UNIVERSITY OF MINNESOTA
ST. ANTHONY FALLS HYDRAULIC LABORATORY

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LAKE DESTRATIFICATION BY AIR BUBBLE COLUMNS

by

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1. INTRODUCTION

Many lakes and reservoirs develop a seasonal temperature stratification. The thermal stratification is associated with a density stratification which effectively controls convection and diffusion in the lake. The most stable stratification conditions exist during the summer months when lakes or reservoirs exhibit large thermal gradients. As a result the water quality of the lower layers may be reduced significantly for example because dissolved oxygen is no longer replenished by surface aeration [Cederwall and Ditmars, 1970].

Artificial mixing may then be an effective way of improving water quality. One of the methods which can be used is mixing by air injection (air bubble column). The air released at some depth will entrain water, and it will create a circulation and mixing.

Schematic presentations of the flow fields induced by air bubble column under isothermal and stratified circumstances are given in Fig. 1.1. As can be seen in Fig. 1.1, the flow field under stratified circumstances is very complicated. One of the possibilities, which is described in this report, is to divide the whole lake into two section [Goossens, 1979].

1. The region in the vicinity of the air column can be treated as a isothermal region, and it is called nearfield.

2. The rest of the lake is treated as a stratified environment and it is called farfield.

The following report is based n the mathematical model and computer programs presented in Goossens [1979]. An effort is made to achieve better convergence of the available computer programs and to incorporate them into the computer program MINLAKE which was already developed at St. Anthony Falls Hydraulic Laboratory. The extended computer program is then applied to two cases, one given in Goossens [1979] and a second one which is the analysis of destratification of Lake Calhoun in 1971.

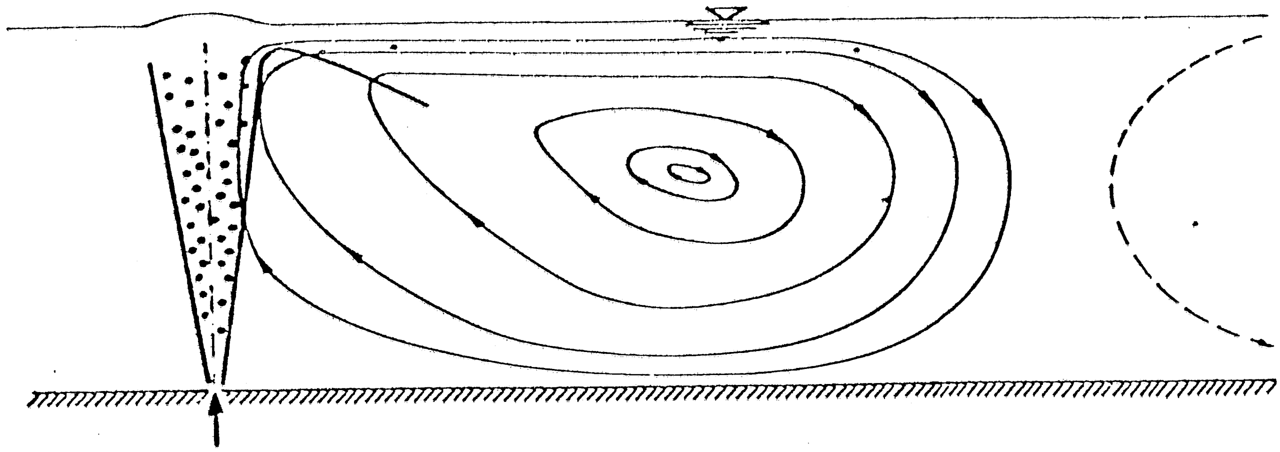


Fig.1.1a. Flow field due to a bubble column under isothermal conditions (after Goossens, 1979).

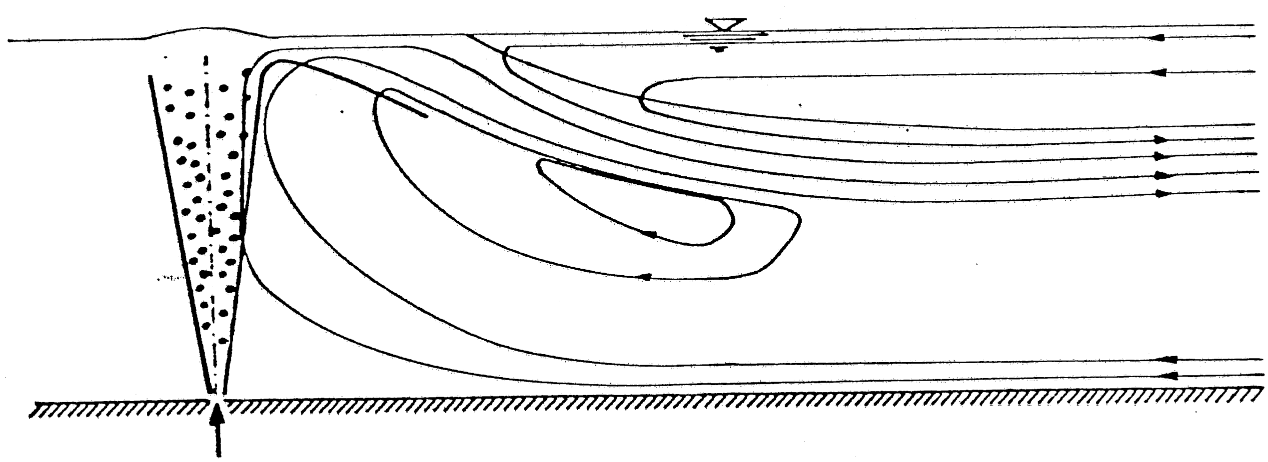


Fig.1.1b. Flow field due to a bubble column under stratified conditions (after Goossens, 1979).

2. NEARFIELD

The basic assumption for the nearfield is that the region in the vicinity of the air column can be treated as isothermal. In the analysis of the nearfield there are two main elements:

1. Bubble plume

2. Radial jet through which the effects of the bubble plume are transmitted to the surrounding.

2.1. Bubble Plume

A bubble plume is a two-phase plume comparable to a single phase buoyant jet. It is found to have an entrainment coefficient equal or slightly greater than that of a single phase free jet.

Plume equations are presented without assuming either the radial profiles of liquid velocity and gas distribution or the proportion of the total momentum flux associated with the mean flow.

The situation of the bubble plume is shown in Fig. 2.1.

As in the free jet it is assumed that the static pressure over a given cross section is approximately constant and equal to the value in the undisturbed liquid outside.

$$p = \rho g^{2} \tag{2.1}$$

where p = static pressure

ρ = liquid density

g = acceleration due to gravity

z = distance in the downward direction measured from a height z_{atm} above the undisturbed liquid surface such that

$$p_{atm} = \rho g z_{atm}$$

This choice of coordinate origin simplifies the inclusion of gas compessibility.

The flow is assumed to be steady, the time scale being large compared to the time scale of turbulent motion.

If we consider the volume enclosed by a control surface s defined by two horizontal planes of diameter d_s separated by a distance Δz (Fig. 2.1) one can write:

a) the weight of the mixture with S

$$\rho g \left(\frac{\pi}{4} d_s^2 - a\right)\Delta z + \rho G \cdot a \cdot g \cdot \Delta z \tag{2.2}$$

where a = mean cross-sectional area occupied by gas

ρ_a = density of gas

b) an upward force

$$\rho g \frac{\pi}{4} d_s^2 \quad \Delta z \tag{2.3}$$

As the mean upward force existed on the fluid within this volume equals the change in the momentum flux M of the plume over the vertical interval Δz, it can be written

$$\Delta M = \rho g \left(\frac{\pi}{4} d_S^2 - a\right)\Delta z - \rho g \frac{\pi}{4} d^2 \; \Delta z \tag{2.4}$$

as $\frac{\Delta M}{\Delta z} \rightarrow \frac{dM}{dz}$ when $\Delta z \rightarrow 0$

$$\frac{dM}{dz} = - \rho g a \tag{2.5}$$

The previous derivation of eq. 2.5 is based on derivation proposed in [Goossens, 1979]. The final result on the right-hand side is the buoyancy force on gas because the weight and buoyancy on water cancel and net force on the water is equal to zero. It is important to note here that M consists of the mean and fluctuating part

$$M = \overline{M} + M' \tag{2.6}$$

In the case of a single phase force turbulent jet M' is of the order of 10% of $\overline{M}$. In the case of a bubble plume the two terms may initially be of the same order and neglect of M' will omit significant terms in the momentum balance.

2.1.1. Mass entrainment

Mass entrainment is defined as

$$- \frac{dm}{dz} = k \cdot g^{1/2} \; M^{1/2} \qquad (2.7)$$

where K = dimensionless parameter

The local mass and momentum fluxes can be used to define uniquely an equivalent diameter d and a coupled velocity v which the jet would have if its velocity profile were rectangular (Fig. 2.2) as it can be written for mass and momentum fluxes

$$m = \rho \frac{\pi}{r} d^2 v \qquad (2.8)$$

$$M = \rho \frac{\pi}{4} d^2 v^2 \qquad (2.9)$$

Equations 2.8 and 2.9 with 2.7 give

$$- \frac{dm}{dz} = K \sqrt{\pi/4} \;\; \rho v d \qquad (2.10)$$

The important assumption at this stage is that of similarity between the bubble plume and a free jet. The effect of bubbles can be taken into account by ascribing an effective value for d to the jet, d_{eff} , given by

$$\frac{\pi}{4} d^2_{eff} = \frac{\pi}{4} d^2 + a \qquad (2.11)$$

In the second place there may be an increase in K due to the presence of bubbles. With Eq. 2.11, Eq. 2.10 can be rewritten in the form.

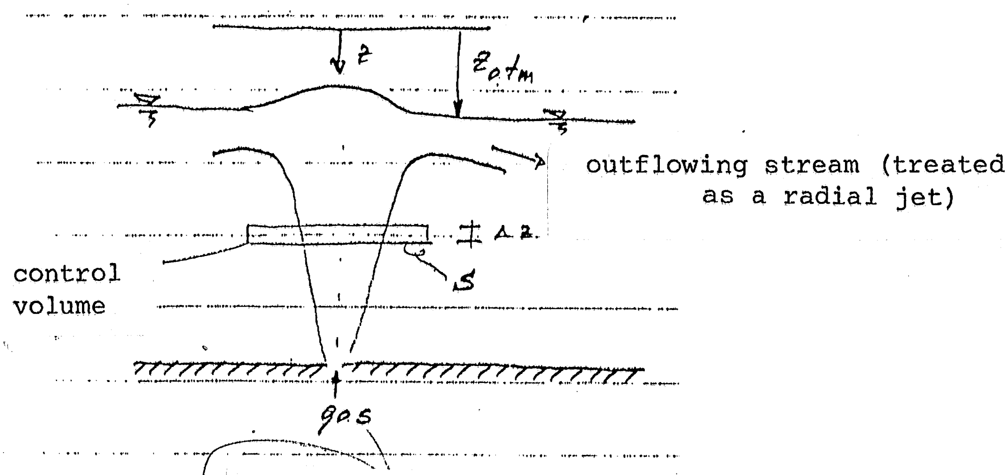
$$- \frac{dm}{dz} = K_1 \cdot \rho v d_{eff} \qquad (2.12)$$

in which $K_1 = \sqrt{\pi/4}$ K is the key parameter in plume development.

The value of the entrainment coefficient K_1 for a bubble plume is found to be similar to the free jet value, $K_1 = 0.25$ for the axysymmetric case and $K_1 = 0.16$ for the plane symmetric case.

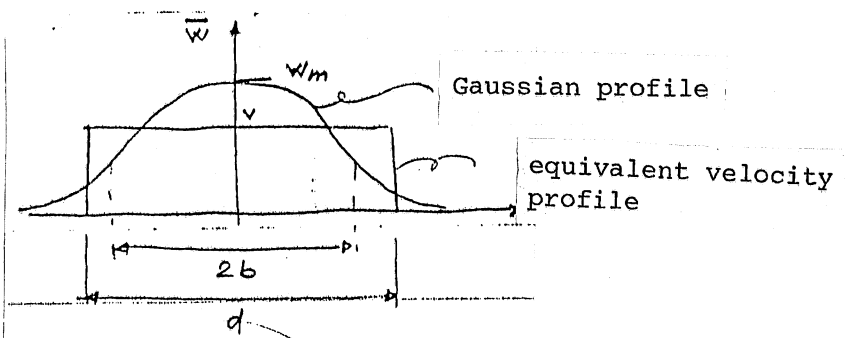
2.1.2. Continuity of gas flow

The magnitude of the mean cross-sectional area (a) occupied by gas, can be obtained from continuity considerations:



z
$z_o + t_m$
outflowing stream (treated as a radial jet)
Δz
S
control volume
gas

FIG. 2.1



$\overline{W}$
W_m
Gaussian profile
V
equivalent velocity profile
2b
d

FIG. 2.2

$$m_G = \rho_G \cdot v_G \cdot a \tag{2.13}$$

where m_G is the mean mass flow rate of gas (assumed to be constant and equal to the injection rate), ρ_G the mean gas density in a given section, and v_G the mean gas velocity. If an isothermal expansion of the bubbles is assumed then

$$\rho_G = \rho_{Go} \cdot \frac{z}{z_o} \tag{2.14}$$

This is replaced by

$$\rho_G = \rho_{Go} \cdot \left(\frac{z}{z_o} \right)^{1/4} \tag{2.15}$$

if the bubble expansion is assumed adiabatic (possibly so with large bubbles) while the bubble velocity v_G may be written as

$$v_G = v + v_r \tag{2.16}$$

The first term represents the mean velocity of the liquid in which the bubbles are situated and v_r is the mean relative velocity of the bubbles in a given section which is of the order of the terminal rise velocity of the single bubble.

Substitution of 2.14 and 2.16 into 2.13 yields:

$$a = \frac{G_o z_o}{z(v+v_r)} \tag{2.17}$$

when G_o denotes the volumetric gas injection rate under orifice conditions.

2.1.3. The final plume equations

From Eqs. 2.5 and 2.7 with substitutions given in the preceding equations two coupled equations for the rate of change of the equivalent diameter and velocity of the plume can be obtained:

$$-\frac{d(d)}{dz} = \frac{\dfrac{-gG_o z_o}{2z(v+v_r)} + k_1 d v^2 \left(1 + \dfrac{4G_o z_o}{\pi d^2 z(v+v_r)} \right)^{1/2}}{} \tag{2.18}$$

$$- \frac{d(d)}{dz} = \frac{\frac{gG_o z_o}{z(v+v_r)} - K_1 d v^2 \left(\frac{1+4\ G_o z_o}{\pi d^2 z(v+v_4)} \right)^{1/2}}{\frac{\pi d^2 v}{4}} \tag{2.19}$$

Equations 2.18 and 2.19 are for the axisymmetric case. The corresponding equations for the plane symmetric case proves to be:

$$- \frac{d(d)}{dz} = \frac{\frac{-g\ G_o z_o}{z(v+v_r)} + 2\ K_1\ v^2}{v^2} \tag{2.20}$$

$$- \frac{dv}{dz} = \frac{\frac{g\ G_o z_o}{z(v+v_r)} - K_1 v^2}{vd} \tag{2.21}$$

The quantities G_o, m, and M are per unit length of curtain:

2.1.4. Volume flux relations

On the basis of Equations 2.18 and 2.19 supported by measurements in the laboratory and the field [1]* the following relation between the depth of the lake H_1 the volumetric gas rate at atmospheric pressure Q and the volumetric outflow Q_w is

$$Q_w = 0.47 \times Q^{1/3} \times H^{4/3} \tag{2.22}$$

2.1.5. Summary

The entrainment coefficient K_1 is the key parameter of the plume. Since K_1 is close to the free jet value the length scale of the energy containing eddies does not seem to be influenced by the presence of bubbles, implying that the entrainment process is fully determined by the shear of the liquid rather than the eddies obtained from the bubble motion. The environment is treated as isothermal. As measurements fit very well with the mathematical model [1]*, it can be concluded that the behavior of the bubble plume is well described.

2.2. Radial Jet

The isothermal nearfield is essentially determined by the radial jet flowing away at the surface from the bubble column. If we assume stationary conditions with radial symmetry, no Coridis influence and a sufficiently turbulent flow to neglect viscous effect, we may rewrite the Navier-Stokes equations as

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* [Goossens, 1979. See references list.]

$$u_r \frac{\partial u_r}{\partial r} + v_z \frac{\partial u_r}{\partial z} + \frac{\partial \overline{u_r' v_z'}}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{1}{r}\frac{\partial \overline{r u_r'^2}}{\partial r} + \frac{\overline{U'\rho^2}}{r} \quad (2.23)$$

$$u_z \frac{\partial u_r}{\partial z} + u_z \frac{\partial u_z}{\partial z} + \frac{1}{r}\frac{\partial r u_r' u_z'}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - \frac{\partial \overline{u_z'^2}}{\partial z} + g \quad (2.24)$$

where r, ψ, z are radial, tangential and vertical coordinates, u_r, u_z are corresponding velocity components, $\overline{u'_1{}^2}$, $\overline{u'_y{}^2}$, $\overline{u'_z{}^2}$ are the turbulent terms; $\overline{u_r' u_z'}$ is a Reynolds stress, ρ the pressure, g the acceleration due to gravity and ρ the density.

Equations 2.23 and 2.24 can be reduced to

$$u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_r}{\partial z} + \frac{\partial u_r' u_z'}{\partial z} = 0 \quad (2.25)$$

Equations 2.25 and 2.26 can be solved if similarity in velocity and shear stress profiles is assumed.

The solution is:

$$u_r = \frac{m'}{\ell} \operatorname{sech}^2 \left(\frac{z}{\ell}\right) \quad (2.27)$$

$$u_z = -\frac{m'}{\ell}\frac{\ell}{r}\left\{\tan h \left(\frac{z}{\ell}\right) - \frac{z}{\ell} \operatorname{sech}^2 \left(\frac{z}{\ell}\right)\right\} \quad (2.28)$$

$$-\overline{u_r' u_z'} = -\left(\frac{m'}{\ell}\right)\frac{\ell}{r} \operatorname{sech}^2 \left(\frac{z}{\ell}\right) \tanh \left(\frac{z}{\ell}\right) \quad (2.29)$$

where m' = momentum coefficient = $0.69\ k^{0.5} Q^{0.35} H^{0.5}$

$\ell = kr$

k = spreading coefficient $0.063 \leqslant k \leqslant 0.078$

Since the theory is valid only for infinitely deep reservoirs, it is extended to reservoirs of finite depth with the assumption that the return flow has no influence on the velocity profile. Therefore superposition of a uniform flow directed towards the bubble column is assumed in the horizontal velocity profile. This yields:

$$u_r = \left(\frac{m'}{\ell}\right) \left\{ \text{sech}^2 \left(\frac{z}{\ell}\right) - \frac{z}{H} \tanh \left(\frac{H}{\ell}\right) \right\} \tag{2.30}$$

$$u_z = -\frac{m'}{\ell} \frac{\ell}{r} \left\{ \tanh \left(\frac{z}{\ell}\right) - \frac{z}{\ell} \text{sech}^2 \left(\frac{z}{\ell}\right) - \frac{z}{H} \tanh \left(\frac{H}{\ell}\right) + \frac{z}{\ell} \text{sech}^2 \left(\frac{H}{\ell}\right) \right\} \tag{2.31}$$

The thickness of the outflowing stream can be found from 2.30 since $u_r = 0$ at $z = z_T$.

$$\frac{T}{H} = \frac{\ell}{H} \tanh^{-1} \left\{ 1 - \frac{\ell}{H} \tan h \left(\frac{H}{\ell}\right) \right\} \tag{2.32}$$

where H is the total depth.

2.3. Nearfield Model

On the basis of the results shown in sections 2.1 and 2.2 the physical model for the nearfield is formulated as shown in Fig. 2.3.

It is clear that an exact analytical solution cannot be derived for the physical situation. Fig. 2.4 shows a simplified model which is used to describe the nearfield.

Five parameters determine the discharge into the interlayer shown in Fig. 2.3.

a) water flow in the bubble column Q_w at the surface
b) water entrained in the discharge Q_o from the epilimnion
c) water entrained from the epilimnion Q_e
d) return flow of mixed water towards the air injection Q_{rt}
e) flow from the hypolimnion Q_h

It can be seen that when the volume of the nearfield is constant, the flow into the interlayer is:

$$Q_i = Q_e + Q_h \tag{2.33}$$

Since the bubble column is the driving force in this mixing process all flow may be related to Q_w as

$$\alpha_x = \frac{Q_x}{Q_w} \tag{2.34}$$

A volume balance over the nearfield therefore shows the return flow to be:

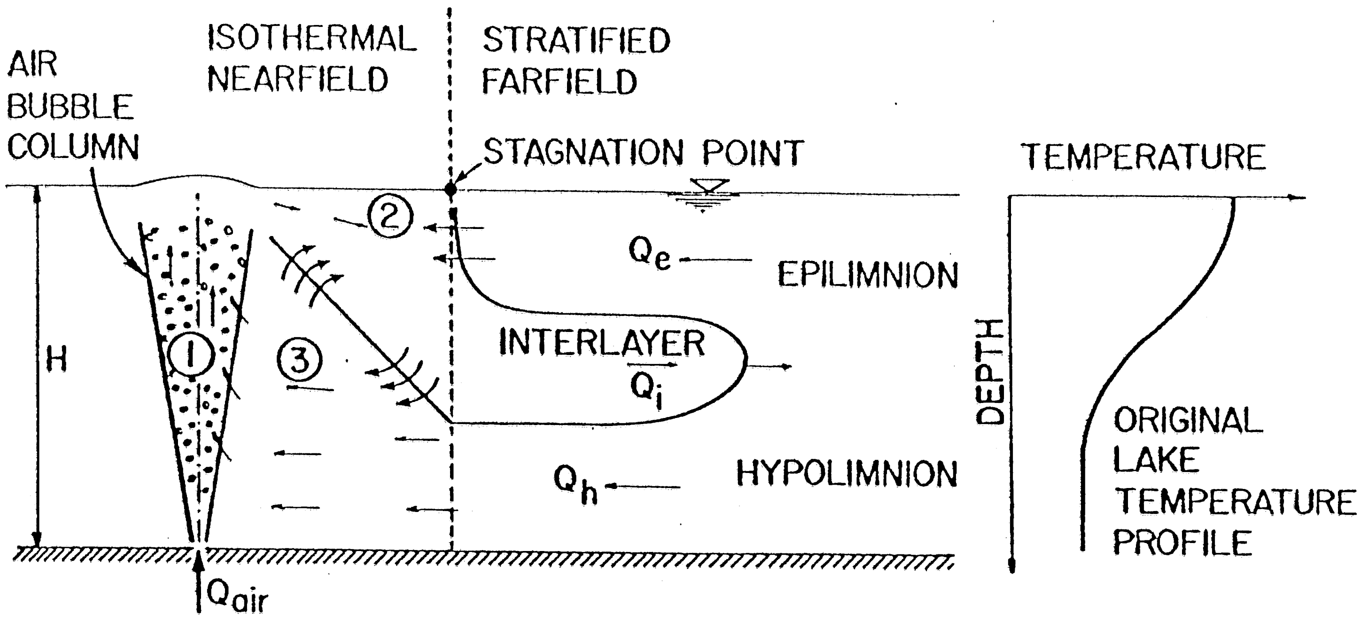

Fig. 2.3. Flow in a nearfield [after Goossens, 1979].

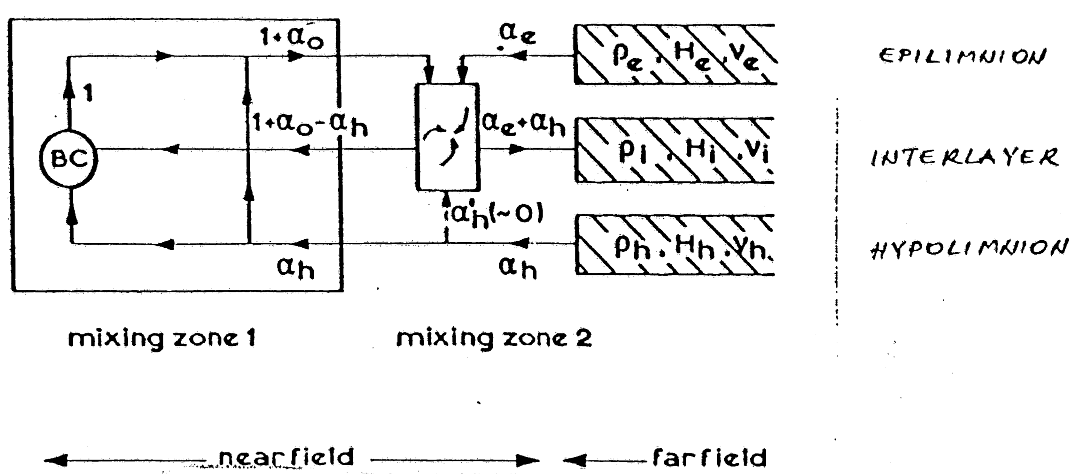

Fig. 2.4. Mathematical stratified near field model Five parameters determine the discharge into the interlayer shown in Fig. 2.3.

$$\alpha_{rt} = 1 + \alpha_o - \alpha_h \tag{2.35}$$

In Fig. 2.4 the nearfield is divided into two mixing zones. Mixing Zone 1 entrains hypolimnion water and return flow and yields a discharge to mixing Zone 2. Mixing Zone 2 entrains the discharge and epilimnion water and yields the return flow to mixing Zone 1 and the interlayer flow to the farfield.

Since both bubble column and discharge are assumed not to be influenced by density differences mixing Zone 1 can be described by the model for the isothermal situation. Mixing Zone 1 is essentially the region of direct mixing out to R_o, the stagnation point.

The original two layers are each assumed to be sharply defined and of uniform density.

2.3.1. Model equations

The mass balances of mixing Zones 1 and 2 yield expressions for the interlayer density ρ_i and the outflowing discharge density ρ_o

$$(1+\alpha_o)\ \rho_o = (1 + \alpha_o - \alpha_h)\rho_i + \alpha_h\ \rho h \tag{2.36}$$

$$(1 + \alpha_o + \alpha_e)\rho_i = (1 + \alpha_o)\rho_o + \alpha_e\ \rho_e \tag{2.37}$$

where ρ_c and ρ_h are the epilimnion and hypolimnion densities, respectively.

Since mixing zone 1 acts as if it were an isothermal radial jet a_o can be found from the theory presented in section 2.2 by multiplying with the upflow due to the air inejction Q_{wl} so that

$$(1 + \alpha_o)\ Q_w = \int_o^{z_T} u_r dz \tag{2.38}$$

where the thickness T measured from the surface to the depth z_T is:

$$T = \ell\ \tanh^{-1}\{ \sqrt{1 - \frac{\ell}{H} \tanh\left(\frac{H}{\ell} \right)} \} \tag{2.39}$$

and

$$u_r = \frac{m'}{\ell} [\, \mathrm{sech}^2 \left(\frac{z}{\ell}\right) - \frac{\ell}{H} \tanh \left(\frac{H}{\ell}\right)] \tag{2.40}$$

where ℓ = kr and k = 0.0725. (2.41)

Furthermore it was shown [Goossens, 1979] that

$$m' = 0.69\ k^{0.5}\ Q^{0.35}\ H^{0.5} \tag{2.42}$$

Inserting Eqs. 2.39 to 2.42 into Eq. 2.38 yields an expression for

$$\alpha_o = f(R_o) \tag{2.43}$$

If mixing zone 2 is assumed to be small compared to mixing zone 1, R_o is at the point of transition from nearfield to farfield. If the velocity distribution in mixing zone 1 is represented by (2.40), the balance between return flow and hypolimnion flow is determined by the layer thicknesses

$$\frac{1 + a_o - a_h}{a_h} = \frac{H - T - H_h}{H_h} \tag{2.44}$$

where H_h is the hypolimnion thickness after some period of air injection. Therefore

$$\alpha_1 = \frac{H_h}{H - T}(1 + a\ ) \tag{2.45}$$

A_e can be estimated if the intake of epilimnion water in mixing zone 2 is assumed to depend on the discharge into mixing zone 2. It is assumed that there is a unique relation between the epilimnion velocity v_e and the discharge velocity u_o (Fig. 2.3)

$$v_e = C_1\ u_o \tag{2.46}$$

where U_o is taken as the surface velocity of the discharge determined by (2.40) at z=0. C_1 should be a constant determined by the turbulence in mixing zone 2 and found to be of order 0.2-0.05. Then

$$\alpha_e = \frac{2\pi R_o\ H_e\ V_e}{Q_w} \tag{2.47}$$

where H_e is the epilimnion thickness after some period of air injection.

2.3.2. Model criteria

The limit of the direct mixing region (the stagnation point) can be determined by assuming equilibrium between the forces in mixing zone 1 and the farfield forces, insulting in

$$F^n = F^f \tag{2.48}$$

at $r = R_o$. Mixing zone 2 is then assumed to be infinitesimally small and located at $r = R_o$. Nearfield forces can be represented as

$$F^n = F^n{}_f + F^n{}_p \tag{2.49}$$

where $F^n{}_f$ = mean flow force and $F^n{}_p$ pressure force, and it can be shown that nearfield forces are equal to:

$$F = 2\pi R_o \left(\frac{m'}{kr}\right)^2 \ell \, \overline{\rho} \left(\frac{2}{3} - \frac{\ell}{H}\right) + \pi_1 R_o \rho_o gT^2 + 2\pi R_o \{\rho_o gT$$

$$+ \frac{1}{2} \rho g (H - H_h - T)\} + 2\pi R \{\rho_o gT + \rho_i g(H - H_h - T)$$

$$+ \frac{1}{2} \rho hg \, Hh\} H_h \tag{2.50}$$

The farfield force is

$$F^f = F^f_f + F^f_p \tag{2.51}$$

where

$$F^f_f = 2\pi R_o\left(\rho_e V_e^2 H_e + \rho_i v_i^2 H_i + \rho_h v_h^2 H_h\right) \tag{2.51}$$

and

$$F^f_p = F^f_{po} + F^f_{\Delta p} \tag{2.53}$$

where F^f_{po} is a normal static head

$F_{\Delta p}$ overpressure term (see Fig. 2.5) = $2\pi R_o \Delta\pi H$

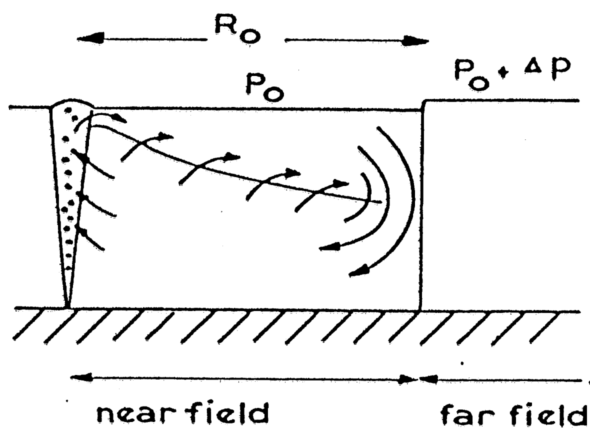

FIG. 2.5. Schematic representation of the nearfield and range of influence R_o (from ref. 1).

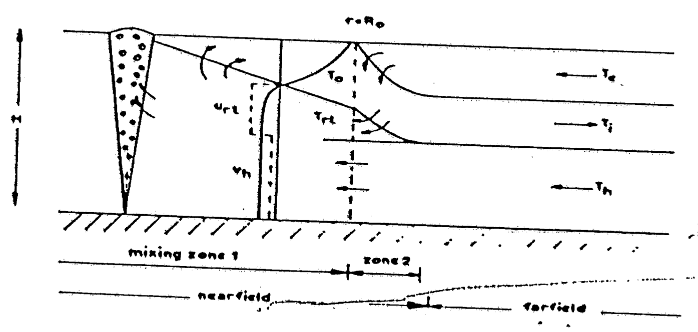

FIG. 2.6. Presentation of the velocities in the nearfield (Ref. 1, Goossens, 1979).

or

$$F^f = 2\pi\ R_o\ \rho_e v_e^2\ H_e + 2\pi\ R_o\ \rho_i v_i^2\ H_i + 2\pi\ R_o\ \rho h\ v_h^2\ H_r + 2\pi R_o\ \Delta pH$$

$$\pi R_o\ \rho_e g\ H_e^2 + 2\pi R_o(\rho_e g H_e + \frac{1}{2}\rho_i g\ H_i)\ H_i + 2\pi R_o(\rho_e g\ H_e$$

$$+ \rho_i g\ H_i + \frac{1}{2}\rho h\ g\ H_i)\ H_h \tag{2.54}$$

Inserting Eqs. 2.50 and 2.54 into Eq. 2.48 allows a solution for the stagnation point. The only unknowns are:

- the average nearfield density $\bar{\rho}$ at R_o to be represented by

$$\rho = \frac{\rho_o T + \rho_i (H - T - H_h) + \rho h\ H_h}{H} \tag{2.55}$$

- the thicknesses of the three layers. If we assume that the mixing layer is located at the position that would result from direct mixing of the appropriate epilimnion and hypolimnion flows, then if one can write

$$H_e = H_{ep} - \frac{\rho_h - \rho_i}{\rho_h - \rho_e} H \tag{2.56}$$

$$H_n = H_{hy} - \frac{\rho_i - \rho_e}{\rho_h - \rho_e} H \tag{2.57}$$

$$He + Hi + H_h = H = H_{cp} + H_{hy} \tag{2.58}$$

where H_{cp} and H_{hy} are the original thicknesses of the two-layered system.

- the velocities in the interlayer and hypolimnion are represented by

$$V_i = \frac{(\alpha_e + \alpha_h) Q_w}{2\pi R_o H_i} \tag{2.59}$$

$$V_h = \frac{a_h Q_w}{2\pi R_o H_h} \tag{2.60}$$

- the overpressure in the farfield can be calculated as

$$\rho_o + \frac{1}{2} \rho_o U_o^2 = \rho_o + \Delta\rho + \frac{1}{2} \rho_e v_e^2$$

yielding

$$\Delta p = \frac{1}{2} \rho_o u_o^2 - \frac{1}{2} \rho_e v_e^2 \tag{2.61}$$

- the thickness of the interlayer H_i which is found under condition that the eplimnion corresponds to the weir and a mixing zone 2 to the hydraulic jump in front of it. Critical flow is present if the following condition is satisfied.

$$F_e^2 F_1^2 + F_i^2 F_b^2 + F_b^2 F_e^2 - \frac{\rho_h - \rho_i}{\rho_h} F_e^2 - \frac{\rho_h - \rho_e}{\rho_h} F_i^2$$

$$- \frac{\rho_i - \rho_e}{\rho_h} F^2 + \frac{(\rho_i - \rho_e)(\rho_h - \rho_i)}{\rho h^2} = \phi \tag{2.62}$$

where

$$F_e = \frac{\rho_h v_e^2}{(\rho_h - \rho_e) g H_e} \tag{2.63}$$

$$F_i = \frac{\rho_h v_i^2}{(\rho_h - \rho_e) g H_i} \tag{2.64}$$

$$F_h = \frac{\rho_h v_h^2}{(\rho_h - \rho_e) g H_h} \tag{2.65}$$

These factors are densimetric Froude numbers. Both Eqs. 2.49 and 2.62 predict results only at a certain c_1 value. From experimental evidence [ ] c_1 is maximal when a_c has a maximum value. It is therefore necessary to investigate which c_1 value will give a_c maximum, and to do the calculations with this fixed value for c_1 (which gives maximum a_c).

A second more complicating feature is the non-uniformity of the flow towards the bubble column, consisting of a return flow velocity from mixing zone 2 u_{rt} and the hypolimnion velocity v_h (Fig. 2.6).

Since the return flow is determined by the critical flow in the interlayer, and the hypolimnion flow more by suction of the bubble column and entrainment in the outflowing jet, these velocities will not necessarily be equal. One can assume

$$u_{rt} = \beta \ v_h \tag{2.66}$$

where β is expected to lie between 1 and 8 and must be determined by experiments. Reference [1]*proposes the value of 4 as the most appropriate because the smaller values give smaller flow rates than measured. In the computer program this is treated through a TO LOOP in which a value of (β) will be chosen which gives maximum d_e (epilimnion flow rate).

2.3.3. Conclusions about the nearfield model

In general the nearfield is treated as a radial jet in a reservoir of finite depth, and under isothermal conditions. The different densities are included through continuity of mass flow in a vertical section through the stagnation point. Additional equations are for the balance of forces of the stagnation point and treatment of the interlayer flow as the critical flow at the stagnation point (the epilimnion is treated as a weir). A schematic representation of the input and output data for the nearfield is given in Fig. 2.7.

As it can be seen in Fig. 2.7, the input values are the initial temperature profile (which is transformed into a two layer system) and the volumetric gas flow rate. Outputs are depths and flow rates of hypolimnion, epilimnion (different from the initial one) and position and flow rate of the new layer (interlayer) which is created due to the air bubble system.

3. FARFIELD

The farfield represents the model for the rest of the lake with information from the nearfield as the boundary condition, as schematically shown in Fig. 3.1. The thermal energy balance in a radially symmetrical system is:

$$\rho \ Cp \frac{\partial T}{\partial t} + \rho \ Cp \ u \ \frac{\partial T}{\partial r} + \rho \ Cp \ u_z \frac{\partial T}{\partial t} = \lambda\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right]$$

$$+ \frac{1}{r}\frac{\partial}{\partial r}\left(r \ \lambda_{vT} \frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(\lambda_{vT} \frac{\partial T}{\partial z}\right) + g(z) \tag{3.1}$$

where t_{1r} and z are time, radial and vertical coordinates, respectively, r = ϕ at the bubble plume, z = ϕ at the free surface, T is the temperature, u_r and u_z are radial and vertical velocity components, ρ is the liquid den-

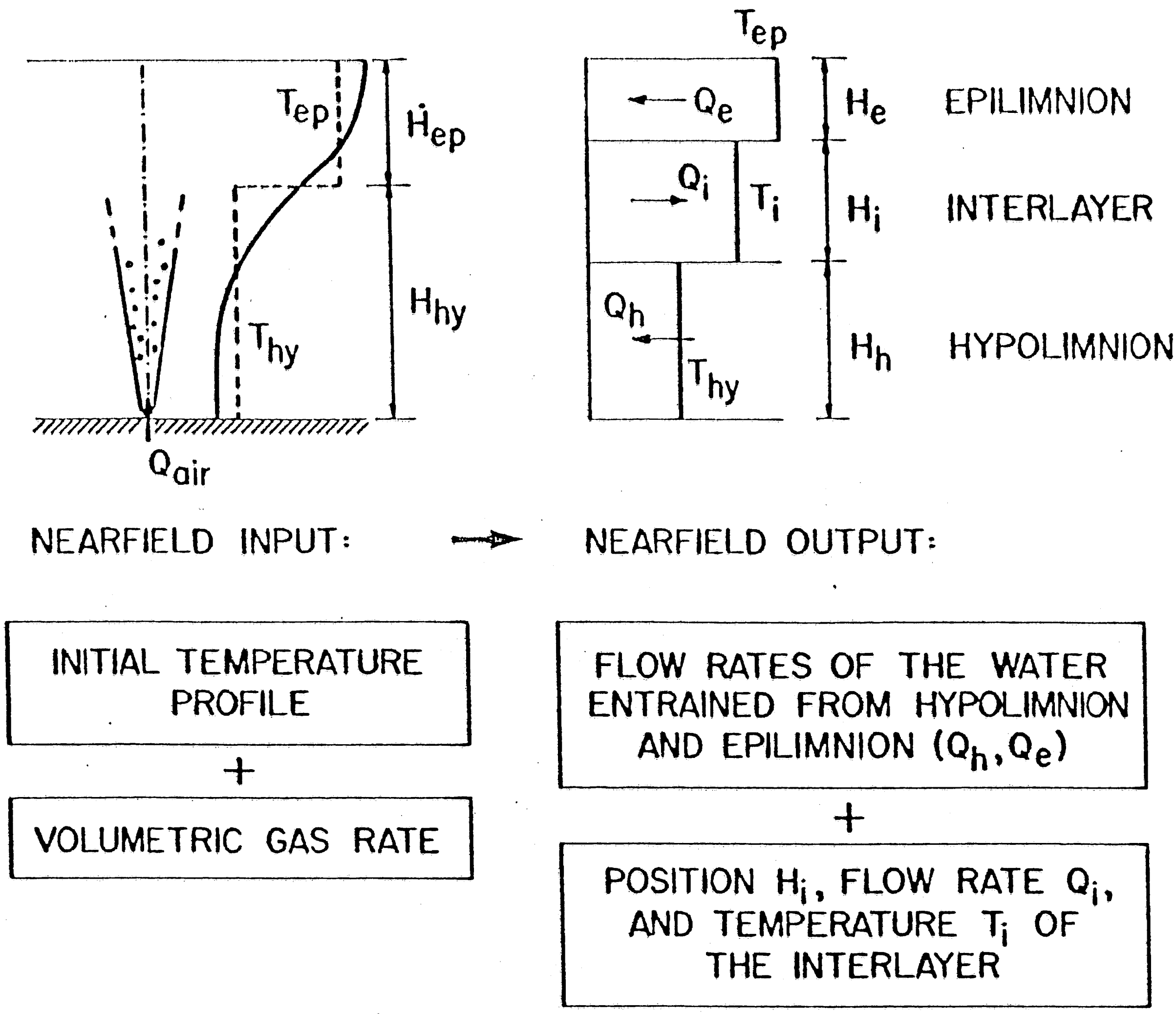

Fig. 2.7. Schematic presentation of the input and output from the nearfield model.

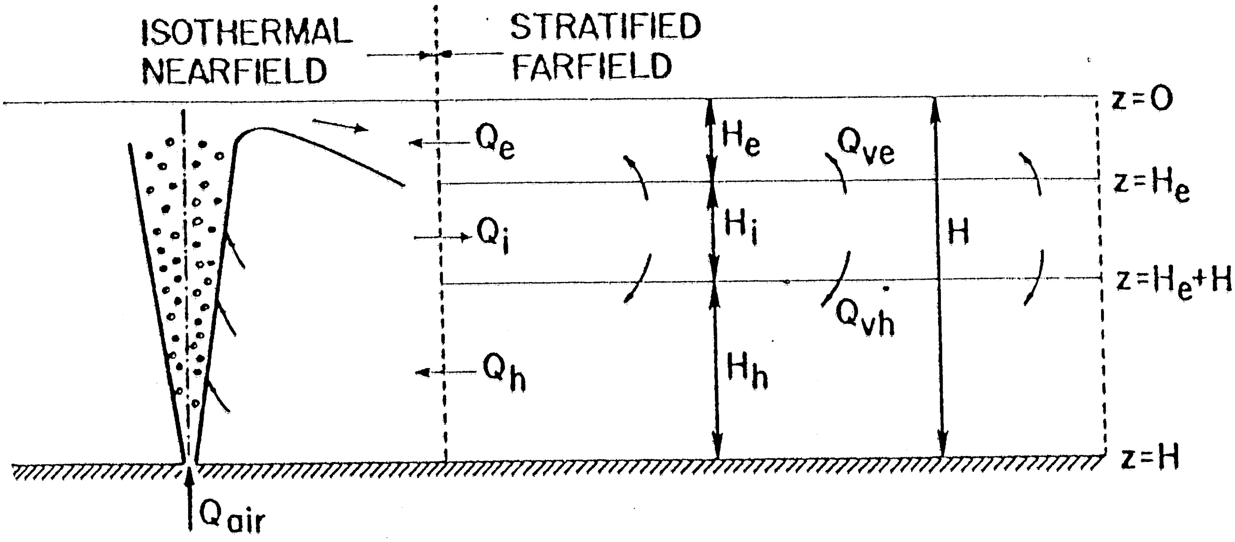

Fig. 3.1. Nearfield and farfield model (after Goossens, 1979).

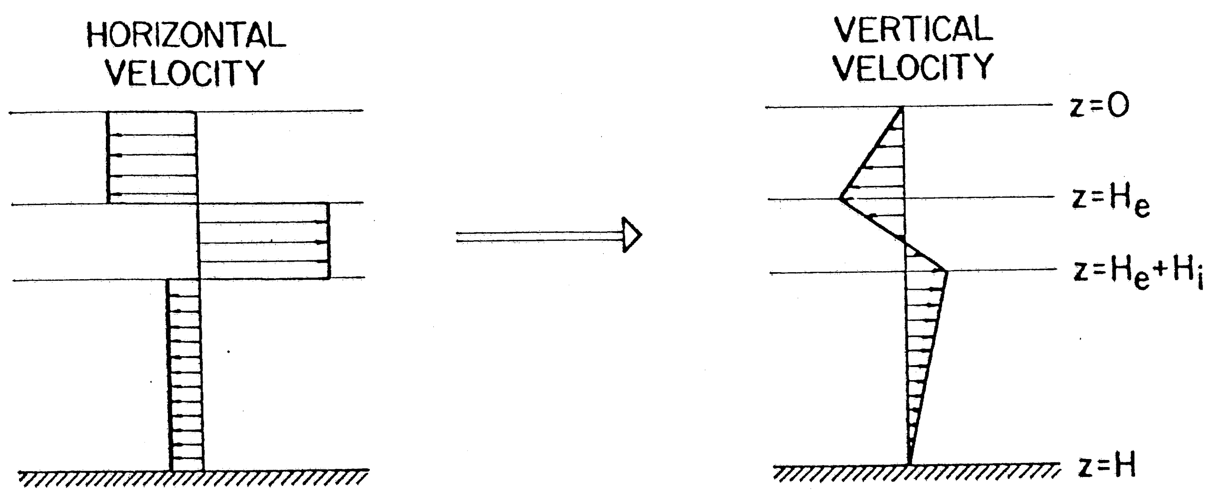

Fig. 3.2. Derivation of vertical velocities (after Goossens, 1979).

sity, cp is the heat capacity per unit mass at constant pressure, λ is the thermal conductivity of water, while λ_{HT}/ρ cp and λ_{VT}/ρ cp are the horizontal and vertical thermal diffisivity and g(z) represents heat generation per unit volume.

For a horizontally homogeneous situation $\partial/\partial r = \phi$ and if the thermal conductivity is neglected when $\lambda_{VT} >> \lambda$ then Eq. 3.1 can be rewritten as

$$\rho \; cp \frac{\partial T}{\partial z} = - \rho \; cp \; u_z \frac{\partial T}{\partial z} + \frac{\partial}{\partial z} \left(\lambda_{vT} \frac{\partial T}{\partial z}\right) + g(z) \tag{3.2}$$

A numerical solution may be obtained if the three terms on the right-hand side of 3.2 are specified and the initial boundary conditions are set.

3.1. Vertical Transport

Figure 3.2 shows the general concept of the farfield model. This is to be as an extension of the nearfield model in Fig. 2.6. The nearfield model predicts the three farfield flows $Q_{e,i,h}$ and their thicknesses $H_{e,i,h}$ at stagnation point Re.

3.1.1. Vertical velocities

The extent of the farfield is determined by the following set of equations (Fig. 3.4) at $r = R_o$.

$$Q_e(r) = Q_e \tag{3.3}$$

$$Q_i(r) = Q_i \tag{3.4}$$

$$Q_h(r) = Q_h \tag{3.5}$$

and at $r = R_1$

$$Q_e(r) = 0 \tag{3.6}$$

$$Q_i(r) = 0 \tag{3.7}$$

$$Q_h(r) = 0 \tag{3.8}$$

Continuity is only preserved if there are two vertical exchange flows from the interlayer into the other two layers denoted by Q_{ve} and Q_{vh}, respectively, so that

$$Q_{re} = \int_{R_o}^{R_1} 2\pi \; r u_z(z_e) dr = Q_e \tag{3.9}$$

$$Q_{vh} = \int_{Re}^{R_1} 2\pi \; r u_z(z_h) dr = Q_h \tag{3.10}$$

at $z = z_e$ and $z = z_h$, respectively.

From the continuity equations applied in horizontal layers, it is possible then to define the vertical velocity profile.

Vertical velocities obtained this way enter into the one-dimensional energy equation 3.2.

3.1.2. Vertical diffusion

Vertical diffusion is incorporated in the main program MINLAKE, but there is some additional shear stress in the flow field induced by the horizontal movement of the three layers with different directions, velocities and densities. The following approach was used to account this effect. The vertical diffusion coefficient is at first defined according to the stability method given in HEC-5Q, Users Manual (U.S. Army Corps of Engineers, 1982), and shown in Fig. 3.3.

Stability E is equal to:

$$E = - \frac{1}{\rho} \frac{d\delta}{dz} \tag{3.11}$$

where δ is density of water. The next step is the definition of an incremental diffusion coefficient in the form

$$D = k' \; k_z \tag{3.12}$$

where k' = coefficient which is calibrated with measured data, and k_z is the original diffusion coefficient.

The coefficient k' in Eq. 3.12 was found equal to approximately 0.05 for some investigated cases.

3.2. Mathematical Model

A numerical model for farfield model applies an explicit finite difference method to solve Eq. (3.2). It should be mentioned here that this model treats only vertical transport and turbulent diffusion. The heat exchange due to the insolation, longwave radiation, convection, evaporation, and fluid mixing are already incorporated in computer programs developed previously.

The reservoir depth is divided into "MBOT' intervals of thicknesses (DZ(I). The notation used corresponds to that used in the program MINLAKE [6]. The time base is divided into equal Δt steps and k = 0 corresponds with $t = \phi$.

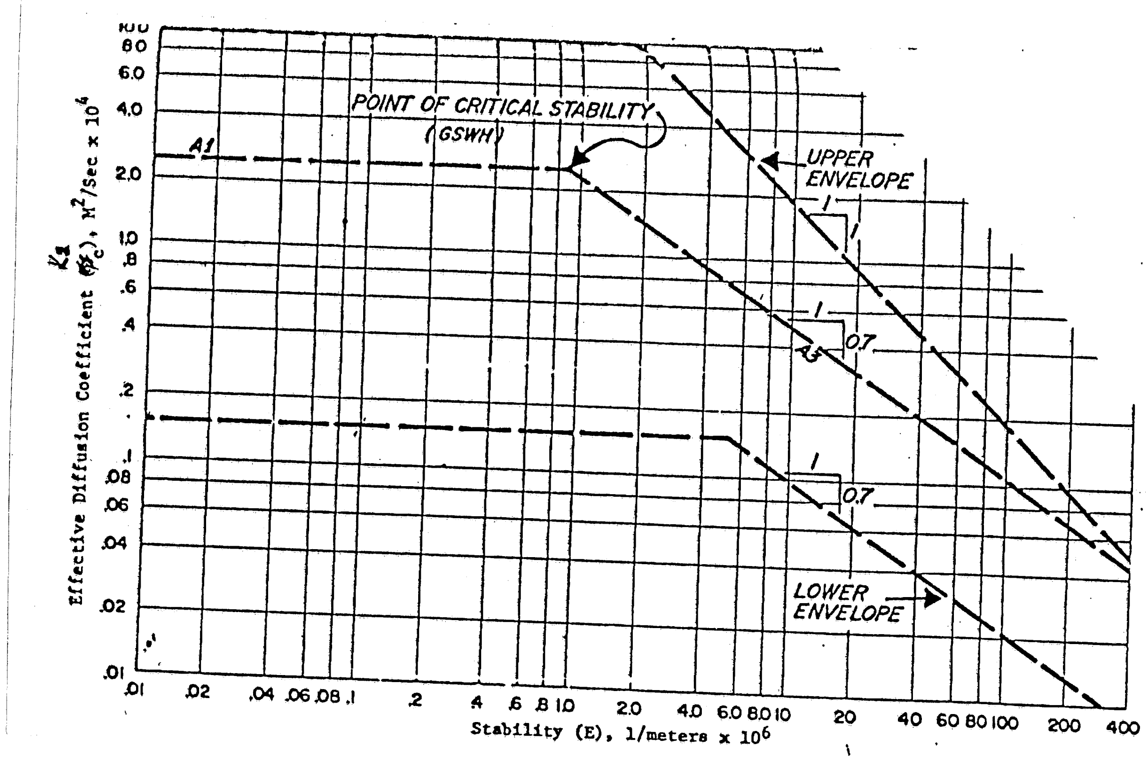

FIG. 3.3. Stability method.

The backward difference in space are represented by

$$\frac{dT}{dz} \sim \frac{\Delta T}{\Delta z} = \frac{T_{z,k} - T_{j-1,k}}{\Delta z} \tag{3.13}$$

and the forward difference in time is represented by

$$\frac{dT}{dt} \sim \frac{\Delta T}{\Delta t} = \frac{T_{j,k+1} - T_{j,k}}{\Delta t} \tag{3.14}$$

where j subscript corresponds to the layer, and k subscript to the time step. With Eqs. 3.13 and 3.14, Eq. 3.2 can be rewritten as (just for vertical transport).

$$T_{j,k+1} = T_{j,k} - u_{zj} \frac{T_{j,k} - T_{j-1,k}}{\Delta z_j} \Delta t \tag{3.15}$$

The derivation of transport terms in finite difference form for different situations depending on the position of the layer is shown in Appendix B.

4. COMPUTER PROGRAM

On the basis of the theory presented in Chapters 2 and 3 a computer program is developed. The program is developed on the basis of two computer programs given in Ref. [1]* and represents a subroutine for the MINLAKE program mentioned previously.

The listing of the subroutine called BUBBLES is given in Appendix E. Subroutine BUBBLES is a part of the subroutine USER which is a subroutine of the MINLAKE program and which gives the opportunity to the user of the program to include some specific requirement apart from those which MINLAKE already treats. Such a specific requirement is a calculation of destratification of the lake due to the air bubble column. USER is suppose to give the following information:

1. H-depth of the lake (m)

2. RH - depth of the diffuser (m)

3. QAIR - volumetric gas rate at atmospheric pressure (m^3/s)

The geometry of the lake, initial conditions and meteorological data are normally treated through the MINLAKE program. A schematic flow chart for the revision of the MINLAKE program which includes subroutine BUBBLES (aeration dueto air bubbles column) is shown in Fig. 4.1.

([Goossens, 1979, see reference page.])

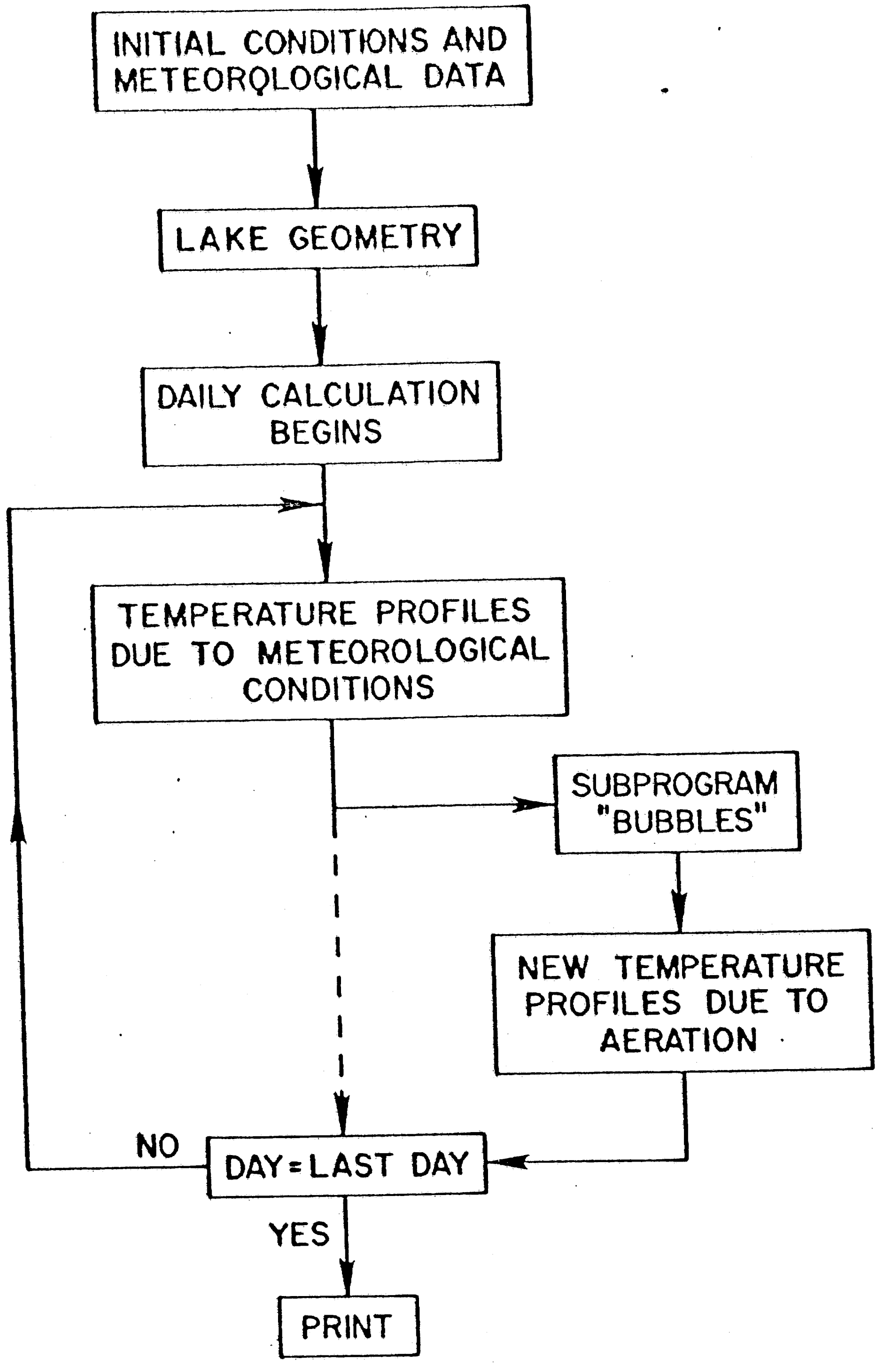

Fig. 4.1 Flow chart of the MINLAKE program with subroutine BUBBLES.

Table 5.2

SURFACE AREA [km²]	MEAN DEPTH [m]	MAXIMUM DEPTH	VOLUME [x10⁶ m³]
1.71	10.0	27.4	17.1

5. APPLICATION

The computer program is applied to two lakes. The first one is Petrusplaat (Netherlands) for which the basic two computer programs for Ref. [1] are developed. The second case is Lake Calhoun (Minneapolis, Minnesota).

5.1. Petrusplaat

Reservoir Petrusplaat is situated in the fourth-west port of The Netherlands. Basic data are shown in Table 5.1.

TABLE 5.1

Depth (m)	Surface Area (km^2)	Volume (m^3)
15.0	1.05	13.0×10^6

Results of the simulation of the artificial mixing of the reservoir by air column are shown in Fig. 5.1. As can be seen the comparison between measured and calculated data is reasonably good. Mean values of absolute difference between measured and calculated temperatures are shown for each temperature profile.

5.2 Lake Calhoun

Lake Calhoun is shown in Fig. 5.2. Basic data of the lake are shown in Table 5.2. The purpose of the air bubble system was "to determine whether artificial circulation of the lake could change the algal populations from primarily bluegreens to greens and thus improve the appearance of the lake as well as improve conditions for fish.." [cited from (from Shapiro and Pfannkuch, 1974).

TABLE 5.2

Surface Area (km^2)	Mean Depth (m)	Maximum Depth	Volume ($x10^6$ m^3)
1.71	10.0	27.4	17.1

Operation was begun August 4, 1971. An estimated 100-125 cfm of air were used (0.047 - 0.059 m^3 s). The aerator feet of 2-inch pvc perforated tube. Results of numerical simulation are shown in Fig. 5.3.

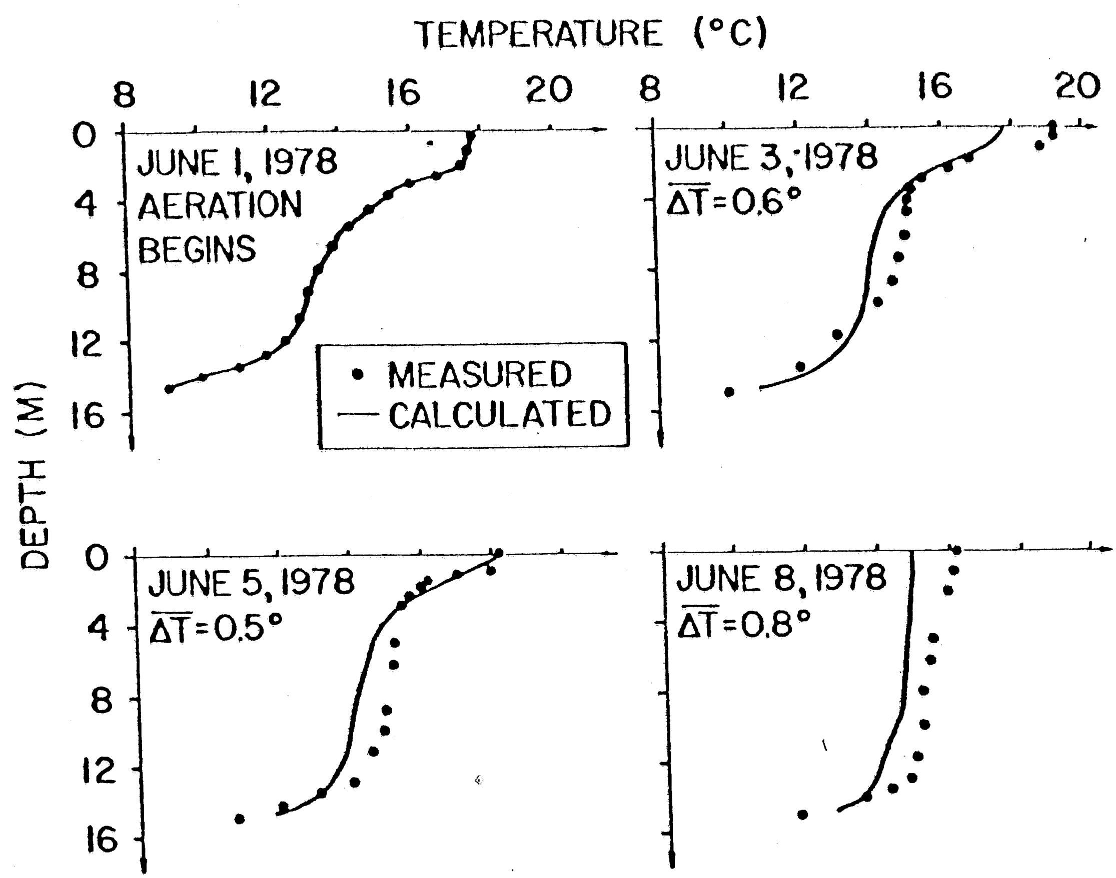

Fig. 5.1.Development of destratification of the reservoir Petrusplaat (data from Goossens, 1979).

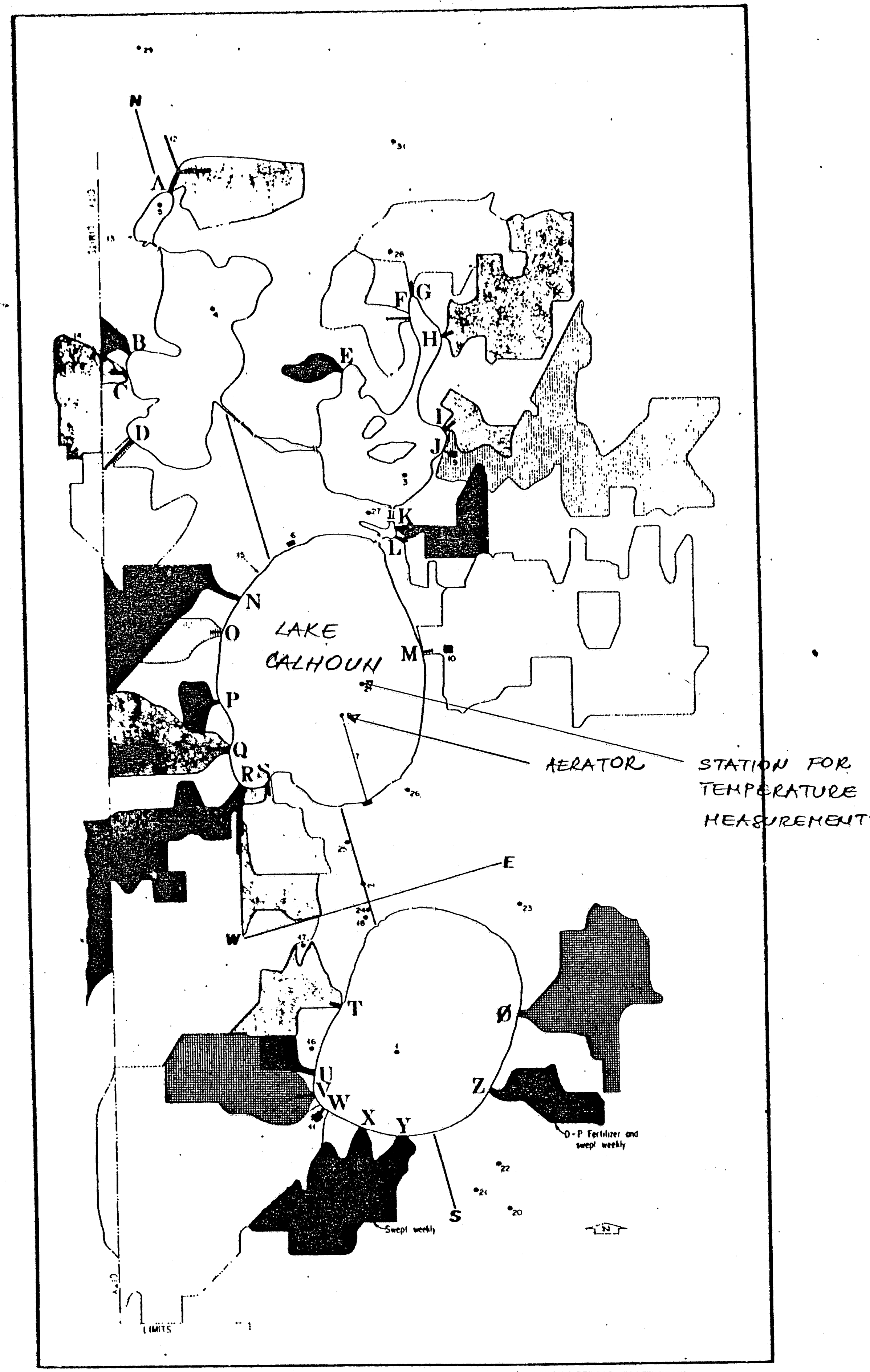

FIG. 5.2. (from Shapiro et al., 1974).

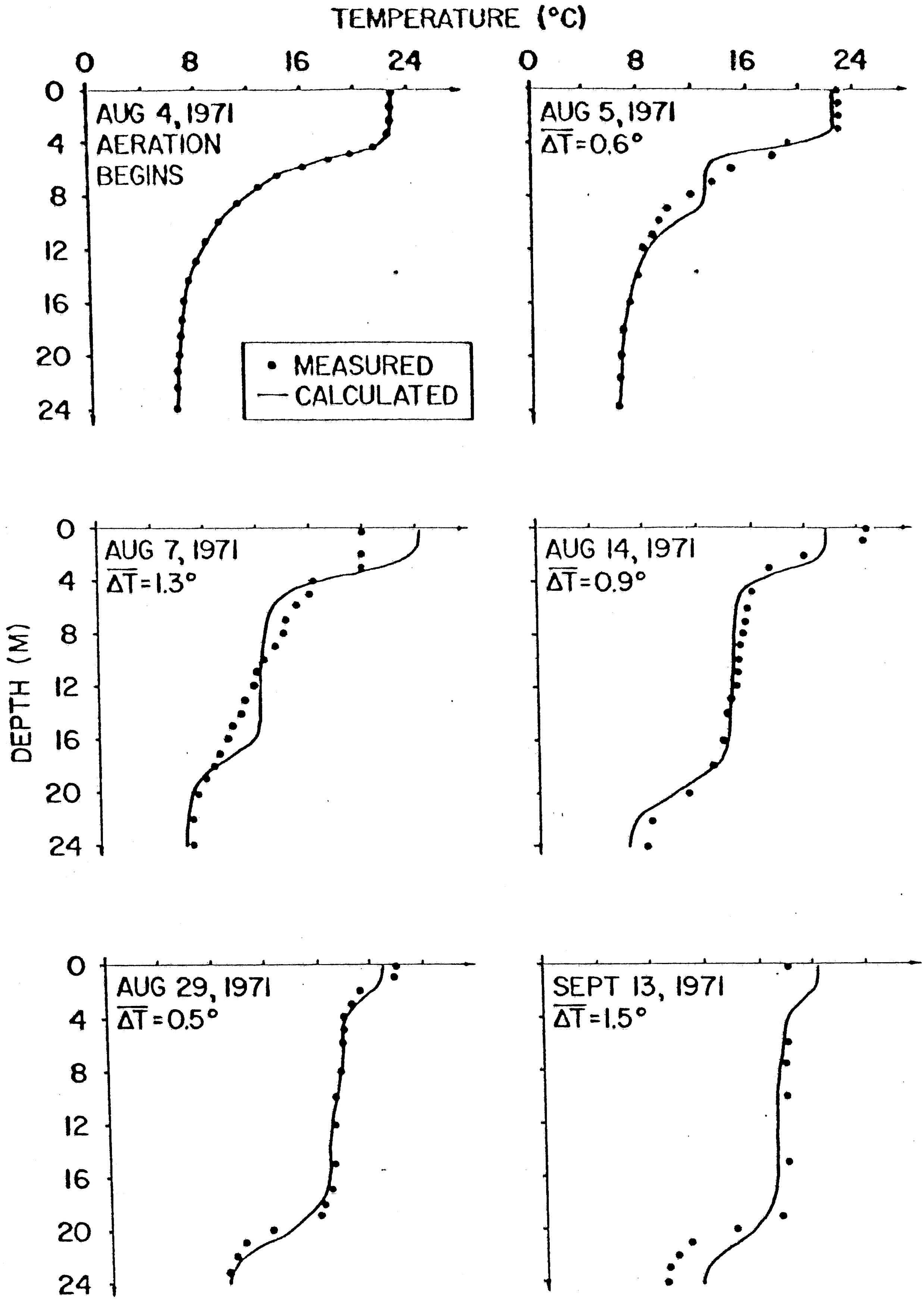

Fig. 5.3. Development of destratification of Lake Calhoun (data from Shapiro and Pfannkuch, 1973).

It can be seen that the agreement is good. It should be noted that it was not possible to find very accurate data about the operation itself. It is therefore assumed in the calculations that the operation ran all the time continuously with a value of 0.059 m^3/s. For more accurate calculations, more accurate data about the operation are needed. In calculations the following values were used. C_1(Eq. 2.46) = 0.05; factor to multiply the diffusion coefficient (Chapt. 3.12) FCT = 0.05.

6. CONCLUSIONS

The mathematical model simulating the progressive destratification of lake by an air bubble curtain has been developed. It consists of two parts:

1. Nearfield which is treated as isothermal and which determines flow rates of the water entrained from the epilimnion and the hypolimnion. An interlayer flow is created.

2. The farfield uses information on flow rates in those three layers (epilimnion, interlayer and hypolimnion), calculates vertical velocities and through a heat balance for each layer calculates a new temperature distribution.

The model is applied to lakes with depths of 15-27 m and with air flow rates of 0.05 - 0.17 m^3/s. Agreement between measured and calculated values is found to be good.

In reference [1] *the proposed method of interaction between near and farfield is that the nearfield is calculated just one at the beginning with the assumption that the process will remain the same during the whole period of destratification. In the model presented here another approach is used. The nearfield is recalculated every day; numerical predictions of temperature profiles are given on a daily basis.

It is found that values for the c_1 coefficient (ratio between surface velocity and epilimnion velocity given in Eq. 2.66) should be in the range of 0.03-0.05.

Numerical results show good agreement with measurements in both investigated cases. In order to get more experience with the behavior of the numerical predictions, it will be necessary to apply this model to some other lakes or reservoirs and to compare simulations with measurements.

(Goossens, 1979; see reference list).

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APPENDIX A

a	mean cross-sectional area occupied by gas (m^2)
b	width
c_p	heat capacity per unit mass at constant pressure (1/kg°C)
c_1	epilimnion-outflow velocity ratio (-)
d	equivalent plume diameter (m)
d_{eff}	equivalent effective plume diameter (m)
d_s	diameter of control surface (m)
D	incremental diffusion coefficient (m^2/s)
E	stability (1/m)
F^f	farfield force (μ)
F^n	nearfield force (μ)
Fe	densimetric epilimnion Froude number (-)
F_f^f	mean flow force of farfield
F_f^n	mean flow force of nearfield
F_h	densimetric hypolimnion Froude number (-)
F_i	densimetric interlayer Froude number (-)
F_p^f	farfield pressure impact (μ)
F_p^n	nearfield pressure impact (μ)
$F_{\Delta p}^p$	farfield overpressure impact (μ)
F_{po}^f	farfield pressure impact based on ρ_o (μ)
g	acceleration due to gravity (m/s^2)
G_o	gas injection rate (m^3/s)
H	liquid height above injector (m)

He	epilimnion thickness (m)
H_{ep}	original epilimnion thickness (m)
H_h	hypolimnion thickness (m)
H_{hy}	original hypolimnion thickness (m)
H_i	interlayer thickness (m)
k	jet spreading coefficient (-)
k'	coefficient (-)
K	parameter defined by equation (-)
K_1	entrainment coefficient (-)
K_z	diffusion coefficient (m^2/s)
ℓ	characteristic vertical length scale (m)
m	liquid mass flux (kg/s)
m'	momentum coefficient (-)
M	liquid momentum flux (μ)
$\overline{M}$	mean liquid momentum flux (μ)
M'	fluctuating liquid momentum flux (μ)
p	pressure (μ/m^2)
Δp	overpressure (μ/m^2)
p_{otm}	atmospheric pressure (μ/m^2)
ρ_o	corrected free suface pressure (μ/m^2)
g(z)	heat generation per unit volume (μ/m^3)
gh	heat transport (m)
Q	volumetric gas rate at atmospheric pressure (m^3/s)
Qe	equilibrium flow at $r = R_o$ (m^3/s)
Qe(r)	epilimnion flow (m^3/s)
Q_h	hypolimnion flow at $r = R_o$ (m^3/s)
$Q_h(r)$	hypolimnion flow (m^3/s)

Q_i	interlayer flow at $r = R_o$ (m^3/s)
$Q_i(r)$	interlayer flow (m^3/s)
Q_{rt}	return flow at $r = R_o$ (m^3/s)
Q_{ve}	vertical flow at epilimnion-interlayer transition (m^3/s)
Q_{vh}	vertical flow at hypolimnion-interlayer transition (m^3/s)
Q_w	flow from bubble column (m^3/s)
Q_x	volumetric flow (m^3/s)
r	radial coordinate ($r=0$ at bubble column centre) (m)
R_o	stagnation point (m)
R_1	equivalent farfield radius (m)
t	temperature (°C)
T	thickness of outflowing stream (m)
$T_{i,k}$	temperature of the layer i in time k (°C)
ΔT	temperature difference (°C)
T_e	epilimnion temperature (°C)
T_{ep}	original epilimnion temperature (°C)
T_h	hypolimnion temperature (°C)
T_{hy}	original hypolimnion temperature (°C)
T_i	interlayer temperature (°C)
u	(horizontal) velocity component (m/s)
u_o	free surface velocity at stagnation point (m/s)
u_r	radial velocity component (m/s)
u_t	return flow velocity at stagnation point (m/s)
u_z	vertical velocity component (m/s)
v	equivalent plume velocity (m/s)
v_e	epilimnion velocity (m/s)
v_G	gas velocity (m/s)

v_h	hypolimnion velocity (m/s)
v_i	interlayer velocity (m/s)
v_r	relative bubble velocity (m/s)
w	vertical velocity (m/s)
$\overline{w}$	mean vertical velocity (m/s)
w'	fluctuating vertical velocity (m/s)
W_m	mean centre line velocity (m/s)
z	vertical coordinate (z=0 at free surface except in chapter 2.1 where z=0 at 10.2 m height above the free surface (m)
Δz	vertical interval (m)
a_{atm}	water column corresponding to 1_{atm} (m)
z_e	position of epilimnion-interlayer transition (m)
z_h	position of hypolimnion-interlayer transition (m)
z_i	position in interlayer where u_z=0 (m)
z_T	position indicating the thickness of the outflowing stream (m)
z_o	position of injector (m)
α	normalized flow defined at $r = R_o$ (-)
α_e	normalized epilimnion flow (-)
α_o	normalized outflow (-)
α_{rt}	normalized return flow (-)
α_x	normalized flow defined by Eq. 2.34 (-)
β	constant defined by Eq. 2.66
λ	thermal conductivity (w/m°C)
$\frac{\lambda HT}{\rho C_p}$	horizontal thermal diffusivity (m^2/s)
$\frac{\lambda vT}{\rho C_p}$	vertical thermal diffusivity (m^2/s)
ρ	(liquid) density (kg/m^3)
$\overline{\rho}$	density defined by Eq. 2.55 (kg/m^3)

ρ_a air density (kg/m^3)

ρ_e epilimnion density (kg/m^3)

ρ_G gas density (kg/m^3)

ρ_{Go} gas density at the injector (kg/m^3)

ρh hypolimnion density (kg/m^3)

ρ_i interlayer density (kg/m^3)

ρ_o outflow density (kg/m^3)

APPENDIX B

Descriptive List of Variables Used in Subroutine Bubbles

A	parameter in DO loop to determine HI
AA	incremental step of A
ATOP	area of layer with index I in subroutine bubbles
C1	epilimnion-outflow velocity ratio
CC1	temporary value to calculate RW
CC11	temporary value to calculate RW
CC12	temporary value to calculate RW
CC13	temporary value to calculate RW
CC14	temporary value to calculate RW
CC2	temporary value
CON1	temporary value
CON2	temporary value
CRAZ	array which controls the accuracy in the computation of the variable RHOI
DEL	controls the accuracy in the computation of the variable RFARF
DRHEH	temporary value
DRHG	temporary value
DRHH	temporary value
DRHOHI	temporary value
DRHOIE	temporary value
DRW	controls the accuracy in the computation of the variable RW
DT	temperature difference

DTOU	time step in FARF
DUM	temporary value
DZ	array width differences in depths
DPS	temporary value of stability parameter
EPS1	stability parameter for density difference between epilimnion and interlay.
EPS2	stability parameter for density difference between interlayer and hypolimnion
EPSCRT	limiting value of the stability parameter
FCT	coefficient which defines the value of incremental diffusion coefficient
FFC	arbitrary parameter
G	acceleration due to gravity
GR	array of vertical temperature gradients
GRMX	maximum temperature gradient
H	total depth of the lake
HA	depth at which the hypolimnion starts (measured from surface)
HB	depth at which the interlayer starts (measured from surface)
HEP	epilimnion thickness
HEPO	original epilimnion thickness
HEPOLD	temporary value of epilimnion thickness
HH	temporary value
HHY	hypolimnion thickness
HHYO	original hypolimnion thickness
HIN	interlayer thickness
HINOLD	temporary interlayer thickness
HMK	array of diffusion coefficients
I	DO LOOP index
IA	index which corresponds to HA

IB	index which corresponds to HB
IDIF	index which corresponds to the depth of the aerator
IDIF1	temporary value
IMAX	index which corresponds to the depth with GRMAX
IZ	index which corresponds to the depth with zero vertical velocity
J	DO LOOP Index
JCS	step counter
K	DO LOOP index
KCON	step counter
KCS	step counter
KDRW	array which controls the accuracy in the computations of the variable RW
KHI	temporary value
KK	DO LOOP index
KKCON	step counter
N	number of layers in the FARF subroutine
N1	N-1
NTS	number of time steps in the
PI	π
QAIR	volumetric gas rate at atmospheric pressure
QEO	epilimnion flow
QE01	temporary value of epilimnion flow
QEOOLD	temporary value of epilimnion flow
QHO	hypolimnion flow
QHOOLD	temporary value of hypolimnion flow
QIO	interlayer flow
QW	flow from bubble column at the surface
RAE	normalized epilimnion flow

RAEAH	temporary value
RAH	normalized hypolimnion flow
RAO	normalized outflow
RAO1	temporary value
RAZ	temporary value
RAZ1	temporary value
RB	constant defined by Eq. 2.66
RB1	temporary value at RB
RCON	temporary value
RDP	temporary value
RFARB	temporary value of the residue of the Eq. 2.62
RFARF	temporary value of the residue of the Eq. 2.62
RFARF1	temporary value to get RFARF
RFARF2	temporary value to get RFARF
RFARF3	temporary value to get RFARF
RFARF4	temporary value to get RFARF
RFARF5	temporary value to get RFARF
RFE	temporary value to get RFARF
RFH	temporary value
RFKRI	resolution of HIN
RFM	densimetric interlayer Froude number
RH	depth of the aerator
RHH	hypolimnion thickness
RHO1	temporary value of density
RHO2	temporary value of density
RHO3	temporary value of density
RHOE	density of the epilimnion

RHOH	density of the hypolimnion
RHOI	density of the interlayer
RHOIOL	temporary value of density
RIA	mean flow force of nearfield
RK	jet spreading coefficient
RKRIT	resolution of farfield equation
RM	momentum coefficient
RO	radius of the stagnation point
RO2PI	temporary value
ROOLD	temporary value
ROP12	temporary value
RPEEN	temporary value
RPTWE	temporary value
RRHOO	temporary value
RRKRI	resolution of RO-LOOP
RRR	temporary value
RSTEP	incremental step in RO LOOP
RSTEP1	temporary value of RSTEP
RUR	characteristic velocity
RURO	free surface velocity at stagnation point
RURR	radial velocity component
RURT	return flow velocity at stagnation point
RVE	epilimnion velocity
RVH	hypolimnion velocity
RVM	interlayer velocity
RW	force balance
RZO	thickness of the outflowing stream

RZZ	temporary value
SM1	temporary sum
SM2	temporary sum
SUM1	temporary sum
SUM2	temporary sum
T	temperature array in BUBBLES
T12	coefficient
T2	temperature array in MINLAKE
T22	coefficient
TANHRH	temporary value
TANHRZ	temporary value
TEP	temperature of the epilimnion
THY	temperature of the hypolimnion
TI	temperature of the interlayer
TIOLD	temporary value of Tl
TM1	temporary value
TM2	temporary value
TM3	temporary value
TP1	temporary value
TT	temporary value
UH	temporary value of horizontal velocity
UZ	vertical velocities
X	temporary value
XA	temporary array
Y	temporary value of RW
Y2	temporary value
YA	temporary array

YP1	temporary value
YPN	temporary value
YY	temporary value
Z	depth (z = 0 at surface)
ZI	depth at which vertical velocity is equal to zero

APPENDIX C

Descriptive List of Subroutine and Functions Called by BUBBLES

CHANGE	transforms temperature array T2(I) from MINLAKE into temperature array T2(I,2) used in BUBBLES
COEF	computer coefficients used in finite differences formulations
DISOLID	computes the concentration profile of dissolved substances
E1	computer part of heat transport in farfield
E11	computer part of heat transport in farfield
FARF	computes temperature distribution in farfield
FUN	computes temperature from density
IAB	computes index of the layer with specific depth
INP	computes input values for NEAR
NEAR	computes nearfield
NEWT	computes diffusion coefficients in farfield
RHO	function in the MINLAKE which calculates density as a function of temperature
SPLINE	interpolation and extrapolastion by spline function
SPLINT	computer coefficients for SPLINE
TEMP2	transforms temperature array T2(I,2) used by BUBBLES into temperature array T(I) used by MINLAKE
TINT	computer influence of the nearfield into farfield
VERVE	computes vertical velocities profile

APPENDIX D

Deviation of the heat transport equations for various layers in the stratified farfield. The derivation of the equation is based on the notation shown in Fig. D.1.

As one can see from Fig. D.1 there is a difference in the position for which temperatures are defined. In the MINLAKE program all the temperatures are calculated for the middle point of each layer and the subroutine BUBBLES was more convenient to define them on the way shown on Fig. D.1. The following equations are derived based on the BUBBLES notation.

Fig. D.2 shows a notation used for a control volume of the layer situated in epilimnion, where $T_{j,k}$ is the temperature at the time step k, Q_j vertical flow rate; Q_e horizontal epilimnion flow rate, Δz_i the thickness of the layer; $ATOP_j$ = surface of the layer.

On the basis of previously defined notation and equations shown in Section 3.2 for temperature of the layer j in the time step k+1 can be written that

$$T_{j,k+1} = T_{j,k} + \frac{1}{\rho\ Cp} \frac{2tr_j}{ATOP_j\ \Delta z_j} \Delta t \tag{D.1}$$

where 2trj represents the heat gain in the control volume due to the heat transport; Δt = time step.

Depending on the position of the layer j different situations with horizontal and vertical flow rates can occur and therefore different 2trj. As all the variables on the right-hand side of Eq. D.1 are for the time step k, the subscript k is dropped in the following equations.

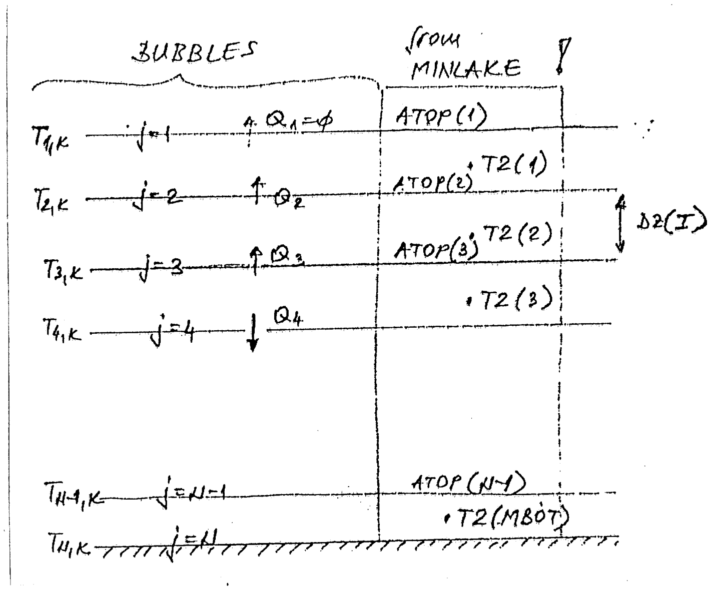

FIG. D.1. Notation used in derivation of the farfield equations.

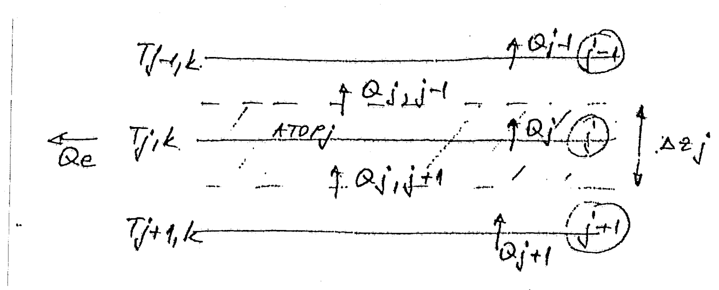

FIG. D.2. Control volume for layer j in epilimnion.

SURFACE

Heat transport balance:

$$2tr = \rho Cp\ (Q_{1-2}\ T_{1-2} - Q_e\ T_1) \tag{D.2}$$

Continuity:

$$Q_{1-2} = Q_e \tag{D.3}$$

$$Q_{1-2} = \frac{Q_1 + Q_2}{2} = \frac{Q + Q_2}{2} = \frac{Q_2}{2} \tag{D.4}$$

$$T_{1-2} = \frac{T_1 + T_2}{2} \tag{D.5}$$

$$2tr = \rho\ Cp\ \frac{1}{4}\ Q_2(T_2 - T_1) \tag{D.6}$$

EPILIMNION

Heat Transport Balance:

$$2tr = \rho Cp \left(Q\ j + \frac{1}{2}\ Tj + \frac{1}{2}\ h - Q\ j - \frac{1}{2}\ Tj - \frac{1}{2},\ k - Q_{e,j}\ T_{jk}\right) \qquad (D.7)$$

Continuity:

$$Q\ j + \frac{1}{2} = Q\ j - \frac{1}{2} + Q_{e,j} \qquad (D.8)$$

as:

$$Q\ j + \frac{1}{2} = \frac{Q_{j+h} + Q_j}{2} \qquad \ldots\ (D.8)$$

$$Q\ j - \frac{1}{2} = \frac{Q_{j-1} + Q_j}{2} \qquad (D.9)$$

$$Q_{e,j} = \frac{Q_{j+1} - Q_{j-1}}{2} \qquad (D.10)$$

$$T\ j + \frac{1}{2},k = \frac{T_{j+1,k} + T_{j,k}}{2} \qquad (D.11)$$

$$T\ j - \frac{1}{2},k = \frac{T_{j-1,k} + T_{j,k}}{2} \qquad (D.12)$$

$$2tr = \frac{1}{4}\rho\ Cp\left[Q_{r,j} + 1(T_{j+1,k} - T_{j,k}) + Q_{r,j}(T_{j,1,k} - T_{j,1,k}) + Q_{r,j-1}(T_{j,k} - T_{j-1,k})\right] \qquad (D.13)$$

TRANSITION BETWEEN EPILIMNION AND INTERLAYER

Heat Transport Balance:

$$2tr = \rho Cp \left(Q\, j + \frac{1}{2} Tj + \frac{1}{2} h - Q_i\, T_i - Q\, j - \frac{1}{2} T\, j - \frac{1}{2} \right) \tag{D.14}$$

Continuity:

$$Q'_i \ + Q\, j + \frac{1}{2} = Q_j + \quad Q\, i - \frac{1}{2} (Q_j - Q_{j+1}) \tag{D.15}$$

$$Q'_e + Q\, j - \frac{1}{2} = Q_j + \quad Q'_e = \frac{1}{2} (Q_j - Q_{j-1}) \tag{D.16}$$

$$2tr = \rho\ Cp \frac{1}{4} [Q_{j+1}(T_{j+1} - T_j) + Q_j(T_{j+1} - T_{j-1}) + Q_{j-1}(T_j - T_{j-1})]$$

$$+ \rho\ Cp \frac{1}{2} (T_j - T_i)(Q_{j+1} - Q_j) \tag{D.17}$$

where

T_i - interlayer temperature

INTERLAYER-UPPER PART

Heat transport balance

$$2tr = \rho\ Cp\ \left(Q\ j + \frac{1}{2}\ T\ j + \frac{1}{2} + Q'\ \ T\ - Q\ j - \frac{1}{2}\ T\ j - \frac{1}{2}\right) \tag{D.18}$$

Continuity:

$$Q\ j - \frac{1}{2} = Q_i' + Q\ j + \frac{1}{2} \tag{D.19}$$

$$Q_i = \frac{Q_{j-1} - Q_{j+1}}{2} \tag{D.20}$$

$$2tr = \rho Cp\ \frac{1}{4}\left[Q_{j+1}(T_{j+1} - T_j) + Q_j(T_{j+1} - T_{j-1}) + Q_{j-1}(T_j - T_{j-1})\right]$$

$$+ \rho\ Cp\ \frac{1}{2}\left(Q_{j+1} - Q_{j-1}\right)(T_j - T_i) \tag{D.20}$$

LAYER IN THE INTERLAYER WHERE $Q_r = \phi$

Heat transport balance:

$$2tr = \rho\ Cp\ \left(Q_i'\ T_i - Q\ j + \frac{1}{2}\ T\ j + \frac{1}{2} - Q\ j - \frac{1}{2}\ T\ j - \frac{1}{2}\right) \tag{D.21}$$

Continuity:

$$Q_i' = Q\ \ j + \frac{1}{2} + Q\ j - \frac{1}{2}\ \rightarrow\ Q_i' = \frac{Q_{j+1} + Q_{j-1}}{2}\ \ /Q_j = \phi/ \tag{D.22}$$

$$2tr = \frac{1}{4}\rho\ Cp\left[Q_{j+1}(T_{j+1} - T_j) + Q_j(T_{j+1} - T_{j-1}) + Q_{j-1}(T_j - T_{j-1})\right]$$

$$+ \frac{1}{2}\rho\ Cp\left[2\ Q_{j+1}(T_i - T_{j+1}) - Q_j(T_j + T_{j+1}) + Q_{j-1}(T_i - T_j)\right] \tag{D.23}$$

INTERLAYER-LOWER PART

Heat transport balance:

$$2tr = \rho\ Cp\ (Q_i T_i + Q\ j - \frac{1}{2}\ T\ j - \frac{1}{2} - Q\ j + \frac{1}{2}\ T\ j + \frac{1}{2} \tag{D.24}$$

Continuity:

$$Q'_i + Q\ j - \frac{1}{2} = Q\ j + \frac{1}{2} \tag{D.25}$$

$$Q'_i = \frac{Q_{j+1} - Q_{j-1}}{2} \tag{D.26}$$

$$2tr = \frac{1}{4}\rho\ Cp\ [Q_{j+1}(T_j - T_{j+1}) + Q_j(T_{j-1} - T_{j+1}) + Q_{j-1}(T_{j-1} - T_j)]$$

$$+ \frac{1}{2}\rho\ Cp\ (Q_{j+1} - Q_{j-1})(T_i - T_j) \tag{D.27}$$

TRANSITION BETWEEN INTERLAYER AND HYPOLIMNION

Heat transport balance:

$$2tr = \rho \ Cp \ (Q_i' \ T_i + Q \ j - \frac{1}{2} T \ j - \frac{1}{2} - Q_h' \ T_j - Q \ j + \frac{1}{2} T \ j + \frac{1}{2}) \quad (D.28)$$

Continuity:

$$Q_h' + Q \ j + \frac{1}{2} = Q_j \rightarrow Q_h' = \frac{1}{2} (Q_j - Q_{j+1}) \quad (D.29)$$

$$Q_i' = Q \ j - \frac{1}{2} = Q_j \rightarrow Q_i' = \frac{1}{2} (Q_j - Q_{j-1}) \quad (D.30)$$

$$2tr = \frac{1}{4} \rho \ Cp[Q_{j+1}(T_j - T_{j+1}) + Q_j(T_{j-1} - T_{j+1}) + Q_{j-1}(T_{j-1} - T_j)]$$

$$+ \frac{1}{2} \rho \ Cp(Q_j - Q_{j-1})(T_i - T_j) \quad (D.31)$$

HYPOLIMNION

Heat transport balance:

$$2tr = \rho\ Cp\ (Q\ j - \frac{1}{2}\ T\ j - \frac{1}{2} - Q'h\ T_j - Q\ j + \frac{1}{2}\ T\ j + \frac{1}{2}) \tag{D.32}$$

Continuity:

$$Q\ j - \frac{1}{2} = Q' \ + Q\ j + \frac{1}{2} \tag{D.33}$$

$$Q'h = \frac{Q_{j-1} - Q_{j+1}}{2} \tag{D.34}$$

$$2tr = \frac{1}{4}\rho\ Cp[Qv_{j+1}(T_{j,k} - T_{j+1}) + Q_{vj}(T_{j-1,k} - T_{j,1,k}) + Q_{v,j-1}(T_{j-1} - T_j)] \tag{D.35}$$

BOTTOM

Head transport balance:

$$2tr = \rho \ Cp[Q_{NN-1} \ T_{NN-1} - Q'_h \ T_N] \tag{D.36}$$

Continuity:

$$Q_{NN-1} = Q'_h = \frac{1}{2} Q_{N-1} \ /Q_N = \phi / \tag{D.37}$$

$$2tr = \rho \ Cp \frac{1}{4} (T_{N-1} - T_N)Q \tag{D.38}$$

APPENDIX E

Version of "USER" subroutine for Program MINLAKE

for Case of Reservoir Petrusplaat Calling Subroutine BUBBLES

User is a name of the subroutine of the MINLAKE program which gives the opportunity to the user of the program to include some specific requirement apart of those features which MINLAKE itself treats. Such a specific requirement is calculation of destratification of the lake due to the air bubbles column. The subroutine which treats that is called BUBBLES and it performs trough USER.

```
$DEBUG
$NOFLOATCALLS
$STORAGE:2
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE USER(ZD,DUM,NFLOW,ID)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     THIS SUBROUTINE PROVIDES GEOMETRY FOR THE APPROPRIATE LAKE AND
C     DIFFERENT SPECIFIC REQUIREMENTS;ONE OF THEM IS AERATION WHICH
C     IS INCLUDED CALLING THE SPECIFIC SUBROUTINE FOR IT CALLED
C     BUBLES
C***********************************************************************
      REAL*8 A,V,TV,ATOP,AK,BK,CK,DK,SVOL,DUM,AREA
      COMMON/WATER/BETA,EMISS,XK1,XK2,HKMAX,WCOEF,WSTR,RHOW,HCAP,HF,TCW
      COMMON/PHYTO/PDEL(3),PMAX(3),PMIN(3),THR(3),THM(3),XKR1(3),
     +XKR2(3),XKM(3),HSCPA(3),HSC1(3),HSC2(3),UPMAX(3),THUP(3),
     +GROMAX(3),TMAX(3),TOPT(3),XNMAX(3),XNMIN(3),UNMAX(3),THUN(3),
     +HSCN(3),HSCNH(3),XNDEL(3),HSCI,IDIATOM,CHLMEAN,CHLMAX,SECCHI
      COMMON/YIELD/YCA,YCHO2,Y2CHO2,YCBOD,YPBOD,YZW,YPZP,YNZP,YZDO,
     +YSCHL,YNHBOD,BRNO,BRNH,XKNNH,THNNH,YPCHLA,BODK20,SB20,BRR
      COMMON/ZOOPL/IZ,MINDAY,MAXDAY,ZP,ZPMIN,PRMIN,PRMAX,PREDMIN,XIMIN,
     +XIMAX,XKRZP,GRAZMAX(3),THGRAZ(3),ASM,THRZP,HSCGRAZ(3),CHLAMIN(3),
     +REPRO,XI,XKMZ,GRAZE(3,40)
      COMMON/SOLV/ AK(40),BK(40),CK(40),DK(40)
      COMMON/MTHD/TAIR(31),TDEW(31),RAD(31),CR(31),WIND(31),
     + PR(31),DRCT(31)
      COMMON/RESULT/ T2(40),CHLATOT(40),PA2(40),PTSUM(40),BOD2(40),
     +DSO2(40),C2(40),CD2(40),XNO2(40),XNH2(40),CHLA2(3,40),
     +PC2(3,40),XNC2(3,40),T20(40),SI2(40)
      COMMON/SOURCE/RM(3,40),PROD(40),XMR(3,40),PRODSUM(40)
      COMMON/FLOW/HMK(41),QE(40),FVCHLA(5),PE(5,41)
      COMMON/TEMP/PARIO(4),PCDUM(3,40),XNHD(40),XNOD(40),
     + CHLADUM(3,40),XNCD(3,40),PADUM(40),SID(40)
      COMMON/VOL/ZMAX,DZ(40),Z(40),A(40),V(40),TV(40),ATOP(41),DBL
      COMMON/SUB/SDZ(60),SZ(60),LAY(40),AVGI(4,60),SVOL(60)
      COMMON/CHOICE/MODEL,NITRO,IPRNT(6),NDAYS,NPRNT(30),NCLASS,PLOT(30)
      COMMON/CHANEL/WCHANL,ELCB,ALPHA,BW,WLAKE
      COMMON/STEPS/DZLL,DZUL,MBOT,NM,NPRINT,MDAY,MONTH,ILAY,DY
      COMMON/CONVAL/GRA,RHOA,CONST
      COMMON/INFLOW/QIN(5),TIN(5),PAIN(5),BODIN(5),DOIN(5),CIN(5),
     +CDIN(5),XNHIN(5),XNOIN(5),CHLAIN(3,5)
      COMMON/STAT/SUMXY(10),SUMX(10),SUMY(10),XSQ(10),
     +YSQ(10),RSQ(10),RMS(10),RELM(10),MTHREL(10),MDAYREL(10),ZREL(10),
     +ZRELM(10),RS(10),REL(10),MTHRMS(10),MDAYRMS(10),ZRS(10),ZRMS(10)
      COMMON/FILE/ DIN,MET,FLO,TAPE8,TAPE1
      COMMON/FIELD/IFLAG(10),FDATA(10,50),DEPTH(50),NFLD(10),SD
C **         COMMON BLOCK FOR BUBBLES ***
      COMMON/BUB1/QAIR,RH,H,RB,IA,IB,IDIF,N
C
      CHARACTER*16 DIN,MET,FLO,TAPE8,TAPE1
      GOTO (100,200,300,400,500,600,700,800,900,1000,1100,1200,1300) ID
C********** AREA COMPUTATION SECTION ***********
100   CONTINUE
      DUM=0.651*1.05e6+0.349*1.05e6*ZD/15.
      RETURN
C********** FETCH COMPUTATION SECTION ******
 200  ZD=1100.
      RETURN
C********** VOLUME COMPUTATION SECTION *****
300   CONTINUE
```

```
      DUM=(0.651*1.05e6+0.349*1.05e6*0.5)*ZD
      RETURN
C***** COMPUTE DEPTH FROM VOLUME *****
 400  CONTINUE
      ZD=DUM/(0.651*1.05e6+0.5*0.349*1.05e6)
      RETURN
C***** TREATMENT SECTION *****
 500  WRITE(*,*) MONTH,MDAY,MBOT
      RETURN
C***** PHOSPHORUS SOURCES/SINKS *****
 600  RETURN
c***** NO2-NO3 SOURCES SINKS
 700  RETURN
C***** NH4 SOURCES/SINKS *****
 800  RETURN
C***** O2 SOURCES/SINKS *****
 900  RETURN
C***** OUTFLOW COMPUTATION *****
 1000 RETURN
C***** GROUNDWATER FLOW COMPUTATION *****
 1100 RETURN
C***** INPUT REQUEST OR MODIFICATION ****
C      AS THIS SUBROUTINE IS CALED JUST ONCE,IN THE BEGINNING,
C      IT IS CONVENIENT TO WRITE HERE ALL THE VALUES WHICH ARE
C      GOING TO BE UNCHANGED
 1200 CONTINUE
      RETURN
C***** POST DAILY SIMULATION TREATMENT AND COMPUTATIONS  *******
C*****    A E R A T I O N    ******
C
 1300 CONTINUE
      N=MBOT+1
      H=15.
      RH=15.
      QAIR=0.17
      CALL BUBLES
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE BUBLES
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C*     THIS SUBROUTINE CALCULATES DESTRATIFICATION OF THE LAKE DUE TO *
C*     AIR BUBBLES COLUMN                                            *
C
      REAL*8 A,V,TV,ATOP,AK,BK,CK,DK,SVOL,DUM,AREA
      COMMON/RESULT/ T2(40),CHLATOT(40),PA2(40),PTSUM(40),BOD2(40),
     +DSO2(40),C2(40),CD2(40),XNO2(40),XNH2(40),CHLA2(3,40),
     +PC2(3,40),XNC2(3,40),T20(40),SI2(40)
      COMMON/VOL/ZMAX,DZ(40),Z(40),A(40),V(40),TV(40),ATOP(41),DBL
      COMMON/BUB1/QAIR,RH,H,RB,IA,IB,IDIF,N
      COMMON/BUB2/RO,QEO,QHO,HEP,HIN,HHY,TEP,THY,RHOI,TI,PI,QIO
      COMMON/AS/T(42,2),UZ(42)
      COMMON/BUB3/HEP0,HHY0
      DATA FFC/0.8/
      PI=3.14159
C-----------------       NEAR FIELD      ------------------------------------
      QEO=0.
      RB=0.
      CALL INP
      I=0
```

```
40    I=I+1
      IF(I.GE.20) GO TO 2
      QEO1=QEO
      IF(I.GT.1) THEN
        HEPOLD=HEP
        ROOLD =RO
        QEOOLD=QEO
        QHOOLD=QHO
        RHOIOL=RHOI
        HINOLD=HIN
      ENDIF
        CALL NEAR
      RB=RB+1
      IF(I.GE.7)            GO TO 50
      IF((QEO+QEO1).EQ.0.)  GO TO 40
      IF(QEO.GT.(1.05*QEO1))GO TO 40
      GO TO 200
50    IF((QEO.GT.0.).AND.(QEO.GT.(FFC*QEO1))) GO TO 200
      GO TO 40
2     WRITE(*,5)
      STOP
C----------------      FAR FIELD      ------------------------------------
200     HEP =HEPOLD
        RO  =ROOLD
        QEO =QEOOLD
        QHO =QHOOLD
        RHOI=RHOIOL
        HIN =HINOLD
        TI  =FUN(RHOI)
        QIO =QEO+QHO
      CALL FARF
C
 5    FORMAT(2X,'** THE FEASIBLE SOLUTION FOR THE NEAR FIELD IS NOT',
     +'FOUND **')
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      FUNCTION FUN(RHOI)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DATA DT/2./
      FUN=30.
      DO 2 I=1,30
      FUN=FUN-DT
      RAZ=RHOI-RHO(FUN,0.,0.)
      IF(I.GT.1) GO TO 1
      RAZ1=RAZ
      GO TO 2
 1    IF(ABS(RAZ).LE.1.E-3) RETURN
      IF((RAZ1*RAZ).GT.0.)  GO TO 2
      RAZ1=RAZ
      DT=-DT/2
 2    CONTINUE
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE INP
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      REAL*8 A,V,TV,ATOP,AK,BK,CK,DK,SVOL,DUM,AREA
      COMMON/RESULT/ T2(40),CHLATOT(40),PA2(40),PTSUM(40),BOD2(40),
     +DSO2(40),C2(40),CD2(40),XNO2(40),XNH2(40),CHLA2(3,40),
```

```
     +PC2(3,40),XNC2(3,40),T20(40),SI2(40)
      COMMON/VOL/ZMAX,DZ(40),Z(40),A(40),V(40),TV(40),ATOP(41),DBL
      COMMON/BUB1/QAIR,RH,H,RB,IA,IB,IDIF,N
      COMMON/BUB2/RO,QEO,QHO,HEP,HIN,HHY,TEP,THY,RHOI,TI,PI,QIO
      COMMON/BUB3/HEP0,HHY0
      COMMON/AS/T(42,2),UZ(42)
      DIMENSION GR(41)
      DATA GRMX/0./
C
      HEP0=0.
      TEP=0.
      THY=0.
      CALL TEMP2
      DO 1 I=2,N
1     GR(I)=ABS((T(I,1)-T(I-1,1))/DZ(I-1))
C**     find gr(i) max
      I=1
 7    I=I+1
      IF(I-N)8,8,6
 8    IF(GR(I).LT.GRMX)GO TO 6
      IF(GR(I).GE.GRMX)THEN
      GRMX=GR(I)
      IMAX=I
      ENDIF
2     GO TO 7
6     CONTINUE
      IF(IMAX.GE.3) IMAX=IMAX-1
      IF(IMAX.GT.(N/2))IMAX=N/2
      DO 3 I=1,IMAX
      IF(I.GT.1)HEP0=HEP0+DZ(I-1)
 3    TEP=TEP+T(I,1)
      TEP=TEP/REAL(IMAX)
      IDIF=N
      IF(RH.NE.H) IDIF=IAB(RH)
      DO 4 I=IMAX+1,IDIF
 4    THY=THY+T(I,1)
      THY=THY/REAL(IDIF-IMAX)
      HHY0=RH-HEP0
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE NEAR
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C**     THIS SUBROUTINE CALCULATES VARIABLES AT THE STAGNATION POINT   *
C**     BETWEEN NEAR FIELD AND FAR FIELD                               *
C**     WHEN DESTRATIFYING A THERMALLY STRATIFIED RESERVOIR            *
C
      COMMON/BUB1/QAIR,RH,H,RB,IA,IB,IDIF,N
      COMMON/BUB2/RO,QEO,QHO,HEP,HIN,HHY,TEP,THY,RHOI,TI,PI,QIO
      COMMON/BUB3/HEP0,HHY0
      DIMENSION CRAZ(60),DEL(60),DRW(60)
      DATA RK/0.0725/,RRKRI/0.004/,RKRIT/1E-09/,RFKRI/0.01/,G/9.81/,
     +    C1/0.04/
C
      TPI=2.*PI
      RB1=RB+1.
C**       check if hep0+hhy0=rh and tep>thy
      IF((HEP0+HHY0).NE.RH)     WRITE(*,300)
      IF(TEP.LT.THY)            WRITE(*,301)
      IF(((HEP0+HHY0).NE.RH).OR.(TEP.LT.THY)) STOP
```

```
C**        calculation starts
      RM=0.67*SQRT(RK*RH)*QAIR**0.333
      QW=0.47*QAIR**0.333*RH**1.333
C**        density-temperature relation
      RHOE=RHO(TEP,0.,0.)
      RHOH=RHO(THY,0.,0.)
      DRHEH=RHOH-RHOE
      DRHG=DRHEH*G
      RRR=RHOH/DRHG
C**        loop to determine ro
C**        initial value for iterations
      RSTEP=4.*RH/30.
      RO   =RH*0.5-RSTEP
      KDRW=0
      JCS=0
      J=1
 201  J=J+1
      IF((J.GT.50).OR.(ABS(RSTEP).LT.1.E-8)) GO TO 25
      RO=RO+RSTEP
      RCON=TPI*RO/QW
      ARG=RH/(RK*RO)
      TANHRH=TANH(RH/(RK*RO))
      RZZ=SQRT(1.-RK*RO/RH*TANHRH)
      RZO=RK*RO*LOG((1.+RZZ)/(1.-RZZ))/2.
      TANHRZ=TANH(RZO/(RK*RO))
      RAO=RCON*RM*(TANHRZ-RZO/RH*TANHRH)-1.
      IF(RAO.LT.0.) GO TO 201
      RUR=RM/(RK*RO)
      RAO1=RAO+1.
      RURR=RAO1/(RCON*RZO)
      RURO=RM/(RK*RO)*(1.-RK*RO/RH*TANHRH)
      RVE=C1*RURO
c**        loop to determine hin and rhoi so that the conditon of
c**        critical flow at ro is satisfied
c**      - initial value for rhoi -
      RHOI=(RHOE*HEP0+RHOH*HHY0)/RH
C**      - initial value for variable a and for step of a -
        A=0.
        AA=0.05
        A=A-AA
c**        initial values of variables which limits number of iterations
c**        do loop to determine hi
        KCS=0
        K=0
        KKCON=0
        KCON=0
 202    K=K+1
        IF(K.GT.50)  GO TO 18
 104    A=A+AA
        HIN=RZO/2.+(RH-RZO/2.)*A
c**        hin is always less or equal to rh/2. if critical flow
        IF(HIN.GT.(RH/2.)) THEN
        HIN=RH/2.
        IF(ABS(RFARF).LE.RFKRI) GO TO 18
        ENDIF
        CRAZ(1)=10.
        DRHH=DRHEH/HIN
          KK=1
 203      KK=KK+1
          IF(KK.GT.20) GO TO 204
```

```
          RHO1=RHOI
          DRHOHI=RHOH-RHOI
          DRHOIE=RHOI-RHOE
          HEP=HEP0-DRHOHI/DRHH
          RHH=HHY0-DRHOIE/DRHH
          IF(RHH.LT.0.) GO TO 202
          IF(HEP.LT.0.) GO TO 202
          CC2=RH-RHH-RZO
          RVH=RAO1/(RCON*(RHH+RB1*CC2))
          RAH=RVH*RHH*RCON
          RAE=RVE*HEP*RCON
          RAEAH=RAE+RAH
c**        rae+rah must always be larger or equal to zero
          IF(RAEAH.LE.0.) GO TO 202
          RVM=RAEAH/(HIN*RCON)
          RHOI=(RAE*RHOE+RAH*RHOH)/RAEAH
          RHO2=RHOI
          CRAZ(KK)=ABS(RHO1-RHO2)
          IF(CRAZ(KK).LE.0.0001) GO TO 204
          IF(KCON.GT.3)          GO TO 204
          IF(CRAZ(KK).GT.CRAZ(KK-1))THEN
          A=A-AA
          AA=AA/5.
          KCON=KCON+1
          A=A-AA
          GO TO 104
          ENDIF
          GO TO 203
  204   RFE=RVE*RVE*RRR/HEP
        RFM=RVM*RVM*RRR/HIN
        RFH=RVH*RVH*RRR/RHH
        RFARF1=DRHOIE*DRHOHI/(RHOH*RHOH)
        RFARF2=DRHOIE*RFH*RFH/RHOH
        RFARF3=DRHOHI*RFM*RFM/RHOH
        RFARF4=DRHOHI*RFE*RFE/RHOH
        RFARF5=RFE*RFE*RFM*RFM+RFM*RFM*RFH*RFH+RFH*RFH*RFE*RFE
        RFARF=RFARF5-RFARF4-RFARF3-RFARF2+RFARF1
        DEL(K)=RFARF
c**        far field equation which controls the critical flow condition
        IF(KCON.GT.3) GO TO 18
        IF(K-3) 202,14,15
  14    RFARB=RFARF
  15    IF(ABS(RFARF).LE.RKRIT) GO TO 18
        IF((RFARB*RFARF).LE.0.) GO TO 17
        IF(KCS.GE.10)           GO TO 202
        IF(KKCON.GT.10)         GO TO 18
        IF(ABS(DEL(K)).GT.ABS(DEL(K-1))) THEN
        A=A-AA
        AA=AA/3.
        A=A-AA
        KKCON=KKCON+1
        ENDIF
        GO TO 202
 17     IF(KCS.LE.10) THEN
        A=A-AA
        AA=-AA/3.
        A=A-AA
        KCS=KCS+1
        RFARB=RFARF
        GO TO 202
```

```
         ENDIF
 18   IF(RHH.LT.0.) RVH=0.
      RURT=(RAO1-RAH)/(RCON*(RH-RHH-RZO))
      RRHOO=((RAO1-RAH)*RHOI+RAH*RHOH)/RAO1
      CC1 =RRHOO*RZO
      CC11=RHOI*CC2
      CC12=RHOH*RHH
      CC13=RHOE*HEP
      CC14=RHOI*HIN
      RPEEN=G*(CC1*0.5*RZO+(CC1+CC11*0.5)*CC2+(CC1+CC11+CC12*0.5)*RHH)
      RDP  =0.5*(RRHOO*RURO*RURO-RHOE*RVE*RVE)
      RPTWE=G*(0.5*CC13*HEP+(CC13+0.5*CC14)*HIN+(CC13+CC14+CC12*0.5)
     +      *RHH)+RDP*RHH
      RIA=(CC1+CC11+CC12)/RH*RM*RUR*(TANHRZ-0.333*TANHRZ**3
     +      -2.*RK*RO*TANHRH*TANHRZ/RH+RK*RO*RZO*TANHRH*TANHRH/(RH*RH))
     +      +CC11*RB1*RVH*RB1*RVH+CC12*RVH*RVH
      RW=RPTWE-RPEEN-RIA+RHOI*HIN*RVM*RVM+RHOE*HEP*RVE*RVE
     +      +RHOH*RHH*RVH*RVH
      DRW(J)=RW
      IF(J.LE.2) THEN
      Y=RW
      GO TO 201
      ENDIF
      IF(ABS(RW).LE.RRKRI) GO TO 25
      IF((RW*Y).LE.0.)     GO TO 23
      IF(JCS.GE.40)        GO TO 201
      IF(KDRW.GT.20)       GO TO 25
      IF(ABS(DRW(J)).GT.ABS(DRW(J-1)))THEN
      RO=RO-RSTEP
      RSTEP=RSTEP/3.
      KDRW=KDRW+1
      RO=RO-RSTEP
      ENDIF
      GO TO 201
 23   IF(JCS.GT.40) GO TO 25
      RO=RO-RSTEP
      RSTEP=-RSTEP/4.
      RO =RO -RSTEP
      JCS=JCS+1
      Y=RW
      GO TO 201
C----------------- calculations are done -------------------------------
 25   CONTINUE
      IF(ABS(RW).GT.1.) GO TO 501
      IF(RO.LT.(RH*5.-2.*RSTEP1).AND.RAE.GT.0.) THEN
      QEO=RAE*QW
      QHO=RAH*QW
      RETURN
      ENDIF
501   QEO=0.
      RETURN
C
300   FORMAT(2X,'* ERROR IN NEAR : HEP+HHY IS NOT EQUAL TO RH *')
301   FORMAT(2X,'* ERROR IN NEAR : TEP > THY *')
      END

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE FARF
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C**   CALCULATION OF THE TEMPERATURE PROFILE IN THE FAR FIELD         **
```

```
C
      REAL*8 A,V,TV,ATOP,AK,BK,CK,DK,SVOL,DUM,AREA
      COMMON/RESULT/ T2(40),CHLATOT(40),PA2(40),PTSUM(40),BOD2(40),
     +DSO2(40),C2(40),CD2(40),XNO2(40),XNH2(40),CHLA2(3,40),
     +PC2(3,40),XNC2(3,40),T20(40),SI2(40)
      COMMON/VOL/ZMAX,DZ(40),Z(40),A(40),V(40),TV(40),ATOP(41),DBL
      COMMON/BUB1/QAIR,RH,H,RB,IA,IB,IDIF,N
      COMMON/BUB2/RO,QEO,QHO,HEP,HIN,HHY,TEP,THY,RHOI,TI,PI,QIO
      COMMON/AS/T(42,2),UZ(42)
      DATA DTOU/720./,NTS/121/
C
      IB=IAB(HEP)
      HHY=HEP+HIN
      IA=IAB(HHY)
c**         vertical velocities profile
            CALL VERVE
      ZI=IB+UZ(IB)/(UZ(IA)+UZ(IB))*(IA-IB+1.E-30)
      IZ=IFIX(ZI)
      T22=0.25*DTOU
      T12=0.5 *DTOU
C---------------------- DIFFERENCE METHOD STARTS -----------------------
      DO 20 J=1,NTS
C**         calculate influence of the near field
      TIOLD=TI
      TI=TINT(TIOLD)
C**         surface
      T(1,2)=T(1,1)+UZ(2)*T22/DZ(1)*(T(2,1)-T(1,1))/ATOP(1)
C**         epilimnion
      IF(IB.LE.2) GO TO 666
      DO 6 I=2,IB-1
6     T(I,2)=T(I,1)+E1(T22,I)/ATOP(I)
C**         transition with I=IB
666   CON1=T12*(T(IB,1)-TI)*(UZ(IB+1)-UZ(IB))/(Z(IB)-Z(IB-1))/ATOP(IB)
      T(IB,2)=T(IB,1)+E1(T22,IB)/ATOP(IB)+CON1
C**         interlayer
      DO 7 I=IB+1,IA-1
      IF(I.LT.IZ) T(I,2)=T(I,1)+(E1(T22,I)+E11(T22,I))/ATOP(I)
      IF(I.EQ.IZ) T(I,2)=T(I,1)+(E1(T22,I)+T22*2*(UZ(I+1)
     +                  *(TI-T(I+1,1))+UZ(I-1)*(TI-T(I,1))
     +                 -UZ(I)*(T(I,1)+T(I+1,1)))/(Z(I)-Z(I-1)))/ATOP(I)
      IF(I.GT.IZ) T(I,2)=T(I,1)-(E1(T22,I)+E11(T22,I))/ATOP(I)
  7   CONTINUE
C**         transition with I=IA
      T(IA,2)=T(IA,1)-(E1(T22,IA)+T12*(T(IA,1)-TI)*(UZ(IA)-UZ(IA-1))/
     +         (Z(IA)-Z(IA-1)))/ATOP(IA)
C**         hypolimnion
      DO 12 I=IA+1,N-1
12    T(I,2)=T(I,1)-E1(T22,I)/ATOP(I)
C**         bottom
      T(N,2)=T(N,1)+(T22*UZ(N-1)/(Z(N-1)-Z(N-2))*(T(N-1,1)
     +        -T(N,1)))/ATOP(N-1)
C
      DO 16 I=2,N
      IF(T(I,2).GT.100.)                   WRITE(*,100)
      IF(T(I,2).LT.0.)                     WRITE(*,101)
      IF(T(I,2).GT.100.OR.T(I,2).LT.0.) STOP
      IF(T(I,2).LE.T(I-1,2))GO TO 16
      T(I-1,2)=(T(I-1,2)+T(I,2))/2
      T(I,2)=T(I-1,2)
 16   CONTINUE
```

```
1               DO 23 I=1,N
2          23   T(I,1)=T(I,2)
1          20   CONTINUE
           C
           100  FORMAT(2X,'** ERROR :WATER BOILS')
           101  FORMAT(2X,'** ERROR :WATER FREEZES')
           C
                CALL CHANGE(N)
                CALL NEWT
           C
                RETURN
                END
           CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
                SUBROUTINE VERVE
           CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
           C**  CALCULATION OF THE VERTICAL VELOCITIES
           C
                REAL*8 A,V,TV,ATOP,AK,BK,CK,DK,SVOL,DUM,AREA
                COMMON/VOL/ZMAX,DZ(40),Z(40),A(40),V(40),TV(40),ATOP(41),DBL
                COMMON/BUB1/QAIR,RH,H,RB,IA,IB,IDIF,N
                COMMON/BUB2/RO,QEO,QHO,HEP,HIN,HHY,TEP,THY,RHOI,TI,PI,QIO
                COMMON/AS/T(42,2),UZ(42)
           C
                DO 20 I=1,N
1          20   UZ(I)=0.
                RO2PI=RO*RO*PI
                ROPI2=RO*PI*2.
                HB=Z(IB)-DZ(IB)/2.
                HA=Z(IA)-DZ(IA)/2.-HB
                UH=QEO/(ROPI2*HB+1.E-30)
                UZ(IB)=QEO/(ATOP(IB)-RO2PI)
                IF(IB.LE.2) GO TO 7
                DO 8 I=IB-1,1,-1
1          8    UZ(I)=1./(ATOP(I)-RO2PI)*(UZ(I+1)*(ATOP(I+1)-RO2PI)-UH*(Z(I+1)-
1              +Z(I))*ROPI2)
           7    UH=QHO/(ROPI2*(RH-HA-HB))
                UZ(IA)=QHO/(ATOP(IA)-RO2PI)
                DO 9 I=IA+1,IDIF
1               UZ(I)=1./(ATOP(I)-RO2PI)*(UZ(I-1)*(ATOP(I-1)-RO2PI)-UH*(Z(I)-Z(I-
1              +      1))*ROPI2)
1               IF(UZ(I).LT.0.)  UZ(I)=0.
1          9    CONTINUE
                CON1=(UZ(IB)+UZ(IA))/HA
                CON2=UZ(IB)/(UZ(IB)+UZ(IA))*HA
                DO 10 I=IB,IA
1          10   UZ(I)=CON1*ABS(CON2-Z(I)+DZ(I)/2.+HB)
                DO 90 I=1,N
1          90   UZ(I)=UZ(I)*ATOP(I)
                RETURN
                END
           CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
                FUNCTION TINT(TT)
           CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
                COMMON/BUB1/QAIR,RH,H,RB,IA,IB,IDIF,N
                COMMON/BUB2/RO,QEO,QHO,HEP,HIN,HHY,TEP,THY,RHOI,TI,PI,QIO
                COMMON/AS/T(42,2),UZ(42)
           C
                SUM1=0.
                DO 1 I=1,IB
1          1    SUM1=SUM1+T(I,1)
```

```
      SM1=SUM1*QEO/real(IB)
      SUM2=0.
      DO 2 I=IA,IDIF
2     SUM2=SUM2+T(I,1)
      SM2=SUM2*QHO/REAL(IDIF-IA+1)
      TINT=(DTOU*(SM1+SM2-QIO*TT)+PI*RO**2*RH*TT)/(PI*RO**2*RH)
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      FUNCTION E1(T2,I)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      REAL*8 A,V,TV,ATOP,AK,BK,CK,DK,SVOL,DUM,AREA
      COMMON/VOL/ZMAX,DZ(40),Z(40),A(40),V(40),TV(40),ATOP(41),DBL
      COMMON/AS/T(42,2),UZ(42)
C
      E1=T2*(UZ(I+1)*(T(I+1,1)-T(I,1))+UZ(I)*(T(I+1,1)-T(I-1,1))
     +   +UZ(I-1)*(T(I,1)-T(I-1,1)))/(Z(I)-Z(I-1)+1.E-30)
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      FUNCTION E11(T2,I)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      REAL*8 A,V,TV,ATOP,AK,BK,CK,DK,SVOL,DUM,AREA
      COMMON/VOL/ZMAX,DZ(40),Z(40),A(40),V(40),TV(40),ATOP(41),DBL
      COMMON/BUB2/RO,QEO,QHO,HEP,HIN,HHY,TEP,THY,RHOI,TI,PI,QIO
      COMMON/AS/T(42,2),UZ(42)
C
      E11=T2*2*((UZ(I+1)-UZ(I-1))*(T(I,1)-TI))/(Z(I)-Z(I-1)+1.E-30)
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      FUNCTION IAB(HH)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      REAL*8 A,V,TV,ATOP,AK,BK,CK,DK,SVOL,DUM,AREA
      COMMON/VOL/ZMAX,DZ(40),Z(40),A(40),V(40),TV(40),ATOP(41),DBL
      COMMON/BUB1/QAIR,RH,H,RB,IA,IB,IDIF,N
C
      DO 1 I=1,N-1
      IF((HH.GT.Z(I)).AND.(HH.LE.Z(I+1))) THEN
      IAB=I+1
      RETURN
      ENDIF
1     CONTINUE
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE TEMP2
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      REAL*8 A,V,TV,ATOP,AK,BK,CK,DK,SVOL,DUM,AREA
      COMMON/RESULT/ T2(40),CHLATOT(40),PA2(40),PTSUM(40),BOD2(40),
     +DSO2(40),C2(40),CD2(40),XNO2(40),XNH2(40),CHLA2(3,40),
     +PC2(3,40),XNC2(3,40),T20(40),SI2(40)
      COMMON/BUB1/QAIR,RH,H,RB,IA,IB,IDIF,N
      COMMON/AS/T(42,2),UZ(42)
      COMMON/VOL/ZMAX,DZ(40),Z(40),A(40),V(40),TV(40),ATOP(41),DBL
C
      NN=N-1
      CALL SPLINE(Z,T2,NN,0.,0.,Y2)
      X=0.
      T(1,1)=T2(1)
```

```
      T(N,1)=T2(N-1)
      DO 2 I=2,N-1
      X=X+DZ(I-1)
      CALL SPLINT(Z,T2,Y2,NN,X,Y)
      IF(Y.GT.T2(I-1).OR.Y.LT.T2(I)) Y=(T2(I-1)+T2(I))/2
2     T(I,1)=Y
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE CHANGE(N)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      REAL*8 A,V,TV,ATOP,AK,BK,CK,DK,SVOL,DUM,AREA
      COMMON/RESULT/ T2(40),CHLATOT(40),PA2(40),PTSUM(40),BOD2(40),
     +DSO2(40),C2(40),CD2(40),XNO2(40),XNH2(40),CHLA2(3,40),
     +PC2(3,40),XNC2(3,40),T20(40),SI2(40)
      COMMON/AS/T(42,2),UZ(42)
      COMMON/VOL/ZMAX,DZ(40),Z(40),A(40),V(40),TV(40),ATOP(41),DBL
      DIMENSION XA(41),YA(41),Y2(41)
C
      XA(1)=0.
      YA(1)=T(1,2)
      DO 1 I=2,N
      XA(I)=XA(I-1)+DZ(I-1)
1     YA(I)=T(I,2)
      CALL SPLINE(XA,YA,N,0.,0.,Y2)
      CALL SPLINT(XA,YA,Y2,N,Z(1),YY)
      T2(1)=T(1,2)
      DO 2 I=2,I-1
      CALL SPLINT(XA,YA,Y2,N,Z(I),YY)
      IF(YY.GT.T(I,2).OR.YY.LT.T(I+1,2)) YY=(T(I+1,2)+T(I,2))/2.
2     T2(I)=YY
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE SPLINE(X,Y,N,YP1,YPN,Y2)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      dimension x(41),y(41),y2(41),u(41)
      if(yp1.gt..99e30) then
         y2(1)=0.
         u(1)=0.
      else
         y2(1)=-0.5
         u(1)=(3./(x(2)-x(1)))*((y(2)-y(1))/(x(2)-x(1))-yp1)
      endif
      do 11 i=2,n-1
         sig=(x(i)-x(i-1))/(x(i+1)-x(i-1))
         p=sig*y2(i-1)+2
         y2(i)=(sig-1.)/p
         u(i)=(6.*((y(i+1)-y(i))/(x(i+1)-x(i))-(y(i)-y(i-1))
     +         /(x(i)-x(i-1)))/(x(i+1)-x(i-1))-sig*u(i-1))/p
   11 continue
      if(ypn.gt..99e30) then
         qn=0.
         un=0.
      else
         qn=0.5
         un=(3./(x(n)-x(n-1)))*(ypn-(y(n)-y(n-1))/(x(n)-x(n-1)))
      endif
      y2(n)=(un-qn*u(n-1))/(qn*y2(n-1)+1.)
      do 12 k=n-1,1,-1
```

```
         y2(k)=y2(k)*y2(k+1)+u(k)
   12 continue
      return
      end
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      subroutine splint(xa,ya,y2a,n,x,y)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      dimension xa(41),ya(41),y2a(41)
      klo=1
      khi=n
 1    if(khi-klo.gt.1)then
      k=(khi+klo)/2
      if(xa(k).gt.x)then
        KHI=K
      else
        klo=k
      endif
      go to 1
      endif
      h=xa(khi)-xa(klo)
      if(h.eq.0.) pause 'bad xa input '
      a=(xa(khi)-x)/h
      bb=(x-xa(klo))/h
      y=a*ya(klo)+b*ya(khi)+
     +     ((a**3-a)*y2a(klo)+(bb**3-bb)*y2a(khi))*(h**2)/6.
      return
      end
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE NEWT
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      REAL*8 A,V,TV,ATOP,AK,BK,CK,DK,SVOL,DUM,AREA
      COMMON/RESULT/ T2(40),CHLATOT(40),PA2(40),PTSUM(40),BOD2(40),
     +DSO2(40),C2(40),CD2(40),XNO2(40),XNH2(40),CHLA2(3,40),
     +PC2(3,40),XNC2(3,40),T20(40),SI2(40)
      COMMON/VOL/ZMAX,DZ(40),Z(40),A(40),V(40),TV(40),ATOP(41),DBL
      COMMON/FLOW/HMK(41),QE(40),FVCHLA(5),PE(5,41)
      COMMON/BUB1/QAIR,RH,H,RB,IA,IB,IDIF,N
      DATA DAY/86400./,EPSCRT/0.9E-6/,TM1/0./,TM2/0./,TM3/0./,FCT/0.05/
C**     epilimnion
      DO 40 I=1,IB
   40 TM1=TM1+T2(I)
      TM1=TM1/IB
      RHO1=RHO(TM1,0.,0.)
C**     interlayer
      DO 41 I=IB,IA
   41 TM2=TM2+T2(I)
      TM2=TM2/(IA-IB+1)
      RHO2=RHO(TM2,0.,0.)
C**     hypolimnion
      DO 42 I=IA,N-1
 42   TM3=TM3+T2(I)
      TM3=TM3/(N-IA)
      RHO3=RHO(TM3,0.,0.)
      EPS1=1./RHO2*ABS(RHO2-RHO1)/(0.5*Z(IA))
      EPS2=1./RHO2*ABS(RHO3-RHO2)/(0.5*(RH-Z(IB)))
      DO 33 I=1,N-1
 33   HMK(I)=0.
      IDIF1=IDIF
      IF(IDIF.NE.N)IDIF1=IDIF+1
      DO 21 I=2,IDIF1
```

```
1          EPS=EPS1
1          IF(I.GE.IB) EPS=EPS2
1          HMK(I)=EXP(ALOG(EPS*1.E6)*(-0.5903)-7.608)*DAY*FCT
1          IF(EPS.LT.EPSCRT)HMK(I)=2.5E-4*DAY*FCT
1      21  CONTINUE
           NN1=N-1
           CALL COEF(1,NN1,0)
           CALL DISOLID(T2,0,1,0)
           RETURN
           END
```