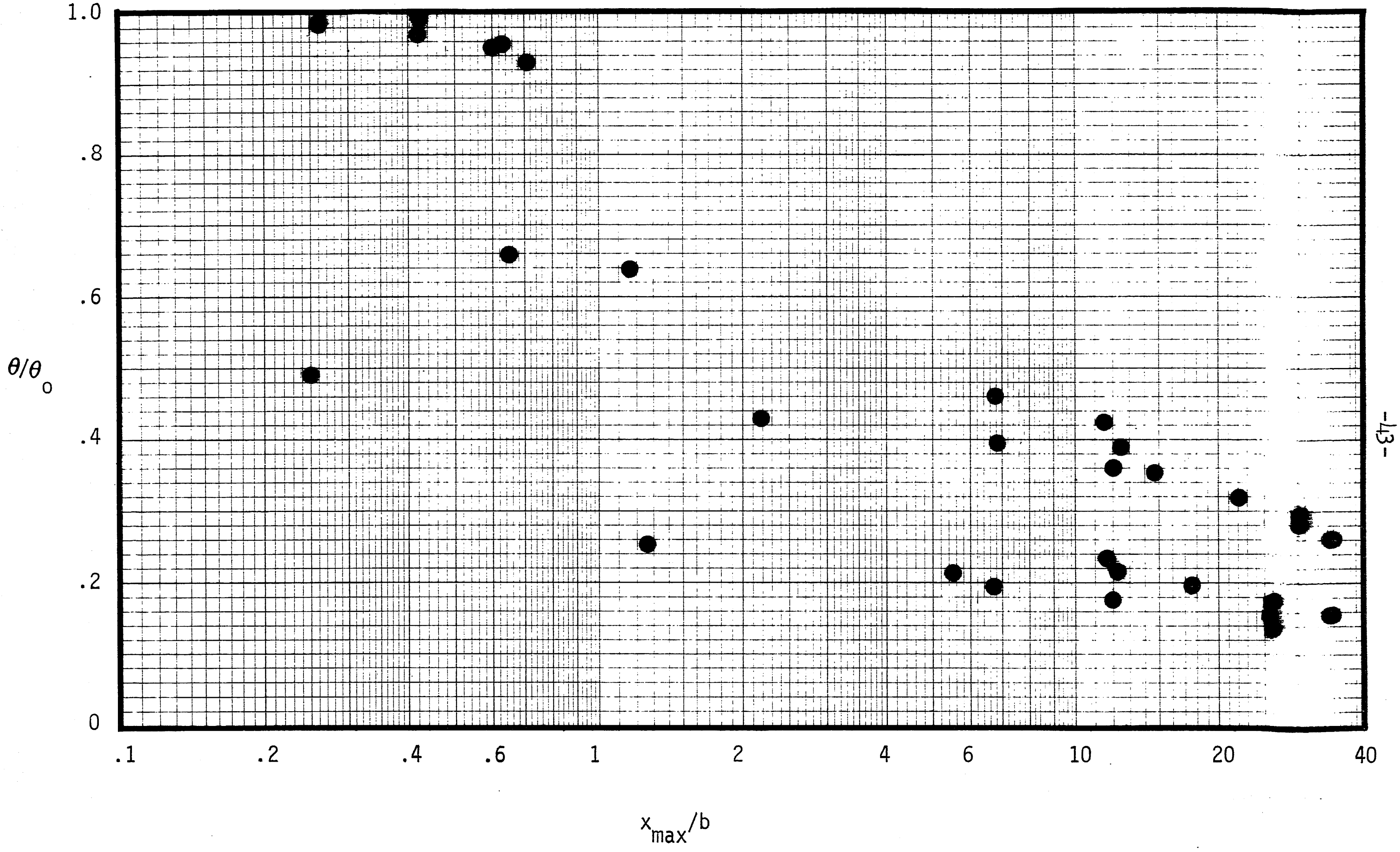

Fig. 7 - Maximum longitudinal spread as a function of excess temperature for winter conditions at $Q_p/Q_R$ from 0.021 to 0.125.

### c. Surface Areas Enclosed by Isotherms

Data plots of $A_sV/Q_p$ versus $\theta/\theta_o$ are shown in Fig. 8a, b, c, and d for summer conditions and Fig. 9 for winter conditions. The data follow roughly an equation of the form

$$\left(\frac{\theta}{\theta_o}\right)\left(\frac{A_sV}{Q_p}\right)^n = p$$

It was noted, however, that the $Q_p/Q_R$ ratio had a large effect on the data. In particular $\theta/\theta_o$ did no longer tend toward zero as $A_sV/Q_p$ increased. The excess temperature ratio was therefore modified so that

$$\left(\frac{\theta}{\theta_o} - \frac{Q_p}{Q_R}\right)\left(1 - \frac{Q_p}{Q_R}\right)^{-1}\left(\frac{A_sV}{Q_p}\right)^n = p$$

The above formulation is arrived at by considering that the river temperature downstream from the outlet must tend towards a temperature in the vicinity of the mixed river water temperature. That temperature is

$$\frac{\theta_m}{\theta_o} = \frac{T_m - T_R}{T_o - T_R} = \frac{Q_p}{Q_R}$$

A plot of the data representative of the above equation is given in Fig. 10. Data points involving surface areas of 5,000 $ft^2$ or less were excluded.

An equation which gives a rough fit to all the data points is

$$\left(\frac{\theta}{\theta_o} - \frac{Q_p}{Q_R}\right)\left(1 - \frac{Q_p}{Q_R}\right)^{-1}\left(\frac{A_sV}{Q_p}\right)^{0.45} = 7$$

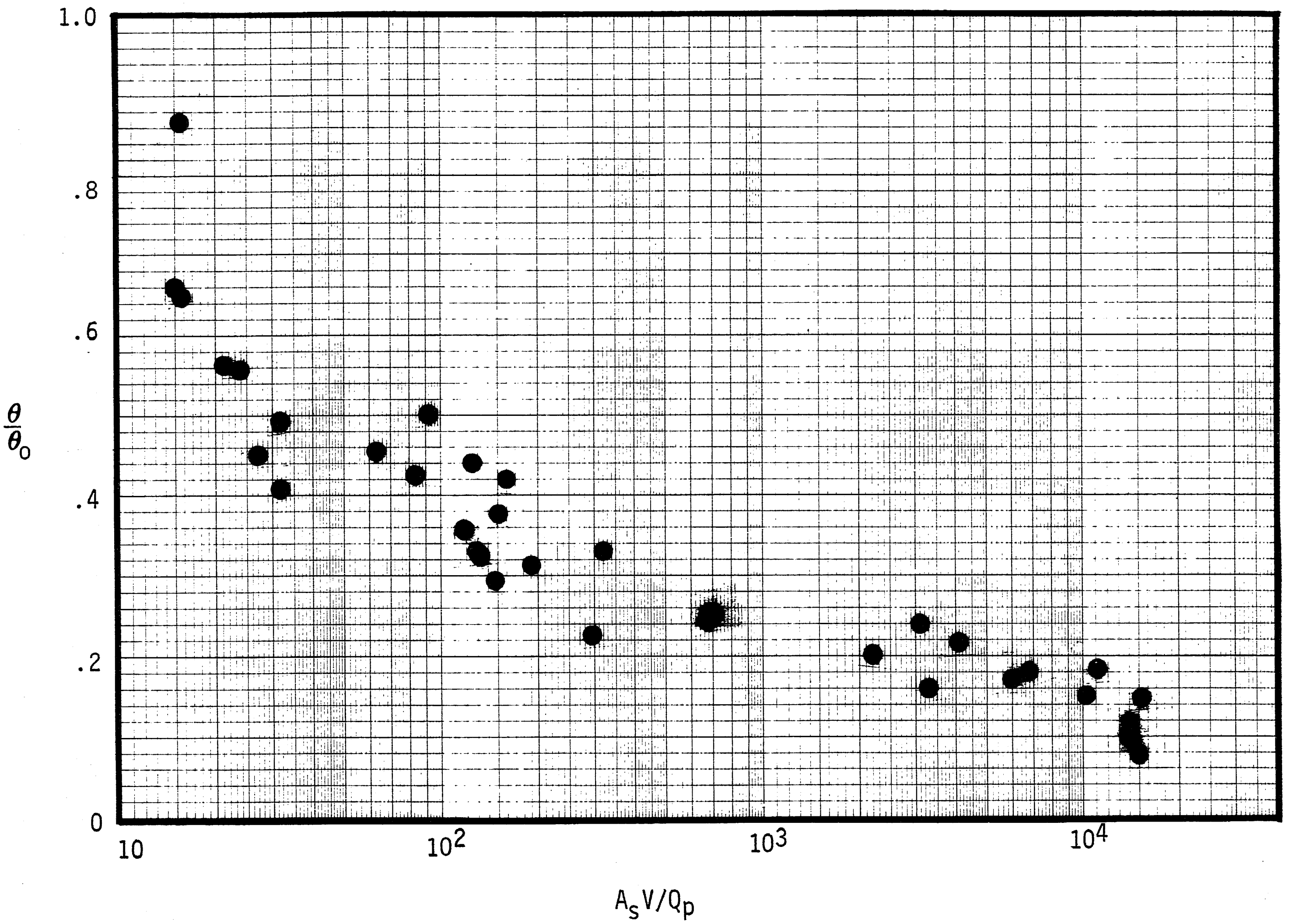

Fig. 8a - Enclosed surface areas vs. excess temperatures at $Q_p/Q_R$ from 0.036 to 0.054.

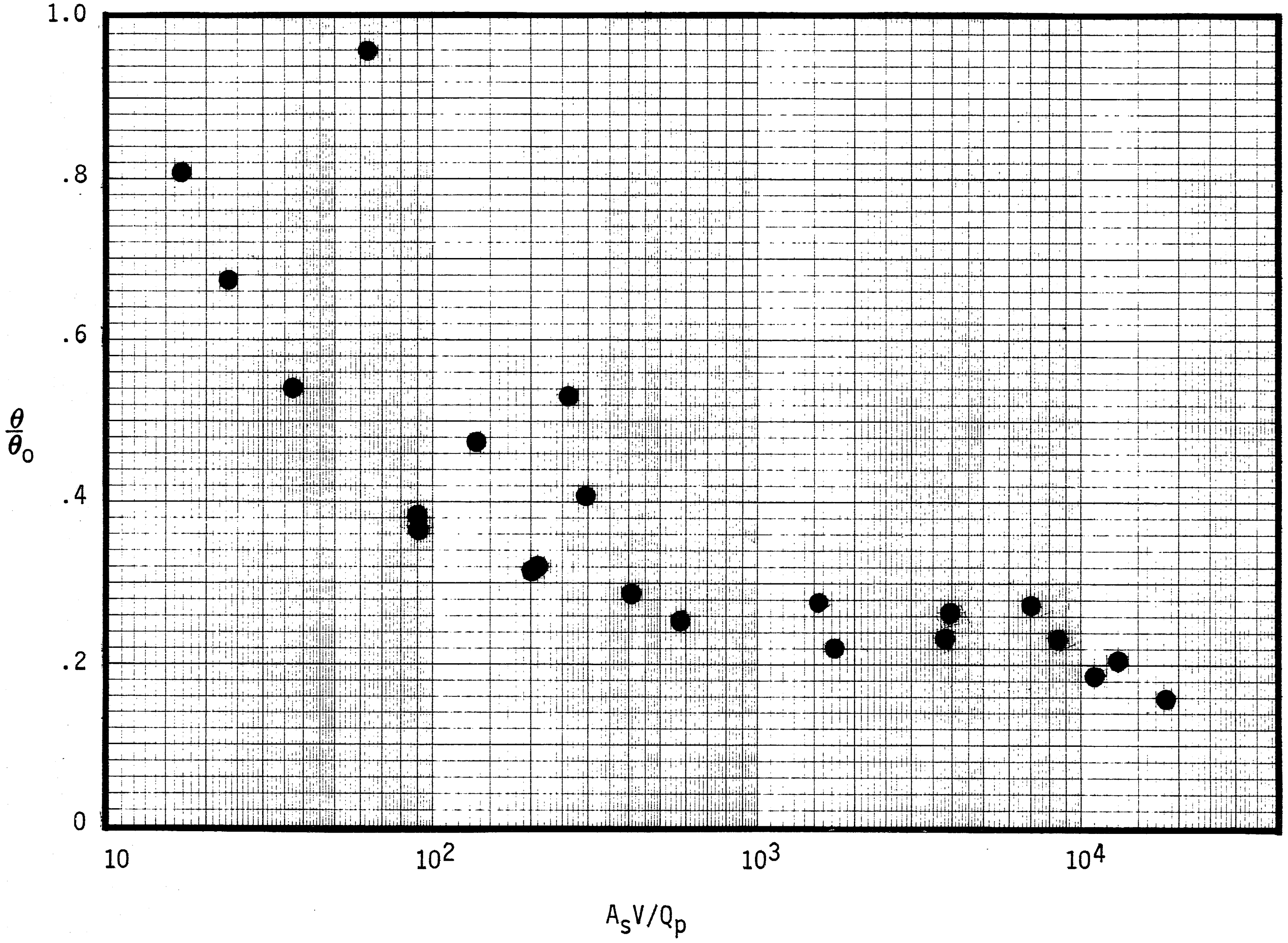

Fig. 8b - Enclosed surface areas vs. excess temperatures at $Q_p/Q_R$ from 0.070 to 0.083.

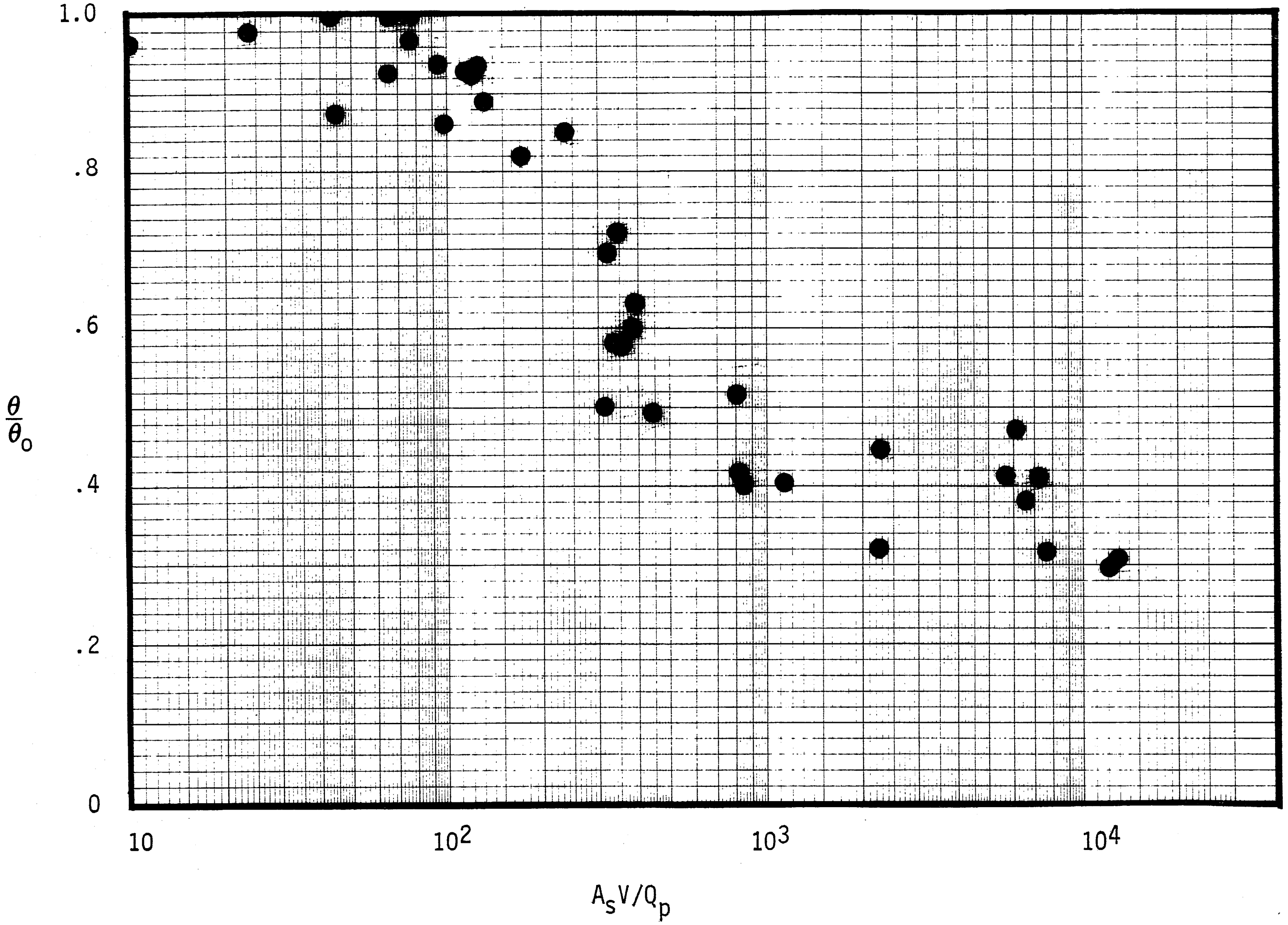

Fig. 8c - Enclosed surface areas vs. excess temperatures at $Q_p/Q_R$ from 0.135 to 0.188.

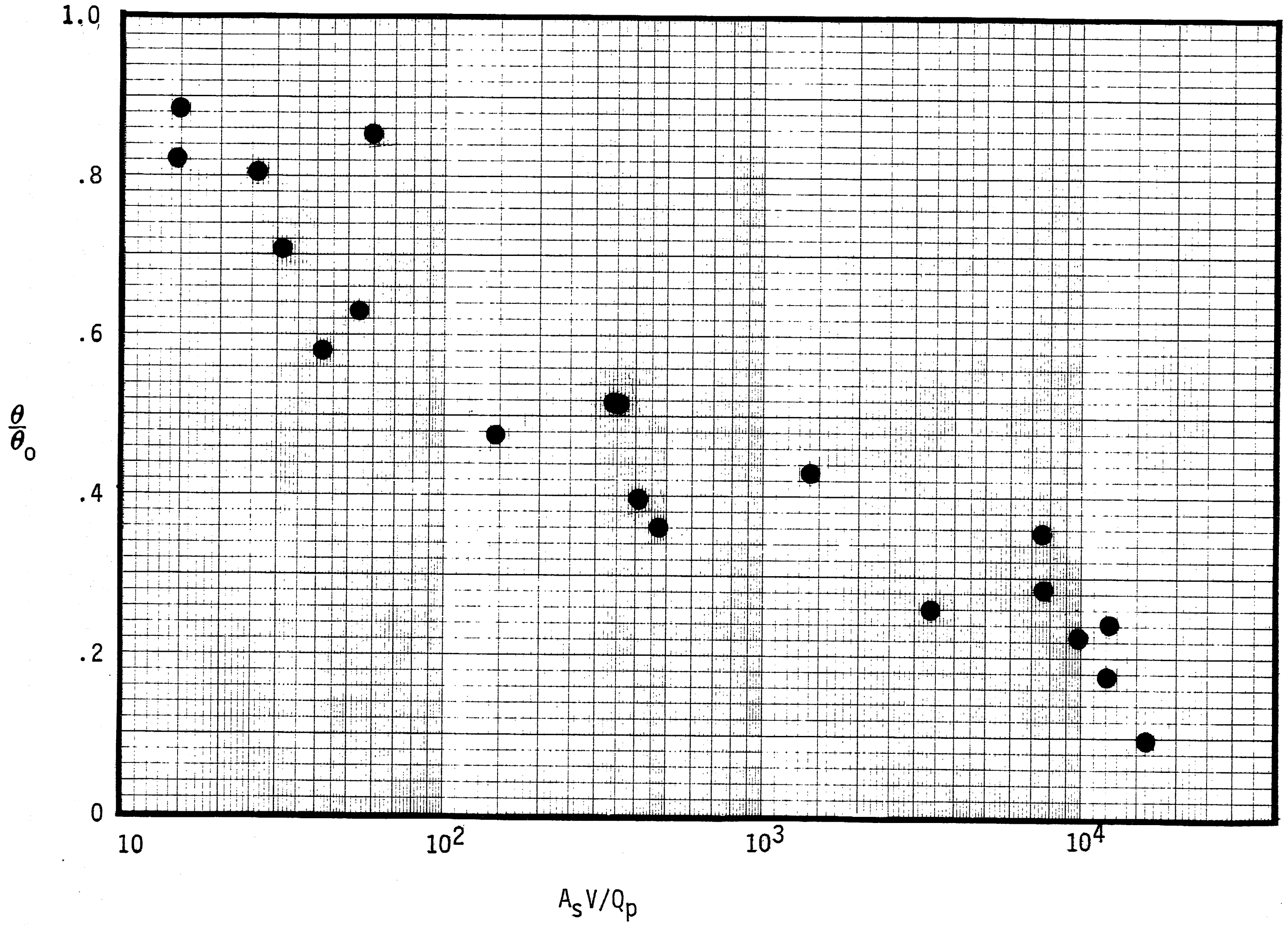

Fig. 8d - Enclosed surface areas vs. excess temperatures at $Q_p/Q_R$ from 0.223 to 0.284.

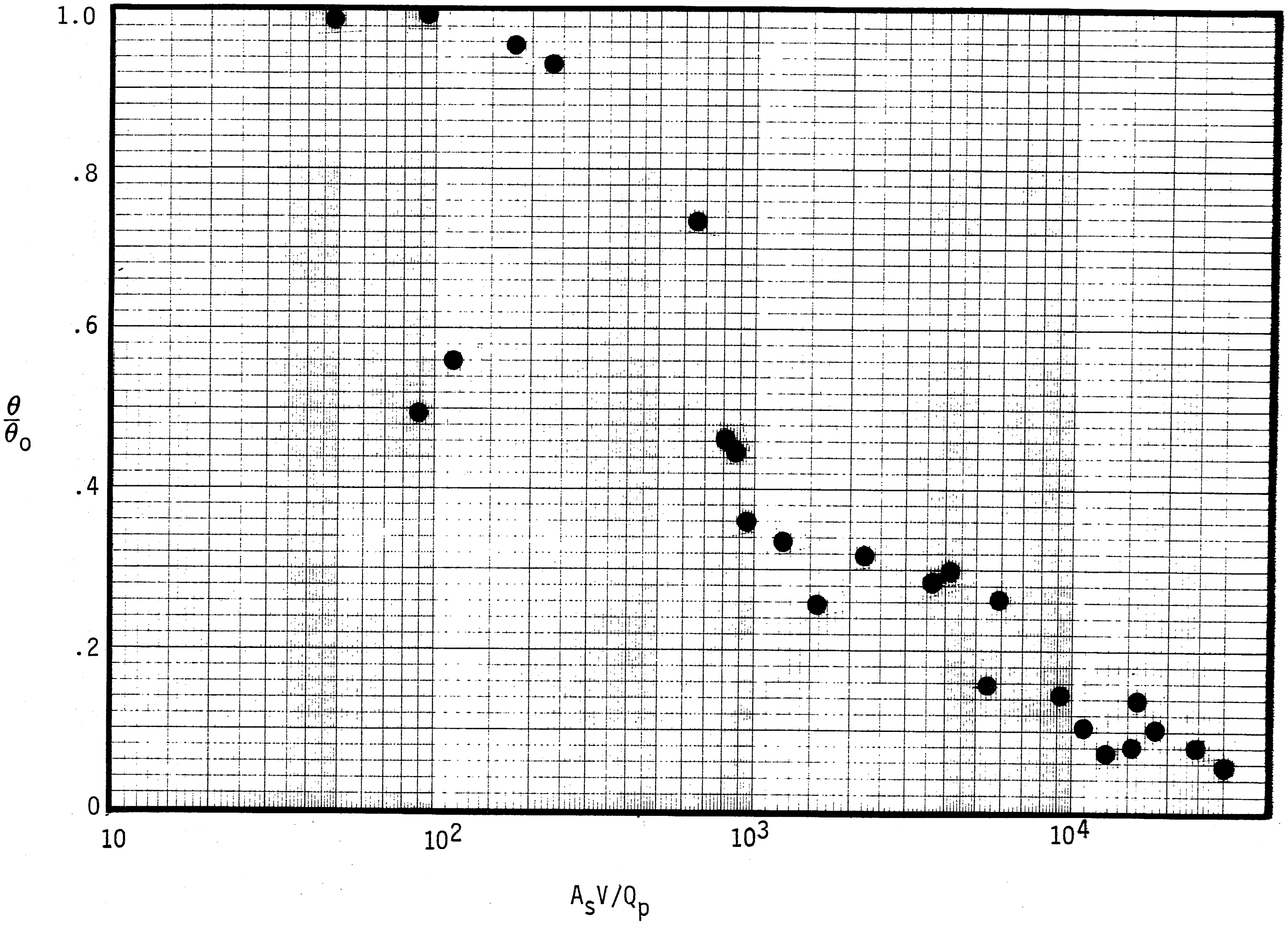

Fig. 9 - Enclosed surface areas vs. excess temperatures for <u>winter conditions</u> at $Q_p/Q_R$ from 0.041 to 0.125.

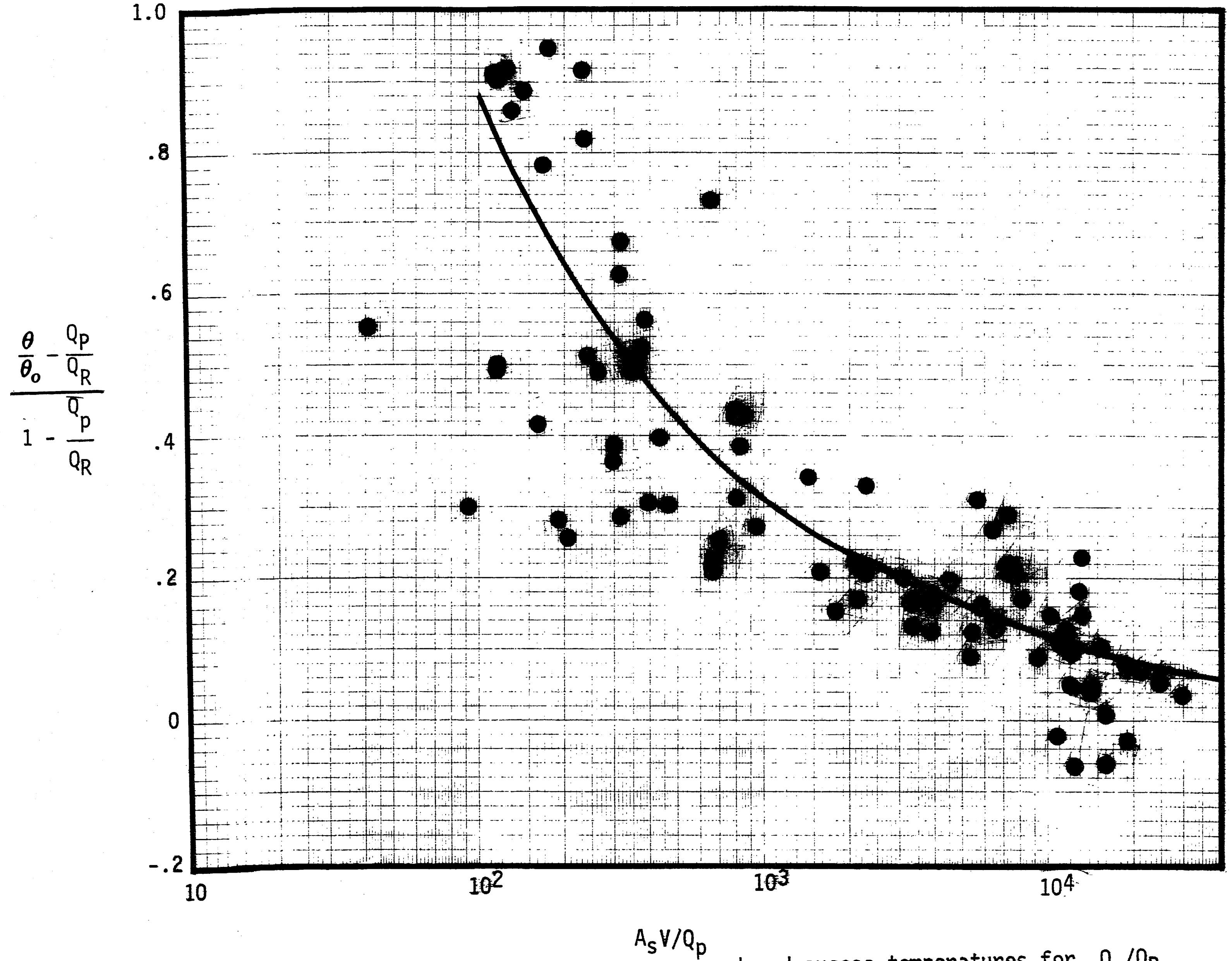

Fig. 10 - Total enclosed surface areas vs. reduced excess temperatures for $Q_p/Q_R$ from 0.036 to 0.284.

Gradients shown in Fig. 10 suggest three somewhat distinct regions:

1) $A_s V/Q_p < 500$; represents the outlet mixing region in the immediate vicinity of the discharge channel and includes some momentum effects.
2) $500 < A_s V/Q_p < 10{,}000$; represents the two-dimensional turbulent diffusion region.
3) $A_s V/Q_p > 10{,}000$; represents the region in which surface heat transfer can make an important contribution; this includes weather effects.

The boundaries between the three regions are obviously somewhat arbitrary.

The above equation does not seem to fit all ranges equally well. It does also not provide a meaningful result when $A_s V/Q_p < 100$.

Two-dimensional, turbulent mixing is the process which dominates most of the river reach under investigation. An equation more appropriate for that process is

$$\left(\frac{\theta}{\theta_o} - \frac{Q_p}{Q_R}\right)\left(1 - \frac{Q_p}{Q_R}\right)^{-1} \left(\frac{A_s V}{Q_p} + 1\right)^{0.19} = 1$$

This equation has no lower theoretical bound for $A_s V/Q_p$. However, it will fit well only if $A_s V/Q_p$ refers to values somewhat beyond the outlet. Inspection of Fig. 8 shows that the lower practical limit for $A_s V/Q_p$ moves up as $Q_p/Q_R$ increases. It is near $A_s V/Q_p = 20$ when $Q_p/Q_R \approx 0.04$ and near $A_s V/Q_p = 400$ when $Q_p/Q_R \approx 0.16$.

Application of the information given in Figs. 8 and 10 and the above equations is straight-forward. In order to find the approximate area $A_s$ enclosed by an isotherm of a given temperature T, one computes the value of $\theta/\theta_o$. This requires knowledge of river temperature $T_R$ and outlet temperature $T_o$ in addition to T. The cooling water discharge rate $Q_p$ and the river flow upstream from the plant $Q_R$ must also be known. With this information, one finds the value of $A_s V/Q_p$ using Fig. 10. The mean river flow velocity V can be computed from river discharge $Q_R$ using the relationship given in a previous section. The absolute value of $A_s$ can then be found.

Example: From $Q_p = 600$ cfs, $Q_R = 3{,}000$ cfs, $T_o = 90^\circ F$, $T_R = 78^\circ F$ find the area enclosed by the isotherm $T = 85^\circ F$.

$$\frac{\theta}{\theta_o} = \frac{85 - 78}{90 - 78} = 0.58$$

$$\frac{Q_p}{Q_R} = \frac{600}{3000} = 0.2$$

$$\left(\frac{\theta}{\theta_o} - \frac{Q_p}{Q_R}\right)\left(1 - \frac{Q_p}{Q_R}\right) = (.58 - .2)(1 - .2) = .475$$

$$A_s V/Q_p \approx 400$$

$$V = 0.056\ Q_R^{0.48} = 2.6 \text{ fps}$$

$$A_s \approx 400\ Q_p/V \approx (400)(600)/(2.6)$$

$$\approx 92{,}000 \text{ ft}^2 \approx 2.2 \text{ acres}$$

This represents a mean value. Since there is considerable scatter in the data, it is also meaningful to find an upper bound. An approximate value is roughly $A_s V/Q_p \approx 800$ or twice the mean or 4.4 acres.

## F. RECIRCULATION

Northern States Power Company has indicated that the permit under which the Monticello generating plant is operated limits the maximum cooling water withdrawal rate to 75 percent of low river flows. It was shown in this study, however, that at withdrawal rates larger than 40 percent of river flow, midstream temperature restrictions and specifications are the limiting factor. At a 40 percent withdrawal rate recirculation will not be a problem, because buoyancy induced forces will not be sufficient to move the cooling water effluent upstream to the intake.

## G. CONCLUSIONS

The maximum relative width, the approximate length, and the approximate total surface area enclosed by a given excess water temperature isotherm in the Monticello cooling water effluent plume can be calculated from the following three relationships.

$$\left(\frac{y}{b}\right)_{max}\left(\frac{\theta}{\theta_o}\right)=\left(\frac{Q_p}{Q_R}\right)^{0.75}$$

$$\left(\frac{x_{max}}{b}\right)\left(\frac{\theta}{\theta_o}\right)^{2.88}\left(\frac{Q_p}{Q_R}\right)^{-1.15}=3.9$$

$$\left(\frac{A_s V}{Q_p}\right)^{0.45}\left(\frac{\theta}{\theta_o}-\frac{Q_p}{Q_R}\right)\left(1-\frac{Q_p}{Q_R}\right)^{-1}=7$$

The basic format of the first two relationships is derived by analysis, the last one is empirical. Suitable coefficients were formed by correlation with extensive field data. The degree of approximation provided is readily apparent in the graphs showing the equations as well as the field data. Figures 3 (a, b, c, d), 5 (a, b, c), 6 (a, b, c, d) and 10 provide the information of interest for the summer months.