Essays in Heterogeneous Agent Macroeconomics

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I dedicate this dissertation to my wife, Kat Townsend.

Abstract.

This dissertation consists of two essays. The first essay revisits a puzzle for productivity driven business cycle theories. A challenge for these models of business cycles is that labor productivity and hours are negatively correlated in the United States, whereas the theory predicts a strong positive correlation. I refine the empirical puzzle and provide a theory to account for it. I emphasize that hours per worker and individual hours relative to *usual* hours move counter to productivity. In the cross-section, I find that almost all workers increase their hours when productivity is low, except for low (residual) wage earners. Furthermore, low wage earners suffer larger wage losses when productivity falls. Based on these findings, I hypothesize that non-neutral movements in productivity exacerbate frictions due to adverse selection. I use recent advances in competitive search theory to imbed Akerlof's Rat Race into a Neo-Classical growth model with search frictions. Firms bundle high earnings with long hours in order to separate more able and willing workers from the less productive. When low productivity workers fall (relatively) farther behind, the firm requires longer hours from everyone else; perversely, many workers work harder even as the market value of their time falls. A calibrated version of the model is used to measure the aggregate volatility generated by productivity shocks when labor markets are burdened by adverse selection.

In the second essay, Jacob Short and I study the behavior of entrepreneurs in order to understand the sources and magnitude of risks that they face. Entrepreneurship is risky; entrepreneurs forgo wages and invest their time and resources into a business with large potential gains, but uninsurable risks. On the other hand, bankruptcy and incorporation may help insure against these risks. We document that incorporated entrepreneurs operate larger businesses, accumulate more wealth, and are on average more productive than unincorporated entrepreneurs. We embed the U.S. bankruptcy and incorporation legal systems in a quantitative macroeconomic theory of occupation, incorporation, and default choices that accounts for the cross-sectional facts. In the model, as in the U.S., incorporation provides insurance via limited liability beyond personal bankruptcy exemptions, at the expense of administrative burdens and an endogenous interest rate premium. Our model suggests that capital embodied shocks, in which catastrophes occur with a small probability, are important entrepreneurial risks. We find the welfare gains for entrepreneurs from eliminating investment risk are huge (5.9% increase in annual consumption). And, the welfare loss from removing limited liability appears to be large.

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1. Chapter: A Quantitative Rat-Race Theory of Labor Market Dynamics

2. Introduction

This paper is a new attempt to reconcile productivity driven theories of the business cycle with the aggregate evidence against them. Specifically, I provide a theory to account for three empirical regularities in macroeconomic data that are puzzling for many business cycle models: 1) aggregate hours (but even more so, individual hours worked) and average labor productivity are negatively or weakly positively correlated, 2) the residual from a representative household's intratemporal trade off between hours and consumption (the "labor wedge" as in Chari et al. [2007]) is strongly positively correlated with hours, and 3) total hours are much more volatile than labor productivity.

Traditional equilibrium theories of business cycles (Kydland and Prescott [1982]) generate tight predictions on the comovement between consumption, labor supply, output, and productivity. In these theories, the substitution effect on leisure dominates in response to a temporary rise in real wages; under complete markets the aggregate response to productivity mirrors the individual response to wages. One finds that 1) the response of hours to productivity is overwhelmingly positive, 2) the intratemporal optimality condition on labor supply holds with equality at all times and states of the world, and 3) a large fraction of output volatility is endogenous only if the elasticity of hours with respect to productivity is substantially larger than empirical micro estimates.

The key mechanism in this paper is that workers are heterogeneous in their productivity but hours are aggregated so that employers cannot observe the marginal product of any given worker. This creates an adverse selection problem in the labor market. Furthermore, good workers are assumed to receive lower disutility for every marginal hour worked, which allows employers to separate good and bad workers by bundling high paying jobs with long hours. In this environment, the productivity of bad workers and the labor supply options available to good workers are linked. When the productivity of bad workers falls, they are more willing to work long hours to escape their low paying jobs. In response, firms require longer hours from good workers even if their productivity is constant. Thus 1) aggregate hours can rise while average labor productivity falls (both because some workers are truly less productive and others are increasing hours under decreasing *average* returns), 2) hours are high when the intratemporal optimality condition indicates an implicit subsidy (the labor wedge is high), and 3) changes in productivity dispersion induce additional movements in hours beyond those due to TFP shocks.

In order to measure the quantitative importance of the theory, I embed Akerlof's Rat Race (Akerlof [1976]) into the labor market of a stochastic neo classical growth model. In order to avoid measurement issues involved with identifying bad workers in the data, I consider the limit of economies as the share of bad workers approaches zero. A well known pathology arises in adverse selection environments when there are few of the bad agents: competitive equilibria fail to exist (see Rothschild and Stiglitz [1976]). I use competitive search theory (as in Guerrieri et al. [2009]) to overcome this problem; when workers and employers match bilaterally, market tightness provides an additional margin with which to sort workers by type. In this economy, competitive equilibria exist for any composition of the workforce. This is the first paper to capitalize on Guerrieri et al. [2009]'s work to study adverse selection in a dynamic business cycle model.

I measure the impact of adverse selection on a benchmark economy in which workers have complete insurance against employment and type risk and in which job posting costs are small enough to ignore unemployment. If the gap between good and bad workers is constant, then this economy mimics a standard frictionless RBC model: the correlation between hours and labor productivity is near 1, the labor wedge is constant, and the standard deviation of output is only 22% greater than that of TFP. [1] When the volatility of the productivity gap between good and bad workers is chosen to match the volatility of the labor wedge relative to output, the correlation between hours and labor productivity falls to -0.23 and the labor wedge is positively correlated with hours. Furthermore, the standard deviation of output rises by 25% relative to the full info economy.

The paper proceeds as follows. After a survey of previous research on this topic, I review the aggregate data and emphasize that individual workers'

[1] I use a Frisch labor supply elasticity of $\epsilon = 0.43$. This is the value used for men in Heathcote et al. [2008], and is within the bounds of micro estimates.

hours respond counter to labor productivity, which is the margin most relevant for the theory. I then look at the cross-section to understand *who's* hours move counter to labor productivity and how do changes in productivity affect these individuals differently. While it is clearly impossible to construct the *unobservable* residual productivity distribution, this exercise is illustrative. Next, I outline a static version of the theory in which ex-ante homogeneous workers are insured against type and employment risk in a large family. I then embed this labor market into the growth model and quantitatively assess the impact of adverse selection on the macroeconomy; intitial findings indicate that the effect is large. I conclude with directions for future research.

2.1. **Related Literature.** Many previous researchers have studied the macroecomic puzzles considered in this paper, and it would be amiss to ignore these contributions. At the same time, my theory of macroeconomic labor dynamics rests on a particular friction at the microeconomic level, and so I also review the relevant literature on adverse selection in labor markets.

2.1.1. *Aggregate Labor Dynamics.* Christiano and Eichenbaum [1992] emphasize that aggregate hours are perfectly positively correlated with labor productivity in vanilla RBC model, since movements in real wage are proportional to movements in labor productivity and the Frisch elasticity is positive. They explore the effects of adding a second shock, which they identify as government spending. Government spending doesn't translate one for one into consumption, so a rise in G has a negative wealth effect: both consumption and leisure fall in response. Their model can dampen the correlation between labor productivity and aggregate hours, but quantitatively cannot change the sign.

A similar line of attack is taken in Hansen and Wright [1992], who explore the effect of indivisible labor, nonseperable leisure, and home production of the comovement between hours and labor productivity. They find that shocks to the value of home production are as successful as government spending. In that model, what matters for aggregate labor supply is not the productivity of time at work, but it's market value relative to value at home. If market productivity falls but home productivity falls even more, hours supplied to market activities can increase. A common criticism of this method is that it adds an unobservable sector to the economy (home production) which comoves negatively with the observable sectors (market production). The theory that I propose, while still relying on unobservable shocks to relative productivities between good and bad workers, generates cross-sectional implications that are absent in the home-production model.

Chang and Kim [2007] attempt to reconcile the comovement between labor productivity and hours in a way most similar to this paper. They consider an economy with heterogeneity, indivisible labor, and incomplete markets. The aggregate elasticity of labor supply is no longer dictated by the value at the micro level, but is instead determined by the fraction of the population who are inframarginal on supplying a fixed quantity of labor. Their model generates a very low positive correlation between hours and labor productivity, but is silent on the intensive margin. Interestingly, they show that an aggregate labor supply elasticity as measured by a representative agent cannot be taken as a structural parameter in an economy with nonconvexities and heterogeneity. My theory suggests that the nonconvexity in labor supply cannot be taken as a technological parameter in a heterogeneous agent economy with adverse selection.

Gali [1999] argues solutions that rely on multiple shocks are unsatisfying. He provides evidence from a structural VAR that hours respond negatively to an innovation in productivity. While this finding is contentious (Francis and Ramey [2009] provide an overview of the debate), it leads Gali [1999] to propose a theory that relies on the interaction between price stickiness in the goods market and changes in productivity: if demand is fixed (because prices are fixed) and workers become more productive, then they must work fewer hours. Common criticisms of ad hoc stickiness aside, the model is less applicable when labor productivity falls. In such a situation, the goods market is exactly the same as when productivity rises, but now the assumption that firms must produce to meet demand is no longer innocuous. Firms would rather just produce at shortage levels without changing their workforce.

2.1.2. *Micro Evidence.* There is little direct evidence on workers' hours being distorted upward. Landers et al. [1996] test whether the hours of some workers are distorted due to a rat race. They ask associates at two law firms which option they would prefer in addition to a 5% wage raise: decrease hours by 5%

(earnings constant), keep hours constant (earnings rise 5%), or increase hours by 5% (earnings rise 10%). They find that two thirds of respondents would prefer to reduce hours and keep earnings constant, while only 12% would want to increase hours. Many associates seem to be working longer hours than would be implied by the first best (with the caveat that the fractions didn't differ by income level and therefore respondents likely weren't reducing hours due to the income effect). Furthermore, partners were substantially more likely to support promotion for a hypothetical associate who worked long hours than one who requested to work short hours, keeping their contribution constant (20% vs 2% levels of very heavy support, respectively).

They also cite a statistic from the Canadian Labor Force Survey, which asked workers if they would reduce hours at the current wage if given the chance. Of the college educated group, about 26% said that they would. This is likely a lower bound on the fraction of workers who work in rat race conditions, since a worker who expects to have higher earnings due to long hours might make consumption commitments (such as mortgages or car payments) that prohibit reducing hours in the hypothetical situation presented in the survey.

The labor market is more likely afflicted by adverse selection if an individual worker's marginal product is difficult for the firm to observe. This is more likely to be the case if workers produce in groups, such as a team of engineers who spend time in brain storming sessions in the process of bringing a new good to market. Lazear and Shaw [2007] provide some evidence that such forms of production are prevalent in the modern US economy. They report that, between 1987 and 1999, over 80% of large firms typically had some production done by workers in teams. Further, the fraction of these firms that had more than 20% of their workers employed in teams was 61% in 1999 (up from 37% in 1987).

It is also widely known that the past thirty years have seen a large increase in residual wage inequality (Autor et al. [2005]), which, under the theory of adverse selection, would generate long-run changes in the variance of labor supply and its covariance with wages. Kuhn and Lozano [2008] show that the share of men working long work weeks (over 50 hours) rose by 6%-points between 1979-2006 and that most of the rise occured during the eighties when residual wage inequality was rising fastest. Furthermore, the rise was greatest in occupations and industries with 1) the largest rise in residual wage inequality, 2) the smallest rise in average real wages, and 3) independently from rising returns to long hours at the individual level. All of these observations are consistent with the micro-economic environment used in this paper. [2]

3. Data

3.1. **Aggregate Accounting.** I organize the data around an aggregate production function of the form $Y = zF(K, H)$, where Y is real output, z is total factor productivity, K is the stock of capital, and H is total hours. Labor productivity is defined as $p = \frac{Y}{H}$. Many previous researchers have found a negative or weakly positive correlation between average labor productivity and aggregate hours, defined as total private real output divided by total private hours. For example, the data collected by Prescott et al. [2005] from 1947 to 2009Q3 exhibit a correlation between these series of -0.34 at the business cycle frequency.

I focus on the comovement between hours per worker and labor productivity. Again, using Prescott et al. [2005], the correlation is calculated as -0.43. Furthermore, the elasticity of total hours with respect to labor productivity is -0.83 and the elasticity of hours per worker is -0.42, hence half of the negative comovement between total hours and labor productivity is due to hours per worker. [3] This cannot be explained entirely by changes in composition, as the ratio of current *usual* hours also moves counter to labor productivity [4], with an elasticity of -0.24. [5] The negative correlation between hours and

[2]Michelacci et al. [2007] give a human capital accumulation theory of this phenomenon in the U.S. relative to Europe.

[3]The elasticities are $\eta_{H,p} = \text{corr}(H, p)\sigma_H/\sigma_p$ and $\eta_{h,p} = \frac{\text{corr}(h,p)\sigma_h}{\text{corr}(H,p)\sigma_H}\eta_{H,p} = 0.51\eta_{H,p}$.

[4]Data concerning usual hours are since 1979Q1

[5]Usual hours are below current hours on average, because the CPS asks about current hours during weeks without a holiday. This affects the level (the mean of the ratio is 1.01, but not the covariance with labor productivity. I thank Sam Shulholfer-Wohl for point this convention out to me.

labor productivity is the reason that the labor wedge is procyclical [6] Detailed moments [7] can be found in Table 1.

TABLE 1. Second Moments at Business Cycle Frequencies, 1947-2009

	p	Y	h
	$\sigma_Y = 1.72\%$		
$\frac{\sigma_x}{\sigma_Y}$	0.60	1.0	0.45
$\sigma_{x,p}$	1.0	0.31	**-0.41**

3.1.1. *Cross-Sectional Dynamics.* I next turn to cross-sectional features of hours per worker and wages for guidance on whether the forces in the model seem relevant. I find that essentially all workers' hours move counter to average labor productivity, but that the magnitude diminishes at the lower tail of the residual wage distribution. To this end, I regress a worker's wage (defined in the CPS as their earnings divided by their usual hours) against available observables, including Mincerian terms, gender, race, and occupation. I then group them by residual deciles and create time series of each group's current relative to usual hours. Table 2 shows the elasticities of each of these time series with respect to average labor productivity. It is clear that all deciles comove negatively with labor productivity, but the lowest decile is smallest in magnitude and is statistically insignificant from zero at the 5% level.

I group workers by their residual because, ideally, I want to compare the comovement of labor productivity with the hours of workers with different *unobservable* productivities. Because the theory always yields a separating equilibrium, such an exercise is conceptually well defined. Practically speaking, however, it is impossible to control for every worker characteristic that employers observe. This means that any given grouping of workers based on measured wages likely contains some workers who differ along a characteristic

[6] I use one minus Shimer [2009]'s wedge, which is the residual from the intratemporal optimality condition of a stand-in household with preferences consistent with balanced-growth and constant Frisch elasticity in an economy with Cobb-Douglas production.

[7] All moments from deseasoned variables at quarterly frequency, detrended via hp-filter with smoothing parameter of 1600

that is observable in the market but not in the CPS data. If there is a distribution of private worker productivity within each of these observable classes, and the empirical group contains members of both classes, then some movements in hours will be due to optimal changes for undistorted workers and some will be due to increases/decreases in the distortion on (privately observed) high productivity workers within a given (publicly observed but unmeasured) class. Nonetheless, this is an illustrative exercise if those at the bottom of the observable residual distribution make up a disproportionate fraction of the lowest unobservable residual groups. This can be seen by considering the worker at the very bottom of the wage distribution: no matter what observable characteristics he shares with with another group of workers, he must be at the bottom of that group as well.

Table 2. Elasticities with Respect to Avg Labor Productivity, 1979-2009

	h_{0-10}	h_{10-20}	h_{20-30}	h_{30-40}	h_{40-50}	h_{50-60}	h_{60-70}	h_{70-80}	h_{80-90}	h_{90-100}
Elasticity	-0.15	-0.25*	-0.21*	-0.17	-0.32*	-0.19*	-0.30*	-0.39*	-0.45*	-0.39*

It can be seen that the hours of high residual wage earners are more negatively correlated (in terms of point estimates, and likelihood of being significantly non-zero) than are low residual wage earners. Given the likely noise previously mentioned, this is remarkable. With the measurement issues in mind, I focus on the difference between the lowest decile and everyone else in Table 3 since it is at the 10th percentile that statistical significance is lost. In addition, I compute the elasticity of each group's real wage with respect to aggregate labor productivity and find support for modeling productivity shocks as non-neutral, with lower productivity workers affected by more than the average worker.

Table 3. Elasticities wrt Average Labor Productivity, 1979-2009

	h_L	h_H	w_L	w_H
Elasticity	-0.10	-0.44*	0.16	0.05

4. Model

4.1. **Static Version.** I start by considering a static model which is similar to the Rat Race example in Guerrieri et al. [2009], except that workers are ex-ante homogeneous and are members of a large family which insures them against type and employment risk. The family comprises a unit mass of workers. They wake up in the morning and τ_H of the members are good workers, while τ_L are bad. An hour from a good worker will generate more output than that of a bad worker. In addition, workers differ in their disutility of labor supply, with high productivity workers *less* averse to supplying a marginal hour than low productivity workers. At this point workers are heterogeneous and enter the labor market in which employers cannot observe their type. Workers direct their search to a job which specifies that they work h hours and receive w in earnings, which they find with some probability. Workers who find work return at the end of the day with their earnings at which point the head distributes the total earnings amongst family members.

There is an aggregate firm with production function $zF(H)$, where H is total *effective* hours used in production, and $F' > 0, F'' < 0, \frac{\partial}{\partial H}F(H)/H < 0$. An hour of the good worker's time translates to ζ_H hours in production, whereas an hour of the low productivity worker is effectively $\zeta_l < \zeta_H$ hours. The aggregation of hours makes it impossible to infer any given worker's marginal contribution to output. The firm comprises a large group of risk neutral capitalists. Capitalists post vacancies at cost κ; these posts specify the hours and earnings paid to a worker who reports being type i. Profits are distributed to capitalists according to their marginal contribution to aggregate profits.

4.1.1. *Time Zero Problem of the Family.* Assuming that the labor market equilibrium allocation is given by earnings, hours, and job finding rates for each type $(w_i, h_i, \bar{f}_i)_{i=L,H}$, the head of the family chooses how to allocate consumption across workers to solve:

$$\max_{\{c_i^1, c_i^0\}_{i=L,h}} \sum_{i=L,H} \tau_i \left(\bar{f}_i (u(c_i^1) - v_i(h_i)) + (1 - \bar{f}_i) u(c_i^0) \right) \text{ s.t.}$$

$$\sum_i \tau_i \left(\bar{f}_i c_i^1 + (1 - \bar{f}_i) c_i^0 \right) = \tau_L \bar{f}_L w_L + \tau_H \bar{f}_H w_H$$

I will assume that u is strictly concave. The first order conditions from this problem ensure that the marginal utility of consumption is equalized across all workers. Thus, because of separability between consumption and labor supply, the actual consumption of each member is independent of employment or type.

In order to map the labor market for this economy into the language of Guerrieri et al. [2009] I will derive a worker's valuation of a job paying w and requiring h hours. Think of a positive mass, δ, of workers of type i, each taking the job (w, h). Family income is increased by $w\delta$, which is then split evenly amongst the unit mass of workers. The family utility gain is $u(c + w\delta) - u(c) \approx u'(c)w\delta$. The utility gain per worker taking the job is then just $u'(c)w$. This is also the marginal value of w for the worker taking the job. The family's disutility of labor supply changes by $-v_i(h)\delta$, which is just $-v_i(h)$ per worker. Thus the worker's valuation of (w, h) (which agrees with the family's) is $u'(c)w - v_i(h)$. With this in mind, I review the environment of competitive search with adverse selection.

4.1.2. *Competitive Search With Adverse Selection.* Call a job $y_i = (h_i, w_i) \in Y$ the hours and earnings paid to a worker who claims to be type i, where $Y = [0, 1] \times \mathbb{R}_+$ is the feasible set of jobs. Define $\mathbf{y} = (y_L, y_H)$ to be a mechanism, which is a pair of jobs depending on the reported type. A worker who is type i and reports for a job y_j gets utility:

$$U_i(y_j) = u'(c)w - v_i(h) \tag{4.1}$$

Now consider a capitalist who posts a mechanism $\mathbf{y}$ and meets with a worker who claims to be type j but is in fact type i. The capitalist gets profits from this worker of:

$$\pi_i(y_j) = zF'(H)\zeta_i h_j - w_j \tag{4.2}$$

By the Revelation Principle, capitalists will only post mechanisms that are incentive compatible. Call this set of mechanisms $\mathbb{C} = \{\mathbf{y} : U_i(y_i) \geq U_i(y_j)\ \forall i, j\}$.

Capitalists post mechanisms in $\mathbb{C}$, to which workers apply and report their types. Since in equilibrium mechanisms will be separating, we can consider *contracts* instead of mechanisms. A contract is a mechanism that specifies the same job for any worker type, so contract y is just the mechanism $\mathbf{y} = (y, y)$. Let $\theta(y)$ be the market tightness (capitalist-worker ratio) for contract y. Then $f(\theta(y))$ is the probability that a worker who applies to y finds a job and $q(\theta(y)) = f(\theta(y))/\theta(y)$ is the probability that a capitalist posting y meets a worker. Let $\gamma_i(y)$ be the share of $i-$type workers out of all workers who apply to y. The expected utility of a worker who applies to contract y is:

$$\hat{U}_i(y) = f(\theta(y))U_i(y) \tag{4.3}$$

and the expected profit of a capitalist who posts contract y is:

$$\hat{\pi}(y) = q(\theta(y)) \sum_i \gamma_i(y) \left(zF'(H)\zeta_i h_i - w_i\right) - \kappa \tag{4.4}$$

With this notation in hand, I define the labor market equilibrium as in Guerrieri et al. [2009].

Definition 4.1. A competitive search equilibrium is a vector $\{\bar{U}_i\}_{i=L,H}$, a measure λ on Y with support $\bar{Y}$, a function θ on Y, and a function $\Gamma(y) = (\gamma_i(y))_{i=L,H}$ on Y satsifying:

I. Capitalists maximize profits and free entry: $\forall y \in Y$,

$$\hat{\pi}(y) \leq 0$$

with equality if $y \in \bar{Y}$.

II. Workers search is optimal and generates $\bar{U}_i$.

$$\bar{U}_i = \max_{y' \in Y} \hat{U}_i(y')$$

with $\bar{U}_i = 0$ if $\bar{Y} = \emptyset$. For any $y \in Y$ and i, $\bar{U}_i \geq f(\theta(y))U_i(y)$ with equality if $\theta(C) < \infty$ and $\gamma_i(C) > 0$. Also, if $U_i(y) < 0$ then either $\theta(y) = \infty$ or $\gamma_i(y) = 0$.

III. The market clears.

$$\int_{\bar{Y}} \frac{\gamma_i(y)}{\theta(y)} d\lambda(\{y\}) \leq \tau_i$$

with equality if $\bar{U}_i > 0$.

The first part of the definition reflects optimizing behavior of capitalists and the competitiveness of the market, so that a given capitalist posts the contract that maximizes his profits but capitalists post a given contract until the cost of doing so κ is equal to the returns. The second condition says that workers maximize their utility when applying to contracts, and must be indifferent between any contracts to which they actually apply in equilibrium. The final condition says that the sum of type i applications over all possible contracts must equal the total number of workers of type i who are supposed to apply (τ_i).

It is important to note the restrictions placed on jobs *not* posted in equilibrium. Specifically, consider a the case where the fraction of bad workers approaches zero. Without competitive search, equilibrium doesn't exist. Any possible equilibrium must be separating. If it wasn't then a capitalist could offer a contract with slightly higher hours that would attract only good workers; underpaying them slightly gives positive profits. On the other hand, the equilibrium cannot be separating because a firm could post a pooling contract in which good worker's hours were not distorted and the (infinitesimally few) bad workers were subsidized. Because this contract would attract all workers, the average product would essentially be equal to the good worker's production. Since neither a separating nor a pooling contract can be an equilibrium, none exists.

With competitive search the separating equilibrium always exists. The key condition is that the share of workers applying to a contract y is zero if y gives less utility than their equilibrium contracts. Starting with the least cost separating contracts as a proposed equilibrium, consider a capitalist who posts a pooling equilibrium that lessens the hours distortion for good workers. It is no longer the case that the average product will calculated according to the population shares of each worker; instead, he must forecast what fraction of applicants for the job will be each type. Since a given bad worker gains more from applying to the pooling contract (he is cross-subsidized by the good workers), the applicant pool will be predominantly bad. Since good workers always have less to gain from applying to the pooling contract, there is always a separating equilibrium.

It is now possible to check that this economy satisfies the conditions required to appeal to Guerrieri et al. [2009]'s existence result in proposition 3 and their algorithm for computing equilibria.

Proposition 4.2. *The functions U_i and π_i satisfy:*

A1 *Monotinicity: For any y: $\pi_L(y) \leq \pi_H(y)$.*

A2 *Local Non-Satiation: Let $B_\delta(y) = \{y' : ||y - y'|| < \delta\}$. Then for any y with $w > 0$, and any $\delta > 0$, there is some $y' \in B_\delta(y)$ such that $\pi_H(y') > \pi_H(y)$ and $U_L(y') < U_L(y)$*

A3 *Sorting: For any $y >> 0$ and any $\delta > 0$, there is some $y' \in B_\delta((h', w'))$ such that:*

$$U_H(y') > U_H(y) \text{ and } U_L(y') < U_L(y)$$

The first condition is obviously satisfied, since having a low productivity worker work h hours and paying him w gives lower profits than if her productivity was instead high. The second condition holds since for any (h, w) with $w > 0$, it is possible to keep h the same and put $w' = w - \delta$, thereby reducing the utility for every worker and raising the capitalist's profits. Under the assumption that $v'_L(h) > v'_H(h)$ for all h, the final condition holds because a good worker is willing to increase her hours for a smaller increase in consumption than would a low productivity worker. Formally, consider taking some $\delta > 0$ and increasing h and w to:

$$h' = \frac{u'(c)}{v'_H(h)}\delta \text{ and } w' = w + \delta$$

Then the change in utility for the good worker is approximately zero (and could be made positive by increasing hours slightly less), whereas the change in utility for the bad worker is:

$$\Delta U_L = u'(c)\delta\left(1 - \frac{v'_L(h)}{v'_H(h)}\right) < 0$$

Where the inequality is due to $v'_L(h) > v'_H(h)$. Thus the perturbed utility is lower for the low productivity worker.

The sorting condition **A3** is fundamental for the existence of seperating equilibria, and is why the assumption that $v'_L(h) > v'_H(h)$ is typically made. This is a strong assumption, and, while plausible, it is very hard to verify empirically. It is plausible due to two mechanisms through which high productivity workers would endogenously provide a marginal hour of labor for less marginal earnings. The first is learning by doing: even if productivity is initially the same, workers with lower disutility of labor supply will be more productive after a short time, since they will work longer hours and accumulate more human capital. Another mechanism is consumption commitments made before contracting with employers. If good workers know that they will have high earnings, then they may commit to payments on durable consumption goods before entering the labor force (car payments, mortgages, etc). If utility over these goods and flow consumption is separable, then good workers will have a higher marginal utility for every dollar in earnings than will bad workers (see Chetty and Szeidl [2007] for how such commitments can be modelled). This would generate preferences over earnings and hours that satisfy assumption **A3**, even without ex-ante differences in preferences over consumption and hours. I leave these implementations for future research, and continue with complete markets and preference heterogeneity, which allows me to appeal to Guerrieri et al. [2009]'s existence result:

Proposition 4.3. *There exists a unique seperating labor market equilibrium and it can be computed by solving the programming problems below.*
For the low productivity worker:

$$\bar{U}_L = \max_{\theta,w,h} f(\theta) U_L(w,h) \tag{4.5}$$

$$s.t. \tag{4.6}$$

$$\kappa \leq q(\theta)\left(F'(H)\zeta_L h - w\right) \tag{4.7}$$

For the high productivity worker (taking $\bar{U}_L$ parametrically):

$$\bar{U}_H = \max_{\theta,w,h} f(\theta) U_H(w,h) \tag{4.8}$$

$$s.t. \tag{4.9}$$

$$\kappa \leq q(\theta)\left(F'(H)\zeta_H h - w\right) \tag{4.10}$$

$$\bar{U}_L \geq f(\theta)\left(u'(c)w_H - v_L(h_H)\right) \tag{4.11}$$

This result is powerful for two reasons. The first is that it makes computation of equilibria relatively easy. The second is more subtle: because it is impossible to identify the workers by type in the data, I want to look at the limit of economies as $\tau_L \to 0$. But this is exactly the situation in which equilibria do not exist without competitive search! Competitive search therefore guarantees well behaved equilibria for the exact parameterizations that allow me to overcome previously unsurmountable measurement issues.

4.1.3. *Labor Market Characterization.* The low productivity worker is entirely undistorted, and therefore behaves as in traditional theory. Specifically, her intratemporal Euler Equation is given by:

$$\frac{\phi_L^{-1} v'(h_L^*)}{u'(c^*)} = F'(H^*)\zeta_L \tag{4.12}$$

Where a variable with $*$ superscript is the first best. If the incentive compatibility constraint doesn't bind, then the high productivity worker's hours also satisfy the standard intratemporal Euler Equation:

$$\frac{\phi_H^{-1} v'(h_H^*)}{u'(c^*)} = F'(H^*)\zeta_H \tag{4.13}$$

On the other hand, the good worker's hours are distorted upwards whenever the incentive compatibility constraint on the low productivity worker binds. This happens if:

$$f(\theta_H^*)\left(u'(c^*)w_H^* - \phi_L^{-1} v(h_H^*)\right) > \bar{U}_L$$

In this case, letting a hat variable denote the distorted adverse selection equilibrium value, the good worker's intratemporal condition becomes:

$$\frac{\phi_H^{-1} v'(\hat{h}_H)}{u'(\hat{c})} = \left(\frac{1-\psi}{1-\psi\frac{v_L'(\hat{h}_H)}{v_H'(\hat{h}_H)}}\right) F'(\hat{H})\zeta_H \tag{4.14}$$

This latter equation drives the results. The term ψ is the Lagrange multiplier on the incentive compatibility constraint, and if the constraint binds then the "wedge" in front of the good worker's marginal product is greater than one. When ζ_L falls, the constraint tightens and ψ rises, as does the wedge. This is the mechanism through which average labor productivity can fall, but some workers increase their hours. In addition, for typical parameterizations, hours per worker and vacancies for that worker move in the same direction. Thus, in response to a fall in the bad worker's productivity, the total hours of good workers (bodies time hours per body) can rise. In the extreme case, the fraction of bad workers can approach zero, so that *every* worker's hours rise, and since $F(H)$ is concave, labor productivity falls. This is summarized in the following proposition.

Proposition 4.4. *If the incentive compatibility constraint binds, then* $\hat{h}_H > h_H^*$ *and* $\frac{\partial h_H}{\partial \zeta_L} < 0$.

At this point the first order conditions on earnings and market tightness, along with the zero profit condition for the firms, give the remaining necessary conditions for equilibrium. Let $\eta_{x,y}$ be the elasticity of x with respect to y. In the appendix I show that, after algebraic manipulation, the conditions become:

$$k = q(\hat{\theta}_i)\left(1 - \eta_{v,h_i}^{-1}\right)\eta_{f,\theta_i}\zeta_i F'(\hat{H})h_i \tag{4.15}$$

$$\hat{w}_L = (1 - \eta_{f,\theta_L})\zeta_L F'(\hat{H})\hat{h}_L + \eta_{f,\theta_L}\frac{v_L(\hat{h}_L)}{u'(\hat{c})} \tag{4.16}$$

$$\hat{w}_H = (1 - \eta_{f,\theta_H})\zeta_H F'(\hat{H})\hat{h}_H + \eta_{f,\theta_H}\frac{v_H(\hat{h}_H)}{u'(\hat{c})}\left(\frac{1 - \psi\frac{v_L'(\hat{h}_H)}{v'(\hat{h}_L)}}{1-\psi}\right)\zeta_H F'(\hat{H})\hat{h}_H \tag{4.17}$$

The earnings equation for the bad worker should be familiar: earnings are a convex combination of the worker's marginal production and their utility cost (expressed in units of the good) from taking the job. The weights for the combination are given by the elasticity of the job finding function, due to competitive search. A similar result attains in full information models of job search in which earnings are determined via generalized Nash bargaining with weights that satisfy the Hosios condition (see Andolfatto [1996], Cheron and Langot [2004], Merz [1995]). Interestingly, the good workers' earnings take a similar form, except that they get η_{f,θ_H} times the distorted cost of their labor supply. Further manipulation yields a unified equation for earnings:

$$\hat{w}_i = (1 - \eta_{f,\theta_i} + \eta_{f,\theta_i}/\eta_{v,h_i})\,\zeta_i F'(\hat{H})\hat{h}_i \tag{4.18}$$

4.1.4. *Equilibrium.* At this point we can define the equilibrium of the static economy. It is:

Definition 4.5. A competitive equilibrium consists of:

i Labor market allocations: per capita $(w_L, h_L, \theta_L, \bar{f}_L, w_H, h_H, \theta_H, \bar{f}_H)$ and aggregate, H

ii Consumption by type and job finding outcome, $(c_L^1, c_L^0, c_H^1, c_H^0)$

Such that:

I Aggregate hours are consistent with per capita hours, ie $H = \tau_H f(\theta_H)\zeta_H h_H + \tau_L f(\theta_L)\zeta_L h_L$

II Given the items in [i] (with $\bar{f}_i = f(\theta_i)$), the consumption in [ii] solve the family's optimization problem. In effect:

$$c_i^1 = c_i^0 = c = \tau_H \bar{f}_H w_H + \tau_L \bar{f}_L w_L, \ \forall i = L, H$$

III Given the value of c, the items in [i] form a labor market equilibrium as previously defined.

Conditions under which the equilibrium exists are forthcoming in an online appendix. I note here that equilibria are readily computable for every parameterization that I have considered.

4.2. **Dynamic Model.** I now build the static model into the neo-classical growth model, period by period. Labor market equilibria are essentially identical to the static case, except that preference terms include expected future values.

4.3. **Environment.** Time is infinite and discrete. There is a unit mass of families, each with a unit mass of ex-ante identical workers, each of whom has one unit of time that can be used in production or as leisure. Workers have preferences over consumption and labor that are additively separable over time and states of the world with $U(c, h) = u(c) - v(h)$. Workers live forever and have lifetime expected utility given by $U_0 = \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t U(c_t, h_t)$.

At the beginning of each period, the aggregate state of the economy is known and is denoted $(\hat{z}, S)$ and will be described below. Each worker then draws a type $i \in L, H$ with probability τ_i, which determines her disutility of labor supply and the efficiency of her hours in production. I assume that more efficient workers find labor less displeasurable, so that $v'_L > v'_H$, but will eventually take the limit of economies as $\phi_L \to \phi_H$.

Search frictions keep some workers from immediately finding work in each period. Each family has n_t employed members at the beginning of a period. At the aggregate there are N_t employed workers. An employed worker's type is observed by firms, whereas an unemployed worker's is not. An employed worker can then choose to take a job immediately, whereas an unemployed worker must apply for a job specifying (h, w), which she finds with probability $f(\theta(h, w)) \leq 1$. I assume that search occurs at the beginning of the period, so that new matches produce immediately.

There is an aggregate firm with neoclassical production function $zF(K, H)$, where H is total *effective* hours used in production and K is rented capital. An hour of the good worker's time translates to $\zeta_H = 1$ hours in production, whereas an hour of the low productivity worker is effectively $\zeta_{L,t} < 1$ hours; hence the exogenous state is $\hat{z} = (z, \zeta_L)$. The firm rents capital on a spot market and thus chooses K to maximize $\Pi(H; \hat{z}) = zF(K, H) - (r + \delta)K$, where r is the rental rate and δ the depreciation rate. The firm comprises a large number of capitalists, each of whom is risk neutral and lives for one period. Capitalists can meet with unemployed workers costlessly in spot markets, taking the linear wage w_i^* as given, or can post a contract for unemployed workers. The profit from an employed worker of type i is given by $\pi_i^n = \max_h zF_H(K, H)\zeta_i h - w_i^* h$.

4.4. **Equilibrium.**

4.4.1. *Family Programming Problem.* There are three outcomes for a worker: those concerning his type, employment, and job finding outcome. The family enters the period with assets a and n workers who are employed. The head takes the outcomes from the labor market as given and decides how much to consume this period as well as how to allocate the consumption across workers. As before, consumption will be equated in equilibrium, and so I start with that result in the definition of the family's problem. At the time of entering the labor market, workers are heterogeneous with different (hidden) types and the competitive search market with adverse selection commences.

$$W(a,n,\hat{z},S) = \max_{c,a',h_L,h_H} u(c) - \tilde{v}(a,\hat{z},S) + \beta \mathbb{E} W(a',\hat{z}',S') \tag{4.19}$$

$$\text{s.t.} \tag{4.20}$$

$$c + a' \leq (1 + r(\hat{z},S))a + n\left(\tau_H w_H^*(\hat{z},S)h_H + \tau_L w_L^*(\hat{z},S)h_L\right)... \tag{4.21}$$

$$... + (1-n)\left(\tau_H \bar{f}_H(\hat{z},S)w_H(\hat{z},S) + \tau_L \bar{f}_L(\hat{z},S)w_L(\hat{z},S)\right) \tag{4.22}$$

$$n' = (1-s)\left(n + (1-n)\sum_i \tau_i \bar{f}_i(\hat{z},S)\right) \tag{4.23}$$

$$S' = G_S(\hat{z},S) \tag{4.24}$$

$$z' = \rho z + \nu_z' \tag{4.25}$$

$$\zeta_L' = G_\zeta(z',\zeta_L) + \nu_\zeta' \tag{4.26}$$

A standard stochastic process is assumed for aggregate TFP. The efficiency of a bad worker's hours, ζ_L, is stochastic and can depend on current z_t, whereas $\zeta_H = 1$ as a normalization. I will consider two specifications of G_ζ in the quantitative section; one in which ζ_L is a stochastic iid random variable in $(0,1)$ and another where ζ_L is a determinitic function of z.

For workers who are employed to start the period, there is no assymetric information and so competition drives $w_i^*(z,S) = zF_H(K,H)\zeta_i$ and the marginal rate of substitution between consumption and leisure equal to $w_i^*(z,S)$. The second utility term is the weighted average of disutilities:

$$\tilde{v}(a,z,S) = N\left(\tau_H v_H(h_H^*) + \tau_L v_L(h_L^*)\right) + (1-N)\left(\tau_H \bar{f}_H v_H(h_H) + \tau_L \bar{f}_L v_L(h_L)\right)$$

from which it is clear that there are utility gains from having an additional worker in the employed pool, since newly employed good workers work longer hours due to the Rat Race.

4.4.2. *Labor Market Equilibrium.* There is now a simple dynamic consideration for workers when searching for a job. The family's value tomorrow rises by the discounted expected value of having another worker employed. Let this value be denoted by $\Lambda(a,n,z,S) = \frac{\partial W(a,n,z,S)}{\partial n}$. An applciation of the envelope theorem give a recursion:

$$V(a,n,z,S) = \Omega_H + \Omega_L + \beta(1-s)(1-\tau_H \bar{f}_H - \tau_L \bar{f}_L)\mathbb{E}\Lambda(a',n',z',S') \tag{4.27}$$

$$\Omega_i = u'(c)\tau_i \left(w_i^* h_i^* - \bar{f}_i w_i^0 - (v_i(h_i^1) - \bar{f}_i v_i(h_i^0))\right) \tag{4.28}$$

Thus, letting $\Lambda(a',n',z',S') = \Lambda'$ and suppressing dependence on family and aggregate states, the value of a job y_j for a worker of type i is now:

$$U_i(y_j) = u'(c)w - v_i(h) + \beta(1-s)\mathbb{E}\Lambda' \tag{4.29}$$

The definitions of the profit function and equilibrium are unchanged, and it is clear that the sorting condition **A3** is still satisfied. The only thing that changes are the programming problems solved in order to find the equilibria. For the low productivity worker:

$$\bar{U}_L = \max_{\theta,w,h} f(\theta) U_L(w,h) \tag{4.30}$$

$$\text{s.t.} \tag{4.31}$$

$$\kappa \leq q(\theta)\left((1-\alpha)\frac{Y}{H}\zeta_L h - w\right) \tag{4.32}$$

For the high productivity worker (taking $\bar{U}_L$ parametrically):

$$\bar{U}_H = \max_{\theta,w,h} f(\theta) U_H(w,h) \tag{4.33}$$

$$\text{s.t.} \tag{4.34}$$

$$\kappa \leq q(\theta)\left((1-\alpha)\frac{Y}{H}\zeta_H h - w\right) \tag{4.35}$$

$$\bar{U}_L \geq f(\theta)\left(u'(c)w_H - v_L(h_H) + \beta(1-s)\mathbb{E}\Lambda'\right) \tag{4.36}$$

5. Calibration and Results

There are a number of parameters and choices that must be made. I parameterize the economy to admit a balanced growth path and retain a constant Frisch elasticity of labor supply, thus:

$$U(c,h) = \log(c) - \phi^{-1}\frac{\epsilon}{1+\epsilon}h^{\frac{1+\epsilon}{\epsilon}}$$

A period is set to a quarter. Production is Cobb-Douglas $F(K,H) = K^\alpha H^{1-\alpha}$ and the matching function is bounded above by 1, as in Hagedorn and Manovskii

[2008]: $f(\theta) = \frac{\theta}{(1+\theta^l)^{1/l}}$. The growth rate is set to $\gamma = 1.015^{1/4}$ to match historical data, the depreciation rate is then $\delta = 1.67\%$ to match the investment to capital ratio given γ. The discount rate is then set to $\beta = 0.993$ so that the quarterly output to capital ratio is $1/12$. The resulting steady state net interest rate is 1.08%.

5.1. **Benchmark Model.** As a benchmark, I want to use the theory to avoid measurement difficulties and to focus on the intensive margin, since it is the focus of my cross-sectional evidence. I therefore make the following restrictions:

1. The economy is homogenous for all practical purposes, ie $\tau_L \to 0$. Note that the distortion due to adverse selection still occurs so long as $\tau > 0$, since workers search after their types are realized. Since it is the good workers' hours that are distorted, and every worker is good, this increases the effect of adverse selection on aggregate hours.
2. Search is cheap enough that unemployment can be ignored, ie $\kappa \to 0$. With my specification of the matching function this ensures that $\theta_i \to \infty$ for $i = L, H$.
3. There is complete turnover of the labor market in every period, ie $s = 1$ so that $N_t \to 0$. This ensures that every worker is *newly* employed in every period, and therefore affected by adverse selection. In effect, this is a frictionless economy in which adverse selection determines hours but workers get paid their marginal product.
4. The steady state value of $\zeta_L = 1$ and the stochastic process is $\zeta_{L,t} = e^{-\nu^2_{\zeta,t}}$. Thus in steady state there is only one type of worker and the hours allocation is first best. Along a sample path, bad workers are always less productive than good workers.
5. Finally, $\phi_L \to \phi_H$. In the theory, so long as there is an infinitesimal difference in the disutilities, the equilibrium is separating. This ensures that the incentive compatibility constraint always holds with equality, which allows me to study the log-linearized economy.

The resulting system of equations can be solved at the detrended steady state and then approximated in order to study the cyclical properties of the economy:

$$\gamma c_t^{-1} = \beta \mathbb{E}_t\{(1+r_{t+1})c_{t+1}^{-1}\} \tag{5.1}$$

$$\phi^{-1} h_{L,t}^{1/\epsilon} c_t = (1-\alpha)\frac{Y_t}{H_t}\zeta_{L,t} \tag{5.2}$$

$$w_{L,t} = (1-\alpha)\frac{Y_t}{H_t}\zeta_{L,t} h_{L,t} \tag{5.3}$$

$$c_t^{-1} w_{H,t} - \phi^{-1} v(h_{H,t}) = c_t^{-1} w_{L,t} - \phi^{-1} v(h_{L,t}) \tag{5.4}$$

$$w_{H,t} = (1-\alpha)\frac{Y_t}{H_t} h_{H,t} \tag{5.5}$$

$$H_t = h_{H,t} \tag{5.6}$$

$$\hat{z}_t \alpha (\frac{K_t}{H_t})^{\alpha-1} = r_t + \delta \tag{5.7}$$

$$c_t = w_{H,t} + (1+r_t)K_t - \gamma K_{t+1} \tag{5.8}$$

The key equation is number 5.4, which is the incentive compatibility condition for the low productivity worker. This gives the distortion on high productivity workers' hours. Notice that, since in steady state the workers have the same productivities there is no distortion. Also, since $s = 1$, the Λ terms vanish.

5.2. **An Analytical Solution.** The model is sufficiently simple that a well known property of the neo-classical growth model carries through: since utility is logarithmic over consumption and separable between consumption and hours, when capital depreciates fully every period ($\delta = 1$), there is a simple analytic solution. The proposition is as follows:

Proposition 5.1. *Under the restrictions of the benchmark model and $\delta = 1$, the dynamics of the economy are fully characterized by:*

$$K' = \alpha\beta z K^{\alpha} H^{1-\alpha} \tag{5.9}$$

$$\left(\frac{1-\alpha}{1-\alpha\beta}\right) H^{1+\epsilon} = \frac{\epsilon}{1+\epsilon}\phi^{-1} H^{\frac{(1+\epsilon)^2}{\epsilon}} + \left(\frac{1-\alpha}{1-\alpha\beta}\right)^{1+\epsilon} \phi^{\epsilon}\frac{\zeta^{1+\epsilon}}{1+\epsilon} \tag{5.10}$$

Furthermore, if $\epsilon = 1$ then:

$$H = \left(1 + (1-\zeta^2)^{1/2}\right)^{1/2} \left(\phi \frac{1-\alpha}{1-\alpha\beta}\right)^{1/2} \tag{5.11}$$

It is especially informative to consider the case when $\zeta = 1$, which I impose in steady state. In that case, equation 5.10 simplifies and hours are set to the full information optimum, so that $H = H^* = \left(\phi \frac{1-\alpha}{1-\alpha\beta}\right)^{\frac{\epsilon}{1+\epsilon}}$. With this in mind, implicit differentiation of equation 5.10 yields (letting $\lambda = \frac{1-\alpha}{1-\alpha\beta}$ for now):

$$\frac{\partial H}{\partial \zeta} = \left(\frac{\lambda^{1+\epsilon}\phi^{\epsilon}\zeta^{\epsilon}}{(1+\epsilon)H^{\epsilon}}\right)\left(\lambda - \phi^{-1}H^{\frac{1+\epsilon}{\epsilon}}\right)^{-1} \tag{5.12}$$

This derivative takes the same sign as the term $\lambda - \phi^{-1}H^{\frac{1+\epsilon}{\epsilon}}$. Since $H > H^* = (\phi\lambda)^{\frac{\epsilon}{1+\epsilon}}$ whenever $\zeta < 1$, this term is negative, and so hours rise when bad workers become less productive, as expected. Furthermore, when $\epsilon = 1$, equation 5.11 ensures that $H = \varphi(\zeta)H^* = \left(1 + (1-\zeta^2)^{1/2}\right)^{1/2} H^*$. Here it is clear that the upward distortion on hours in never greater than $2^{1/2}$. What is important for the model's ability to generate large variation quantitatively, however, is the elasticity of the distortion near $\zeta = 1$. As can be seen in figure 2, the slope is greatest around this point.

Note also that, as can be seein in Figure 2, the Frisch elasticity of labor supply ϵ is not a particularly important parameter for determining the labor response to relative productivity shocks. Only when labor supply is very inelastic ($\epsilon = 0.1$) is the curve flat over most of the productivity gap region. Once the Frisch elasticity is above 0.5 (which is the realm of recent estimates), hours respond essentially identical to when $\epsilon = 1$. This is especially striking since TFP shocks do not affect labor supply in this example, regardless of the Frisch elasticity. This indicates that market structure and the source of shocks can drive aggregate dynamics largely independent of the micro parameters.

5.3. **Quantitative Model.** The value of ϕ is chosen so that the average labor supply is 1/3 in steady state and $\epsilon = 0.48$. Higher values of ϵ generate greater volatility in both hours and output, but an interesting test of the model is whether adverse selection further amplifies or dampens these volatilities for a low value of the Frisch elasticity.

The most important parameters are those that govern the steady state value of N, the fraction of bad workers (τ_L), and the relationship between $\hat{z}$ and ζ_L. My benchmark is to get rid of unemployment altogether by taking κ to be small, which also gets rid of l as a parameter. I then vary the fraction of bad

Table 4. Relative Volatilities

Variable	Data	Full Info	Adverse Selection
σ_Y	1.72	1.11	1.39
σ_H/σ_Y	0.82	0.24	0.90
$\sigma_{Y/H}/\sigma_Y$	0.60	0.76	0.67
$\sigma_H/\sigma_{Y/H}$	1.64	0.32	1.34
$\sigma_{\text{wedge}}/\sigma_Y$	0.92	0.35	0.92
σ_C/σ_Y	0.45	0.28	0.22
σ_I/σ_Y	2.41	3.08	3.02

workers and restrict the innovations to $\hat{z}$ and ζ_L to be uncorrelated. I choose the persistence and standard deviation of innovations to TFP to be $\rho = 0.95$ and $\sigma_\nu = 0.007$. The standard deviation of ν_ζ is unmeasurable, but, as is explained below, determines the standard deviation of the labor wedge. In the data, the wedge, when measured from the perspective of a representative household with $\epsilon = 1.5$, is 92% as volatile as output (Chang and Kim [2007]). This necessitates that I set $\sigma_{\nu_\zeta} = 0.018$.

5.3.1. *Benchmark Results.* In Table 4 I compare the volatilities of the data to the full information version of the benchmark model and to the adverse selection version. Relative to the full information version of the model, adverse selection can generate substantial additional volatility in hours, output, and the labor wedge without affecting consumption or investment very much. In some sense, this is to be expected because the full information economy is not affected by shocks to the bad worker's productivity (since they are negligible in the population), whereas these shocks do affect hours and output in the adverse selection economy via their distortion on the good workers. In essence, the adverse selection economy sometimes experiences changes in hours and output for no apparent reason, which increases the volatilities of these variables.

The more relevant moments to consider are the cross-correlations and elasticities of output, hours, the labor wedge and labor productivity. These moments from the data and each economy are in Table 5. Here we see that the full information model does a very poor job of capturing the correlations and elasticities seen in the data. Labor productivity is strongly positively correlated with both output and hours and the elasticity of output with respect to

TABLE 5. Correlations and Elasticities

Variable	Data	Full Info	Adverse Selection
$\rho_{H,Y/H}$	-0.43	0.98	-0.23
$\beta_{H,Y/H}$	-0.83	0.32	-0.31
$\rho_{Y,Y/H}$	0.31	1.0	0.46
$\beta_{Y,Y/H}$	0.51	1.32	0.69
$\rho_{\text{wedge},Y}$	0.55	-0.99	0.33
$\beta_{\text{wedge},Y}$	0.60	-0.33	0.36

labor productivity is nearly three times what is in the data. When the economy is burdened by adverse selection, however, there are times when hours and output rise due to a fall in productivity of bad workers. These expansions cause labor productivity to fall due to the diminishing marginal product of hours. From the perspective of a representative household, it appears that hours must get subsidized, which generates the positive correlation between the labor wedge and output.

5.3.2. *Changing Population Share of Bad Workers.* I now consider the effects of having the share of bad workers positive (100% is the benchmark, 0% is all bad workers). The informational friction per good worker is unchanged, but now bad workers contribute to aggregate hours. The resulting economy is essentially a convex combination of the adverse selection benchmark and a standard RBC economy. The volatilities for the adverse selection economy with all parameters the same as the benchmark except for different values of $\tau_H = 1 - \tau_L$ can be found in Table 6. As the economy fills with bad workers, things look more like a standard RBC world. The volatility of output and hours fall because adverse selection generates additional fluctuations in the good workers' hours, but these make up a smaller fraction of the total. The wedge becomes less volatile since the classical intratemporal optimality condition is always satisfied for bad workers; in the limit, as they populate the entire economy, the wedge is only volatile due to misspecfication of the Frisch elasticity at the aggregate (1.5 vs 0.48 at the micro).

A similar pattern emerges for comovements and can be seen in Table 7. As bad workers make up the entirety of the population, the response of hours to

TABLE 6. Relative Volatilities

Variable	Data	Share of Good Workers 100%	90%	50%	10%	0%
σ_Y	1.72	1.39	1.34	1.20	1.11	1.11
σ_H/σ_Y	0.82	0.90	0.85	0.57	0.27	0.25
$\sigma_{Y/H}/\sigma_Y$	0.60	0.67	0.68	0.73	0.76	0.75
$\sigma_H/\sigma_{Y/H}$	1.64	1.34	1.24	0.78	0.36	0.33
$\sigma_{\text{wedge}}/\sigma_Y$	0.92	0.92	0.87	0.61	0.37	0.35
σ_C/σ_Y	0.45	0.22	0.23	0.26	0.28	0.28
σ_I/σ_Y	2.41	3.02	3.03	3.05	3.07	3.07

TABLE 7. Correlations and Elasticities

Variable	Data	Share of Good Workers 100%	90%	50%	10%	0%
$\rho_{H,Y/H}$	-0.43	-0.23	-0.16	0.18	0.87	0.98
$\beta_{H,Y/H}$	-0.83	-0.31	-0.20	0.14	0.31	0.32
$\rho_{Y,Y/H}$	0.31	0.46	0.54	0.54	0.99	1.0
$\beta_{Y,Y/H}$	0.51	0.69	0.79	1.14	1.31	1.32
$\rho_{\text{wedge},Y}$	0.55	0.33	0.25	-0.20	-0.91	-0.99
$\beta_{\text{wedge},Y}$	0.60	0.36	0.22	-0.12	-0.34	-0.35

productivity is overwhelmingly positive [8]. Since hours move strongly positively with productivity, the model overstates output's response to productivity relative to the data.

6. CONCLUSION

This paper makes three contributions. Empirically, I use micro data to extend the hours/labor productivity comovement puzzle to the hours of individuals, both on average and across the wage distribution. Theoretically, I use the recent breakthrough of Guerrieri et al. [2009] in order to embed adverse selection into the labor market of an equilibrium macro model. Competitive search is vital, as it allows me to study an economy in which there are essentially zero bad workers; this is exactly the situation in which equilibrium

[8] The correlation becomes positive with $\tau_H \approx 2/3$

do not exist without competitive search, and the one of interest due to measurement problems. Finally, I use the model to measure the extent to which adverse selection can account for the observed comovements between labor market variables and labor productivity. I find that the benchmark model exhibits labor market dynamics that are very similar to the data.

7. Chapter: Bankruptcy, Incorporation, and the Nature of Entrepreneurial Risk

7.1. **Introduction.** [9]

Entrepreneurship is inherently risky; the macroeconomic literature on entrepreneurship (Evans and Jovanovic [1989],Quadrini [2000],Cagetti and De Nardi [2006],etc) focuses on how individuals make decision in the face of uninsurable risks, and how these risk influence the savings behavior of entrepreneurs. These risks, along with interest rate premia on external financing, encourage savings by entrepreneurs and allow these models to generate the high level of wealth inequality in the United States.

While acknowledging the successes of these theories, it is important to recognize that there is substantial heterogeneity amongst entrepreneurs. Furthermore, entrepreneurs' choices differ systematically by the amount of insurance against business risks available to them. In particular, the U.S. legal system provides two tiers of insurance for entrepreneurs. The first is that, according to bankruptcy law, the entrepreneur may unilaterally default on liabilities while keeping personal assets up to an exemption level. Second, an entrepreneur may attain limited liability by incorporating her business, and thereby eliminate personal exposure to business risk. This allows the incorporated entrepreneur to effectively choose the level of insurance she desires.

We include bankruptcy and incorporation margins for two reasons. First, the data suggest that incorporated entrepreneurs are *better* entrepreneurs: on average, they enjoy higher incomes, accumulate more wealth, and pay lower interest rates on debt than their unincorporated peers. On the other hand, the insurance provided by limited liability induces incorporated entrepreneurs to default more frequently, thereby increasing their interest rates, both in the data (table 11) and in our theory. Modeling entrepreneurial bankruptcy therefore generates financing frictions emphasized in the literature, but via an endogenous mechanism that relies on realistic contracts. Modeling incorporation gives cross-sectional restrictions on the model, as well as increasing the model's ability to generate wealth inequality.

[9] This chapter is coauthored with Jacob Short.

Second, since incorporated entrepreneurs operate at sizes in between unincorporated and large corporate firms, they are a natural springboard for studying the transition between the entrepreneurial and corporate sectors. Models of entrepreneurship are unable to study corporate tax reform, because they are silent on this transition. This model opens the door to studying such a reform; as an initial application, we use it to predict the effects of making incorporation cheaper on aggregate output, entrepreneurship rates, income and wealth inequality, and welfare.

7.1.1. *Data and Legal Structure.*

In the United States, an individual entrepreneur can operate her business under one of three legal regimes. The first is a sole-proprietorship, in which all profits are passed-through to the entrepreneur's income and there is no legal distinction between liabilities of the business and owner. The second consists of Sub-Chapter C Corporations, in which business profits are taxed at corporate rates and can be either retained or distributed, and in which each owner is liable only up to the equity injected into the business. The final type consists of Sub-Chapter S Corporations, in which profits are passed-through as income, but liability is limited. We will call sole-proprietors "unincorporated entrepreneurs" and single-owner corporations "incorporated entrepreneurs."

We segment aggregate activity with these definitions in mind. The entrepreneurial sector as defined composed 18% of private non-farm GDP in 2003[10]. Of this, 8% was produced by unincorporated entrepreneurs and 10% by incorporated entrepreneurs. These shares are similar to what one would get using national income accounts, although it is impossible to slice NIPA in the same way.

We next look at the cross-section of entrepreneurs. Using data from the 2003 waves of the Survey of Small Business Finances (SSBF) and Survey of Consumer Finances (SCF), we find that, while demographically very similar, incorporated entrepreneurs were much more successful and wealthier than unincorporated entrepreneurs: revenue for was seven times greater, profits were

[10]We use IRS SOI data to construct value added, with some extrapolations from the SSBF. Full details are available upon request

three times greater, wealth to income was 40% greater, and interest rates on debts were on average 70 basis points lower. Importantly, however, incorporated entrepreneurs paid a $40 - 70$ basis point interest rate premium on business debts. These observations inform our inclusion of unilateral default and and incorporation as key modeling ingredients.

7.1.2. *Description of Theory.*

We model entrepreneurs as households who have a persistent ability to produce, but face idiosyncratic shocks and forgo labor income if they choose to do so. An entrepreneur faces shocks to both their ability, as well as shocks to installed capital. The first represents shocks that affect management ability (such as illness) or shifts in demand for the entrepreneur's product. The second represents a combination of shocks, such as business specific investments that turn sour (e.g. building and designing a restaurant which turns out to be in a poor location) or judgements against the entrepreneur due to negligence. Note that the second source of shocks can be catastrophic, thereby causing losses greater than revenue. As detailed below, these are the types of shocks that entrepreneurs identify as the causes for bankruptcy.

As a result of these shocks, entrepreneurs sometimes find it optimal to default. In our theory this resembles Chapter 7 bankruptcy. If entrepreneurs ever exercise this option, then banks charge interest rate premia above the risk free rate on business loans (as in Eaton and Gersovitz [1981], Chatterjee et al. [2007]. For unincorporated entrepreneurs, bankruptcy also entails seizure of personal assets up to the exemption level and exclusion from financial markets for a period (in this way, our bankruptcy resembles a "fresh start" as in Livshits et al. [2007]). Incorporated entrepreneurs, on the other hand, can keep the entirety of personal assets, and maintain access to financial markets.

Since incorporated entrepreneurs are "punished" less in the event of bankruptcy, they naturally default more frequently than unincorporated entrepren--eurs. Therefore, they pay even higher interest rates on debt than would an identical unincorporated entrepreneur, as we see in the data. This also implies that a given incorporated entrepreneur saves more than if she were unincorporated. First, since fewer personal assets are seized in the event of bankruptcy, the implicit rate of return on personal savings is higher for incorporated entrepreneurs. Second, the covariance between income and returns on personal savings is zero for incorporated entrepreneurs, while it is positive for unincorporated entrepreneurs. Finally, debt is more expensive for incorporated entrepreneurs, and so self financing is more attractive.

Whether the theory can generate greater revenue and lower average interest rates for incorporated entrepreneurs hinges on whether more productive entrepreneurs select into incorporation. For a fixed level of resources, a more productive entrepreneur wishes to install more capital than an unproductive entrepreneur, and so more expensive debt discourages them from incorporating. On the other hand, more productive entrepreneurs have more income and therefore wish to hold more personal savings than what is protected by personal exemptions, which may not be the case for low productivity entrepreneurs. Thus the model can generate positive selection if the financing motive is weaker than the insurance motive, which makes the calibration in table 14 non-trivial.

7.1.3. *Applying the Model.*

We emphasize that the world faced by entrepreneurs in our model is radically different than in the rest of the literature. Previous researchers think of entrepreneurial risk as individual specific productivity shocks, so that in the worst states of the world an entrepreneur who fully funds production via debt still has non-negative net equity in her business. In our world, net equity after the most adverse shocks is equal to the level of debt. We find that our specification of risk is necessary to get the wealthiest entrepreneurs to incorporate, as we observe in the data. We also use the model to measure how large these risks are in consumption equivalence terms, and find that an annual increase in consumption of 5.9% is needed to make an entrepreneur indifferent between starting a business in the presence of investment risk and a world with none. Indicating that investment risk is a *big* source of risk for entrepreneurs. For comparison, Storesletten et al. [2001] find the welfare gains from eliminating idiosyncratic labor income risk to be 1.17% annual consumption.

We use the model to conduct two policy experiments that are relevant to many actual legal changes made in the last twenty years. We first ask what happens when incorporation becomes cheaper. Since the only form of incorporation in our model has pass-through income, this resembles the introduction of recent legal forms such as Limited Liability Corporations, as well as bestowing more and more benefits to S-Corporations that were once reserved for C-Corporations. We find that this increases the entrepreneurship rate, which increases aggregate income by 0.5%, but also increases income and wealth inequality.

A similar exercise, which has been the focus of previous papers on entrepreneurship and default (Meh and Terajima [2008], Athreya et al. [2009]), explores the effects of changing personal bankruptcy exemptions. Since in our model an entrepreneur can attain full exemptions by incorporating, we find that changing personal exemptions has a much smaller impact than either of these previous studies.

It is important to note that our model gives indeterminant welfare conclusions with respect to these policies. This is because some fraction of the expenditure shocks are modeled as damages to third parties, and entrepreneurs are absolved of reimbursing these expenses in bankruptcy. Furthermore, while lenders will price their expected losses due to bankruptcy, damages due to negligence will still not be internalized. This is because secured creditors are paid first in bankruptcy proceedings.

Because entrepreneurs can generate large expenses to third parties (in 2003, 6% of total entrepreneurial output, one third of which was uninsured). This creates a trade-off for society. On one hand, the the insurance provided by incorporation encourages entrepreneurial activity, which has high marginal product. On the other hand, limited liability also enables these entrepreneurs to avoid accountability. Because the average net damages to each household depends on the number of entrepreneurs, as well as their incorporation status and level of exemptions, we find a non-monotonic relationship between welfare and entrepreneurship rates.

7.1.4. *Literature Review.*

Our theory bridges the literature on entrepreneurship in macroeconomics with the literature on personal default. The default decision increases the interest rate on business debt, which creates the fundamental friction in Quadrini [2000] and Cagetti and De Nardi [2006], but is an improvement over their modeling choices since it is endogenous (as opposed to Quadrini [2000]) and is an observable feature of contracts (as opposed to Cagetti and De Nardi [2006]). We are not the first researchers to realize this, however: Meh and Terajima [2008] study an environment in which default risk generates such a premium. Our additional value over their paper is to use the incorporation choice to inform us of the risks faced by entrepreneurs.

We also focus on the tradeoff between insuring entrepreneurs from business risk, versus allowing them to avoid responsibility for negligence. While we are the first to develop this tradeoff, previous researchers have studied the effect of insurance on entrepreneurial behavior in these models. Meh and Terajima [2008] conduct policy experiments in which the exemption levels of assets under personal bankruptcy affect the extensive margin on entrepreneurial choice, and Athreya et al. [2009] look at the individual trade-off between insurance through limited liability and costly financing. Neither paper, however, allows the entrepreneur to incorporate and receive full limited liability.

To our knowledge, the paper that comes closest to allowing for an incorporation decision is Herranz et al. [2009]. While not giving entrepreneurs the option of incorporating, they point out that less risk averse entrepreneurs, because they operate larger more risky projects and therefore would gain the most from limited liability, are those who would most likely incorporate if given the option. We differ from them in that we explicitly have the decision to incorporate, and we stay more loyal to the conventions of the literature on entrepreneurs.

The rest of the paper is organized as follows: Section 2 documents the important differences between unincorporated and incorporated entrepreneurs, and provides evidence supporting the specific margins in the model. Sections 3 through 6 develop a quantitative model of occupational choice, choice of legal form of business and default decisions, which respects the U.S. legal framework of incorporation and bankruptcy. Section 7 describes the calibration procedure. Section 8 uses the calibrated model to assess the value of limited liability by performing policy experiments on bankruptcy exemptions and direct costs of incorporation. Section 9 summarizes the results and concludes.

7.2. **Data.**

We study the characteristics of owners and businesses using the Survey of Small Business Finances (SSBF) and the Survey of Consumer Finances (SCF). The SSBF provides information about a nationally representative sample of non-financial, non-farm, non-subsidiary businesses with fewer than 500 employees. We focus primarily on the 2003 survey, which contains information from a sample of 4,420 businesses, which represent 6.3 million small businesses in the United States.

Since we focus only on how limited liability affects the decision to incorporate, we further restrict the sample to businesses in which there is a single owner who owns and operates the business. Therefore, for the purpose of this paper we define an entrepreneur as an individual who owns a business and has an active management role in that business. Incorporated entrepreneurs are entrepreneurs who have undertaken a costly legal procedure to establish their business as a separate legal entity, thereby creating a legal separation of business and personal assets (i.e. limited liability). The restriction to solely owned businesses mitigates the influence of financial benefits, such as issuing equity, which might lead individuals to incorporate above and beyond access to limited liability. As a result of this restriction the sample size is reduced to 1,846 firms, of which 37% are incorporated.

7.2.1. *Legal Differences and Costs.*

Unincorporated entrepreneurs are individuals operating sole-proprietorships; a business structure in which the individual and business are considered a legal entity for both tax and liability purposes, that is the owner is inseparable from the business and is liable for any business debts. In contrast, incorporated entrepreneurs are individuals operating corporations[11], which are separate legal entities. There are three distinct characteristics of a corporation: (i) Legal existence - a corporation can own property, enter into contracts, can bring suits against others and be sued by them. (ii) Limited liability - owners liability is limited to the value of their investment in the corporation. (iii) Continued existence - the corporations' existence is not limited to that of its owners.

The cost of incorporation can be significant; the direct costs of state fees and franchise taxes range from $\$1,050$ to $\$1,450$ depending on the state in which the entrepreneur incorporates[12]. In addition to these taxes and fees, there are initial costs associated with the necessary accounting and legal paperwork (tax filing, articles of incorporation, bylaws, etc.) involved in establishing the corporation. Estimates of these costs range from $500 to $\$5,000$[13]. The Small Business Administration (SBA) estimates yearly time and financial costs of tax filing for home-based corporations to be approximately $\$3,250$ (approximately 3.5 times larger than unincorporated home-based businesses). To give a sense of the magnitude, these direct costs are upwards of 10.2% of 2003 median earnings in the U.S.

7.2.2. *Limited Liability and Bankruptcy.*

Given the inherent riskiness of operating a business, entrepreneurs must consider their position in the contingency that the business fails. In particular, as Meh and Terajima [2008] and Sullivan et al. [1999] document, these businesses can often fail with large amounts of debt. The extent to which the entrepreneur is liable for these debts is a first order concern. State and Federal bankruptcy law stipulates certain exemptions on personal wealth, the largest being the homestead exemption; these exemption levels vary widely across states. Using the variation in homestead exemptions across states, Fan

[11]This includes both subchapter S and C corporations. In our sample the majority (73%) of the incorporated entrepreneurs are S corporations.
[12]Bebchuk and Cohen [2003] find that firms display a substantial home-state preference when incorporating.
[13]Estimates of the administrative (accounting and legal) are provided by allbusiness.com, a subsidiary of Dunn and Bradstreet Company.

and White [2003] examine how homestead exemptions affect the decision for families to become entrepreneurs. They find the probability of homeowners owning a business is 35% higher in states with unlimited homestead exemptions. Suggesting that limited liability is an important consideration when individuals are deciding whether to operate a business.

Not only do bankruptcy exemptions influence the decision to own a business; once an individual is operating a business, these exemptions will affect the bankruptcy choice of the entrepreneur. Agarwal et al. [2005] use panel data on 43,000 small business credit card holder over a two year period to estimate a proportional hazard model to measure the impact of bankruptcy exemption laws on the likelihood of bankruptcy. They find that an increase of $\$10,000$ in homestead exemptions increases the likelihood of small business owners declaring bankruptcy by 8%; and, $\$1,000$ increase in personal property exemptions increases the risk of bankruptcy by 4%. In a related study, Agarwal et al. [2003] find that bankruptcy exemptions are also positively related to individuals decisions to declare bankruptcy.

Lawless and Warren [2005] estimate that 17.4% of bankruptcy filings in 2003 involve the failure of a business[14]. Since entrepreneurs are only 12.4%[15] of the population, they are over-represented in bankruptcy filings. Businesses may file bankruptcy for a variety of reasons, and knowing the source of these bankruptcies is important in guiding modeling choices about the type of shocks that entrepreneurs face. Sullivan et al. [1999] survey a sample of non-farm businesses which filed bankruptcy in 1994 to determine the causes of bankruptcy. They find that the most common reasons given for bankruptcy where associated with business conditions; such as new competition, a bad location, mismanagement of business and inability to collect accounts receivables. In particular, they find that 11.4% of businesses report a pending legal action, such as a lawsuit, as the reason for filing. In addition, Dunn and Bradstreet report that 6.0% of business failures in 1994 resulted from disasters, such as .

[14] The 17.4% includes businesses with single and multiple owners.

[15] SCF 2004

7.2.3. *Aggregate.*

Output

Using data from the Internal Revenue Service, we construct private non-farm GDP by computing the value-added for businesses across four groups: (i) sole-proprietors (ii) partnerships (iii) incorporated entrepreneurs (single owner corporations) and (iv) multiple shareholder corporation. Not surprisingly, we find that corporations with multiple shareholders (corporate sector) produce 70% of private non-farm GDP. Sole-proprietors produce 8%, and incorporated entrepreneurs another 10%. The remaining portion, 12%, is produced by partnerships. Together, single-owner businesses produce 18% of private non-farm GDP. In addition, we find that for young businesses (under 5 years) about 3% of sole-proprietors incorporate their business (i.e. transition to incorporated entrepreneurs). This percent falls over the age of the business. Considering the contribution to GDP, the stark legal differences and position in the transition between entrepreneur and corporate sector, we find it surprising that the literature has largely ignored incorporated entrepreneurs, either pooling them with sole-proprietors or large corporations.

Damages

The operation of businesses can often negatively impact third parties; for example, faulty wiring in pacemakers or use of lead based paint or asbestos. In 2003, aggregate tort costs for small businesses were 6% of total small business output; roughly one-third of these costs are uninsured[16]. These tort costs consist of (i) payments to third parties (settlements) (ii) defense costs (iii) administrative expenses. Comparing the tort costs of small businesses to the aggregate tort costs of the U.S., which are 1.5% of total output of all businesses, suggests that small businesses are particularly damaging. The Bureau of Justice Statistics reports that for 2001, 31% of all tort trials involved a

[16]Towers Perrin, a consulting firm, provides estimates of aggregate tort costs for the U.S. in their annual report.

business defendant (only 3% of tort trials involved a business plaintiff). Litigation can be very costly for businesses; the median final damage awards for tort trials involving business defendants was approximately $\$75,000$ in 2001. In addition, the median length from filing to judgment for tort litigation is 22 months; for product liability cases, the median is 29 months.

7.2.4. *Cross-Section.*

We now focus on the cross-sectional differences between unincorporated and incorporated entrepreneurs. We find that incorporated entrepreneurs differ significantly from their unincorporated peers in three dimensions: (i) size of their business (ii) accumulated wealth (iii) interest rates on external financing.

In regards to the business operations of incorporated entrepreneurs relative to unincorporated, table 1 provides details on how they differ. Specifically, we look at size by sales, profits, assets, equity, and employment. We see that the single-owner incorporated businesses are considerably larger across all of these measures. In particular, they exhibit higher sales (by a factor of seven), they are 2.6 times as profitable and on average employ three times as many employees as sole-proprietorships. In addition, these businesses have over three times as much assets and twice as much net equity.

Table 8. Firm Characteristics by Legal Form

	UE	IE
Firm Age	15.1	13.6
Sales (000s)	181	1,264
Profits	68	176
Assets	124	438
Equity	87	171
Employees		
–Total	3.0	9.6

In addition to operating larger businesses, incorporate entrepreneurs accumulate greater levels of wealth. Table 9 provides the mean net worth and income by legal form of business. Incorporated entrepreneurs have a wealth to income ratio 40% times greater than the unincorporated entrepreneur.

Table 9. Mean Net Worth and Income by Legal Form

	UE	IE
- Net Worth	753	2,741
- Income	91	224
Wealth to Income Ratios		
	UE	IE
- Net Worth/Income	8.29	12.24

Source: SCF 2003

Interestingly unincorporated and incorporated entrepreneurs do not differ significantly in their demographics. Table 10 provides a demographic comparison of the owners. We find that IE are somewhat more likely to have a college degree, whereas experience and age are approximately equal. This suggests that differences within the firm are a consequence of the legal form, rather than specific attributes of the owner.

Table 10. Demographics of Owners by Legal Form

Education		
	U	I
- share with associate degree or higher	0.59	0.62
Experience and Age		
	U	I
- ave. years of experience	19.0	19.5
- ave. age of owners	51.7	50.7

Lastly, we look at the different interest rates that these entrepreneurs face. We find that the unconditional interest rates for incorporated entrepreneurs are lower than for unincorporated. However, if we condition on a characteristics of the firm, loan and owner; we find that incorporated entrepreneurs pay higher interest rates. In particular, the interest rate premium for business loans is 0.40 percentage points, and for credit cards the premium is 0.79 percentage points. Table 11 contains these findings.

The difference in conditional interest rates is found by regressing the interest rate on the most recent loan that the firm received on a host of control variables, and the full regression results can be found in tables 18 and 19. A

Table 11. Interest Rates by Legal Form

	UE	IE
Unconditional Interest Rates	7.0	6.3
	UE	IE
Conditional Interest Rates		
- business loans	-	+0.40 (0.19)
- business credit cards	-	+0.79 (0.23)

complete discussion of the control variables that are included is provided in the appendix.

In the model, this interest rate premium will be determined endogenously. In particular, our theory suggests that this interest rate premium paid by incorporated entrepreneurs results from a contraction in the supply of credit associated with incorporation. In a perfectly competitive banking industry, banks make zero expected profits on each loan. The interest rate charged is thus a function of (1) the risk of the entrepreneur defaulting on the loan and (2) the fraction of the loan that the bank can recover upon default. We expect an increase in (1) to be associated with an increase in the interest rate, and just the opposite for (2). Thus, limited liability induces entrepreneurs to default more frequently and reduces the amount of assets the bank can recover in the event of default leading to higher interest rates for incorporated businesses.

On the demand side, the protection provided by limited liability means that at a given interest rate, an entrepreneur will demand a greater amount of debt upon incorporation. Therefore, if we observe a fall in the amount of debt a business has upon incorporation, this provides evidence that supply contracts upon incorporation, supporting our theory. We test for this evidence using panel data on firms from the Kauffmann Firm Survey (KFS).

Formally, we are interested in the estimation of the simultaneous system of supply and demand for debt and the shift effect of incorporation on these two curves. A linear specification of this problem is standard:

$$d_{i,t} = \beta_1 o_{i,t} + \beta_2 r_{i,t} + \beta_3 \pi_{i,t} + \beta_4 A_{i,t} + \beta_5 d_{i,t-1} + \beta \mathbb{X}_i + \epsilon_{i,t} \tag{7.1}$$

$$r_{i,t} = \alpha_1 o_{i,t} + \alpha_2 d_{i,t} + \alpha_3 \pi_{i,t} + \alpha_4 A_{i,t} + \alpha \mathbb{X}_i + \eta_{i,t} \tag{7.2}$$

Where d is debt, o is legal status of business, r is interest rate, π is profit, A is assets, X is a vector of observable and unobservable traits that do not change over time, and (ϵ, η) are errors that are uncorrelated with regressors from their equations.

We do not have information on the interest rates in the KFS, and therefore estimating the above model is impossible. However, we are really interested in the sign of β_1. Fortunately, under reasonable assumptions on α_1, α_2, and β_2, specifically that incorporation increases demand and that the supply and demand curves have the "correct" slopes, we can test whether $\beta_1 > 0$ from the reduced form:

$$d_{i,t} = \frac{\beta_1 + \beta_2\alpha_1}{1 - \beta_2\alpha_2} o_{i,t} + \tilde{\beta}_3\pi_{i,t} + \tilde{\beta}_4 A_{i,t} + \beta_5 d_{i,t-1} + \tilde{\beta}\mathbb{X}_i + \nu_{i,t} \tag{7.3}$$

Where coefficients of form $\tilde{\beta}$ are found via algebra and are uninteresting to us. Now, we still have the issue of unobserved $\mathbb{X}$ variables possibly being correlated with o, so we take differences to get:

$$\Delta d_{i,t} = \frac{\beta_1 + \beta_2\alpha_1}{1 - \beta_2\alpha_2} \Delta o_{i,t} + \tilde{\beta}_3\Delta\pi_{i,t} + \tilde{\beta}_4 \Delta A_{i,t} + \beta_5 \Delta d_{i,t-1} + \Delta\nu_{i,t} \tag{7.4}$$

So, if we assume that limited liability increases loan demand ($\beta_1 > 0$) as well as downward sloping demand and upward sloping supply curves ($\beta_2 < 0$ and $\alpha_2 > 0$) then an estimate of a negative coefficient on the change in legal status implies an estimate of $\alpha_1 > 0$.

Since we do not have the continuous variables available in the public release of the KFS, we reformulate the model as a discrete choice of whether or not to increase bank debts. We estimate a logit model on the odds of an entrepreneur decreasing their bank debt of the form:

$$\text{odds of } \hat{d}_{i,t+1} \leq \hat{d}_{i,t} = \frac{e^{z_{i,t+1}}}{1 + e^{z_{i,t+1}}}$$

and

$$z_{i,t+1} = \gamma_0 + \gamma_1\Delta o_{i,t+1} + \gamma_2\hat{d}_{i,t} + \gamma_3\hat{A}_{i,t} + \gamma_4\hat{A}^{\uparrow}_{i,t+1} + \gamma_5\hat{A}^{\downarrow}_{i,t+1} + \gamma_6\hat{\pi}_{i,t} + \gamma_7\hat{\pi}^{\uparrow}_{i,t+1} + \gamma_8\hat{\pi}^{\downarrow}_{i,t+1}$$

Where hat variables are the categorical versions of their continuous counterparts and superscripted up or down arrows indicate whether or not the variable moved up a category or down a category. For each age, we restrict attention to those entrepreneurs who start that year unincorporated, and so $\Delta o_{i,t+1}$ takes value zero if the business remains unincorporated and value 1 if it becomes incorporated.

We find that incorporating significantly increases the probability that an entrepreneur reduces her level of bank debt between each age of the business. The estimates are in table 7.2.4.

Estimation Results

Age	Coeff on Incorp	P-Value
0	1.38	0.9%
1	2.05	0.5%
2	1.26	13.5%

We therefore find evidence in favor of the limited liability default risk theory of incorporation using the publicly available KFS. Estimates indicate that, for each age, incorporation increases the odds of an entrepreneur decreasing or maintaining her bank debt. Furthermore, for the first two years of existence these estimates are significant at the 1% level.

7.3. **Theory.**

There is a unit measure of households in the economy. In each period households decide whether to operate a risky entrepreneurial project or to inelastically supply their labor to the market. In addition to the households' occupational choice, upon deciding to operate the entrepreneurial project the household must choose the projects legal form, unincorporated or incorporated. The latter requires a one time fix cost and provides limited liability in the case of bankruptcy. Finally, upon realization of idiosyncratic shocks to the entrepreneurial project, the entrepreneur decides whether to unilaterally default on debt obligations.

7.3.1. *Preferences.*

Households receive utility from consumption and maximize the expected lifetime utility:

$$E_0\{\sum_{t=o}^{\infty}\beta^t u(c_t)\}$$

and discount the future at rate β. The utility function, $u(c_t)$, is assumed to be continuous, strictly concave and satisfy $\lim_{c\to\infty} u'(c) = 0$ and $\lim_{c\to 0} u'(c) = \infty$.

7.3.2. *Endowments.*

Households are endowed with 1 unit of labor, which they either supply to the market and receive wage, w, or used in their entrepreneurial project. In addition to labor, the household is endowed with an entrepreneurial project, denoted by $z \in \mathbb{Z}$, which governs the average and marginal returns on capital (modeled as Lucas's "span of control") invested in the project as specified below. The project quality, z, follows a first-order Markov process with transition matrix, $\Gamma(z'|z)$.

7.3.3. *Technology.*

As in Quadrini (2000), we model production in two sectors both producing a homogeneous good. The first sector comprises smaller units of production operated by individual entrepreneurs, which face uninsurable idiosyncratic risk and face individual-specific prices on debt, specifics of these contracts are described below. The second, corporate, sector consists of large firms which can borrow and invest capital at the exogenous interest rate $\bar{r}$. In addition to the financial differences, these two sectors differ in the technologies.

Entrepreneurial Technology

If a household decides to implement their entrepreneurial project, they choose a level of capital input $k > 0$. It is important to note that if a household chooses capital input $k = 0$, then she is considered a worker, receives wage w and is not exposed to any risk associated with operating a business. If the household implements a positive level of capital, then they receive revenue plus undepreciated capital given by:

$$y(z,k,\eta) = z \cdot k^{\nu} + (1-\delta-\eta)k \qquad 0 < \nu < 1$$

where η is an idiosyncratic capital embodied shock which is unknown at the time of the investment decision. The shock η is independent over time and individuals and has a two parameter cdf $H(\eta)$ with mean μ_η and variance σ_η^2. The specification of the capital shock has the important feature that additional investment in capital does not provide insurance from the downside risk of the business; that is, the entrepreneur can not accumulate her way out of risk. This feature is crucial in matching the cross sectional feature of the data. In a model without investment risk, the largest and wealthiest entrepreneurs can insure themselves against productivity/demand risk and therefore would not value the insurance provided by incorporation, counter to the data. Notice that the technology does not depend on the legal form of the entrepreneur.

Corporate Technology

The technology in the corporate sector is given by a standard constant returns to scale technology:

$$F(K_c, N_c) = K_c^{\alpha} N_c^{1-\alpha}$$

where K_c and N_c are the capital and labor inputs, respectively. Corporate sector capital depreciates deterministically at rate δ.

7.3.4. *Financial Intermediation.*

The financial intermediation sector consists of a large number of banks that price loans competitively. Households can save, $a \geq 0$, at the exogenous risk free rate $\bar{r}$. The price financial intermediaries charge on debt are individual-specific, as in Chatterjee et al. [2007] and Livshits et al. [2007]. That is, intermediaries can observe the households savings, capital, debt ($S = (a,k,D) \in \mathbb{R}_+^3$), current period project quality and the legal form of the business, $n \in \{U, I\}$.

When an entrepreneur purchases a loan value of D at price $q^n(S,z)$, the bank must borrow $q^n(S,z) \cdot D$ at the risk free interest rate $1+\bar{r}$. Hence, making such a loan costs the bank $(1+\bar{r}) \cdot q^n(S,z) \cdot D$. The bank expects to get receipts of D in the event that the entrepreneur does not default, which occurs if $\eta \in \mathbb{B}(S,z)$ which occurs with probability $\Omega^n(S,z)$. If default occurs, which happens with probability $1 - \Omega^n(S,z)$, then the bank receives liquidation value $\Phi^n(S,z,\eta)$. The zero profit condition for a loan contract indexed by (S,z,n) is therefore:

$$(1+r) \cdot q^n(S,z) \cdot D = \Omega^n(S,z) \cdot D + (1 - \Omega^n(S,z)) \cdot \mathbb{E}[\Phi^n(S,z,\eta)|\eta \in \mathbb{B}(S,z)]$$

Differences in the interest rates for unincorporated and incorporated entrepreneurs will emerge through two channels, the probability of default, $\Omega^n(S,z)$, and the recoverable amount in bankruptcy. We impose structure on the Φ function in accordance with bankruptcy law and explore the properties that this equation imposes on interest rates below.

7.3.5. *Involuntary Claims on Entrepreneurial Resources.*

Our specification of η leaves open the interpretation of what happens to the resources when an entrepreneur experiences the shock. Specifically, motivated by the reasons for entrepreneurial default discussed above, we will assume that some fraction λ of this shock is due to damages caused to third parties by the entrepreneur's actions. We model it as a constant amount of damages incurred by each household in the economy, with payments to cover these damages received after financial intermediaries recover resources. The net transfer, which includes incorporation fees, damages and payments, appear in each household's budget constraint as T and will be defined in equilibrium. They will depend on the number of entrepreneurs in the economy, the bankruptcy decisions of the entrepreneurs, and the incorporation rate.

7.4. **Individual Decisions.**

7.4.1. *Timing.*

A household enters the period with resources R, historical project quality

z, legal form n, and a flag b for whether or not they filed bankruptcy in the previous period. The legal form is held regardless of whether or not the household installs capital, so that the incorporation status is maintained even during periods as a worker. The first decision is then whether or not to operate a business as an entrepreneur, or to be a worker and receive the market wage.

If the household elects to operate a business, then they must decide what legal form to operate under. A previously unincorporated household must pay legal and administrative fees, f, in order to become incorporated, whereas a previously incorporated household can operate as either legal form without incurring these costs. The incorporation decision will determine which pricing function for debt that the household faces.

The household must then decide how to allocate the available resources. An unincorporated entrepreneur chooses between consumption, capital, and personal assets, as well as how much to borrow. Note that, with bankruptcy exemptions as exist in most states, personal assets and debt must be accounted for separately even for unincorporated entrepreneurs. If the entrepreneur is incorporated, then they instead have two distinct choices. On the personal side they choose between consumption, savings, and how much equity to inject into the business. On the business side, they decide how much capital to install, with anything beyond their own equity being financed via debt.

After capital is installed, a new value of z is realized and production occurs. In the process of production the η shock is realized. At this point, given the gross return on capital, the entrepreneur must decide whether or not to file for bankruptcy. For an unincorporated entrepreneur, bankruptcy entails a complete seizure of all business and personal assets up to the exemption level. In addition, they will carry the bankruptcy flag into the next period and be excluded from financial markets, a consequence of which is that they have to be a worker for one period. For an incorporated entrepreneur only business assets are seized. They also lose their incorporation status, but they enter the next period without the bankruptcy flag, thereby maintaining access to financial markets.

The timing is illustrated in figure 1:

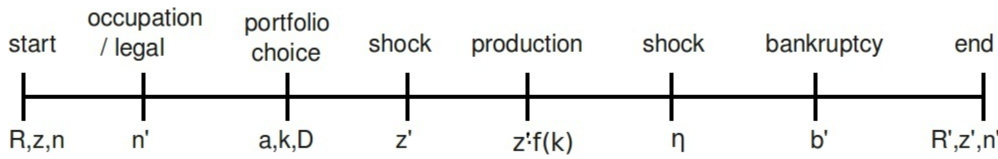

FIGURE 1. Timing of Events

7.4.2. *Decision Problem.*

We describe the household decision problems recursively. Consider first the problem of an unincorporated household which filed for bankruptcy in the previous period. The household cannot access financial markets, and so must consume whatever is on hand. They enter the next period with zero resources and a fresh start without the bankruptcy flag:

$$V^U(R, z, 1) \quad = \quad u(R + w + T) + \beta \sum_{z'} \Gamma(z'|z) V^U(0, z', 0)$$

Which brings us to the unincorporated entrepreneur who remained solvent in the previous period. For convenience, we define two intermediate problems conditional upon whether or not the entrepreneur decides to incorporate. If they remain unincorporated, then they face q^U, but in the event of bankruptcy are left with only an exemption level of assets, given by the function $\chi(a)$.

$$\widehat{V}^U(R, z) = \max_{c,S,b(\cdot)} u(c) + ...$$

$$\beta \int_{\underline{\eta}}^{\bar{\eta}} \sum_{z'} \big(b(z', \eta) V^U(\chi(a), z', 1) + (1 - b(z', \eta)) V^U(R', z', 0)\big) \, \Gamma(z'|z) dH(\eta)$$

s.t.

$$\begin{aligned} R &= c + \frac{a}{1 + \bar{r}} - q^U(S, z) D + T \\ R' &= z' k^\nu + (1 - \delta - \eta) k + a - D \end{aligned}$$

If a previously unincorporated entrepreneur incorporates then they pay a fixed cost, face the price function q^I, and keep the entirety of personal assets in bankruptcy.

$$\widehat{V}^I(R,z) = \max_{c,S,b(\cdot)} u(c) + ...$$

$$\beta \int_{\underline{\eta}}^{\bar{\eta}} \sum_{z'} \left(b(z',\eta)V^U(a,z',0) + (1-b(z',\eta))V^I(R',z') \right) \Gamma(z'|z)dH(\eta)$$

$$\begin{aligned} &\text{s.t.} \\ R &= c + \frac{a}{1+\bar{r}} - q^I(S,z)D + T + f \\ R' &= (z')k^\nu + (1-\delta-\eta)k + a - D \end{aligned}$$

Using these intermediate value functions allows us to write the final function as the max over remaining unincorporated or incorporating. It is:

$$V^U(R,z,0) = \max_{n\in\{U,I\}} \widehat{V}^n(R,z)$$

The incorporated entrepreneur has similar intermediate value functions, with the exception that they do not pay the fixed cost f if they choose to remain incorporated. The problems are similar enough that we omit them here.

The value functions have been defined assuming a solution. We now state the proposition that this is a valid assumption, with the proof forthcoming using standard techniques from dynamic programming.

Proposition 7.1. *Given pricing functions and an exemption function χ, there exists a unique couplet of value functions V^U, V^I that solve the above problems. These functions are increasing in R and z, and have the property that $V^I(R,z) \geq V^U(R,z)$ for all (R,z) ($>$ if the support of η is unbounded above). Furthermore, the policies that attain the maximum values of the Bellman equations are compact, upper hemi-continuous correspondences. In addition, if the utility from zero consumption is assumed small enough and $w+T>0$ then consumption will always be strictly positive, since default is always feasible.*

Full proofs of the existence and uniqueness of the value functions, along with the properties of the policy correspondences, are forthcoming but involve standard dynamic programming techniques. The interesting result is that it is always better to enter a period incorporated. The reason for this is straight forward. The unincorporated value cannot be strictly higher, because the incorporated entrepreneur can costlessly unincorporate and attain the exact same value. Hence, $V^I \geq V^U$. Suppose instead that the unincorporated entrepreneur would file bankruptcy at this state (such a state is guaranteed by the assumption that η can take arbitrarily high values). If $a = \chi(a)$ then, by also filing, the incorporated attains exactly the same value. If instead $a > \chi(a)$ then the incorporated entrepreneur must be strictly better off. This is because she retains at least as much of her assets (such an a exists by the assumption that $\frac{d\chi}{da} < 1$) and also retains the ability to save. In short, because the incorporated entrepreneur can always at least mimic the unincorporated entrepreneur and faces a more favorable future when filing for bankruptcy, limited liability gives rise to a positive value of incorporation.

7.5. **Equilibrium.**

We are now ready to define equilibrium given exemption functions χ, $(\Phi^n)_{n\in\{U,I\}}$. For this economy, a stationary recursive competitive equilibrium consists of:

(1) Value functions as described in the household problems.
(2) Policies:
   - $g_S = (g_a, g_k, g_D) : \mathbb{R} \times \mathbb{Z} \times \{U, I\} \to \mathbb{S} = \{(a, k, D) \in \mathbb{R}^3_+\}$
   - $g_n : \mathbb{R} \times \mathbb{Z} \times \{U, I\} \to \{U, I\}$
   - $g_b : \mathbb{R} \times \{U, I\} \times \mathbb{S} \times \mathbb{Z} \to \{0, 1\}$
(3) Solvency function: $\Omega : \mathbb{S} \times \mathbb{Z} \times \{U, I\} \to [0, 1]$
(4) Pricing functions: $q : \mathbb{S} \times \mathbb{Z} \times \{U, I\} \to \left[0, \frac{1}{1+r}\right]$
(5) A stationary distribution: $\psi : \mathbb{R}_+ \times \mathbb{Z} \times \{U, I\} \times \{0, 1\} \to \mathbb{R}_+ \times \mathbb{Z} \times \{U, I\} \times \{0, 1\}$
(6) Net transfers: T
(7) Wage: w
(8) Labor and capital in corporate sector: N_c, K_c

Equilibrium requires that the policies attain the values of the value functions taking pricing functions and exemptions as given. The solvency function is generated by the bankruptcy policy according to the condition:

$$\Omega^n(S,z) = \int_{\underline{\eta}}^{\bar{\eta}} \sum_{z'} \left(1 - g_b^n(\eta, S, z)\right) \Gamma(z'|z) dH(\eta)$$

Pricing functions must solve the intermediary zero profit condition given the above solvency functions and the recovery functions Φ.

The stationary distribution is intuitive. We must first map the true states, (R, z, n), to the choice of legal form and allocation of resources across business and personal accounts in the current period. The g_S and g_n functions accomplish this. We then have to integrate over η and z' to get the flows into bankruptcy, and then final resources, legal form, and bankruptcy flag at the end of the period.

$$\psi(R', z', n', b') = \int_{R,z,\eta} \sum_{n,b} \mathbb{I}_{\{(R',n',b')' \in g(S,z,\eta,n,b)\}} d\Gamma(z'|z) dH(\eta) \psi(dR, dz, n, b)$$

Using the stationary distribution we can define the components of net transfers. Denoting $\mathbb{E}_\psi$ as the average over the stationary distribution gives:

$$\begin{aligned}
T_d &= \mathbb{E}_\psi[g_k^n(S,z) \int_{\underline{\eta}}^{\bar{\eta}} \lambda \cdot \eta dH(\eta)] \\
T_p &= \mathbb{E}_{\psi,z',\eta}[(1 - g_b^n(\eta, S, z'))\lambda\eta g_k^n(S,z) + \\
&\quad g_b^n(\eta, S, z') \max(0, y(z', k, \eta) + (1-n)(a - \chi(a)) - \Phi^n(S, z', \eta))] \\
T_f &= \mathbb{E}_\psi[\mathbb{I}_{\{g_n^U(S,z)=I\}}]
\end{aligned}$$

The formula for T_d just says that if each unit of entrepreneurial capital causes on average $\lambda \times \eta$ units of negligent damages, then the total such damages is the sum of these damages over all entrepreneurs. The formula for the paid damages, T_p, says that if an entrepreneur doesn't file for bankruptcy then she pays the entire damage caused, whereas if she files for bankruptcy then the only payments made are those less than the remaining resources in the business after exemptions and bank's recovery. Lastly, T_f is the amount of fees paid by the unincorporated entrepreneurs who incorporate.

Notice two things about these transfers. First, T_d is increasing in the fraction of entrepreneurs. Second, T_p is decreasing in the incorporation rate, in so far as incorporated entrepreneurs file for bankruptcy more and are able to protect personal savings from seizure.

The corporate sector faces the exogenous risk free rate $\bar{r}$. Hence, $K_c = N_c \left(\frac{\alpha_c A_c}{\delta+\bar{r}}\right)^{\frac{1}{1-\alpha_c}}$. The labor utilized in the corporate sectors is just the measure of households not operating businesses:

$$N_c = 1 - \mathbb{E}_\psi[\mathbb{I}_{\{g_k^n(S,z)>0\}}]$$

Thus we can normalize the wage to 1 by choosing A_c correctly. We therefore measure everything in average wage units.

7.6. **Characterization.**

At this point we do not have a full proof of existence, although we can compute equilibria readily. We believe that we can get a general proof following the strategies in Chatterjee et al. [2007]. Existence boils down to getting a fixed point for the correspondence that maps the space of cross-sectional distributions and prices into itself. An important obstacle in way of doing so is the discrete choice of whether or not to file for bankruptcy, which can create discontinuities of the stationary distribution with respect to the debt prices.

7.7. **Calibration.**

7.7.1. *Households.*

We assume households' period utility function is of the form

$$u(c_t) = \frac{c^{1-\sigma}}{1-\sigma},$$

and assume the risk aversion coefficient σ to be 1.5. We also assume that the discount factor β to be 0.96. These values are taken from Cagetti and De Nardi (2006) and are in the range of those typically used.

For the baseline model, we assume that there are three possible values for project quality, $\{z_0, z_1, z_2\}$ with transition matrix Γ_z. We assume that the lowest project quality is $z_0 = 0$ (i.e. no project). Furthermore, we restrict the transition matrix to be of the following form:

$$\Gamma_z = \begin{bmatrix} \gamma_0 & 1-\gamma_0 & 0 \\ 1-\gamma_1-\gamma_{12} & \gamma_1 & \gamma_{12} \\ 1-\gamma_2 & 0 & \gamma_2 \end{bmatrix}$$

We make the assumption of three productivity levels to allow for the possibility of positive selection that we observe in the cross-section of unincorporated and incorporated entrepreneurs. We therefore need to have two positive values of the project quality in order to allow for such a phenomenon. We follow Quadrini by assuming the ladder structure of the transition matrix, with the exception that we allow for the lower quality project to fall to zero with some probability. We calibrate these parameters by targeting the cross-sectional differences in output, interest rates and wealth to incomes of unincorporated and incorporated entrepreneurs, as well as aggregate entrepreneur share of output, fraction of entrepreneurs and bankruptcy and incorporation rates.

7.7.2. *Technology.*

Corporate Technology

The corporate sector is represented by a constant returns to scale technology of the form $F(K,N) = A_c K_c^{\alpha} N_c^{1-\alpha}$. We set the capital income share parameter α to be .33, and choose A to normalize the wage to 1.

Entrepreneurial Technology

For the entrepreneurial technology, we need to choose the decreasing returns parameter, ν. Cagetti and De Nardi [2006] calibrate the capital income share in a model where entrepreneurial technology uses both capital and labor. However, assuming labor is hired in spot markets, there is a one to one mapping between their parameters. Therefore, we use their calibrated values, and set ν to be 0.708[17].

[17]See the appendix for details.

Lastly, following standard business cycle literature, we set the depreciation rate δ to be 0.068.

7.7.3. *Financial Intermediation.*

We will assume that the losses associated with η are covered before debt is repaid to banks. Therefore, the recoverable amount, Φ, is given by

$$\Phi^U(S, z, \eta) = \min\{D, \max\{z \cdot f(k) + (1 - \delta)k - \eta \cdot k + (a - \xi(a)), 0\}\},$$

for unincorporated entrepreneurs, and

$$\Phi^I(S, z, \eta) = \min\{D, \max\{z \cdot f(k) + (1 - \delta)k - \eta \cdot k, 0\}\}$$

for incorporated entrepreneurs. The assumption on the order of payment implies that all risk in the economy will be priced by financial intermediaries.

McGrattan et al. [2000] estimate the average return on capital to be 4%, therefore set the risk-free rate, $\bar{r}$, to be 4%.

7.7.4. *Bankruptcy.*

The U.S. bankruptcy laws impose the following functional form for the exemption function:

$$\xi(a) = \min\{\xi, a\}.$$

Following Meh and Terajima [2008], we use a weighted average of state homestead and personal bankruptcy exemptions to calibrate the the exemption level ξ. To calibrate the fixed cost of incorporation, f, we target the fraction of incorporated entrepreneurs, 3.1%.

7.7.5. *Business Shocks.*

For calibration, we will interpret the capital shocks as disasters in which the capital is lost. We will restrict the distribution to have support $\{0, \bar{\eta}\}$; therefore, we have two parameters to calibrate, $\bar{\eta}$, the severity of the disaster and, p_η, the probability of a disaster occurring. In addition, since the capital is destroyed, we will assume $\lambda = 0$ and there will be no transfers associated with the capital shocks. We target the bankruptcy rate of entrepreneurs and the fraction of bankruptcies resulting from disaster reported by Dunn and Bradstreet.

Table 12. Calibrated Parameters

Parameters		Values
Fixed Parameters		
σ	relative risk aversion	2.0
β	discount rate	0.96
α	capital income share (corporate)	0.33
ν	returns to scale (entrepreneur)	0.71
δ	depreciation rate	0.068
$\bar{r}$	risk free rate	0.04
ξ	bankruptcy exemption	0.15
Calibrated Parameters		
z_1	project scale	0.34
z_2	project scale	0.45
γ_0	persistence of no project, z_0	0.989
γ_1	persistence of z_1	0.969
γ_{12}	$\Gamma(z_2\|z_1)$	0.02
γ_2	persistence of z_2	0.99
f	fixed cost of incorporation	3.35
p_η	probability of $\bar{\eta}$	0.01
$\bar{\eta}$	capital shock	1.00

7.7.6. *Benchmark Results.*

Table 14 present the current calibration of the model. While the fit is currently being improved, the current calibration captures many of the features in the data. In particular, the positive selection of higher productivity entrepreneurs into incorporation. In the benchmark model, incorporated entrepreneurs have larger businesses and accumulate greater wealth. In addition, table 15 shows the unconditional interest rates in the benchmark model. The unconditional interest rate for incorporated entrepreneurs, which we did not target in the calibration, is lower than the unincorporated, as it is in the data.

Table 13. Targeted Moments

Moment	Values
Cross-Sectional Moments	
Wealth to Incomes	
Unincorporated	8.30
Incorporated	12.20
Ratio of IE to UE Output	6.98
Unconditional Interest Rate	
Unincorporated	7.00
Aggregate Moments	
Fraction of Entrepreneurs	0.083
Fraction of Incorporated Entrepreneurs	0.031
Entrepreneur Share of Output	0.180
Entrepreneur Bankruptcy Rate	0.023
Incorporation Rate	0.01
Bankruptcies due to Disaster	0.063

Table 14. Baseline Economy

Moment	Data	Model
Cross-Sectional Moments		
Wealth to Incomes		
Unincorporated	8.30	6.33
Incorporated	12.20	12.41
Ratio of IE to UE Output	6.98	4.14
Unconditional Interest Rate		
Unincorporated	7.00	7.31
Aggregate Moments		
Fraction of Entrepreneurs	0.083	0.089
Fraction of Incorporated Entrepreneurs	0.031	0.022
Entrepreneur Share of Output	0.180	0.325
Entrepreneur Bankruptcy Rate	0.023	0.005
Incorporation Rate	0.009	0.010
Bankruptcies due to Disaster	0.063	0.043

Table 15. Interest Rates by Legal Form

	UE	IE
Unconditional Interest Rates	7.3	7.2

7.8. **Investment Risk and Consumption.**

We use the model to measure the importance of the investment risk and the value of incorporation to the entrepreneurs. We do this by following Lucas Jr [1987] and Storesletten et al. [2001], and measure the risk and value of incorporation by comparing the welfare across alternative economies.

How much investment risk?

Consider two economies, one in which the entrepreneur faces both the output risk from movements in z and the investment risk, η; and the the other an economy in which the entrepreneur faces only the output risk from z. In both economies, the entrepreneurs has the option to incorporate their business. We calculate the annual % increase in consumption required to make an entrepreneur indifferent between starting her business in the presence of the investment risk and in a world with none. Let $V_{\text{start-up}}(\cdot)$ denote the value function for an individual enters the period as a worker and installs capital for the following period (i.e. becomes an entrepreneur this period). Furthermore, let $V^{\eta}_{\text{start-up}}(\cdot)$ denote the start-up value in the economy with the investment risk, η, and $V^{0}_{\text{start-up}}(\cdot)$, the start-up value in the economy with no investment risk. We compute the necessary adjustment to consumption in the economy with the investment risk, ρ, which equates the average utility in the two economies:

$$\mathbb{E}_{\psi}[\big(V^{\eta}_{\text{start-up}}(R,z) - V^{0}_{\text{start-up}}(R,z)\big)\mathbb{I}_{\{g^{n}_{k}(S,z)>0|k=0\}}] = 0$$

Note that an individual who would start a business in the economy with investment risk, would also do so in the economy without investment risk. Therefore, we average over the stationary distribution in the economy with investment risk. We find that that the entrepreneur would require an additional 5.9% of consumption annually in order to make her indifferent. This suggests that investment risk is a large source of risk for entrepreneurs. Using

2003 U.S. per capita consumption, in absolute terms, 5.6% of consumption is approximately $1,600 per year. In comparison, Lucas Jr [1987] finds that the welfare gains from eliminating business cycles amounts to less than a fraction of a percent of consumption annually. Similarly, Storesletten et al. [2001] consider the welfare gains from eliminating idiosyncratic labor income risk and find that the necessary adjustment to consumption is 1.17% annually.

How much insurance is provided by limited liability?

We perform a similar exercise to measure the value of limited liability to entrepreneurs. Consider two economies, one in which the entrepreneur has the option to incorporate her business, and in the other economy, the entrepreneur can not incorporate. In both economies, the entrepreneur faces the output and investment risks. We again focus on the start-up value of a business for new entrepreneurs. We compute the annual consumption an entrepreneur would demand at start-up in response to eliminating the ability to incorporate. Since more individuals start businesses in the economy where they can incorporate, we average over the stationary distribution of the economy *without* incorporation. Thus, our calculation ignores the extensive margin associated with allowing for incorporation, and is a lower bound on the consumption equivalent value of limited liability. We find the required additional annually consumption from eliminating incorporation to be 0.81%.

7.9. **Policy.**

We use the model to conduct two policy experiments that are relevant to many actual legal changes made in the last twenty years. We first ask what happens when incorporation becomes cheaper. Since the only form of incorporation in our model has pass-through income, this resembles the introduction of recent legal forms such as Limited Liability Corporations, as well as bestowing more and more benefits to S-Corporations that were once reserved for C-Corporations. In particular, we reduce the fixed costs by 50% and find that this reduction increase aggregate income of 0.5%, but also increases income and wealth inequality. However, we find that the fraction of entrepreneurs does not change very much. We believe that this is a result of the coarseness of our grid when solving the model. The effects of lowering the fixed costs are in Table 16.

TABLE 16. 50% Reduction in Fixed Costs

Moment	Baseline	Lower f
Cross-Sectional Moments		
Wealth to Incomes		
Unincorporated	3.85	1.71
Incorporated	7.12	6.67
Ratio of IE to UE Output	2.52	2.04
Unconditional Interest Rate		
Unincorporated	14.53	14.75
Aggregate Moments		
Fraction of Entrepreneurs	0.159	0.159
Fraction of Incorporated Entrepreneurs	0.081	0.108
Entrepreneur Share of Output	0.263	0.265
Entrepreneur Bankruptcy Rate	0.089	0.100
Incorporation Rate	0.05	0.13
Uninsured Tort Costs (Share of Entrepreneur Output)	0.015	0.015

A similar exercise, which has been the focus of previous papers on entrepreneurship and default (Athreya et al. [2009], Meh and Terajima [2008]), explores the effects of changing personal bankruptcy exemptions. Since in our model an entrepreneur can attain full exemptions by incorporating, we find that changing personal exemptions has a much smaller impact than either of these previous studies. We find that doubling the exemption level decreases aggregate income by .1%; again we believe these results to be sensitive to the coarseness of our grid. The effects of lowering the fixed costs are in Table 17.

It is important to note that our model gives indeterminant welfare conclusions with respect to these policies. This is because some fraction of the expenditure shocks are modeled as damages to third parties, and entrepreneurs are absolved of reimbursing these expenses in bankruptcy. Furthermore, while lenders will price their expected losses due to bankruptcy, damages due to negligence will still not be internalized. This is because secured creditors are paid first in bankruptcy proceedings.

TABLE 17. Doubling of Exemption Levels

Moment	Baseline	Double ξ
Cross-Sectional Moments		
Wealth to Incomes		
Unincorporated	3.85	4.48
Incorporated	7.12	6.35
Ratio of IE to UE Output	2.52	2.58
Unconditional Interest Rate		
Unincorporated	14.53	15.06
Aggregate Moments		
Fraction of Entrepreneurs	0.159	0.159
Fraction of Incorporated Entrepreneurs	0.081	0.084
Entrepreneur Share of Output	0.263	0.263
Entrepreneur Bankruptcy Rate	0.089	0.087
Incorporation Rate	0.05	0.06
Uninsured Tort Costs (Share of Entrepreneur Output)	0.015	0.015

Because entrepreneurs can generate large expenses to third parties (in 2003, 6% of total entrepreneurial output, one third of which was uninsured). This creates a trade-off for society. On one hand, the the insurance provided by incorporation encourages entrepreneurial activity, which has high marginal product. On the other hand, limited liability also enables these entrepreneurs to avoid accountability. Because the average net damages to each household depends on the number of entrepreneurs, as well as their incorporation status and level of exemptions, we find a non-monotonic relationship between welfare and entrepreneurship rates.

7.10. **Conclusion.**

In this paper, we embed the U.S. bankruptcy and incorporation legal systems in a quantitative macroeconomic theory of occupational, incorporation, and default choices that accounts for the cross-sectional facts. In the model, as in the U.S., incorporation provides insurance via limited liability beyond personal bankruptcy exemptions, at the expense of administrative burdens and an endogenous interest rate premium. We discipline the model using cross-sectional differences by legal form and several aggregate moments of the U.S.

economy. The model is able to capture all of these features. In addition, our model suggests that capital shocks are important entrepreneurial risks.

We use the model to conduct two policy experiments that are relevant to many actual legal changes made in the last twenty years: (i) A reduction in the costs of incorporation, meant to capture the introduction of recent legal forms such as Limited Liability Corporations, as well as bestowing more and more benefits to S-Corporations that were once reserved for C-Corporations. (ii) Increasing the personal bankruptcy exemption levels. We find that reductions in the cost of incorporation increase aggregate income by 0.5%, but also increases income and wealth inequality. In regards to the second policy, we find that doubling the exemption level decreases aggregate income by .1%.

Lastly, by studying the decision of an entrepreneur to incorporate their business, this paper contributes towards developing a theory that can bridge the entrepreneurial and corporate sectors; and allow us to better understand and study the process in which individuals start businesses and how some of those businesses subsequently transition to the corporate sector. Furthermore, a deeper theory of these extensive margins will allow for more comprehensive answers to questions regarding firm entry and exit; in particular, questions regarding financial markets and tax policies; for example, changes to the personal and/or corporate tax structures will influence the decision of individuals to move between the different legal forms, and therefore the environments in which these businesses operate.

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Appendix A. Appendix to Chapter 1

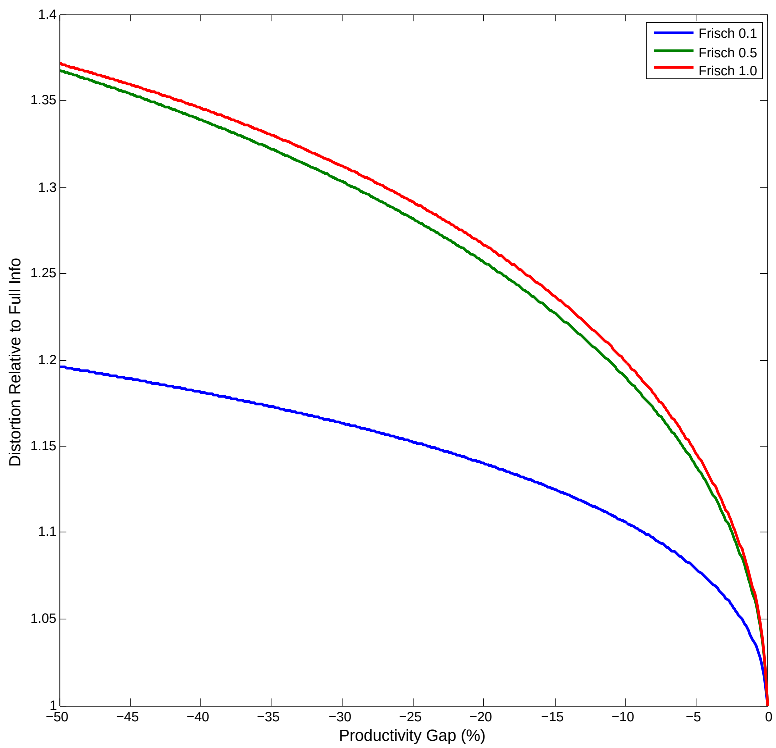

Figure 2. Distortion of Hours, Analytical Solution Case

A.1. **Figures.**

A.2. **Proofs.**

Proof. Proof of Proposition 4.1. Via substitution of the earnings functions and interest rate, it is clear that the budget constraint simplifies to $C + K' = Y$. Guessing that $K' = \alpha\beta Y$ gives $C = (1 - \alpha\beta)Y$. It is straight forward to verify the intertemporal Euler Equation at this point, noting the importance of separability. Using this function for C in equation 5.2 gives the hours of the low productivity worker, $h_L = \left(\phi\frac{1-\alpha}{1-\alpha\beta}\right)^{\epsilon}\left(\frac{\zeta}{H}\right)^{\epsilon}$. Then using those hours in the incentive compatibility equation gives the equation to be solved for H, which after some simple manipulation yields equation 5.10.

Let $\lambda = \frac{1-\alpha}{1-\alpha\beta}$ for the time being. Then plugging $\epsilon = 1$ turns 5.10 into the quadratic form:

$$H^4 - 2\lambda\phi H^2 + (\lambda\zeta\phi)^2 = 0$$

which gives the solution:

$$H^2 = \lambda\phi\left(1 + / - (1 - \zeta^2)^{1/2}\right)$$

Since H must be greater or equal to $H^* = (\phi\lambda)^{1/2}$, there is only one economically interesting root. Since hours must be positive, the solution is as given in the proposition. □

Appendix B. Appendix to Chapter 2

B.1. **Figures.**

B.2. Mapping $f(z,k,n)$ to $F(z,k) = f(z,k,n^*(z,k))$. We want to use an entrepreneurial technology without labor, but use parameters from Cagetti and De Nardi [2006]. This can be seen as a shortcut, since if labor is hired in spot markets then it is easy to solve for the optimal level of labor demand and substitute out, making a new production function that depends only on capital. The only thing that needs to change is the labor market clearing condition. However, care must be taken when using previous calibrated parameters for the scale of production, since profits and optimal levels of capital are extremely sensitive to this parameter.

They use a production function of the form: $y = f(z,k,n) = z(n^\theta k^{1-\theta})^\nu$. Thus profits are given by $\pi(z,k,n) = f(z,k,n) - (r+\delta)k - wn$. Take the first order condition wrt labor to get that $n^*(z,k) = \left(\frac{z\nu\theta k^{\nu(1-\theta)}}{w}\right)^{\frac{1}{1-\nu\theta}}$. Substituting this into the production function gives $\pi(z,k,n^*(z,k)) = \tilde{z}k^\alpha - (r+\delta)k$, where the transformed variables take the values $\tilde{z} = (1-\nu\theta)\left(\frac{\nu\theta}{w}\right)^{\frac{\nu\theta}{1-\nu\theta}} z^{\frac{1}{1-\nu\theta}}$ and $\alpha = \frac{\nu(1-\theta)}{1-\nu\theta}$.

Cagetti and De Nardi use $\nu = 0.88$ and $\theta = 0.67$. Using their calibration in a model without capital would imply a value of $\nu = \frac{0.88(1-0.67)}{1-0.67\times 0.88} = 0.708$, which is what we use.

B.3. Interest Rate Regressions. A preview of the theoretical loan supply schedule is useful for understanding why each control variable was included. In a perfectly competitive banking industry, banks make zero profits on each loan. The interest rate charged is thus a function of (1) the risk of the entrepreneur defaulting on the loan and (2) the fraction of the loan that the bank can recover upon default. We expect an increase in (1) to be associated with an increase in the interest rate, and just the opposite for (2). We explore what variables would affect these two items and whether or not the correlations uncovered by our regressions confirm these expectations. We emphasize that these regressions are not meant to be structural in nature, but rather descriptive of what happens to the equilibrium interest rate when keeping quantity constant and changing factors that would shift supply or demand.

We first include proxies of the business's current productivity or demand. Higher levels of either should decrease (1) and increase (2) since the levels of these variables at the time the loan comes due are likely positively correlated with their levels at the time the loan is made. These considerations lead us to include sales, whether the owner went to college, and the age of the firm as controls. We find that each is associated with a decrease in interest rates, as we would expect, but that the coefficient on sales is not significantly different from zero in neither magnitude or the statistical sense.

While we do not model reputation directly, we think it is important to include proxies for such a variable in light of (1). We therefore include the length of the relationship between the entrepreneur and the lending bank, the entrepreneur's credit score, and whether or not the owner/business has made late payments or filed bankruptcy in the last three years. The regression supports our expectations of negative coefficients on relationship length and credit score and a positive coefficient for payment delinquency. Bankruptcy could theoretically have either a positive or negative sign, but turns out to be positive, which echoes previous studies by Berkowitz and White [2004].

We also include controls that directly affect how much the bank could recover in the event of bankruptcy. These include whether or not the entrepreneur owns a home as well as whether a guarantee or collateral were provided by the entrepreneur. We would expect each of these to have a negative coefficient, which is confirmed strongly in both magnitudes and statistical significance.

Finally, we include many things that we do not try to model but are likely to affect both (1) and (2). These include industry and location variables, aspects of the loan contract such as whether or not it is fixed or variable rate, and a measure of how competitive the banking industry is in the city in which the loan originated. Clearly industry and urbanicity are likely to affect the liquidation value of the firm, as well as the variance of demand and technology shocks. It is also clear that variable interest rate loans should have lower rates than fixed rates whenever there is inflation risk. We include the measure of competitiveness because we model markets as competitive out of convenience, not faith. Interestingly, a more competitive banking sector is associated with nearly a full interest point reduction, which suggests that introducing monopolistic competition to models of default may be fruitful in future work.

Table 18. Estimation results : Regression of Business Loan Interest Rate on Observables

Variable	**Coefficient**	**(Std. Err.)**
Incorporation	0.398	(0.190)
log(sales)	-0.007	(0.078)
log(capital)	-0.021	(0.044)
log(employment)	-0.337	(0.091)
length of relationship with bank	-0.164	(0.056)
Loan Type (Line of Credit)	0.899	(0.161)
In Person	-0.429	(0.144)
Guarantee	-0.618	(0.141)
Collateral	-0.677	(0.143)
Fixed Maturity	-1.779	(0.341)
Fixed Interest Rate	1.249	(0.145)
Age of Owner	-0.022	(0.008)
College Education	-0.317	(0.141)
Homeowner	-1.868	(0.373)
D&B Credit Score	-0.141	(0.177)
Bankruptcy	0.109	(0.411)
Late Payment	0.987	(0.239)
Herfindahl-Hirschman Index	-0.934	(0.171)
Urban Area	0.086	(0.174)
Industry - Mining	-1.769	(0.305)
Industry - Construction	-0.967	(0.283)
Industry - Manufacturing	-0.092	(0.232)
Industry - Trans./Utilities	0.907	(0.319)
Industry - Wholesale Trade	-0.490	(0.289)
Industry - Retail Trade	-0.427	(0.216)
Industry - Finance/Insurance/RealEst	-1.332	(0.246)
Intercept*	12.578	(1.389)

*The baseline industry is services.

Table 19. Estimation results : Regression of Business Credit Card Interest Rate on Observables

Variable	**Coefficient**	**(Std. Err.)**
Incorporation	0.738	(0.230)
log(sales)	-0.253	(0.095)
log(capital)	-0.105	(0.068)
log(employment)	0.334	(0.129)
Monthly Charges	-0.008	(0.002)
Rolling Balance	0.152	(0.079)
Pay Balance	-0.390	(0.243)
Age of Owner	-0.015	(0.010)
College Education	0.253	(0.193)
Homeowner	-0.975	(0.287)
Urban Area	-1.142	(0.254)
D&B Credit Score	-0.182	(0.254)
Bankruptcy	1.435	(0.431)
Late Payment	3.513	(0.414)
Industry - Mining	-1.181	(0.343)
Industry - Construction	1.294	(0.345)
Industry - Manufacturing	-0.597	(0.388)
Industry - Trans./Utilities	0.411	(0.450)
Industry - Wholesale Trade	0.586	(0.368)
Industry - Retail Trade	0.398	(0.276)
Industry - Finance/Insurance/RealEst	0.145	(0.359)
Intercept*	17.427	(1.238)

*The baseline industry is services.