Essays in Macroeconomics and Asset Pricing

A DISSERTATION

SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL

OF THE UNIVERSITY OF MINNESOTA

BY

Daniil Manaenkov

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

Patrick Kehoe

July, 2009

Acknowledgments

I am extremely grateful to my advisor Patrick Kehoe for his continued guidance, encouragement and support. I also thank V.V. Chari, Ellen McGrattan, Maria Ponomareva and Warren Weber

Dedication

This dissertation is dedicated to my family.

Abstract

In this dissertation I study the role recursive preferences due to Epstein and Zin (1989) play in macroeconomics and asset pricing. First I combine recursive preferences with long-run productivity growth risk and study the implications for asset pricing. Second, I focus on the preference for the timing of resolution of uncertainty that arises when one uses Epstein-Zin recursive utility, and the interaction of such preference with incentives to invest into technology that could cause uncertainty to be realized early.

In the first part of this dissertation I setup a monetary production economy with capital accumulation and recursive preferences and evaluate model's implications for pricing of equity and nominal default-free bonds. Plausibly parameterized model generates equity premium of about 1%, large and positive nominal bond term premium. Equity and nominal bond excess returns are forecastable, but considerably less so than in the data. Model generates large inflation premium, that is fairly sensitive to the parameters of interest rate rule.

In the second part I investigate the interaction between government policy and incentives to invest in risk-control technology in a heterogenous preference setting. Empirical studies show that intertemporal elasticity of substitution varies a great deal within population. I setup a stylized model where such heterogeneity leads to difference in preference for the timing of the resolution of uncertainty. The uncertainty in the model is about the future productivity of a risky technology. Investors can choose to observe an early signal about their individual future productivity (hence shifting the resolution of uncertainty to the earlier date) and cut exposure in case of a bad signal via conversion of a part of risky technology investment into safe investment. Government in the model has the power to influence the cost of borrowing and the return of the safe investment. I show that government policy has important implications both for the individual choice of whether to observe a signal about future productivity and for the aggregate output.

Contents

Appendices **111**

List of Tables

List of Figures

Part I

Introduction

This thesis focuses on the role of recursive preference specification in macroeconomics. Usual time-separable preferences bundle together into a single parameter two distinct concepts of aversion to risk and the desire to smooth consumption over time. Unbundling these concepts has important consequences: first, economic agents start caring about the risk of sudden revisions of future consumption growth expectations; second, separation of risk aversion and consumption smoothing motives creates preference for the timing of the resolution of uncertainty. If an economic agent is not very risk-averse, but prefers relatively smooth consumption paths then it is in his best interest to push the time of the resolution of uncertainty as far as possible into the future. On the other hand, if an agent is very risk averse but can tolerate somewhat unequal consumption profile it is best to have any uncertainty realized as soon as possible. In part 2 of this thesis I explore the role aversion to risk in expected future consumption growth has for asset pricing in a production economy, and in part 3 I study how government policy could affect the decisions of agents given a choice of time at which uncertainty is resolved.

In Part 2 I combine recursive preferences with long-run risks to productivity growth and explore asset pricing implications of such set-up. This work fits into the line of production-based models of asset pricing, that have until recently struggled to account for unconditional moments of asset prices without sacrificing too much of a fit of macroeconomic variables. Recently, with the advance of a long-run risk theory in the field of

Finance by Bansal and Yaron (2004) [12] and its adoption into a production economy, there is a new hope in the task of accounting for the asset pricing puzzles within a model that would be suitable for subsequent policy analysis. Fortunately or not, the field of economics is judged by the general public by the quality of forecasts, especially macroeconomic ones. Structural macroeconomic models' inability to account for the movements of asset prices could be the reason they still lag behind reduced form analysis that dominates most of the applied work, and improving the fit of asset pricing dynamics could be the much needed boost to the out-of-sample forecasting ability of the structural macroeconomic models. With this very ambitious objective I set out to build a model that would be suitable for policy analysis and would be able to account for the asset pricing puzzles. In order to do it I augment a standard production economy with long-run risks to productivity growth and with recursive preferences for consumption and leisure. To make the model suitable for policy (only monetary for now) analysis I add a nominal side, which takes the form of a simple monetary model with pricing frictions. I explore asset pricing implications of this model. Results go a long way towards accounting for asset pricing dynamics in a macroeconomic model: I generate sizable equity premium, nominal term premium and sizable predictability of bond and stock returns. There is a room for improvement, most notably the model under-predicts variances of interest rates, which I plan to tackle in the future extensions.

In part 3 I study the interaction of the government policy with private incentives to institute internal risk controls for the future use. I am motivated by recent crisis where it appears that a number of influential market participants chose to forgo the exercising of the due diligence which resulted in large losses for the public. While a typical explanations emphasize moral hazard problems, sub-optimal executive compensation schemes and too-big-to-fail arguments, I explore another possible explanation: investors' heterogeneity towards the timing of the resolution of uncertainty interacted with government's commitment to an accommodative monetary policy in a financial crisis and caused agents to rationally forgo installing/maintaining project risk control technology, that would have allowed early liquidation of projects expected to be very unprofitable in the future. I set-up a very stylized model of this mechanism and evaluate its importance. The channel through which government policy affects investment outcomes that I explore in this paper - is the cost of borrowing that investors do in order to finance their investments. I evaluate the argument that the government's actions during the previous potentially systematic financial crisis that could have signalled monetary authority's commitment to a very accommodative policy in the next potential crisis could have caused enough agents to forgo risk control investment, which might have contributed to current events. The model does generate sizable variation in the number of agents that choose to install internal project risk control technology, however in the present model implications for the aggregate output are quite small. I suggest ways to improve that in the future research.

Part II

Asset Pricing, Long-run Risk and Monetary Policy

1 Introduction

At least since Fama and Bliss [34] and Campbell and Shiller [22] for bond returns[1] and since Fama and French [35], Campbell and Shiller [40] for stock returns it has been known that asset returns are forecastable. In other words current financial data contains information about future expected returns and excess returns of financial instruments. Presence of such information about future returns suggests that information contained in the asset prices can potentially be used to predict evolution of the aggregate economy, and hence should be important in shaping the policy.[2] But what is the nature of this information? Can it be successfully recovered from asset prices? These are fundamental questions that needed to be answered before one could proceed to analyze importance of return predictability for policy design.

Recent advances in the field of Finance put forward a theory of long-run risks, proposed by Bansal and Yaron [12]. This theory states that consumption growth contains a small but very persistent component. Information about this future long-run consumption

[1] For more recent empirical analysis see, for example, Cochrane-Piazzesi [27]

[2] Rudebusch et at [48] show that macro-finance VAR evidence suggests that decline in term premium is associated with subsequent GDP growth. Beaudry and Portier [14] show that shocks that shift stock market today are the same shocks that move long-run productivity growth.

growth is contained in the asset prices, and is quite capable of explaining a large portion of asset return predictability puzzles as well as the equity premium puzzle and a number of related asset-pricing puzzles.

However, it remains to be established whether long-run risks model would survive the transition to a general equilibrium production economy with monetary frictions, as those are the minimum requirements for a model to be suitable for policy analysis. If it does, one can start analyzing interaction of the monetary policy and asset pricing puzzles. The most straightforward question to ask is whether monetary policy has large implications for the unconditional moments of asset prices and for predictability of asset pricing dynamics. This is particularly important, because if the answer to this question is 'yes', one has to be extra careful about fully taking into account these effects when trying to extract information from financial data.

Thus in this paper I study two questions outlined above: whether long-run risk explanation asset pricing puzzles can be easily transferred into a production economy with non-trivial monetary side, and if it could, what implications does the monetary policy have for unconditional moments and dynamics of asset prices. Results suggest that long-run risks model does work in generating key unconditional moments (although not quite as large as in the data) of the asset pricing, and it does generate sizable return predictability, however quantitatively explaining predictability of bond returns while maintaining good fit for the rest of macroeconomic and asset pricing moments is still a challenge. I also show that monetary policy has important implications for the predictability of returns and, although less so, for unconditional moments of asset prices, so attempts at recovering long-run risks from financial data could benefit from taking monetary policy into account.

I now outline these results in more detail and discuss my contribution to the literature.

In this paper I setup a framework suitable for analysis of monetary policy that at the same time can explain asset pricing puzzles. My key building blocks are: long-run productivity risk, variable economic uncertainty, risk sensitive preferences that depend on both consumption and leisure and a nominal rigidity - staggered price setting by producers. First three ingredients are motivated by long-run risk model of Bansal and Yaron [12] that has very similar key features: small but very persistent component of consumption growth, stochastic economic uncertainty that is also very persistent and risk-sensitive preferences due to Epstein and Zin [33], Weil (1989) [54] and Kreps-Porteus [43]. Recently Bansal and Yaron [12] were able to generate large equity premium and predictable excess returns using a model of long-run risk and Bansal-Shaliastovich [11] augmented the same model with exogenous nominal side and were able to generate large and predictable movements in nominal bond yields for empirically plausible levels of risk-aversion.

A natural next step from the endowment economy, that serves as a basis of long-run risk model in finance literature, is to move to a production economy and look for evidence of long-run component in productivity, so this is what I do. I am not the first to make a switch to long-run productivity risks, for example Croce [28] finds evidence of long-run productivity component in the annual data and uses production model to show that small but persistent movements in long-run productivity are capable of delivering equity premium puzzle. I extend the analysis of Croce to include endogenous dynamics of leisure and explore the role of time-variable uncertainty in expected productivity growth rates.

Leisure in recursive utility is a key ingredient of my model. As shown by Uhlig [52], leisure movements can have sizable effects on the pricing kernel. So it is not clear whether predictive power of long-run productivity risks is affected by movements in leisure.

In the model these endogenous movements of leisure are caused by two forces: capital accumulation dynamics in response to shocks affecting the economy and a monetary friction. Following New-Keynesian literature I assume that only a fraction of producers can re-optimize their prices in any given period, requiring firms to set prices that could possibly be in effect for a while. This nominal rigidity causes dynamic distortions and labor movements in response to both long- and short-run shocks. This usual friction can have unusual implications in the model with recursive preferences and shocks to expected growth rates. These shocks affect stochastic discount factors that firms use to discount future marginal costs and can have large implications for short-run dynamics of the model.

The key result is that long-run risk's ability to explain asset pricing puzzles survives a transition to a production economy with capital accumulation and inclusion of leisure into utility function. However, quantitatively accounting for all the asset pricing moments considered in this paper is still a challenge and getting close to the data requires assuming high risk aversion. With effective risk aversion of about 25 my monetary model with long-run productivity risks generates only modest equity premium, bond returns are forecastable, but much less so than in the data. While value of risk aversion is higher than what is considered consistent with microeconomic evidence, it is still not as high as frequently assumed in the production-based asset-pricing literature[3]. Excess returns on equity are very forecastable, and the model replicates nominal bond term premium. The most important moments the model is unable to replicate are the shape of US real

[3] See for example Tallarini [50]

yield curve and the volatility of interest rates. US real yield curve is upward sloping while the model implies mildly downward sloping real term structure. The interest rates are too smooth in the model, and this is a major problem, since this means that the model under-predicts the volatility of the stochastic discount factor. I propose ways to improve this.

My second contribution is to the literature that studies interaction of monetary policy and the shape of the yield curve and movements in term spreads. Rudebusch and Swanson [49] show that standard DSGE monetary models augmented by "habit persistence" preferences[4] can explain the size of the term premium only at the cost of assuming very unrealistic behavior of macroeconomic variables. They conclude than using risk-sensitive preferences of Epstein and Zin [33] and Kreps-Porteus [43] seems like a more promising approach in accounting for asset pricing puzzles in DSGE models. My model follow this recent recommendation and is capable of accounting for term premium. I show that monetary policy rule parameters have large implications for the shape of nominal yield curve, as well as implications for unconditional expectations of price of real assets. The major reason behind this - policy rules can influence the sign and the size of inflation premium, that is included in all "real" prices of financial instruments, which usually take out realized values of inflation. In my model inflation premium is due to persistent inflation movements in response to long-run shocks, and which are predictable. This is the reason it can affect moments of "real" asset prices and predictability of their returns. For example, the benchmark calibration of my model suggests that at 5 year horizon 99 basis points of the spread between real and nominal bond yields reflects inflation premium.

[4] See Campbell and Cochrane (1999) [21]

Ang et al [3], using a reduced-from factor model find large inflation premium that reaches 1% at long maturities, with volatility of around 30 basis points. My results confirm this view.

My results have important implications for empirical literature that tries to back out values of long-run shocks from financial data.[5] Since monetary policy can influence unconditional moments of asset prices, when estimating the size of long-run components from "real" financial data, one should explicitly account for effects of monetary policy.

And finally, my work fits well into new research agenda proposed by Atkeson and Kehoe [4], emphasizing the role of risk and its interaction with monetary policy as an important channel through which risks could affect the economy.

The rest of the paper proceeds as follows: Section two presents the data, asset pricing moments and predictability regressions and empirical results concerning the size of long-run productivity risks, section three presents the model, implied bond pricing formulas, theoretical predictions for bond predictability regression slope coefficients. Section four includes calibration, and discussion of results, section five concludes.

2 Data, Predictability Puzzles, Long-run productivity

In this section describe the predictability puzzles that I am going to aim to account for and then present empirical analysis of productivity, that motivates my model set-up. In this section I am using bond price database constructed in Fama and Bliss [34], that is available from CRSP. Stock data comes from Campbell handbook chapter [23],

[5]See for example Bansal, Kiku and Yaron [10] and Bansal, Khachatrian and Yaron,[9]

macroeconomic data comes from BEA, and I am using post-war quarterly data sample. I also use BLS annual multi-factor productivity data that covers 1947-2006, and quarterly Industrial Production data from the FED and hours data from the BLS.

2.1 Bond predictability puzzle

Denote $P_t^{(n)}$-period t price of a nominal risk-free discount bond that matures and pays 1\$ at time $t+n$. Annualized rate of return then is given by $r_t^{(n)} = \frac{-\log P_t^{(n)}}{n}$. Denote $p_t^n = \log P_t^{(n)}$. Consider buying an $n-$period bond, holding it for m periods, and selling as $(m-n)$-period bond. Gross return from doing so is $P_{t+m}^{(n-m)}/P_t^{(n)}$. Compare this return to a simple return from buying an m-period bond: $1/P_t^{(m)}$. The difference in return is an excess return from holding $n-$period bond for m periods. If expectation hypothesisof bond pricing was true, expected excess returns would be constant.[6] If expected excess returns are constant, then current long-short bond yield spread should contain no information about future excess returns.

I use the Fama-Bliss bond yields database available from CRSP to show that in the data this is not true and long-short spread has considerable forecasting power. I use monthly data for nominal default-free bond yields.

Sample is from June 1952 to September 2006. Descriptive statistics are given in Table 1. I construct a measure of excess returns with holding period set to 1 year:

$$\begin{aligned} rx_{t,12}^{(n)} &= -p_{t+12}^{(n-12)} + p_t^{(n)} - p_t^{(12)} \\ rx_{t,12}^{(n)} &= nr_t^{(n)} - (n-12)\, r_{t+12}^{(n-12)} - 12 r_t^{(12)} \qquad (1) \end{aligned}$$

[6]Expectation Hypothesis says that price of a long maturity bond is given by an expectation of future price of shorter maturity bonds.

and regress it on the relevant long-short spread $\left(r_t^{(n)} - r_t^{(12)}\right)$. Regression coefficients are given in the upper portion of Table 2. They are positive and statistically significant, and increase with maturity, that is high long-short spread forecasts sizable expected excess returns. Expectation hypothesis is clearly violated, since it implies slope coefficients of zero.

To assess the role of inflation in the predictability of excess returns I construct a measure of real interest rates: I subtract a measure of inflation expectations from nominal yields. Following Bansal-Shaliastovich [11] I smooth inflation process, fitting it to AR(2) dynamics and use filtered values to adjust nominal returns for inflation expectations. Similarly to (1), I construct a measure of real excess returns and regress it on real term spread. Results are presented in the lower panel of Table 2. As can be seen from the table, predictability is stronger for nominal returns, suggesting importance of non-trivial monetary side while modeling excess returns.[7]

2.2 Equity Returns and Predictability

Historically equity has been yielding quite a bit more than safe government instruments. First column of table 14 documents the key moments of the data. These moments are notoriously hard to replicate in a production economy[8]. Another set of moments con-

[7] Furthermore, some simple excess return regressions using 5, 10 year constant maturity TIPS series available from the Treasury with 5 year holding period suggest the same, when compared to identical regressions using nominal constant maturity excess returns, although the data is very limited at this point.

[8] see for example Jermann (1998) [39], Campbell and Cochrane (1999) [21], Boldrin et al (2001) [19] and Tallarini (2000) [50]

cern dynamics of asset prices: Campbell and Shiller (1998) [40] and Fama and French (1988)[35] document that stock returns over extended periods of time are forecastable. If table 3 I replicate their results. I regress T-period forward looking sum of log real stock returns $\sum_{j=1}^{T}(r^e_{t+j})$, excess real stock returns $\sum_{j=1}^{T}(r^e_{t+j} - r^f_{t+j})$, consumption growth $\sum_{j=1}^{T}(\Delta c_{t+j})$ and dividend growth $\sum_{j=1}^{T}(\Delta d_{t+j})$ on current log of price-dividend ratio $p^e_t - d_t$. Results show that equity returns are predicted by price-dividend ratio at long horizons, and that the reason behind such predictability is price-dividend ratio's predictive power for excess returns, not because they forecast future fundamentals such as consumption or dividend growth. In these regressions I use the stock return data from Campbell (1999) [23], which covers 1947-1998. I will repeat the same regressions for stock returns, excess returns, consumption and dividend growth, using the simulated data. Since all the equity analysis is usually done with "real" equity returns and dividends, where "real" means subtracting realized inflation, possibly leading to inflation premium being counted as a part of the measured real returns, I run these regression on true real equity returns as well as "real", as implied by typical procedure done in empirical analysis.

2.3 Long-run productivity

Croce [28] finds sizable persistent component in productivity growth using annual post-war data for US. The reason for the use of annual data is absence of seasonal adjustment and better signal-to-noise ratio. I replicate analysis by Croce using annual data and also construct a simple measure of quarterly labor productivity using data on quarterly index of Industrial Production in the manufacturing sector available from the Board of Governors, and a measure of manufacturing hours available from BLS.[9] I focus on manufacturing sector, since I expect manufacturing output to be a better signal of improvements in labor productivity.

Properties of annual and quarterly measures of productivity are given in Table 4. In tables 5 and 6 I report results of fitting GARCH(1,1) dynamics to the data,

$$\begin{aligned} \Delta z_{t+1} &= (1-\rho)\,\mu_x + \rho\Delta z_t - b\sigma_t\varepsilon_{x,t} + \sigma_{t+1}\varepsilon_{x,t+1} \\ \sigma^2_{t+1} &= \sigma^2 + \rho_\sigma\sigma^2_t + b_\sigma\sigma^2_t\varepsilon^2_{x,t} \end{aligned} \tag{2}$$

as advocated by Croce [28]. This econometric specification allows separation of long-run productivity growth component and identification of time-varying economic uncertainty, changes in which have been a prominent feature of recent business cycles around the world.[10]

My annual results in Table 5 are consistent with those reported by Croce. Quarterly results in Table 6 reinforce those of the annual analysis. There is a sizable and persistent long-run component in productivity growth and evidence of variable economic uncertainty.

However, as in most of the long-run risk literature, parameter bounds are quite wide leaving a lot of freedom in calibration of productivity process, so in my calibrations I will use parameters that are close to Bansal-Shaliastovich [11], even though their model is an endowment economy.

[9]Even though the data is available at monthly frequency, I aggregate it to reduce noise.

[10]See, for example Fernandez-Villaverde and Rubio-Ramirez (2007) [36]

3 The Model

To study effects of long-run risks in productivity I setup a model where productivity, monetary policy shock and variable economic uncertainty are exogenously specified driving processes.

There is a representative consumer that has risk-sensitive preferences over streams of consumption and leisure. Agent's consumption is subject to cash-in-advance constraint with Lucas timing, where wages and profits are not available to spend until the period after they are actually earned.

Final goods are produced by competitive firms using intermediate goods as inputs. Intermediate goods are produced by monopolistically competitive firms. Their production function is Cobb-Douglas in labor and capital inputs. I assume that there exist competitive markets for both factors of production, even though capital accumulation is done by the firms. Intermediate producers are also subject to Calvo [20] pricing friction: every period they get a chance to re-optimize their price with a fixed probability. If they can't re-optimize I assume that producers can index their last period's price to a long-run inflation average.

There is a government in the model, that actively adjusts short-term nominal interest rate in response to movements in macroeconomic variables.

3.1 Stochastic processes

There are three key driving processes: monetary policy shock, productivity and stochastic economic uncertainty.

Monetary policy shock $\varkappa_{t+1}$ is assumed to follow a simple AR(1) process:

$$\varkappa_{t+1} = \rho_{\varkappa}\varkappa_t + \sigma_{\varkappa}\epsilon_{\varkappa,t+1} \tag{3}$$

Choice of processes for productivity and variable economic uncertainty is augmented by GARCH regressions from the empirical part of the paper. Logarithm of productivity growth $x_{t+1} = \log X_{t+1} = \log Z_{t+1} - \log Z_t$ has two parts: a random walk with time variable volatility of innovations (representing stochastic economic uncertainty) and persistent long-run component that follows AR(1) process, innovations to which also have time-variable volatility.

$$x_{t+1} = g_t + \mu_x + \sigma_x \upsilon_t^{1/2}\epsilon_{x,t+1} \tag{4}$$

$$g_{t+1} = \rho_g g_t + \sigma_g \upsilon_t^{1/2}\epsilon_{g,t+1} \tag{5}$$

$$\upsilon_{t+1} = (1-\rho_\upsilon)\upsilon + \rho_\upsilon \upsilon_t + \sigma_\upsilon \epsilon_{\upsilon,t+1} \tag{6}$$

Notice also that g_t gives a conditional expectation of the future productivity growth deviation from the mean $g_t = E_t x_{t+1} - \mu_x$. It will be convenient to refer to all the driving shocks in vector form, so I define vector of process values and innovations:

shock vector: $s_t = \begin{bmatrix} x_t & g_t & \upsilon_t & \varkappa_t \end{bmatrix}^T,$

Innovations vector $\omega_{t+1} = \begin{bmatrix} \epsilon_{x,t+1} & \epsilon_{g,t+1} & \epsilon_{\upsilon,t+1} & \epsilon_{\varkappa,t+1} \end{bmatrix}^T$

and their evolution in matrix form:

$$s_{t+1} = (I - A)\, s + As_t + \Sigma^{1/2}(s_t)\omega_{t+1} \tag{7}$$

where $I-$is an identity matrix, and

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \rho_g & 0 & 0 \\ 0 & 0 & \rho_\upsilon & 0 \\ 0 & 0 & 0 & \rho_\varkappa \end{bmatrix}, \Sigma^{1/2}(s_t) = \begin{bmatrix} \sigma_x \upsilon_t^{1/2} & 0 & 0 & 0 \\ 0 & \sigma_g \upsilon_t^{1/2} & 0 & 0 \\ 0 & 0 & \sigma_\upsilon & 0 \\ 0 & 0 & 0 & \sigma_\varkappa \end{bmatrix}$$

And, as always, s^t will denote a complete history of the process up to time t: $s^t = \{s^\varsigma\}_{\varsigma=-\infty}^{t}$. While all the prices, quantities and value functions below depend on history of shock realizations S^t, to save on notation in most cases I will not write it explicitly.

3.2 Consumers

I consider a discrete time production economy populated with a continuum of identical households. Representative household has preferences over consumption and leisure given by a utility function along the lines of Kreps-Porteus [33] and Epstein-Zin [43], that separates risk aversion and intertemporal elasticity of substitution:

$$U_t = \left[(1-\beta)\{C_t^{\nu}(H - L_t)^{1-\nu}\}^{\frac{1-\gamma}{\rho}} + \beta \left(E_t U_{t+1}^{1-\gamma} \right)^{1/\rho} \right]^{\rho/(1-\gamma)} \tag{8}$$

where β is the rate of time preference, γ is a parameter that governs risk-aversion , ψ is an intertemporal elasticity of substitution (IES), ν is an elasticity of substitution between consumption and leisure and parameter ρ is defined as $\rho = \frac{1-\gamma}{1-\frac{1}{\psi}}$. H is a maximum share of time endowment that a consumer could devote to work. Notice that if $\psi = \dfrac{1}{\gamma}$ these preferences reduce to the case of standard time separable preferences. As argued by Epstein and Zin [33] the value of parameter ψ determines household's preference for the timing of the resolution of uncertainty. If $\psi > 1/\gamma$ they prefer early resolution of uncertainty and if $\psi < 1/\gamma$ they prefer late resolution. Throughout the paper I am going to assume that agents prefer early resolution to stay consistent with most of the literature, results of which I am aiming to replicate in a production economy.

Also note that due to presence of leisure in the felicity function my value of γ is not directly comparable to risk-aversion parameter estimated in the literature using endowment economies. Most meaningful comparison would be in terms of price of short-term consumption growth risk that is given by γ in an endowment economy and will be equal to $(1 - \nu(1 - \gamma))$ in mine.

Consumption of households is subject to a cash-in-advance constraint:

$$P_t C_t \leq M_t \tag{9}$$

The timing of event in the period follows Lucas-Stokey [45]. Consumers start the period with bond holdings B_t, asset holdings A_t, receive government transfers T_t, then asset markets open, consumers split their beginning of period wealth into state-contingent bonds $\{B_{t+1}\}$ at prices $\{Q_{t+1,t}\}$ and money M_t.

$$M_t + \sum Q_{t+1,t} B_{t+1} \leq A_t + B_t + T_t \quad (10)$$

Then they rent out labor services at real wage W_t, that they do not receive until the beginning of the next period.

So given current decisions of household next period asset holdings are equal to unspent money balances, current period wages and current period profits of intermediate goods firms:

$$A_{t+1} = (M_t - P_t C_t) + P_t W_t L_t + \Upsilon_t \quad (11)$$

where Υ_t - nominal profits of intermediate goods producers in period t.

Given the beginning of the period asset values A_t, B_t and facing final goods prices, bond prices and wages of P_t, $\{Q_{t+1,t}\}$, W_t respectively consumers choose consumption C_t labor input L_t, next period bond holdings $\{B_{t+1}\}$ and money holdings to be spent this period M_t to maximize recursive utility in (8) subject to cash-in-advance constraint (9),asset market constraint (10) and a set of borrowing constraints preventing consumers from borrowing too much. Asset evolution for next period is given by (11).

Since productivity grows exponentially this problem is non-stationary. To convert it into a stationary-recursive form I divide all non-stationary variables by labor productivity $\bar{C}_t = C_t/Z_t$, $\bar{M}_t = M_t/Z_t, \bar{W} = W_t/Z_t, \bar{\Upsilon}_t = \Upsilon_t/Z_t, \bar{U}_t = U_t/Z_t, \bar{B}_t = B_t/Z_t, \bar{A}_t = A_t/Z_t$ and as reminder $Z_{t+1}/Z_t = X_{t+1}$.

AssumptionCash-in-advance constraint always holds as equality.

Consumer's problem rewrites as:

$$Z_t^{\nu}\bar{V}_t(\frac{\bar{B}_t}{P_t}, \frac{\bar{A}_t}{P_t}, S_t) =$$

$$\underset{\bar{C}_t, L_t, \bar{M}_t, \{\bar{B}_{t+1}\}}{Max} \left[\begin{array}{c} Z_t^{\frac{\nu(1-\gamma)}{\rho}}(1-\beta)\{\bar{C}_t^{\nu}(H-L_t)^{1-\nu}\}^{\frac{1-\gamma}{\rho}} \\ +\beta\left\{E_t(Z_{t+1}^{\nu(1-\gamma)}\left[\bar{V}_{t+1}\left(\frac{\bar{B}_{t+1}}{P_{t+1}}, \frac{\bar{A}_{t+1}}{P_{t+1}}, S_{t+1}\right)\right]^{1-\gamma}\right\}^{1/\rho} \end{array} \right]^{\rho/(1-\gamma)}$$

s.t.

$$\left\{ \begin{array}{c} Z_t P_t \bar{C}_t \leq Z_t \bar{M}_t \\ Z_t \bar{M}_t + \sum Q_{t+1,t} Z_{t+1} \bar{B}_{t+1} \leq Z_t \bar{A}_t + Z_t \bar{B}_t + Z_t \bar{T}_t \\ Z_{t+1} = X_{t+1} Z_t \\ Z_{t+1}\bar{A}_{t+1} = Z_t\left(P_t \bar{W}_t L_t + \bar{\Upsilon}_t\right) \\ 0 \leq L_t \leq 1 \\ 0 \leq \bar{C}_t, \bar{M}_t \\ \bar{B}_{t+1} \geq \underline{B} \end{array} \right.$$

where $S_t = \exp s_t$ and $\underline{B}$ is a bound on borrowing. Here I used the fact the recursive utility inherits homogeneity properties of instantaneous utility, namely homogeneity of degree ν in bond and assets holdings.

So consumer problem in stationary recursive form:

$$\bar{V}_t(\frac{\bar{B}_t}{P_t}, \frac{\bar{A}_t}{P_t}, S_t) =$$

$$\underset{\bar{C}_t, L_t, \bar{M}_t, \{\bar{B}_{t+1}\}}{Max} \left[\begin{array}{c} (1-\beta)\{\bar{C}_t^{\nu}(H-L_t)^{1-\nu}\}^{\frac{1-\gamma}{\rho}} \\ +\beta \left\{ E_t(X_{t+1}^{\nu(1-\gamma)} \left[\bar{V}_{t+1}\left(\frac{\bar{B}_{t+1}}{P_{t+1}}, \frac{\bar{A}_{t+1}}{P_{t+1}}, S_{t+1} \right) \right]^{1-\gamma} \right\}^{1/\rho} \end{array} \right]^{\rho/(1-\gamma)} \quad (12)$$

s.t.

$$\begin{cases} P_t \bar{C}_t \leq \bar{M}_t \\ \bar{M}_t + \sum Q_{t+1,t} X_{t+1} \bar{B}_{t+1} \leq \bar{A}_t + \bar{B}_t + \bar{T}_t \\ \bar{A}_{t+1} = \dfrac{\left(P_t \bar{W}_t L_t + \bar{\Upsilon}_t\right)}{X_{t+1}} \\ 0 \leq L_t \leq 1 \\ 0 \leq \bar{C}_t, \bar{M}_t \\ \underline{B} \leq \bar{B}_{t+1} \end{cases} \quad (13)$$

With the exception of presence of productivity growth in asset evolution equation this is a normal Bellman problem. So I proceed to solving it using conventional methods.

3.2.1 First Order conditions

Let μ be a Lagrange multiplier on CIA constraint and λ - a Lagrange multiplier on asset market constraint. To save on notation denote

$$\Omega_t = (1-\beta)\{\bar{C}_t^{\nu}(H-L_t)^{1-\nu}\}^{\frac{1-\gamma}{\rho}} + \beta \left\{ E_t(X_{t+1}^{\nu(1-\gamma)} U_{t+1}^{1-\gamma}) \right\}^{1/\rho}$$

Then envelope conditions are:

$$\begin{aligned} V_{\bar{B}_t} &= \lambda_t & (14) \\ V_{\bar{A}_t} &= \lambda_t & (15) \end{aligned}$$

First order conditions: With respect to $\bar{C}_t$

$$\Omega_t^{\frac{\rho+\gamma-1}{1-\gamma}} \frac{(1-\beta)\nu\{\bar{C}_t^{\nu}(H-L_t)^{1-\nu}\}^{\frac{1-\gamma}{\rho}}}{\bar{C}_t} - P_t\mu_t = 0 \tag{16}$$

With respect to L_t :

$$\begin{gathered} -(1-\beta)\{\bar{C}_t^{\nu}(H-L_t)^{1-\nu}\}^{\frac{1-\gamma}{\rho}} (1-\nu)(H-L_t)^{-1} + \\ +\beta\left\{E_t(X_{t+1}^{\nu(1-\gamma)}U_{t+1}^{1-\gamma})\right\}^{\frac{1-\rho}{\rho}} E_t\left[X_{t+1}^{\nu(1-\gamma)}U_{t+1}^{-\gamma}\left(U_{At+1}\frac{P_t\bar{W}_t}{X_{t+1}}\right)\right] = 0 \end{gathered} \tag{17}$$

With respect to $\bar{M}_t$:

$$-\mu_t + \lambda_t = 0 \tag{18}$$

With respect to $\bar{B}_{t+1}$:

$$\Omega_t^{\frac{\rho+\gamma-1}{1-\gamma}} \left\{\beta\left\{E_t(X_{t+1}^{\nu(1-\gamma)}U_{t+1}^{1-\gamma})\right\}^{\frac{1-\rho}{\rho}} \left[X_{t+1}^{\nu(1-\gamma)}U_{t+1}^{-\gamma}U_{\bar{B}t+1}\right]\right\} - \mu_t Q_{t+1,t}X_{t+1} = 0 \tag{19}$$

Using (18) with (16) and (14), and substituting into (19) gives us the nominal pricing kernel:

$$Q_{t+1,t} = \beta \left[\frac{\left\{ E_t(X_{t+1}^{\nu(1-\gamma)} U_{t+1}^{1-\gamma}) \right\}}{X_{t+1}^{\nu(1-\gamma)-1} \left[U_{t+1}^{1-\gamma} \right]} \right]^{\frac{1-\rho}{\rho}} \left[\frac{\{\bar{C}_{t+1}^{\nu}(H-L_{t+1})^{1-\nu}\}^{\frac{1-\gamma}{\rho}} \bar{C}_{t+1}^{-1}}{\{\bar{C}_t^{\nu}(H-L_t)^{1-\nu}\}^{\frac{1-\gamma}{\rho}} \bar{C}_t^{-1}} \frac{P_t}{P_{t+1}} \right] \tag{20}$$

Combining (18) with (16) and (15) and substituting into (17) yields a condition optimal choice of consumption and leisure has to satisfy:

$$\frac{\bar{C}_t\,(1-\nu)}{\nu(H-L_t)} = \frac{\bar{W}_t}{R_t} \tag{21}$$

where R_t is the 1 period risk-free rate:

$$\frac{1}{R_t} = E_t\left(Q_{t+1,1}\right) \tag{22}$$

3.3 Producers

Setup of production side follows the standard differentiated product market for intermediate inputs due to Dixit and Stiglitz [31] and a perfectly competitive final goods market. There is a unit measure both of final and intermediate goods producers. Final goods producers are competitive, so without loss of generality I will use a representative final goods producer that is a price taker both in input and output markets. Intermediate goods produce differentiated goods, thus having some degree of monopoly power over their own price. Intermediate producers are be indexed by $i \in [0, 1]$

3.3.1 Final goods producers

The only inputs into final good production are intermediate goods, which are imperfect substitutes in production. Final good output is given by:

$$Y_t = \left[\int_0^1 Y_t(i)^\theta di \right]^{\frac{1}{\theta}} \tag{23}$$

where $Y_t(i)$ is the amount of intermediate of type i used in production and parameter θ controls the elasticity of substitution between different intermediate goods. Since final goods representative producer is a price taker, its maximization problem is to maximize the expression below given final good price P_t and intermediate good prices$\{P(i, \cdot)\}$.

$$\max_{\{Y_t, Y_t(i)\}} P_t Y_t - \int_0^1 P(i, t) Y_t(i) di \tag{24}$$

s.t.

$$Y_t = \left[\int Y_t(i)^\theta di \right]^{\frac{1}{\theta}}$$

First order condition of the above problem gives final producer's demand for individual intermediate goods:

$$Y_t(i) = \left[\frac{P(i,t)}{P_t}\right]^{\frac{1}{\theta-1}} Y_t \tag{25}$$

Substituting (25) into objective function (24) yield a zero profit condition:

$$P_t^{\frac{\theta}{\theta-1}} = \int_0^1 [P(i,t)]^{\frac{\theta}{\theta-1}} \, di \tag{26}$$

3.3.2 Intermediate goods producers

Intermediate goods produce using the two factors: labor and capital. The production function is a standard Cobb-Douglas technology:

$$F(K_t(i), L_t(i)) = (K_t(i))^{\kappa}(Z_t L_t(i))^{1-\kappa} \tag{27}$$

, where Z_t is a current value of stochastic labor productivity. Intermediate producers purchase investment goods, hire labor services in a competitive markets at a nominal wage rate $P_t W_t$ and set prices of their own goods. Following King and Wolman [42] I assume that there are competitive markets for factors of production and capital and labor are not restricted from being re-allocated from one firm to another. Firms maximize present discounted value of future profits:

$$\max_{P_{s,t}(i),Y_t(i),K_{t+1}(i),I_t(i),L_t(i)} E_t \sum_{\tau=t}^{\infty} Q_{\tau,t} \left[P_{s,t}(i) Y_t^d(i) - P_t W_t L_t(i) - P_t I_t(i) \right] \tag{28}$$

$$K_{t+1}(i) = K_t(i)[(1-\delta) + G\left(\frac{I_t(i)}{K_t(i)}\right)] \tag{29}$$

$$Y_t(i) = (K_t(i))^{\kappa} (Z_t L_t(i))^{1-\kappa} \tag{30}$$

$$Y_t^d(i) = \left[\frac{P_{s,t}(i)}{P_t}\right]^{\frac{1}{\theta-1}} Y_t \tag{31}$$

Where $G\left(\frac{I_t(i)}{K_t(i)}\right)$ is an adjustment cost function given by:

$$G\left(\frac{I_t(i)}{K_t(i)}\right) = \left[\left(\frac{I_t}{K_t}\right)^{\eta} \left(\frac{I}{K}\right)^{1-\eta} - (1-\eta)\frac{I}{K}\right] / \eta \tag{32}$$

While this problem is also possibly non-stationary, normalization technique identical to the one used in analysis of consumer problem applies, so in derivation of first order conditions I skip the normalization step. As King and Wolman [42], I assume that every intermediate goods firm consists of investment department, production department and pricing department, all acting independently of each other, taking actions of the other departments as given. Production department faces market wage, rental rate of capital, production demand from pricing department and chooses inputs of capital and labor to minimize production costs:

$$TC(Y_t(i)) = \min_{K_t^d(i), L_t^d(i)} \left[R_{k,t} K_t^d(i) + W_t L_t^d(i) \right] \tag{33}$$

s.t.

$$Y_t(i) = (K_t^d(i))^{\kappa} (Z_t L_t^d(i))^{1-\kappa} \tag{34}$$

This is a standard cost minimization problem, and its first order conditions imply:

$$
\begin{aligned}
TC(Y_t(i)) &= MC(K_t, L_t)Y_t(i) && (35) \\
\frac{W_t}{R_{k,t}} &= \frac{(1-\kappa)K_t}{\kappa L_t} && (36) \\
\frac{K_t(i)}{L_t(i)} &= \frac{K_t}{L_t} && (37)
\end{aligned}
$$

Pricing department of a producer of type i takes as given investment decision, marginal cost of production and faces a constant probability $(1-\alpha)$ of getting a chance to re-optimize its price in current period. In periods where producers don't re-optimize they index their past price to a long run inflation average Π, that is determined by the long-run nominal interest rate target of the monetary authority $\bar{R}$ and long-run productivity growth μ_x. Producers who get a chance to re-optimize face a residual demand for their product given by (25), marginal cost of production $MC(K_t, L_t)$ and choose to maximize expected profits (ignoring parts that pricing decision does not affect), conditional on future price being the same price set today (save for indexation to long-run inflation Π).

$$
\max_{P_{s,t}(i)} E_t \sum_{\tau=t}^{\infty} \alpha^{\tau-t} \left[Q_{\tau,t} \left\{ P_{s,t}(i) \Pi^{\tau-t} y_\tau^d(i) - P_\tau MC_\tau y_\tau^d(i) \right\} \right] \tag{38}
$$

s.t.

$$
Y_t(i) = \left[\frac{P_{s,t}(i)}{P_t} \right]^{\frac{1}{\theta-1}} Y_t \tag{39}
$$

First order condition of this problem with respect to price yields:

$$E_t \sum_{\tau=t}^{\infty} \alpha^{\tau-t} Q_{\tau,t} \left\{ \begin{array}{c} \Pi^{(\tau-t)\frac{\theta}{\theta-1}} \frac{\theta}{\theta-1} \left[P_{s,t}(i)\right]^{\frac{1}{\theta-1}} \left[P_\tau\right]^{\frac{1}{1-\theta}} Y_\tau - \\ -\frac{1}{\theta-1} \Pi^{(\tau-t)\frac{1}{\theta-1}} MC_\tau \left[P_{s,t}(i)\right]^{\frac{2-\theta}{\theta-1}} \left[P_\tau\right]^{\frac{2-\theta}{1-\theta}} Y_\tau \end{array} \right\} = 0 \tag{40}$$

Which with some rearranging and substitution yields:

$$P_{s,t}(i) = \frac{\sum_{\tau=t}^{\infty} \alpha^{\tau-t} \left(E_t \left[Q_{\tau,t} \Pi^{(\tau-t)\frac{1}{\theta-1}} \left[\frac{P_\tau}{P_t}\right]^{\frac{2-\theta}{1-\theta}} MC_\tau Y_\tau \right]\right)}{\theta \sum_{\tau=t}^{\infty} \alpha^{\tau-t} \left(E_t \left[Q_{\tau,t} \Pi^{(\tau-t)\frac{\theta}{\theta-1}} \frac{1}{P_t} \left[\frac{P_\tau}{P_t}\right]^{\frac{1}{1-\theta}} Y_\tau \right]\right)} \tag{41}$$

which is the usual condition of price being set to a mark-up over expectation of marginal cost.

Finally, investment department of an intermediate producer of type i faces market capital rental rate $R_{k,t}$, a price for new investment P_t and chooses investment and next period capital stock to maximize:

$$\max_{K_{t+1}(i), I_t(i)} E_t \sum_{\tau=t}^{\infty} Q_{\tau,t} \left[R_{k,t} K_t(i) - P_t I_t(i)\right] \tag{42}$$

s.t.

$$K_{t+1}(i) = K_t(i)[(1-\delta) + G(I_t(i)/K_t(i))] \tag{43}$$

First order conditions of this problem are:

$$\frac{1}{G'\left(\frac{I_t(i)}{K_t(i)}\right)} = E_t \left[Q_{t+1,t} \frac{P_{t+1}}{P_t} \left[R_{k,t+1} + \frac{(1-\delta) + G\left(\frac{I_{t+1}(i)}{K_{t+1}(i)}\right)}{G'\left(\frac{I_{t+1}(i)}{K_{t+1}(i)}\right)} - \frac{I_{t+1}(i)}{K_{t+1}(i)} \right]\right] \tag{44}$$

Which is the usual Euler equation for capital for a problem with capital adjustment costs.

3.4 Government

Government in the model changes monetary policy via interest rate rule, adjusting short-term nominal rate of interest in response to deviations of inflation rate and output from target and potential levels respectively. Also government smoothes fluctuations in the short-term interest rate itself.

The government in the model has perfect information about long-run productivity and economic uncertainty and evaluates potential output and a natural rate of interest using the same model save for perfectly flexible prices and monetary friction. Perfect information assumption, while clearly pretty unrealistic, is a natural starting point and I leave the investigation of implications of limiting the information the government possesses for future research. So the monetary authority reaction takes the following form:

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\left(r_t^{pot} + \bar{\pi} + r_\pi\left(\pi_t^* - \bar{\pi}\right) + r_y\left(\bar{y}_t - \bar{y}_t^{pot}\right)\right) + \varkappa_t \tag{45}$$

where r_π is reaction coefficient to the deviations of the log of inflation from long-run target $\bar{\pi}$, r_y is reaction coefficient to a log of ratio of output to potential and $\bar{y}_t - \bar{y_t^{pot}}$ is a measure of this ratio,r_t^{pot} is a natural rate of interest from a model with monetary frictions removed, ρ_r is a degree of interest rate smoothing, π_t^* is log of inflation over the past 12 months and $\varkappa_t$ is a monetary policy shock beyond the control of a monetary authority, that was specified in the section describing stochastic processes that affect this economy.

While a direct measure of annual inflation is possible, it introduces a few additional state variables into the mix, so instead I approximate annual inflation with the following moving average:

$$\pi_t^* = (1-\rho_\pi)\pi_t + \rho_\pi \pi_{t-1}^* \tag{46}$$

Using this measure of annual inflation adds only 1 extra state variable.

Government is assumed to run a balanced budget every period, it injects new money balances into economy via lump-sum transfers.

$$M_t = M_{t-1} + T_t \tag{47}$$

The amount of money to be injected is determined endogenously by interest rate rule.

3.5 Competitive Equilibrium

Equilibrium prices and quantities in period t naturally depend on history of shock realizations S^t, to save on notation in what follows I suppress this dependence.

Competitive equilibrium in this economy consists of sequences of final good prices $\{P_t\}$, intermediate good prices $\{P_t(i)\}$, nominal bond prices $\{Q_{t+1,t}\}$, real wages $\{W_t\}$, rental rate of capital $\{R_{k,t}\}$; sequences of quantities of final consumption $\{C_t\}$, labor $\{L_t\}$, capital $\{K_t\}$, investment $\{I_t\}$, money balances held by households $\{M_t\}$, nominal bond holdings $\{B_{t+1,t}\}$, final producer's output $\{Y_t\}$, consumers assets $\{A_t\}$, intermediate producer's outputs $\{Y_t(i)\}$, their labor and capital demands $\{L_t(i)\}$ and $\{K_t(i)\}$, profits $\{\Upsilon_t\}$ and government transfers $\{T_t\}$. Such that:

1. given prices $\{P_t\}$, bond prices $\{Q_{t+1,t}\}$, wages $\{W_t\}$ profits $\{\Upsilon_t\}$ and transfers $\{T_t\}$ quantities $\{C_t\}, \{L_t\}, \{M_t\}, \{B_{t+1,t}\}, \{A_t\}$-solve consumer's problem in $(12), (13)$

2. Given prices $\{P_t\}$, intermediate producer prices $\{P(i,t)\}$ quantities of intermediate good $\{Y(i,t)\}$, and final good$\{Y_t\}$ solve final producer's problem in (24).

3. Given final good prices $\{P_t\}$,bond prices $\{Q_{t+1,t}\}$,wages W_t, rental rate of capital $\{R_{k,t}\}$ and demand schedule $\left\{Y^d(i,t)\right\}$ as a function of final good prices and production $\{L_\tau(i)\}$ $\{K_t(i)\}$, $\{I_t(i)\}$ and $\{P(i,t)\}$ solve problem of intermediate producers in $(38), (28), (33)$ and (42).

4. Government budget is balanced every period (47).

5. Short-term interest rate follows (45).

6. Markets clear:

$$L_t = \int_0^1 L_t(i)di \tag{48}$$

$$K_t = \int_0^1 K_t(i)di \tag{49}$$

$$M_t^s = M_t \tag{50}$$

$$Y_t = C_t + I_t \tag{51}$$

$$B_{t+1,t} = 0 \tag{52}$$

3.6 Solution Technique

To solve the model I use log-linearization technique of Christiano [25] and combine it with Binder and Pesaran [17] method of solving linear rational expectations models. In what follows level variables, such as consumption C_t are denoted as capital letter with time subscript, level variables scaled by productivity are denotes as a capital letter with a bar: $\bar{C}_t = C_t/Z_t$, unconditional means of stationary variables are denoted with capital letter with no time subscript. Small letters denote natural logarithms of level variables scaled by productivity $c_t = log(\bar{C}_t)$, small letters with a hat denote deviations of logged scaled variables from logs of unconditional means: $\hat{c}_t = c_t - log\bar{C}$. Key log-linearized equations are labor FOC in (21):

$$\hat{w}_t = \hat{c}_t + \frac{L}{H-L}\hat{l}_t + \hat{r}_t \tag{53}$$

FOC of cost-minimization problem of intermediate producer in (36):

$$\hat{r}_{k,t} - \hat{w}_t = \hat{l}_t + \hat{x}_t - \hat{k}_t \tag{54}$$

where $\hat{x}_t$ is $log(Z_t/Z_{t-1} - E(Z_t/Z_{t-1}))$ - first element of $\hat{s}_t$

Expectation of the pricing kernel in (22):

$$-\hat{r}_t = \lambda_1 \left(E_t\hat{c}_{t+1} - \hat{c}_t\right) + \lambda_2 \left(E_t\hat{l}_{t+1} - \hat{l}_t\right) - E_t\hat{\pi}_{t+1} + \lambda_3'\hat{s}_t \tag{55}$$

where λ_1 λ_2 are coefficients that depend on parameters of the model and λ_3 depends both on parameters and on the equilibrium dynamics as well.

Interest rate rule in (45):

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1-\rho_r)\left(r_\pi \hat{\pi}_t^*\right) + \gamma_1 \hat{k}_t + \gamma_2 \hat{l}_t + \delta_s \hat{s}_t \tag{56}$$

where γ_1, γ_2 are parameters of log-linearization, δ_s is a vector of coefficients to be solved for, and $\hat{s}_t$ is a vector of current values of the driving shocks.

A version of the Phillips curve implied by producer maximization in (41):

$$\hat{\pi}_t = \frac{1-\alpha}{\alpha}\left(1-\alpha\hat{\beta}\right)\hat{mc}_t + \hat{\beta}E_t\hat{\pi}_{t+1} + \frac{1-\alpha}{\alpha}\left(1-\alpha\hat{\beta}\right)\kappa'\hat{s}_t \tag{57}$$

where $\hat{\beta} = Q\Pi X$, and Q, Π, X are unconditional averages of stochastic discount factor, gross inflation and productivity growth rates. κ' is a vector of coefficients that depends on evolution of the system.

Evolution of the annual inflation proxy in (46):

$$\hat{\pi}_t^* = (1-\rho_\pi)\hat{\pi}_t + \rho_\pi \hat{\pi}_{t-1}^* \tag{58}$$

Log-linearized aggregate feasibility constraint (51) with aggregated law of motion for capital from (43) substituted in:

$$\bar{K}\hat{k}_{t+1} = \left(\bar{K}/X\right)^\kappa (L)^{1-\kappa}\left(\kappa\hat{k}_t + (1-\kappa)\hat{l}_t - \kappa\hat{x}_t\right) - \bar{C}\hat{c}_t + (1-\delta)\frac{\bar{K}}{X}\left(\hat{k}_t - \hat{x}_t\right) \tag{59}$$

And finally Euler equation for capital in (44):

$$E_t\left\{\hat{q}_{t+1,t} + \hat{p}_{t+1} + \hat{r}_{k,t+1} + \alpha_1\hat{k}_{t+1} + \alpha_2\hat{l}_{t+1} + \alpha_3\hat{c}_{t+1}\right\} = \alpha_4\hat{k}_t + \alpha_5\hat{l}_t + \alpha_6\hat{c}_t + \alpha_7\hat{s}_t \tag{60}$$

where again α_1 through α_6 are parameters of log-linearization and α_7 is a parameter to be solved for.

Using (53) and (54) together with properties of cost-minimization in (33) I substitute marginal cost out of (57):

$$\hat{\pi}_t = \frac{1-\alpha}{\alpha}\left(1-\alpha\hat{\beta}\right)\left(\hat{c}_t + \frac{L(1-\kappa)+\kappa}{(H-L)}\hat{l}_t + \hat{r}_t\right) + \hat{\beta}E_t\hat{\pi}_{t+1} + \frac{1-\alpha}{\alpha}\left(1-\alpha\hat{\beta}\right)\kappa'\hat{s}_t \quad (61)$$

Again using (53) and (54) with a pricing kernel in (20) I substitute rental rate of capital and the kernel out of (60). What I'm left with is a system of 6 equations linking values of endogenous 6 variables $\hat{h}'_t = \left(\hat{c}_t, \hat{l}_t, \hat{k}_t, \hat{\pi}_t, \hat{r}_t, \hat{\pi}^*_t\right)$ in periods $t-1, t, t+1$:

$$A_1\hat{h}_t = A_2\hat{h}_{t-1} + A_3E_t\hat{h}_{t+1} + A_4\hat{s}_t \quad (62)$$

The key difference between standard set-up and my model is that values of λ_3, δ_s κ and α_7 depend on dynamics of endogenous variables and dynamics of endogenous variables in turn depends on values of λ_2, λ_1 α_{1-6} which turn out to depend on α_7, λ_3, δ_s and κ. This happens because of time-variable uncertainty has to be taken into account during log-linearization of expectations. And more generally moving uncertainty affects endogenous variables such as capital and labor through precautionary saving motive, affecting aggregate dynamics in significant ways. So in order to find a solution to the system above I solve a fixed point problem, so that α_7,λ_3, δ_s and κ and dynamics of endogenous variables are consistent with each other.

Solution to the fixed point problem yields the following dynamic system:

$$h_t = Bh_{t-1} + C\hat{s}_t \tag{63}$$

where $h_t' = \left(\hat{c}_t, \hat{l}_t, \hat{k}_t, \hat{\pi}_t, \hat{r}_t, \hat{\pi}_t^*\right)$, and $\hat{s}_t$ is vector of deviations of forcing stochastic processes from their respective means. Any other variable of interest can be expressed as a linear combination of $\left(h_t', \hat{s}_t{}'\right)$

3.7 Equity, Bond Prices and Excess Return Regressions

Once I have solved for the dynamics of the model what I have is essentially a multi-factor Duffie-Kan(1996) [32] affine asset pricing model adopted to discreet time along the lines of Backus at al.(1998,2001) [8], [7] so I use their recursive approach to the calculation of asset prices and yields of multi-period bonds, nominal and real.

3.7.1 Bond prices

Let Γ_t be an extended state vector $\Gamma_t = \begin{bmatrix} h_{t-1} & s_t \end{bmatrix}^T$. Its evolution follows from what I just derived

$$\Gamma_{t+1} = \widetilde{A}\Gamma_t + \Sigma^{1/2}\left(\Gamma_t\right)\Omega_{t+1} \tag{64}$$

where $\widetilde{A} = \begin{bmatrix} B & C \\ 0_{6x4} & A \end{bmatrix}$, $\Sigma^{1/2}\left(\Gamma_t\right) = \begin{bmatrix} 0_{6x6} & 0_{4x6} \\ 0_{6x4} & \Sigma^{1/2}\left(s_t\right) \end{bmatrix}$, $\Omega_{t+1} = \begin{bmatrix} 0_{1x6} \\ \omega_{t+1} \end{bmatrix}$

Pricing kernel given by the following decomposition:

$$q_{t+1,t} = q + q_{\Gamma}\hat{\Gamma}_t + q_{\Omega}\Sigma^{1/2}\left(\Gamma_t\right)\Omega_{t+1} \tag{65}$$

where $q_S = \begin{bmatrix} q_h & q_s \end{bmatrix}$, $q_{\Omega} = \begin{bmatrix} 0 & q_{\omega} \end{bmatrix}$. Denote $b_t^{(n)} = \log B_t^{(n)}$-log price of n-period bond default-free discount bond. Its price can be determined recursively

$$b_t^{(n)} = \log E_t\left(\exp\left(q_{t+1,t} + b_{t+1}^{(n-1)}\right)\right) \tag{66}$$

with price of current consumption being one, so $b_t^{(0)} = 0$. Backus et all (2001) show that bond prices for all maturities can be expressed as linear combination of factors:

$$b_t^{(n)} = a_0^{(n)} + a_{\Gamma}^{(n)}\hat{\Gamma}_t \tag{67}$$

Derivation of coefficients is given in appendix A. Period yields are given by the formula:

$$r_t^{(n)} = \frac{-\log B_t^{(n)}}{n} = \frac{-b_t^{(n)}}{n} \tag{68}$$

Completely analogously I derive pricing formulas for real bonds using real pricing kernel.

$$\psi_{t+1,t} = \psi + \psi_{\Gamma}\hat{\Gamma}_t + \psi_{\Omega}\Sigma^{1/2}\left(\Gamma_t\right)\Omega_{t+1} \tag{69}$$

3.7.2 Excess bond return regressions

Excess return of buying a bond of maturity n and selling it after m periods as a bond of maturity $n-m$ is

$$rx_{t,m}^{(n)} = \log\left(\frac{B_{t+m}^{(n-m)}}{B_t^{(n)}} B_t^{(m)}\right) = b_{t+m}^{(n-m)} - b_t^{(n)} + b_t^{(m)} \tag{70}$$

Using (67) I write expected average annual excess return as

$$E_t rx_{t,m}^{(n)} = a_0^{(n-m)} - a_0^{(n)} + a_0^{(m)} + \left(a_\Gamma^{(n-m)} \widetilde{A}^m - a_\Gamma^{(n)} + a_\Gamma^{(m)}\right) \hat{\Gamma}_t \tag{71}$$

Current period long-short bond yield spread $r_t^{(n)} - r_t^{(m)}$ can be written using factors:

$$r_t^{(n)} - r_t^{(m)} = a_0^{(n)} - a_0^{(m)} + \left(\frac{-a_\Gamma^{(n)}}{n} + \frac{a_\Gamma^{(m)}}{m}\right) \hat{\Gamma}_t \tag{72}$$

So the slope coefficient in regression of expected excess returns $E_t rx_{t,m}^{(n)}$ on yield spread $r_t^{(n)} - r_t^{(m)}$ is given by

$$C_{n,m} = \frac{\left[a_\Gamma^{(n-m)} \widetilde{A}^m - a_\Gamma^{(n)} + a_\Gamma^{(m)}\right] V(\Gamma) \left[\frac{-a_\Gamma^{(n)}}{n} + \frac{a_\Gamma^{(m)}}{m}\right]^T}{\left[\frac{-a_\Gamma^{(n)}}{n} + \frac{a_\Gamma^{(m)}}{m}\right] V(\Gamma) \left[\frac{-a_\Gamma^{(n)}}{n} + \frac{a_\Gamma^{(m)}}{m}\right]^T} \tag{73}$$

where $V(\Gamma)$ is a variance-covariance matrix of process Γ_t and solves discrete Lyapunov equation:

$$V(\Gamma) = \widetilde{A} V(\Gamma) \widetilde{A}^T + \Sigma(\Gamma) \tag{74}$$

So the key to getting slope coefficient close to where it is in the data is to generate enough covariance in the numerator without generating too much spread variance in the denominator.

3.8 Equity

First, the claim to single intermediate firm's profits is priced as follows:

$$P_t^e(i) = E_t\left\{\sum_{\tau=t+1}^{\infty} Q_{\tau,\tau-1}\left(\left[P_\tau Y_\tau(i) - P_\tau W_\tau L_\tau(i) - P_\tau I_\tau(i)\right]\right)\right\} \tag{75}$$

Real price is given by:

$$\frac{P_t^e(i)}{P_t} = E_t\left\{Q_{t+1,t}\frac{P_{t+1}}{P_t}\left[\frac{P_{t+1}^e(i)}{P_{t+1}} + Y_{t+1}(i) - W_{t+1}L_{t+1}(i) - I_{t+1}(i)\right]\right\} \tag{76}$$

$$\begin{aligned}\frac{P_t^e(i)}{P_t} =& E_t\left\{Q_{t+1,t}\frac{P_{t+1}}{P_t}\left[Y_{t+1}(i) - W_{t+1}L_{t+1}(i) - I_{t+1}(i)\right]\right\} + \\ & E_t\left\{Q_{t+1,t}\frac{P_{t+1}}{P_t}\left[\frac{P_{t+1}^e(i)}{P_{t+1}}\right]\right\}\end{aligned} \tag{77}$$

To get the price of a claim on aggregate dividends of intermediate firms I integrate:

$$\begin{aligned}\frac{P_t^e}{P_t} =& E_t\left\{Q_{t+1,t}\frac{P_{t+1}}{P_t}\int\left[Y_{t+1}(i) - W_{t+1}L_{t+1}(i) - I_{t+1}(i)\right]di\right\} + \\ & E_t\left\{Q_{t+1,t}\frac{P_{t+1}}{P_t}\left[\frac{P_{t+1}^e}{P_{t+1}}\right]\right\}\end{aligned} \tag{78}$$

Define aggregate real dividend:

$$D^r_{t+1} = \int \left[Y_{t+1}(i) - W_{t+1}L_{t+1}(i) - I_{t+1}(i)\right] di \tag{79}$$

Real return on equity then is given by

$$R^e_{t+1} = \frac{\frac{P^e_{t+1}}{P_{t+1}} + D^r_{t+1}}{\frac{P^e_t}{P_t}} \tag{80}$$

And it satisfies

$$E_t\left\{Q_{t+1,t}\frac{P_{t+1}}{P_t}R^e_{t+1}\right\} = 1 \tag{81}$$

I use the model's solution to express real equity return as a function of aggregate state at time t and innovation at time $t+1$:

$$r^e_{t+1,t} = r^e + r^e_\Gamma\hat{\Gamma}_t + r^e_\Omega\Sigma^{1/2}(\Gamma_t)\,\Omega_{t+1} \tag{82}$$

4 Calibration, Results and Discussion

First I will describe my choice of the key parameters of the model and present the results of my baseline calibration, next I will evaluate sensitivity of results to capital adjustment costs, IES and parameters of Taylor rule.

4.1 Baseline Calibration

As shown in Bansal et al.[10], if decision interval in the true model is shorter than data sampling interval, estimating the model using sampling interval leads to a bias in parameter estimates. For this reason I calibrate the model using monthly decision interval. I set consumption leisure elasticity parameter ν to 0.4, so that implied share of time endowment spent working is about one third. For baseline calibration I set γ, parameter controlling risk aversion, to 50, so that my price of short-term consumption risk $1 - \nu(1 - \gamma)$ and is quite a bit higher than the price of risk that is estimated by Bansal et al. [10] and the one used by Bansal-Shaliastovich [11], but on par with the one used by Croce (2008) [28] in production setting. Depreciation rate is set a 0.005, to match approximately 6% annual depreciation of capital. Adjustment cost parameter is set at 0.9, which is quite low[11]. Capital share is set 0.3. In all calibrations, except those explicitly changing it, I am going to set intertemporal elasticity of substitution parameter ψ to 1.5. Rate of time preference β is set at 0.999 for baseline calibration and is adjusted for some calibrations to make sure that steady state short-term real rate of interest stays as close as possible to 2% per annum.

Productivity process is calibrated as follows: Size of long-run productivity component is set to 5%, which is common in the long-run risk literature. Long-run productivity component persistence is set at 0.991, that (when converted to quarterly persistence) matches those estimated in Croce [28] and in the empirical section of this paper, and is very close to parameters used by Bansal-Shaliastovich [11]. Level of monthly productivity volatility is set 0.0020 so that the quarterly consumption growth volatility in the calibrated model is about 1.3%. Persistence of time-variable economic uncertainty is set at 0.986 close to the one used in Bansal-Yaron [12].

Parameter θ, that governs producer's mark-up is set at 0.7, implying 30% steady state mark-up over the marginal costs. Parameter α governing the frequency of price adjustment is set at 0.92, which means that on average firms re-optimize prices once

[11]Which corresponds to about 0.04% of output cost

every 12 months.[12] This might be somewhat higher than some estimates of the actual frequency of price adjustments but the parameter could be easily lowered if I modify my model to include real rigidities.[13]

In baseline calibration I assume that monetary authority does some interest rate smoothing: ($\rho_r = 0.5$), and that interest rate sensitivity to inflation gap and output gap are $r_\pi = 2.0$ and $r_y = 1.0$.[14] Monetary shocks are assumed to follow AR(1) process, with persistence and standard deviation set at 0.9 and 0.002 respectively. Table 7 summarizes parameters and their values in the baseline calibration.

4.2 Results and discussion

4.2.1 Impulse Responses

Figures 1 through 6 show model impulse responses of consumption, labor supply, capital and prices to one-time innovations in the four driving processes. Innovations are equal to one standard deviation of the respective shocks. Figures 3 and 5 give short horizon "impact view" of model's reaction to long-run productivity growth and uncertainty shocks, while figures 4 and 6 give long-run responses to the same shocks. Response to transitory productivity *growth* shock on Figure 1 is what one would expect: consumers start consuming and working more and slowly accumulate a little more capital. Picture

[12] While Bills and Klenow [16] find that median duration of prices is 4.3 months, more recently Nakamura and Steinsson [46] find that non-sale price duration is 8 to 11 months

[13] See, for examples of read rigidities Woodford(2003) [55]

[14] Rule parameters chosen so that they are loosely consistent with those estimated by Clarida at al [26] , Judd and Rudebusch [41] and Taylor [51] among others.

2 is a response to a positive interest rate shock, and it is unremarkable[15]. Persistence is entirely due to interest rate smoothing and persistence of the monetary shock itself. Figures 3 and 4 show model response to long-run productivity growth shock during 5 and 50 year periods after the shock. So impact is quite long-lasting and is well beyond the business-cycle frequency. While long-run response is natural, short-run dynamics is worth explaining. The explanation for short-run fall in capital after a long-run productivity growth shock is more intuitive if one thinks about permanent increase in the rate of growth of productivity in a stationary problem that was normalized by the rate of productivity growth. In such normalized setting permanently higher productivity translates into higher investment required just to keep the steady state normalized capital stock constant, effectively working like increased capital depreciation, so the steady state normalized capital decreases in response to a permanently higher growth rate of productivity, but the overall capital stock quickly picks up the new rate of productivity growth and exceeds the original level. This is exactly what is happening here, long-run productivity growth shock is very persistent, and it is optimal to decrease capital level relative to new expected level of productivity growth. To do that agents eat a lot and cut working hours. Inflation dynamics in response to long-run shock will be the key for the shape of nominal yield curve. In the model inflation follows the discounted forward looking marginal costs, where marginal cost in the short-run is dominated by labor dynamics and the discount depends on the stochastic discount factor of consumers. When the long-run shock arrives

[15]Perhaps except for strong the response of labor, which is explained by quite large Frisch labor elasticity that is hard to get around with Epstein-Zin preferences and $IES > 1$. Frisch elasticity is approximately given by $\frac{H-L}{L}\frac{1}{1-(1-\nu)(1-1/\psi)}$ which exceeds 2 in my calibration.

consumers will generally want to borrow against higher future income and interest rates go up, while the stochastic discount factor declines. So the weights in the discounted sum of future marginal costs shift into the near future, and the short-run response of marginal cost determines the direction of price movements. To explain the positive slope of the nominal yield curve exposure to inflation has to carry a risk premium. So price of nominal bonds has to positively co-move with good shocks to consumption growth, thus for inflation to carry an inflation risk premium inflation has to come down after a positive long-run productivity growth shock. Short-run dynamics of marginal cost is driven mostly by dynamics of labor, and depends on capital adjustment costs. With low costs of adjusting capital stock it is optimal to reduce capital in the short-run via working less, so wages come down and so dow prices. Figures 5 and 6 and show response of the model to innovations in economic uncertainty. Effects of uncertainty on model dynamics are quite strong and rival those of the short-run productivity growth shocks and monetary shocks. Explanation is simple - precautionary investment. Higher uncertainty exposes agents to more long-run productivity risks, and they want to accumulate more capital as a cushion to protect themselves. So this desire to invest explains decrease in consumption and increase in hours. This endogenous dynamics has important implications for most of predictability regressions below.

4.2.2 Growth and Business Cycle Statistics

Table 8 gives statistics on growth rates and standard deviations of growth rates of the key variables. Tables 9 and 10 contain basic business cycle statistics of the model-implied data. To calculate those I simulated the model, aggregated the data to quarterly frequency and HP filtered it. In the growth rates of macro variable the model is calibrated to match the consumption volatility. Matching both consumption and growth volatility is extremely hard in this model while keeping consumption growth predictability low and investment volatility similar to the data. The reason is that consumers want to smooth consumption and if capital adjustment costs are low they will quickly smooth out long-run shocks resulting in investment being very volatile, but even with zero adjustment costs consumption is still more volatile relative to output than in the data; on the other hand, if one assumes high costs of adjusting capital investment becomes too smooth and consumption volatility becomes closer to the volatility of output and consumption growth becomes very predictable. Investment is naturally very volatile in the model with no adjustment costs, so getting investment volatility in line with the data makes consumption volatility pretty large relative to volatility of output. Lower panel of 8 gives consumption data moments that Bansal and Yaron (2004) [12] and later Bansal et. al (2007) [10] use to estimate and test long-run consumption risk model. My calibrated model implies a bit more consumption growth predictability than assumed by Bansal-Yaron, and somewhat less of variability in the level of uncertainty. The latter is an intentional choice since matching volatility of volatility would make dividends super-volatile, due to endogenous response of intermediate producers to changes in stochastic discount factor caused by movements in uncertainty. HP filtered RBC statistics from the model again show that consumption is more volatile relative to output than in the data for the same reason as above, labor, however, does not move much despite the high Frisch elasticity that is implied by Epstein-Zin preferences and the value of intertemporal elasticity of substitution in my calibration.

4.2.3 Nominal and Real Yields

Table 11 shows nominal and real yield curves, as implied by the model. On the nominal side model does a good job of accounting for term structure observed in the data. As explained above, nominal bonds increase in price after long-run shock due to decrease in dynamic producer mark-up, that causes inflation to slowdown too. So nominal bonds are risky relative to real bonds, and yields carry a premium for inflation risk. This premium increases for longer maturities. Real term structure, on the other hand is mildly downward sloping, which is not consistent with US data.[16] The reason real term spread is negative is the fact that real bonds hedge long-run expected consumption growth risk in this model. When long-run shock arrives consumers want to borrow against future income, so interest rates go up, and prices of long-term bonds come down, acting as a hedge. Longer-maturity bonds are, naturally, more exposed to long-run risk so they hedge long-run consumption risk better. In other words real term premia is negative[17]. However, the effect is milder in a production economy with capital, since agents can self-insure via capital accumulation.

Table 12 shows the size of inflation term premium as implied by the model. Inflation term premium is modest at short maturities, and gets quite large at 5 year maturity, but it is not very volatile.

Volatility of the yields is too small compared to the data, and this is the major challenge of the current model. While it is possible to increase the volatilities via increasing the variance of economic uncertainty, unfortunately it would have very undesirable effects on the other variables in the model, namely dividends. Higher uncertainty implies higher

[16] In UK however real term structure is downward sloping.

[17] This fact has been pointed out by Piazzesi and Schneider (2005)[47]

desire to invest for precautionary reasons, and causes dividends to inherit the properties of the economic uncertainty process, leading to very unrealistic values of dividend growth volatility. One possible way to improve the model is to increase currently low volatility of labor supply. A natural extension of present model is the inclusion of sticky wages, which would increase the labor variability, increasing the variance of stochastic discount factor as well.

4.2.4 Equity Market Statistics

Table 14 contains key moments of stock market variables. Equity returns and equity premium is sizable but both are quite a bit less than those observed in the data and those reported by Croce(2008) for a model with long-run risk and capital accumulation. It is worth noting that equity return in my model is un-levered, unlike that of Croce (2008) [28], and my model includes endogenous labor and nominal rigidities, which could explain the difference in the size of equity premium. Sharpe ratio, which is the ratio of excess return on equity to volatility of this excess return is 0.41, which is quite high, in fact higher than in the data, which is one of the good features of the model. Equity return and excess return exhibit similar amount of autocorrelation in the model as in the data, but the dividends in the model are substantially more autocorrelated than in the data. Large negative autocorrelation for the data is partially explained by the use of the quarterly data, since dividends have significant seasonal patterns, but for longer time horizons autocorrelation of dividends in the data is quite low. In the model dividends are heavily influenced by the value of long-run productivity growth component and thus are fairly autocorrelated. Higher price-dividend ratio in the model is also driven by autocorrelation of dividends. Claim to aggregate dividend is risky in the model, since dividends go up when long-run shock hits, so the price of equity goes up at the same time as expected consumption goes up, and due to this positive correlation commanding a premium over a safe assets. Reaction of dividends to long-run growth shock is determined by reaction of investment to such shock. In the calibrated model investment falls on impact, unless adjustment costs are very high, increasing profits of intermediate firms.

4.2.5 Predictability of Bond and Equity Returns

Table 13 bond predictability regression coefficients and R^2 statistics that come out of the model and compares them to the data. Baseline calibration does not generate enough predictable variation in bond prices to fully account for the data. The model explains about a third of the slope of predictability regression and only a quarter in terms of regression R^2, however the deviations from the "Expectation Hypothesis" are pronounced. Due to the presence of inflation premium nominal excess holding period returns in the model are more predictable than the real excess returns. If one compares predictability implications of this model to those of Bansal-Yaron endowment economy augmented by exogenous inflation studied by Bansal and Shaliastovich [11], relative lack of predictability here is explained both by endogeneity of inflation risk, that does not produce quite as high "loading" of inflation on long-run shock innovations, and by endogenous response of quantities to the innovations of economic uncertainty and long-run productivity, which are not possible in an endowment economy. Basically movements in bond risk premia depend on how sensitive the prices of long maturity bonds are to long-run productivity growth shocks, and in the production economy with capital this sensitivity is lower because the agents can smooth consumption over time via capital accumulation. Table 15 repeats equity market predictability regressions using simulated data and compares results to the data. The model does a very good job of generating predictability in excess returns, and the predictability does not come from predictability of the returns themselves, which are about half as forecastable in the model as in the data. Unfortunately the model also has price-dividend ratio predicting consumption growth and dividend growth, both of which are not in the data. The latter two regression results are not surprising since in the model most of the movements of variable relevant to asset pricing are related to long-run risk and thus forecast each other. Overall the model generates quite a bit of predictability of bond and equity returns, but bond predictability is only a fraction of what it is in the data, and price-dividend ration ends up predicting more variables than in does in the data.

4.2.6 Sensitivity: risk-aversion, different interest rate rule parameters.

In this section I explore model sensitivity to key parameters: risk aversion, intertemporal elasticity of substitution, capital adjustment cost and Taylor rule parameters. Table 16 summarizes the results of sensitivity analysis by showing selected moments of the consumption and equity data. First column contains data, second column shows moments implied by the baseline calibration of the model. Third column lowers the risk-aversion parameter to 20, so that the price of consumption risk $(1-\nu(1-\gamma))$ is about 10. Impact is obvious, term spread and equity premium and predictability regression statistics decline, while consumption data moments are almost unchanged. In the next column I present moments for the model with IES set at 3. Most notably, consumption volatility declines relative to output, and investment volatility increases. Predictability in consumption goes down, along with predictability in bond returns and level of term premium. Real equity excess returns are not really affected and are still very predictable, despite decreased predictability of consumption growth. Nominal facts are explained by the relative fragility of the mechanism that determines inflation premium in the model, and I will discuss it below. Equity markets facts are real, and the forecastability of consumption growth process is not very relevant since the model is driven by productivity dynamics. The change in equity premium is explained by the increase in investment volatility, that increases dividend volatility and makes equity claim riskier. Fourth column considers effect of lowering IES to 0.5. Consumers now prefer much smoother consumption process, so the amount of predictability in consumption goes up. Nominal bond excess returns are a little more predictable, but most notably equity premium is negative. This is due to the change in agent's attitude towards consumption smoothing. While with IES larger than one consumption today and tomorrow are substitutes and loosely speaking one can think about asset pricing implications in terms of hedging wealth. With IES below one consumption today and tomorrow are complements, so now, when a long-run productivity growth shock arrives and drives up the value of dividend claim, it allows holders of such claim sell it and consume more today too, so holding stocks provides insurance, thus the equity premium can go negative. Last two columns report effects of changing parameters of Taylor rule. What these parameters do - they determine the strength and the sign of response of inflation to a long-run shock. The mechanism explained before still applies, but if the monetary authority ends up cutting interest rates in response to a long-run shocks too much, inflation might be induced to actually increase as a result of such a shock and make nominal bonds a good hedge for consumption risk. This is what increasing sensitivity of interest rate rule does. Suppose we start from inflation going down 1% in response to long-run shock; now if we increase inflation sensitivity of interest rate rule to inflation, interest rates would be cut, and inflation would end up declining less than initial 1%; make the policy response strong enough, and the resulting inflation premium could end up being negative, which of course will imply downward sloping nominal yield curve. Output gap parameter has similar role except that output gap is more sensitive to long-run risk, so changes in this parameter have more drastic effects. Changes in Taylor rule have real effects, since they affect inflation premium, which in turn affects real decisions of intermediate producers by changing stochastic discount factor of consumers. But more importantly inflation premium is counted in real returns typically analyzed in the finance literature, since those are usually obtained by subtracting realized inflation. This last result has important implications: non-trivial monetary side of the model matters even for analysis of real asset pricing data and could have sizable effects on estimates of key parameters, especially ones so hard-to-detect as long-run components of consumption or productivity.

5 Conclusion and Future Research

I set out with a broad objective to set-up a simple model that is suitable for the analysis of conduct of monetary policy and could also be used to explain asset pricing puzzles. As the key building block I used a model of long-run consumption risks due to Bansal and Yaron [12] that has been shown to be quite successful in accounting for asset pricing puzzles in an endowment economy. As a monetary side I used a simple version of New-Keynesian model with nominal rigidity in the form of staggered price adjustment. As shown above, the model has non-trivial implications for price, consumption, capital and leisure dynamics in response to long-run productivity and uncertainty shocks.

I have explored the performance of the above model's in accounting for a wide range of asset pricing puzzles. While results fall short of the data, overall they are promising. I am able to generate sizable deviations from the "expectation hypothesis" of bond pricing, account for nominal term premium, generate sizable and predictable equity premium. Overall my results imply that monetary policy can contribute both to levels of asset prices and to the predictable movements in prices of those assets, thus suggesting that nominal side asset prices should be taken into account in attempts to extract information about long-run shocks contained in the financial variables.

The model is well suited for further research of interaction between monetary policy and financial variables. There are several fruitful research directions. First one can relax assumptions of perfect information and study implications of monetary policy when the monetary authority does not have complete information about long-run shocks. In such a setting monetary policy can have even larger effects on asset prices, since policy rules could induce persistent reactions to long-run shocks during the time it takes the government to realize that a shock to long-run growth rate has occurred and adjust the potential output. Second, one can use the model to study the design of optimal monetary policy in the presence of long-run productivity risks. Another possible research venue is to study the interaction between monetary policy and long-run productivity and economic uncertainty risks in an international setting. I have shown that long-run productivity innovations and changes in economic uncertainty induce sizable in capital accumulation dynamics that could have a important implications for exchange movements and international business cycle statistics.

6 Tables and Figures

Table 1: Sample statistics for nominal interest rates

Maturity	1y	2y	3y	4y	5y
Mean	5.55	5.75	5.92	6.05	6.13
St. deviation	(2.89)	(2.83)	(2.77)	(2.73)	(2.70)

Data is from Fama-Bliss CRSP June:1952-Sept:2006

Table 2: Excess return regression, nominal and real

Maturity:	2y	3y	4y	5y
Slope, Nominal Returns	1.68	2.02	2.41	2.39
st.deviation	(0.43)	(0.51)	(0.57)	(0.63)
R^2	0.11	0.12	0.15	0.13
Slope, Real Returns	1.14	1.58	1.61	1.56
st.deviation	(0.41)	(0.53)	(0.62)	(0.67)
R^2	0.11	0.07	0.06	0.05

Table reports slope coefficients from regression $E_t rx_{t,12}^{(n)} = const + \beta\left(r_t^{(n)} - r_t^{(12)}\right) + residual$, where $rx_{t,12}^{(n)} = -p_{t+12}^{(n-12)} + p_t^{(n)} - p_t^{(12)}$ is a holding period (12 months) excess return on default-free nominal (real) bond. Inflation adjustment using past inflation taken from AR(2)-filtered inflation process.[18] Standard errors are Newey-West adjusted with 10 lags.

Table 3: Equity return predictability

T:	1year	2y	3y	4y	5y	6y	7y
Equity Return	-.21	-.36	-.41	-.55	-.7	-.79	-.87
R^2	0.07	0.12	0.13	0.19	0.23	0.24	0.27
Equity Excess Return	-.22	-.39	-.47	-.62	-.77	-.87	-.94
R^2	0.09	0.14	0.15	0.19	0.26	0.31	0.33
Consumption Growth	.01	.009	.01	-.01	0.003	0.005	0.009
R^2	0.06	0.04	0.03	0.018	0.01	0.00	0.00
Dividend Growth	.06	.05	.07	.06	.05	.03	.01
R^2	0.07	0.05	0.03	0.04	0.01	0.01	0.00

Table reports slope coefficients from regressing T-period forward sum of real stock returns $\sum_{j=1}^{T}(r^e_{t+j})$, excess real stock returns $\sum_{j=1}^{T}(r^e_{t+j}-r^f_{t+j})$, consumption growth $\sum_{j=1}^{T}(\Delta c_{t+j})$ and dividend growth $\sum_{j=1}^{T}(\Delta d_{t+j})$ on current Price-Dividend ratio. Standard errors are Newey-West adjusted with 10 lags. Stock data is from Campbell (1999) [23].

Table 4: Measures of productivity

	Frequency	Sample	Mean	St.dev
BLS productivity growth	Annual	1948-2007	2.3%	1.5%
Constructed measure	Quarterly	1971:4-2007:4	0.8%	0.7%

BLS productivity is "Private Non-Farm Business Sector Multi-factor Productivity". Constructed measure is an (Index of Industrial production, manufacturing) divided by total hours in manufacturing, which are given by (average weekly hours, manufacturing) x (payroll, manufacturing). IP data is from Federal Reserve Board, Hours data from BLS Establishment Survey, Payroll data from BLS Establishment Survey also.

Table 5: Annual productivity regression

Model Estimated: GARCH(1,1)

$$\Delta z_{t+1} = (1-\rho)\,\mu_x + \rho \Delta z_t - b\sigma_t \varepsilon_{x,t} + \sigma_{t+1}\varepsilon_{x,t+1}$$

$$\sigma_{t+1}^2 = \sigma^2 + \rho_\sigma \sigma_t^2 + b_\sigma \sigma_t^2 \varepsilon_{x,t}^2$$

	ρ	b	ρ_σ	b_σ
Coefficient	0.87	0.96	0.61	-0.12
st.errors	(0.24)	(0.20)	(0.28)	(0.08)

Table 6: Quarterly productivity regression

Model Estimated: GARCH(1,1)

$$\Delta z_{t+1} = (1-\rho)\,\mu_x + \rho \Delta z_t - b\sigma_t \varepsilon_{x,t} + \sigma_{t+1}\varepsilon_{x,t+1}$$

$$\sigma_{t+1}^2 = \sigma^2 + \rho_\sigma \sigma_t^2 + b_\sigma \sigma_t^2 \varepsilon_{x,t}$$

	ρ	b	ρ_σ	b_σ
Coefficient	0.79	0.85	0.8	0.05
st.errors	(0.14)	(0.2)	(0.21)	(0.07)

Table 7: Calibrated parameters

	description:	value
β	rate of time preference	0.999
ν	consumption-leisure elasticity	0.4
θ	producer mark-up 30%	0.7
ψ	IES	1.5
γ	risk aversion parameter	50
α	price adjustment frequency	0.92
μ_x	mean monthly productivity growth	0.0025
σ_g	relative st.dev of long-run shock	5%
ρ_g	persistence of L-R shock	0.991
$\upsilon^{1/2}$	st. deviation of productivity	0.0020
ρ_υ	persistence of economic uncertainty	0.996
σ_υ	st.dev of uncertainty	$1.0e-07$
$\bar{r}$	nominal rate target	0.0055
$\rho_\varkappa$	persistence of monetary shock	0.9
$\sigma_\varkappa$	monetary shock variance	0.0015
r_π	policy rule reaction to inflation	2.0
r_y	policy rule reaction to output gap	1.2
ρ_r	policy rate smoothing	0.5
δ	depreciation	0.005
η	capital adjustment cost parameter	0.9
κ	capital share in income	0.30

Table 8: Business cycle growth statistics

	Data Sample	Model
$E(\Delta Y)$	2.07%	2.0%
$\sigma(\Delta Y)$	2.62%	1.6%
$E(\Delta C)$	2.03	2.0
$\sigma(\Delta C)/\sigma(\Delta Y)$	.49	.95
$\sigma(\Delta I)/\sigma(\Delta Y)$	3.3	3.5
$E(\Pi)$	2.5%	2.90%
$\sigma(\Pi)$	1.6%	1.41%
Bansal-Yaron moments		
$\sigma\left(E_t\left[\Delta c_{t+1}\right]\right)$	0.17	0.25
$Autocorrel\left(\sigma\left(E_t\left[\Delta c_{t+1}\right]\right)\right)$	0.94	0.97
$\sigma\left[\hat{\sigma}_t^2\left(\Delta c\right)\right]$	0.17	0.09
$Autocorrel\left[\hat{\sigma}_t^2\left(\Delta c\right)\right]$	0.94	0.90

Bansal-Yaron moments come from fitting ARMA(1,1) process to consumption growth Δc. Predictions from this fitted model give $E_t\left[\Delta c_{t+1}\right]$. $\hat{\sigma}_t^2\left(\Delta c\right)$ is a non-parametric measure of consumption volatility constructed by Bansal et. al (2007)[10]. $\hat{\sigma}_t^2\left(\Delta c\right) = \sum_{j=t}^{t-T} \varepsilon_j^2$ where ε_j is a residual from the above ARMA(1,1) model.

Table 9: Business cycle statistics: HP filtered simulations

	Data Sample	Model
st.dev(Y)	1.7%	1.50%
st.dev(C)/st.dev(Y)	.49	1.1
st.dev(L)/st.dev(Y)	0.98	0.37
st.dev(I)/st.dev(Y)	3.3	5.21
st.dev(Z)/st.dev(Y)	0.63	0.66
st.dev(Inflation)	1.5%	1.02%
st.dev(R_{3m})/R_{3m}	0.61	0.15
st.dev($R_{3m,real}$)/$R_{3m,real}$	0.86	0.2

Table 10: Correlations

Correlation with Output:	Data Sample	Model
Consumption	0.91	0.75
Labor	0.95	0.69
Investment	0.91	0.83
Productivity	0.47	0.76
Inflation	0.35	0.59
3month Interest Rate	0.18	0.47
3month Real Interest Rate	-0.07	-0.63

Tables 9 and 10 report model's business cycle statistics calculated from simulated data, that was quarterly averaged and HP filtered. Data moments are calculated using post-war US quarterly data.

Table 11: Nominal and real yield curves

Maturity	3m	6m	1y	2y	3y	4y	5y
Nominal Interest Rate	5.55	5.59	5.66	5.79	5.93	6.06	6.19
Standard Deviation	0.97	0.97	0.96	0.95	0.94	0.93	0.92
Real Interest Rate	2.30	2.29	2.26	2.20	2.14	2.08	2.02
Standard Deviation	0.57	0.57	0.56	0.56	0.55	0.54	0.53

Table 12: Inflation term premium

Maturity	3m	6m	1y	2y	3y	4y	5y
Inflation Term Premium	0.07	0.11	0.21	0.40	0.60	0.79	0.99
Standard Deviation	0.01	0.03	0.05	0.09	0.13	0.16	0.20

Inflation Term Premium in Table 9 is given by

$$itp_n = r_{nom}^{(n)} - r_{\text{real}}^{(n)} - \frac{1}{n} E\left(\pi_{t+n,t}\right)$$

where n-is maturity, $r_{nom}^{(n)}$ is a yield of n-period nominal discount bond, $r_{\text{real}}^{(n)}$ is a yield of n-period real discount bond, $\pi_{t+n,t}$ - is an log of n-period inflation rate.

Table 13: Predicted excess return regression, nominal and real

Maturity	2y	3y	4y	5y
Data, nominal	1.70	2.03	2.41	2.39
R^2	0.11	0.12	0.15	0.13
Model, nominal	0.64	0.70	0.72	0.71
R^2	0.01	0.02	0.04	0.04
Model, real	0.59	0.73	0.85	0.95
R^2	0.00	0.01	0.02	0.02

Table 13 reports predicted slope coefficients from regression

$E_t rx_{t,12}^{(n)} = const + \beta \left(r_t^{(n)} - r_t^{(12)} \right) + residual$, where $rx_{t,12}^{(n)} = -p_{t+12}^{(n-12)} + p_t^{(n)} - p_t^{(12)}$

is a holding period (12 months) excess return on default-free nominal (real) bond.

Table 14: Equity market moments

	Data Sample	Model
$E(r_e)$	8.08%	3.2%
$\sigma(r_e)$	15.6%	2.13%
$Autocorr(r_e)$	0.08	0.04
$E(r_e - r_f)$	7.1%	0.90%
$\sigma(r_e - r_f)$	20.5%	2.19%
$Autocorr(r_e - r_f)$	0.04	0.03
$E(\Delta d)$	2.15%	1.98%
$\sigma(\Delta d)$	28.2%	5.7%
$Autocorr(\Delta d)$	-0.54	0.27
$E(p - d)$	3.34	5.8
$\sigma(p - d)$	0.3	0.4
$Autocorr(p - d)$	0.94	0.99

Table 15: Equity return predictability

T:	1year	2y	3y	4y	5y	6y	7y
Equity Return, data	-.21	-.36	-.41	-.55	-.7	-.79	-.87
R^2	0.07	0.12	0.13	0.19	0.23	0.24	0.27
Equity Return, model	-.06	-.14	-.21	-.28	-.34	-.40	-.46
R^2	0.03	0.06	0.08	0.10	0.11	0.11	0.12
Equity Excess Return, data	-.22	-.39	-.47	-.62	-.77	-.87	-.94
R^2	0.09	0.14	0.15	0.19	0.26	0.31	0.33
Equity Excess Return, model	-.12	-.26	-.39	-.52	-.64	-.75	-.86
R^2	0.11	0.21	0.29	0.36	0.40	0.44	0.47
Consumption Growth, data	.01	.009	.01	-.01	0.003	0.005	0.009
R^2	0.06	0.04	0.03	0.018	0.01	0.00	0.00
Consumption Growth, model	-.15	-.30	-.45	-.62	-.7	-.8	-.9
R^2	0.25	0.25	0.24	0.24	0.23	0.23	0.23
Dividend Growth, data	.06	.05	.07	.06	.05	.03	.01
R^2	0.07	0.05	0.03	0.04	0.01	0.01	0.00
Dividend Growth, model	.24	.39	.53	.67	.79	0.91	1.02
R^2	0.04	0.04	0.04	0.05	0.05	0.05	0.05

Table 16: Sensitivity: Interest rate rule, IES, adjustment cost

	Data	Model	$\gamma = 20$ $\psi = 1.5$ $\eta = 0.9$ $r_\pi = 2.1$ $r_y = 1.2$	$\gamma = 50$ $\psi = 3$ $\eta = 0.9$ $r_\pi = 2.1$ $r_y = 1.2$	$\gamma = 50$ $\psi = 0.5$ $\eta = 0.9$ $r_\pi = 2.1$ $r_y = 1.2$	$\gamma = 50$ $\psi = 1.5$ $\eta = 0.9$ $r_\pi = 3$ $r_y = 1.2$	$\gamma = 50$ $\psi = 1.5$ $\eta = 0.9$ $r_\pi = 2.1$ $r_y = 0.9$
$R_{5y} - R_{1y}$	0.58%	0.64%	0.22%	-0.7%	0.55%	0.48%	0.30%
$r_{5y} - r_{1y}$	n/a	-0.28%	-0.1%	-0.3%	-0.35%	-0.3%	-0.29%
$E(R^e - R^f)$	7.1%	0.9%	0.33%	1.15%	-0.3%	0.8%	0.89%
$\sigma(R^e - R^f)$	20.5%	2.19%	2.04%	2.58%	1.4%	2.08%	2.15%
β_{5y}^{bond}	2.39	0.7	0.15	1.17	1.0	0.6	0.2
$R^2_{b,5y}$	0.13	0.04	0.001	0.02	0.05	0.02	0.01
β_{5y}^{eq}	-0.94	-0.86	-0.47	-0.78	0.001	-0.82	-0.86
$R^2_{eq,7y}$	0.33	0.47	0.25	0.47	0.0	0.45	0.41
$\sigma(\Delta c)/\sigma(\Delta y)$	.49	0.95	0.94	0.8	0.95	0.95	0.93
$\sigma(I)/\sigma(\Delta y)$	3.3	3.5	3.4	5.98	4.04	4.8	4.6
$\sigma(E_t[\Delta c_{t+1}])$	0.17	0.26	0.24	0.11	0.28	0.2	0.2

Tables 16 reports key statistics for different specifications of interest rate rules. $R_{5y} - R_{1y}$ is a nominal term spread, $r_{5y} - r_{1y}$ -real spread, $E(R^e - R^f)$ and $\sigma(R^e - R^f)$ are an equity premium and its variance, β_{5y}^{bond} and $R^2_{b,5y}$ and β_{5y}^{eq} & $R^2_{eq,7y}$ are respectively bond and equity predictability regressions slope and R^2, $\sigma(\Delta c)/\sigma(\Delta y)$ and $\sigma(I)/\sigma(\Delta y)$ are ratios of consumption and investment growth volatilities to output growth volatility, $\sigma(E_t[\Delta c_{t+1}])$ is a measure of how predictable is consumption growth.

Figure 1: Impulse response to temporary productivity growth shock

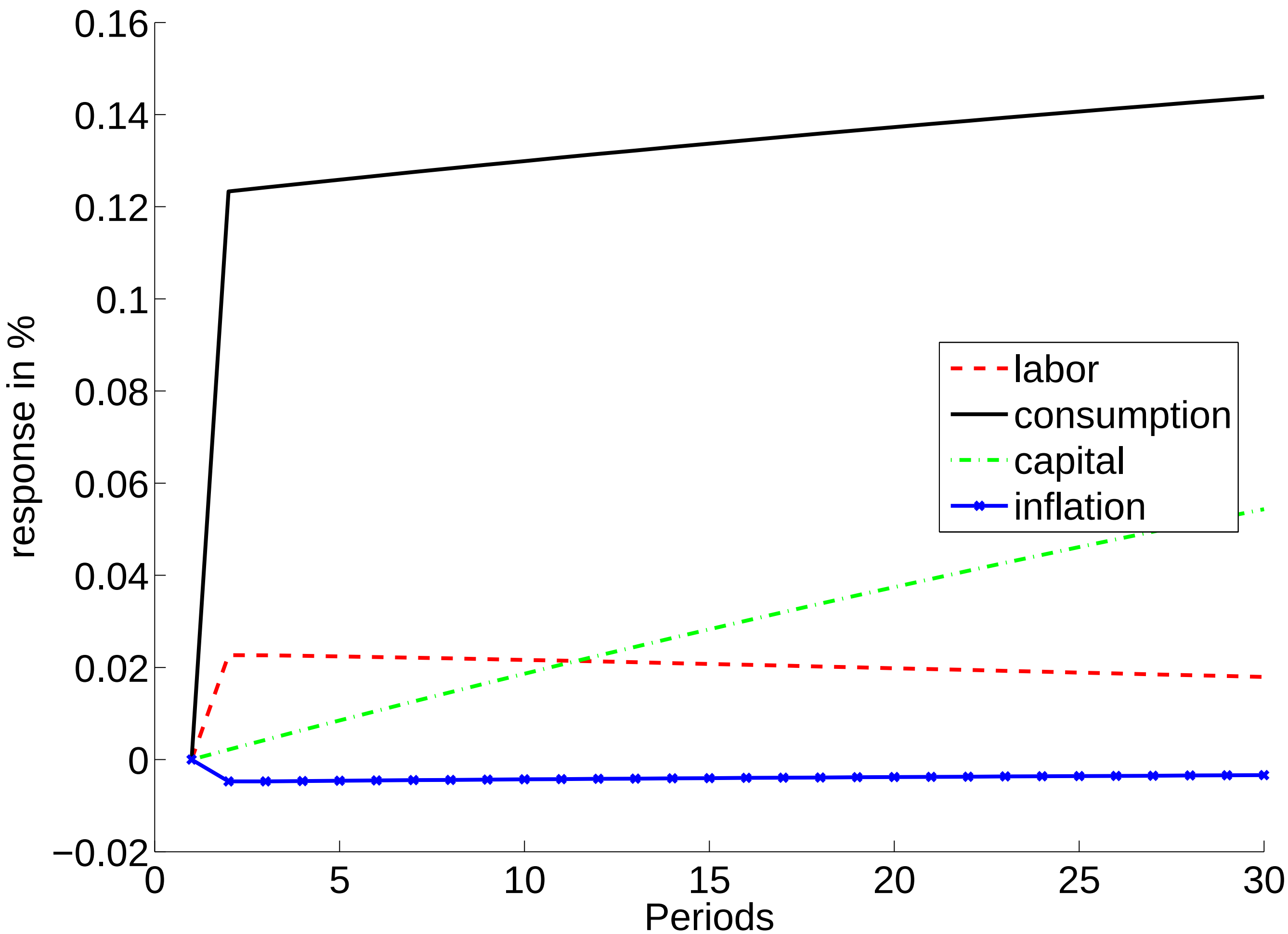


Figure 2: Impulse response to monetary shock

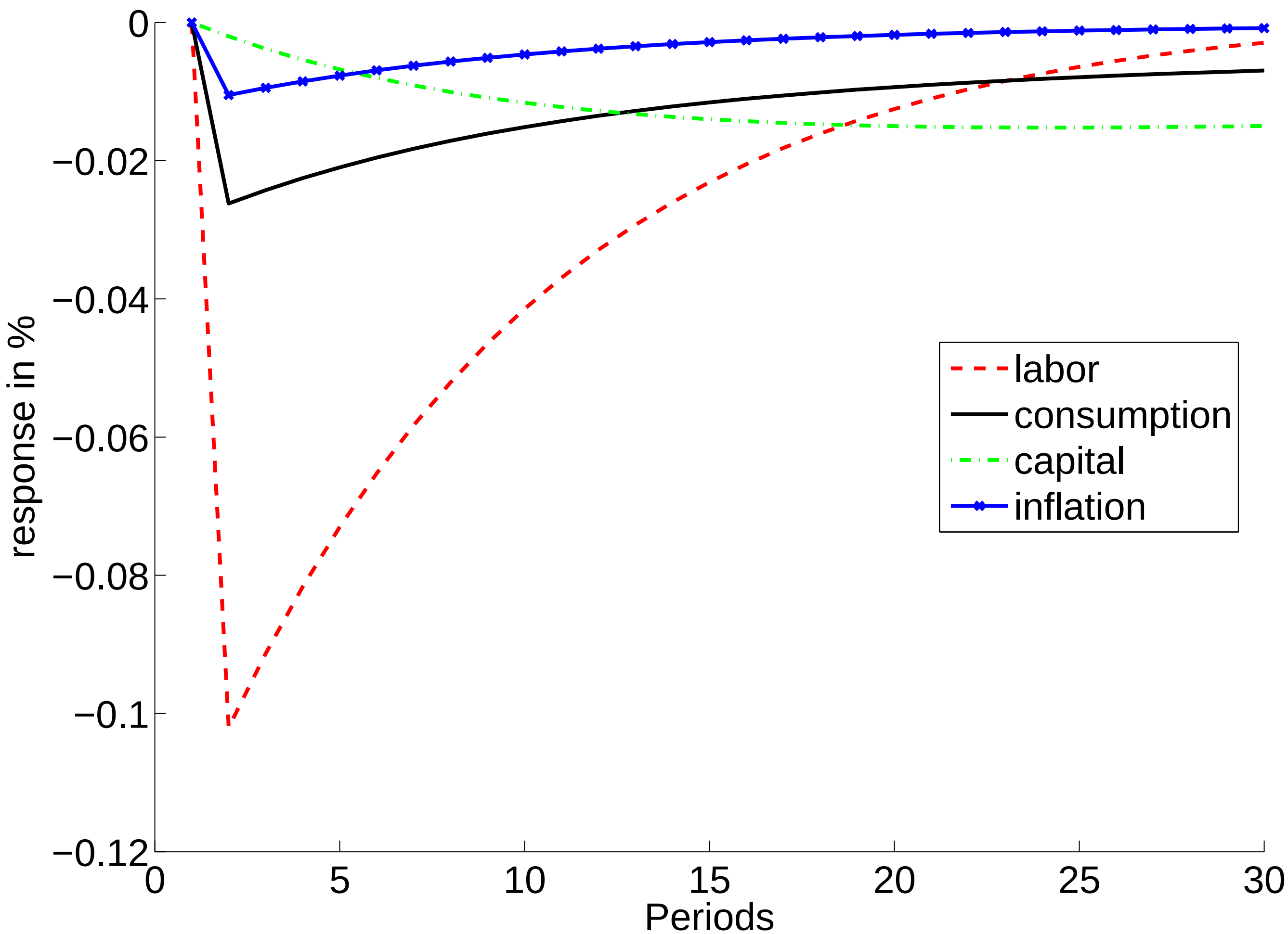


Figure 3: Impulse response to persistent productivity growth shock, impact view

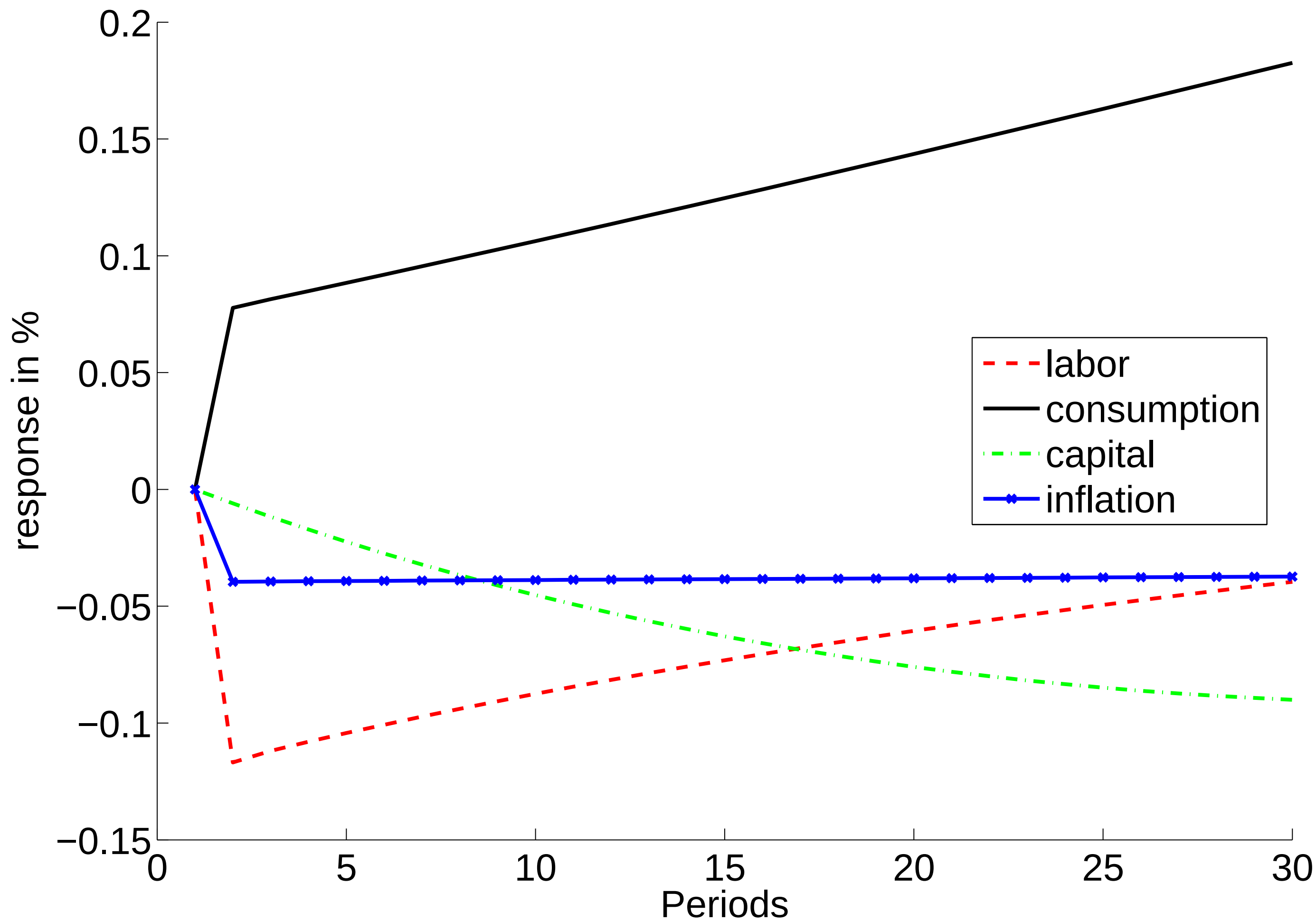


Figure 4: Impulse response to persistent productivity growth shock

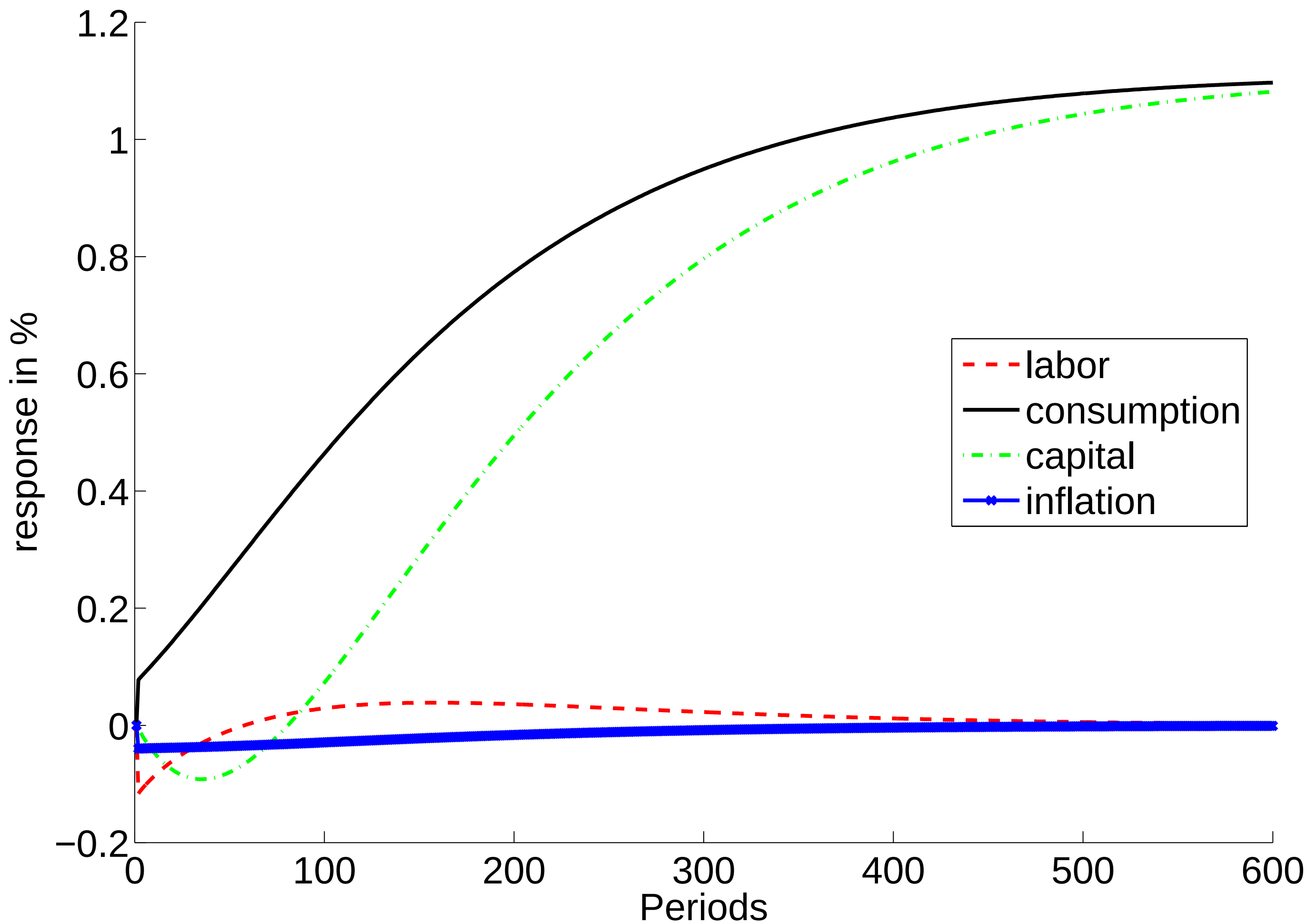


Figure 5: Impulse response to uncertainty shock, impact view

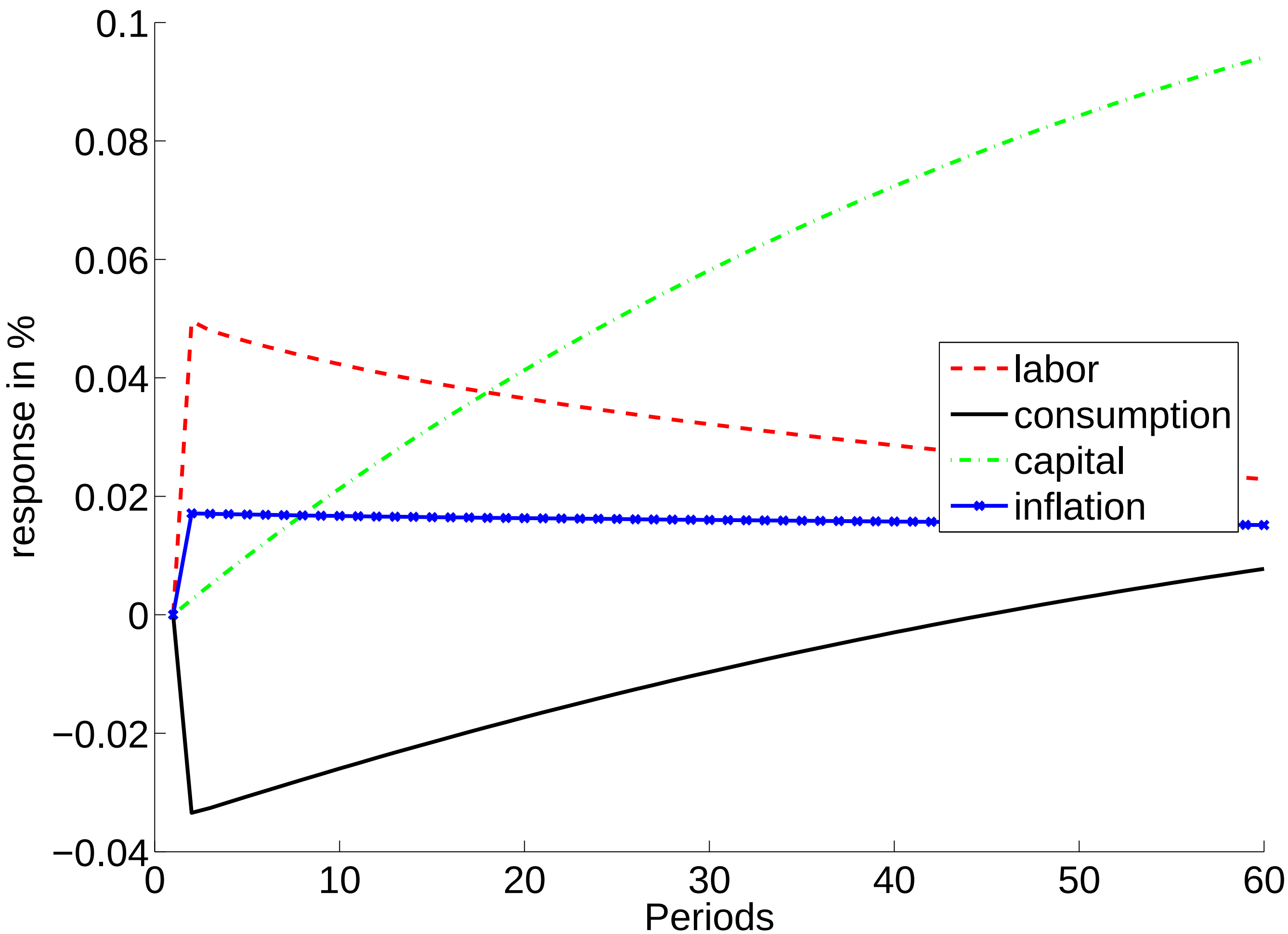
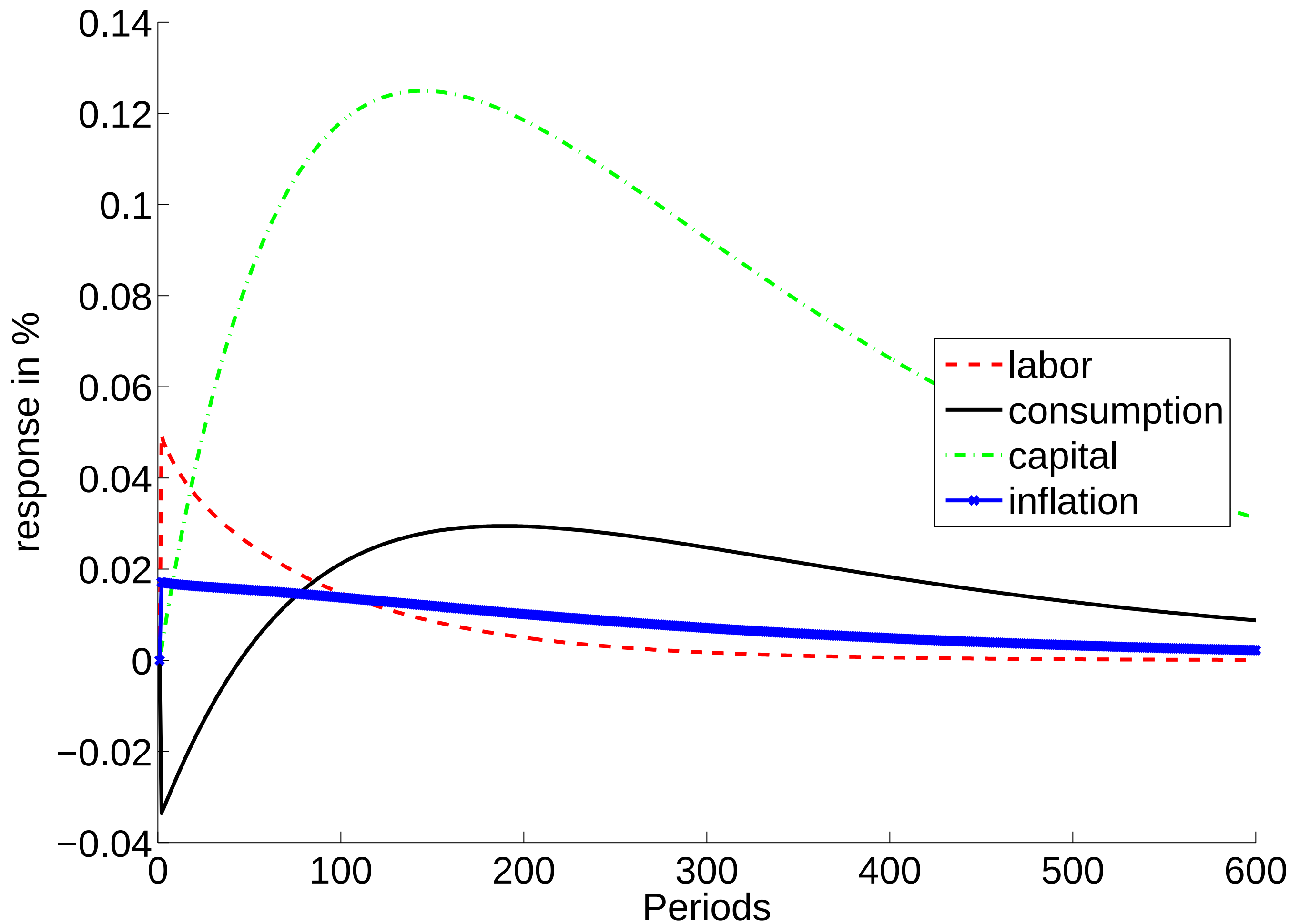


Figure 6: Impulse response to uncertainty shock



Part III

On Preference Heterogeneity and Internal Risk-control Technology Investment

7 Introduction

Recent financial and economic crisis and as well as government reaction to it are bound to fuel the thought of researchers for many years to come. While there is unlikely to be a single cause of the crisis, some commentators point out that the readiness of the Fed to avoid disorderly liquidation of LTCM in 1998, which could have led to large losses to its counterparties could have contributed to current crisis via creating a belief that the authorities would not allow a collapse of large players because of potentially large ripple effects it could cause. Usual argument in such story emphasizes moral hazard concerns: given that some losses would be adsorbed by the public one can increase risk exposure and pocket the proceeds until the bad shock arrives. I explore a different but related line of reasoning. It is often possible to institute internal risk controls that would signal the upcoming problems in advance, but these controls should be installed and properly operated long before possible period of turbulence is in sight. Incentives to install such risk controls clearly depend on what would happen to the economic agent if the worst comes to pass. If one knows that in the situation where there is a prospect of major losses to key players in the marketplace the authorities would perform an economy-wide intervention to help the troubled players, one's incentives to install risk controls weaken. In this paper I set-up a very stylized model of the above mechanism and evaluate its importance. The particular motive to install internal risk controls that I explore in this paper is the preference for the timing of the resolution of uncertainty that arises when one separates concepts of risk-aversion and preference for consumption smoothing. When an investor is sufficiently risk averse relative to his attitude towards consumptions smoothing he would prefer to know realization of uncertainty as soon as possible, while investors who prefer smoother consumption paths and are not too risk-averse would prefer to push the realization of uncertainty into the distant future. The former would have much stronger incentive to invest into internal risk control technology and institute internal risk controls. The benefit of such risk control is that recognizing in advance a bad future shock allows early liquidation of the unprofitable investment or at least a part of it and minimizing the loss. The channel through which government policy affects investment outcomes that I explore in this paper - is the cost of borrowing that investors do in order to finance their investments. The monetary authority usually has the power to affect the interest rates charged in the market via monetary policy. I am motivated by two strands of research. One of them is recent work in the field of Finance, by Bhamraa and Uppal (2006) [15] and Chacko and Viceira (2005) [24], who study portfolio problem of an investor with recursive preferences facing stochastic investment opportunity set and find the investor preference for the timing of the resolution of uncertainty has important effects on portfolio choice. The second one documents large heterogeneity in intertemporal elasticity of substitution (IES). Blundell et al. (1994) [18] and Attanasio and Browning (1995) [6] find that the IES is increasing in income. Attanasio et. al. (2002) [5] use U.K survey data and find that stockholders have values of IES close to unity, while non-stockholders have IES around 0.2. Vissing-Jørgensen (1998) [53] arrives at comparable estimates for the US using CEX data. While there is disagreement in the literature on the average values of IES, heterogeneity is undeniable.

In general my work fits into the growing literature that explores importance of heterogeneity in macroeconomics. There has been a lot of research on the issue of heterogeneity, with important contributions by Aiyagari(1994) [1], Krussel and Smith (1998) [44], who emphasized heterogeneity in private shocks or productivity. While early results were generally modest, more recently with the incorporation of incomplete market or limited participation assumptions results are more encouraging (see Heathcote at all (2009) [38] for a survey of recent literature). Key papers on the interaction of this kind of uncertainty and monetary policy are da Costa and Werning (2008) [29], who study optimal inflation tax in the economy with heterogeneity in productivity and find that Friedman rule is still optimal given that labor income taxes are non-decreasing; and Albanesi (2005) [2], who finds that the presence of heterogeneity has significant implications for time-consistency of optimal policies. Recently, perhaps in light of the recent crisis, heterogeneity in preferences started to attract more attention of the researchers. Diamond and Spinnewijn [30] study implications of heterogeneity in time preference for optimal taxation of capital income. Beaudry and Lahiri [13] use a simple model with heterogeneity in risk-aversion and limited production opportunities to explain key features of recent macroeconomic history. Guvenen [37] is perhaps the most related to my work. Using Epstein and Zin ([33]) preferences Guvenen studies implications of heterogeneity in intertemporal elasticity of substitution for asset pricing in a limited participation model and finds that the model is capable of generating key moments of the macroeconomic and asset pricing data. I am also motivated by heterogeneity in IES, but I focus on interaction between monetary policy and investor's preference for the timing of resolution of uncertainty. To the best of my knowledge I am the first to investigate the role government policy can have on internal risk control technology investment decisions of agents in a heterogenous preference setting.

The key features of my model are investors with Epstein-Zin preferences, heterogeneity in IES, and the risk control technology that one has to invest into well in advance, that provides information about future idiosyncratic productivity shocks. Conditional on such information investors could liquidate part of the unprofitable investment (at a cost of loss of a fraction of a current output) and convert it into safe investment. Investment projects are debt financed and the government policy can influence the cost of borrowing as well as the return of the safe investment. Government in the model commits to a rule that aims to cut interest rates if bad aggregate productivity shock arrives. Important feature of the government policy is that it moves the cost of borrowing and the return of the safe investment in the same direction. The former makes all investors better off while the latter decreases the value of information about future idiosyncratic shocks, making investing in risk control less attractive. How aggressive the government is expected to be in the case of adverse aggregate shock has significant implications for the risk control investment decision of investors. However quantitative impact of this decision on aggregate expected output dynamics turns out to be very modest. The rest of the paper proceeds as follows: Second section sets up the model, third section constructs a particular equilibrium that is aimed to illustrate the main point, fourth section presents parameter choices, results and discussion and the last section concludes.

8 Setup

There are consumers that save and consume, and investors that produce and consume. Consumers have normal time-separable preferences, while investors have recursive preferences advocated by Epstein and Zin [33], Kreps and Porteus [43] and Weil [54] and are heterogenous in attitude towards consumption smoothing. Time lasts 3 periods: 1 ,2 and 3. Production is risky in the period 3 from the individual investor perspective. In period 1 investors can choose whether to invest in risk control technology that will allow them to get an advance signal of their idiosyncratic productivity and possibly liquidate part of their ongoing project in period 2 if returns for period 3 are known to be poor in advance (in period 2). In period 2 part of the risky project can be converted into safe production opportunity. So the option to liquidate projects with bad future productivity serves as a boost to the present value of an investment project and for some investors that, all else equal, would prefer late resolution of uncertainty (i.e. no investment in risk control), it is enough to induce them to make risk control technology investment.

8.1 Physical Environment

In the model time lasts three periods.

8.1.1 Preferences

There are two classes of agents. First class - measure one of identical consumers/savers, who have usual time-separable CRRA preferences with risk-aversion γ and who only care about consumption in period 2 and 3. Second class - measure one of investors that have recursive preferences due to Epstein and Zin [33], and also only care about consumption in periods 2 and 3. Investors have common aversion to risk but are heterogeneous with respect to intertemporal elasticity of substitution. Individual investors' preferences are characterized by their IES $\psi_i \in \left[\underline{\psi}; \overline{\psi}\right]$, $i \in [0;1]$, with probability distribution over possible values of IES given by $F(\psi)$.

8.1.2 Endowments and production technologies

Consumer-savers have endowment of zero in period 1, A_{2c} in period 2 and zero in period 3, so they save part of endowment from period 2 using a risk-free bond. At the start of time investors are pre-committed to a unit-sized risky investment project that generates period 2 output A_{2I} , which is a random variable, that is the same for every investor. For this technology to generate output in period 3 investors must commit an additional unit of investment in period 2. If investment is made this project yields $A_{3I}\alpha_k$ units of consumption good in period 3. Where A_{3I} is an aggregate level of productivity in period 3 and α_k is an idiosyncratic multiplicative shock to individual productivity and $k \in [0;1]$. Its support is $\alpha_k \in [\underline{\alpha}; \overline{\alpha}]$ and probability distribution is $G(\alpha)$. Aggregate shocks A_{2c}, A_{2I} and A_{3I} have a joint probability distribution Φ. Individual productivity shock distribution is independent of investor type and of aggregate shocks. So the probability of getting a particular realization of individual productivity does not depend on investor's IES or on the overall level productivity. In period 2 investors can choose to liquidate a fraction of the risky investment project up to a maximum fraction $\bar{\delta}$, $0 \leq \bar{\delta} < 1$ and convert it into safe investment. Non-liquidated part requires proportional investment of $1 - \delta$ units of consumption good to keep producing in period 3. Liquidation is costly in the present period. If fraction δ is scheduled for liquidation and conversion into safe investment after period 2, it only generates a share $0 \leq \lambda < 1$ of output in period 2. So if fraction δ is liquidated then output in period 2 is given by $(1 - \delta) A_{2I} + \delta\lambda A_{2I}$. Return of safe investment in period 3 is given by A_{3S}.

8.1.3 Risk Control technology, running away, information

In period 3, which is a final period of the model, any investor can run away with a fixed amount of income $\hat{c}$ and avoid repaying debts in full if the rest of his income are not enough to cover for the debt.

In period 1 investors can install a risk control technology that will allow them to know the realization of their idiosyncratic period 3 productivity one period in advance. This investment is costless in this model.

Information about realizations of A_{2c}, A_{2I} and A_{3I} is public and becomes available at the beginning of period 2. Distributions $F(\psi)$, $G(\alpha)$ are known by all market participants. Individual preference for consumption smoothing, choice of risk control investment and period 3 idiosyncratic productivity are all private information.

8.2 Consumer and Investor problem

8.2.1 Consumer problem

Consumers have no meaningful decisions to make in period 1. They maximize their consumption in period 2 and 3, given their endowment in period 2 and information about aggregate productivity. They can save in a competitive market at an interest rate R.

$$\max_{B} (C_2)^{1-\gamma} + \beta (C_3)^{1-\gamma} \tag{83}$$

s.t.

$$C_2 + B \leq A_{2c} \tag{84}$$

$$C_3 \leq BR \tag{85}$$

The problem implies the usual optimality condition

$$C_2^{-\gamma} = \beta R C_3^{-\gamma} \tag{86}$$

8.2.2 Investor problem

There are two types of investor heterogeneity: preference heterogeneity that is given and heterogeneity induced by idiosyncratic productivity shocks. For convenience I use two dimensional index $j = (i, k)$ to describe a particular investor whether he will ever observe his complete type or not, where i indexes possible values of IES and k indexes possible values of the idiosyncratic productivity shocks. Hence $\alpha(j) = \alpha_k$ and $\psi(j) = \psi_i$.

Investors of type j make the following decisions: whether to invest in risk control $m(j) = \{0,1\}$, how much to consume $c_2(j)$ and $c_3(j)$, what share of investment to liquidate $\delta(j) \leq \bar{\delta}$ in the period 2, how much to borrow or save $b(j)$ in period 2.

The problem in period 1:

$$\max_{m(j),c_2(j),c_3(j),\delta(j),b(j)} U_1(j, m(j)) \tag{87}$$

s.t.

$$
\begin{aligned}
U_1(j, m(j)) &= E_1\left(U_2^{1-\gamma}(j, m(j))\right)^{\frac{1}{1-\gamma}}, \quad m(j) = \{0,1\} && (88)\\
U_2(j, m(j)) &= \left[(1-\beta)\left[c_2(j)\right]^{(1-1/\psi(j))} + \beta\left[E_2\left(U_3^{1-\gamma}(\cdot)\,|I_2(\cdot)\right)\right]^{\frac{1-1/\psi(j)}{1-\gamma}}\right]^{\frac{1}{1-1/\psi(j)}} && (89)\\
I_2(j, m(j)) &= \begin{cases} \text{if } m(j) = 0: A_{2c}, A_{2I}, A_{3I} \\ \text{if } m(j) = 1: A_{2c}, A_{2I}, A_{3I}, \alpha(j) \end{cases} && (90)\\
U_3(j, m(j)) &= \left[(1-\beta)\left[c_3(j)\right]^{(1-1/\psi(j))}\right]^{\frac{1}{1-1/\psi(j)}} && (91)\\
0 &< \delta(j) \leq \bar{\delta} && (92)\\
c_2(j) &\leq (1-\delta(j))\,A_{2I} + \delta(j)\,\lambda A_{2I} + b(j) - (1-\delta(j)) + \delta(j) && (93)\\
c_3(j) &\leq \max\left[(1-\delta(j))\,A_{3I}\alpha(j) - Rb(j) + \delta(j)\,A_{3S}, \hat{c}\right] && (94)\\
0 &\leq c_2(j), c_3(j) && (95)\\
\bar{b} &\leq b(j) && (96)
\end{aligned}
$$

First line (88) just specifies future utility, so in period one maximization is just choosing a maximum out of two numbers: $E_1\left(U_2^{1-\gamma}(j,0)\right)$ and $E_1\left(U_2^{1-\gamma}(j,1)\right)$. Second and third lines (89) and (90) specify problem facing an agent who already made a risk control investment decision, that is reflected in the information set for conditional expectation operator. If agents chose to invest in risk control $m(j) = 1$ then they know their full type $j = \{\psi_j, \alpha_j\}$, and, hence know their productivity in period 3. If agents chose not to invest $m(j) = 0$, then they only know their IES and aggregate productivity shock realizations. Fourth line (91) states that period 3 utility, which is a terminal period, is just given by consumption in period 3, scaled by time discount factor to stay consistent with period 2 formulation of utility. Conditions (92) and (93) define the budget constraint of investors. Second and third period output depends on the extend of project liquidation. Condition (94) reflects the option to run away with fixed part of resources, that investors have in period 3. The last condition, that constrains borrowing is going to be crucial for the model's results. To keep things simple and to abstract from possible risk-sharing arrangements I will make a very restrictive assumption that borrowing limits are determined in such a way, so that in the worst possible outcome lenders get what they are owed with certainty. While this assumption is not very unreasonable to make in an environment where type, risk control investment decision and idiosyncratic shock are all private information and agents can take off with the share of output, it leaves a lot of possibilities for relaxing informational assumptions and studying how efficient risk-sharing arrangement would affect the results. I leave that to future research.

Assumption

$\bar{b}$ is set so that in equilibrium all debts are repaid for sure

8.2.3 Market clearing

Market clearing conditions make sure that total consumption and investment in periods 2 and 3 is equal to total output.

$$C_2+\int c_2(j)\,h(j)\,dj+\int(1-\delta(j))\,h(j)\,dj=A_{2c}+\int\left[(1-\delta(j))\,A_{2I}+\delta(j)\,\lambda A_{2I}\right]h(j)\,dj \tag{97}$$

$$C_3+\int c_3(j)\,h(j)\,dj=\int\left[A_{3I}\alpha(j)\,(1-\delta(j))+A_{3S}\delta(j)\right]h(j)\,dj \tag{98}$$

Where A_{3S} is a return of safe production technology.

8.2.4 Competitive equilibrium definition

A competitive equilibrium in this model consists of aggregate shocks $\{A_{2c}, A_{2I}, A_{3I}\}$, an interest rate R, allocations of consumers $\{C_2, C_3, B\}$, allocations of investors $\{m(j), \delta(j), b(j), c_2(j), c_3(j)\}_{j\in[\underline{\psi};\overline{\psi}]\times[\underline{\alpha};\overline{\alpha}]}$ and such that:

1. Given interest rate R representative consumer allocations $\{A_{2c}, A_{2I}, A_{3I}\}$ solve maximization problem in (83)

2. Given interest rate R, allocations $m(j), \delta(j), b(j), c_2(j), c_3(j)$ solve investor problem for every $j\in[\underline{\psi};\overline{\psi}]\times[\underline{\alpha};\overline{\alpha}]$

3. Markets clear

In the definition above I suppressed explicit dependence of all the quantities save for risk control investment decision on the realization of aggregate shocks.

8.3 Adding government

Now assume there is a government that is concerned with output drops caused by project liquidation in period 2 and tries to interfere via influencing short-term interest rate. So at the beginning of time government commits to an interest rate tax/subsidy rule $\tau(A_{2I})$ as a function of period 2 public shock realization. Tax revenues (subsidy spending) are distributed to (taken from) consumers in lump-sum form.

$$T = \tau(A_{2I}) BR \tag{99}$$

After the government has committed to the above policy agents in the economy behave as above with the exception of consumers, whose intertemporal budget constraint is modified to account for interest rate tax or subsidy.

$$C_2 + B \leq A_{2c} \tag{100}$$

$$C_3 \leq (1 + \tau(A_{2I})) BR + T \tag{101}$$

I will study the impact government tax/subsidy policy has on a particular equilibrium that I construct below. Intuitively lowering interest rates makes those who borrowed better off, hence increasing the value of non-liquidated project. At the same time lower interest rates make liquidation option less attractive because it affects the return on safe investment. In an economy without investor heterogeneity this generally leads to less liquidations and higher output in period 2 but lower output in period 3 due to lower fraction of projects with poor realizations of individual productivity being liquidated. In the model with investor heterogeneity this government action makes the value of investing in risk control less appealing, resulting in larger share of investors choosing not to invest in it, affecting the response of output to government policy both in period 2 and 3.

8.4 A particular equilibrium

To illustrate my main point I construct an equilibrium, making explicit assumptions about distributions of shocks. I assume that consumer wealth A_{2c} is constant. Aggregate productivity in periods 2 and 3 is the same and takes 2 values: $\{A_H > A_L\}$ with equal probability $\pi_L = \pi_H = 1/2$. Investor elasticity of intertemporal substitution is distributed uniformly on $[\underline{\psi}; \overline{\psi}]$, hence the probability density function is constant $\frac{1}{\overline{\psi}-\underline{\psi}}$. I assume that

$$\frac{1}{\gamma} \in [\underline{\psi}; \overline{\psi}] \tag{102}$$

Idiosyncratic multiplicative productivity shock also has a uniform distribution on $[\underline{\alpha}; \overline{\alpha}]$ with a pdf given by $\frac{1}{\overline{\alpha}-\underline{\alpha}}$. I assume that safe technology yields the same return as risk-free bond.

$$A_{3S} = R \tag{103}$$

To make the problems interesting I assume that safe investment technology yields more than the worst possible return on risky investment:

Assumption

$$A_L \underline{\alpha} < 1 < R \tag{104}$$

Government policy is described by a single parameter τ, so that the government policy takes the following form:

$$\tau\left(A_{2I}\right)=\begin{cases}\text{if } A_{2I}=A_L:\tau\\ \\ \text{if } A_{2I}=A_H:0\end{cases} \tag{105}$$

Now I describe the equilibrium I am constructing starting from period 3 decision and working my way backward. In period 3 investors have one decision - to run away or not. Outcome depends on borrowing constraints imposed. I assume that borrowing constraints are set so that any lender supplying money to any investor always gets his savings back with interest. Additionally, I construct an equilibrium where borrowing constraints bind for all the investors and do not depend on investor type.

Proposition 1 *If $\hat{c} < A_L\underline{\alpha}$ Equilibrium borrowing constraints are determined by the lowest possible risky investment output $A_{3I}\underline{\alpha}$ and amount investors can run away with:*

$$\bar{b}\left(A_{3I}\right)=\frac{A_{3I}\underline{\alpha}-\hat{c}}{R} \tag{106}$$

Proof: in Appendix B

To make sure that these constraints bind for all investors I make the following joint assumption on parameters:

Assumption

$$\left[\frac{\hat{c}}{A_H-1+A_H\underline{\alpha}-\hat{c}}\right]^{1/\overline{\psi}} \geq \left[\left(\frac{A_H\underline{\alpha}-\hat{c}}{A_{2c}-A_H\underline{\alpha}+\hat{c}}\right)^{\gamma}\right] \tag{107}$$

The following proposition shows why this assumption is sufficient for borrowing constraints to bind.

Proposition 2 *If condition (107) holds, borrowing constraints bind for all investors.*

Proof: In Appendix B

Agents who did not choose to invest in risk control face a blind decision to liquidate part of risky project. In the equilibrium I construct those that do not invest in risk control do not find it optimal to liquidate risky project without advance information. The following restrictions on parameters ensures that:

Proposition 3 *There exists γ^* such that for every $\gamma < \gamma^*$ there exists $0 < \lambda^* < 1$ so that for all $\lambda < \lambda^*$ investors that choose not to invest in risk control do not blindly liquidate their risky investments in period 2, so $\delta(j) = 0$ for $m(j) = 0$*

Proof: In Appendix B

Intuitively, the benefit of liquidation is the spread between R and risk-adjusted expected return of risky project, if risk-aversion is too high there will always be blind liquidation. Also if λ is to close to unit value, meaning not too much output is lost in liquidation, then effective return on such saving could be quite high, and investors could choose to effectively save via liquidating risky projects blindly. In parametrizations I use $\lambda^* > 0.9$.

Given that those who do not invest in risk control do not liquidate it is straightforward to calculate the value of doing so:

$$U_1(j,0) = E_1\left(U_2^{1-\gamma}(j,0)\right)^{\frac{1}{1-\gamma}} \tag{108}$$

$$U_2(j,0) = \left[\begin{array}{c} (1-\beta)\left[A_{2I} + \frac{A_{3I}\underline{\alpha}-\hat{c}}{R} - 1\right]^{(1-1/\psi(j))} + \\ +\beta\left[\frac{[A_{3I}(\overline{\alpha}-\underline{\alpha})+\hat{c}]^{2-\gamma}}{A_{3I}(2-\gamma)(\overline{\alpha}-\underline{\alpha})} - \frac{[\hat{c}]^{2-\gamma}}{A_{3I}(2-\gamma)(\overline{\alpha}-\underline{\alpha})}\right]^{\frac{1-1/\psi(j)}{1-\gamma}} \end{array}\right]^{\frac{1}{1-1/\psi(j)}} \tag{109}$$

I now turn to the value of investing in risk control. First I look at the period 2 liquidation decision of investors that chose to invest in risk control. For realized idiosyncratic productivity second period utility as a function of R and $\delta(i)$ is given by:

$$\left[\begin{array}{c} \left[A_{2I}-\delta(j)\,A_{2I}(1-\lambda)+\frac{A_{3I}\underline{\alpha}-\hat{c}}{R}\right]^{(1-1/\psi(j))}+ \\ \beta\left[A_{3I}\left(\alpha(j)-\underline{\alpha}\right)+\hat{c}+\delta(j)\left(R-A_{3I}\alpha(j)\right)\right]^{1-1/\psi(j)} \end{array}\right]^{\frac{1}{1-1/\psi(j)}} \tag{110}$$

To focus on the equilibrium of interest I select parameter λ so that investors with low realizations of $\alpha(j)$ find it optimal to liquidate and investors with high realizations - do not. The following proposition says that its possible to choose parameters so that the liquidation problem is non-trivial.

Proposition 4 *If condition (107) holds there exist* $\underline{\lambda}$ *and* $\overline{\lambda}$*:* $\underline{\lambda}<\overline{\lambda}$ *so that for every value of* $\lambda\in\left(\underline{\lambda};\overline{\lambda}\right)$ *there exist* j_1 *and* j_2*,* $j_1\neq j_2$*, so that* $\delta(j_1)=0$ *and* $\delta(j_2)=\bar{\delta}$*.*

Proof: In Appendix B

It is possible to construct an equilibrium with binding borrowing constraints, investors with no risk control technology not liquidating blindly and some investors that chose positive risk control investment liquidating some investments if $\underline{\lambda}<\lambda^{*}$, the condition that I verify numerically.

The following is needed to be able to do comparative statics:

Proposition 5 *The utility functions of investing in risk control and not investing are continuously differentiable functions of interest rate* R *for* $R\in[1;A_{3I}\overline{\alpha}]$

Proof: in Appendix B

8.4.1 Comparative statics

I will be interested in studying the change in utilities of investors and in measures of output in period 2 and 3, as functions of government subsidy policy. Output in period 2 is given by

$$Y_2(\tau) = A_{2I} - \int A_{2I}(1-\lambda)\,\delta(j)\,h(j)\,dj \tag{111}$$

Output in period 3 is given by:

$$Y_3(\tau) = (1-\delta(j))\,A_{3I}\alpha(j) + \delta(j)\,R \tag{112}$$

I now turn to parameter choices and study model properties numerically.

9 Parameters, Results and Discussion

9.1 Parameter choices

I study two parametrizations, in the first one I set risk aversion of investors above one and keep IES values below one; in the second I choose low risk aversion and high IES. First I describe parameters that are common for both parameterizations. The rate of time preference is set at 0.95. Endowment of consumers is set at $A_{2c} = 0.55$. Aggregate productivity in period 2 and 3 takes two values $A_L = 1.1$, $A_H = 1.15$ with equal probability. Idiosyncratic productivity is uniformly distributed in $[\underline{\alpha}; \overline{\alpha}] = [0.65; 1.35]$. The amount investor can take-off with is set at 0.43. I evaluate impact of government interest rate policy in the interval $\tau \in [0, \hat{\tau})$, with $\hat{\tau}$ such that $R(\hat{\tau}) = 1$. In the high risk-aversion parametrization I set risk-aversion to $\gamma = 2.01$, and in low risk-aversion to $\gamma = 0.5$. In the case of heterogenous investors interval of values for IES is set so that $1/\gamma$ is in the interval. So in high risk-aversion parametrization it is set to $[\underline{\psi}; \overline{\psi}] = [0.02; 0.98]$. and in low risk-aversion to $[\underline{\psi}; \overline{\psi}] = [1.2; 2.8]$. When I evaluate policy response, I compare output impact of government policy in the model with heterogenous investors to the model where preferences are homogenous. In that model I set IES at $\psi = 1/\gamma$. The values for a share of risky project that could be liquidated and converted into safe investment is set at 30% (at 50%) , and an output loss in period 2 is set at 15% (10%) of liquidated output in the low (high) risk-aversion parametrization. The difference in these last two parameters is necessary, since in high risk-aversion calibration IES is low and demand for consumption smoothing is very high. Given that project liquidations increase the desire to smooth consumption, it requires a larger increase in lifetime wealth to overcome the desire to smooth consumption and induce investors to liquidate investments. Table 1 17 summarizes values of the high and low risk-aversion calibrations of the model.

9.2 Results

Figures 7 through Figure 11 refer to high risk-aversion low IES case, figures 12 through 16 refer to low risk-aversion high IES calibration. Since both sets of pictures are qualitatively similar I discuss them simultaneously, pointing out the differences and the reasons behind those. Figures 7 and 12 show the ratio of values of not investing in risk control to the value of investing. As expected, for low values of IES not investing in risk control is better, the shape of the ratio however depends in the case considered. While for low risk-aversion the ratio is downward sloping in IES, it has a more complicated shape for high risk-aversion parametrization. The latter is driven by the preference for consumption smoothing that gets very strong for low values of IES and this effect overshadows the small difference in utility due to timing of resolution of uncertainty. For both calibrations the cut-off below which investors choose to forgo risk control lies to the left of the point where $\psi = 1/\gamma$. The option to convert risky project into safe one increases present value of income for some realizations of individual productivity, hence increases the value of risk control technology, shifting the point where the utility of investing into it is equal the utility of not investing to the left of $1/\gamma$. Overall the size of the difference in expected utilities is very modest, suggesting that fairly small costs can have big impact on risk control investment decision. For low values of IES government interest rate policy has very small effect on the share of investors that choose risk control, the effect gets bigger for low risk-aversion and high IES parametrization. Increase in τ means that the government is subsidizing savings, and lowering equilibrium interest rate faced by investors. This has two effects. First, it makes all risky investment returns higher, since the cost of borrowing that is needed to cover investment goes down, second it decreases the value of liquidation, since the return on safe project drops. This second effect affects only the value of investing in risk control. So, overall, higher subsidy for savings makes not investing in risk control better relative to the option of investing, and some investors with borderline values of IES switch. But high value of IES is essential for this effect to be sizable since liquidation option increases present value of income only making it even more uneven across periods, and with low IES values forgoing consumption in period 2 in exchange for increased consumption in period 3 is very costly in terms of utility.

Figures 8, 9 and 13, 14 show impact investor preference heterogeneity has on aggregate output in periods 2 and 3, as a function of government policy. Solid line gives impact in the economy with homogeneous preference as a reference and dashed line gives response in the economy with heterogenous preferences. The values on the vertical axis reflect the change of output in respective period relative to the case of no government policy $\tau = 0$. The parameters of the model are chosen so that the government policy stimulates the output in period 2 via lowering liquidations and that this benefit[19] comes at a cost of lower output in period 3. This lowered output comes about due to fewer projects with low productivity being liquidated, resulting in lower average output in period 3. This is the logic of economy with homogeneous preferences. The difference in responses due to heterogeneity is very modest at about 10-15% of the response in the homogeneous investor economy at low values of τ, and growing smaller with higher levels of savings subsidy. The reason for this difference is that while in homogeneous investor economy lower interest rates simply lead to fewer liquidations, in heterogenous investor economy decrease in interest rates does decrease the volume of liquidations, but at the same time increases the proportion of investors who choose to forgo investing in risk control. So additional investors without information about future productivity lower output somewhat more due to the expected output of a risky project with worst realizations being substituted for safe return is being replaced with the full expected output of a risky project, which is calibrated to be lower at market interest rates that prevail with no government interference. However, with increasing subsidy and decreasing interest rates, at some point the expected values of risky project and of risky project with liquidations become equal and for further subsidy the difference between heterogenous and homogenous economy starts decreasing. In period 2 the difference in responses is accounted for by the increased share of investors with no risk control investment, that do not liquidate in period 2. Absolute size of effects again depends on the size of IES. With low values of IES government policy change does not induce enough of impact on the volume of liquidations.

Figures 10, 11 and 15, 16 show the effect government's commitment to a particular

[19]from the point of view of the government. I do not evaluate welfare here.

policy has on the measure of investors that invest in risk control and liquidate, invest in risk control and do not liquidate and do not invest in risk control at all for heterogenous economy and for homogenous investor economy as a reference. As stated above, high-risk aversion parametrization exhibits much lower sensitivity to government policy than that of low risk-aversion/high IES calibration.

9.3 Discussion

Results presented above are quite modest in size at best but show some interesting effects government policy can have on the risk control investment behavior of investors. First thing I want to emphasize is that the difference in level of utilities of installing risk control and not doing so shows that quite small costs are required to push investors with strong preference for early resolution of uncertainty into no risk control mode. Second important result of the paper - interest rate policy of the government can have very sizable effects on the composition of those who choose to invest in risk control, liquidate or do neither of those. It is worth noting that all the results here are obtained for the case of zero risk control investment cost and uniformly distributed preference parameters and idiosyncratic shocks. While quantitatively the changes in responses of expected output to government policy are fairly modest, by affecting the risk control investment decision of investors the government actions can have sizable impact on output and consumption variance. It would be interesting to model response of aggregate uncertainty to risk control decisions of investors. Another possibility for future research is to relax very restrictive financing and informational assumptions to allow for some risk-sharing arrangements for those who chose not to invest in risk control and more consumption-smoothing options for those investors who did.

10 Conclusions

In this paper I have presented a very stylized model of interaction between government policy that affects interest rates and private decisions to install project risk control technology. Such technology could reveal information about future profitability in advance and could be used to terminate the most unprofitable projects in advance. While it is natural to expect this kind of technology to be of value to investors, once one allows for preference heterogeneity and for separation of risk-aversion and intertemporal elasticity of substitution, that creates preference for the timing of resolution of uncertainty, such technology is no longer valuable to all investors. Some prefer not to know their future productivity in advance, and would like to delay resolution of uncertainty. The composition of those who choose not to invest into this risk control technology can be significantly affected by government's commitment to a policy that aims to move interest rates to satisfy short-term output stabilization goals. Such government actions affect value of investing into risk control and value of not investing differently because of impact on the value of liquidations. Presence of investor heterogeneity can have modest influence on the output response to government policy actions. Current paper studied the environment where distributional considerations only affected the means of output, future research is needed to evaluate impact of risk control investment decisions on output variance.

11 Tables and Figures

Table 17: Parameters

	description:	high risk-aversion	low risk-aversion
β	rate of time preference	0.95	0.95
γ	risk aversion parameter	2.01	.5
$\underline{\psi}$	lower bound of IES	0.02	1.2
$\overline{\psi}$	upper bound of IES	0.98	2.8
$\underline{\alpha}$	lowest private productivity shock	.65	.65
$\overline{\alpha}$	highest private productivity shock	1.35	1.35
A_{2c}	Period 2 endowment of consumers	.63	.55
$\hat{c}$	Period 3 consumption investors can steal	.43	.43
A_L	Low aggregate productivity	1.1	1.1
A_H	High aggregate productivity	1.15	1.15
$1-\lambda$	Period 2 % Output loss of liquidated project	.1	.15
$\hat{\delta}$	Share of investment liquidated	.5	.3

Figure 7: Ratio of value of investing in risk control to value of not investing, $\gamma = 2$

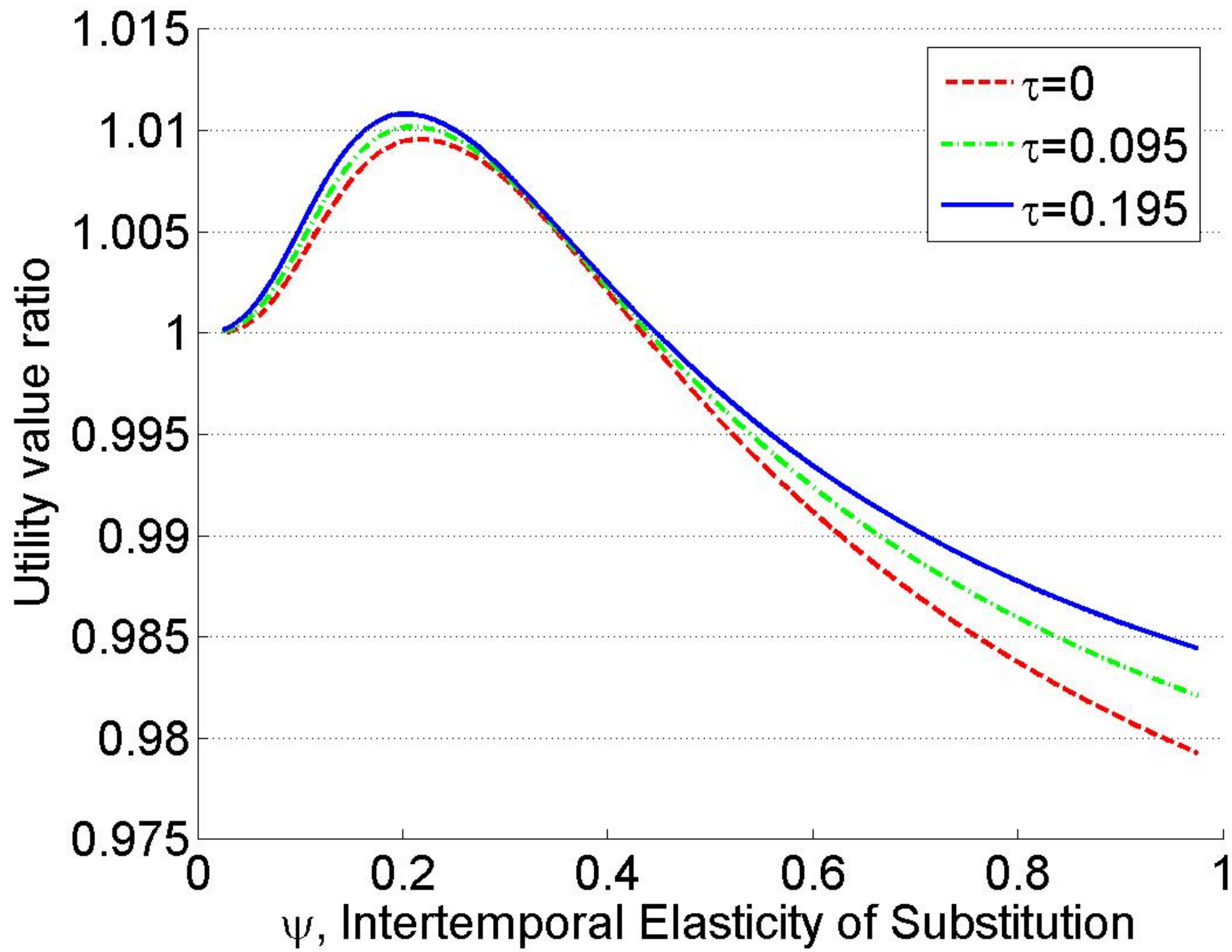


Figure 8: Period 2 Output Response to Subsidy, $\gamma = 2$

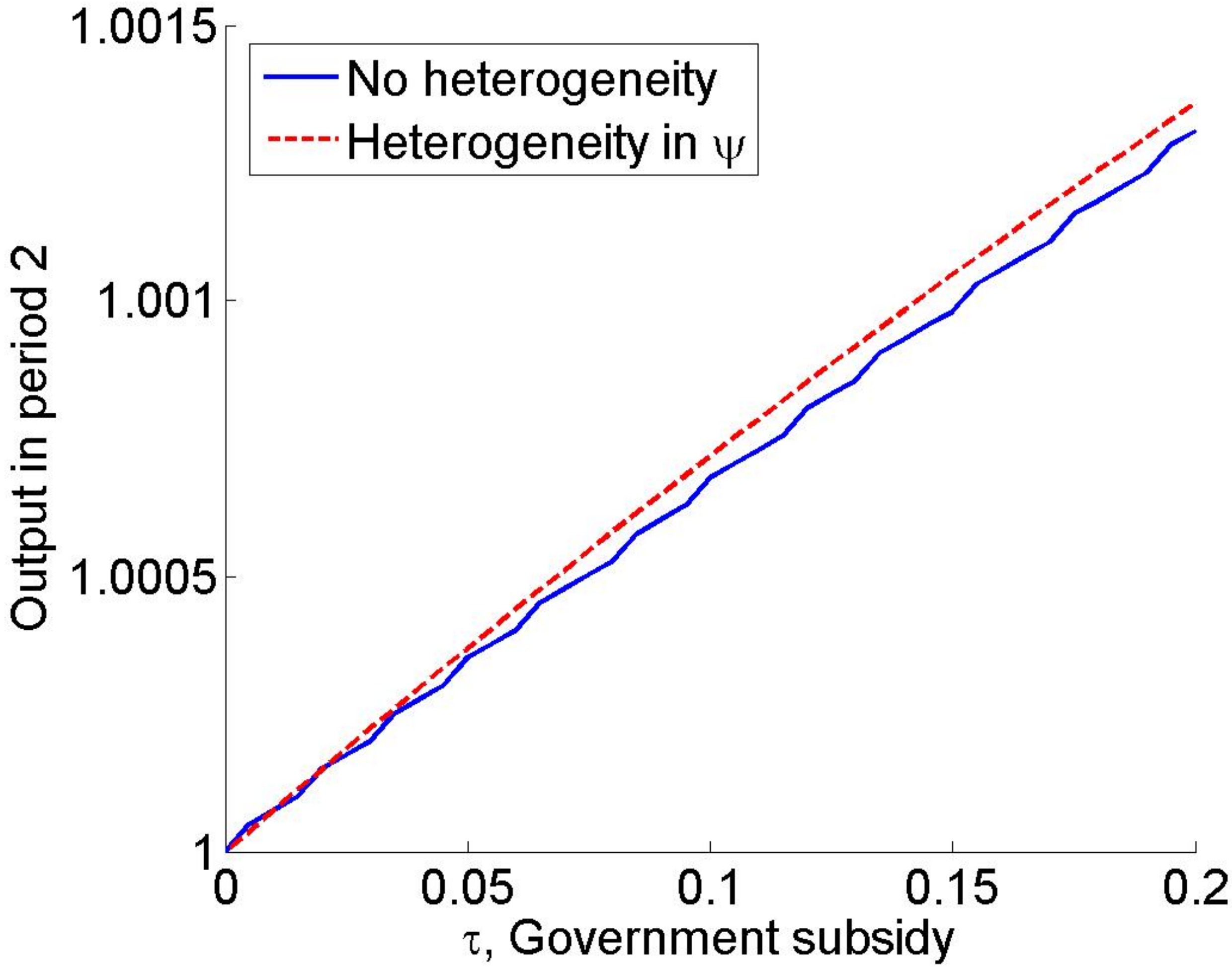


Figure 9: Period 3 Output Response to Subsidy, $\gamma = 2$

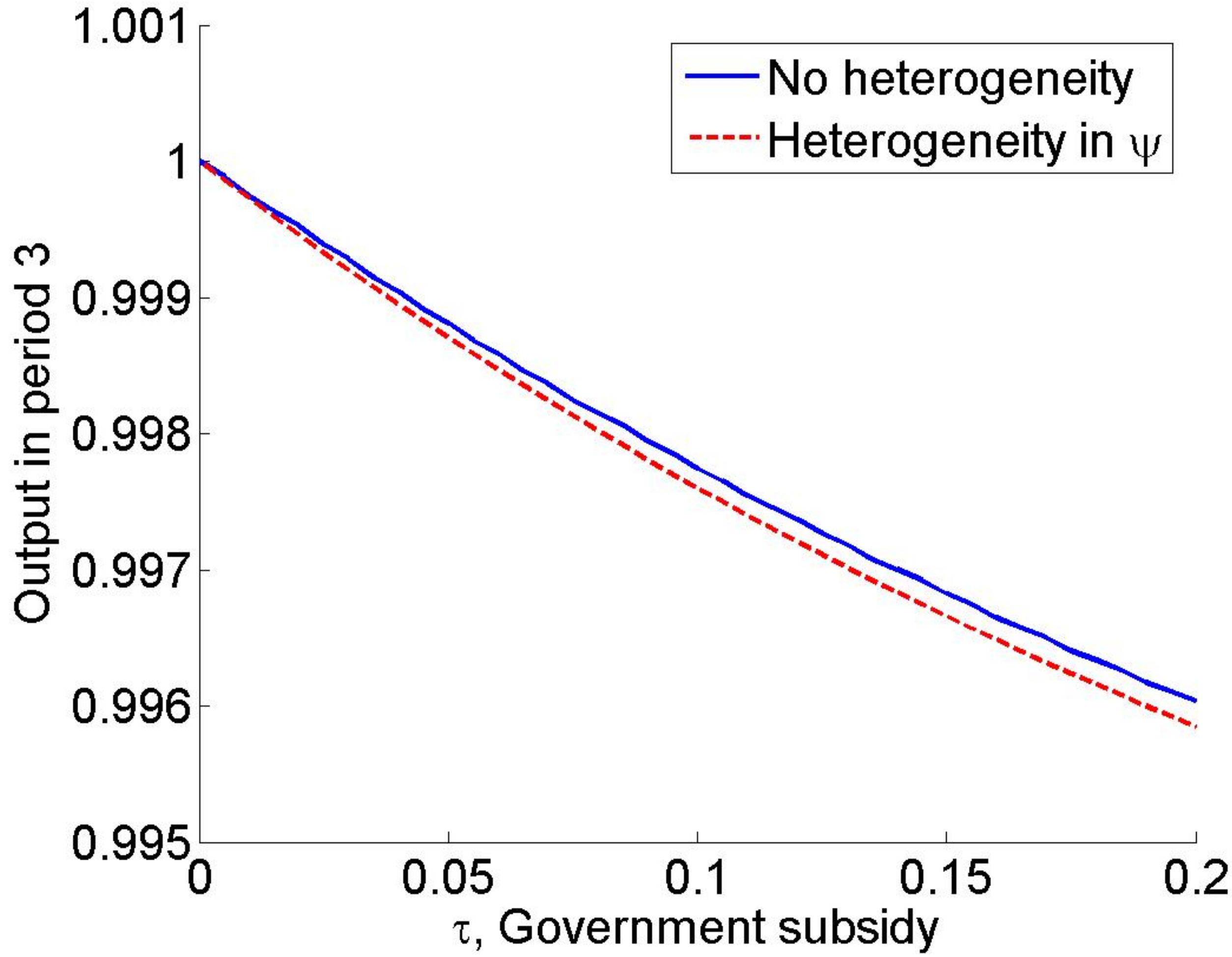


Figure 10: Fractions of investors, heterogeneous model, $\gamma = 2$

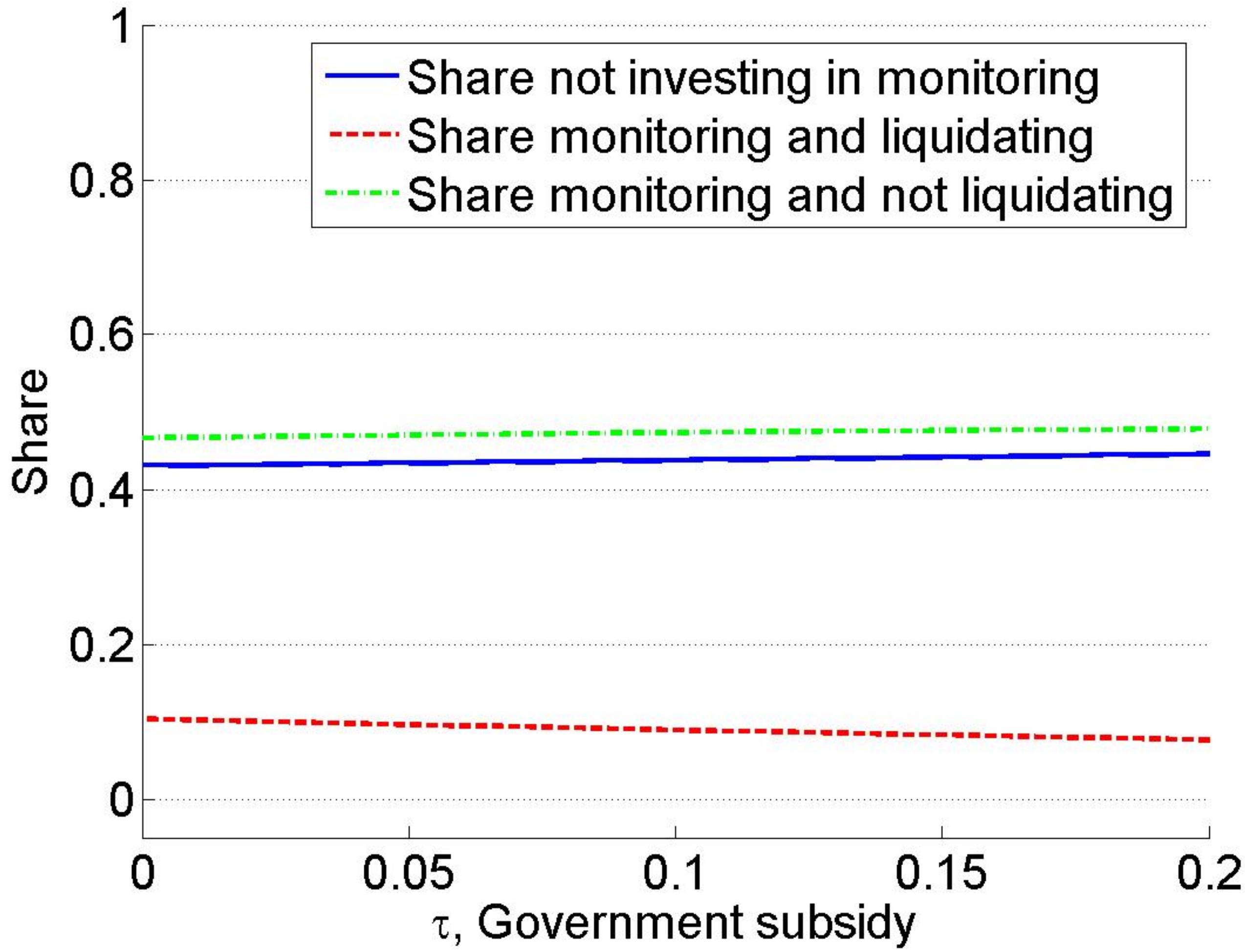


Figure 11: Fractions of investors, homogeneous model, $\gamma = 2$

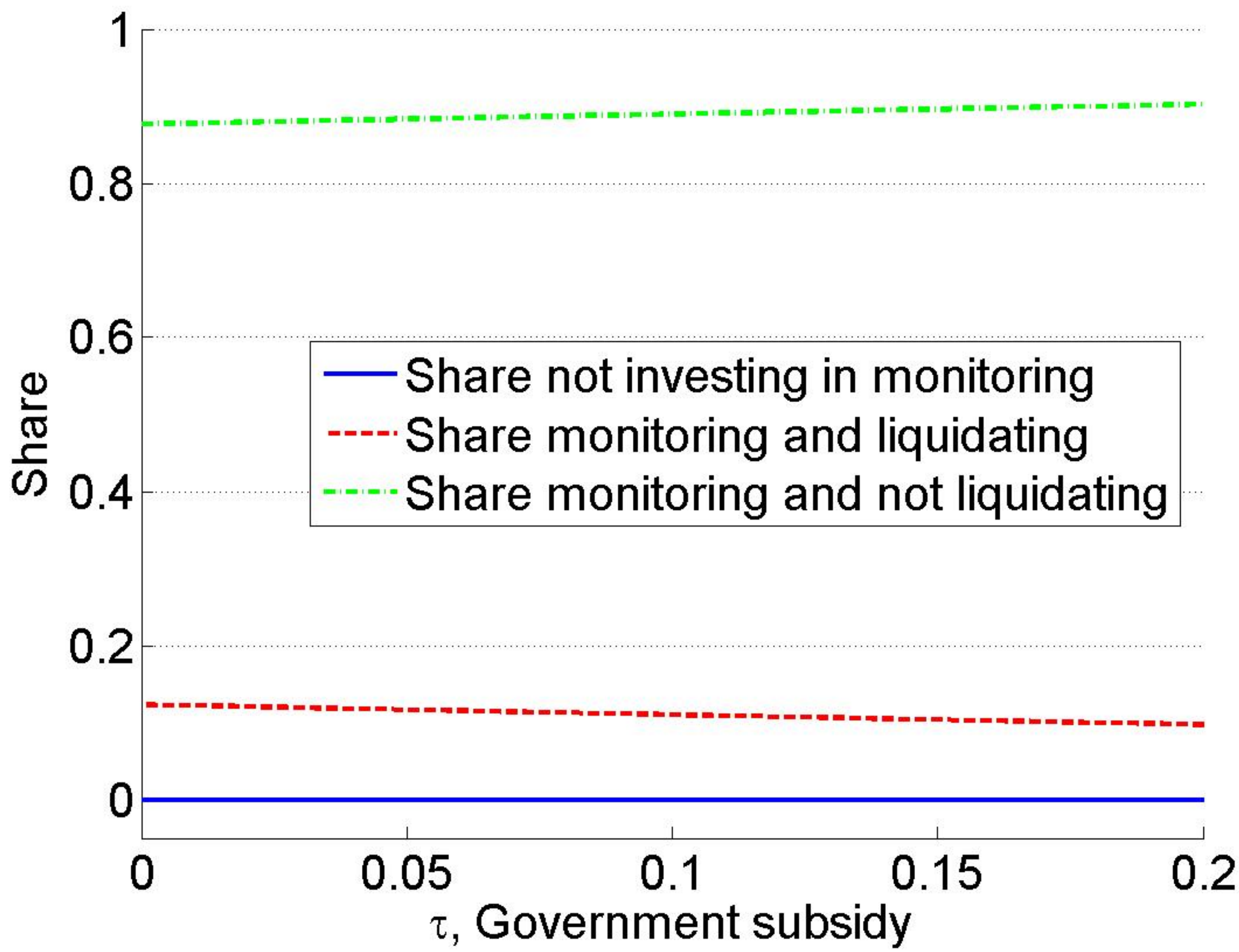


Figure 12: Ratio of value of no risk control to value of having risk control, $\gamma = .5$

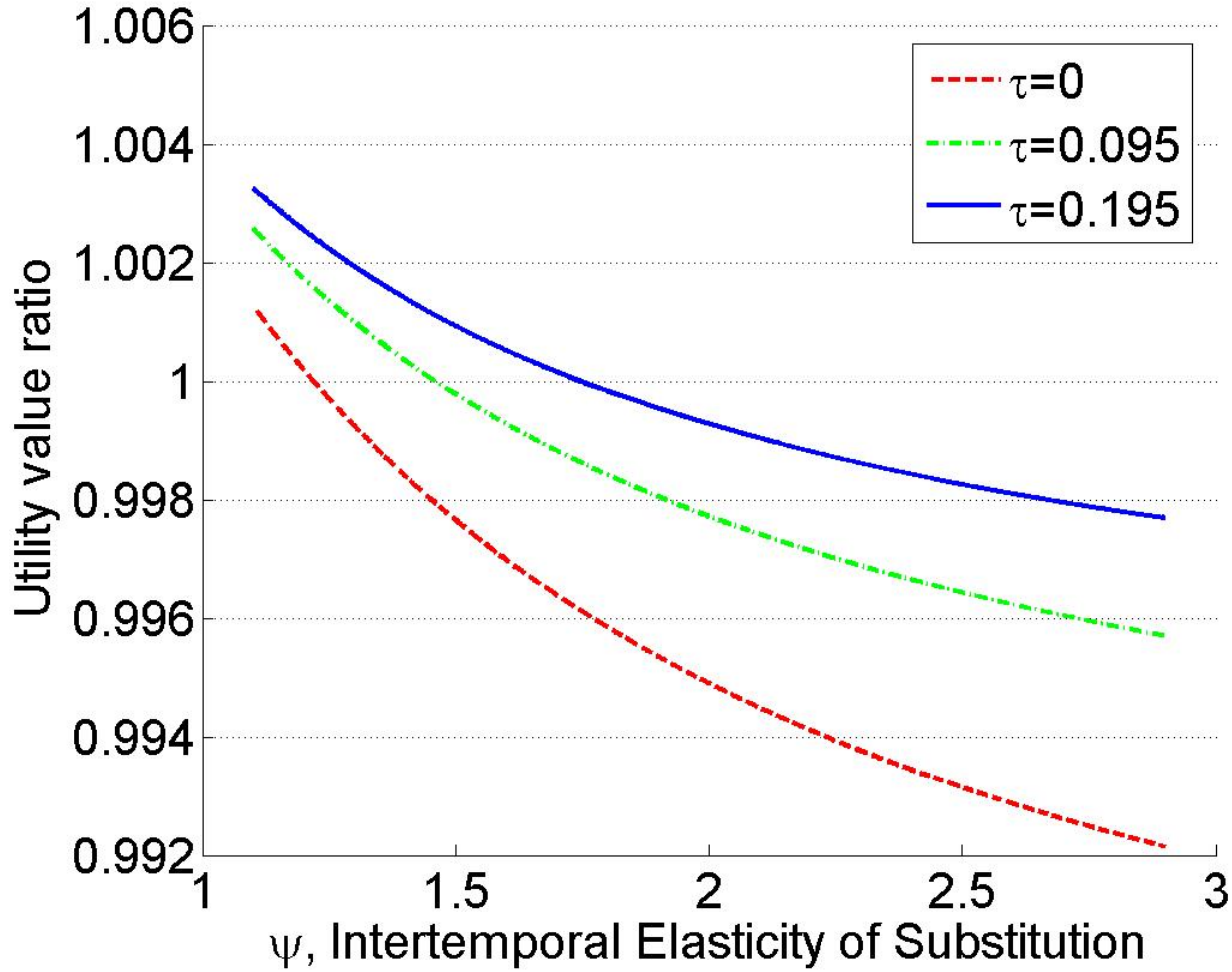


Figure 13: Period 2 Output Response to Subsidy, $\gamma = .5$

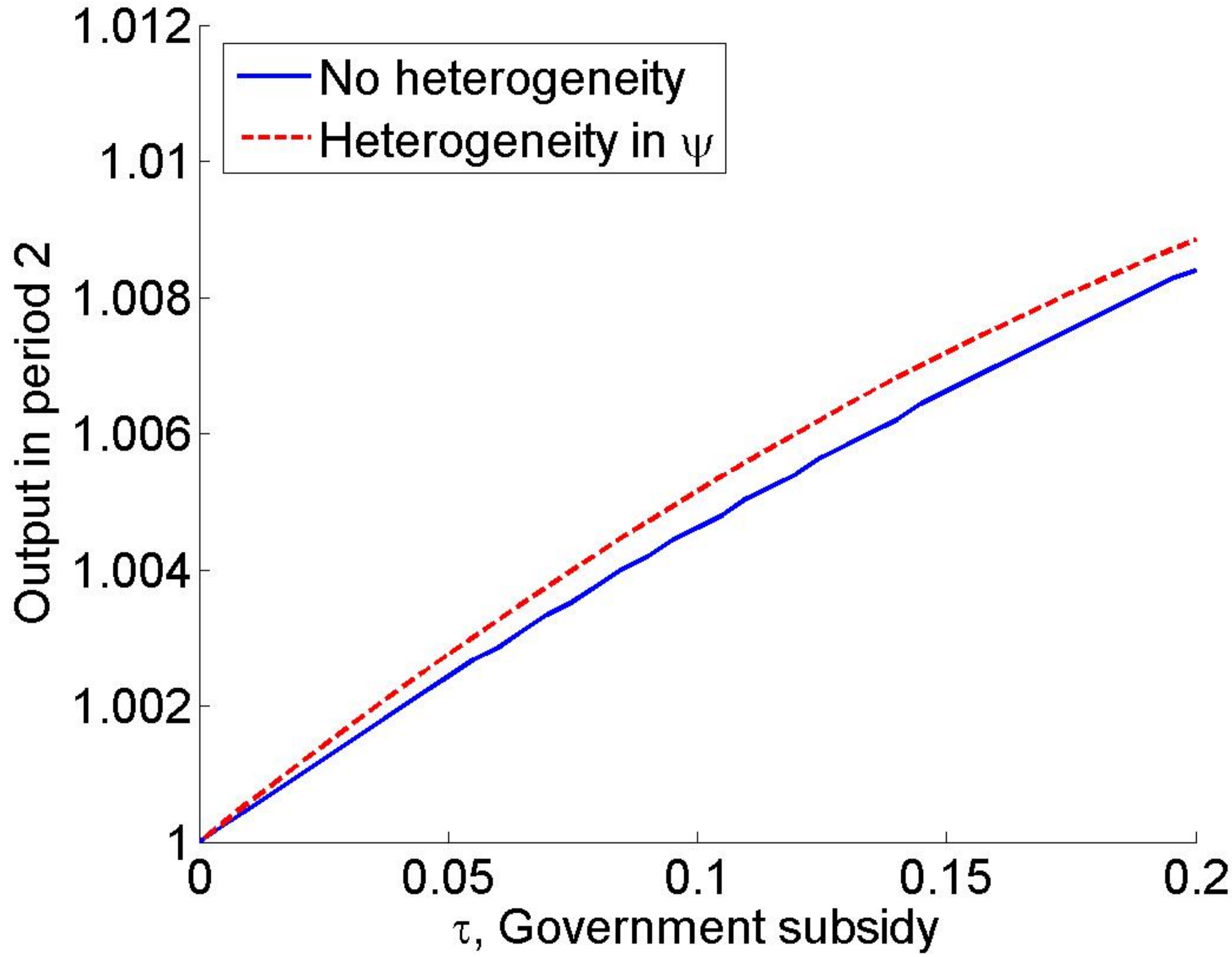


Figure 14: Period 3 Output Response to Subsidy, $\gamma = .5$

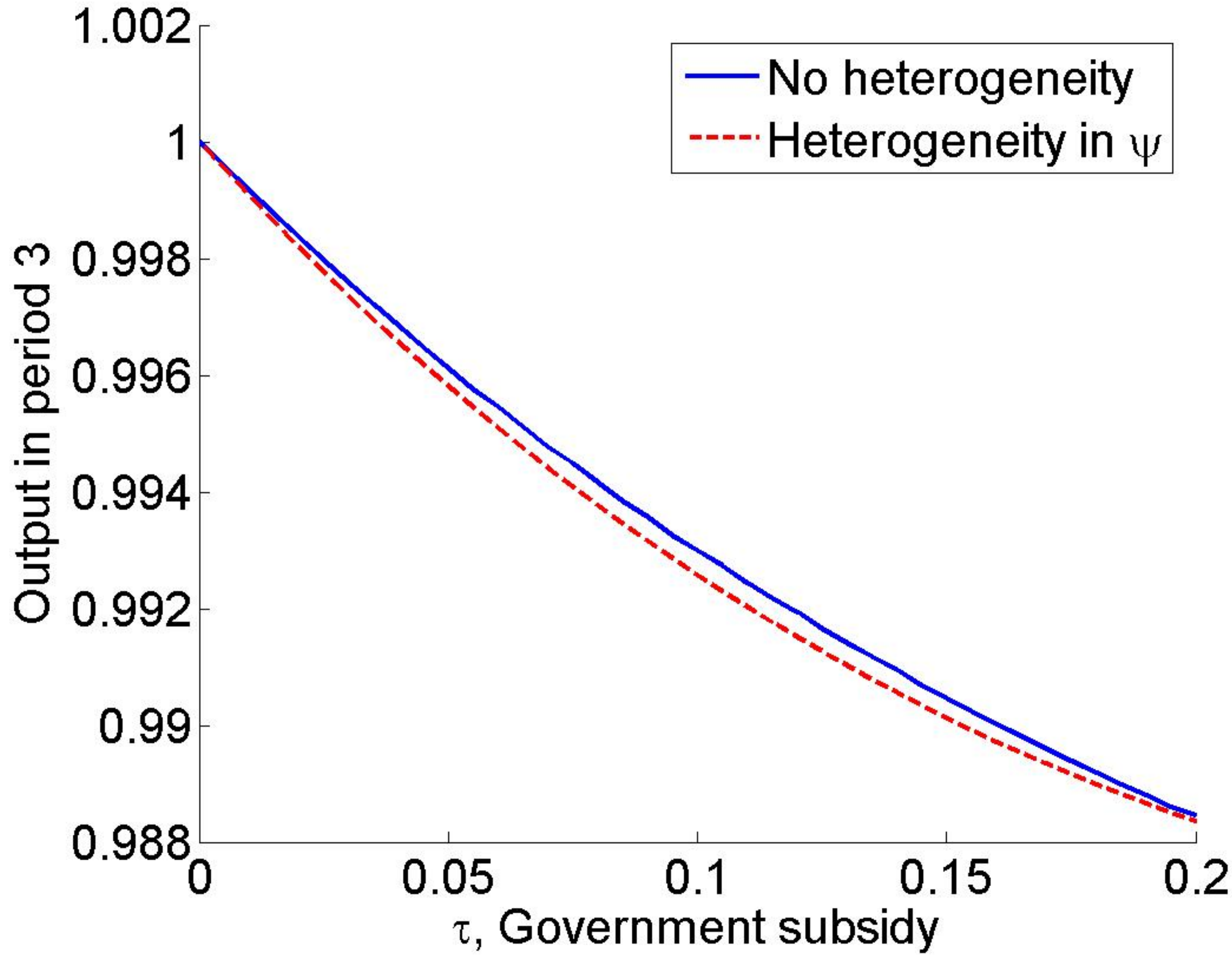


Figure 15: Fractions of investors, heterogeneous model, $\gamma = .5$

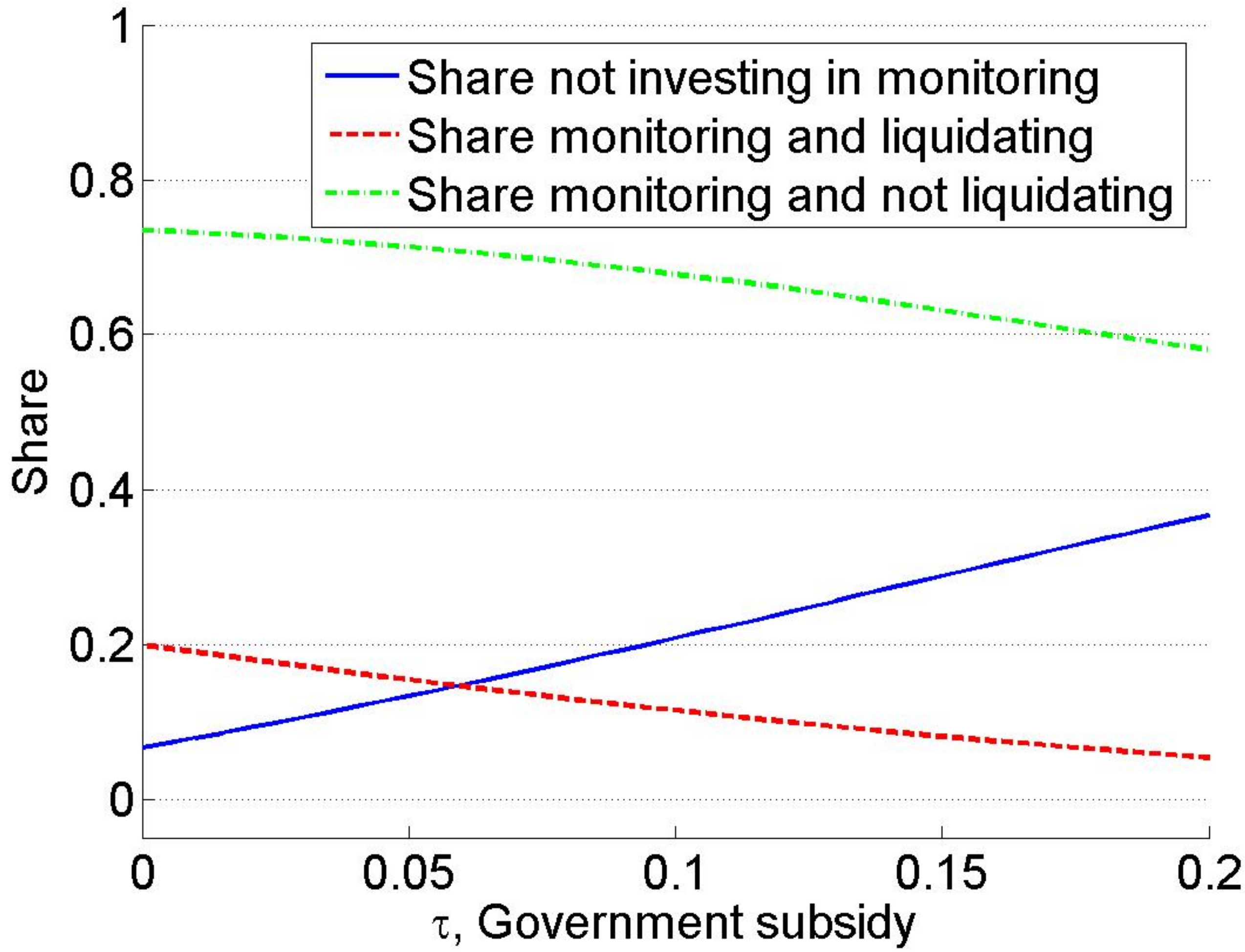
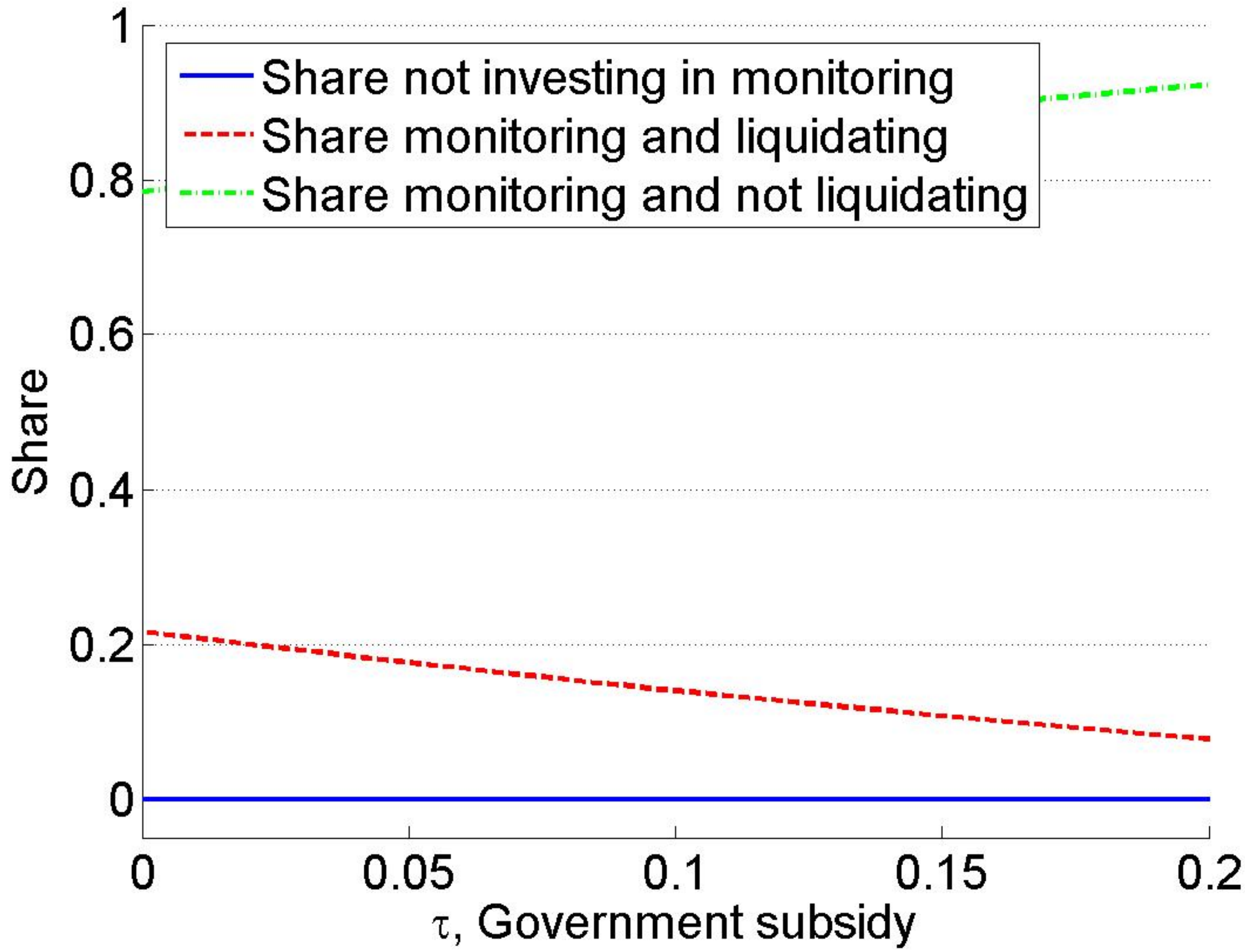


Figure 16: Fractions of investors, homogeneous model, $\gamma = .5$



References

[1] S. R. Aiyagari, *Uninsured idiosyncratic risk and aggregate saving*, Quarterly Journal of Economics **109** (1994), 659–684.

[2] Stefania Albanesi, *Optimal and time consistent monetary and fiscal policy with heterogeneous agents*, Unpublished Manuscript (2005).

[3] Andrew Ang, Geert Bekaert, and Min Wei, *The term structure of real rates and expected inflation*, Bank of Finland Research Discussion Papers **63-2** (2008), 797–849.

[4] Andrew Atkeson and Patrick J. Kehoe, *On the need for a new approach to analyzing monetary policy*, FRB Minneapolis Working Paper 662 (2008).

[5] Orazio P. Attanasio, James Banks, and Sarah Tanner, *Asset holding and consumption volatility*, Journal of Political Economy **110** (2002), no. 4, 771–792.

[6] Orazio P Attanasio and Martin Browning, *Consumption over the life cycle and over the business cycle*, American Economic Review **85** (1995), no. 5, 1118–37.

[7] David K. Backus, Silverio Foresi, and Chris I. Telmer, *Discrete time models of bond pricing*, NBER Working paper 6736 (1998).

[8] ______, *Affine term structure models and the forward premium anomaly*, Journal of Finance **56** (2001), 279–304.

[9] Ravi Bansal, Varoujan Khatchatrian, and Amir Yaron, *Interpretable asset markets?*, European Economic Review **49** (2005), 531–560.

[10] Ravi Bansal, Dana Kiku, and Amir Yaron, *Risks for the long run: Estimation and inference*, working paper, Duke.

[11] Ravi Bansal and Ivan Shaliastovich, *A long-run risks explanation of predictability puzzles in bond and currency markets*, working paper (2008).

[12] Ravi Bansal and Amir Yaron, *Risks for the long run: A potential resolution of asset pricing puzzles*, Journal of Finance **59** (2004), 1481–1509.

[13] Paul Beaudry and Amartya Lahiri, *Risk allocation, debt fueled expansion and financial crisis*, Unpublished Manuscript (2009).

[14] Paul Beaudry and Franck Portier, *Stock prices, news, and economic fluctuations*, American Economic Review **96** (2006), no. 4, 1293–1307.

[15] Harjoat S. Bhamraa and Raman Uppal, *The role of risk aversion and intertemporal substitution in dynamic consumption-portfolio choice with recursive utility*, Journal of Economic Dynamics & Control **30** (2006), 967–991.

[16] Mark Bills and Peter K. Klenow, *Some evidence on the importance of sticky prices*, Journal of Political Economy **112** (2004), no. 5, 541–585.

[17] Michael Binder and M. Hashem Pesaran, *Multivariate linear rational expectations models: characterization of the nature of the solutions and their fully recursive computation*, Econometric Theory **13** (1997), 877–888.

[18] Richard Blundell, Martin Browning, and Costas Meghir, *Consumer demand and the life-cycle allocation of household expenditures*, Review of Economic Studies **61** (1994), no. 1, 57–80.

[19] Michele Boldrin, Lawrence J. Christiano, and Jonas D. M. Fisher, *Habit persistence, asset returns, and the business cycle*, American Economic Review **91** (2001), no. 1, 149–166.

[20] Guillermo A. Calvo, *Staggered prices in a utility-maxllmizing framework*, Journal of Monetary Economics **12** (1983), 383–398.

[21] John Campbell and John Cochrane, *By force of habit: A consumption-based explanation of aggregate stock market behavior*, Journal of Political Economy **107** (1999), 205–251.

[22] John Campbell and Robert Shiller, *Yield spreads and interest rate movements: a birds eye view*, Review of Economic Studies **58** (1991), 495–514.

[23] John Y. Campbell, *Asset prices, consumption, and the business cycle*, Handbook of Macroeconomics (J. B. Taylor and M. Woodford, eds.).

[24] George Chacko and Luis M. Viceira, *Dynamic consumption and portfolio choice with stochastic volatility in incomplete markets*, Review of Financial Studies **18** (Winter 2005), no. 4.

[25] Lawrence J. Christiano, *Solving the stochastic growth model by linear-quadratic approximation and by value-function iteration*, Journal of Business and Economic Statistics **8** (1990), 23–26.

[26] Richard Clarida, Jordi Galí, and Mark Gertler, *Monetary policy rules and macroeconomic stability: Evidence and some theory*, Quarterly Journal of Economics **115** (2000), 147–180.

[27] John Cochrane and Monika Piazzesi, *Bond risk premia*, American Economic Review **95** (2005), 138–160.

[28] Mariano M. Croce, *Long-run productivity risk: A new hope for production-based asset pricing*, working paper (2008).

[29] Carlos da Costa and Iván Werning, *Optimality of the friedman rule with heterogeneous agents and non-linear income taxes*, Journal of Political Economy **116, issue 1** (2008), 82–112.

[30] Peter A. Diamond and Johannes Spinnewijn, *Capital income taxes with heterogeneous discount rates*, NBER Working Paper 15115 (2009).

[31] Avinash K. Dixit and Joseph E. Stiglitz, *Monopolistic competition and optimum product diversity*, American Economic Review **67** (1977), 297–308.

[32] Darrell Duffie and Rui Kan, *A yield factor model of interest rates*, Mathematical Finance **6** (1996), 379–406.

[33] Larry G. Epstein and Stanley Zin, *Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework*, Econometrica **57** (1989), 937–969.

[34] Eugene Fama and Robert Bliss, *The information in long-maturity forward rates*, American Economic Review **77** (1987), 680–692.

[35] Eugene F. Fama and Kenneth R. French, *Dividend yields and expected stock returns*, Journal of Financial Economics **22** (1988), no. 1, 3–25.

[36] Jesus Fernandez-Villaverde and Juan F. Rubio-Ramirez, *Estimating macroeconomic models: A likelihood approach*, Review of Economic Studies **74** (2006), 1059–1087.

[37] Fatih Guvenen, *A parsimonious macroeconomic model for asset pricing*, Unpublished Manuscript (2008).

[38] Jonathan Heathcote, Kjetil Storesletten, and Giovanni L. Violante, *Quantitative macroeconomics with heterogeneous households*, NBER Working Paper 14768 (2009).

[39] Urban J. Jermann, *Asset pricing in production economies*, Journal of Monetary Economics **41** (1998), no. 2, 257–275.

[40] Robert J. Shiller John Y. Campbell, *The dividend-price ratio and expectations of future dividends and discount factors*, Review of Financial Studies **1** (1988), no. 3, 195–228.

[41] John F. Judd and Glenn D. Rudebusch, *Taylor's rule and the fed: 1970-1997*, Federal Reserve Bank of San Francisco Economic Review **3** (1998), 3–14.

[42] Robert G. King and Alexander L. Wolman, *Inflation targeting in a st. louis model of the 21st century*, Federal Reserve Bank of St. Louis Review **May** (1996), 83–107.

[43] David M Kreps and Evan L. Porteus, *Temporal resolution of uncertainty and dynamic choice theory*, Econometrica **46** (1978), 185–200.

[44] Per Krusell and Anthony A. Smith, *Income and wealth heterogeneity in the macroeconomy*, Journal of Political Economy **106** (1998), no. 5, 867–896.

[45] Jr Lucas, Robert E and Nancy L Stokey, *Money and interest in a cash-in-advance economy*, Econometrica **55** (1987), no. 3, 491–513.

[46] Emi Nakamura and Jón Steinsson, *Five facts about prices: A reevaluation of menu cost models*, Quarterly Journal of Economics **123** (2008), no. 4, 1415–1464.

[47] Monika Piazzesi and Martin Schneider, *Equilibrium yield curves*, NBER Working paper 12609 (2005).

[48] Glenn D. Rudebusch, Brian Sack, and Eric Swanson, *Macroeconomic implications of changes in the term premium*, FRB of St. Louis Review **89** (2007), 241–269.

[49] Glenn D. Rudebusch and Eric Swanson, *Examining the bond premium puzzle with a dsge model*, FRB of San Francisco working paper (2008).

[50] Thomas Tallarini, *Risk-sensitive real business cycles*, Journal of Monetary Economics **45** (2000), 507–532.

[51] John B. Taylor, *A historical analysis of monetary policy rules*, Monetary Policy Rules (J.B.Taylor, ed.), University of Chicago Press, Chicago, 1999.

[52] Harald Uhlig, *Leisure, growth and long run risk.*, (2007), Working Paper.

[53] Annette Vissing-Jorgensen, *Limited asset market participation and the elasticity of intertemporal substitution*, Journal of Political Economy **110** (2002), no. 4, 825–853.

[54] Philippe Weil, *The equity premium puzzle and the risk-free rate puzzle*, Journal of Monetary Economics **24** (1989), 401–421.

[55] Michael Woodford, *Interest and prices: Foundations of a theory of monetary policy*, Princeton University Press, 2003.

Appendix A: Recursive bond pricing formulas

Assume $a_0^{(n)} = a_\Gamma^{(n)} = 0$. Guess that all bonds can be written as $b_t^{(n)} = a_0^{(n)} + a_\Gamma^{(n)}\hat{\Gamma}_t$ and use recursive formula

$$b_t^{(n)} = \log E_t\left(\exp\left(q_{t+1,t} + b_{t+1}^{(n-1)}\right)\right) \tag{113}$$

$$\begin{aligned}
b_t^{(n+1)} =& \log E_t\left(\exp\left(q_{t+1,t} + a_0^{(n)} + a_\Gamma^{(n)}\hat{\Gamma}_t\right)\right) = && (114)\\
=& \log E_t\left(\exp\left(\begin{array}{l} q + q_\Gamma\hat{\Gamma}_t + q_\Omega\Sigma^{1/2}\left(\Gamma_t\right)\Omega_{t+1} + \\ a_0^{(n)} + a_\Gamma^{(n)}A\hat{\Gamma}_t + a_\Gamma^{(n)}q_\Omega\Sigma^{1/2}\left(\Gamma_t\right)\Omega_{t+1}\end{array}\right)\right) = && (115)\\
=& \log E_t\left(\exp\left(q + a_0^{(n)} + \left(q_\Gamma + a_\Gamma^{(n)}A\right)\hat{\Gamma}_t + \left(q_\Omega + a_\Gamma^{(n)}\right)\Sigma^{1/2}\left(\Gamma_t\right)\Omega_{t+1}\right)\right) = && \\
& && (116)\\
=& q + a_0^{(n)} + \left(q_\Gamma + a_\Gamma^{(n)}A\right)\hat{\Gamma}_t + \frac{\left(q_\Omega + a_S^{(n)}\right)\Sigma(S)\left(q_\Omega + a_\Gamma^{(n)}\right)'}{2} + && (117)\\
& \frac{\left(q_\Omega + a_\Gamma^{(n)}\right)B_{\Sigma(\Gamma)}\left(q_\Omega + a_\Gamma^{(n)}\right)'}{2}e_{S6}\hat{\Gamma}_t && (118)
\end{aligned}$$

So coefficients are given by recursive formulas

$$a_0^{(n+1)} = q + a_0^{(n)} + \frac{\left(q_\Omega + a_\Gamma^{(n)}\right)\Sigma(\Gamma)\left(q_\Omega + a_\Gamma^{(n)}\right)'}{2} \tag{119}$$

$$a_\Gamma^{(n+1)} = \left(q_\Gamma + a_\Gamma^{(n)}A\right) + \frac{\left(q_\Omega + a_\Gamma^{(n)}\right)B_{\Sigma(\Gamma)}\left(q_\Omega + a_\Gamma^{(n)}\right)'}{2}e_{S\upsilon} \tag{120}$$

Where

$$e_{S\upsilon} = [0,0,0,0,0,0,0,0,1,0]$$

$$\Sigma\left(S\right) = \begin{bmatrix} 0_{6x6} & 0_{4x6} \\ 0_{6x4} & \Sigma\left(s\right) \end{bmatrix}, B_{\Sigma(\Gamma)} = \begin{bmatrix} 0_{6x6} & 0_{4x6} \\ 0_{6x4} & B_\Sigma \end{bmatrix}$$

Appendix B: Proofs of Propositions

Proof of Proposition 1:

Investors run away if their consumption in period two implied by allocation

$$(1-\delta(j))\, A_{3I}\alpha(j) - Rb(j) + \delta(j)\, R$$

is less than $\hat{c}$. There is no loss in considering only two cases: $\delta(j) = 0$ and $\delta(j) = \bar{\delta}$. So the lowest possible 3rd period income (given 2nd period information about aggregate productivity level) is given by $\min\left[A_{3I}\underline{\alpha}, \left(1-\bar{\delta}\right) A_{3I}\underline{\alpha} + \bar{\delta}R\right]$. Using condition (104) I conclude that minimum income is $A_{3I}\underline{\alpha}$, hence borrowing limit is set so that:

$A_{3I}\underline{\alpha} - R\bar{b} \geq \hat{c}$, or

$$\bar{b}(A_{3I}) \leq \frac{A_{3I}\underline{\alpha} - \hat{c}}{R} \tag{121}$$

■ **Proof** of Proposition 2:

First consider investors that have chosen to invest in risk control, know their future idiosyncratic shock and maximize

$$[c_2(j)]^{(1-1/\psi(j))} + \beta\,[c_3(j)]^{1-1/\psi(j)} \tag{122}$$

They would prefer to borrow more if $\left[\frac{c_3(j)}{c_2(j)}\right]^{1/\psi(j)} \geq \beta R$. Investors with low IES want to smooth consumption more, so to make sure the everyone's borrowing constraint is binding I have to make sure that the investor that is least willing to smooth consumption still would like to do it.

$$c_2(j) = (1-\delta(j))\, A_{2I} + \delta(j)\, \lambda A_{2I} + b(j) - 1 \leq A_{3I} - 1 + \frac{A_{3I}\underline{\alpha} - \hat{c}}{R} \tag{123}$$

Since $R \geq 1$[20]

$$c_2(j) \leq A_{2I} - 1 + A_{3I}\underline{\alpha} - \hat{c} = A_{2I} - 1 + A_{3I}\underline{\alpha} - \hat{c} \tag{124}$$

$c_3(j) \geq \hat{c}$ by assumption, so the lowest consumption growth profile is given by $\frac{\hat{c}}{A_H - 1 + A_H\underline{\alpha} - \hat{c}}$ and is achieved for those who chose not to liquidate, since liquidation increases consumption growth via reducing current consumption and possibly (depending on realization) increasing consumption in the future.

So everyone's borrowing constraint binds if

$$\left[\frac{\hat{c}}{A_H - 1 + A_H\underline{\alpha} - \hat{c}}\right]^{1/\overline{\psi}} \geq \beta \sup(R) \tag{125}$$

The problem implies the usual optimality condition

$$R = \left[\frac{1}{(1 + \tau(A_{2I}))\,\beta}\left(\frac{C_2}{C_1}\right)^{\gamma}\right] \tag{126}$$

Since $\gamma > 0$, above expression is maximized when all investors borrow up to a borrowing limit so that

$$\mu B = \frac{A_{3I}\underline{\alpha} - \hat{c}}{R} \tag{127}$$

So

$$R = \left[\frac{1}{\beta}\left(\frac{\frac{A_H\underline{\alpha} - \hat{c}}{\mu}}{A_{2c} - \frac{A_H\underline{\alpha} - \hat{c}}{R\mu}}\right)^{\gamma}\right] \leq \left[\frac{1}{\beta}\left(\frac{A_H\underline{\alpha} - \hat{c}}{\mu A_{2c} - A_H\underline{\alpha} + \hat{c}}\right)^{\gamma}\right] = \hat{R} \tag{128}$$

So if

$$\left[\frac{\hat{c}}{A_H - 1 + A_H\underline{\alpha} - \hat{c}}\right]^{1/\overline{\psi}} \geq \left[\left(\frac{A_H\underline{\alpha} - \hat{c}}{\mu A_{2c} - A_H\underline{\alpha} + \hat{c}}\right)^{\gamma}\right] \tag{129}$$

[20] Formally there's no assumption in the model to prevent interest rate from becoming less than 1, but I could modify the model, granting consumers access to storage, which would effectively put a lower bound on R

Every investor who invested in risk control wants to borrow more. Now consider investors who did not invest in risk control. Above I have essentially analyzed the case of investors who do not liquidate. So in problem of investors who did not invest is to maximize

$$[c_2(j)]^{(1-1/\psi(j))} + \beta\left[\Upsilon\left[c_3(j)\right]\right]^{1-1/\psi(j)} \tag{130}$$

where $\Upsilon[c_3(j)]$ is certainty equivalent of consumption in period 3. Since $\Upsilon[c_3(j)] \geq \hat{c}$, if all investors who chose risk control want to borrow more, those who did not want to do so as well. ■

Proof of Proposition 3: Expectation is just an integral (and note that $\delta(j)$ is independent of idiosyncratic productivity shock)

$$E_2\left[[c_3(j)]^{1-\gamma}\right] = \int_{\underline{\alpha}}^{\overline{\alpha}} \left[(1-\delta(j)) A_{3I}x - A_{3I}\underline{\alpha} + \hat{c} + \delta(j) R\right]^{1-\gamma} \frac{1}{\overline{\alpha}-\underline{\alpha}} dx$$

If $\gamma \neq 2$:

$$E_2\left[[c_3(j)]^{1-\gamma}\right] =$$

$$= \frac{1}{(2-\gamma)(\overline{\alpha}-\underline{\alpha})(1-\delta(j)) A_{3I}} \left[\begin{array}{c} \left[A_{3I}(\overline{\alpha}-\underline{\alpha}) + \hat{c} + \delta(j)(R - A_{3I}\overline{\alpha})\right]^{2-\gamma} - \\ - \left[\hat{c} + \delta(j)(R - A_{3I}\underline{\alpha})\right]^{2-\gamma} \end{array}\right]$$

To make sure that $\delta^*(j)|_{m(j)=0} = 0$ it is sufficient to have marginal benefit of liquidation to be negative at $\delta = 0$ since at $\delta > 0$ consumption profile is even more uneven, but the marginal benefit in terms of expected present value increase from liquidation is the same. Investors who chose no risk control are those with $\psi(j) < 1/\gamma$, so $\gamma - 1/\psi(j) < 0$.

So it is sufficient to find the conditions under which the most willing to substitute consumptions smoothing for higher present value does not wish to do so. So consider Investors who chose no risk control with $\psi(j) = 1/\gamma$

They chose $\delta(j)$ to maximize the following:

$$\hat{U}_2 = \left[[c_2(j)]^{(1-\gamma)} + \beta E_2 \left[c_3^{1-\gamma}(j) \right] \right]^{\frac{1}{1-\gamma}}$$

Subject to

$$c_2(j) + 1 \leq A_{2I} - \delta(j) A_{2I} (1-\lambda) + \frac{A_{3I}\underline{\alpha} - \hat{c}}{R}$$

$$c_3(j) = A_{3I}(\alpha(j) - \underline{\alpha}) + \hat{c} + \delta(j)(R - A_{3I}\alpha(j))$$

$$\hat{U}_2 =$$

$$\left[[c_2(j)]^{(1-\gamma)} + \frac{\beta}{(2-\gamma)(\overline{\alpha} - \underline{\alpha})(1-\delta(j)) A_{3I}} \left[\begin{array}{c} \left[\begin{array}{c} A_{3I}(\overline{\alpha} - \underline{\alpha}) + \hat{c} \\ +\delta(j)(R - A_{3I}\overline{\alpha}) \end{array} \right]^{2-\gamma} - \\ -\left[\hat{c} + \delta(j)(R - A_{3I}\underline{\alpha}) \right]^{2-\gamma} \end{array} \right] \right]^{\frac{1}{1-\gamma}}$$

$$\frac{\Delta \hat{U}_2}{\Delta \delta} = -\frac{U^{\frac{\gamma}{1-\gamma}}}{1-\gamma} A_{2I} (1-\lambda)(1-\gamma)\left[c_2(j)\right]^{-\gamma} +$$

$$\frac{U^{\frac{\gamma}{1-\gamma}}}{1-\gamma} \frac{\beta}{(\overline{\alpha}-\underline{\alpha})(1-\delta(j)) A_{3I}} \left[\begin{array}{c} (R - A_{3I}\overline{\alpha})\left[A_{3I}(\overline{\alpha}-\underline{\alpha}) + \hat{c} + \delta(j)(R - A_{3I}\overline{\alpha})\right]^{1-\gamma} \\ -(R - A_{3I}\underline{\alpha})\left[\hat{c} + \delta(j)(R - A_{3I}\underline{\alpha})\right]^{1-\gamma} \end{array} \right] +$$

$$\frac{U^{\frac{\gamma}{1-\gamma}}}{1-\gamma} \frac{\beta}{(2-\gamma)(\overline{\alpha}-\underline{\alpha})(1-\delta(j))^2 A_{3I}} \left[\begin{array}{c} \left[A_{3I}(\overline{\alpha}-\underline{\alpha}) + \hat{c} + \delta(j)(R - A_{3I}\overline{\alpha})\right]^{2-\gamma} - \\ -\left[\hat{c} + \delta(j)(R - A_{3I}\underline{\alpha})\right]^{2-\gamma} \end{array} \right]$$

$$\left(\frac{\Delta \hat{U}_2}{\Delta \delta}\right) \frac{1}{U^{\frac{\gamma}{1-\gamma}}} |_{\delta(j)=0} = -A_{2I}(1-\lambda)\frac{1}{c_2(j)^{\gamma}} +$$

$$+\frac{1}{1-\gamma} \frac{\beta}{(\overline{\alpha}-\underline{\alpha}) A_{3I}} \left[\begin{array}{c} A_{3I}\underline{\alpha}\left[\hat{c}\right]^{1-\gamma} - A_{3I}\overline{\alpha}\left[A_{3I}(\overline{\alpha}-\underline{\alpha}) + \hat{c}\right]^{1-\gamma} \\ +R\left(\left[A_{3I}(\overline{\alpha}-\underline{\alpha}) + \hat{c}\right]^{1-\gamma} - \left[\hat{c}\right]^{1-\gamma}\right) \end{array} \right] +$$

$$+\frac{1}{1-\gamma} \frac{\beta}{(2-\gamma)(\overline{\alpha}-\underline{\alpha}) A_{3I}} \left[\begin{array}{c} \left[A_{3I}(\overline{\alpha}-\underline{\alpha}) + \hat{c}\right]^{2-\gamma} - \\ -\left[\hat{c}\right]^{2-\gamma} \end{array} \right]$$

Using

$$c_2(j)|_{\delta(j)=0} \leq A_{2I} - 1 + A_{3I}\underline{\alpha} - \hat{c}$$

and

$$R \leq \hat{R}$$

I get:

$$\left(\frac{\Delta\hat{U}_2}{\Delta\delta}\right)\frac{1}{U^{\frac{\gamma}{1-\gamma}}}|_{\delta(j)=0} \leq -\frac{A_{2I}(1-\lambda)}{\left[A_{2I}-1+A_{3I}\underline{\alpha}-\hat{c}\right]^{\gamma}}+$$
$$+\frac{1}{1-\gamma}\frac{\beta}{(\overline{\alpha}-\underline{\alpha})A_{3I}}\left[\begin{array}{c} A_{3I}\underline{\alpha}\left[\hat{c}\right]^{1-\gamma}-A_{3I}\overline{\alpha}\left[A_{3I}(\overline{\alpha}-\underline{\alpha})+\hat{c}\right]^{1-\gamma} \\ +\hat{R}\left(\left[A_{3I}(\overline{\alpha}-\underline{\alpha})+\hat{c}\right]^{1-\gamma}-\left[\hat{c}\right]^{1-\gamma}\right)\end{array}\right]+$$
$$+\frac{1}{1-\gamma}\frac{\beta}{(2-\gamma)(\overline{\alpha}-\underline{\alpha})A_{3I}}\left[\begin{array}{c}\left[A_{3I}(\overline{\alpha}-\underline{\alpha})+\hat{c}\right]^{2-\gamma}- \\ -\left[\hat{c}\right]^{2-\gamma}\end{array}\right]$$

Denote RHS as Ξ

If $\Xi < 0$ then $\frac{\Delta\hat{U}_2}{\Delta\delta} < 0$ So $\frac{\Delta\hat{U}_2}{\Delta\delta} < 0$ if

$$\lambda < \lambda^* = 1-\frac{\Phi\left[A_{2I}-1+A_{3I}\underline{\alpha}-\hat{c}\right]^{\gamma}}{A_{2I}}$$

And γ^* is the smallest positive solution of:

$$1=\frac{\Phi\left[A_{2I}-1+A_{3I}\underline{\alpha}-\hat{c}\right]^{\gamma}}{A_{2I}}$$

■

Proof of Proposition 4:

Liquidation decision problem

$$U_2(j,1)=\max_{\delta(j)}\left[\begin{array}{c}\left[A_{2I}-\delta(j)A_{2I}(1-\lambda)+\frac{A_{3I}\underline{\alpha}-\hat{c}}{R}\right]^{(1-1/\psi(j))}+ \\ +\beta\left[A_{3I}(\alpha(j)-\underline{\alpha})+\hat{c}+\delta(j)(R-A_{3I}\alpha(j))\right]^{1-1/\psi(j)}\end{array}\right]^{\frac{1}{1-1/\psi(j)}} \tag{131}$$

s.t.

$$\delta(j) < \bar{\delta} \tag{132}$$

$$0 < \delta(j) \tag{133}$$

FOC, ignoring bounds on $\delta(j)$

$$U_2^{\frac{1/\psi(j)}{1-1/\psi(j)}} \begin{bmatrix} -A_{2I}(1-\lambda)[c_1(\delta(j))]^{(1/\psi(j))} + \\ \beta(R - A_{3I}\alpha(j))[c_2(\delta(j), j)]^{-1/\psi(j)} \end{bmatrix}^{\frac{1}{1-1/\psi(j)}} = 0 \tag{134}$$

But because of condition (104) the sum in square brackets is negative for $\alpha(j) = \overline{\alpha}$, so for high realizations of productivity investors that know that in advance never liquidate. so $\overline{\lambda} = 1$ and $j_1 : \alpha(j_1) = \overline{\alpha}$

There exists j_2, so that $\delta(j_2) = \bar{\delta}$ if at least one type of investor chooses to liquidate the worst realization of investment. The most willing to liquidate is the investor with highest IES $\overline{\psi}$.

$\delta(j) = \bar{\delta}$ is an interior solution for $\alpha(j) = \underline{\alpha}$ for investor that is the most willing to substitute consumption between periods if

$$\begin{bmatrix} -A_{2I}(1-\lambda)\left[A_{2I} - \bar{\delta}A_{2I}(1-\lambda) + \frac{A_{3I}\underline{\alpha} - \hat{c}}{R}\right]^{1/\overline{\psi}} \\ +\beta(R - A_{3I}\underline{\alpha})\left[\hat{c} + \bar{\delta}(R - A_{3I}\underline{\alpha})\right]^{-1/\overline{\psi}} \end{bmatrix} \geq 0$$

or

$$\left[-1 + \frac{\beta(R - A_{3I}\underline{\alpha})}{A_{2I}(1-\lambda)}\left(\frac{\hat{c} + \bar{\delta}(R - A_{3I}\underline{\alpha})}{A_{2I} - \bar{\delta}A_{2I}(1-\lambda) + \frac{A_{3I}\underline{\alpha} - \hat{c}}{R}}\right)^{-1/\overline{\psi}}\right] \geq 0$$

Using condition (107) $\delta(j) = \bar{\delta}$ is optimal if:

$$-1 + \frac{(1 - A_{3I}\underline{\alpha})}{\hat{R}A_{2I}(1-\lambda)} \geq 0$$

so

$$\underline{\lambda} = 1 - \frac{(1 - A_{3I}\underline{\alpha})}{\hat{R}A_{2I}}$$

■

Proof of Proposition 5:

Continuity of $U_1(j,0)$ is straightforward.

From the proof of proposition 4 it follows that for every j optimal liquidation decision $\delta^*(j)$ is a continuous function of R but not necessarily continuously differentiable: $\delta^*(j)$ is either interior solution an is continuously differentiable, or a boundary solution, so is a constant. So there is a possibility of kinks in $\delta^*(j)$. Likewise, maximized objective $U_2^*(j,1)$ is also continuous in R, but not necessarily continuously differentiable. However, period 1 utility is continuously differentiable in R, since it involves integration of a continuous function over a compact set with density that is positive everywhere.

$$U_1(j,1) = E_1\left(U_2^{1-\gamma}(j,1)\right)^{\frac{1}{1-\gamma}} \tag{135}$$

■