Essays in Macroeconomics and Taxation

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Dedication

A la H, por creer en mi.

A mi familia, por estar aquí sin estar aquí.

A los amigos de México, si leen esto . . . es porque me acordé de ustedes.

Abstract

In these essays, I explore the business cycle implications of changes in expectations about the course of future fiscal policy. Some conventional models of fiscal policy assume that tax rates and government consumption follow an exogenous stochastic process, so that expectations of future policy are determined by current policy only. I consider two ways in which expectations about future policy change even when current policy does not. First, I construct a DSGE model where agents receive news about changes in future policy every period, and compare it to an economy where agents receive no information whatsoever but is identical in every other detail. Comparison of the models' impulse-response functions show that, in the economy with news, model variables move—and can even trigger a recession—before the policy change takes place. Second, I construct a DSGE model where agents' expectations about future policy are derived from a simple tax rule which ties future tax rates to the current deficit and debt; I compare this economy to a similar economy which lacks these tax rules. Analysis of the impulse-response functions shows that the qualitative response is the same in both economies; however, the quantitative response is amplified in the economy with tax rules. This result is robust to the choice of policy variable used. I conclude that deviations from the traditional strategies for including fiscal policy in DSGE models can create drastically different implications for the business cycle.

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Chapter 1

Introduction

Over the last years, the policy literature has been increasingly interested in studying the implications of changes in expectations about future policy. These essays contribute to the literature by analyzing how variations over the conventional modeling strategy can change the business cycle implications of the model.

News about future policy

The first essay, "News and Expectations," allows for changes in expectations about future policy by considering the case where agents receive news about changes in future policy every period.[1] These news directly influence the agents' expected value for fiscal policy variables, so that expectations of future policy can change even though actual policy remains the same.

The main result of the essay is that when agents receive news about future policy, the variables in the economy can vary before a policy change takes place; these movements can be consistent with expansions or recessions, depending on the particular policy variable which is expected to change.

[1] The idea of "news shocks" goes back a long way; see the work by Cochrane [1].

Tax expectations, debts, deficits, and rules

The second essay, "Tax Rules and Expectations," allows for changes in expectations about future policy by introducing a particular tax rule, which derives the value of next period's tax rate from today's debt and deficit values (and other state variables).

When including these tax rules as a part of the economy's equilibrium requirements, the response of the economy's variables to an exogenous shock is amplified relative to the case where no tax rule is present.

Summary

The results obtained in both essays confirm that changes in expectations matter: either news about future policy or tax rules which incorporate expected debt or deficits can change the results that would be obtained from a standard model. In this sense, fiscal policy may have more than one way of affecting the economy.

Chapter 2

News and Expectations

In this essay, I explore the business cycle implications of changes in expectations about the course of future fiscal policy. Some conventional models of fiscal policy—see the work of Braun [2] or McGrattan [3] as an example—assume that tax rates and government consumption follow an exogenous stochastic process. In such models, expectations of future policy are determined by current policy only.

Recently, the fiscal policy literature has started to explore the implications of changes in expectations about future policy. This essay contributes to the literature by analyzing how variations over the conventional modeling strategy can change the business cycle implications of the model.

In particular, I model changes in expectations about future policy by considering the case where agents receive news about changes in future policy every period. These news—which will be precisely defined in Section 2.3—directly affect the agents' expected value for fiscal policy variables; in this way, expectations of future policy can change even though actual policy remains the same.

To find out the importance of news about future fiscal policy, I compare this environment to an economy where agents receive no information whatsoever but is identical in every other respect. The logic underlying the availability of news about future policy is straightforward: changes in the future path of tax rates or government consumption are rarely surprises, as it is often the case that firms and households have a clear idea about the future path of policy variables. Proposed changes to existing tax laws and new pieces of legislation are subject to lengthy debates in Congress before they are approved; moreover, these debates are frequently highlighted in the media and their pros and cons are usually explained in detail.[1] Also, considerable lags can be introduced from the time the proposals are signed into law and the moment they go into effect. While it is true that the original proposal may not be exactly the same as the one that gets signed and implemented, it seems difficult to believe that the no news assumption holds in the real world.

The comparison is done relative to a simple DSGE model which is used as a benchmark. This model has three agents: a representative household, a representative firm, and a government. The representative household has habit formation in consumption, indivisible labor supply, variable capacity utilization, and includes investment adjustment costs. The representative firm is standard in every respect. Finally, the government levies taxes on capital and labor income, makes consumption purchases, and uses lump-sum transfers to balance the budget every period. As can be expected from the previous discussion, the expectation of the policy variables—tax rates and government consumption—is a function of the current value of these variables.

To accommodate the case where agents receive news about future policy, I modify the benchmark model to include the following informational assumption: agents receive news about changes in fiscal policy three periods before they occur. (Naturally, in the benchmark economy agents do not receive any news until the policy change gets implemented.)

In addition to solving both models, I use Bayesian techniques to estimate a subset of the parameter values, including the volatilities of the news terms. This is particularly important as the standard deviation of these terms has a first-order effect on the ability of the model with news to account for U.S. data.

[1] A clear example of this is the recent attention to the Bush Administration tax cuts, which were originally scheduled to expire at the beginning of 2011.

With both models and parameter value estimates at hand, I perform the following experiments. I first compare the impulse-response functions derived from the benchmark and news economy in this particular way: I assume that both economies rest at the steady state and derive the theoretical impulse-response function resulting from a one-time, one-percent increase in a policy variable which is set to occur three periods into the future; a comparison of the dynamic paths resulting from the impulse-response functions allows me to determine the effect of news. In particular, I focus on the evolution of a subset of variables (output, hours, consumption, and investment) during the periods prior to the policy change. (The actual policy is exactly the same in both economies; the only difference consists of the information set that the agents possess.)

The exercises I perform show that in the model economy where agents receive no news about the future, the relevant model variables stay at the steady state until the policy change occurs. On the contrary, in the model economy where agents receive news about changes in future policy, numerical examples show that, in the periods prior to the policy change,

(a) news about a future increase in the government consumption to output ratio makes output, investment and hours increase;

(b) news about a future increase in the labor income tax makes output, consumption, and hours fall, and

(c) news about a future increase in the capital income tax creates a recession.

I also calculate and compare the statistical properties of U.S. and model data. My results do not show a clear advantage of the news economy over the benchmark economy in terms of accounting for the statistical properties of U.S. data.

The rest of the essay is structured as follows. Section 2.1 briefly discusses how this paper is related to the current literature. Section 2.2 describes the benchmark economy, introduces a notion of equilibrium, and characterizes the optimality conditions of the agents. Section 2.3 analyzes the case where agents receive news about future changes in policy. Section 2.4 explains the estimation procedure in detail, and Section 2.5 presents the results of the quantitative exercise.

2.1 Relation to the literature

Research on the effects of changes in expectations about future policy is fairly recent.[2] In terms of models with news about changes in future policy, one of the earliest results was provided by Yang [9]. In her paper, she constructs a simple DSGE model with one and four-period quarter anticipation horizons which allow for agents to receive news about future changes to capital and labor income taxes. Qualitatively, she arrives at the same pattern of pre-implementation changes when news about future increases to the labor income tax arrive. However, the lack of real frictions—habit persistence, indivisible labor supply, variable capacity utilization, and investment adjustment costs—and use of calibrated parameter values does not allow her to reach the same quantitative results as I do.

A more recent paper is that of Mertens and Ravn [10], who estimate a VAR to obtain empirical impulse-responses to announced, exogenous changes in tax liabilities. (For this, they use the methodology of Romer and Romer [11]; they focus on 70 tax liability changes which they classify as exogenous.) They then construct a stylized DSGE model which is estimated by matching moments with the VAR. Contrary to my results, they find that news about future increases in capital income taxes are expansionary.[3]

The idea of using Bayesian methods to estimate a model with news shocks can be traced to Leeper et al. [15]; to the best of my knowledge, there has been no work which estimates the statistical properties of the fiscal policy news

[2] A related—and also recent—line of research explores the business cycle implications of changes in expectations about future total factor productivity. A (non-exhaustive) list of references should include Beaudry and Portier [4, 5], Christiano et al. [6], Jaimovich and Rebelo [7], and Schmitt-Grohe and Uribe [8].

[3] Other related papers which have been published in recent months are those of Leeper, Richter, and Walker [12] and Mertens and Ravn [13, 14].

terms using econometric methods.[4] The work of Schmitt-Grohe and Uribe [8] is the closest one to this idea; they construct a DSGE model with a similar set of frictions and allow for news over four different stochastic processes: permanent and stationary neutral productivity shocks, permanent investment-specific shocks, and a government spending shock. (The model lacks distortionary taxes.) They conclude that the government spending shock can account for a negligible fraction of the variance of output growth.

2.2 The benchmark model

The model economy contains three kinds of agents: a representative household, a representative firm, and a government. I discuss each of these in turn and then present a notion of equilibrium.[5]

2.2.1 Household

The representative household chooses sequences of consumption C_t, investment I_t, labor supply n_t, and capital utilization u_t to solve

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^t [\log(C_t - \nu C_{t-1}) + \phi(1 - n_t)] \qquad (2.1)\\
\text{s.t.} \quad & C_t + I_t = r_t u_t K_t + W_t n_t + V_t - \tau_{kt}(r_t - \delta(u_t))K_t - \tau_{nt} W_t n_t \\
& K_{t+1} = (1 - \delta(u_t))K_t + I_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] \\
& \delta(u) = \delta_0 + \delta_1(u-1) + \frac{\delta_2}{2}(u-1)^2 \\
& S(\iota) = \frac{\kappa}{2}(\iota - \mu)^2.
\end{aligned}$$

In the above, $\beta \in (0,1)$ is a discount factor, $\nu \in [0,1)$ is a parameter governing habit persistence in consumption, and $\phi \geq 0$ controls the relative weight of

[4] The use of Bayesian methods is considered "safe" given the problems associated with using VARs to identify the properties of the news components. See Leeper et al. [16] for details.

[5] The model below is loosely based in the one contained in Schmitt-Grohe and Uribe [8].

consumption and leisure. The stock of physical capital is denoted by K_t, which is rented to the representative firm at a rental rate r_t (adjusted by the capital utilization rate). The real wage rate is denoted by W_t. In addition, V_t denotes government lump-sum transfers/taxes, and the nonnegative variables τ_{kt} and τ_{nt} represent tax rates levied over capital and labor income, respectively.

The depreciation rate $\delta(u_t)$ is an increasing and convex function of the utilization rate; $\{\delta_0, \delta_1, \delta_2\}$ are nonnegative parameters which characterize this function. The household also faces adjustment costs for changing the level of investment as in Christiano et al. [17]; in this specification, $\kappa > 0$ is a parameter and μ denotes the steady-state growth rate of investment. I assume that the household behaves competitively and takes the processes for prices $\{r_t, W_t\}$ and fiscal policy $\{V_t, \tau_{kt}, \tau_{nt}\}$ as given.

2.2.2 Firm

Each period, the representative firm rents capital and labor inputs K_{Ft} and n_{Ft} from the household to solve the profit maximization problem

$$\begin{aligned} \max \quad & Y_t - r_t u_t K_{Ft} - W_t n_{Ft} \\ \text{s.t.} \quad & Y_t = z_t (u_t K_{Ft})^{\alpha} (X_t n_{Ft})^{1-\alpha}. \end{aligned} \tag{2.2}$$

In the above, $\alpha \in (0, 1)$ denotes the capital share, z_t is a stationary neutral productivity shock and X_t is a nonstationary neutral productivity shock. I assume that z_t satisfies

$$\log z_t = (1 - \rho_z) \log z + \rho_z \log z_{t-1} + \epsilon_{zt}, \tag{2.3}$$

where z is the steady-state level of z_t, $\rho_z \in (0, 1)$ is a persistence parameter and ϵ_{zt} is an i.i.d. process with mean zero and standard deviation σ_z. The logarithm of the permanent productivity shock X_t follows

$$\log X_t = \log X_{t-1} + \log \mu_t, \tag{2.4}$$

where $\log \mu_t$ is a stationary process of the form

$$\log \mu_t = (1 - \rho_m) \log \mu + \rho_m \log \mu_{t-1} + \epsilon_{mt}. \tag{2.5}$$

In equation (2.5), $\rho_m \in (0,1)$ is a persistence parameter and ϵ_{mt} is an i.i.d. process with mean zero and standard deviation σ_m. I assume that the firm behaves competitively and takes the processes for prices $\{r_t, W_t\}$ as given.

2.2.3 Government

The government levies taxes over capital and labor income at rates τ_{kt} and τ_{nt}, respectively, to finance an exogenous sequence of consumption G_t. It then sets lump-sum taxes/transfers V_t to balance the constraint

$$G_t + V_t = \tau_{kt}(r_t - \delta(u_t))u_t K_t + \tau_{nt} W_t n_t.$$

Denote the ratio of government consumption to output by g_t:

$$g_t = \frac{G_t}{Y_t}$$

so that the government budget constraint can be expressed as

$$g_t Y_t + V_t = \tau_{kt}(r_t - \delta(u_t))u_t K_t + \tau_{nt} W_t n_t. \tag{2.6}$$

I assume that the processes $\{\tau_{kt}, \tau_{nt}, g_t\}$ are stationary and follow

$$\tau_{kt} = (1-\rho_k)\tau_k + \rho_k \tau_{k,t-1} + \epsilon_{kt} \tag{2.7}$$

$$\tau_{nt} = (1-\rho_n)\tau_n + \rho_n \tau_{n,t-1} + \epsilon_{nt} \tag{2.8}$$

$$g_t = (1-\rho_g)g + \rho_g g_{t-1} + \epsilon_{gt}. \tag{2.9}$$

In the above, $\{\tau_k, \tau_n, g\}$ denote steady-state values for the policy variables. Also, for $j \in \{k, n, g\}$ each $\rho_j \in (0,1)$ is a persistence parameter and the ϵ_{jt} terms are i.i.d. disturbances with mean zero and standard deviation σ_j, uncorrelated across time and with each other.

2.2.4 Detrending

Given the stochastic trend in process X_t, I need to perform a transformation of the model's variables to induce stationarity. It is easy to verify that all non-stationary variables (denoted with capital letters) are affected by the stochastic trend.

To achieve stationarity, consider the transformations

$$c_t = \frac{C_t}{X_t},\ i_t = \frac{I_t}{X_t},\ k_t = \frac{K_t}{X_{t-1}},\ w_t = \frac{W_t}{X_t},\ v_t = \frac{V_t}{X_t},\ y_t = \frac{Y_t}{X_t},\ k_{Ft} = \frac{K_{Ft}}{X_{t-1}}$$

and let

$$\mu_t = \frac{X_t}{X_{t-1}}. \tag{2.10}$$

Using these equivalences, I now adjust the household's problem (2.1), firm's problem (2.2), and government budget (2.6).

Household's problem

I start with the household's objective function. Substituting for C_t I get

$$E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_t X_t - \nu c_{t-1} X_{t-1}) + \phi(1 - n_t)].$$

Now use (2.10):

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(c_t X_t - \frac{\nu c_{t-1} X_t}{\mu_t}\right) + \phi(1 - n_t)\right]$$

and after a bit of algebra,[6] I get that the stationary objective function for the household is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(c_t - \frac{\nu c_{t-1}}{\mu_t}\right) + \phi(1 - n_t)\right].$$

I can follow the same logic to obtain the household's stationary budget constraint:

$$c_t + i_t = \frac{r_t u_t k_t}{\mu_t} + w_t n_t + v_t - \frac{\tau_{kt}(r_t - \delta(u_t)) u_t k_t}{\mu_t} - \tau_{nt} w_t n_t$$

and the household's law of motion for capital

$$k_{t+1} = (1 - \delta(u_t)) k_t + i_t \left[1 - S\left(\frac{\mu_t i_t}{i_{t-1}}\right)\right].$$

[6] I drop off the term $E_0 \sum_{t=0}^{\infty} \beta^t \log X_t$ as it does not affect the ordering of preferences.

Note that the depreciation function $\delta(u_t)$ need not be adjusted as it is already stationary. Altogether, the household's problem (2.1) can be rewritten as

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left[\log \left(c_t - \frac{\nu c_{t-1}}{\mu_t} \right) + \phi(1 - n_t) \right] \\ \text{s.t.} \quad & c_t + i_t = \frac{r_t u_t k_t}{\mu_t} + w_t n_t + v_t - \frac{\tau_{kt}(r_t - \delta(u_t))u_t k_t}{\mu_t} - \tau_{nt} w_t n_t \\ & k_{t+1} = (1 - \delta(u_t))k_t + i_t \left[1 - S\left(\frac{\mu_t i_t}{i_{t-1}} \right) \right] \end{aligned} \tag{2.11}$$

where the functions $\delta(u_t)$ and $S(\mu_t i_t / i_{t-1})$ are defined in problem (2.1).

Firm's problem

The firm's objective function is transformed to

$$y_t - \frac{r_t u_t k_{Ft}}{\mu_t} - w_t n_{Ft}$$

while the production function is now

$$y_t = z_t \left(\frac{u_t k_{Ft}}{\mu_t} \right)^{\alpha} n_{Ft}^{1-\alpha}.$$

Hence, the firm's problem (2.2) can be expressed as

$$\begin{aligned} \max \quad & y_t - \frac{r_t u_t k_{Ft}}{\mu_t} - w_t n_{Ft} \\ \text{s.t.} \quad & y_t = z_t \left(\frac{u_t k_{Ft}}{\mu_t} \right)^{\alpha} n_{Ft}^{1-\alpha}. \end{aligned} \tag{2.12}$$

Government budget constraint

It is straightforward to verify that the government's stationary budget constraint equals

$$g_t y_t + v_t = \frac{\tau_{kt}(r_t - \delta(u_t))u_t k_t}{\mu_t} + \tau_{nt} w_t n_t. \tag{2.13}$$

2.2.5 State vector

Given the stochastic processes described in (2.3), (2.5), and (2.7)–(2.9), the exogenous state vector is given by $\{z_t, \mu_t, \tau_{kt}, \tau_{nt}, g_t\}$. The description of the model's state vector is completed by adding the endogenous state variables, which in this case correspond to the household's capital stock k_t and last period's consumption c_{t-1}. I will use $\mathbf{s}_t$ as shorthand for the state vector, so that

$$\mathbf{s}_t \equiv \{k_t, c_{t-1}; z_t, \mu_t, \tau_{kt}, \tau_{nt}, g_t\}.$$

In what follows, the history of states up to period t will be denoted by $\mathbf{s}^t$.

2.2.6 Characterizing the equilibrium

I now proceed to characterize the equilibrium conditions of the model. I first start by defining a competitive equilibrium in this economy:

Definition 2.2.1. A (tax-distorted) competitive equilibrium consists of sequences of prices $\{r(\mathbf{s}^t), w(\mathbf{s}^t)\}$, household allocations $\{c(\mathbf{s}^t), i(\mathbf{s}^t), n(\mathbf{s}^t), u(\mathbf{s}^t)\}$, firm allocations $\{k_F(\mathbf{s}^t), n_F(\mathbf{s}^t)\}$, and government transfers $\{v(\mathbf{s}^t)\}$ such that, for all periods,

1. The sequences $\{c(\mathbf{s}^t), i(\mathbf{s}^t), n(\mathbf{s}^t), u(\mathbf{s}^t)\}$ solve the household's problem (2.11).
2. The sequences $\{k_F(\mathbf{s}^t), n_F(\mathbf{s}^t)\}$ solve the firm's problem (2.12).
3. The government constraint (2.13) holds.
4. The capital and labor markets clear:

$$k(\mathbf{s}^t) = k_F(\mathbf{s}^t), \;\; n(\mathbf{s}^t) = n_F(\mathbf{s}^t).$$

To characterize the equilibrium, I start from the firm's problem; straightforward calculations give that the first-order conditions with respect to capital and labor inputs are, respectively,

$$\begin{aligned} \frac{r_t u_t k_t}{\mu_t} &= \alpha z_t \left(\frac{u_t k_t}{\mu_t}\right)^{\alpha} n_{Ft}^{1-\alpha} \\ w_t n_t &= (1-\alpha) z_t \left(\frac{u_t k_{Ft}}{\mu_t}\right)^{\alpha} n_{Ft}^{1-\alpha}. \end{aligned}$$

If I impose market-clearing and use that y_t denotes aggregate output, the firm's equilibrium conditions become

$$y_t = z_t \left(\frac{u_t k_t}{\mu_t}\right)^{\alpha} n_t^{1-\alpha} \tag{2.14}$$

$$\alpha y_t = \frac{r_t u_t k_t}{\mu_t} \tag{2.15}$$

$$(1-\alpha) y_t = w_t n_t. \tag{2.16}$$

To tackle the household's problem, I set up the Lagrangian

$$\begin{aligned}
\mathscr{L} = \; & E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \log\left(c_t - \frac{\nu c_{t-1}}{\mu_t}\right) + \phi(1-n_t) \\
& + \lambda_t \left[\frac{r_t u_t k_t}{\mu_t} + w_t n_t + v_t - \frac{\tau_{kt}(r_t - \delta(u_t)) u_t k_t}{\mu_t} - \tau_{nt} w_t n_t - c_t - i_t\right] \\
& + \lambda_t q_t \left[(1-\delta(u_t)) k_t + i_t \left[1 - S\left(\frac{\mu_t i_t}{i_{t-1}}\right)\right] - k_{t+1}\right] \Bigg\},
\end{aligned}$$

where λ_t and $\lambda_t q_t$ are nonnegative Lagrange multipliers and the functions $\delta(u_t)$ and $S(\mu_t i_t / i_{t-1})$ are defined in problem (2.1). The household's first-order conditions with respect to consumption, investment, labor supply, utilization, and capital stock are given by, respectively,

$$\lambda_t = \frac{1}{c_t - \nu c_{t-1}/\mu_t} - \beta \nu E_t \frac{1}{\mu_{t+1} c_{t+1} - \nu c_t} \tag{2.17}$$

$$\begin{aligned}
\lambda_t = \; & \lambda_t q_t \left[1 - S\left(\frac{\mu_t i_t}{i_{t-1}}\right) - \kappa\left(\frac{\mu_t i_t}{i_{t-1}} - \mu\right)\frac{\mu_t i_t}{i_{t-1}}\right] \\
& + \beta \kappa E_t \lambda_{t+1} q_{t+1} \mu_{t+1} \left(\frac{\mu_{t+1} i_{t+1}}{i_t} - \mu\right)\left(\frac{i_{t+1}}{i_t}\right)^2
\end{aligned} \tag{2.18}$$

$$\phi = \lambda_t (1 - \tau_{nt}) w_t \tag{2.19}$$

$$r_t = \tau_{kt}[r_t - \delta(u_t) - u_t(\delta_1 + \delta_2(u_t - 1))] + q_t(\delta_1 + \delta_2(u_t - 1)) \tag{2.20}$$

$$\lambda_t q_t = \beta E_t \frac{\lambda_{t+1}}{\mu_{t+1}} [r_{t+1} u_{t+1} - \tau_{k,t+1}(r_{t+1} - \delta(u_{t+1})) + q_{t+1}(1 - \delta(u_{t+1})]. \tag{2.21}$$

I also need to add the budget constraint and the law of motion for capital:

$$c_t + i_t = \frac{r_t u_t k_t}{\mu_t} + w_t n_t + v_t - \frac{\tau_{kt}(r_t - \delta(u_t)) u_t k_t}{\mu_t} - \tau_{nt} w_t n_t \tag{2.22}$$

$$k_{t+1} = (1 - \delta(u_t)) k_t + i_t \left[1 - S\left(\frac{\mu_t i_t}{i_{t-1}}\right)\right]. \tag{2.23}$$

I can solve the government's constraint (2.13) for v_t and combine it with the household's budget constraint (2.22) to get

$$c_t + x_t + g_t y_t = \frac{r_t u_t k_t}{\mu_t} + w_t n_t.$$

If I use (2.15) and (2.16) in the expression above, I obtain the aggregate feasibility constraint

$$(1 - g_t) y_t = c_t + x_t. \tag{2.24}$$

Thus, the equilibrium in this economy is described by equations (2.14)–(2.21), (2.23), and (2.24); I also need to add the laws of motion (2.3), (2.5), and (2.7)–(2.9).

2.3 A model with news about future fiscal policy

In this section I review the case where agents receive news about changes in future policy every period. I modify the benchmark model to include the following informational assumption: agents receive news about changes in fiscal policy three periods *before* they occur. This can be contrasted with the benchmark economy, where agents do not receive any news until the policy change gets implemented.

In order to accommodate for news about future policy, I first detail how the laws of motion for fiscal policy change to include news. Then, I redeclare the state vector and adjust my definition of competitive equilibrium.

2.3.1 News about future fiscal policy

Let $x_t \in \{\tau_{kt}, \tau_{nt}, g_t\}$ be a fiscal policy process, which according to (2.7)–(2.9) follows

$$x_t = (1 - \rho)x + \rho x_{t-1} + \epsilon_{xt}.$$

News about future fiscal policy enter the ϵ_{xt} term as

$$\epsilon_{xt} = \mu_{xt} + \xi^t_{x,t-1} + \xi^t_{x,t-2} + \cdots + \xi^t_{x,t-p} \tag{2.25}$$

where μ_{xt} and each of the $\xi^t_{x,t-j}$ terms ($j = 1, \ldots, p$) are i.i.d. disturbances with mean zero and finite variance, uncorrelated across time and with each other. In this case the μ_{xt} term is an unanticipated shock; the $\xi^t_{x,t-j}$ terms are news about future policy, representing a j-period anticipated change in x_t (i.e., $\xi^t_{x,t-j}$ is a change in period t policy that is revealed in period $t - j$).

Remark 2.3.1. The variable $p \geq 1$ represents the anticipation horizon. In what follows, I will set $p = 3$ for all of the fiscal policy processes (2.7)–(2.9).

The key departure from the standard informational framework is the assumption that agents have an information set much larger than the one containing current and past realizations of ϵ_{xt}: in period t, agents are assumed to observe current *and* past values of the innovations μ_{xt}, ξ^{t+1}_{xt}, ξ^{t+2}_{xt}, and ξ^{t+3}_{xt}. Agents can use this information to forecast ϵ_x in period t, up to a finite horizon, since

$$\begin{aligned} E_t\epsilon_{x,t+1} &= \xi^{t+1}_{xt} + \xi^{t+2}_{x,t-1} + \xi^{t+3}_{x,t-2} \\ E_t\epsilon_{x,t+2} &= \xi^{t+2}_{xt} + \xi^{t+3}_{x,t-1} \\ E_t\epsilon_{x,t+3} &= \xi^{t+3}_{xt}. \end{aligned}$$

Note that $E_t\epsilon_{x,t+s} = 0$ for $s \geq 3$.

2.3.2 State vector

Given the stochastic processes described in (2.3), (2.5), and (2.7)–(2.9), the state vector $\mathbf{s}_t$ from Section 2.2.5 can be updated to

$$\tilde{\mathbf{s}}_t \equiv \{k_t, c_{t-1}; z_t, \mu_t, \tau_{kt}, \{\xi^{t+j}_{k,t-j+1}\}^3_{j=1}, \tau_{nt}, \{\xi^{t+j}_{n,t-j+1}\}^3_{j=1}, g_t, \{\xi^{t+j}_{g,t-j+1}\}^3_{j=1}\}.$$

The history of states up to period t will be denoted by $\tilde{\mathbf{s}}^t$.

2.3.3 Characterizing the equilibrium

It is straightforward to verify that Definition 2.2.1 is valid, provided the state vector now equals $\tilde{\mathbf{s}}_t$. The equilibrium in this economy is still described by equations (2.14)–(2.21), (2.23) and (2.24). While the laws of motion (2.3) and (2.5) still hold, I need to modify (2.7)–(2.9) to include the news terms as described in (2.25). Clearly, this changes the approximated laws of motion for the model variables.

2.4 Estimating the models' parameters

In this section I estimate some of the parameters of the models. I calibrate a subset of the parameters and use Bayesian techniques to estimate the remaining values. Section 2.4.1 discusses the data used in the estimation; Section 2.4.2 presents the values for the calibrated parameters, and Section 2.4.3 shows the details of the estimation procedure.

2.4.1 Data

I use U.S. data from 1955Q1 to 2006Q4. The series used as observables are the gross growth rates of output and consumption, the ratio of government consumption to output, and a measure of market hours as a fraction of total household time. I assume that all observables have measurement error, denoted ϵ_{jt}^{me} for $j \in \{c, y, g, n\}$. The vector of observable variables is

$$\begin{bmatrix} C_t/C_{t-1} \\ Y_t/Y_{t-1} \\ G_t/Y_t \\ n_t \end{bmatrix} + \begin{bmatrix} \epsilon_{ct}^{me} \\ \epsilon_{yt}^{me} \\ \epsilon_{gt}^{me} \\ \epsilon_{nt}^{me} \end{bmatrix} = \begin{bmatrix} \mu_t c_t/c_{t-1} \\ \mu_t y_t/y_{t-1} \\ g_t \\ n_t \end{bmatrix} + \begin{bmatrix} \epsilon_{ct}^{me} \\ \epsilon_{yt}^{me} \\ \epsilon_{gt}^{me} \\ \epsilon_{nt}^{me} \end{bmatrix},$$

where the left hand side of the expression corresponds to U.S. data and the right hand side corresponds to model variables. Details on the construction of the data series are contained in Appendix A.

2.4.2 Calibrated parameter values

Table 2.1 shows the calibrated parameters and their values. The discount factor β is calibrated to match an annual risk-free rate of 5%. The capital income share

α is set to 0.36, and the depreciation parameter δ_0 is consistent with an annual depreciation rate of 8%. I normalize the steady state level of technology z to unity and use historical averages to set the steady state levels of the capital (τ_k) and labor (τ_n) income tax rates to 0.42 and 0.26, respectively. This historical data also allows me to set the steady state government consumption to output ratio g at 0.21 and to pin down the average growth rate of the economy μ at 1.005 per quarter.

Table 2.1: Calibrated model parameters

Parameter	Description	Value
β	Discount factor	0.9879
α	Capital income share	0.36
δ_0	Depreciation rate	0.0194
z	Steady state technology level	1
τ_k	Steady state capital income tax rate	0.42
τ_n	Steady state labor income tax rate	0.26
g	Steady state government consumption to output ratio	0.15
μ	Steady state growth rate	1.005

2.4.3 Bayesian estimation

I use Bayesian estimation techniques to calculate values for the rest of the parameters; the prior distributions for these parameters are presented in Table 2.2.

Prior distributions

In Table 2.2 both the leisure weight (ϕ) and the adjustment cost (κ) parameters are assigned a gamma prior distribution with mean 4 and standard deviation 1. The habit persistence (ν) and autocorrelation (σ_j for $j \in \{z, m, k, n, g\}$) parameters have a beta prior distribution, which is subject to a linear transformation to avoid numerical instability (see Appendix A). Given lack of guidance, the remaining variables have a wide uniform prior; I set the standard deviations of the anticipated and unanticipated shocks to have an uniform distribution with an upper bound of 10%; I follow Schmitt-Grohe and Uribe [8] and set the volatility of the unanticipated shocks (σ_j for $j \in \{k, n, g\}$) to be higher than that of the anticipated shocks (σ_j^i for $i = 1, 2, 3$ and $j \in \{k, n, g\}$), so that the prior makes all anticipated shocks as important as the unanticipated shock (this insight follows Beaudry and Portier [18]). Finally, measurement errors (σ_j^{me} for $j \in \{c, y, g, n\}$) are set to have a volatility equal to 25% of that of the empirical variables.

Table 2.2: Prior distributions

Parameter	Description	Distribution	Mean	SD	LB	UB
ϕ	Leisure weight	Gamma	4	1		
κ	Adjustment cost	Gamma	4	1		
ν	Habit persistence	Beta*	0.5	0.1	0	0.99
$\rho_z, \rho_k, \rho_n, \rho_g$	Autocorrelation	Beta*	0.7	0.2	0	0.99
ρ_m	Autocorrelation	Beta*	0.0	0.1	-0.5	0.5
δ_2	Depreciation function	Uniform			0	0.5
σ_z, σ_m	Standard deviation	Uniform			0	50
$\sigma_k, \sigma_n, \sigma_g$	Standard deviation	Uniform			0	$50\sqrt{3}$
σ_j^i	Standard deviation	Uniform			0	50
σ_h^{me}	Measurement error	Uniform			0	$0.25\hat{\sigma}_h$

Notes: SD = standard deviation, LB = lower bound, UB = upper bound; $i = 1, 2, 3$; $j \in \{k, n, g\}$; $h \in \{c, y, g, n\}$. Values in the lower part of the table are expressed as percentages. The symbol $\hat{\sigma}_h$ denotes the sample standard deviation of the empirical measure of variable h. Beta* indicates that a linear transformation of the parameter has a beta prior distribution.

Posterior distributions

I use Dynare to get the parameter estimates for the benchmark economy and the economy with news. The results reported below correspond to 4 chains of 500,000 draws, discarding the first 100,000 draws.

Table 2.3 contains the results of the estimation procedure for the benchmark economy.[7] Some things are worth noting: first, the autocorrelation parameter

[7] In order to successfully implement the estimation routine, I drop the growth rate of consumption as an observable.

of the capital and labor income tax (and that of the government consumption to output ratio too) are close to unity; in particular, the mean value for the income tax autocorrelations is 0.99, even after considering the linear transformation to the estimated parameter.

Table 2.3: Posterior distributions, benchmark economy

	Prior			Posterior		
Parameter	Distrib.	Mean	SD	Mean	Conf. Interval	Description
ϕ	Gamma	4	1	4.24	$[3.95, 4.92]$	Leisure weight
κ	Gamma	4	1	3.54	$[2.99, 3.83]$	Adjustment cost
ν	Beta*	0.5	0.1	0.64	$[0.59, 0.69]$	Habit persistence
ρ_z	Beta*	0.7	0.2	0.80	$[0.72, 0.85]$	AR, technology
ρ_m	Beta*	0.0	0.1	0.08	$[0.05, 0.11]$	AR, growth
ρ_k	Beta*	0.7	0.2	0.99	$[0.99, 0.99]$	AR, capital tax
ρ_n	Beta*	0.7	0.2	0.99	$[0.99, 0.99]$	AR, labor tax
ρ_g	Beta*	0.7	0.2	0.97	$[0.96, 0.99]$	AR, government
δ_2	Uniform	2.5	1.44	0.21	$[0.19, 0.25]$	Depreciation
σ_z	Uniform	5	2.89	0.71	$[0.54, 0.89]$	SD, ϵ_{zt}
σ_m	Uniform	5	2.89	0.68	$[0.48, 0.86]$	SD, ϵ_{mt}
σ_k	Uniform	8.7	5	2.77	[1.98,3.60]	SD, ϵ_{kt}
σ_n	Uniform	8.7	5	1.48	$[1.23, 1.73]$	SD, ϵ_{nt}
σ_g	Uniform	8.7	5	0.19	$[0.17, 0.20]$	SD, ϵ_{gt}
σ_y^{me}	Uniform	0.1	0.06	0.19	$[0.16, 0.20]$	SD, ϵ_{yt}^{me}
σ_g^{me}	Uniform	0.1	0.06	0.08	$[0.07, 0.09]$	SD, ϵ_{gt}^{me}
σ_n^{me}	Uniform	0.2	0.1	0.01	$[0.00, 0.02]$	SD, ϵ_{nt}^{me}

Notes: AR = autoregresive, SD = standard deviation. Values in the lower part of the table are expressed as percentages. Beta* indicates that a linear transformation of the parameter has a beta prior distribution.

Table 2.4 contains the results of the estimation procedure for the economy with news. Relative to the benchmark economy case, only the autocorrelation parameter for the labor income tax rate hits the upper bound of 0.99. The volatility of the unanticipated shock to the capital income tax rate process is higher than in the benchmark economy (12.84 versus 2.77), and the volatilities of the news components are also somewhat high: the standard deviation of the two-period ahead news component is 6.72%, while that of the three-period ahead component is almost 10%. Also, several of the standard deviations have a zero lower bound in their confidence intervals (σ_n, σ_n^1, σ_n^2, and all of the government consumption to output ratio news terms.)

2.5 The quantitative effect of news

In this section I present two different ways of quantifying the effect of news. First, given the parameter values I estimated in Section 2.4, I derive the impulse-response functions of the model with news and compare them to the impulse-response functions of the benchmark economy. In addition, I provide a comparison between the statistical properties of the U.S. data and those generated by the benchmark economy and the model with news.

2.5.1 Impulse-response functions

The experiment I perform is as follows. I first assume that both economies rest at the steady state, and then derive the theoretical impulse-response function resulting from a one-time, one-percent increase in a policy variable (government consumption to output,[8] labor income tax rate, or capital income tax rate) which is set to occur three periods into the future. In particular, I focus on the evolution of a subset of model variables (output, consumption, investment, hours, and capital) during the time periods prior to the policy change. It is important to realize that the actual policy is exactly the same in both economies; the only difference consists of the information set that the agents possess.

The numerical exercises I perform show that in the model economy where agents receive no news about the future, the relevant model variables remain at the steady state until the policy change occurs. On the contrary, in the model

[8] The actual variable which is increased is the government consumption to output ratio, which in turn affects the government consumption variable. This follows the particular structure of the model, where the unanticipated and anticipated shocks affect the ratio and not the overall value of government consumption.

Table 2.4: Posterior distributions, economy with news

	Prior			Posterior		
Parameter	Distrib.	Mean	SD	Mean	Conf. Interval	Description
ϕ	Gamma	4	1	3.86	$[3.25, 4.49]$	Leisure weight
κ	Gamma	4	1	1.55	$[0.98, 2.31]$	Adjustment cost
ν	Beta*	0.5	0.1	0.69	$[0.57, 0.76]$	Habit persistence
ρ_z	Beta*	0.7	0.2	0.70	$[0.61, 0.84]$	AR, technology
ρ_m	Beta*	0.0	0.1	0.34	$[0.29, 0.38]$	AR, growth
ρ_k	Beta*	0.7	0.2	0.84	$[0.78, 0.89]$	AR, capital tax
ρ_n	Beta*	0.7	0.2	0.99	$[0.99, 0.99]$	AR, labor tax
ρ_g	Beta*	0.7	0.2	0.97	$[0.96, 0.99]$	AR, government
δ_2	Uniform	2.5	1.44	4.35	$[3.73, 5.00]$	Depreciation
σ_z	Uniform	5	2.89	0.64	$[0.55, 0.72]$	SD, ϵ_{zt}
σ_m	Uniform	5	2.89	0.46	$[0.38, 0.53]$	SD, ϵ_{mt}
σ_k	Uniform	8.7	5	12.84	[9.11,16.39]	SD, μ_{kt}
σ_k^1	Uniform	5	2.89	8.78	$[7.29, 10.00]$	SD, $\xi_{k,t-1}^t$
σ_k^2	Uniform	5	2.89	6.72	$[4.78, 8.75]$	SD, $\xi_{k,t-2}^t$
σ_k^3	Uniform	5	2.89	9.15	$[8.06, 10.00]$	SD, $\xi_{k,t-3}^t$
σ_n	Uniform	8.7	5	0.24	$[0.00, 0.48]$	SD, μ_{nt}
σ_n^1	Uniform	5	2.89	0.30	$[0.00, 0.56]$	SD, $\xi_{n,t-1}^t$
σ_n^2	Uniform	5	2.89	0.45	$[0.00, 0.85]$	SD, $\xi_{n,t-2}^t$
σ_n^3	Uniform	5	2.89	1.03	$[0.43, 1.71]$	SD, $\xi_{n,t-3}^t$
σ_g	Uniform	8.7	5	0.07	$[0.00, 0.14]$	SD, μ_{gt}
σ_g^1	Uniform	5	2.89	0.08	$[0.00, 0.15]$	SD, $\xi_{g,t-1}^t$
σ_g^2	Uniform	5	2.89	0.08	$[0.00, 0.15]$	SD, $\xi_{g,t-2}^t$
σ_g^3	Uniform	5	2.89	0.08	$[0.00, 0.14]$	SD, $\xi_{g,t-3}^t$
σ_c^{me}	Uniform	0.1	0.04	0.09	$[0.06, 0.11]$	SD, ϵ_{ct}^{me}
σ_y^{me}	Uniform	0.1	0.06	0.22	$[0.20, 0.22]$	SD, ϵ_{yt}^{me}
σ_g^{me}	Uniform	0.1	0.06	0.09	$[0.07, 0.12]$	SD, ϵ_{gt}^{me}
σ_n^{me}	Uniform	0.2	0.1	0.07	$[0.05, 0.10]$	SD, ϵ_{nt}^{me}

Notes: AR = autoregresive, SD = standard deviation. Values in the lower part of the table are in percent. Beta* indicates that a linear transformation of the parameter has a beta prior distribution.

economy where agents receive news about changes in future policy, numerical examples show that, in the periods prior to the policy change,

(a) news about a future increase in the government consumption to output ratio makes output, investment and hours increase;

(b) news about a future increase in the labor income tax makes output, consumption, and hours fall, and

(c) news about a future increase in the capital income tax creates a recession.

News shock to the government consumption to output ratio

Figures 2.1 shows the effect of a news shock to the government consumption to output ratio. To begin, note that the top left panel shows the evolution of the policy variable throughout the impulse-response exercise. For both economies, the realized path of g is the same (except for the effect of a slightly different autocorrelation coefficient); the differences between the benchmark economy and the economy with news emerge only from the different informational assumption.

In the benchmark economy, the model variables remain at their steady state level until the change in policy gets implemented in period 3. The theoretical impulse-responses are consistent with standard results: output and hours increase while consumption falls. In this case, the adjustment cost makes investment increase after the policy is implemented. At their maximum deviation, output, investment, and hours jump to 1.2, 1.8, and 1.2% above their steady state level, respectively, while consumption falls by about 0.4%.

In the economy where agents receive news about future policy, the announcement of a 1% increase in g which will take place three periods into the future generates immediate responses in output, investment, and hours. While the change in output and hours is marginal, investment jumps by 0.6% on impact, and increases to about 1.3% at its peak.

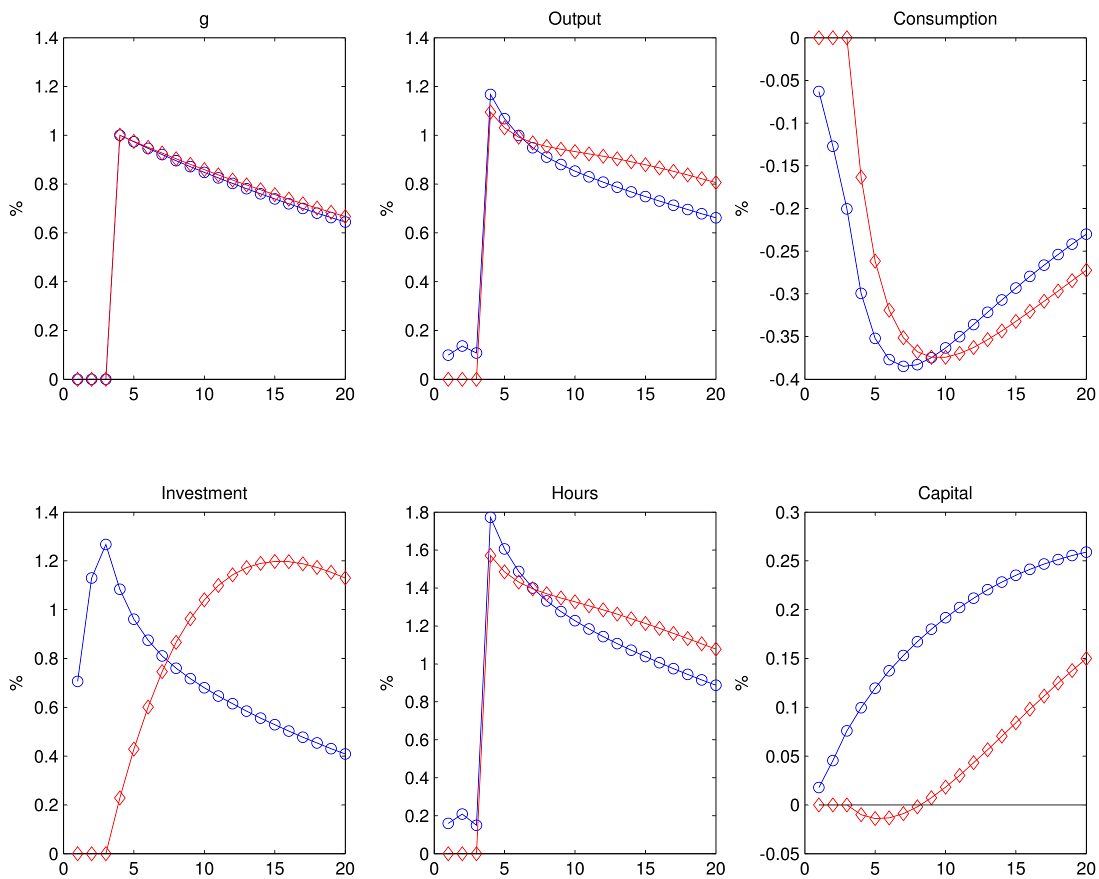

Figure 2.1: Shock to g.
Blue line (circles): economy with news. Red line (diamonds): benchmark economy.

News shock to the labor income tax rate

The effect of a news shock to the labor income tax rate is presented in Figure 2.2. I first point out to the fact that the model variables in the benchmark economy remain at their steady state until period 3.

When considering the economy with news, output, consumption, and hours fall on impact while investment slightly increases. In detail, hours are reduced marginally on impact, yet they fall to about 0.5% below their steady state level by period 3. The evolution of output and consumption is similar: a marginal reduction on impact and then a gradual decline over time. Investment increases to about 0.5% above the steady state during the first two periods, only to begin a consistent slide which reaches a trough at about 2.5% below the steady state level. Note that the trough of investment is actually reached at a smaller deviation from steady state, relative to the benchmark model (which hits the trough at about 3%).

News shock to the capital income tax rate

The case of a news shock to the capital income tax rate is presented in Figure 2.3. In the benchmark model, the increase in the tax rate moves investment and hours below their steady state level after period 3 while output and consumption increase as soon as the unanticipated shock is realized. The jump in output, however, is considerably smaller than the fall in investment or hours.

In the economy with news, the news shock to the capital income tax rate is consistent with a recession. Output, investment, and hours fall before the policy change takes place, and while consumption increases on impact, the size of the movement is minimal and the variable begins a decline by period 5. Note that the peaks and troughs of the benchmark and news economies are different: adjustment speed (back to steady state) is faster in the economy with news (which is evident from the top left panel, since the autocorrelation coefficients are different); moreover, the benchmark economy has deeper troughs and higher peaks

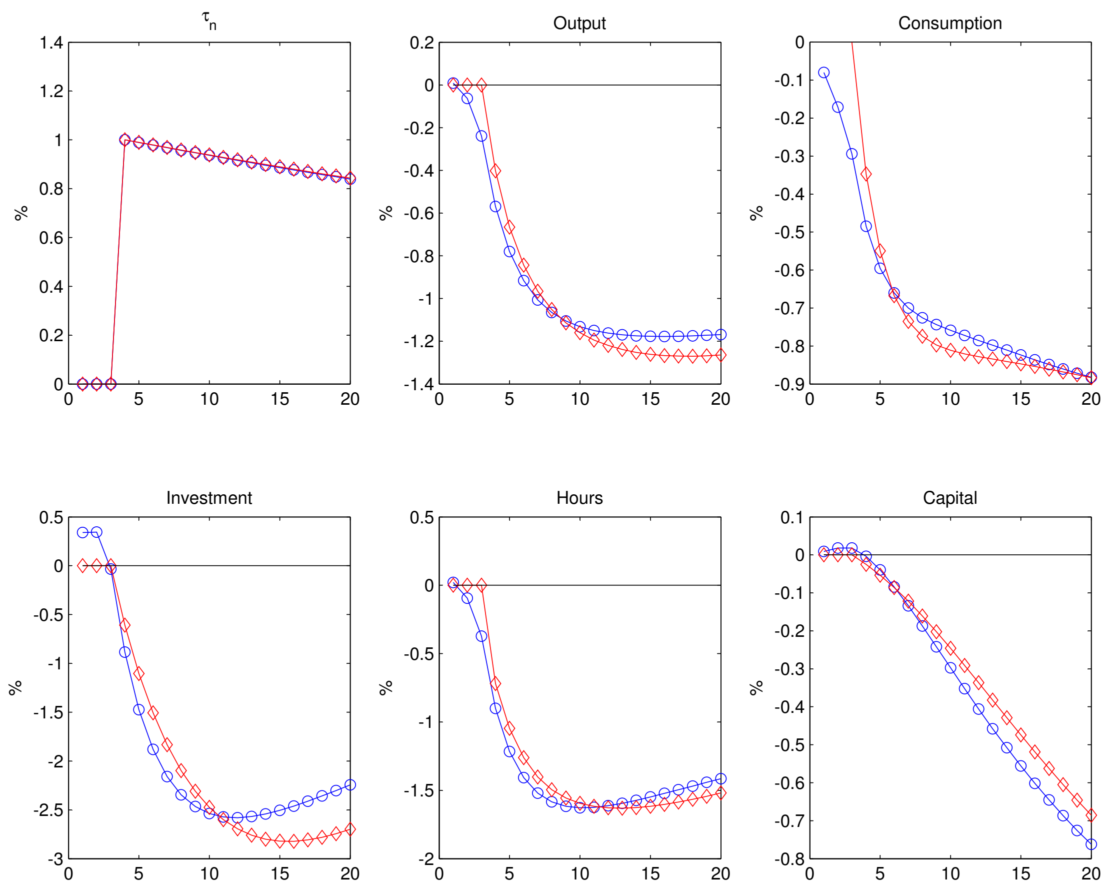

Figure 2.2: Shock to τ_n.
Blue line (circles): economy with news. Red line (diamonds): benchmark economy.

when compared to the economy with news.

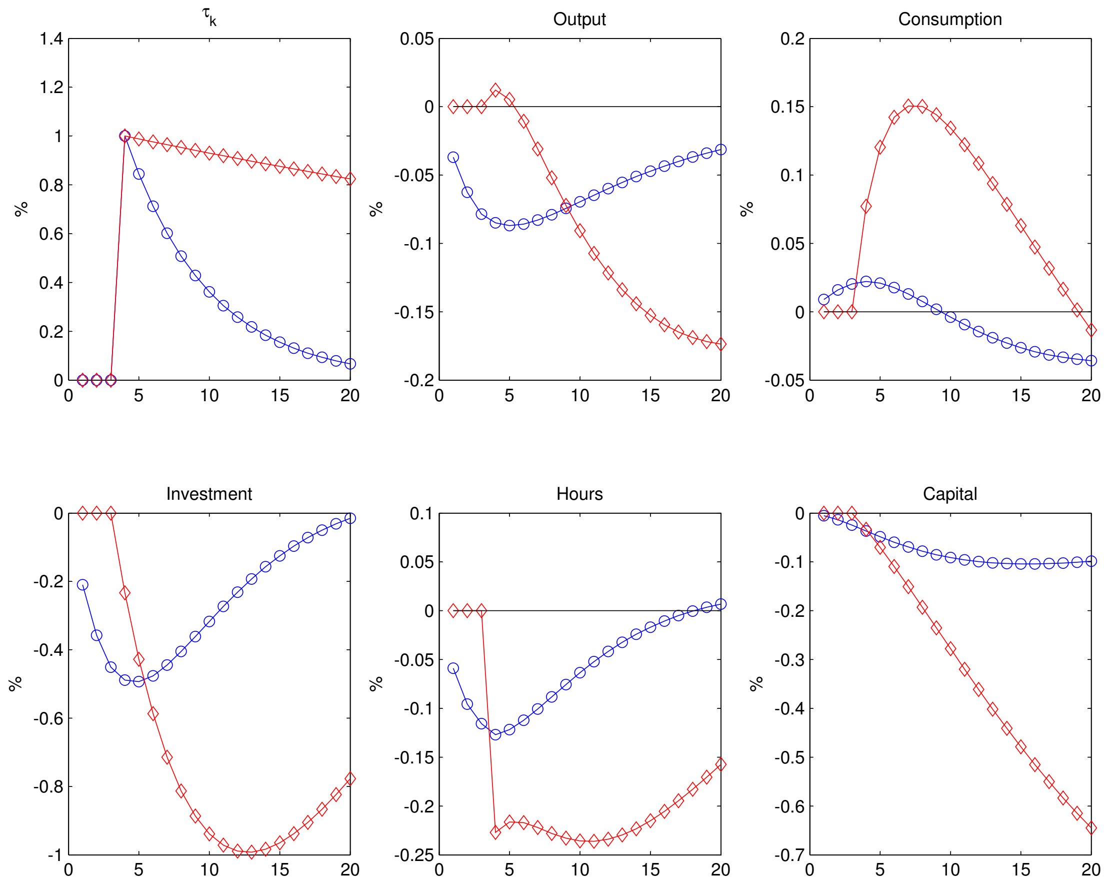

Figure 2.3: Shock to τ_k.
Blue line (circles): economy with news. Red line (diamonds): benchmark economy.

2.5.2 Statistical properties of the model

This section calculates the statistical properties of the U.S. and model data. The summary of these values is presented in Table 2.5. The top portion of the table shows the properties of the (actual) U.S. data. The second column displays the standard deviation of the observed variable relative to output; columns 3 to 7 present the correlation of the observed variable with output at different lags and leads.

Table 2.5: Statistical properties: data vs. models

		Correlation with output at lag $k =$				
Variable	Relative SD	$+2$	$+1$	0	-1	-2
Data						
Output	1.000					
Consumption	0.588	0.549	0.718	0.819	0.807	0.712
Investment	2.935	0.601	0.805	0.944	0.764	0.531
Government	0.962	0.021	-0.005	0.020	0.020	0.027
Hours	0.916	0.518	0.700	0.839	0.823	0.720
Benchmark						
Output	1.000					
Consumption	0.793	0.607	0.749	0.871	0.715	0.540
Investment	2.625	0.521	0.685	0.797	0.748	0.646
Government	1.498	0.556	0.667	0.793	0.638	0.488
Hours	1.481	0.648	0.762	0.839	0.723	0.555
News						
Output	1.000					
Consumption	0.510	0.034	0.030	0.014	-0.044	-0.094
Investment	5.361	0.632	0.813	0.913	0.871	0.741
Government	1.098	0.710	0.833	0.886	0.760	0.565
Hours	1.611	0.741	0.885	0.947	0.864	0.676

The mid part of the table contains the statistical properties of artificial data generated by the benchmark model. In terms of the relative standard deviation measure, consumption, government consumption, and hours are more volatile than their counterparts in the U.S. data, while investment is slightly lower. In terms of correlations the benchmark model seems to do a good job at capturing the interaction between output and model variables, yet the model is unable to account for the near-zero correlations of output and government consumption.

The bottom part of the table shows the statistical properties derived from data simulated in the news economy. Compared to the benchmark model, the relative volatility of consumption and government consumption has a better fit, but the values for investment and hours are much higher than those in the U.S. data. In terms of correlations, the news economy fails to match the relationship between consumption and output at all leads and lags, and it experiences the same problem as the benchmark economy, in being unable to account for the correlations between government consumption and output.

Overall, the results from Table 2.5 do not show a clear advantage of the news economy over the benchmark economy, in terms of accounting for the statistical properties of the U.S. data.

Chapter 3

Tax Rules and Expectations

In this essay, I analyze how fiscal policy affects the economy when agents' expectations about future policy are derived from a simple tax rule which ties the expectation of future tax rates to the current deficit and government debt levels.

To find out how fiscal policy works in such an environment, I compare the theoretical impulse-response functions of the model to those of a standard one where expectations of future policy are determined by current policy only: I build a simple DSGE model to be used as a benchmark upon which I can compare the two cases detailed above. This model has three agents: a representative household, a representative firm, and a government. The representative household has habit formation in consumption, indivisible labor supply, includes capital adjustment costs, and trades bonds with the government. The representative firm is standard in every respect. Finally, the government levies taxes on capital and labor income, makes consumption purchases and lump-sum transfers, and trades bonds with the household. As can be expected from the previous discussion, the expectation of the policy variables in the benchmark model—tax rates, government consumption, and transfers—is a function of the current value of the variables.

I then construct a particular rule which ties the expected capital income tax rate next period to the current period's deficit, debt, and other state variables. I assume that both economies rest at the steady state, and then derive the theoretical impulse-response function resulting from a one-time, one-percent increase in the economy's exogenous variables (productivity, government consumption, and transfers) occurring in period zero. I focus on the evolution of a subset of variables (output, capital, hours, consumption, investment, government debt, and the tax rate) in the periods posterior to the shock.

The numerical exercises show that the behavior of an economy where capital income tax expectations are derived from a tax rule is significantly different to that where expectations depend only on the current value of the tax rate. In particular,

(a) an increase in productivity yields the same qualitative response in the benchmark economy and in the economy with a tax rule—output, consumption, and investment increase—yet the quantitative response is amplified when the tax rule is present;

(b) an increase in government consumption produces the standard crowding-out response in both economies in the short run, but there is a long-lasting recession in the case where a tax rule is present, and

(c) an increase in transfers has no effect on the benchmark economy but can create a persistent recession when a tax rule is present.

The rest of the essay is structured as follows. Section 3.1 briefly discusses how this paper is related to the current literature. Section 3.2 describes the benchmark economy, introduces a notion of equilibrium and characterizes the optimality conditions of the agents, and then Section 3.3 reviews the required changes to allow expectations of future policy to be based on a tax rule. Finally, Section 3.4 presents some quantitative results.

3.1 Relation to the literature

The closest work to this essay is the recent paper by Uhlig [19]. He constructs a simple DSGE model with a tax rule which ties the labor income tax rate to the current deficit. The goal of Uhlig's paper, however, is to show that the government multipliers for an economy calibrated to the U.S. are misleading when this rule is in effect and a long-run horizon is considered.

Relative to his results, I change the structure of the tax rules to consider the capital income tax rate (as opposed to the labor income tax rate), and allow for a richer model which includes a set of real frictions which are not present in his contribution. Moreover, I modify the rules in such a way that the *expected* tax rates (and not the current ones) depend on the current deficit and debt levels, and these expected levels are incorporated to the economy's equilibrium.

3.2 The benchmark model

The model economy is subject to three kinds of agents: a representative household, a representative firm, and a government. I discuss each of these in turn and then present a notion of equilibrium.

3.2.1 Household

The representative household includes three real rigidities: habit formation in consumption, indivisible labor supply, and capital adjustment costs. In particular, the household chooses sequences of consumption c_t, labor supply n_t, investment x_t, and bond holdings b_t to solve

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_t - \nu c_{t-1}) + \phi(1 - n_t)] \qquad (3.1) \\ \text{s.t.} \quad & c_t + x_t + b_t = r_t k_t + w_t n_t + R_t^b b_{t-1} + v_t - \tau_{kt}(r_t - \delta)k_t - \tau_{nt} w_t n_t \\ & k_{t+1} = (1-\delta)k_t + x_t - \frac{\eta}{2}\left(\frac{x_t}{k_t} - \delta\right)^2 k_t. \end{aligned}$$

In the above, $\beta \in (0,1)$ is a discount factor, $\nu \in [0,1)$ is a parameter governing habit persistence, and $\phi \geq 0$ controls the relative weight of consumption and leisure. The stock of physical capital is denoted by k_t, which is rented to the representative firm at a rental rate r_t. The real wage rate is denoted by w_t. In addition, R_t^b denotes the gross rate of return on government bonds (the rate is fixed in period $t-1$), v_t denotes government lump-sum transfers/taxes, and the nonnegative variables τ_{kt} and τ_{nt} represent tax rates levied over capital income (net of depreciation allowances) and labor income, respectively. Finally, $\delta \in (0,1)$ is the depreciation rate of the capital stock and $\eta \geq 0$ is a parameter which characterizes the adjustment costs. I assume that the household behaves competitively and takes the processes for prices $\{r_t, w_t, R_t^b\}$ and fiscal policy $\{v_t, \tau_{kt}, \tau_{nt}\}$ as given.

3.2.2 Firm

The representative firm rents capital and labor inputs, K_t and N_t, from the household to solve

$$\begin{aligned} \max \quad & z_t K_t^{\alpha} N_t^{1-\alpha} - r_t K_t - w_t N_t \\ \text{s.t.} \quad & K_t \geq 0,\ N_t \geq 0. \end{aligned} \tag{3.2}$$

In the above, $\alpha \in (0,1)$ represents the capital share and z_t is a stationary technology disturbance. I assume that z_t follows

$$\log z_t = (1-\rho_z)\log z + \rho_z \log z_{t-1} + \epsilon_{zt}, \tag{3.3}$$

where z is the steady-state level of z_t, $\rho_z \in (0,1)$ is a persistence parameter and ϵ_{zt} is an i.i.d. process with mean zero and standard deviation σ_z. I assume that the firm behaves competitively and takes the processes for prices $\{r_t, w_t\}$ as given.

3.2.3 Government

The government trades bonds B_t with the household and levies taxes over capital and labor income (at rates τ_{kt} and τ_{nt}, respectively), to finance bond payments

$R_t^b B_{t-1}$ and exogenous sequences of consumption g_t and transfers v_t. The government's budget constraint is

$$g_t + v_t + R_t^b B_{t-1} = B_t + \tau_{kt}(r_t - \delta)k_t + \tau_{nt} w_t n_t \tag{3.4}$$

where the processes for taxes, government consumption, and transfers satisfy

$$\begin{aligned}
\tau_{kt} &= (1-\rho_k)\tau_k + \rho_k \tau_{k,t-1} + \epsilon_{kt} && (3.5)\\
\tau_{nt} &= (1-\rho_n)\tau_n + \rho_n \tau_{n,t-1} + \epsilon_{nt} && (3.6)\\
g_t &= (1-\rho_g)g + \rho_g g_{t-1} + \epsilon_{gt} && (3.7)\\
v_t &= (1-\rho_v)v + \rho_v v_{t-1} + \epsilon_{vt}. && (3.8)
\end{aligned}$$

In the above, $\{\tau_k, \tau_n, g, v\}$ denote steady-state values for the policy variables. Also, for $j \in \{k, n, g, v\}$ each $\rho_j \in (0,1)$ represents the persistence of the stochastic process and the ϵ_{jt} terms are i.i.d. disturbances with mean zero and standard deviation σ_j, uncorrelated across time and with each other.

3.2.4 State vector

Given the stochastic processes described in (3.3) and (3.5)–(3.8), the exogenous state vector is given by $\{z_t, \tau_{kt}, \tau_{nt}, g_t, v_t\}$. The description of the model's state vector is completed by adding the endogenous state variables, which in this case correspond to the household's capital stock k_t and last period's consumption c_{t-1}. I will use $\mathbf{s}_t$ as shorthand for the state vector, so that

$$\mathbf{s}_t \equiv \{k_t, c_{t-1}; z_t, \tau_{kt}, \tau_{nt}, g_t, v_t\}.$$

In what follows, the history of states up to period t will be denoted by $\mathbf{s}^t$.

3.2.5 Characterizing the equilibrium

I now proceed to characterize the equilibrium conditions of the model. I first start by defining a competitive equilibrium in this economy:

Definition 3.2.1. A (tax-distorted) competitive equilibrium consists of sequences of prices $\{r(\mathbf{s}^t), w(\mathbf{s}^t), R^b(\mathbf{s}^t)\}$, household allocations $\{c(\mathbf{s}^t), n(\mathbf{s}^t), x(\mathbf{s}^t), b(\mathbf{s}^t)\}$, firm allocations $\{K(\mathbf{s}^t), N(\mathbf{s}^t)\}$, and government bonds $\{B(\mathbf{s}^t)\}$ such that, for all periods,

1. The sequences $\{c(\mathbf{s}^t), n(\mathbf{s}^t), x(\mathbf{s}^t), b(\mathbf{s}^t)\}$ solve the household problem (3.1).
2. The sequences $\{K(\mathbf{s}^t), N(\mathbf{s}^t)\}$ solve the firm problem (3.2).
3. The government constraint (3.4) holds.
4. The bond, capital, and labor markets clear:

$$b(\mathbf{s}^t) = B(\mathbf{s}^t),\ k(\mathbf{s}^t) = K(\mathbf{s}^t),\ n(\mathbf{s}^t) = N(\mathbf{s}^t).$$

To characterize the equilibrium, I start from the firm's problem; straightforward calculations give that the first-order conditions with respect to capital and labor inputs are, respectively,

$$\begin{aligned} \alpha z_t K_t^{\alpha-1} N_t^{1-\alpha} &= r_t \\ (1-\alpha) z_t K_t^{\alpha} N_t^{-\alpha} &= w_t. \end{aligned}$$

Let $y_t \equiv z_t K_t^{\alpha} N_t^{1-\alpha}$ denote aggregate output. If I impose the market-clearing conditions for the capital and labor markets, the firm's equilibrium conditions become

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha} \tag{3.9}$$

$$r_t k_t = \alpha y_t \tag{3.10}$$

$$w_t n_t = (1-\alpha) y_t. \tag{3.11}$$

To tackle the household's problem, I set up the Lagrangian

$$\mathscr{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \log(c_t - \nu c_{t-1}) + \phi(1-n_t) + \lambda_t[(1-\tau_{kt}) r_t k_t + (1-\tau_{nt}) w_t n_t$$
$$+ \delta \tau_{kt} k_t + R_t^b b_{t-1} + v_t - c_t - x_t - b_t] + \chi_t \left[(1-\delta) k_t + x_t - \frac{\eta}{2} \left(\frac{x_t}{k_t} - \delta \right)^2 k_t - k_{t+1} \right] \Bigg\},$$

where λ_t and χ_t are nonnegative Lagrange multipliers. The household's first-order conditions with respect to consumption, labor supply, investment, bond holdings, and capital stock are given by, respectively,

$$\begin{aligned}
\lambda_t &= \frac{1}{c_t - \nu c_{t-1}} - \beta \nu E_t \frac{1}{c_{t+1} - \nu c_t} && (3.12)\\
\phi &= \lambda_t (1 - \tau_{nt}) w_t && (3.13)\\
\lambda_t &= \chi_t \left[1 - \eta \left(\frac{x_t}{k_t} - \delta \right) \right] && (3.14)\\
\chi_t &= \beta E_t \lambda_{t+1} R^b_{t+1} && (3.15)\\
\chi_t &= \beta E_t \lambda_{t+1} [(1 - \tau_{k,t+1}) r_{t+1} + \delta \tau_{k,t+1}] \\
&\quad + \beta E_t \chi_{t+1} \left[1 - \delta + \eta \left(\frac{x_{t+1}}{k_{t+1}} - \delta \right) \frac{x_{t+1}}{k_{t+1}} - \frac{\eta}{2} \left(\frac{x_{t+1}}{k_{t+1}} - \delta \right)^2 \right]. && (3.16)
\end{aligned}$$

I also need to add the real budget constraint and the law of motion for capital:

$$\begin{aligned}
c_t + x_t + b_t &= (1 - \tau_{kt}) r_t k_t + (1 - \tau_{nt}) w_t n_t + \delta \tau_{kt} k_t + R^b_t b_{t-1} + v_t && (3.17)\\
k_{t+1} &= (1 - \delta) k_t + x_t - \frac{\eta}{2} \left(\frac{x_t}{k_t} - \delta \right)^2 k_t. && (3.18)
\end{aligned}$$

I can solve the government's constraint (3.4) for v_t and combine it with the household's budget constraint (3.17) to get

$$c_t + x_t = r_t k_t + w_t n_t - g_t.$$

If I use (3.10) and (3.11) in the expression above, I obtain the aggregate feasibility constraint

$$c_t + x_t + g_t = y_t. \tag{3.19}$$

The equilibrium in this economy is described by equations (3.4), (3.9)–(3.16), (3.18), and (3.19); I also need to add the laws of motion (3.3) and (3.5)–(3.8). As is standard in the literature, I perform a (first-order) log-linear approximation of the equilibrium conditions around the model's steady state and then solve for the model's decision rules by following the methodology of Uhlig [20]. Appendix B contains a detailed account of these steps.

3.3 A model with tax rules

In this section I analyze the case where agents' expectations about future policy are derived from a simple tax rule which ties future tax rates to the current deficit.

In particular, I let the capital income tax rate τ_{kt} be adjusted by the rule while fixing the labor income tax rate to τ_n for all t. I show how the benchmark model of Section 3.2 and the notion of equilibrium should be adjusted, and present some qualitative results. (Quantitative results are contained in Section 3.4.)

3.3.1 Deficits and the capital tax rule

I follow Uhlig [19] and define d_t as "deficit remaining," where

$$d_t \equiv g_t + v_t + R_t^b b_{t-1} - \tau_n w_t n_t$$

which also implies, given equation (3.4),

$$d_t = b_t + \tau_{kt}(r_t - \delta)k_t. \tag{3.20}$$

In the steady state, equation (3.20) is

$$d = b + \tau_k(r - \delta)k.$$

Subtracting the above from (3.20) gives

$$d_t - d = b_t - b + \tau_{kt}(r_t - \delta)k_t - \tau_k(r - \delta)k.$$

I now add and subtract the term $E_t[\tau_{k,t+1}(r_{t+1} - \delta)k_{t+1}]$ in the right-hand side of the equation; paired with minor cosmetic changes on the left-hand side, the equation becomes

$$\begin{aligned} d_t - d = b_t - b &+ \{\tau_{kt}(r_t - \delta)k_t - E_t[\tau_{k,t+1}(r_{t+1} - \delta)k_{t+1}]\} \\ &- \{\tau_k(r - \delta)k - E_t[\tau_{k,t+1}(r_{t+1} - \delta)k_{t+1}]\}. \end{aligned} \tag{3.21}$$

With equation (3.21) in hand, it is trivial to show the following result.

Proposition 3.3.1. *Let $\psi \in [0,1]$ denote the fraction of the current deficit that the government commits to paying with capital taxes in the next period. If the government follows the tax rules*

$$\begin{aligned} \psi(d_t - d) &= E_t[\tau_{k,t+1}(r_{t+1} - \delta)k_{t+1}] - \tau_k(r - \delta)k \\ b_t - b &= (1 - \psi)(d_t - d) \\ &\quad + E_t[\tau_{k,t+1}(r_{t+1} - \delta)k_{t+1} - \tau_{kt}(r_t - \delta)k_t] \end{aligned}$$

then the government budget constraint is satisfied each period.

It is also possible to interpret ψ as the fraction of the current deficit that is to be paid with next period's capital income taxes; in this same logic, $(1 - \psi)$ is the fraction of the current deficit that is paid (today) via increased bond sales. Note that the bonds already include the expected change in tomorrow's tax receipts.

3.3.2 Equilibrium

It is straightforward to define an equilibrium given the changes outlined above. Equilibrium conditions from the household and the firm are still determined by (3.9)–(3.16), (3.18), and (3.19). However, I need to include the new equilibrium conditions for the government, which are given by

$$d_t = g_t + v_t + R_t^b b_{t-1} - \tau_n w_t n_t \tag{3.22}$$

$$\psi(d_t - d) = E_t[\tau_{k,t+1}(r_{t+1} - \delta)k_{t+1}] + \tau_k(r - \delta)k \tag{3.23}$$

$$\begin{aligned} b_t - b &= (1 - \psi)(d_t - d) \\ &\quad + E_t[\tau_{k,t+1}(r_{t+1} - \delta)k_{t+1} - \tau_{kt}(r_t - \delta)k_t] \end{aligned} \tag{3.24}$$

as well as the laws of motion (3.3), (3.7), and (3.8).

3.3.3 Interpreting the tax rules

(In what follows, let variables with hats denote percentage deviations from steady-state, variables with tildes denote level deviations from steady-state, and variables without time subscripts denote steady-state values.) Here I take the tax rules (3.23) and (3.24) and provide a brief interpretation. As a first step, I log-linearize both equations; the key point to consider is that d_t and b_t are log-linearized relative to steady-state output.[1]

I begin with the following result:

Lemma 3.3.2. *The log-linearized version of equations (3.23) and (3.24) is given by*

$$\begin{aligned}
\psi \hat{d}_t &= \gamma E_t \tilde{\tau}_{k,t+1} + \alpha \tau_k E_t \hat{r}_{t+1} + \gamma \tau_k \hat{k}_{t+1} && (3.25) \\
\hat{b}_t &= (1-\psi)\hat{d}_t + \gamma E_t(\tilde{\tau}_{k,t+1} - \tilde{\tau}_{kt}) \\
&\quad + \alpha \tau_k E_t(\hat{r}_{t+1} - \hat{r}_t) + \gamma \tau_k (\hat{k}_{t+1} - \hat{k}_t), && (3.26)
\end{aligned}$$

where $\gamma \equiv \alpha - x/y$.

Proof. See Appendix B. □

With the results of Lemma 3.3.2 at hand, I now solve equations (3.25) and (3.26) for the tax terms $E_t \tilde{\tau}_{k,t+1}$ and $E_t(\tilde{\tau}_{k,t+1} - \tilde{\tau}_{kt})$:

$$\begin{aligned}
E_t \tilde{\tau}_{k,t+1} &= \frac{\psi}{\gamma}\hat{d}_t - \frac{\alpha \tau_k}{\gamma} E_t \hat{r}_{t+1} - \tau_k \hat{k}_{t+1} && (3.27) \\
E_t[\tilde{\tau}_{k,t+1} - \tilde{\tau}_{kt}] &= \frac{1}{\gamma}\hat{b}_t - \frac{1-\psi}{\gamma}\hat{d}_t \\
&\quad - \frac{\alpha \tau_k}{\gamma} E_t(\hat{r}_{t+1} - \hat{r}_t) - \tau_k(\hat{k}_{t+1} - \hat{k}_t). && (3.28)
\end{aligned}$$

Equation (3.27) shows how the expectation of next period's capital income tax rate depends on the current deficit, the expectation of tomorrow's rental rate,

[1] For example, the percent deviation of the deficit relative to steady-state output, $\hat{d}_t$, is defined as

$$\hat{d}_t \equiv \frac{d_t - d}{y},$$

which implies that I can substitute for d_t in all the expressions above using

$$d_t = d + y\hat{d}_t.$$

(Even though they are not used in the material below, g_t and v_t are also log-linearized relative to steady-state output.)

and the capital stock. In equation (3.28), the left-hand side can be interpreted as the expected change in the capital income tax rate, which depends both on bond holdings, the current deficit, and the terms $E_t(\hat{r}_{t+1} - \hat{r}_t)$ and $(\hat{k}_{t+1} - \hat{k}_t)$, which can be interpreted as the expected changes in the tax base.

I now present some results about the expected capital income tax rate and the expected *change* in the capital income tax rate. I first need

Lemma 3.3.3. *The value of γ is pinned down by the model's parameters and is such that $\gamma \leq \alpha$. In particular,*

$$\gamma = \alpha \left[1 - \frac{\delta\beta(1-\tau_k)}{1-\beta(1-\delta(1-\tau_k))} \right].$$

Proof. See Appendix B. □

Proposition 3.3.4. *The expected capital income tax rate*

(a) increases with $\hat{d}_t$ (the increase is greater than one-to-one if $\psi > \gamma$);

(b) increases with ψ, and

(c) decreases when there is an expected increase in either of the tax base components.

Proof. Follows directly from equation (3.27). □

Proposition 3.3.5. *The expected* change *in the capital income tax rate*

(a) increases (by more than one-to-one) with $\hat{b}_t$;

(b) decreases with $\hat{d}_t$ (the reduction is smaller than -1 if $1 > \psi + \gamma$);

(c) decreases with ψ, and

(d) decreases when there is a positive expected change in either of the tax base components.

Proof. Statement (a) follows from Lemma 3.3.3 since $\gamma \leq \alpha < 1$. Statement (b) is trivial after a bit of algebra. The others follow directly from equation (3.28). □

3.4 The quantitative effect of tax rules

The experiment I perform is the following: I assume that both the benchmark economy and the economy with a tax rule rest at the steady state[2] and then derive the theoretical impulse-response function resulting from a one-time, one-percent increase in (a) productivity, (b) government consumption, and (c) transfers, occurring in period 0. I focus on the evolution of a subset of variables (output, hours, consumption, investment, capital, bond holdings, and tax rates) in the periods posterior to the shock. The path of the exogenous variable is the same in both economies; the only difference consists of the fact that in one economy, the evolution of the expectations about future tax rates depends on the tax rules (3.22)–(3.24).

The numerical exercises show that the behavior of an economy where capital income tax expectations are derived from a tax rule is significantly different to that of the benchmark economy. In particular,

(a) an increase in productivity yields the same qualitative response in the benchmark economy and in the economy with a tax rule—output, consumption, and investment increase—yet the quantitative response is amplified when the tax rule is present;

(b) an increase in government consumption produces the standard crowding-out response in both economies in the short run, but there is a long-lasting recession in the case where a tax rule is present, and

(c) an increase in transfers has no effect on the benchmark economy but can create a persistent recession when a tax rule is present.

Presenting the results

In what follows, for each exogenous variable (z_t, g_t, or v_t), I present four graphs.

[2] A bit of algebra can be used to show that the steady states of the benchmark economy and the economy with a tax rule are the same. See Appendix B.

The first one shows the short-run impulse-response functions for output, consumption, investment, hours, and capital; the second one is similar but increases the time range to see the same variables over a longer horizon. The third graph shows the evolution of household bond holdings and the capital income tax rate in the same long-run horizon as the second graph.

The fourth graph shows the short-run evolution of the exogenous variable, together with the impulse-response functions for the inputs of the tax rule (3.28): deficit, bond holdings, and tax base components (rental rate and capital stock). For completeness, I plot the evolution of the capital tax rate as well. In this way, it is possible to use (3.28) to see how the components of the tax rule influence the rate itself.

Parameter values

The parameter values contained in Table 3.1 are standard. The discount factor β is consistent with an annual risk-free rate of 5%. The leisure weight parameter ϕ is set so that the steady state allocation of time to market activities equals one third. The depreciation rate δ is set to an annual rate of 8%. I fix the habit persistence parameter ν to 0.9, while the capital income share α is set to 0.36. I follow Chari et al. [21] to set the adjustment cost parameter η to 3.22.

I set the deficit rollover percentage ψ equal to 0.15.[3] I normalize the steady state level of technology z to equal 1 and use historical averages to set the steady state levels of the capital and labor income tax rates to 0.42 and 0.26, respectively. This historical data also allows me to set the steady state government consumption to output ratio at 0.21, and the steady state debt to output ratio at 0.63. Without loss of generality, I set the autocorrelation parameters for all the stochastic processes at 0.95 and fix a standard deviation of 1% on the stochastic disturbances of said processes. (Note that the autocorrelation parameters only influence the speed at which the exogenous process returns to the steady state, and thus the persistence of the model's variables to the original shock. The size of

[3] As is expected, not all values of ψ are consistent with a competitive equilibrium.

Table 3.1: Model parameters

Parameter	Description	Value
β	Discount factor	0.9879
ϕ	Leisure weight parameter	2.55
δ	Depreciation rate	0.0194
ν	Habit persistence parameter	0.9
α	Capital income share	0.36
η	Adjustment cost parameter	3.22
ψ	Deficit rollover percentage	0.15
z	Steady state technology level	1
τ_k	Steady state capital income tax rate	0.42
τ_n	Steady state labor income tax rate	0.26
g/y	Steady state government consumption to output ratio	0.21
b/y	Steady state debt to output ratio	0.63
ρ_j	Autocorrelation parameter, $j \in \{k, n, g, v\}$	0.95
σ_j	Standard deviation, $j \in \{k, n, g, v\}$	0.01

the standard deviation matters for determining the size of the initial disturbance, but does not change the direction of the impulse-response functions.)

3.4.1 Productivity shock

For the short-run effects, I refer to Figure 3.1. From the figure, it is straightforward to verify that the path of productivity is the same in both economies (see the top-left panel). The response of model variables to a one-time increase in productivity is consistent with standard theory: output, consumption, and investment increase. The effect on hours depends on whether there is a tax rule in place: in the benchmark economy, hours fall slightly on impact and remain about 0.1% below the steady state for the first 40 periods; in the economy with a tax rule, hours jump to 0.6% above the steady state and then decline steadily, eventually converging to a level below the steady state value.

The effect of the tax rule is most notorious in the long run. Figure 3.2 shows the same exercise as above but expands the time range to observe a longer horizon. The deviation from the steady state for output, consumption, and investment is

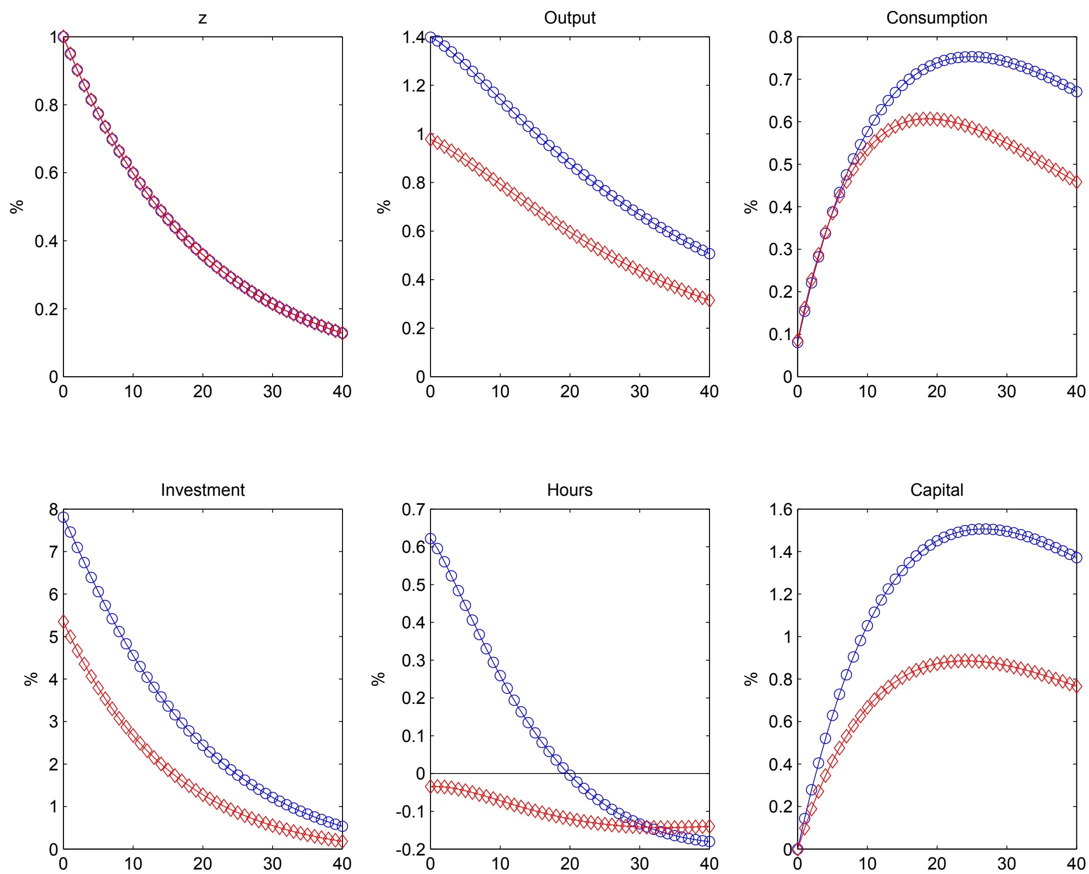

Figure 3.1: Shock to z, short-run path.
Blue line (circles): economy with tax rule. Red line (diamonds): benchmark economy.

amplified in the model with a tax rule, relative to the benchmark economy.

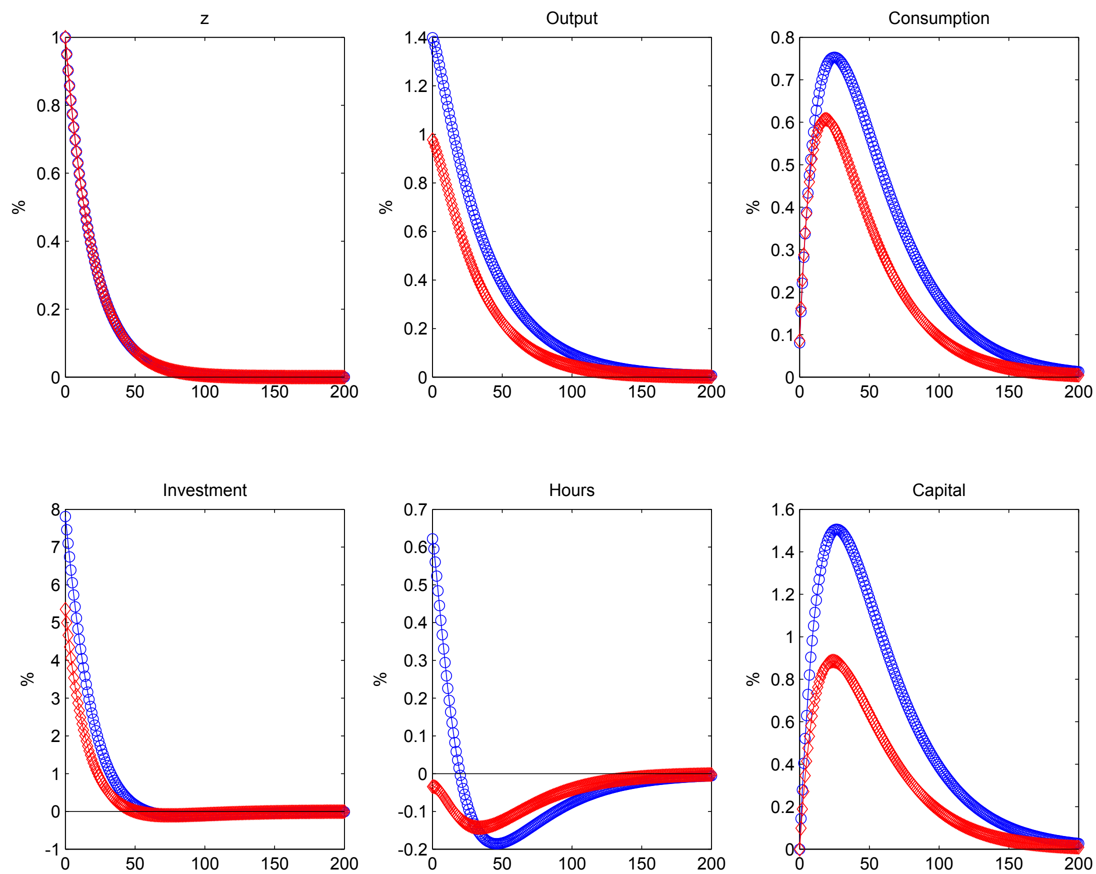

Figure 3.2: Shock to z, long-run path.
Blue line (circles): economy with tax rule. Red line (diamonds): benchmark economy.

Figure 3.3 shows the evolution of bonds held by the household and the capital income tax rate. In the benchmark economy, the shock to productivity makes the household increase its bond holdings by about 5% relative the steady state level (recall that bond holdings are expressed relative to steady state output); the capital income tax rate remains at its steady state level. When there is a tax rule present, households reduce bond holdings, reaching a trough at about 1% below the steady state level. In turn, the capital income tax rate falls by 1.5 percentage points and gradually returns to its steady state value.

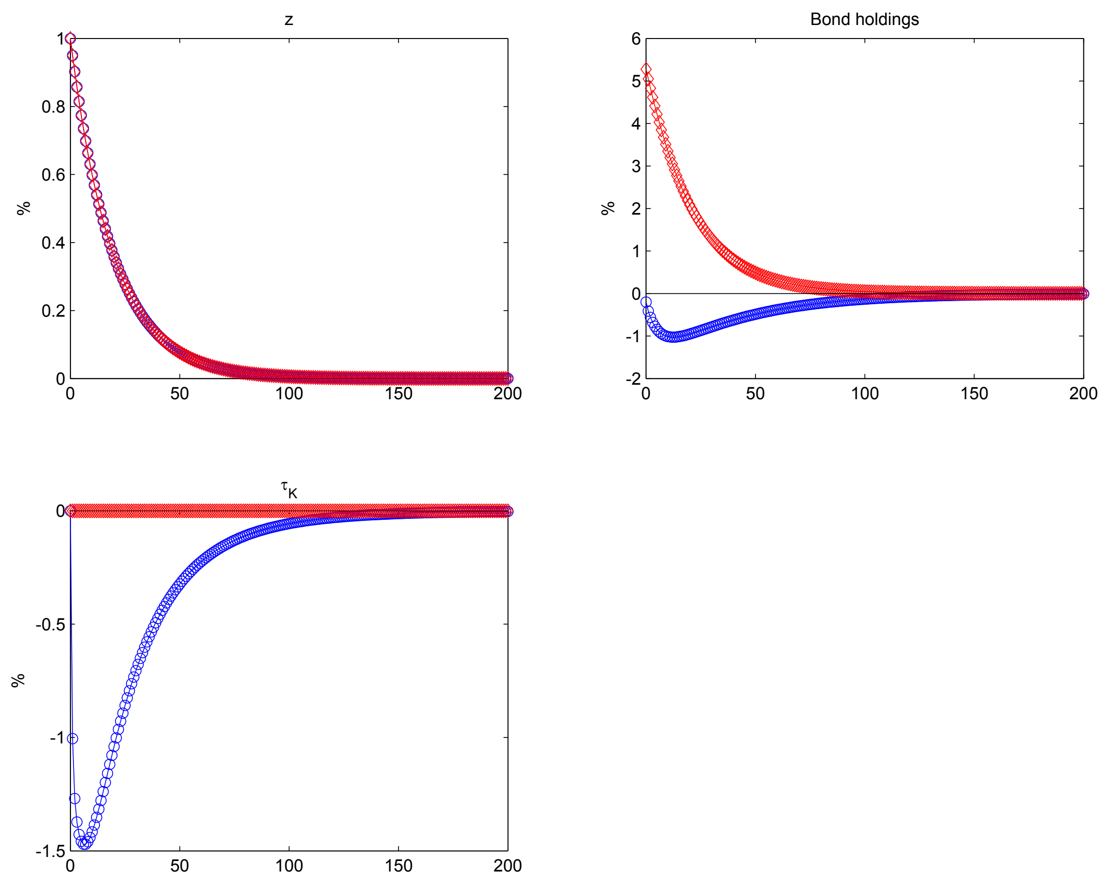

Figure 3.3: Shock to z, long-run household bond holdings and capital tax rate. Blue line (circles): economy with tax rule. Red line (diamonds): benchmark economy.

Figure 3.4 shows the evolution of the components of tax rule (3.28) after an exogenous shock to productivity. First, note that the bottom right panel is consistent with the dynamic path for τ_k shown in Figure 3.3. The rest of the panels are helpful in understanding why the tax rate follows a U-shaped path; from Proposition 3.3.5, the expected *change* in the tax rate has

(a) A positive relationship with $\hat{b}_t$ (with coefficient $1/\gamma = 5.33$);

(b) A negative relationship with $\hat{d}_t$ (with coefficient $-(1-\psi)/\gamma = -4.53$);

(c) A negative relationship with $E_t(\hat{r}_{t+1} - \hat{r}_t)$ (with coefficient $-\alpha\tau_k/\gamma = -0.81$), and

(d) A negative relationship with $(\hat{k}_{t+1} - \hat{k}_t)$ (with coefficient $-\tau_k = -0.42$).

The bottom mid panel of Figure 3.4 shows a small negative deviation from trend for bond holdings at period 0; however, this value gets multiplied by a factor of 5.3 when calculating the expected change in the tax rate from period 0 to period 1. At period 0 the deficit is still at its steady state level (see the bottom left panel), so that the tax rate is not influenced by that component. The last two panels in the top row of the figure show that the expected change in the rental rate is negative, while that of the capital stock is positive. These effects work in opposite directions; given the size of the coefficients the rental rate effect should be larger and pull the deviation of the tax rate towards the steady state. Overall the effect of bond holdings dominates and thus the change in the tax rate from period 0 to period 1 is of about 1%. A similar logic can be used to trace the evolution of τ_k from any period to the next one.

3.4.2 Government consumption shock

Figure 3.5 presents the short-run effects of a one-time, one percent increase in government consumption. Theory predicts an increase in output and hours, paired with a decrease in consumption and investment; qualitatively, this happens in

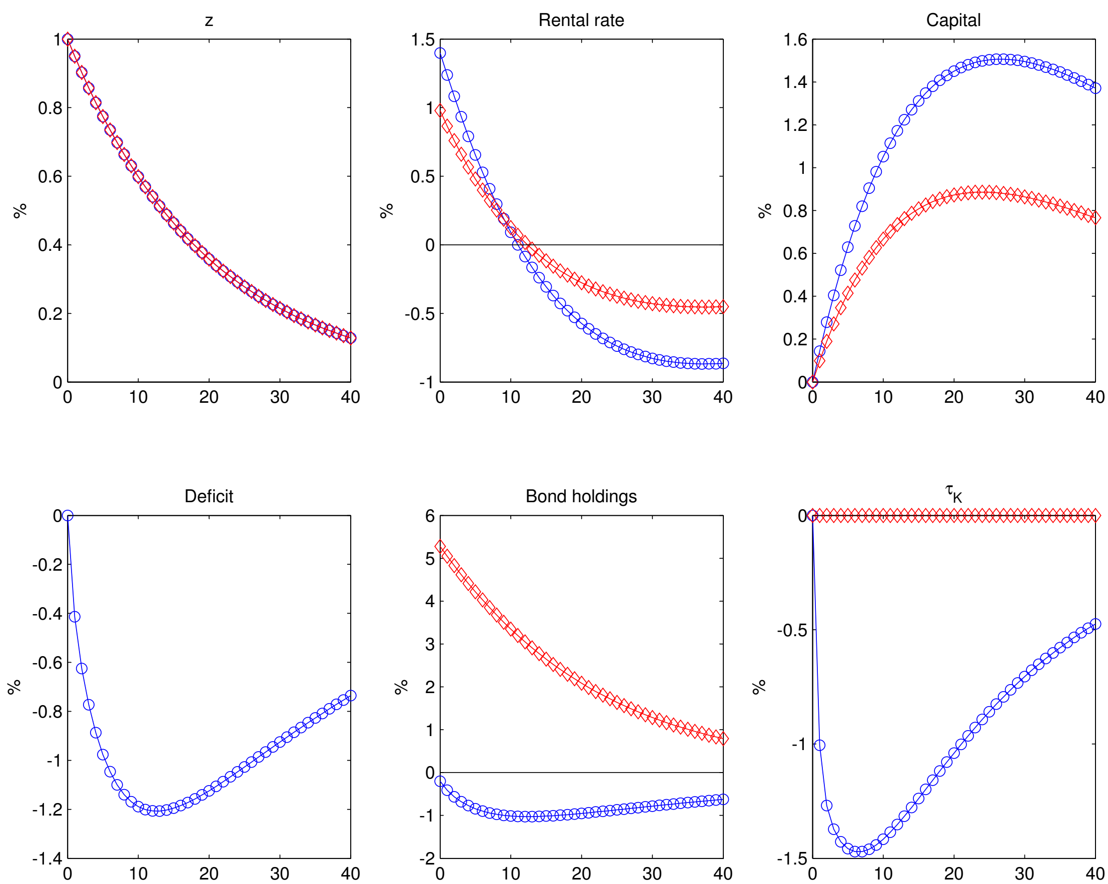

Figure 3.4: Shock to z, components of the tax rule.
Blue line (circles): economy with tax rule. Red line (diamonds): benchmark economy.

both economies on impact. There is, however, a quantitative difference between economies. In the benchmark economy, hours increase by 1.4% on impact and output does the same by 0.9%, while investment falls by 0.8%. Consumption is slightly reduced on impact and then follows a U-shaped path, with a trough at about 0.2% below its steady state value. In the economy with a tax rule, hours and output increase slightly on impact; however, both series quickly fall below their steady state levels. While hours return to the steady state rather quickly (about 15 periods) and stay above this value for a while, the drop in output is highly persistent. The fall in investment is more pronounced, at about 6% below the steady state level on impact; the path for consumption follows that of the economy without tax rule for the first periods but fails to return to the steady state with the same speed.

Figure 3.6 shows that the effect of an increase in government consumption is relatively short-lived in the benchmark economy; the same shock has a completely different effect when the tax rule is in effect. Output, consumption, and investment follow a U-shaped path, yet the variables reach a trough in different periods. Investment does so around period 10 (this can be seen in detail in Figure 3.5), output around period 15, and consumption only after period 40. Note that capital and consumption do not return to their steady state level even after period 200, and that the troughs are reached at significantly lower levels compared to the benchmark case. Hours follow an S-shaped path, increasing on impact, reaching a trough in period 10, overshooting to a peak around period 50, and gradually returning to the steady state afterwards.

Figure 3.7 shows that for the benchmark economy, the increase in G makes the household increase its bond holdings by 17% relative the steady state level, while the capital income tax rate remains at its steady state level. When there is a tax rule present, households gradually increase bond holdings up to a peak of about 4%; the capital tax rate increases in period 1 and follows an inverse U-shaped path, peaking at about 3.5%.

Figure 3.8 shows the evolution of the components of tax rule (3.28) after an

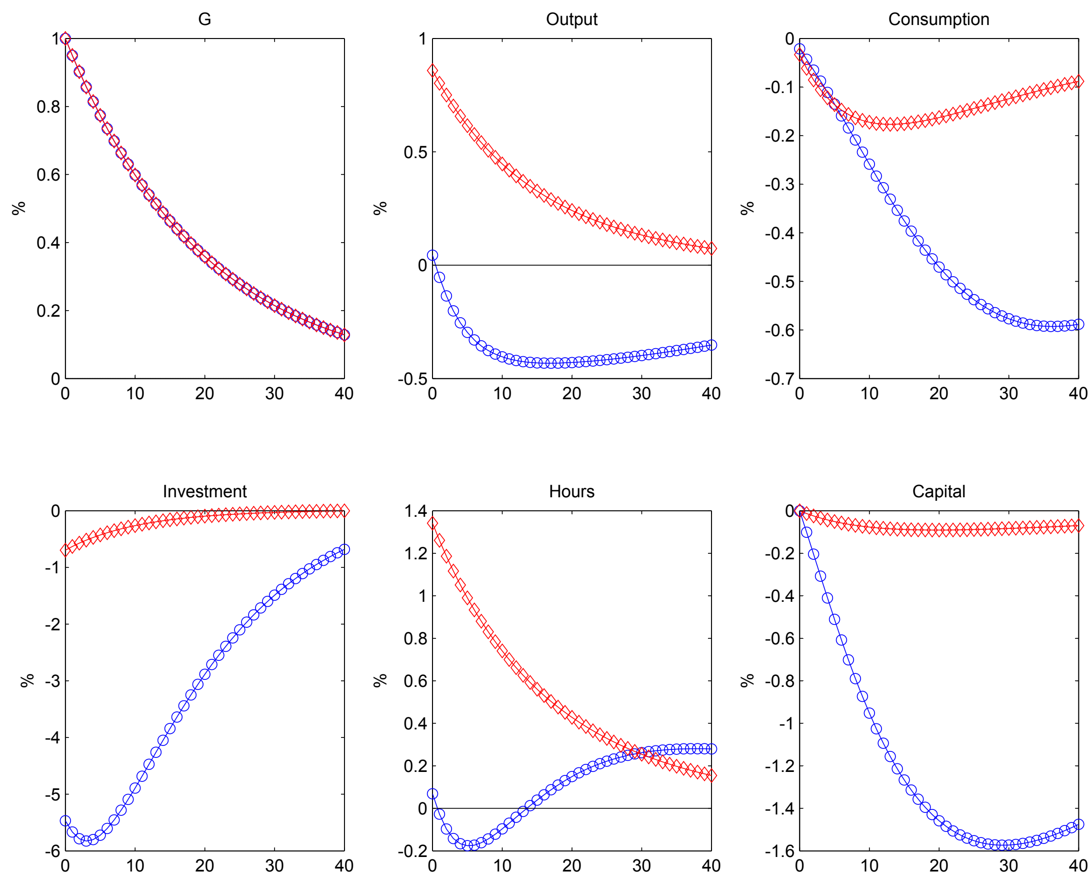

Figure 3.5: Shock to g, short-run path.
Blue line (circles): economy with tax rule. Red line (diamonds): benchmark economy.

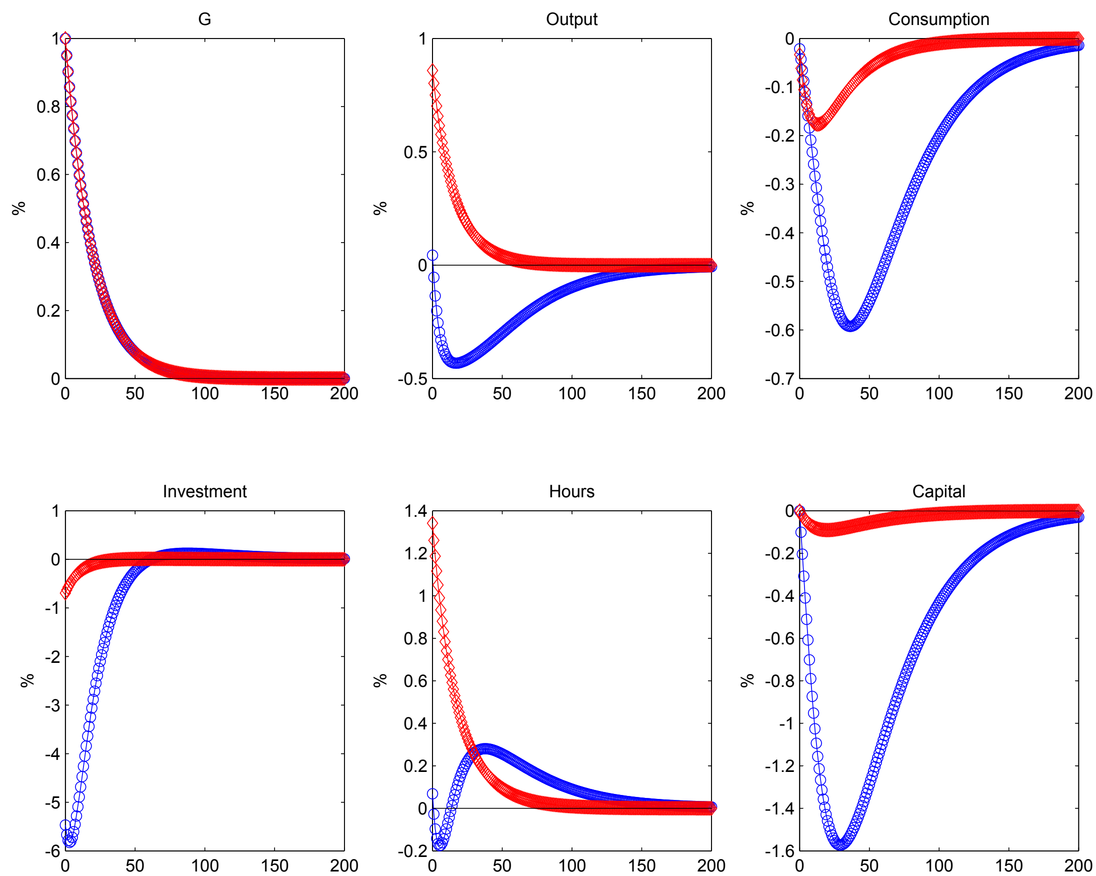

Figure 3.6: Shock to g, long-run path.
Blue line (circles): economy with tax rule. Red line (diamonds): benchmark economy.

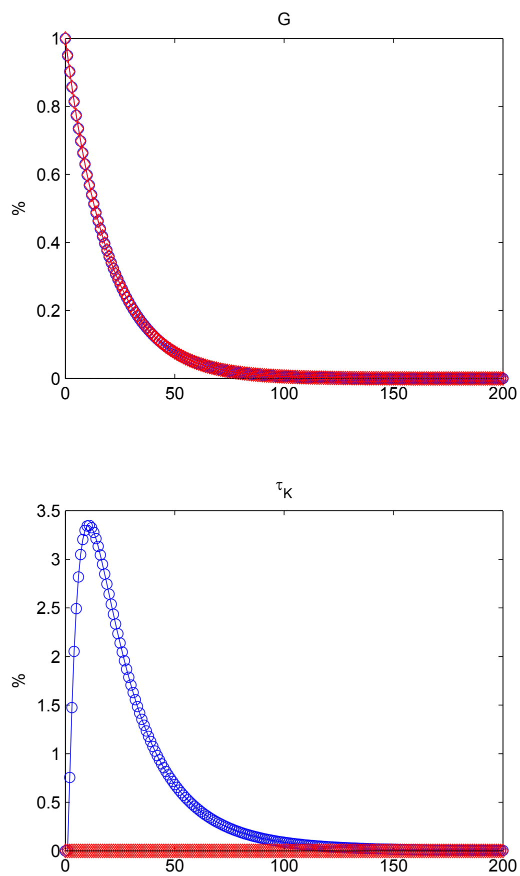

Figure 3.7: Shock to g, long-run household bond holdings and capital tax rate. Blue line (circles): economy with tax rule. Red line (diamonds): benchmark economy.

exogenous shock to government consumption. Both the bottom right panel and the corresponding series in Figure 3.7 show that the dynamic path of τ_k follows an inverse U-shape after the increase in government consumption. From the figure, the path of bond holdings and the capital stock push the expected change in the capital tax rate upwards, while the path of the deficit and the rental rate pulls the expected change in the rate downwards. (Note that the value for the coefficients in the tax rule is the same as in the previous section.) Overall, the first effect dominates throughout the first 40 periods and the capital tax rate remains above its steady state level.

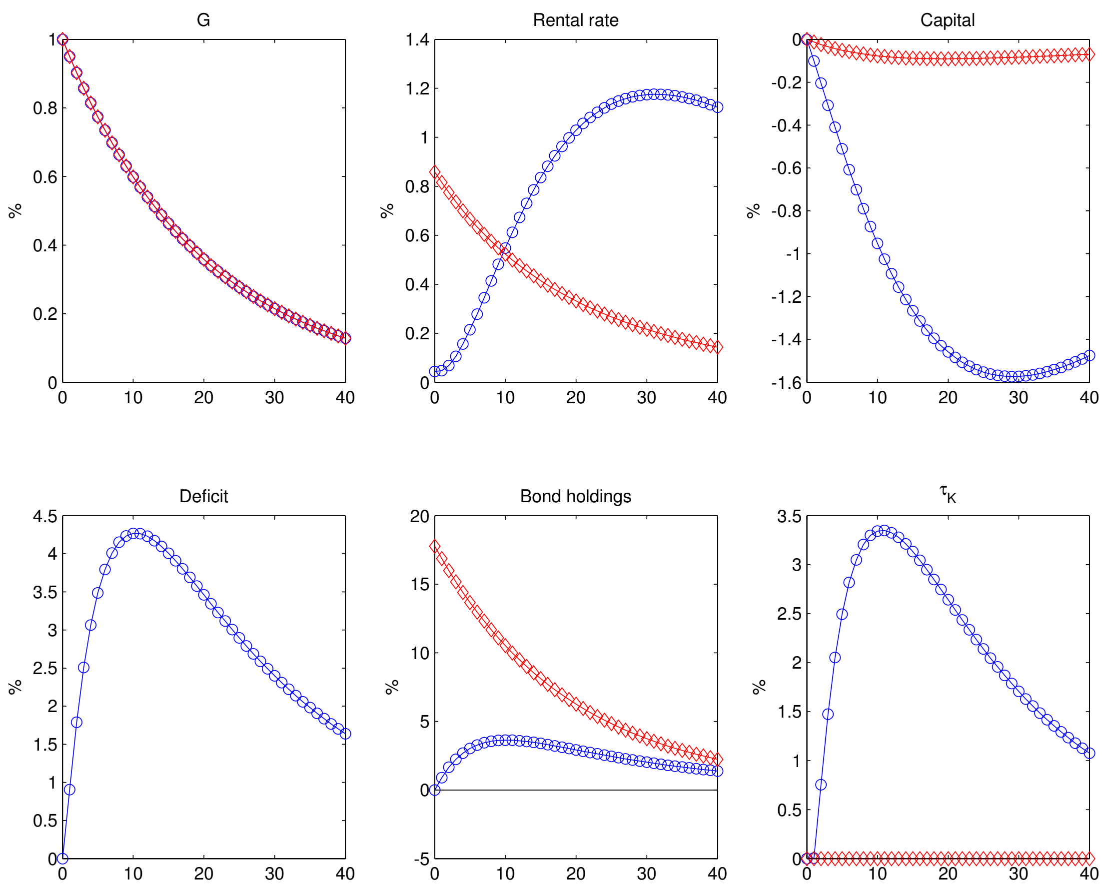

Figure 3.8: Shock to g, components of the tax rule.
Blue line (circles): economy with tax rule. Red line (diamonds): benchmark economy.

3.4.3 Transfers shock

As can be seen from figures 3.9 and 3.10, a one-time, one percent increase in transfers in the benchmark economy has no effect over output, consumption, investment, hours, and capital. Figure 3.11 shows, however, that household bond holdings do increase on impact by 15% relative to its steady state level and then decrease gradually. As expected, the capital income tax rate remains constant.

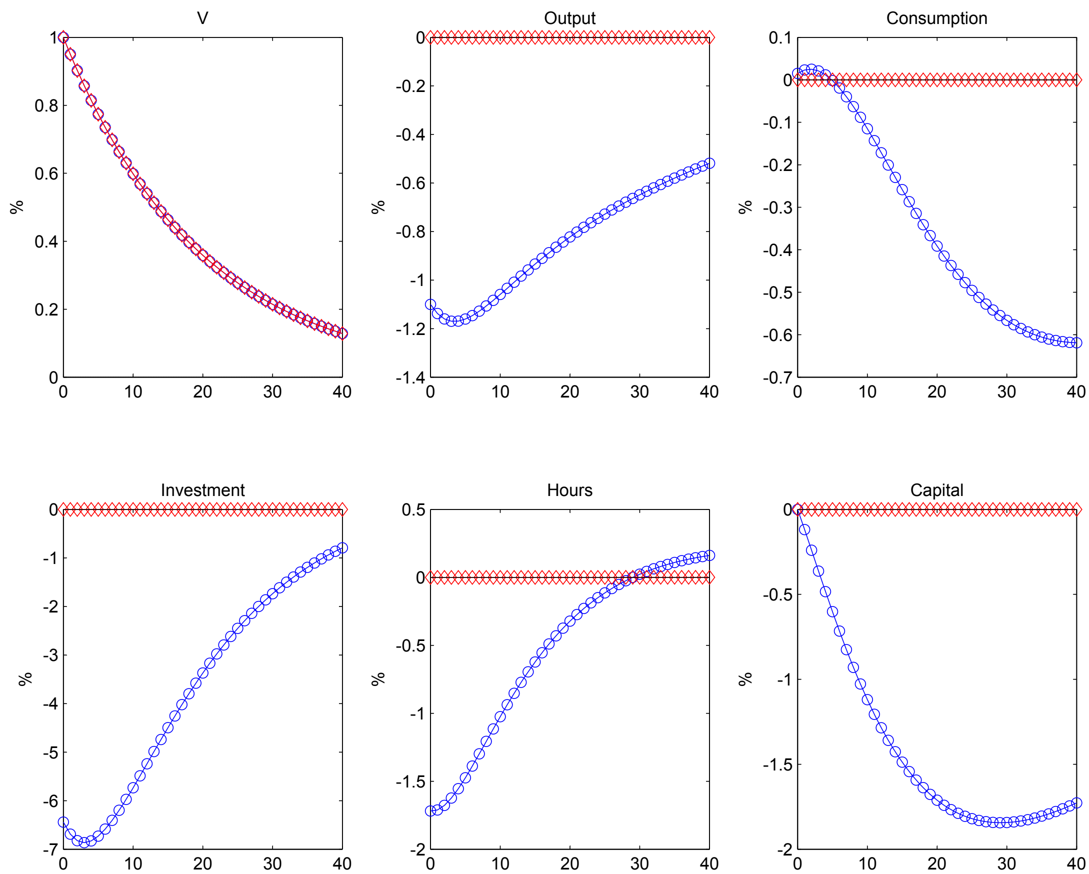

Figure 3.9: Shock to v, short-run path.
Blue line (circles): economy with tax rule. Red line (diamonds): benchmark economy.

The response of economy variables when a tax rule is present is different. From figures 3.9 and 3.10, hours, output, investment, and capital fall on impact;

consumption marginally increases for a few periods only to decrease afterwards. After reaching their troughs, output, consumption, investment, and capital return to their steady state values; hours increase gradually, eventually overshooting the steady state level and then returning to it. Figure 3.11 shows that household bond holdings increase gradually, reaching a peak of 5% and then returning to their steady state levels. The capital income tax rate also increases gradually, up to a maximum deviation of about 4.5% above its steady state level, returning to its steady state value afterwards.

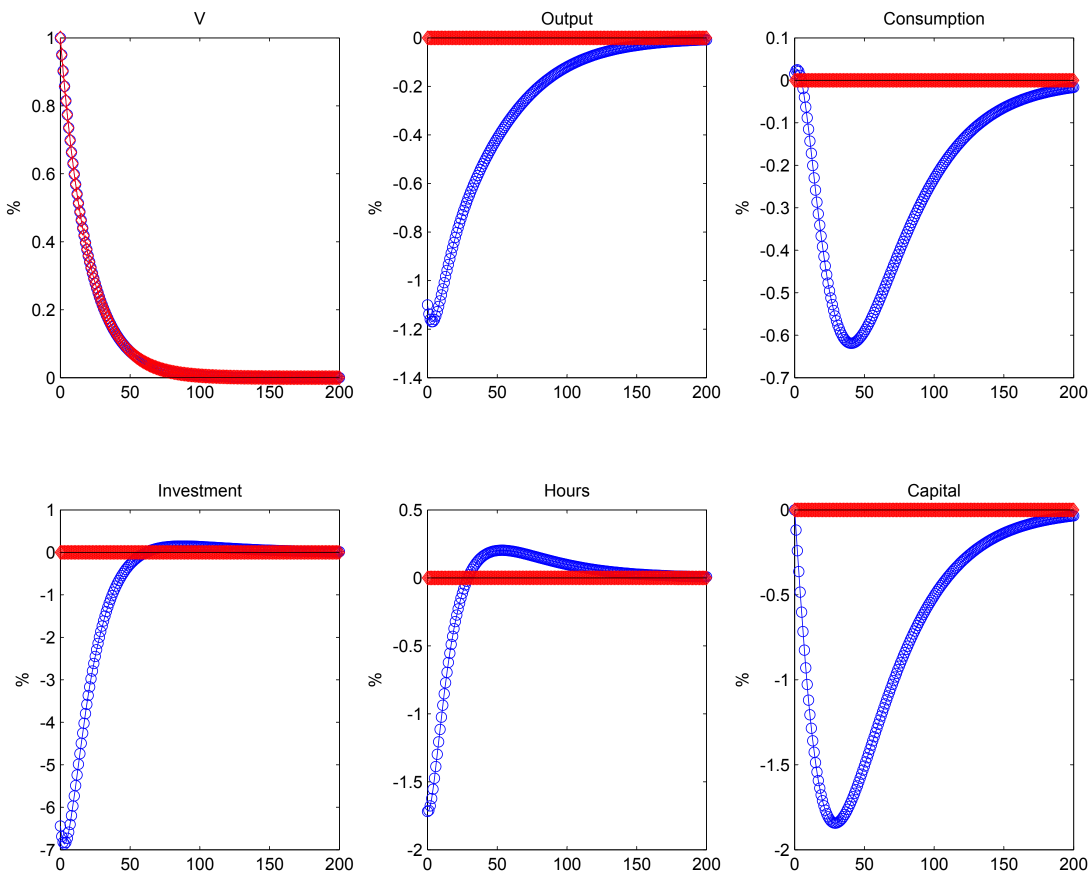

Figure 3.10: Shock to v, long-run path.
Blue line (circles): economy with tax rule. Red line (diamonds): benchmark economy.

Finally, Figure 3.12 shows the evolution of the components of tax rule (3.28)

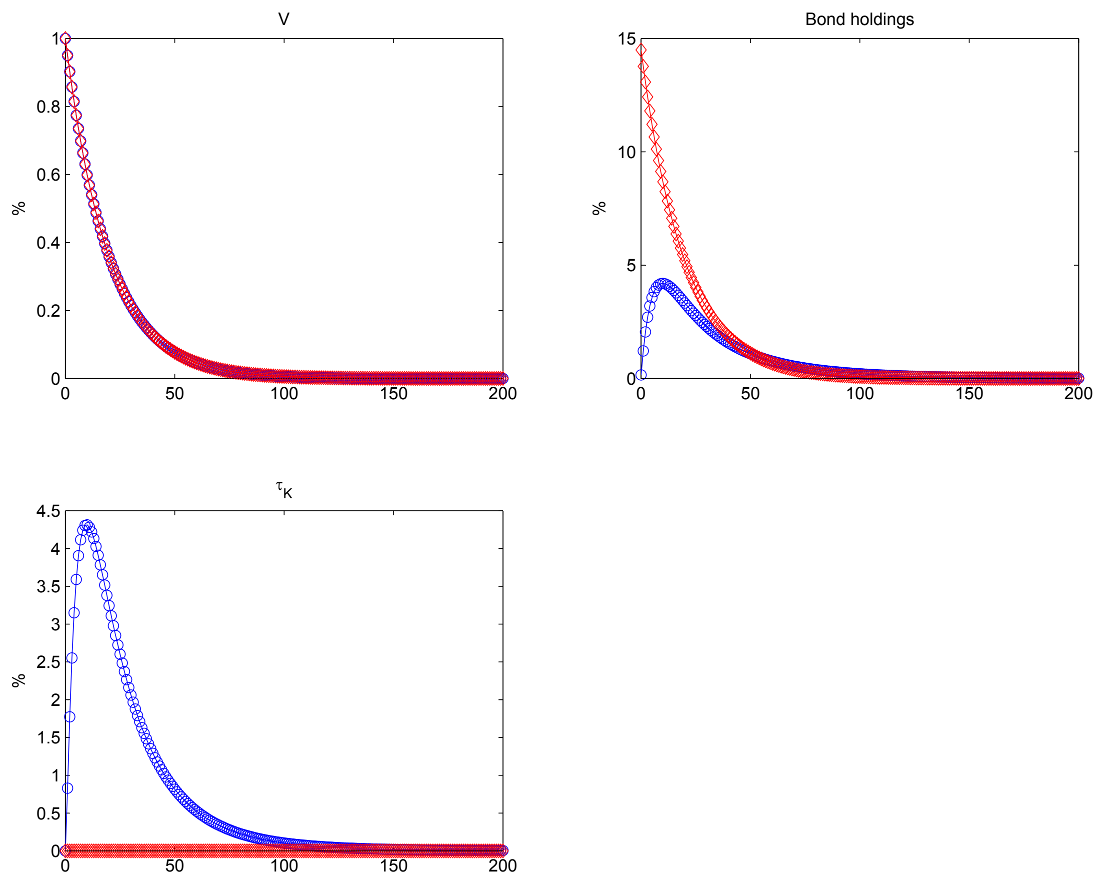

Figure 3.11: Shock to v, long-run household bond holdings and capital tax rate. Blue line (circles): economy with tax rule. Red line (diamonds): benchmark economy.

after an exogenous shock to lump-sum transfers.

Qualitatively, a shock to transfers is similar to a shock to government consumption: first, both the rental rate and the capital stock work in opposite directions (the rental rate pulls the expected change in the tax rate towards the steady state, while the capital stock pushes it away). Second, the deviation in bond holdings pushes the tax rate away from the steady state, while the deficit pulls it towards zero. As was the case in the previous exercise, overall the first effect dominates and the capital tax rate stays above its steady state level throughout the first 40 periods shown in the figure.

However, in this case the combination of all component dynamics creates a higher level of the tax rate, about 1% higher than in the case of an increase in government consumption. Even after 40 periods, the capital tax rate is about 0.25% higher in this case.

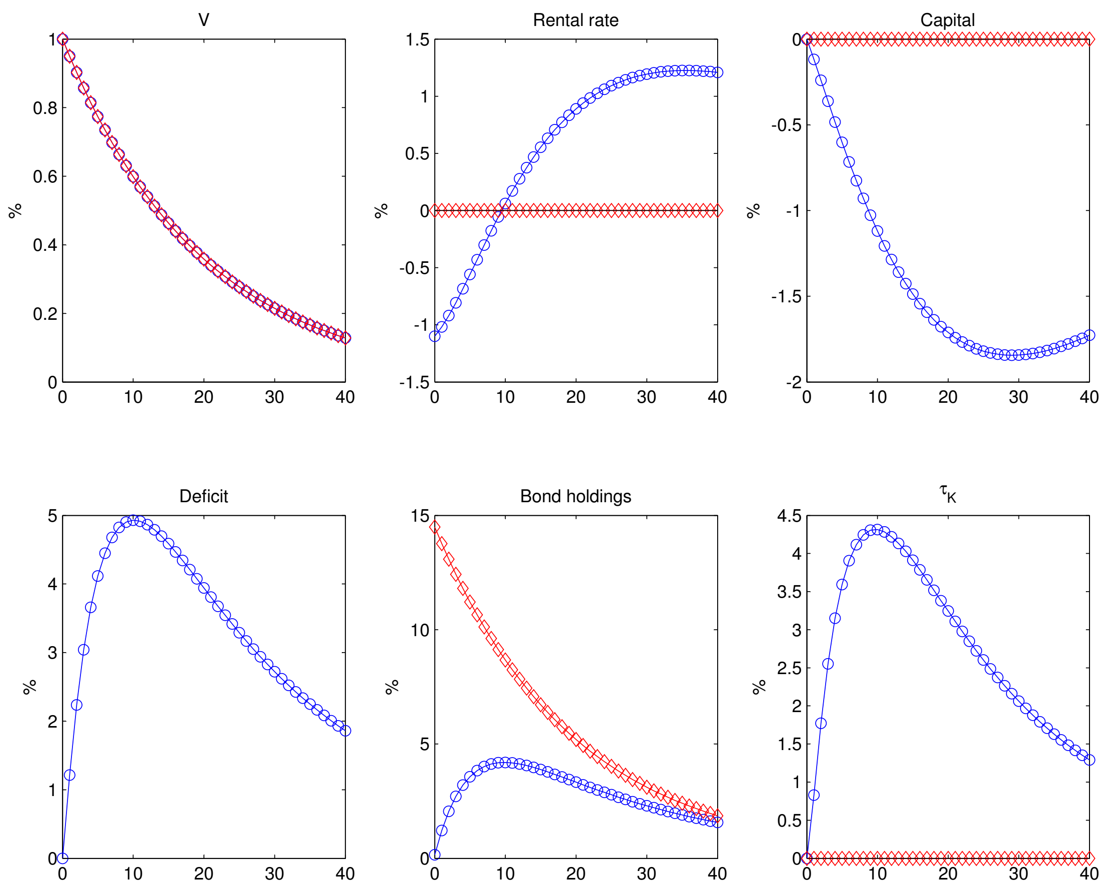

Figure 3.12: Shock to v, components of the tax rule.
Blue line (circles): economy with tax rule. Red line (diamonds): benchmark economy.

Chapter 4

Conclusion and Discussion

In these essays, I explored the business cycle implications of changes in expectations about the course of future fiscal policy. In particular, I considered two variations—relative to standard models—in which expectations about future policy change even when current policy does not.

First, I reviewed the case where agents receive news about changes in future policy every period, and compared this environment to an identical economy economy except for the fact that agents receive no information about future policy. Second, I analyzed the case where agents' expectations about future policy are derived from a simple tax rule which ties future tax rates to the current deficit. In both cases, the theoretical impulse-responses from my models show that changes in expectations about future policy can create drastically different implications for the business cycle.

There are several avenues in which these results could be expanded. An obvious and most immediate one is a battery of robustness analysis applied to the results of Chapter 2; experimenting with a different set of observable variables, prior distributions, and bigger chains (the last one being time-intensive) could potentially improve the estimates (and in turn, the results) of the essay. Also, several authors have brought up the convenience of modeling news in a different way;[1] altering the way in which agents receive news would be a straightforward (yet again, time-consuming) way to add to the literature.

Finally, the tax rules framework could be applied to several (and interesting) economic questions. A rather straightforward extension would be to modify the environment to allow the tax rule to endogenously derive the labor income tax rate (as opposed to the capital income tax). A second extension—which was slightly covered in Chapter 3—was that of the value of the deficit rollover percentage ψ. An interesting exploration would be to derive a correspondence between the values of ψ and the existence of equilibrium; my experience working with the numerical examples suggests that the existence of equilibrium doesn't have a monotonic relation with ψ.

[1] See, for example, Walker and Leeper [22].

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Appendix A

Additional Material (News)

A.1 Steady states

Since the only difference between the benchmark economy and the economy with news are the exogenous stochastic processes, the steady states of the model variables are identical.

To obtain the steady state values for the model's variables, I set $z = u = q = 1$, $\tau_{kt} = \tau_k$, $\tau_{nt} = \tau_n$, $g_t = g$, and $\mu_t = \mu$. Then equations (2.14)–(2.21), (2.23) and (2.24) become

$$y = \left(\frac{k}{\mu}\right)^{\alpha} n^{1-\alpha} \tag{A.1}$$

$$\alpha y = \frac{rk}{\mu} \tag{A.2}$$

$$(1-\alpha)y = wn \tag{A.3}$$

$$\lambda = \frac{\mu - \beta\nu}{(\mu - \nu)c} \tag{A.4}$$

$$\phi = \lambda(1-\tau_n)w \tag{A.5}$$

$$\delta_1 = (1-\tau_k)r + \tau_k(\delta_0 - \delta_1) \tag{A.6}$$

$$\mu = \beta[(1-\tau_k)r + \delta_0\tau_k + 1 - \delta_0] \tag{A.7}$$

$$1 = 1 \tag{A.8}$$

$$(1-g)y = c+i \tag{A.9}$$

$$i = \frac{(\mu - 1 + \delta_0)k}{\mu} \tag{A.10}$$

which is a system of 10 equations in $\{y, k, \mu, n, r, w, \lambda, c, \delta_1, i\}$. Take equation (A.7) and solve for r:

$$r = \frac{\mu - \beta(1 - \delta_0(1 - \tau_k))}{\beta(1 - \tau_k)}$$

so I can solve equation (A.2) for y as a function of k

$$y = \left(\frac{r}{\alpha\mu}\right) k \equiv \omega_y k$$

and equation (A.6) for δ_1

$$\delta_1 = r + \frac{\tau_k}{1 - \tau_k}\delta_0.$$

Equation (A.10) already has i as a function of k:

$$i = \frac{(\mu - 1 + \delta_0)k}{\mu} \equiv \omega_i k$$

and with the expressions for y, I can take equation (A.9) and solve for c as a function of k:

$$c = [(1-g)\omega_y - \omega_i]k \equiv \omega_c k.$$

Use the above in (A.4) to express λ as a function of k:

$$\lambda = \frac{\mu - \beta\nu}{(\mu - \nu)\omega_c k}$$

and plug this expression in (A.5), solving for w as a function of k:

$$w = \frac{\phi(\mu - \nu)\omega_c k}{(\mu - \beta\nu)(1 - \tau_n)} \equiv \omega_w k.$$

I use this in (A.3) to solve for n:

$$n = \frac{(1-\alpha)\omega_y}{\omega_w}$$

and from equation (A.1), solve for k:

$$k = \frac{n}{(\mu^\alpha \omega_y)^{1/(1-\alpha)}}.$$

All other values follow directly.

A.2 Model data

Here I describe how I construct the data series used to estimate the models.

Gross domestic product I define gross domestic product as GDP (NIPA Table 1.5.5 line 1) less sales taxes plus imputed capital rents (4.1% of stock of consumer durables, FA Table 1.1 plus fixed government capital in FA Table 1.1) plus depreciation of consumer durables (FA Table 1.3).[1]

Consumption I let consumption be equal to personal consumption expenditures (NIPA Table 1.5.5 line 2) less PCE durables (NIPA Table 1.5.5 line 4) plus imputed capital rents (4.1% of stock of consumer durables, FA Table 1.1 plus fixed government capital in FA Table 1.1) plus depreciation of consumer durables (FA Table 1.3) less a prorated portion of sales taxes on nondurables and services.

Investment I set investment equal to gross private domestic investment (NIPA Table 1.5.5 line 26) plus net exports (NIPA Table 1.5.5 line 42) plus government investment (NIPA Table 1.5.5 lines 53, 56, and 59) plus PCE durables (NIPA Table 1.5.5 line 4) less a prorated portion of sales taxes on durables.

Government consumption I define this series to be equal to government consumption (NIPA Table 1.5.5 lines 52, 55, and 58).

[1] Sales taxes are not available on a quarterly basis from 1948 to 1959. I use the detailed annual values (NIPA Table 3.5 lines 3 and 14) relative to the totals (NIPA Table 3.5 lines 2 and 13) to calculate the ratio, assumed constant throughout all quarters.

Hours I take average hours worked at annual rate (relative to 5200 hours) directly from the Cociuba et al. [23] data project.

Data adjustments I convert to real dollars by using the GDP deflator (NIPA Table 1.5.4 line 1). I also divide by the population at midperiod (NIPA Table 2.1 line 39).

A.3 Parameter transformations

A linear transformations is applied to a subset of the model's parameters in order to reduce numerical instability when reaching limit values. Following Table 2.2, the parameters' values (under a Beta distribution prior) are

Table A.1: Prior distributions (Beta)

Parameter	Mean	Standard deviation	Lower bound	Upper bound
ν	0.5	0.1	0	0.99
$\rho_z, \rho_k, \rho_n, \rho_g$	0.7	0.2	0	0.99
ρ_m	0.0	0.1	-0.5	0.5

Consider parameter ν. According to Table A.1, ν has a Beta prior with mean 0.5, standard deviation 0.1, and an upper bound of 0.99. The problem is that the estimated value for the parameter could be arbitrarily close to the upper bound of 0.99, in which case Dynare will run into trouble. A solution is to perform a linear transformation; in particular,

$$\nu \sim \text{Beta}(0.5, 0.1),\ \nu \in (0, 0.99)$$
$$\iff \frac{\nu}{0.99} \equiv \nu^{[0.99]} \sim \text{Beta}\left(\frac{0.5}{0.99}, 0.1\right),\ \nu^{[0.99]} \in (0, 1).$$

Let $\widehat{\nu}^{[0.99]}$ be Dynare's estimate of $\nu^{[0.99]}$. Then, the estimate of ν can be backed out via

$$\widehat{\nu} = 0.99\,\widehat{\nu}^{[0.99]}.$$

The transformations for $\{\rho_z, \rho_k, \rho_n, \rho_g\}$ follow a similar logic.

A different transformation is required for ρ_m: while we want the parameter to have a Beta prior, the problem is that we want to impose a zero mean—the mean of a Beta distribution is strictly positive—and lower/upper bounds which are different from $(0, 1)$. In this case, a linear transformation which helps solve this problem is given by

$$\rho_m \sim \text{Beta}(0, 0.1),\ \rho_m \in (-0.5, 0.5)$$
$$\iff \rho_m + 0.5 \equiv \rho_m^{[0.5]} \sim \text{Beta}(0.5, 0.1),\ \rho_m^{[0.5]} \in (0, 1).$$

As in the previous case, Dynare estimates $\widehat{\rho}_m^{[0.5]}$, yet we can back up $\widehat{\rho}_m$ via

$$\widehat{\rho}_m = \widehat{\rho}_m^{[0.5]} - 0.5.$$

A.4 Dynare codes

This section contains the Dynare `mod` files which are used to produce the results of Chapter 2. Both files offer the option to estimate the parameters using Bayesian methods, and whether to print out the statistical properties of the model data or the impulse-response functions.

To facilitate the computation, both codes have initialized the parameter values to the calibration *and* estimation results.

A.4.1 Benchmark economy

```
// benchmark.mod
// Benchmark economy (no fiscal policy news)

// Coded with Dynare 4.2.1
// Last updated: 07/12/2011

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Housekeeping

close all
```

```
// Choose whether to estimate parameters (yes) or not (no)
estvar = 'no';

// Choose simulation (sim) or impulse-response (irf)
simvar = 'irf';

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% A. Defining variables and parameters

// MODEL VARIABLES (29; 16 endogenous, 4 observables, 9 exogenous)

// 11 endogenous variables
    var y u k n r w c lambda q i gy;

    predetermined_variables k;

// 5 stochastic disturbances
    var z mu tk tn g;

// 3 observable variables
    var growth_y_obs gy_obs n_obs;

// 5 exogenous variables
    varexo ez_0 em_0 ek_0 en_0 eg_0;

// 4 measurement errors
    varexo em_c em_y em_gy em_n;


// PARAMETERS (17)

// 3 household parameters
    parameters phi beta nu99;

// 7 firm and technology parameters
    parameters alpha del0 del1 del2 kappa zss rho_z99;

// 6 policy parameters
    parameters tnss tkss gyss rho_n99 rho_k99 rho_g99;

// 2 growth rate parameters
    parameters muss rho_m50;

// Initial values

    phi     = 4.244;
```

```
    beta    = 1.05^(-0.25);
    nu99    = 0.6388/0.99;

    alpha   = 0.36;
    del0    = 1.08^(1/4)-1;
    del1    = 0.1710;
    del2    = 0.2124;
    kappa   = 3.5399;
    zss     = 1;
    rho_z99 = 0.7952/0.99;

    tnss    = 0.26;
    tkss    = 0.42;
    gyss    = 0.15;
    rho_n99 = 0.9894/0.99;
    rho_k99 = 0.9880/0.99;
    rho_g99 = 0.9750/0.99;

    muss    = 1.005;
    rho_m50 = 0.0768+0.5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% B. Model block

model;

# nu    = nu99*0.99;
# rho_z = rho_z99*0.99;
# rho_m = rho_m50-0.5;
# rho_k = rho_k99*0.99;
# rho_n = rho_n99*0.99;
# rho_g = rho_g99*0.99;
// Parameter transformation as in Schmitt-Grohe and Uribe (2008)

// Equation 1: production function
y-z*(u*k/mu)^alpha*n^(1-alpha);

// Equation 2: firm's capital FOC
alpha*y-r*u*k/mu;

// Equation 3: firm's labor FOC
    (1-alpha)*y-w*n;

// Equation 4: household's consumption FOC
    1/(c-nu*c(-1)/mu)-lambda-beta*nu*(1/(mu(+1)*c(+1)-nu*c));

// Equation 5: household's labor FOC
```

```
    -phi+lambda*(1-tn)*w;

// Equation 6: household's utilization FOC
r-tk*(r-del0-del1*(u-1)-(del2/2)*(u-1)^2-u*(del1+del2*(u-1)))
        -q*(del1+del2*(u-1));

// Equation 7: household's capital FOC
    -lambda*q+beta*(lambda(+1)/mu(+1))*(r(+1)*u(+1)
        -tk(+1)*(r(+1)-del0-del1*(u(+1)-1)-(del2/2)*(u(+1)-1)^2)
        +q(+1)*(1-del0-del1*(u(+1)-1)-(del2/2)*(u(+1)-1)^2));

// Equation 8: household's investment FOC
-lambda+lambda*q*(1-(kappa/2)*(mu*i/i(-1)-muss)^2
        -kappa*(mu*i/i(-1)-muss)*mu*i/i(-1))
        +beta*kappa*lambda(+1)*q(+1)*mu(+1)*(mu(+1)*i(+1)/i-muss)
        *(i(+1)/i)^2;

// Equation 9: aggregate feasibility
    y-c-i-g;

// Equation 10: law of motion for capital
    (1-del0-del1*(u-1)-(del2/2)*(u-1)^2)*k/mu
        +i*(1-(kappa/2)*(mu*i/i(-1)-mu)^2)-k(+1);

// Equation 11: government consumption to output ratio
    gy-g/y;

// Equations 12-16: stochastic processes
    z = (1-rho_z)*zss+rho_z*z(-1)+ez_0;
    mu = (1-rho_m)*muss+rho_m*mu(-1)+em_0;
    tk = (1-rho_k)*tkss+rho_k*tk(-1)+ek_0;
    tn = (1-rho_n)*tnss+rho_n*tn(-1)+en_0;
    gy = (1-rho_g)*gyss+rho_g*gy(-1)+eg_0;

// Equations 17-20: observables
    growth_y_obs    = muss*y/y(-1)+em_y;
    gy_obs          = gy+em_gy;
    n_obs           = n+em_n;

end ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% C. Steady state

steady_state_model;
    u   = 1;
    q   = 1;
```

```
    z   = 1;
    tn  = tnss;
    tk  = tkss;
    gy  = gyss;
    mu  = muss;

    r   = (mu-beta*(1-del0*(1-tk)))/(beta*(1-tk));

    ome_y   = r/(alpha*mu);
    ome_i   = (mu-1+del0)/mu;
    ome_c   = (1-gy)*ome_y-ome_i;
    del1    = r+(tk/(1-tk))*del0;
    ome_w   = phi*(mu-nu99*0.99)*ome_c/((mu-beta*nu99*0.99)*(1-tn));

    n   = (1-alpha)*ome_y/ome_w;
    k   = n/(mu^alpha*ome_y)^(1/(1-alpha));
    w   = ome_w*k;
    c   = ome_c*k;
    i   = ome_i*k;
    y   = ome_y*k;

    g   = gyss*y;

    lambda  = phi/((1-tn)*w);

    // Observables are defined as

    // growth_y_obs    = muss*y/y(-1);
    // gy_obs          = g/y;
    // h_obs           = h;

    growth_y_obs    = muss;
    gy_obs          = gy;
    n_obs           = n;

end;

steady;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% D. Estimation

// Checking user input
if (strcmp(estvar,'yes') ~= 1 && strcmp(estvar,'no') ~= 1)
    display(estvar)
    error('Set estvar to yes or no')
end
```

```
if strcmp(estvar,'yes') == 1

    varobs growth_y_obs gy_obs n_obs;

    estimated_params;

    // 2 household parameters (fix beta)
        phi, gamma_pdf, 4, 1;
        nu99, beta_pdf, 0.5/0.99, 0.1;

    // 3 firm and technology parameters (fix alpha, del0, del1, zss)
        del2, uniform_pdf, , ,0.0,5;
        kappa, gamma_pdf, 4, 1;
        rho_z99, beta_pdf, 0.7/0.99, 0.2;

    // 3 policy parameters (fix tnss, tkss, gyss)
        rho_n99, beta_pdf, 0.7/0.99, 0.2;
        rho_k99, beta_pdf, 0.7/0.99, 0.2;
        rho_g99, beta_pdf, 0.7/0.99, 0.2;

    // 1 growth rate parameter (fix muss)
        rho_m50, beta_pdf, 0.0+0.5, 0.1;

    // 5 unanticipated shocks
        stderr ez_0, uniform_pdf, , ,0.0,0.1;
        stderr em_0, uniform_pdf, , ,0.0,0.1;
        stderr ek_0, uniform_pdf, , ,0.0,0.1*sqrt(3);
        stderr en_0, uniform_pdf, , ,0.0,0.1*sqrt(3);
        stderr eg_0, uniform_pdf, , ,0.0,0.1*sqrt(3);

    // 3 measurement errors
        stderr em_y,  uniform_pdf, , ,0.0,0.25*0.009;
        stderr em_gy, uniform_pdf, , ,0.0,0.25*0.009;
        stderr em_n,  uniform_pdf, , ,0.0,0.25*0.014;

    end ;

    estimation(order=1, datafile=newsdata, mh_replic=100000, mh_nblocks=4,
    lik_init=2, mh_drop=0.5, mh_jscale=0.825, mode_compute=6);

end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% E. Simulation

// Checking user input
```

```
if (strcmp(simvar,'sim') ~= 1 && strcmp(simvar,'irf') ~= 1)
    display(simvar)
    error('Set simvar to sim or irf')
end

if strcmp(simvar,'sim') == 1

    // Simulation block

    if strcmp(estvar,'no') == 1

        shocks;
            var ez_0; stderr 0.0071;
            var em_0; stderr 0.0068;
            var ek_0; stderr 0.0277;
            var en_0; stderr 0.0148;
            var eg_0; stderr 0.0019;
            var em_y; stderr 0.0019;
            var em_gy; stderr 0.0008;
            var em_n; stderr 0.0001;
        end;

    end

    stoch_simul(order=1,periods=209,nograph,noprint) y c n i g ;

    // Transforming percent deviations to log levels
    lny     = log(y+oo_.steady_state(1));
    lnc     = log(c+oo_.steady_state(7));
    lnn     = log(n+oo_.steady_state(4));
    lni     = log(i+oo_.steady_state(10));
    lng     = log(g+oo_.steady_state(16));

    YY = lny;
    XX = [ lnc lni lng lnn ];
    [sd_own,sd_rel,clead2,clead1,cc,clag1,clag2] = statistics(XX,YY);

    tab11   = sd_own;
    tab12   = sd_rel;
    tab21   = clead2;
    tab22   = clead1;
    tab23   = cc;
    tab24   = clag1;
    tab25   = clag2;

    // Construct the statistical properties table
    tab1    = [tab11 tab12];
```

```
    tab2    = [tab21 tab22 tab23 tab24 tab25];
    tab     = [tab1 [NaN NaN NaN NaN NaN ; tab2]];

    lab     = ['Output     ';
               'Consumption';
               'Investment ';
               'Government ';
               'Hours      '];

    SL      = size(lab);

    disp(' ')
    disp('MODEL PROPERTIES: STANDARD DEVIATIONS AND CORRELATIONS')
    disp('Statistics based on logged and HP-filtered series')
    disp(' ')
    disp('------------------------------------------------------------------')
    disp('A. Standard deviations                                            ')
    disp('------------------------------------------------------------------')
    disp('                       Standard deviations                        ')
    disp('                       -------------------------------------------')
    disp('Variable               A1. Percent              A2. Relative to Y ')
    disp('------------------------------------------------------------------')
    for i = 1:SL(1)
        disp(sprintf([lab(i,:), ...
            '              %6.3f                       %6.3f    '],...
            tab(i,(1:2))))
    end
    disp('------------------------------------------------------------------')
    disp('B. Cross correlations with Y(t-k)                                 ')
    disp('------------------------------------------------------------------')
    disp('                  k =                                             ')
    disp('                  ------------------------------------------------')
    disp('Variable             -2        -1         0         1         2   ')
    disp('------------------------------------------------------------------')
    for i = 2:SL(1)
        disp(sprintf([lab(i,:), ...
            '        %6.3f    %6.3f    %6.3f    %6.3f    %6.3f'],...
            tab(i,(3:7))))
    end
    disp('------------------------------------------------------------------')
    disp(' ')

else

    // Impulse-response function block
    shocks;
        var ek_0; stderr 0.01;
```

```
    var en_0; stderr 0.01;
    var eg_0; stderr 0.01;
end;

stoch_simul(order=1,periods=209,irf=50,nograph,noprint)
    y c n k i tk tn gy ;

/* Plotting the variables in percent deviation from steady state:
recall that the endogenous variable ordering is

y u k n r w c lambda q i  gy z  mu tk tn g
1 2 3 4 5 6 7 8      9 10 11 12 13 14 15 16

I will need this to locate the steady states in the Dynare file */

// Shock to ek_0
ypct    = oo_.irfs.y_ek_0/oo_.steady_state(1)*100;
cpct    = oo_.irfs.c_ek_0/oo_.steady_state(7)*100;
npct    = oo_.irfs.n_ek_0/oo_.steady_state(4)*100;
kpct    = oo_.irfs.k_ek_0/oo_.steady_state(3)*100;
ipct    = oo_.irfs.i_ek_0/oo_.steady_state(10)*100;
tkpct   = oo_.irfs.tk_ek_0*100;

// Adjusting the periods
ypctk   = [zeros(1,3) ypct(1:17)];
cpctk   = [zeros(1,3) cpct(1:17)];
npctk   = [zeros(1,3) npct(1:17)];
kpctk   = [zeros(1,3) kpct(1:17)];
ipctk   = [zeros(1,3) ipct(1:17)];
tkpctk  = [zeros(1,3) tkpct(1:17)];

save ..\kshock_nonews ypctk cpctk npctk kpctk ipctk tkpctk

time    = maketime(ypctk);
O       = zeros(1,length(time));

// Plotting
figure
subplot(2,3,1)
plot(time,tkpctk,'b-',time,O,'k-')
title('\tau_k')
ylabel('%')
subplot(2,3,2)
plot(time,ypctk,'b-',time,O,'k-')
title('Output')
ylabel('%')
subplot(2,3,3)
```

```
plot(time,cpctk,'b-',time,0,'k-')
title('Consumption')
ylabel('%')
subplot(2,3,4)
plot(time,ipctk,'b-',time,0,'k-')
title('Investment')
ylabel('%')
subplot(2,3,5)
plot(time,npctk,'b-',time,0,'k-')
title('Hours')
ylabel('%')
subplot(2,3,6)
plot(time,kpctk,'b-',time,0,'k-')
title('Capital')
ylabel('%')

// Shock to en_0
ypct    = oo_.irfs.y_en_0/oo_.steady_state(1)*100;
cpct    = oo_.irfs.c_en_0/oo_.steady_state(7)*100;
npct    = oo_.irfs.n_en_0/oo_.steady_state(4)*100;
kpct    = oo_.irfs.k_en_0/oo_.steady_state(3)*100;
ipct    = oo_.irfs.i_en_0/oo_.steady_state(10)*100;
tnpct   = oo_.irfs.tn_en_0*100;

// Adjusting the periods
ypctn   = [zeros(1,3) ypct(1:17)];
cpctn   = [zeros(1,3) cpct(1:17)];
npctn   = [zeros(1,3) npct(1:17)];
kpctn   = [zeros(1,3) kpct(1:17)];
ipctn   = [zeros(1,3) ipct(1:17)];
tnpctn  = [zeros(1,3) tnpct(1:17)];

save ..\nshock_nonews ypctn cpctn npctn kpctn ipctn tnpctn

// Plotting
figure
subplot(2,3,1)
plot(time,tnpctn,'b-',time,0,'k-')
title('\tau_n')
ylabel('%')
subplot(2,3,2)
plot(time,ypctn,'b-',time,0,'k-')
title('Output')
ylabel('%')
subplot(2,3,3)
plot(time,cpctn,'b-',time,0,'k-')
title('Consumption')
```

```
ylabel('%')
subplot(2,3,4)
plot(time,ipctn,'b-',time,0,'k-')
title('Investment')
ylabel('%')
subplot(2,3,5)
plot(time,npctn,'b-',time,0,'k-')
title('Hours')
ylabel('%')
subplot(2,3,6)
plot(time,kpctn,'b-',time,0,'k-')
title('Capital')
ylabel('%')

// Shock to eg_0
ypct    = oo_.irfs.y_eg_0/oo_.steady_state(1)*100;
cpct    = oo_.irfs.c_eg_0/oo_.steady_state(7)*100;
npct    = oo_.irfs.n_eg_0/oo_.steady_state(4)*100;
kpct    = oo_.irfs.k_eg_0/oo_.steady_state(3)*100;
ipct    = oo_.irfs.i_eg_0/oo_.steady_state(10)*100;
gpct    = oo_.irfs.gy_eg_0*100;

// Adjusting the periods
ypctg   = [zeros(1,3) ypct(1:17)];
cpctg   = [zeros(1,3) cpct(1:17)];
npctg   = [zeros(1,3) npct(1:17)];
kpctg   = [zeros(1,3) kpct(1:17)];
ipctg   = [zeros(1,3) ipct(1:17)];
gypctg  = [zeros(1,3) gpct(1:17)];

save ..\gshock_nonews ypctg cpctg npctg kpctg ipctg gypctg

// Plotting
figure
subplot(2,3,1)
plot(time,gypctg,'b-',time,0,'k-')
title('g/y')
ylabel('%')
subplot(2,3,2)
plot(time,ypctg,'b-',time,0,'k-')
title('Output')
ylabel('%')
subplot(2,3,3)
plot(time,cpctg,'b-',time,0,'k-')
title('Consumption')
ylabel('%')
subplot(2,3,4)
```

```
    plot(time,ipctg,'b-',time,0,'k-')
    title('Investment')
    ylabel('%')
    subplot(2,3,5)
    plot(time,npctg,'b-',time,0,'k-')
    title('Hours')
    ylabel('%')
    subplot(2,3,6)
    plot(time,kpctg,'b-',time,0,'k-')
    title('Capital')
    ylabel('%')

end
```

A.4.2 Economy with news

```
// news.mod
// Economy with fiscal policy news

// Coded with Dynare 4.2.1
// Last updated: 07/12/2011

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Housekeeping

close all

// Choose whether to estimate parameters (yes) or not (no)
estvar = 'no';

// Choose simulation (sim) or impulse-response (irf)
simvar = 'irf';

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% A. Defining variables and parameters

// MODEL VARIABLES (47; 25 endogenous, 4 observables, 18 exogenous)

// 11 endogenous variables
    var y u k n r w c lambda q i gy;

    predetermined_variables k;

// 5 stochastic disturbances
    var z mu tk tn g;
```

```
// 4 observable variables
    var growth_c_obs growth_y_obs gy_obs n_obs;

// 9 anticipated variables
    var ek1 ek2 ek3 en1 en2 en3 eg1 eg2 eg3;

// 5 exogenous variables
    varexo ez_0 em_0 ek_0 en_0 eg_0;

// 4 measurement errors
    varexo em_c em_y em_gy em_n;

// 9 exogenous anticipated variables
    varexo ek_1 ek_2 ek_3 en_1 en_2 en_3 eg_1 eg_2 eg_3;


// PARAMETERS (17)

// 3 household parameters
    parameters phi beta nu99;

// 7 firm and technology parameters
    parameters alpha del0 del1 del2 kappa zss rho_z99;

// 6 policy parameters
    parameters tnss tkss gyss rho_n99 rho_k99 rho_g99;

// 2 growth rate parameters
    parameters muss rho_m50;

// Initial values

    phi     = 3.6818;
    beta    = 1.05^(-0.25);
    nu99    = 0.6852/0.99;

    alpha   = 0.36;
    del0    = 1.08^(1/4)-1;
    del1    = 0.1710;
    del2    = 4.3509;
    kappa   = 1.5549;
    zss     = 1;
    rho_z99 = 0.6992/0.99;

    tnss    = 0.26;
    tkss    = 0.42;
    gyss    = 0.15;
```

```
    rho_n99 = 0.9891/0.99;
    rho_k99 = 0.8441/0.99;
    rho_g99 = 0.9729/0.99;

    muss    = 1.005;
    rho_m50 = 0.3367+0.5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% B. Model block

model;

# nu    = nu99*0.99;
# rho_z = rho_z99*0.99;
# rho_m = rho_m50-0.5;
# rho_k = rho_k99*0.99;
# rho_n = rho_n99*0.99;
# rho_g = rho_g99*0.99;
// Parameter transformation as in Schmitt-Grohe and Uribe (2008)

// Equation 1: production function
y-z*(u*k/mu)^alpha*n^(1-alpha);

// Equation 2: firm's capital FOC
alpha*y-r*u*k/mu;

// Equation 3: firm's labor FOC
    (1-alpha)*y-w*n;

// Equation 4: household's consumption FOC
    1/(c-nu*c(-1)/mu)-lambda-beta*nu*(1/(mu(+1)*c(+1)-nu*c));

// Equation 5: household's labor FOC
    -phi+lambda*(1-tn)*w;

// Equation 6: household's utilization FOC
r-tk*(r-del0-del1*(u-1)-(del2/2)*(u-1)^2-u*(del1+del2*(u-1)))
        -q*(del1+del2*(u-1));

// Equation 7: household's capital FOC
    -lambda*q+beta*(lambda(+1)/mu(+1))*(r(+1)*u(+1)
        -tk(+1)*(r(+1)-del0-del1*(u(+1)-1)-(del2/2)*(u(+1)-1)^2)
        +q(+1)*(1-del0-del1*(u(+1)-1)-(del2/2)*(u(+1)-1)^2));

// Equation 8: household's investment FOC
-lambda+lambda*q*(1-(kappa/2)*(mu*i/i(-1)-muss)^2
        -kappa*(mu*i/i(-1)-muss)*mu*i/i(-1))
```

```
        +beta*kappa*lambda(+1)*q(+1)*mu(+1)*(mu(+1)*i(+1)/i-muss)
        *(i(+1)/i)^2;

// Equation 9: aggregate feasibility
    y-c-i-g;

// Equation 10: law of motion for capital
    (1-del0-del1*(u-1)-(del2/2)*(u-1)^2)*k/mu
        +i*(1-(kappa/2)*(mu*i/i(-1)-mu)^2)-k(+1);

// Equation 11: government consumption to output ratio
    gy-g/y;

// Equations 12-16: stochastic processes
    z = (1-rho_z)*zss+rho_z*z(-1)+ez_0;
    mu = (1-rho_m)*muss+rho_m*mu(-1)+em_0;
    tk = (1-rho_k)*tkss+rho_k*tk(-1)+ek_0+ek1(-1)+ek2(-2)+ek3(-3);
    tn = (1-rho_n)*tnss+rho_n*tn(-1)+en_0+en1(-1)+en2(-2)+en3(-3);
    gy = (1-rho_g)*gyss+rho_g*gy(-1)+eg_0+eg1(-1)+eg2(-2)+eg3(-3);

// Equations 17-20: observables
    growth_c_obs    = muss*c/c(-1)+em_c;
    growth_y_obs    = muss*y/y(-1)+em_y;
    gy_obs          = gy+em_gy;
    n_obs           = n+em_n;

// Equations 21-29: anticipated shocks
    ek1 = ek_1;
    ek2 = ek_2;
    ek3 = ek_3;
    en1 = en_1;
    en2 = en_2;
    en3 = en_3;
    eg1 = eg_1;
    eg2 = eg_2;
    eg3 = eg_3;

end ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% C. Steady state

steady_state_model;
    u   = 1;
    q   = 1;
    z   = 1;
    tn  = tnss;
```

```
    tk  = tkss;
    gy  = gyss;
    mu  = muss;

    r   = (mu-beta*(1-del0*(1-tk)))/(beta*(1-tk));

    ome_y   = r/(alpha*mu);
    ome_i   = (mu-1+del0)/mu;
    ome_c   = (1-gy)*ome_y-ome_i;
    del1    = r+(tk/(1-tk))*del0;
    ome_w   = phi*(mu-nu99*0.99)*ome_c/((mu-beta*nu99*0.99)*(1-tn));

    n   = (1-alpha)*ome_y/ome_w;
    k   = n/(mu^alpha*ome_y)^(1/(1-alpha));
    w   = ome_w*k;
    c   = ome_c*k;
    i   = ome_i*k;
    y   = ome_y*k;

    g   = gyss*y;

    lambda  = phi/((1-tn)*w);

    // Observables are defined as

    // growth_c_obs    = muss*c/c(-1);
    // growth_y_obs    = muss*y/y(-1);
    // gy_obs          = g/y;
    // h_obs           = h;

    growth_c_obs    = muss;
    growth_y_obs    = muss;
    gy_obs          = gy;
    n_obs           = n;

end;

steady;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% D. Estimation

// Checking user input
if (strcmp(estvar,'yes') ~= 1 && strcmp(estvar,'no') ~= 1)
    display(estvar)
    error('Set estvar to yes or no')
end
```

```
if strcmp(estvar,'yes') == 1

    varobs growth_c_obs growth_y_obs gy_obs n_obs;

    estimated_params;

    // 2 household parameters (fix beta)
        phi, gamma_pdf, 4, 1;
        nu99, beta_pdf, 0.5/0.99, 0.1;

    // 3 firm and technology parameters (fix alpha, del0, del1, zss)
        del2, uniform_pdf, , ,0.0,5;
        kappa, gamma_pdf, 4, 1;
        rho_z99, beta_pdf, 0.7/0.99, 0.2;

    // 6 policy parameters (fix tnss, tkss, gyss)
        rho_n99, beta_pdf, 0.7/0.99, 0.2;
        rho_k99, beta_pdf, 0.7/0.99, 0.2;
        rho_g99, beta_pdf, 0.7/0.99, 0.2;

    // 1 growth rate parameter (fix muss)
        rho_m50, beta_pdf, 0.0+0.5, 0.1;

    // 5 unanticipated shocks
        stderr ez_0, uniform_pdf, , ,0.0,0.1;
        stderr em_0, uniform_pdf, , ,0.0,0.1;
        stderr ek_0, uniform_pdf, , ,0.0,0.1*sqrt(3);
        stderr en_0, uniform_pdf, , ,0.0,0.1*sqrt(3);
        stderr eg_0, uniform_pdf, , ,0.0,0.1*sqrt(3);

    // 4 measurement errors
        stderr em_c,  uniform_pdf, , ,0.0,0.25*0.005;
        stderr em_y,  uniform_pdf, , ,0.0,0.25*0.009;
        stderr em_gy, uniform_pdf, , ,0.0,0.25*0.009;
        stderr em_n,  uniform_pdf, , ,0.0,0.25*0.014;

    // 9 anticipated shocks
        stderr ek_1, uniform_pdf, , ,0.0,0.1;
        stderr ek_2, uniform_pdf, , ,0.0,0.1;
        stderr ek_3, uniform_pdf, , ,0.0,0.1;
        stderr en_1, uniform_pdf, , ,0.0,0.1;
        stderr en_2, uniform_pdf, , ,0.0,0.1;
        stderr en_3, uniform_pdf, , ,0.0,0.1;
        stderr eg_1, uniform_pdf, , ,0.0,0.1;
        stderr eg_2, uniform_pdf, , ,0.0,0.1;
        stderr eg_3, uniform_pdf, , ,0.0,0.1;
```

```
    end ;

    estimation(order=1, datafile=newsdata, mh_replic=100000, mh_nblocks=4,
    lik_init=2, mh_drop=0.5, mh_jscale=0.525, mode_compute=6);

end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% E. Simulation

// Checking user input
if (strcmp(simvar,'sim') ~= 1 && strcmp(simvar,'irf') ~= 1)
    display(simvar)
    error('Set simvar to sim or irf')
end

if strcmp(simvar,'sim') == 1

    // Simulation block

    if strcmp(estvar,'no') == 1

        shocks;
            var ez_0; stderr 0.0064;
            var em_0; stderr 0.0046;
            var ek_0; stderr 0.1284;
            var en_0; stderr 0.0024;
            var eg_0; stderr 0.0007;
            var em_y; stderr 0.0022;
            var em_gy; stderr 0.0009;
            var em_n; stderr 0.0007;
            var ek_1; stderr 0.0878;
            var ek_2; stderr 0.0672;
            var ek_3; stderr 0.0915;
            var en_1; stderr 0.0030;
            var en_2; stderr 0.0045;
            var en_3; stderr 0.0103;
            var eg_1; stderr 0.0008;
            var eg_2; stderr 0.0008;
            var eg_3; stderr 0.0008;
        end;

    end

    stoch_simul(order=1,periods=209,nograph,noprint) y c n i g ;
```

```
// Transforming percent deviations to log levels
lny     = log(y+oo_.steady_state(1));
lnc     = log(c+oo_.steady_state(7));
lnn     = log(n+oo_.steady_state(4));
lni     = log(i+oo_.steady_state(10));
lng     = log(g+oo_.steady_state(16));

YY = lny;
XX = [ lnc lni lng lnn ];
[sd_own,sd_rel,clead2,clead1,cc,clag1,clag2] = statistics(XX,YY);

tab11   = sd_own;
tab12   = sd_rel;
tab21   = clead2;
tab22   = clead1;
tab23   = cc;
tab24   = clag1;
tab25   = clag2;

// Construct the statistical properties table
tab1    = [tab11 tab12];
tab2    = [tab21 tab22 tab23 tab24 tab25];
tab     = [tab1 [NaN NaN NaN NaN NaN ; tab2]];

lab     = ['Output     ';
           'Consumption';
           'Investment ';
           'Government ';
           'Hours      '];

SL      = size(lab);

disp(' ')
disp('MODEL PROPERTIES: STANDARD DEVIATIONS AND CORRELATIONS')
disp('Statistics based on logged and HP-filtered series')
disp(' ')
disp('------------------------------------------------------------------')
disp('A. Standard deviations                                            ')
disp('------------------------------------------------------------------')
disp('                       Standard deviations                        ')
disp('                       -------------------------------------------')
disp('Variable               A1. Percent              A2. Relative to Y ')
disp('------------------------------------------------------------------')
for i = 1:SL(1)
    disp(sprintf([lab(i,:), ...
        '              %6.3f                      %6.3f    '],...
        tab(i,(1:2))))
```

```
    end
    disp('-----------------------------------------------------------------')
    disp('B. Cross correlations with Y(t-k)                                ')
    disp('-----------------------------------------------------------------')
    disp('                 k =                                             ')
    disp('                 ------------------------------------------------')
    disp('Variable             -2        -1         0         1         2    ')
    disp('-----------------------------------------------------------------')
    for i = 2:SL(1)
        disp(sprintf([lab(i,:), ...
            '       %6.3f    %6.3f    %6.3f    %6.3f    %6.3f'],...
            tab(i,(3:7))))
    end
    disp('-----------------------------------------------------------------')
    disp(' ')

else

    // Impulse-response function block
    shocks;
        var ek_3; stderr 0.01;
        var en_3; stderr 0.01;
        var eg_3; stderr 0.01;
    end;

    stoch_simul(order=1,periods=209,irf=20,nograph,noprint)
        y c n k i tk tn gy ;

    /* Plotting the variables in percent deviation from steady state:
    recall that the endogenous variable ordering is

    y u k n r w c lambda q i  gy z  mu tk tn g
    1 2 3 4 5 6 7 8      9 10 11 12 13 14 15 16

    I will need this to locate the steady states in the Dynare file */

    // Shock to ek_3
    ypctnk  = oo_.irfs.y_ek_3/oo_.steady_state(1)*100;
    cpctnk  = oo_.irfs.c_ek_3/oo_.steady_state(7)*100;
    npctnk  = oo_.irfs.n_ek_3/oo_.steady_state(4)*100;
    kpctnk  = oo_.irfs.k_ek_3/oo_.steady_state(3)*100;
    ipctnk  = oo_.irfs.i_ek_3/oo_.steady_state(10)*100;
    tkpctnk = oo_.irfs.tk_ek_3*100;

    save ..\kshock_news ypctnk cpctnk npctnk kpctnk ipctnk tkpctnk

    time    = maketime(ypctnk);
```

```
O       = zeros(1,length(time));

// Plotting
figure
subplot(2,3,1)
plot(time,tkpctnk,'b-',time,O,'k-')
title('\tau_k')
ylabel('%')
subplot(2,3,2)
plot(time,ypctnk,'b-',time,O,'k-')
title('Output')
ylabel('%')
subplot(2,3,3)
plot(time,cpctnk,'b-',time,O,'k-')
title('Consumption')
ylabel('%')
subplot(2,3,4)
plot(time,ipctnk,'b-',time,O,'k-')
title('Investment')
ylabel('%')
subplot(2,3,5)
plot(time,npctnk,'b-',time,O,'k-')
title('Hours')
ylabel('%')
subplot(2,3,6)
plot(time,kpctnk,'b-',time,O,'k-')
title('Capital')
ylabel('%')

// Shock to en_3
ypctnn  = oo_.irfs.y_en_3/oo_.steady_state(1)*100;
cpctnn  = oo_.irfs.c_en_3/oo_.steady_state(7)*100;
npctnn  = oo_.irfs.n_en_3/oo_.steady_state(4)*100;
kpctnn  = oo_.irfs.k_en_3/oo_.steady_state(3)*100;
ipctnn  = oo_.irfs.i_en_3/oo_.steady_state(10)*100;
tnpctnn = oo_.irfs.tn_en_3*100;

save ..\nshock_news ypctnn cpctnn npctnn kpctnn ipctnn tnpctnn

// Plotting
figure
subplot(2,3,1)
plot(time,tnpctnn,'b-',time,O,'k-')
title('\tau_n')
ylabel('%')
subplot(2,3,2)
plot(time,ypctnn,'b-',time,O,'k-')
```

```
title('Output')
ylabel('%')
subplot(2,3,3)
plot(time,cpctnn,'b-',time,0,'k-')
title('Consumption')
ylabel('%')
subplot(2,3,4)
plot(time,ipctnn,'b-',time,0,'k-')
title('Investment')
ylabel('%')
subplot(2,3,5)
plot(time,npctnn,'b-',time,0,'k-')
title('Hours')
ylabel('%')
subplot(2,3,6)
plot(time,kpctnn,'b-',time,0,'k-')
title('Capital')
ylabel('%')

// Shock to eg_3
ypctng  = oo_.irfs.y_eg_3/oo_.steady_state(1)*100;
cpctng  = oo_.irfs.c_eg_3/oo_.steady_state(7)*100;
npctng  = oo_.irfs.n_eg_3/oo_.steady_state(4)*100;
kpctng  = oo_.irfs.k_eg_3/oo_.steady_state(3)*100;
ipctng  = oo_.irfs.i_eg_3/oo_.steady_state(10)*100;
gypctng = oo_.irfs.gy_eg_3*100;

save ..\gshock_news ypctng cpctng npctng kpctng ipctng gypctng

// Plotting
figure
subplot(2,3,1)
plot(time,gypctng,'b-',time,0,'k-')
title('g/y')
ylabel('%')
subplot(2,3,2)
plot(time,ypctng,'b-',time,0,'k-')
title('Output')
ylabel('%')
subplot(2,3,3)
plot(time,cpctng,'b-',time,0,'k-')
title('Consumption')
ylabel('%')
subplot(2,3,4)
plot(time,ipctng,'b-',time,0,'k-')
title('Investment')
ylabel('%')
```

```
    subplot(2,3,5)
    plot(time,npctng,'b-',time,0,'k-')
    title('Hours')
    ylabel('%')
    subplot(2,3,6)
    plot(time,kpctng,'b-',time,0,'k-')
    title('Capital')
    ylabel('%')

end
```

Appendix B

Additional Material (Tax Rules)

B.1 Steady states with and without tax rules

The analysis below is done for the case where capital income taxes are included in the tax rule. The other case (labor income taxes) can be trivially derived by following a similar logic.

B.1.1 Economy with tax rule

It is easy to verify that in the steady state, the equilibrium conditions become

$$\begin{aligned}
\lambda &= \frac{1-\beta\nu}{(1-\nu)c} \\
\chi &= \lambda \\
\phi &= \lambda(1-\tau_n)w \\
\chi &= \beta\lambda[(1-\tau_k)r + \delta\tau_k] + \beta\chi(1-\delta) \\
1 &= \beta R^b \\
x &= \delta k \\
y &= c + x + g \\
y &= k^\alpha n^{1-\alpha}
\end{aligned}$$

$$\begin{aligned} rk &= \alpha y \\ wn &= (1-\alpha)y \\ d &= g + v + R^b b - \tau_n wn \end{aligned}$$

since the last two equilibrium conditions vanish in the steady state. Note that I have combined the household and government budget constraints to obtain the seventh equation. Moreover, the structure of the system is such that the fifth and last equations are independent of the others: R^b equals $1/\beta$ and d can be obtained once values for R^b, y, w, and n are derived. (The fact that I need y comes from the assumption that the ratios d/y, b/y, g/y, and v/y are given.)

The third equation implies $\chi = \lambda$; I substitute this to get

$$\lambda = \frac{1-\beta\nu}{(1-\nu)c} \tag{B.1}$$

$$\phi = \lambda(1-\tau_n)w \tag{B.2}$$

$$1 = \beta[(1-\tau_k)r + \delta\tau_k + 1 - \delta] \tag{B.3}$$

$$x = \delta k \tag{B.4}$$

$$y = c + x + g \tag{B.5}$$

$$y = k^\alpha n^{1-\alpha} \tag{B.6}$$

$$rk = \alpha y \tag{B.7}$$

$$wn = (1-\alpha)y. \tag{B.8}$$

The above is a system of 8 equations in $\{c, \phi, \lambda, w, r, k, x, y\}$. To find its solution, first take (B.3) and solve for r directly:

$$r = \frac{1-\beta(1-\delta(1-\tau_k))}{\beta(1-\tau_k)}. \tag{B.9}$$

From (B.7) I can solve for y as a function of k:

$$y = \frac{r}{\alpha}k \equiv \omega_y k \tag{B.10}$$

and from (B.4) I solve for x as a function of k:

$$x = \delta k.$$

Recall that the ratio g/y is given; call it g_0. Then

$$g = g_0 y = g_0 \omega_y k.$$

Using this in (B.5) gives c as a function of k:

$$c = \omega_y k - \delta k - g_0 \omega_y k = [\omega_y(1 - g_0) - \delta]k \equiv \omega_c k$$

and then I can plug this in (B.1) to get λ as a function of k:

$$\lambda = \frac{1 - \beta\nu}{(1 - \nu)\omega_c k}.$$

From (B.8), solve for w as a function of k:

$$w = \frac{(1 - \alpha)\omega_y}{n} k \equiv \omega_w k$$

and use this in (B.2) to get ϕ as a function of k

$$\phi = \frac{(1 - \beta\nu)(1 - \tau_n)\omega_w k}{(1 - \nu)\omega_c k} = \frac{(1 - \beta\nu)(1 - \tau_n)\omega_w}{(1 - \nu)\omega_c}.$$

Finally, use (B.6) to solve for k:

$$k = \frac{n}{\omega_y^{1/(1-\alpha)}}.$$

All other values follow directly.

B.1.2 The case without tax rules

I now adjust the model above to consider the case where the tax rules are not present; the only differences come from the government equilibrium conditions. First, the government budget constraint is still

$$g_t + v_t + R_t^b b_{t-1} = b_t + \tau_{kt}(r_t - \delta)k_t + \tau_n w_t n_t$$

yet the capital income tax rate now follows the AR(1) process

$$\tau_{kt} = (1-\rho_k)\tau_k + \rho_k \tau_{k,t-1} + \epsilon_{kt}.$$

Equilibrium conditions from the household are the same as in Section 3.2.5

$$\begin{aligned}
\lambda_t &= \frac{1}{c_t - \nu c_{t-1}} - \beta \nu E_t \frac{1}{c_{t+1} - \nu c_t} \\
\lambda_t &= \chi_t \left[1 - \eta \left(\frac{x_t}{k_t} - \delta \right) \right] \\
\phi &= \lambda_t (1-\tau_n) w_t \\
\chi_t &= \beta E_t \lambda_{t+1} [(1-\tau_{k,t+1}) r_{t+1} + \delta \tau_{k,t+1}] \\
&\quad + \beta E_t \chi_{t+1} \left[1 - \delta + \eta \left(\frac{x_{t+1}}{k_{t+1}} - \delta \right) \frac{x_{t+1}}{k_{t+1}} - \frac{\eta}{2} \left(\frac{x_{t+1}}{k_{t+1}} - \delta \right)^2 \right] \\
\lambda_t &= \beta E_t \lambda_{t+1} R^b_{t+1}
\end{aligned}$$

plus

$$y_t = c_t + x_t + g_t.$$

The same applies for those of the firm:

$$\begin{aligned}
y_t &= z_t k_t^{\alpha} n_t^{1-\alpha} \\
r_t k_t &= \alpha y_t \\
w_t n_t &= (1-\alpha) y_t.
\end{aligned}$$

For the government, the tax rules get substituted by

$$g_t + v_t + R^b_t b_{t-1} = b_t + \tau_{kt}(r_t - \delta) k_t + \tau_n w_t n_t$$

and the capital income tax rate is now an exogenous variable.

B.1.3 Steady states

By inspection, the system above is essentially similar to that in Section B.1.1 when taken to the steady state. The only differences correspond to the equilibrium conditions of the government, which still do not influence the solution of the remaining steady state values. Thus, the equations above form a system of 9 equations in $\{c, \lambda, \phi, w, \chi, r, k, x, y\}$. Given the same parameter values, steady states should be identical.

B.2 Log-linearization

B.2.1 Benchmark model

It is straightforward to verify that the equilibrium conditions of the benchmark model (3.4), (3.9)–(3.16), (3.18), and (3.19) can be log-linearized to

$$\begin{aligned}
\lambda\hat{\lambda}_t &= \kappa_0\hat{c}_{t-1} + \kappa_1\hat{c}_t + \kappa_2 E_t\hat{c}_{t+1} \\
\hat{\lambda}_t &= \hat{\chi}_t - \delta\eta\hat{x}_t + \delta\eta\hat{k}_t \\
\hat{\lambda}_t &= -\hat{w}_t \\
\hat{\chi}_t &= [1-\beta(1-\delta)]E_t\hat{\lambda}_{t+1} - \beta(r-\delta)E_t\tilde{\tau}_{K,t+1} + \beta(1-\tau_k)rE_t\hat{r}_{t+1} \\
&\quad + \beta(1-\delta)E_t\hat{\chi}_{t+1} + \beta\delta^2\eta E_t\hat{x}_{t+1} - \beta\delta^2\eta\hat{k}_{t+1} \\
\hat{\lambda}_t &= E_t\hat{\lambda}_{t+1} + E_t\hat{R}^b_{t+1} \\
\hat{k}_{t+1} &= (1-\delta)\hat{k}_t + \delta\hat{x}_t \\
\hat{y}_t &= \frac{c}{y}\hat{c}_t + \frac{x}{y}\hat{x}_t + \hat{g}_t \\
\hat{y}_t &= \hat{z}_t + \alpha\hat{k}_t + (1-\alpha)\hat{n}_t \\
\hat{y}_t &= \hat{r}_t + \hat{k}_t \\
\hat{y}_t &= \hat{w}_t + \hat{n}_t \\
\hat{b}_t &= \hat{g}_t + \hat{v}_t + \frac{1}{\beta}\frac{b}{y}\hat{R}^b_t + \frac{1}{\beta}\hat{b}_{t-1} - \gamma\tilde{\tau}_{kt} \\
&\quad - \alpha\tau_k\hat{r}_t - \gamma\tau_k\hat{k}_t - (1-\alpha)\tau_n\hat{w}_t - (1-\alpha)\tau_n\hat{n}_t
\end{aligned}$$

where

$$\kappa_0 \equiv \frac{\nu}{(1-\nu)^2 c}, \quad \kappa_1 \equiv -\frac{1+\beta\nu^2}{(1-\nu)^2 c}, \quad \kappa_2 \equiv \frac{\beta\nu}{(1-\nu)^2 c},$$

plus the laws of motion for $\{\hat{z}_t, \hat{g}_t, \hat{v}_t, \tilde{\tau}_{kt}\}$, assumed to be standard AR(1) processes.

B.2.2 Capital income tax rule

In the case where the capital income tax rule is in place, the full system is given by

$$\begin{aligned}
\lambda\hat{\lambda}_t &= \kappa_0\hat{c}_{t-1} + \kappa_1\hat{c}_t + \kappa_2 E_t\hat{c}_{t+1} \\
\hat{\lambda}_t &= \hat{c}_t - \delta\eta\hat{x}_t + \delta\eta\hat{k}_t \\
\hat{\lambda}_t &= -\hat{w}_t \qquad\qquad \text{(B.11)} \\
\hat{\chi}_t &= [1-\beta(1-\delta)]E_t\hat{\lambda}_{t+1} - \beta(r-\delta)E_t\tilde{\tau}_{K,t+1} + \beta(1-\tau_k)rE_t\hat{r}_{t+1} \\
&\quad + \beta(1-\delta)E_t\hat{\chi}_{t+1} + \beta\delta^2\eta E_t\hat{x}_{t+1} - \beta\delta^2\eta\hat{k}_{t+1} \\
\hat{\lambda}_t &= E_t\hat{\lambda}_{t+1} + E_t\hat{R}^b_{t+1} \\
\hat{k}_{t+1} &= (1-\delta)\hat{k}_t + \delta\hat{x}_t \\
\hat{y}_t &= \frac{c}{y}\hat{c}_t + \frac{x}{y}\hat{x}_t + \hat{g}_t \\
\hat{y}_t &= \hat{z}_t + \alpha\hat{k}_t + (1-\alpha)\hat{n}_t \\
\hat{y}_t &= \hat{r}_t + \hat{k}_t \\
\hat{y}_t &= \hat{w}_t + \hat{n}_t \\
\hat{d}_t &= \hat{g}_t + \hat{v}_t + \frac{1}{\beta}\frac{b}{y}\hat{R}^b_t + \frac{1}{\beta}\hat{b}_{t-1} - (1-\alpha)\tau_n\hat{w}_t - (1-\alpha)\tau_n\hat{n}_t \qquad \text{(B.12)} \\
\psi\hat{d}_t &= \gamma E_t\tilde{\tau}_{k,t+1} + \alpha\tau_k E_t\hat{r}_{t+1} + \gamma\tau_k\hat{k}_{t+1} \qquad \text{(B.13)} \\
\hat{b}_t &= (1-\psi)\hat{d}_t + \gamma E_t(\tilde{\tau}_{k,t+1} - \tilde{\tau}_{kt}) \\
&\quad + \alpha\tau_k E_t(\hat{r}_{t+1} - \hat{r}_t) + \gamma\tau_k(k_{t+1} - k_t) \qquad \text{(B.14)}
\end{aligned}$$

plus the laws of motion for $\{\hat{z}_t, \hat{g}_t, \hat{v}_t\}$, assumed to be standard AR(1) processes.

B.2.3 Log-linearization of tax rules

Here I prove Lemma 3.3.2, reproduced below for convenience:

Lemma. *The log-linearized version of equations (3.23) and (3.24) is given by*

$$\begin{aligned}\psi \hat{d}_t &= \gamma E_t \tilde{\tau}_{k,t+1} + \alpha \tau_k E_t \hat{r}_{t+1} + \gamma \tau_k \hat{k}_{t+1} \\ \hat{b}_t &= (1-\psi)\hat{d}_t + \gamma E_t(\tilde{\tau}_{k,t+1} - \tilde{\tau}_{kt}) \\ &\quad + \alpha \tau_k E_t(\hat{r}_{t+1} - \hat{r}_t) + \gamma \tau_k (\hat{k}_{t+1} - \hat{k}_t),\end{aligned}$$

where $\gamma \equiv \alpha - x/y$.

Proof. Begin with equation (3.23); applying the usual rules yields

$$\begin{aligned}\psi(d + y\hat{d}_t - d) &= E_t \tau_k r k \left(1 + \frac{1}{\tau_k}\tilde{\tau}_{k,t+1} + \hat{r}_{t+1} + \hat{k}_{t+1}\right) \\ &\quad - \delta E_t \tau_k k \left(1 + \frac{1}{\tau_k}\tilde{\tau}_{k,t+1} + \hat{k}_{t+1}\right) - \tau_k (r-\delta) k \\ \psi y \hat{d}_t &= \tau_k r k + r k E_t \tilde{\tau}_{k,t+1} + \tau_k r k E_t \hat{r}_{t+1} + \tau_k r k k_{t+1} \\ &\quad - \delta \tau_k k - \delta k E_t \tilde{\tau}_{k,t+1} - \delta \tau_k k \hat{k}_{t+1} - \tau_k (r-\delta) k \\ &= (r-\delta) k E_t \tilde{\tau}_{k,t+1} + \tau_k r k E_t \hat{r}_{t+1} + \tau_k (r-\delta) k \hat{k}_{t+1}.\end{aligned}$$

Now divide by y:

$$\psi \hat{d}_t = \left(\frac{rk}{y} - \frac{\delta k}{y}\right) E_t \tilde{\tau}_{k,t+1} + \tau_k \left(\frac{rk}{y}\right) E_t \hat{r}_{t+1} + \tau_k \left(\frac{rk}{y} - \frac{\delta k}{y}\right) \hat{k}_{t+1}$$

and note that from equation (B.4), $\delta k = x$, and from (B.7), $rk = \alpha y$. Substituting in the above:

$$\psi \hat{d}_t = \left(\alpha - \frac{x}{y}\right) E_t \tilde{\tau}_{k,t+1} + \alpha \tau_k E_t \hat{r}_{t+1} + \tau_k \left(\alpha - \frac{x}{y}\right) \hat{k}_{t+1}.$$

Setting $\gamma \equiv \alpha - x/y$ yields (3.23). For (3.24), apply the rules:

$$\begin{aligned}
b + y\hat{b}_t - b &= (1-\psi)(d + y\hat{d}_t - d) + E_t\tau_k rk\left(1 + \frac{1}{\tau_k}\tilde{\tau}_{k,t+1} + \hat{r}_{t+1} + \hat{k}_{t+1}\right) \\
&\quad - \delta E_t\tau_k k\left(1 + \frac{1}{\tau_k}\tilde{\tau}_{k,t+1} + \hat{k}_{t+1}\right) - E_t\tau_k rk\left(1 + \frac{1}{\tau_k}\tilde{\tau}_{kt} + \hat{r}_t + \hat{k}_t\right) \\
&\quad + \delta E_t\tau_k k\left(1 + \frac{1}{\tau_k}\tilde{\tau}_{kt} + \hat{k}_t\right) \\
y\hat{b}_t &= (1-\psi)y\hat{d}_t + (r-\delta)kE_t\tilde{\tau}_{k,t+1} + \tau_k rkE_t\hat{r}_{t+1} + \tau_k(r-\delta)k\hat{k}_{t+1} \\
&\quad - (r-\delta)k\tilde{\tau}_{kt} - \tau_k rk\hat{r}_t - \tau_k(r-\delta)k\hat{k}_t \\
&= (1-\psi)y\hat{d}_t + (r-\delta)kE_t(\tilde{\tau}_{k,t+1} - \tilde{\tau}_{kt}) \\
&\quad + \tau_k rkE_t(\hat{r}_{t+1} - \hat{r}_t) + \tau_k(r-\delta)k(\hat{k}_{t+1} - \hat{k}_t).
\end{aligned}$$

Divide by y and use the same steady-state relations as above to get

$$\hat{b}_t = (1-\psi)\hat{d}_t + \gamma E_t(\tilde{\tau}_{k,t+1} - \tilde{\tau}_{kt}) + \alpha\tau_k E_t(\hat{r}_{t+1} - \hat{r}_t) + \gamma\tau_k(\hat{k}_{t+1} - \hat{k}_t)$$

which is (3.24). □

B.3 Deriving the value of γ

Here I prove Lemma 3.3.3, reproduced below for convenience:

Lemma. *The value of γ is pinned down by the model's parameters and is such that $\gamma \leq \alpha$. In particular,*

$$\gamma = \alpha\left[1 - \frac{\delta\beta(1-\tau_k)}{1-\beta(1-\delta(1-\tau_k))}\right].$$

Proof. By definition, $\gamma \equiv \alpha - x/y$. I can use (B.4) and (B.10) to expand γ as

$$\gamma = \alpha - \frac{x}{y} = \alpha - \frac{\delta k}{\omega_y k} = \alpha - \frac{\delta}{\frac{r}{\alpha}} = \alpha - \frac{\alpha\delta}{r} = \alpha\left[1 - \frac{\delta}{r}\right].$$

Now use (B.9) to substitute for r:

$$\gamma = \alpha\left[1 - \frac{\delta}{\frac{1-\beta(1-\delta(1-\tau_k))}{\beta(1-\tau_k)}}\right] = \alpha\left[1 - \frac{\delta\beta(1-\tau_k)}{1-\beta(1-\delta(1-\tau_k))}\right]$$

as needed. Since the second term in the brackets satisfies

$$0 \leq \frac{\delta\beta(1-\tau_k)}{1-\beta(1-\delta(1-\tau_k))} < 1$$

it follows that $\gamma \leq \alpha$. □

B.4 Matlab codes

The following codes reproduce all the figures contained in Chapter 3. There are some other codes which are used in the scripts below; these can be obtained in my webpage or via email.

```
% rule_tk.m script
% Model with tax rule

% The model's variables are assigned to matrices x, y and z following
%
% x = [ K C(-1) d B(-1) tk ]'
% y = [ lambda w chi X r Rb Y N ]'
% z = [ z G V ]'

% NOTES
% Created 10/30/2010; last revised 07/05/2010
% Prepared by Mario Solis-Garcia

%% STARTING UP

clear
clc
tic

%% EXPERIMENT PARAMETERS

% Impulse-response: number of periods in the model economy (41 or 201)
TIR     = 41;

% Choose variable to be shocked: z, G, or V
shockvar    = 'z';

%% MODEL PARAMETERS

% Household parameters
```

```
beta    = 1.05^(-1/4);
nu      = 0.9;
phi     = 2.55;
eta     = 3.22;

% Technology parameters
alpha   = 0.36;
delta   = (1.08)^(1/4)-1;
zss     = 1;
rho_z   = 0.95;
sig_z   = 0.01;

% Government parameters
psi     = 0.15;
tkss    = 0.42;
tnss    = 0.26;
BY      = 0.63;
GY      = 0.21;

rho_G   = 0.95;
sig_G   = 0.01;
rho_T   = 0.95;
sig_T   = 0.01;
rho_K   = 0.95;
sig_K   = 0.01;

% Checking user inputs: policy variable
if (strcmp(shockvar,'z') ~= 1 && strcmp(shockvar,'G') ~= 1 && ...
        strcmp(shockvar,'V') ~= 1)
    display(shockvar); error('Set shockvar to: z, G, or V');
end

% Choosing variable for impulse-response and AR order
switch shockvar
    case 'z'
        vvarR   = [ 1 0 0 ];
        vvarN   = [ 1 0 0 0 ];
    case 'G'
        vvarR   = [ 0 1 0 ];
        vvarN   = [ 0 1 0 0 ];
    case 'V'
        vvarR   = [ 0 0 1 ];
        vvarN   = [ 0 0 1 0 ];
end

% Creating labels for the figures
line    = shockvar;
```

```
%% STEADY STATES

% Model steady states - implied values
rss     = (1-beta*(1-delta*(1-tkss)))/(beta*(1-tkss));
ome_Y   = rss/alpha;
ome_C   = (1-GY)*ome_Y-delta;
ome_w   = (phi*(1-nu)*ome_C)/((1-tnss)*(1-beta*nu));
NSS     = (1-alpha)*ome_Y/ome_w;
KSS     = NSS/(ome_Y^(1/(1-alpha)));
lss     = (1-beta*nu)/((1-nu)*ome_C*KSS);
wss     = ome_w*KSS;
CSS     = ome_C*KSS;
XSS     = delta*KSS;
YSS     = ome_Y*KSS;

% Steady state values and ratios
CY      = CSS/YSS;
XY      = XSS/YSS;
kap0    = nu/((1-nu)^2*CSS);
kap1    = -(1+beta*nu^2)/((1-nu)^2*CSS);
kap2    = beta*nu/((1-nu)^2*CSS);
gamma   = alpha-XY;

%% SYSTEM MATRICES

% I first set the matrix equations of the economy with a tax rule
% First matrix equation
AR  = [ zeros(2,5) ;
        -1 0 0 0 0 ;
        0 -CY 0 0 0 ;
        zeros(4,5) ];
BR  = [ 0 0 0 0 0 ;
        delta*eta 0 0 0 0 ;
        1-delta 0 0 0 0 ;
        0 0 0 0 0 ;
        -alpha 0 0 0 0 ;
        1 0 0 0 0 ;
        0 0 0 0 0 ;
        0 0 1 -1/beta 0 ];
CR  = [ 1 1 0 0 0 0 0 0 ;
        -1 0 1 -delta*eta 0 0 0 0 ;
        0 0 0 delta 0 0 0 0 ;
        0 0 0 -XY 0 0 1 0 ;
        0 0 0 0 0 0 1 alpha-1 ;
        0 0 0 0 1 0 -1 0 ;
        0 1 0 0 0 0 -1 1 ;
```

```
        0 (1-alpha)*tnss 0 0 0 -(1/beta)*BY 0 (1-alpha)*tnss ];
DR  = [ zeros(3,3) ;
        0 -1 0 ;
        -1 0 0 ;
        zeros(2,3) ;
        0 -1 -1 ];

% Second matrix equation
FR  = [ 0 kap2 0 0 0 ;
        zeros(4,5) ];
GR  = [ 0 kap1 0 0 0 ;
        -beta*delta^2*eta 0 0 0 -beta*(rss-delta) ;
        0 0 0 0 0 ;
        -gamma*tkss 0 0 0 -gamma ;
        gamma*tkss 0 0 -1 gamma ];
HR  = [ 0 kap0 0 0 0 ;
        zeros(2,5) ;
        0 0 psi 0 0 ;
        -gamma*tkss 0 1-psi 0 -gamma ];
JR  = [ 0 0 0 0 0 0 0 0 ;
        1-beta*(1-delta) 0 beta*(1-delta) beta*delta^2*eta ...
            beta*(1-tkss)*rss 0 0 0 ;
        1 0 0 0 0 1 0 0 ;
        0 0 0 0 -alpha*tkss 0 0 0 ;
        0 0 0 0 alpha*tkss 0 0 0 ];
KR  = [ -lss 0 0 0 0 0 0 0 ;
        0 0 -1 0 0 0 0 0 ;
        -1 0 0 0 0 0 0 0 ;
        0 0 0 0 0 0 0 0 ;
        0 0 0 0 -alpha*tkss 0 0 0 ];
LR  = zeros(5,3);
MR  = zeros(5,3);
NR  = [ rho_z 0 0 ; 0 rho_G 0 ; 0 0 rho_T ];
dWR = [ sig_z sig_G sig_T ];
WR  = diag(dWR);

% Now the matrix equations for the economy without tax rule
% First matrix equation
AN  = [ zeros(2,3) ;
        -1 0 0 ;
        0 -CY 0 ;
        zeros(3,3) ;
        0 0 -1 ];
BN  = [ 0 0 0 ;
        delta*eta 0 0 ;
        1-delta 0 0 ;
        0 0 0 ;
```

```
        -alpha 0 0 ;
        1 0 0 ;
        0 0 0 ;
        -gamma*tkss 0 1/beta ];
CN  = [ 1 1 0 0 0 0 0 0 ;
        -1 0 1 -delta*eta 0 0 0 0 ;
        0 0 0 delta 0 0 0 0 ;
        0 0 0 -XY 0 0 1 0 ;
        0 0 0 0 0 0 1 alpha-1 ;
        0 0 0 0 1 0 -1 0 ;
        0 1 0 0 0 0 -1 1 ;
        0 (alpha-1)*tnss 0 0 -alpha*tkss (1/beta)*BY 0 (alpha-1)*tnss ];
DN  = [ zeros(3,4) ;
        0 -1 0 0 ;
        -1 0 0 0 ;
        zeros(2,4) ;
        0 -1 -1 -gamma ];

% Second matrix equation
FN  = [ 0 kap2 0 ;
        zeros(2,3) ];
GN  = [ 0 kap1 0 ;
        -beta*delta^2*eta 0 0 ;
        0 0 0 ];
HN  = [ 0 kap0 0 ;
        zeros(2,3) ];
JN  = [ 0 0 0 0 0 0 0 0 ;
        1-beta*(1-delta) 0 beta*(1-delta) beta*delta^2*eta ...
            beta*(1-tkss)*rss 0 0 0 ;
        1 0 0 0 0 1 0 0 ];
KN  = [ -lss 0 0 0 0 0 0 0 ;
        0 0 -1 0 0 0 0 0 ;
        -1 0 0 0 0 0 0 0 ];
LN  = [ 0 0 0 0 ;
        0 0 0 -beta*(rss-delta) ;
        0 0 0 0 ];
MN  = zeros(3,4);
NN  = [ rho_z 0 0 0 ; 0 rho_G 0 0 ; 0 0 rho_T 0 ; 0 0 0 rho_K ];
dWN = [ sig_z sig_G sig_T sig_K ];
WN  = diag(dWN);

%% SOLUTION

% Economy with tax rule
[PR,QR,RR,SR]   = UCsystem(AR,BR,CR,DR,FR,GR,HR,JR,KR,LR,MR,NR,0);

display(PR)
```

```
display(QR)
display(RR)
display(SR)

% Economy without tax rule
[PN,QN,RN,SN]   = UCsystem(AN,BN,CN,DN,FN,GN,HN,JN,KN,LN,MN,NN,0);

display(PN)
display(QN)
display(RN)
display(SN)

%% IMPULSE-RESPONSE

% Economy with tax rule
[xir,yir,zir]   = impres_n(PR,QR,RR,SR,NR,WR,TIR,vvarR,0);

% Economy with tax rule
[xin,yin,zin]   = impres_n(PN,QN,RN,SN,NN,WN,TIR,vvarN,0);

% Series for graphs
T2              = TIR+1;
time            = maketime(ones(TIR,1));
time            = time-1;
O               = zeros(1,TIR);

% Here I need to adjust the time indices for the graph since the
% consumption state variable is lagged
KIR = xir(1,1:TIR);
CIR = xir(2,2:T2);
wir = yir(2,1:TIR);
XIR = yir(4,1:TIR);
rir = yir(5,1:TIR);
Rir = yir(6,1:TIR);
YIR = yir(7,1:TIR);
NIR = yir(8,1:TIR);

BIR = xir(4,2:T2);
TXR = xir(5,1:TIR);

KIN = xin(1,1:TIR);
CIN = xin(2,2:T2);
win = yin(2,1:TIR);
XIN = yin(4,1:TIR);
rin = yin(5,1:TIR);
Rin = yin(6,1:TIR);
YIN = yin(7,1:TIR);
```

```
NIN = yin(8,1:TIR);

BIN = xin(3,2:T2);
TXN = zin(4,1:TIR);

% This part decides which exogenous variable is being shocked
switch shockvar
    case 'z'
        ZIR = zir(1,1:TIR);
        ZIN = zin(1,1:TIR);
    case 'G'
        % Since z is always an AR(1) process, the series for G is always
        % located in row 2 of zi
        ZIR = zir(2,1:TIR);
        ZIN = zin(2,1:TIR);
    case 'V'
        ZIR = zir(3,1:TIR);
        ZIN = zin(3,1:TIR);
end

% Plotting the impulse-response functions (order: Z, Y, C, X, N, K)
figure(1)
if TIR < 100
    subplot(2,3,1)
    plot(time,ZIR,'bo-',time,ZIN,'rd-',time,0,'k:')
    title(line)
    ylabel('%')
    subplot(2,3,2)
    plot(time,YIR,'bo-',time,YIN,'rd-',time,0,'k')
    title('Output')
    ylabel('%')
    subplot(2,3,3)
    plot(time,CIR,'bo-',time,CIN,'rd-',time,0,'k')
    title('Consumption')
    ylabel('%')
    subplot(2,3,4)
    plot(time,XIR,'bo-',time,XIN,'rd-',time,0,'k')
    title('Investment')
    ylabel('%')
    subplot(2,3,5)
    plot(time,NIR,'bo-',time,NIN,'rd-',time,0,'k')
    title('Hours')
    ylabel('%')
    subplot(2,3,6)
    plot(time,KIR,'bo-',time,KIN,'rd-',time,0,'k')
    title('Capital')
    ylabel('%')
```

```
else
    subplot(2,3,1)
    plot(time,ZIR,'bo-',time,ZIN,'rd-',time,O,'k')
    title(line)
    ylabel('%')
    subplot(2,3,2)
    plot(time,YIR,'bo-',time,YIN,'rd-',time,O,'k')
    title('Output')
    ylabel('%')
    subplot(2,3,3)
    plot(time,CIR,'bo-',time,CIN,'rd-',time,O,'k')
    title('Consumption')
    ylabel('%')
    subplot(2,3,4)
    plot(time,XIR,'bo-',time,XIN,'rd-',time,O,'k')
    title('Investment')
    ylabel('%')
    subplot(2,3,5)
    plot(time,NIR,'bo-',time,NIN,'rd-',time,O,'k')
    title('Hours')
    ylabel('%')
    subplot(2,3,6)
    plot(time,KIR,'bo-',time,KIN,'rd-',time,O,'k')
    title('Capital')
    ylabel('%')
end

figure(2)
if TIR < 100
    subplot(2,2,1)
    plot(time,ZIR,'bo-',time,ZIN,'rd-',time,O,'k:')
    title(line)
    ylabel('%')
    subplot(2,2,2)
    plot(time,BIR,'bo-',time,BIN,'rd-',time,O,'k')
    title('Bond holdings')
    ylabel('%')
    subplot(2,2,3)
    plot(time,TXR,'bo-',time,TXN,'rd-',time,O,'k')
    title('\tau_K')
    ylabel('%')
else
    subplot(2,2,1)
    plot(time,ZIR,'bo-',time,ZIN,'rd-',time,O,'k:')
    title(line)
    ylabel('%')
    subplot(2,2,2)
```

```
    plot(time,BIR,'bo-',time,BIN,'rd-',time,0,'k')
    title('Bond holdings')
    ylabel('%')
    subplot(2,2,3)
    plot(time,TXR,'bo-',time,TXN,'rd-',time,0,'k')
    title('\tau_K')
    ylabel('%')
end

toc
```