Essays in Uninsurable Income Risk and Household Behavior

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Dedication

I dedicate this dissertation to my loved husband, Futoshi Narita.

Abstract

This dissertation consists of two essays. The first essay examines a key driving force of the recent decline in the U.S. homeownership rate by investigating the effect of each factor on the housing and mortgage decisions. The second paper assesses how efficiently households in the U.S. economy insure their cohort-specific income risk. Although the applications differ, they have a unifying theme: macroeconomic implications of households' behavior in the presence of uninsurable income risk. Both essays address this theme using household-level data and quantitative macroeconomic models.

In the first essay, I ask if a mortgage credit crunch caused the recent decline in the homeownership rate in the U.S. To answer this question, I develop a life-cycle model that accommodates the expansion of alternative mortgages that featured delayed amortization. I use the model to measure the distributional consequences of two factors: (1) the fade-out of alternative mortgages and (2) declines in labor earnings. I find that the declining labor income is the main driving force behind the cross-sectional feature of the housing bust: the proportionally larger decrease in homeownership among non-college educated households. The fade-out of alternative mortgages, however, predicts the opposite. This is because college educated households, who have high future earnings, are more likely to utilize alternative mortgages when those mortgages are available.

In the second essay, we propose an observation-based approach to measuring what percentage of household's income risk is insured and how much welfare cost is generated by the departure from full-insurance. Using a synthetic panel data set from the Consumer Expenditure Survey (CEX) for the period of 1980-2009, we investigate how efficiently U.S. households insure cohort-specific income risk. There are two main findings. First, on average, U.S. households insured 64% of their cohort-level income risk, and the welfare cost of uninsured risk was 1% of their annual expenditure on nondurables and services. Second, households who faced a higher risk tended to insure a larger portion of their risk. This observation implies either or both of the following: (1) they made more effort to hedge their risk (Kocherlakota (1996) and Krueger and Perri (2006)), or (2) the dispersion in income risk mostly came from transitory shocks.

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Chapter 1

Disentangling the Mortgage Credit Crunch and the Recession

1.1 Introduction

After a decade of expansion, the U.S. housing market has seen a continuous decline in the rate of homeownership. In addition, there have been noticeably different behaviors across education groups of households. According to data from the Current Population Survey (CPS), the decline in homeownership during the late 2000s was larger among non-college educated households, while the rise during the early 2000s was larger among college educated households. What can explain the housing bust that involves the distributional change in homeownership across education groups?

There are two possible explanations. One is the contraction of supply for alternative mortgages that featured delayed amortization, such as interest-only mortgages.[1] The origination share of those mortgages sharply decreased after 2007 (Amromin et al. (2010) and Corbae and Quintin (2010)).[2] Alternative explanation is a recession: declines in labor earnings.[3] There were substantial declines in real labor earnings

[1] Mortgage products that involve unambiguously back-loaded payments are option-ARMs, balloon mortgages, and graduated payment mortgages.

[2] It is relevant to consider a mortgage credit crunch in the supply side during the late 2000s. For example, underwriting guidelines for non-traditional mortgages effective in October 2006 and a new consumer protection agency which is currently being created can be seen as evidence of regulation tightening that affects the supply of mortgage credits.

[3] Bajari et al. (2010) showed that one of the key driving forces of the recent increase in mortgage

from 2005 to 2009, especially among non-college educated households, based on the CPS data.

In this essay, I measure to what extent each of these factors can account for the observed cross-sectional features of the housing bust. The main objective of this essay is to determine which factor was quantitatively more important for the housing bust, based on their effects on the distribution of homeownership.

To answer the question, I construct a life-cycle model that accommodates a mortgage innovation. The mortgage innovation here is modeled as the introduction of an interest-only mortgage (IO),[4] which has a jump in the payment amount after the initial interest-only periods. The reason why I focus on the IO mortgage is that it was the most widely used product among delayed-amortization mortgage products in 2005, according to Barlevy and Fisher (2010). In the model without the mortgage innovation, only a standard fixed-rate mortgage (FRM), which has a constant payment over time, is available to households. Households are heterogeneous in terms of uninsurable income shocks and education levels, which determine the shape of earning profiles. I calibrate the model without the mortgage innovation so that key model statistics match the actual statistics in the year 2000.

The mortgage innovation in the calibrated model successfully captures the cross-sectional feature of the housing boom: a larger increase in homeownership among college educated households. This feature is driven by the different characteristics of earning profiles for college and non-college educated households. Since college educated households face steeper earning profiles than non-college educated households, they find the lower initial payments on the IO mortgage more useful for consumption-smoothing. Therefore, when the mortgage innovation is introduced, college educated households are more likely to utilize the innovation. This implication is consistent with recent findings from micro data. Amromin et al. (2010) document that IO mortgages were more prevalent in the area of highly educated population. Also, a typical user of IO mortgages was not a subprime borrower, but a safe and relatively rich borrower.[5]

defaults was the increase of borrowers with high payment to income ratios.

[4] Mian and Sufi (2010) provided evidence that the credit expansion from 2002 to 2006 was supply driven. Chambers et al. (2009) and Corbae and Quintin (2010) also model the financial innovation as the introduction of new types of mortgages.

[5] Amromin et al. (2010) show that the average income and the average FICO credit score among IO mortgage borrowers were both higher than those among traditional mortgage borrowers based on the

Using this model, I measure the relative importance of two factors in accounting for the housing bust: (1) fade-out of the IO mortgage supply and (2) declines in labor earnings. Specifically, I quantify the effects of each factor on the distribution of homeownership by conducting comparative steady state analyses for these two scenarios.

The main finding is that the mortgage credit crunch per se cannot account for the cross-sectional feature of the housing bust. The fade-out of IO mortgage supply results in a proportionally larger decrease in homeownership among college educated households, contrary to the observation. This is because there are more homeowners who use IO mortgages among college educated households. On the other hand, the observed income declines generate a proportionally larger decline in homeownership among non-college educated households, which is consistent with the observation. The observed income decline was about four times larger for non-college educated households, according to the CPS data.

To the best of my knowledge, this is the first study that quantifies the effects of the mortgage innovation on the distribution of homeownership across education groups. The previous literature has shown that the mortgage innovation has a sizable impact on the aggregate homeownership rate and the aggregate foreclosure boom.[6] I show that the mortgage innovation generates a proportionally larger increase in homeownership among college educated households. My results shed light on the housing wealth concentration during the recent housing boom and bust.

This essay is related to the literature that studies other aspects of the housing boom and bust in the United States. Kiyotaki et al. (2010) and Sommer et al. (2010) study house prices, rents, and homeownership by examining the roles of changes in fundamentals, such as declines in downpayment requirements and interest rates. Ríos-Rull and Sánchez-Marcos (2006), Arslan(2008), and Favilukis et al. (2010) study house price dynamics by examining the effects of financial constraints and aggregate uncertainties

Lender Processing Services data from 1998 to 2008. Also, Landier et al. (2010) report that New Century Financial Corporation, which was a large mortgage lender in the U.S., massively issued alternative mortgages to younger, safer, and richer households from 2004 to 2006. Cocco (2010) provides empirical evidence that IO mortgages are extensively used by households who expect higher future labor earnings using the UK data.

[6] Chambers et al. (2008, 2009) show that the expansion of alternative mortgages accounts for a large fraction of the observed increase in the aggregate homeownership rate from 1994 to 2005. Corbae and Quintin (2010) demonstrate that the contraction of alternative mortgages significantly amplify a foreclosure boom following an unexpected house price decline.

on earnings and interest rates. Chatterjee and Eyigungor (2009) study the effect of an unexpected increase in the housing supply on house prices and foreclosures.

This essay is also related to the literature that studies housing decisions over the life-cycle. Bennett et al. (2001), Hurst and Stafford (2004), and Nakajima and Telyukova (2009) find that the home equity plays an quantitatively important role in achiving consumption-smoothing and that the recent innovations in mortgage markets are important to understanding the changes in housing and non-housing consumption in recent years in the U.S. The importance of the first-time buyers' housing finances are also documented by Ortalo-Magné and Rady (2003) and Duca et al. (2011). Nakajima (2005), Fernández-Villaverde and Krueger (2010), Fisher and Gervais (2010), and Iacoviello and Pavan (2010) demonstrate the effects of uninsurable income risk on the life-cycle pattern of housing investment and provide their macroeconomic implications. Yang (2009) and Li and Yao (2007) show the importance of borrowing constraints and housing transaction costs in explaining the distribution of homeownership. My essay examines mortgage decisions in home purchasing and finds the importance of the steepness of income profiles in those decisions.

The rest of this chapter consists of 4 sections. Section 2 demonstrates the distributional changes in homeownership across education groups during the 2000s. Section 3 describes the model economy and defines equilibrium. Section 4 explains how I calibrate the model and shows main results. Finally, section 5 concludes.

1.2 Changes in U.S. Homeownership

This section documents the distributional change in homeownership across education groups, as well as the change in the homeownership rate.

Table 1 shows the homeownership rate and the share of college educated homeowners in the U.S. from 1995 to 2010. These statistics are calculated for households whose householder is aged 20-64, using the data from the March CPS Supplement.[7]

[7] This essay investigates housing and mortgage decisions of households with different education levels, focusing on the different characteristics of their earning profiles. Therefore, the statistics are calculated for the working population.

The definition of householder is provided by the CPS. It refers to the person in whose name the housing unit is owned or rented.

Table 1.1: Change in the Composition of Homeowners

	Homeownership Rate	% of College Educated among Homeowners
1995	61 %	39 %
2000	**65 %**	**41 %**
2005	67 %	45 %
2010	**64 %**	**48 %**

Note: Data source is the March CPS Supplement.

The homeownership rate increased over the period of 1995-2005 but decreased thereafter. The share of college educated homeowners, on the other hand, has continuously increased since 1995. The increase in housing wealth concentration was especially evident during the 2000s. The share of college educated homeowners increased from 41% to 48%, while the homeownership rate in 2010 (64%) was almost the same as in 2000 (65%).

What contributed to the changes in the homeownership rate and the composition of homeowners? They can be driven by demographic changes and/or the changes in individual participation behaviors. To answer this question, I calculate the contributions of the following four factors: change in homeownership among non-college educated households, change in homeownership among college educated households, change in the share of college educated households, and the change in the age structure in the population. I use the age groups by 10 years and the education groups of non-college and college educated households. In the appendix A-1, I describe how to decompose the change in the aggregate homeownership rate into the contribution of each factor.

Table 2 shows the results for the periods of 1995-2000, 2000-2005, 2005-2010. The first row shows the difference between the homeownership rates in the year t_0 and t. The second to fifth rows show the contributions of the four factors to the change in the homeownership rate from t_0 to t. The sixth row shows the contribution of interactions among the four factors. Finally, the last row indicates that contributions of all factors sum up to 100%.

Table 1.2: Change in the Homeownership Rate

	1995-2000	2000-2005	2005-2010
Change in the homeownership rate Δh_t	3.8 %	1.9 %	-3.2 %
<u>Contribution:</u>			
Homeownership (non-college)	**47 %**	4 %	**-92%**
Homeownership (college)	19 %	**58 %**	-47 %
Share of college educated	7 %	13 %	11 %
Age structure of population	29 %	23 %	24 %
Covariance terms	0 %	2 %	4 %
	100 %	100 %	-100 %

Source: March CPS Supplement

There are three messages from Table 2. First, in all periods, the changes in the group specific homeownership rates accounted for a large fraction of the change in the homeownership rate. That is, the changes in behavior were important for the changes in the aggregate homeownership rate.

Second, the decrease in the homeownership rate from 2005-2010 was mainly driven by the decline in the homeownership rate among non-college educated households (-92% vs -47%), while the increase in the homeownership rate from 2000-2005 was mainly driven by the increase in the homeownership rate among college educated households (4% vs 58%). These different behaviors of education groups can also be confirmed by looking at the homeownership rate by age and education. Table A-1 in the appendix shows that there was a proportionally larger increase in homeownership among college educated households during the early 2000s (0.9 percentage point increase vs 4.7 percentage point increase), but there was a proportionally larger decrease in homeownership among non-college educated households during the late 2000s (-3.7 percentage point decrease vs -3.4 percentage point decrease).

Third, a larger contribution of the increase in the homeownership rate among college educated households was specific to the period of 2000-2005. During the period of 1995-2000, the increase in the homeownership rate among non-college educated households contributed more to the increase in the aggregate homeownership rate (47% vs 19%). This reveals a different nature of the housing boom after 2000 and suggests the importance of factors which are specific to this period, such as the expansion of delayed-amortization mortgages.

In summary, there have been noticeably different behaviors across education groups during the recent housing boom and bust in the U.S. The empirical analysis in this section showed that those different behaviors of different education groups had sizable impacts on the aggregate homeownership rate. It also documented that the fall in the homeownership rate was mainly driven by non-college educated households during 2005-2010, while the rise in the homeownership rate was mainly driven by college educated households during 2000-2005.

1.3 Model

In this section, I present a life-cycle model which demonstrates how a financial innovation affects households with different education levels. Specifically, I consider the introduction of alternative mortgages with delayed amortization as a financial innovation. The key ingredients of the model are education-specific earning profiles and mortgages. Different characteristics of earning profiles induce heterogeneous mortgage choices across education groups. The mechanism behind the heterogeneous decisions is crucial for understanding the distributional effects of the financial innovation and reversion.

In the model, there are continuously many households with two levels of education, a representative production firm, a representative housing rental agency, and a government. Houses are indivisible and illiquid, play the dual role of consumption and investment goods. There are two types of mortgages with different amortization schedules, the fixed rate mortgage (FRM) and the interest-only mortgage (IO). The financial market is open to the rest of the world and thus agents face the world interest rates for financial asset and mortgage debt.

In the presence of uninsurable income risk, households make decisions on housing tenure, housing size, and mortgage type in addition to consumption and savings. Future income profiles play an important role in mortgage decisions, because mortgage repayments continue over long periods of time. The production firm produces consumption goods and housing investment goods using two different technologies. Therefore, the house price and the rental price are endogenously determined in the model. The housing rental agency rents housing units to renting households. The government collects income taxes to finance its consumption. It also runs a social security system.

1.3.1 Houses

Houses are traded at price P per unit of housing space, but there is a minimum size of a house to own $\bar{h}$. The minimum size of houses captures the indivisible nature of houses and generates some households who cannot own a house. Such households have no house ($h = 0$), but they rent an apartment h^R from the housing rental agency at rate R.[8]

Transaction costs of housing are introduced in the model in order to capture the nature of houses as an illiquid asset. Households have to pay transaction costs to a financial intermediary firm when they buy or sell a house. The cost function of housing transactions is

$$\chi^T(h_{-1}, h, P) = \begin{cases} \chi_0^T + \chi_b^T P h + \chi_s^T P h_{-1} & \text{if } h_{-1} \neq h \\ 0 & \text{otherwise.} \end{cases}$$

where h_{-1} and h are the house to buy and the house to sell, respectively. Note that these costs are associated with transactions of owned houses.

Houses are assumed to depreciate at different rates based on their status. If a house is owned by a household, it depreciates at rate δ_h. If it is rented by a household, it depreciates at rate δ_R.

1.3.2 Mortgages

In the model, households must make a long-term mortgage contract. There are two types of mortgages: the FRM and the IO mortgage. The type of a mortgage is represented by

[8] Therefore, $h > 0$ means "own" and $h = 0$ means "rent."

$z \in \{0, 1(\text{FRM}), 2(\text{IO})\}$, where $z = 0$ means that no mortgage is held. $z = 0$ is relevant for homeowners who have paid off their mortgage in full and renters. The two types of mortgages are different in two dimensions: the rate of downpayment $\lambda(z)$ and the amortization schedule $A(z)$. The length of contract N is assumed to be common across mortgage types and to be corresponding to 30 years, for simplicity.

For both types of mortgage, the mortgage payment $m(n, z)$ consists of two parts;

$$m(n, z) = r^m D(n, z) + A(n, z) \tag{1.1}$$

where the interest payment part $r^m D(n, z)$ and the amortization part $A(n, z)$. The debt outstanding $D(n, z)$ follows the law of motion below;

$$D(n-1, z) = D(n, z) - A(n, z) \tag{1.2}$$

where n is the remaining periods of mortgage payments and $A(n, z)$ is the amortization part of the mortgage payment.

Fixed Rate Mortgage (z=1)

A key feature of the fixed rate mortgage is a constant payment over the contract periods N. That is, $m(n-1, 1) = m(n, 1)$ for any remaining periods of payments $n = 1, \cdots, N$. Together with (1.1) and (1.2), this gives

$$m(n, 1) = \frac{r^m}{1 - (1 + r^m)^{-N}} D(N, 1) \quad \text{for } n = 1, \cdots, N.$$

where the initial amount of debt is $D(N, 1) = \lambda(1) Ph$.

Interest-Only Mortgage (z=2)

A key feature of the interest-only mortgage is a low initial payment. For the first N^{IO} periods, the borrower pays only the interest part of the payment. Therefore, the debt outstanding $D(n, 2)$ starts declining after N^{IO} periods, when the borrower starts paying the amortization part as well as the interest prat. Together with (1.1) and (1.2), this gives

$$m(n, 2) = \begin{cases} r^m D(N, 2) & \text{if } n = N - N^{IO}, \cdots, N \\ \frac{r^m}{1-(1+r^m)^{-(N-N^{IO})}} D(N, 2) & \text{if } n = 1, \cdots, N - N^{IO} \end{cases}$$

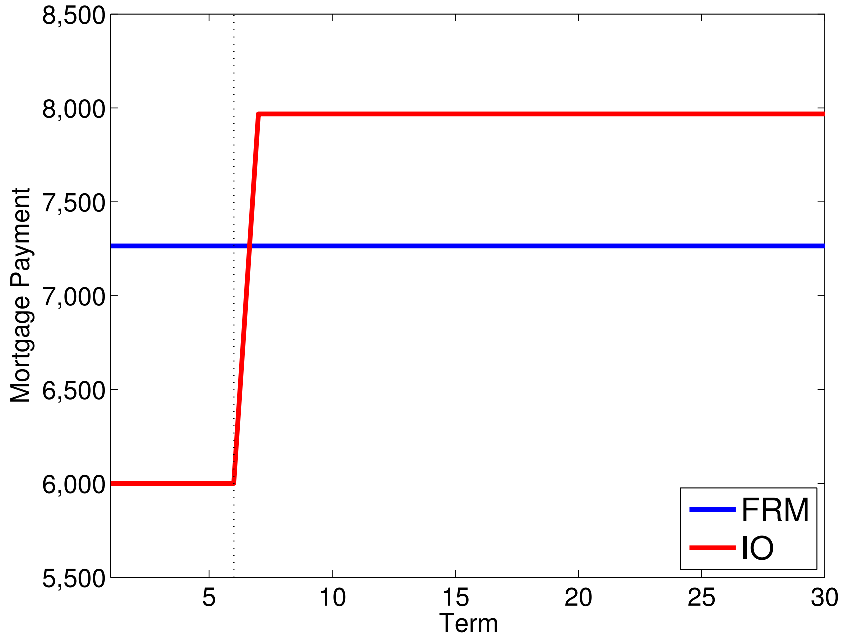

Figure 1.1: Payment Schedules of FRM and IO Mortgage

Note: Mortgage payment schedules for the FRM and the IO mortgage are calculated for the initial debt of \$100,000 and the annual mortgage interest rate of 6%. The mortgage maturity is 30 years for both mortgages and the interest only period is 6 years.

where the initial amount of debt is $D(N, 1) = \lambda(2)Ph$.

The payment schedules of these two mortgages have different characteristics due to their different amortization schedules. Figure 1 depicts typical mortgage payment schedules for the 30-year FRM and the 30-year IO mortgage. The initial loan balance is set to \$100.000, the annual mortgage interest rate is set to 6%, and the initial interest-only periods are 6 years in the example. As Figure 1 shows, the payments on the IO mortgage is low during the interest-only phase and becomes high thereafter. The payments on the FRM are constant over the term of the mortgage.

The characteristics of payment schedules and earning profiles are important for mortgage decisions. For example, households who expect to have higher future earnings find the low initial payments on the IO mortgage useful for a consumption-smoothing purpose. Households use mortgages that work best with their earning profiles in home purchasing. This is the key for the heterogeneity in mortgage choices across education groups, because earning profiles exhibit different shapes for different education groups.

1.3.3 Households

In each period, a mass of households are born with an education level $e \in \{1\text{ (noncollege)},\ 2\text{ (college)}\}$, some asset ($a_0$), but no house ($h_0 = 0$). They live J periods at longest, but there is a possibility of unexpected death in every period. A household at age j survives to age $j+1$ with probability $\psi^s_{j+1} \in [0,1]$ where $\psi^s_1 = 1$.

Households derive utility from consumption goods c and housing services. Housing services come from an owned house h or a rental housing unit h^R, and the relationship between the amount of housing service flow and housing units is assumed to be linear. Therefore, the utility function takes housing units (h or h^R) as an argument.

The expected discounted utility of a household is

$$E_0 \sum_{j=1}^{J} \beta^{t-1} \psi^s_{j+1} u(c_j, h_j), \quad \beta \in (0,1). \tag{1.3}$$

Each household inelastically supplies its labor but its efficiency is heterogeneous among households. Labor efficiency of a household $\varphi_{ej}\epsilon$ consists of a deterministic component φ_{ej} and a stochastic component ε. The deterministic component depends on education e and age j and it captures a hump shape of life-cycle earnings. The stochastic component captures variation in earnings within an age-education group. Specifically, it is assumed as

$$\begin{aligned} \ln \epsilon' &= \nu' + \varepsilon' \qquad (1.4) \\ \text{where} \quad \nu' &= \rho\nu + \iota' \end{aligned}$$

where $\varepsilon' \sim N(0, \sigma^2_\varepsilon)$ and $\iota' \sim N(0, \sigma^2_\iota)$. I assume that labor efficiency becomes zero a mandatory retirement age J_r.

Households earn labor income $w\varphi_{eij}$ at working ages, and they receive a social security benefit $\bar{y}_e$ after retirement. In addition to labor earnings, households receive interest earnings, the per-capita profit from the representative firm, and the transfer from the government. However, households have to pay income taxes and the social security tax to the government. Therefore, the after-tax income (y) of a household is given by

$$y = \begin{cases} (1+r-\delta)a + (1-\tau_p)w\varphi_{ej}\epsilon + \Pi - T(\tilde{y}) & \text{if } \; j \leq J_r \\ (1+r-\delta)a + \bar{y}_e + \Pi - T(\tilde{y}) & \text{if } \; j = J_r \cdots J \end{cases}$$

where $(1+r-\delta)$ is the gross rate on saving, τ_p is the social security tax rate and $T(\tilde{y})$ is the amount of income tax, and $\tilde{y}$ is the taxable household income.

The taxable household income $\tilde{y}$ is defined as

$$\tilde{y} = \begin{cases} (1+r-\delta)a + w\varphi_{ej}\epsilon + \Pi - \Gamma & \text{if } \; j \leq J_r \\ (1+r-\delta)a + \bar{y}_e + \Pi - \Gamma & \text{if } \; j = J_r \cdots J \end{cases} \tag{1.5}$$

where Π is the per-capita profit of the representative firm, Γ is a deduction, and $\bar{y}_e$ is the social security benefit. As a deduction Γ, I consider the mortgage interest deduction, which is one of the important feature of the current U.S. tax code.[9]

This formulation of taxable income also captures the asymmetric treatment of housing in the U.S.; the implicit income from housing capital are tax-deductible, while returns on financial asset, $(1+r-\delta)a$, are taxable. Gervais (2002) argues that the preferential tax treatment of housing distorts composition of households' saving and finds that its welfare cost is substantial. Nakajima (2010) considers the optimal capital income tax rate given the U.S. housing tax provisions and finds that it should be close to zero in order to nullify the distortion.

1.3.4 Households' Problem

Households make decisions on consumption c, savings a', the size of a owned house h, and the type of mortgage z' based on their budget constraints. The state variables that determine their budget constraint is the asset holding a, the previous size of the owned house h_{-1}, the type of the mortgage currently held z, the remaining periods of mortgage payments n, the shock in labor efficiency ϵ, education e and age j.

Among these state variables, the previous housing position h_{-1} needs special attention, because the budget constraint takes different forms for a renter ($h_{-1} = 0$) and for a homeowner ($h_{-1} > 0$). This is because the set of housing choices differs depending on

[9] In the U.S. tax code, interest payments on up to \$ 1 million of home acquisition debt are deductible from federal income taxes.

the previous housing position h_{-1}. The housing positions available are summarized as follows:

$$\text{Renter } (h_{-1}=0) \quad : \quad \begin{cases} (1) \text{ continue renting} \\ (2) \text{ purchase a house} \end{cases}$$

$$\text{Homeowner } (h_{-1}>0) \quad : \quad \begin{cases} (3) \text{ keep the same house} \\ (4) \text{ sell the house} \\ (5) \text{ resize the house} \\ (6) \text{ default on the mortgage} \end{cases}$$

In addition to the discrete choice in homeownership, households have to deal with another discrete choice on the mortgage type because every household have to take out a mortgage in home purchases. Households compare the value functions of all available options and choose the option which associates with the highest value. The value function of each option will be defined below.

Renter Yesterday ($h_{-1} = 0$)

If a household was a renter in the previous period ($h_{-1} = 0$), it has two housing choices: (1) continue renting and (2) buy a house and become a homeowner. Since the household did not have a house yesterday, at the beginning of the current period, it has no mortgage $z = 0$ and zero remaining period of mortgage payments $n = 0$. Renters lives in a space h^R rented from a rental housing retailer at a price R.

The problem of a renter is defined as follows.

$$W(a,0,0,0,\epsilon,e,j) := \max_{\theta\in\{1,2\}} \Big\{ V_\theta(a,0,0,0,\epsilon,e,j) \Big\}$$

where V_1 and V_2 will be defined below.

(1) Continue Renting

The value associated with continued renting is the expected present-value of life-time utility which is maximized over consumption c and asset holding tomorrow a'.

$$V_1(a,0,0,0,\epsilon,e,j) := \max_{c\geq 0,\ a'\geq 0} \left\{ u(c,h^R) + \beta\psi^s_{j+1} \int_{\epsilon'} W(a',0,0,0,\epsilon',e,j{+}1)\ \mathrm{d}F(\epsilon'|\epsilon) \right\}$$
$$\text{s.t.} \quad c + Rh^R + a' = y$$

where $F(\cdot|\cdot)$ is a conditional distribution function of income shock ϵ and y is the after-tax income, which is defined in the section 3.3.

(2) Purchase a House

A renter who chooses to become a homeowner have to decide on the size of the house h and the type of a mortgage z. Since mortgage payments last for N periods, the household considers possible future streams of earnings and chooses a mortgage with a suitable payment schedule. The value to purchase a house and become a homeowner solves

$$\begin{aligned} V_2(a,0,0,0,\epsilon,e,j) &:= \max_{c\geq 0, a'\geq 0, h, z'} \left\{ u(c,h) + \beta\psi^s_{j+1} \int_{\epsilon'} W(a',h,z',N{-}1,\epsilon',e,j{+}1)\ \mathrm{d}F(\epsilon'|\epsilon) \right\} \\ \text{s.t.} \quad & c + a' + \chi^T(0,h,P) + \lambda(z')Ph + m(z',N) + \delta_h Ph = y \end{aligned}$$

where $\chi^T(\cdot)$ is the transaction cost of housing, which is defined in section 3.1, and $\lambda(\cdot)$ and $m(\cdot)$ are the rate of the downpayment and the mortgage payment, which are defined in section 3.2. The household also pays the maintenance cost for the owned house, $\delta_h Ph$, at the end of the period.

Homeowner Yesterday ($h_{-1} > 0$)

If a household was a homeowner in the previous period ($h_{-1} > 0$), it has four choices: (3) keep the same house, (4) sell the house and become a renter, (5) sell the house and buy a different house, and (6) default on the mortgage and become a renter.[10]

The problem of a homeowner, i.e. $(h_{-1}, z, n) \neq (0,0,0)$, is

$$W(a,h_{-1},z,n,\epsilon,e,j) := \max_{\theta\in\{3,4,5,6\}} \left\{ V_\theta(a,h_{-1},z,n,\epsilon,e,j) \right\}$$

where V_3, V_4, V_5, and V_6 will be defined below.

(3) Keep the Same House

A homeowner who chooses to stay in the same house also keep the same mortgage contract. Therefore, it does not choose the housing size nor the mortgage type. The

[10] Refinancing the mortgage while keeping the same house is a reasonable option, but I abstract it from the current model for simplicity.

value to keep the same house solves

$$V_3(a, h_{-1}, z, n, \epsilon, e, j) := \max_{c\geq 0, a'\geq 0} \left\{ u(c, h_{-1}) + \beta\psi^s_{j+1} \int_{\epsilon'} W(a, h_{-1}, z, n{-}1, \epsilon', e, j{+}1) \ \mathrm{d}F(\epsilon'|\epsilon) \right\}$$
$$\text{s.t.} \quad c + a' + m(z, n) + \delta_h P h_{-1} = y,$$

where $m(z, n)$ is the mortgage payment and $\delta_h P h_{-1}$ is the maintenance cost of the owned house.

(4) Sell the Same House

A homeowner who chooses to sell the house and become a renter has to repay all remaining mortgage loan. The value to keep the same house solves

$$V_4(a, h_{-1}, z, n, \epsilon, e, j) := \max_{c\geq 0, a'\geq 0} \left\{ u(c, h^R) + \beta\psi^s_{j+1} \int_{\epsilon'} W(a', 0, 0, 0, \epsilon', e, j{+}1) \ \mathrm{d}F(\epsilon'|\epsilon) \right\}$$
$$\text{s.t.} \quad c + Rh^R + a' + \chi^T(h_{-1}, 0, P) = y + Ph_{-1} - D(z, n)$$

where Rh_R is the rental cost, $\chi^T(h_{-1}, 0, P)$ is the selling cost, and $D(z, n)$ is the outstanding debt on the house h_{-1}.

(5) Sell the House and Buy a Different House

A homeowner who chooses this option have to sell the house and repay the all remaining mortgage loan for the house. Then, it chooses the size of a new house h and the type a new mortgage z. The value to sell the same house and buy another house solves

$$V_5(a, h_{-1}, z, n, \epsilon, e, j) := \max_{c\geq 0, a'\geq 0, h, z'} \left\{ u(c, h) + \beta\psi^s_{j+1} \int_{\epsilon'} W(a, h, z', N{-}1, \epsilon', e, j{+}1) \ \mathrm{d}F(\epsilon'|\epsilon) \right\}$$
$$\text{s.t.} \ \ c + a' + \chi^T(h_{-1}, h, P) + \lambda(z')Ph + m(z', N) + \delta_h Ph = y + Ph_{-1} - D(z, n).$$

(6) Default on the Mortgage

A homeowner who defaults on the mortgage obtains a discharge of debts, but it has to pay a default cost $\chi^D P\tilde{h}_{-1}(n)$ and abandon the house. The value to default on the mortgage solves

$$V_6(a, h_{-1}, z, n, \epsilon, e, j) := \max_{c\geq 0, a'\geq 0} \left\{ u(c, h^R) + \beta\psi^s_{j+1} \int_{\epsilon'} W(a', 0, 0, 0, \epsilon', e, j{+}1) \ \mathrm{d}F(\epsilon'|\epsilon) \right\}$$
$$\text{s.t.} \quad c + Rh^R + \chi^D P h_{-1} + a' = y$$

It is assumed that households in the final period J must rent for the non-Ponzi condition to be satisfied. Therefore, every homeowner at the beginning of the final period have to choose either to sell its house or to default on its mortgage.

1.3.5 Production Firm

The representative firm produces consumption goods and housing investment goods using a constant returns to scale technology for each production. The production function for consumption goods is

$$Y_t = A_{Yt} K_{Yt}^{\gamma} N_{Yt}^{1-\gamma}, \quad \gamma \in (0,1) \tag{1.6}$$

where A_{Yt} is aggregate productivity of consumption goods production, and K_{Yt} and N_{Yt} are the capital and the labor inputs used in consumption goods production.

The production function for housing investment goods is

$$Y_H = A_H K_H^{\alpha_K} N_H^{\alpha_N} L^{1-\alpha_K-\alpha_N}, \quad (\alpha_K + \alpha_N) \in (0,1) \tag{1.7}$$

where A_{Ht} is aggregate productivity of housing investment goods production, and K_{Ht}, N_{Ht}, and L are the capital and the labor inputs and the new supply of land used in consumption goods production. Note that housing investment goods production requires land. I assume that aggregate supply of land is fixed as in Davis and Heathcote (2005).

Given wage rate w_t, capital rental rate r_t, and house price p_t, the firm solves the following problem:

$$\tilde{\Pi} := \max_{\{K_Y, K_H, N_Y, N_H\}} \Big\{ Y + PY_H - r_t(K_Y + K_H) - w_t(N_Y + N_H) \Big\} \tag{1.8}$$
$$s.t. \quad (1.6) \quad \text{and} \quad (1.7)$$

Since aggregate supply of land is exogenously given, the housing production technology is decreasing returns to scale with respect to capital and labor. Therefore, the profit $\tilde{\Pi}$ is positive. I assume that households own the representative firm and its profit is equally distributed across all households.

1.3.6 Housing Rental Agency

There is a representative firm which supplies rental housing, which is called a housing rental agency. At the beginning of each period, it sells all properties held in the previous period H^R_{-1} and purchases properties to be rented in this period H^R. Then, it obtains revenue by renting them RH^R and pays the maintenance cost $\delta_R PH^R$. Therefore, the problem of the housing rental agency is

$$V(H^R_{-1}) := \max_{H^R} \left\{ RH^R - P(H^R - H^R_{-1}) - \delta_R PH^R + \frac{1}{1+r} V(H^R) \right\}$$

where δ is the rate of depreciation, R is the rental price and P is the house price. I assume that housing transactions by the housing rental agency do not incur a transaction cost $\chi^T(\cdot)$.

The solution to the problem gives the following no-arbitrage condition,

$$R = \left(\frac{r}{1+r} + \delta^R \right) P. \tag{1.9}$$

This condition also imply that the agency makes zero profit and it is indifferent about how much rental housing to supply.

1.3.7 Government

The government in this model collects income taxes and consume them all in each period. Therefore, the government budget constraint is $G = \int T(\ \tilde{y}\ )\ \mathrm{d}\mu$, where G is the government expenditure and $\tilde{y}$ is the household taxable income defined by (1.5).

The government also runs a social security system, that is, it collects the social security taxes from the working households and provide benefits to retired households. Given a replacement ratio ς, the government set a social security benefit $\bar{y}_e$ to the average labor earnings multiplied by the replacement ratio ς, for each education group. The social security benefits are assumed to depend on the education level, because the benefits are moderately increasing with the contributions in the U.S. system and because the average earnings are sizably different across education groups (Figure 2).

The social security tax rate τ_p is set so that the social security budget constraint is satisfied for each education groups in each period.

$$\int I_R \cdot I_e \cdot \bar{y}_e \ \mathrm{d}\mu = \int (1 - I_R) \cdot I_e \cdot \tau_p w \varphi_{ej} \epsilon \ \mathrm{d}\mu \tag{1.10}$$

where I_R is the indicator function for retirement and I_e is the indicator function of education levels. By substituting the benefit $\bar{y}_e := \int \varsigma w \varphi_{ej} \epsilon (1-I_R) \cdot I_e \, \mathrm{d}\mu$, the equation (1.10) gives

$$\tau_p = \varsigma \cdot \frac{\int I_R \, \mathrm{d}\mu}{\int (1 - I_R) \, \mathrm{d}\mu}. \tag{1.11}$$

That is, the tax rate τ_p is the replacement ratio times the ratio of the retired population to the working population.

1.3.8 Housing Market Clearing

In this model, the housing market clearing condition is complicated because there are many actions involved. The supply of housing investment goods in each period is given by

$$Y_H + \int h_{-1} \cdot 1_{(\theta \in \{4,5,6\})} \, \mathrm{d}\mu + \int \frac{1-\psi_j^s}{\psi_j^s} h_{-1} \, \mathrm{d}\mu + H_{-1}^R.$$

The first term Y_H is the new housing invest goods produced by the production firm. The second term is the sum of the housing sold by homeowners who sell or resize their houses and the housing of homeowners who default on a mortgage. The third term is the housing of homeowners who unexpectedly die at the beginning of the period and the fourth term is the housing sold by the housing rental agency.

The demand of housing in each period is given by

$$\int h \cdot 1_{(\theta \in \{2,4\})} \, \mathrm{d}\mu + H^R + \delta_h \int h \cdot 1_{(\theta \in \{2,3,5\})} \, \mathrm{d}\mu + \delta_R H^R.$$

The first term captures demand from renters who decide to buy a house and homeowners who decide to resize their houses. The second term is demand from the housing rental agency. The last two terms capture the housing investment goods demanded for a maintenance purpose by homeowners and the housing rental agency.

1.3.9 Equilibrium

Definition:

An equilibrium consists of prices (w, P, R), value and policy functions, and a stationary distribution μ such that

(1) Value and policy functions solve the agents' problems.

(2) Markets clear.

(3) Government budget constraint is satisfied.

(4) μ is implied by the decision rules.

1.4 Quantitative Analysis

This section describes calibration of the model and main results of the analysis. The main objective is to quantify the effects of a mortgage credit crunch and a recession and determine which factor is quantitatively more important for understanding the current housing bust. In the model, a mortgage credit crunch is implemented as the fade-out of IO mortgages and a recession is implemented as declines in labor earnings.

1.4.1 Calibration

The model parameters are calibrated so that key statistics of the model economy in the 2000 steady state match the actual statistics in the year 2000. There are parameters that can be directly specified from their implication. There are also parameters that need to be jointly determined to match a set of statistics in the model with the corresponding statistics in the data.

One period in the model corresponds to three years. Households in the model start their life at age 20 and live until age 79. The exogenous survival probabilities $\{\psi_j^s\}_{j=1}^J$ are taken from "United States Life Tables 2000" issued by the National Center for Health Statistics. The mandatory retirement age is set to age 65.

I use the following utility function

$$u(c,h) = (1-\gamma_h)\frac{c^{1-\sigma_c}}{1-\sigma_c} + \gamma_h\left(1_{own}\cdot\frac{(\theta h)^{1-\sigma_h}}{1-\sigma_h} + (1-1_{own})\cdot\frac{h^{1-\sigma_h}}{1-\sigma_h}\right)$$

where $\theta \cdot 1_{own}$ capture the owner-occupied premium. The coefficients (σ_c, σ_h) are set to (2,1). The share of the utility from housing service γ_h and the owner-occupied premium θ will be jointly determined. The time discounting factor β is set to 0.98.

One of the most important parameters for the quantitative analysis is the deterministic component of labor efficiency $\{\varphi_{ej}\}_{j=1}^J$ for each education level e. This is because

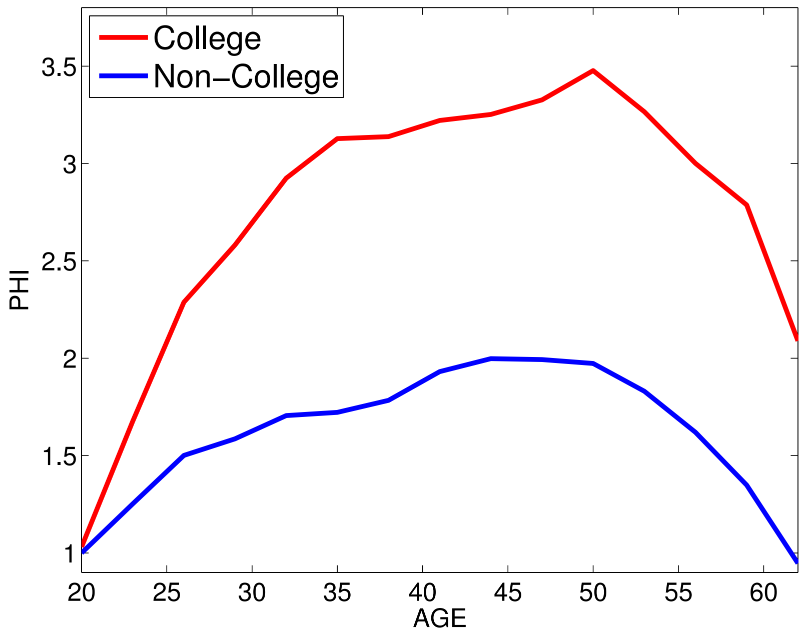

Figure 1.2: Calibrated Labor Efficiency φ_{ej}

Note: The data source is the March CPS supplement in 2000.

the shape of earning profiles are crucial for the decisions on mortgage, which have different payment schedules. As estimates of $\{\varphi_{ej}\}_{j=1}^{J}$, I take the average household labor income by age and education group in 2000 and normalize them by $\varphi_{11} = 1$. The average labor income by age and education group is a reasonable estimate of φ_{ej} under the assumption that idiosyncratic income shocks across large number of households.

Figure 2 shows the estimated profiles of the deterministic component of labor efficiency $\{\varphi_{ej}\}_{j=1}^{J}$ for non-college and college educated households. They clearly exhibit different shapes: the deterministic component of labor efficiency increases at a higher rate for college educated households than non-college educated households.

For the parameters of the stochastic component of labor efficiency, I use the estimates by Guvenen (2007).[11] However, since those estimates are of the annual frequency, I need to transform them into the values of the tri-annual frequency. Since there is

[11] I used the estimates for the "restricted income profiles" process in Guvenen (2007) because it uses the same specification as in this essay.

Table 1.3: Parameters Determined Jointly (Annual Values)

Parameter		Target	
Owner-occupied premium (θ)	10	Homeownership rate (college)	72 %
Min size of house ($\underline{h}$)	1.20	Homeownership rate (non-college)	61 %
Size of rental the housing unit (h_R)	0.43	Rent to Income Ratio for Low-Income Households	40 %
Utility weight on housing services (γ_h)	0.79	Housing exp. to GDP ratio	12 %
Relative productivity of housing production (A_H)	0.41	Housing inv. to GDP ratio	5 %
Tax function coefficient (η_0)	0.33	Govt. exp. to GDP ratio	15 %

no analytical relationship between the parameters in the annual model and those in the tri-annual model, I estimated them using the simulated data based on Guvenen's estimates.[12] Then, I discretize the income process into a seven state Markov chain using the method by Tauchen (1986).

The length of the mortgage N is set at 10 which corresponds to 30 years, and the rate of downpayment λ^{FRM} is set at 20 %. The mortgage interest r_m is set so that the annual rate becomes 5.5%, following Sommer et al. (2010). The default cost χ_D is set to 10%. The parameter values of housing transaction costs are also taken by Sommer et al. (2010) and they are $(\chi_0^T, \chi_b^T, \chi_s^T)$=(0,0.25,0.7).

In order to represent the income tax code in the U.S., I use the tax function originally given by Gouveia and Strauss (1994).

$$T(\tilde{y}) = \eta_0 \left\{ \tilde{y} - (\tilde{y}^{\eta_1} + \eta_2)^{\frac{-1}{\eta_2}} \right\}$$

where $\tilde{y}$ is the taxable income. I set η_1 to 0.768 as in Gouveia and Strauss (1994) and set η_2 to 0.371 so that the measurement units in the model become relevant. η_0 will be

[12] The annual model is $\nu_{t+1} = \rho\nu_t + \iota_{t+1}$, where $\iota_{t+1} \sim N(0, \sigma_\iota^2)$. The estimates in Guvenen (2007) are $(\rho, \sigma_\iota^2) = (0.988, 0.123)$. I simulated this annual model for 60,000 years and estimated $(\tilde{\rho}, \tilde{\sigma}_\iota^2)$ in the following tri-annual model:
$\nu_{t+3} + \nu_{t+4} + \nu_{t+5} = \tilde{\rho}(\nu_t + \nu_{t+1} + \nu_{t+2}) + \tilde{\iota}_{t+3,t+4,t+5}$, where $\tilde{\iota}_{t+3,t+4,t+5} \sim N(0, \tilde{\sigma}_\iota^2)$.
The OLS estimates of $(\tilde{\rho}, \tilde{\sigma}_\iota^2)$ are (0.97, 0.53).

Table 1.4: Steady State Statistics

Target	Data	Model
Homeownership rate (college)	72 %	74 %
Homeownership rate (non-college)	61 %	63 %
Rent to Income Ratio for Low-Income Households	40 %	40 %
Housing expenditure to GDP ratio	12 %	15 %
Housing investment to GDP ratio	5 %	5 %
Gov. expenditure to GDP ratio	15 %	15 %

determined jointly.

The replacement ratio of the social security benefit to the average labor earnings is set to 30%. The social security tax rate τ_p is determined so that aggregate social security tax revenue is equal to aggregate social security benefits. The resulting tax rate is 6.6%.

Including those already mentioned, there are five housing related parameters and one tax parameter that need to be jointly determined. They are the utility premium from owning a house θ, the rental unit size h_R, the minimum housing size $\underline{h}$, the utility weight on housing services γ_h, the relative productivity of housing A_H, and the tax function coefficient η_0.

I determine these parameters so that the following six statistics in the model are close to the 2000 values in the data: the homeownership rate among college educated households, the homeownership rate among non-college educated households, the rent-to-income ratio for low-income renters, the housing expenditure to GDP ratio, the housing investment to GDP ratio, and the government expenditure to GDP ratio.

The homeownership rates are calculated from the CPS March supplement data. The value for the rent-to-income ratio for low-income renters are taken from Green and Malpezzi (1993, p11). The low-income renter is defined as renting households whose income is below 30th percentile of the income distribution. The last three statistics are calculated from the national income and product account (NIPA) tables.

Table 3 reports the calibrated values of the six parameters and Table 4 shows the targeted statistics and those generated by the model. According to Table 4, the calibrated model overall well captures the targeted statistics. Also, it captures the gap between the homeownership rates across education groups reasonably well, given that the only source for the gap is the shapes of earning profiles $\{\varphi_{ej}\}_{j=1}^{J}$: the labor earning profiles are higher and steeper for college educated households (Figure 2).

1.4.2 Model Performance for the Period of 2000-2005

In this section, I examine the effects of the introduction of the IO mortgages, using the calibrated model.

There are two objectives for this exercise. The first objective is to demonstrate the main mechanism through which the mortgage innovation generates a larger increase in homeownership among college educated households. The second objective is to check if the mechanism can quantitatively account for the observed distributional change in homeownership across education groups during the early 2000s. If the calibrated model does a good job in accounting for the effects of the mortgage innovation, we can use this model to assess the effects of the mortgage credit crunch in the late 2000s, which is the main objective in this essay.

I conduct comparative steady state analyses considering the steady states in 2000 and 2005. There are only FRMs available to households in the 2000 steady state while both FRMs and IO mortgages are available to households in the 2005 steady state. That is, the only difference between the two steady states is the availability of the IO mortgage.

In the analysis, we keep the population distribution constant over the five years. This is because five years are too short for a stationary population distribution to converge to a new stationary population distribution.[13] Therefore, for the quantitative analysis, we look at the change in the homeownership rate, net of the population changes, in the data.

$$\Delta\tilde{h}_t := \sum_j \sum_e \mu_{jt_0} s_{jt_0} \mathbf{h^e_{jt}} - \sum_j \sum_e \mu_{jt_0} s_{jt_0} \mathbf{h^e_{jt_0}}. \tag{1.12}$$

[13] Since the main focus of this essay is about the change in decisions, instead of the change in the composition of the population, the cost of

Table 1.5: Change in the Homeownership Rate (2000-2005)

	Data	Model
Change in the homeownership rate $\Delta\tilde{h}_t$	1.2 %	1.2 %
Contribution:		
Homeownership (non-college)	6 %	7 %
Homeownership (college)	**94 %**	**93 %**
	100 %	100 %

Note: $\Delta\tilde{h}_t$ is the change in the homeownership rate, net of population changes, which is defined in (1.12).

This is the change in the aggregate homeownership rate driven by the changes in the homeownership rate in each age and education groups. In this way, we can compare the model results with the corresponding data.

Table 5 shows the change in the homeownership rate and the contribution of each factor[14] in the data and in the model for the period of 2000-2005. The results in the model are calculated from the statistics in the 2000 steady state and in the 2005 steady state. Since the demographic changes are not considered in the model, the whole change in the homeownership rate is explained by the changes in the homeownership rate among non-college educated households and college educated households.

Table 5 shows that the introduction of the IO mortgage accounts for the observed increase in the homeownership rate (1.2%) during the early 2000s. More importantly, it also accounts for the pattern in the contributions of education-specific homeownership rates. In the model, the contribution of the increase in homeownership among college educated households is 93%, which is close to the contribution observed in the data (94%). This result confirms that the calibrated model successfully captures the distributional change in homeownership during the early 2000s.

In order to demonstrate what happens behind these aggregate results, Figure 3 shows age profiles of the homeownership rate and the fraction of IO mortgage users

[14] The measure of the contribution is explained in Appendix A-1.

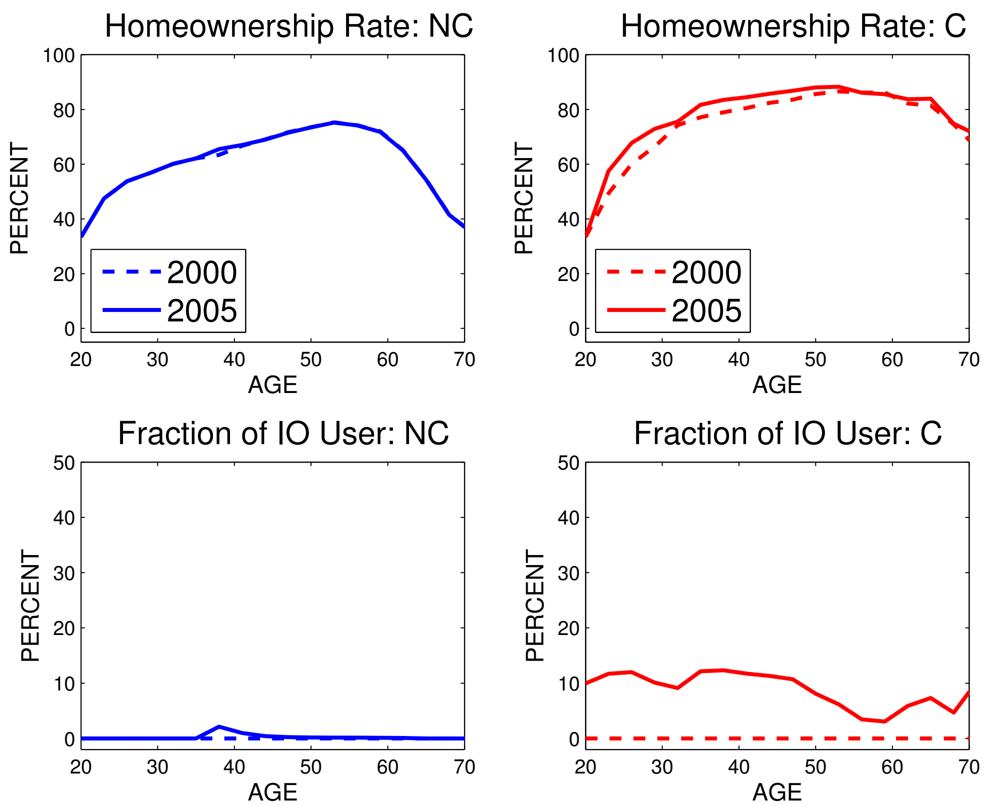

Figure 1.3: Changes from 2000 to 2005

Note: "NC" stands for non-college educated households and "C" stands for college educated households.

in each education group. Each panel shows the age profiles in the 2000 steady state and in the 2005 steady state. In the two upper panels, we see large increases in the homeownership rate among young college educated households. In addition, the bottom two panels show that IO mortgages are most extensively used among young college educated households. This means that the introduction of IO mortgages induced new homeowners among young college educated households the most.

The shape of earning profiles is the key to understand the higher usage of IO mortgages among college educated households. As we saw in Figure 2, the income growth rate is much higher for college educated households than non-college educated households. This means that young college educated households desire more strongly to increase housing and non-housing consumption to smooth out their consumption paths over lifetime. Therefore, the initial lower payments on IO mortgages are more attractive to young college educated households than young non-college educated households.[15]

In summary, this subsection showed that the mortgage innovation accounts for the distributional feature of the housing boom during the early 2000s. Given this performance of the calibrated model, I use this model to figure out the main driving force of the housing bust during the late 2000 in the next subsection.

1.4.3 Results

The main objective of this essay is to disentangle a mortgage credit crunch and a severe recession during the housing bust. For this purpose, I consider three scenarios for the 2010 steady state and conduct comparative steady state analysis for each scenario. In the first scenario, the IO mortgage becomes unavailable to households again (mortgage credit crunch). In the second scenario, I feed the model with the observed decline in the average labor income by age and education groups (recession). The income declines are measured using the labor earnings data from the CPS in 2005 and 2009[16] , which are deflated by the CPI. These income declines are the observed median declines in real labor earnings from 2005 to 2009 in the CPS data (Table 8 in Appendix A-3). As the third scenario, I consider the case where both happen; labor income declines and the supply of IO mortgages vanishes.

Table 6 shows the change in the homeownership rate, net of the population changes, and the contribution of each factor in the data and in the model. The results in the model are calculated using the statistics in the 2005 steady state and in the 2010 steady state of each scenario. First of all, from the third and the forth columns of the table, we see that the recession generates a larger decrease in the aggregate homeownership rate than the mortgage credit crunch (-1.2% vs -1.7%). More importantly, Table 6 shows that these two factors predict different patterns of contributions across education groups.

[15] Doms and Krainer (2007) demonstrate this idea clearly using a simple 2 period model. They also document that there was a larger change in the housing consumption to income ratio among college educated young households during the period of 1995-2005.

[16] The CPS March supplement provides the amount of household labor earnings during the last 12 months. Therefore, the March supplement in the year t provides the income information in the year $t-1$. Since the March supplement in 2011 is not available at this point, I use the income declines during 2005-2009.

Table 1.6: Change in the Homeownership Rate (2005-2010)

	Data	Model: Mortgage Credit Crunch	Model: Recession	Model: Both
$\Delta\tilde{h}_t$	-4.4 %	-1.2 %	-1.7 %	-2.7 %
Contribution:				
Homeownership (non-college)	**-66 %**	-7 %	**-83 %**	**-52 %**
Homeownership (college)	-34 %	**-93 %**	-17 %	-48 %
	-100 %	-100 %	-100 %	-100 %

Note: $\Delta\tilde{h}_t$ is the change in the homeownership rate, net of population changes, which is defined in (1.12).

The mortgage credit crunch predicts that the decline in homeownership among college educated households contributes the most (-7% vs -93%), but the recession predicts that the decline in homeownership among non-college educated households contributes the most (-83% vs -17%).

The credit contraction affects college educated households more because their usage of the IO mortgage is higher when it is available. Since college educated young households expect to have higher future earnings, they have more incentive to use the IO mortgage with initial lower payments. On the other hand, the recession affects non-college educated households more because they get larger income declines than college educated households. Also, non-college educated households tend to be more sensitive to income declines regarding the rent-or-own decision due to their lower income level and the presence of the minimum house size. The results suggest the importance of the observed heterogeneous income shocks to understand the housing bust.

The last column of Table 6 shows the results in the third scenario where the mortgage credit crunch and the recession both happen. Not surprisingly, the decline in the aggregate homeownership rate is the largest in this case (-2.7%). This scenario also successfully predicts that the decrease in homeownership among non-college educated households contributed the most (-52%), although it underestimates the gap between those contributions.

In summary, there are two main findings in this subsection. The first one is that the recession was the main driving force behind the distributional feature of the housing bust. The recession generates a proportionally larger decline in homeownership among non-college educated households, which is consistent with the data. This is because the observed real income decline was larger for those households. The last finding is that the mortgage credit crunch cannot account for the distributional feature of the housing bust. This is because IO mortgages are most extensively used among college educated households when they are available.

1.5 Conclusion

This essay documented that there has been a proportionally larger decline in homeownership among non-college educated households than college educated households since 2005 in the United States. I then investigated if either a mortgage credit crunch or a severe recession accounts for the observed pattern of declines in homeownership across education groups. In order to identify the effects of each factor, I constructed a general equilibrium life-cycle model with housing and mortgages and conducted counterfactual exercises. In the analysis, the mortgage credit crunch was implemented as the contraction of supply for delayed-amortization mortgages and the severe recession was implemented as declines in real labor earnings.

The main finding is that the mortgage credit crunch alone is not able to account for the distributional feature of the housing bust. It predicts a proportionally larger decrease in homeownership among college educated households, contrary to the observation. This is because delayed-amortization mortgages are more extensively used among college educated households, who have steeper earning profiles and find the lower initial payments on those mortgages more useful for consumption-smoothing.

The observed declines in real labor income, on the other hand, generate a larger decrease in homeownership among non-college educated households, consistent with the data. This is because the observed decline in real labor earnings was about four times larger for those households, based on the CPS data, which reflects a sharper increase in their unemployment rate.

Chapter 2

Risk and Risk-Sharing

Many studies rejected the implications of full-insurance to income shocks.[1] The focus in the literature has shifted to investigating how large and how important departures from full-insurance are.[2] In this paper, we propose an observation-based approach for measuring the welfare cost from the lack of full-insurance and the degree of partial insurance on the realized history of aggregate and idiosyncratic shocks. Then, we estimate them by using a synthetic panel data set constructed from the Consumer Expenditure Survey (CEX) in the U.S. for the period of 1980-2009.

Measuring the welfare cost from the lack of full-insurance requires the information about the state space of the economy: the set of all possible histories of income shocks and the probabilities over all histories. Given this information, one can measure the welfare cost and the degree of partial insurance by comparing the allocations in the complete market economy and in the actual economy under same income shocks. However, such information is not available and it has to be essentially assumed.

In this essay, we provide a welfare cost measure without assuming any specific state space of the economy, by focusing on the consumption-smoothing performance over time on the realized history of income shocks. Our approach exploits the fact that, under complete markets, the efficient consumption growth rate over time in every history of shocks does not depend on the information about the entire state space of the economy.

[1] For example, see Cochrane (1991), Mace (1991), Nelson (1994), Attanasio and Davis (1996), Brav et al. (2002), and Guvenen (2007).

[2] Attanasio and Davis (1996), Heathcote et al. (2008), Schulhofer-Wohl (2007), and Blundell et al. (2008) investigated these questions.

Our welfare cost measure is based on the difference between the expenditures of the observed consumption path in the data and the "smooth" consumption path. The "smooth" consumption path satisfies the Euler equation under full-insurance and achieves the utility level from the observed consumption path at the lowest cost.[3] Thus, the welfare cost measure takes zero if the observed consumption path satisfies the Euler equation under full-insurance every period, and it takes a positive value otherwise. We define the welfare cost in a way that we can interpret it as a uniform percentage variation of consumption that the household could have saved under full-insurance in achieving the same utility level on the realized history of shocks.

The notion of the degree of partial insurance is the percentage of welfare cost that a household reduced under the partial insurance available in the actual economy.[4] Specifically, we apply the same way of measuring the welfare cost to the observed income path, which is considered as the consumption path in autarky. The difference between the welfare costs based on the income path and the consumption path is defined as the measure of the degree of partial insurance. By construction, it takes zero if the observed consumption path is equal to the observed income path.

In order to apply these measures to the U.S. economy, we construct a panel data set for synthetic cohorts, which are defined based on birth years and education attainments of household-heads, using the data from the CEX during the period of 1980-2009. The use of the synthetic panel data enables us to analyze risk-sharing performance in the U.S., although there is no panel data that provides information on both consumption and income.[5]

There are two main findings. First, on average, households in the U.S. reduced 64% of their welfare cost due to cohort-specific income shocks, using risk-sharing mechanisms available. This reduction corresponds to the 2.7% of their annual expenditure on nondurables and services. The welfare cost from the remaining uninsured risk was 0.9% of their annual expenditure on nondurables and services. Note that our results are

[3] We control for aggregate shocks and some preference shocks in calculating the "smooth" consumption path. When we apply this welfare cost measure to the U.S. economy, we also consider measurement errors in data in calculating its standard error.

[4] Blundell et al. (2008) also propose a measure of partial insurance. They measure how much of unpredicted income shocks are reflected in consumption growth. That is, their measure captures changes in consumption, whereas ours captures changes in utility due to partial insurance.

[5] Although the Panel Study of Income Dynamics (PSID) is a good source of earnings information, the consumption information is limited to food-consumption.

based on the cohort-specific income shocks, which are uncorrelated with individual-level preference shocks, such as changes in health status and family structure, because we use the synthetic panel data.

The second finding is on the relationship between the degree of partial insurance and the amount of income risk at the cohort-level. The amount of income risk is measured by the welfare cost from cohort-specific income shocks. We find a positive correlation between them; households who face a higher risk tend to insure a larger portion of their risk. This result implies either or both of the following. The first implication is that households with a considerable amount of income risk make more of an effort to hedge their risk. This is a prediction of the models with limited enforcement, such as Kocherlakota (1996) and Krueger and Perri (2006). The second one is that the dispersion in income risk mostly comes from transitory shocks, which are easily insured.

The rest of this essay consists of four sections. Section 2 derives measures of the welfare gain from achieving full-insurance, the amount of income risk, and the degree of partial insurance. Section 3 describes how we construct a synthetic panel data set from the CEX. It also explains the definitions of consumption and income, and the estimation of cohort-level consumption and income. Section 4 demonstrates the results and Section 5 concludes.

2.1 Risk-Sharing Measures

In this section, we define our measures of the welfare cost from the lack of full-insurance and the degree of partial insurance, on the realized history of income shocks. We also discuss the relationship to the other existing measures in the literature.

2.1.1 Welfare Cost from the Lack of Full-Insurance

Our notion of the welfare cost from the lack of full-insurance is the percentage consumption variation required to compensate a household for living without full-insurance on the realized history of aggregate and idiosyncratic income shocks. We ask how effectively a household smooth out its consumption path over time on the realized history of shocks.

Specifically, we consider the expenditure of the observed consumption path of a household i, controlling for aggregate shocks and some preference shocks on the realized history of shocks. We also consider the expenditure of the "smooth" consumption path on the realized history, which satisfies the Euler equation under full-insurance and achieves the utility level from the observed consumption path at the lowest cost. Then, the welfare cost from the lack of full-insurance on the realized history is measured based on the difference between the expenditures of the observed consumption path and the "smooth" consumption path of the household i. We consider the deviation from the "smooth" consumption, which remains after controlling for aggregate shocks and some preference shocks, as a result of the lack of full-insurance.

Formally, we define the welfare cost from the lack of full-insurance on the realized history of shocks $\mathbf{s}$ as

$$\delta(\mathbf{c}^i, \mathbf{s}) := \frac{\rho^{obs}(\mathbf{c}^i, \mathbf{s}) - \rho^*(\mathbf{c}^i, \mathbf{s})}{\rho^{obs}(\mathbf{c}^i, \mathbf{s})} \tag{2.1}$$

$$\text{where} \quad \rho^{obs}(\mathbf{c}^i, \mathbf{s}) := \sum_t^T P_t(\mathbf{s}) \; c_{it}^{obs}, \tag{2.2}$$

where $\mathbf{c}^i := \{c_{it}^{obs}\}_t^T$, c_{it}^{obs} denotes the observed consumption for household i at time t in data, $P_t(\mathbf{s})$ is the state price at time t, and $\rho^*(\mathbf{c}^i, \mathbf{s})$ is the minimum expenditure, which is defined by

$$\rho^*(\mathbf{c}^i, \mathbf{s}) := \min_{\{c_{it}(\mathbf{s})\}_{t=1}^T} \sum_{t=1}^T P_t(\mathbf{s}) c_{it}(\mathbf{s}) \tag{2.3}$$

$$s.t. \quad \sum_{t=1}^T \beta_i^{t-1} u_i(c_{it}(\mathbf{s}); b_{it}) \geq \sum_{t=1}^T \beta_i^{t-1} u_i(c_{it}^{obs}; b_{it}),$$

where b_{it} is the taste shifter.[6] This minimized expenditure, $\rho^*(\mathbf{c}^i, \mathbf{s})$, is a possible minimum cost needed to attain the realized utility level if households can fully smooth out its consumption path over time. In other words, the consumption path that achieves $\rho^*(\mathbf{c}^i, \mathbf{s})$ satisfies the Euler equation under complete markets.

[6] For the empirical analysis in section 3 and 4, we estimate the taste shifters $\{b_{it}\}_{i,t}$ from the shape of the average age-consumption profile in the data, as in Attanasio and Davis (1996). Since we will use group consumption and income data in the empirical analysis, individual preference shocks, such as changes in health status and family structure, are canceled out by taking the average of a large number of households in each group.

Note that the observed expenditure $\rho^{obs}(\mathbf{c}^i, \mathbf{s})$ and the minimum expenditure $\rho^*(\mathbf{c}^i, \mathbf{s})$ are evaluated at the same state prices on the realized path, $\{ \ P_t(\mathbf{s}) \ \}_{t=1}^{T}$. Appendix 1 provides the role of the state prices $P_t(\cdot)$ in a complete market economy and explain the relationship between the state prices $P_t(\cdot)$ and the Arrow-Debreu prices.

The difference between $\rho^{obs}(\mathbf{c}^i, \mathbf{s})$ and $\rho^*(\mathbf{c}^i, \mathbf{s})$, which is the core of the welfare cost $\delta(\mathbf{c}^i, \mathbf{s})$, measures the cost that the household i could have saved by smoothing out its consumption path on the history $\mathbf{s}$ if they have an access to complete markets. In other words, the difference is the loss from the failure of consumption-smoothing over time on the history $\mathbf{s}$, due to the lack of access to a complete market.

The welfare cost $\delta(\mathbf{c}^i, \mathbf{s})$ evaluates the loss from the failure of consumption-smoothing, $\rho^{obs}(\mathbf{c}^i, \mathbf{s}) - \rho^*(\mathbf{c}^i, \mathbf{s})$, in terms of a ratio of the observed expenditure. Therefore, $\delta(\mathbf{c}^i, \mathbf{s})$ can be interpreted as the percentage compensating variation from the partial insurance in the actual economy to the full-insurance in a hypothetical complete market economy, on the realized history $\mathbf{s}$. The household i is willing to give up $\delta(\mathbf{c}^i, \mathbf{s})\%$ of the annual consumption expenditure if the household can get full-insurance on the realized history of shocks $\mathbf{s}$. In other words, the welfare cost of the household i from giving up the "smooth" consumption path is its $\delta(\mathbf{c}^i, \mathbf{s})\%$ of the annual consumption expenditure.

$\delta(\mathbf{c}^i, \mathbf{s})$ takes zero if the household i perfectly smooth out its consumption path on the history of shocks $\mathbf{s}$. That is , if the household-i's consumption path satisfies the Euler equation under full-insurance on $\mathbf{s}$, $\rho^{obs}(\mathbf{c}^i, \mathbf{s}) = \rho^*(\mathbf{c}^i, \mathbf{s})$, and hence $\delta(\mathbf{c}^i, \mathbf{s}) = 0$. On the other hand, if the household i poorly smooth out its consumption path over time, $\delta(\mathbf{c}^i, \mathbf{s})$ takes a large positive number. Since c_{it}^{obs} is always affordable given the constraint of the cost minimization problem, $\rho^*(\mathbf{c}^i, \mathbf{s})$ is always smaller than $\rho^{obs}(\mathbf{c}^i, \mathbf{s})$. This assures that $\delta(\mathbf{c}^i, \mathbf{s}) \geq 0$.

Discussion

Ideally, the welfare cost from the lack of full-insurance is measured by comparing the allocations in a complete market economy and in the actual economy under the same income risk. However, this requires assumptions on (1) the set of all possible histories of income shocks and the probabilities over all histories and (2) the market structure in the actual economy. These assumptions are not verifiable with data, although measures are sensitive to those assumptions.

Attanasio and Davis (1996) assume the following, regarding the state space of the economy (1): (a) the probability that a household is assigned to a household group i is equal to the fraction of the group i in the data in each period, (b) there are only two possible histories of within-group income shocks $\{s^1, s^2\}$, and (c) the associated probabilities are $(0.5, 0.5)$. They also assume that the optimal allocation in the actual economy is $\{c_{it}(s^1), c_{it}(s^2)\}_{t=1}^T = \{c_{it}^{obs}, 2c_{it}^* - c_{it}^{obs}\}_{t=1}^T$ where $\{\{c_{it}^*\}_{t=1}^T\}_i$ is the allocation under full-insurance. In other words, they implicitly assume a market structure under which the optimal allocation is $\{c_i(s^1), c_i(s^2)\} = \{c_{it}^{obs}, 2c_{it}^* - c_{it}^{obs}\}$, for the actual economy (2). Then, they measure the welfare cost by comparing the expected utilities from the allocation in the "actual" economy and that in the full-insurance economy.

Heathcote et al. (2008) use a different set of assumptions on (1) and (2). Regarding the state space of the economy (1), they assume income shocks which follow a log-normal distribution and estimate the distribution using data. For the actual economy (2), they assume that there exist complete markets for transitory shocks and no markets for permanent shocks. Then, they measure the welfare cost by comparing the allocations in the partial insurance economy and in the complete market economy.

As Attanasio and Davis (1996), we consider the observed consumption path as the optimal path under partial insurance available in the actual economy. However, we do not use any assumptions on (1). What we do is to evaluate the welfare cost based on the performance of consumption-smoothing over time on the realized path of aggregate and idiosyncratic shocks. It is possible because the optimal consumption growth rate under full-insurance does not depend on the assumption on (1). In Appendix 1, we prove that this is true under a state-separable utility function. Assumptions on the un-realized histories of shocks and their probabilities are needed to determine the level of consumption on each history.

Since we focus on the performance of consumption-smoothing over time, we do not have to take a specific stand point about (1) the set of all possible histories of shocks and the associated probabilities and (2) the market structure of the actual economy, which are essentially un-verifiable with the data. This is an advantage of our approach. However, there is a drawback. Without additional assumptions for (1) and (2), our welfare cost measure is silent about consumption-smoothing over histories of shocks.

2.1.2 Degree of Partial Insurance

We ask what percentage of the welfare cost from the lack of full-insurance in the autarky economy is reduced in the actual economy, in measuring the degree of partial insurance. In the autarky economy, the consumption of a household i is the income of the household, because there is no risk-sharing mechanisms available. Therefore, the welfare cost in the autarky economy represents the amount of income risk that the household i originally faces. On the other hand, the actual consumption of the household i is different from its income. The consumption data reflects the partial insurance available in the actual economy. Therefore, we measure the degree of partial insurance based on the difference in the welfare costs from the income and the consumption paths of the household i.

Formally, the measure of the degree of partial insurance is defined as

$$\hat{\phi}(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s}) := \delta(\mathbf{y}^i, \mathbf{s}) - \delta(\mathbf{c}^i, \mathbf{s}), \tag{2.4}$$

where $\delta(\mathbf{y}^i, \mathbf{s})$ is the welfare cost in the autarky economy, which is defined by

$$\delta(\mathbf{y}^i, \mathbf{s}) := \frac{\rho^{obs}(\mathbf{y}^i, \mathbf{s}) - \rho^*(\mathbf{y}^i, \mathbf{s})}{\rho^{obs}(\mathbf{c}^i, \mathbf{s})}, \tag{2.5}$$

where $\mathbf{y}^i := \{y_{it}^{obs}\}_t^T$, and y_{it}^{obs} is the income data of the household i at time t. $\rho^*(\mathbf{y}^i, \mathbf{s})$ and $\rho^{obs}(\mathbf{y}^i, \mathbf{s})$ are calculated by replacing observed consumption $\mathbf{c}^i$ by income $\mathbf{y}^i$, which is considered as the autarky consumption, in equations (2.3) and (2.2). Note that the denominator is the observed expenditure of consumption $\rho^{obs}(\mathbf{c}^i, \mathbf{s})$, instead of $\rho^{obs}(\mathbf{y}^i, \mathbf{s})$. Therefore, $\hat{\phi}(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$ is also in terms of the observed expenditure of consumption.

$\hat{\phi}(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$ is the welfare cost reduced by the risk-sharing mechanisms available in the actual economy, represented in terms of the observed consumption expenditure. In other words, the household i needs to receive $\hat{\phi}(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})\%$ of its annual consumption expenditure in order for the household i to achieve the same utility level in the autarky economy.

By construction, $\hat{\phi}(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$ takes zero if the observed consumption path $\mathbf{c}^i$ is equal to the observed income path $\mathbf{y}^i$. It takes a positive value if the degree of partial insurance is positive; the welfare cost from consumption fluctuations $\delta(\mathbf{c}^i, \mathbf{s})$ is lower than that from income fluctuations $\delta(\mathbf{y}^i, \mathbf{s})$.

If we take the ratio of $\hat{\phi}(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$ to $\delta(\mathbf{y}^i, \mathbf{s})$, we obtain a percentage measure of the degree of partial insurance, which takes a value between zero and one[7] :

$$\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s}) := \frac{\hat{\phi}(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})}{\delta(\mathbf{y}^i, \mathbf{s})}. \tag{2.6}$$

Since $\delta(\mathbf{y}^i, \mathbf{s})$ represents the income risk that the household i originally face, we can interprete $\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$ as the percentage of the household income risk hedged by the risk-sharing mechanisms available in the actual economy. Although $\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$ does not have a direct monetary value implication, it is useful to see what percentage of the original income risk is hedged through partial insurance.

Discussion

There is the following relationship among the measures of the welfare cost of *uninsured* income risk $\delta(\mathbf{c}^i, \mathbf{s})$, the welfare cost of *original* income risk $\delta(\mathbf{y}^i, \mathbf{s})$, and the degree of partial insurance $\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$:

$$\delta(\mathbf{c}^i, \mathbf{s}) = \delta(\mathbf{y}^i, \mathbf{s}) \times (1 - \phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})). \tag{2.7}$$

This equation explicitly shows that the welfare cost from the lack of full-insurance depends on two factors. One is the original amount of income risk $\delta(\mathbf{y}^i, \mathbf{s})$, and another is the degree of partial insurance $\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$.

This expression helps us to get a better understanding of risk-sharing. Although a high welfare cost comes from either or both of a large income risk that the household originally faces and a low degree of partial insurance, the welfare cost itself does not distinguish those cases. For example, even though households are able to share most of their income risk, the welfare cost from the lack of full-insurance is large if the magnitude of the income risk is large. By calculating $\delta(\mathbf{y}^i, \mathbf{s})$ and $\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$, we can tell the source of the welfare cost from the lack of full-insurance. This is an advantage of measuring the degree of risk-sharing based on the welfare cost, relative to the other existing approaches, such as Blundell et al. (2008).

[7] Although it rarely happens, consumption fluctuations are larger than income fluctuations in the data. In this case, $\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$ (and $\hat{\phi}(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$) takes a negative value, meaning a negative degree of partial insurance.

2.1.3 State Prices

In order to calculate the risk-sharing measures, we need to impute the state prices, $P_t(s)$. In the following subsection, we propose a way to impute the state prices on the observed history, $P_t(s)$, from data.

How to impute the state prices is the main issue in calculating our measures of risk-sharing. For our measures of risk-sharing, the price, $P_t(s)$, is the only factor that captures interdependency among households in I. Therefore, the result of the analysis depends on the way we impute the state prices.

In general, the state prices are not directly observed in data. Moreover, $P_t(s)$'s are the prices in a *hypothetical* complete market. Thus, we need some additional assumptions to derive a tractable way to impute the prices from data.

The first order conditions for the household maximization problem (HH) in Appendix 1 are

$$\beta_i^{t-1} u_i'(c_{it}^*(s)) = \lambda_i P_t(s), \tag{2.8}$$

for all i, t, and s, where λ_i is a Lagrange multiplier for household i's budget constraint. Then we have

$$\beta_i^{t-1} \frac{u_i'(c_{it}^*(s))}{u_i'(c_{i1}^*(s))} = \frac{P_t(s)}{P_1(s)}, \tag{2.9}$$

for all i, t, and s. We cannot use this equation directly to impute the state prices because the observed consumption path, c_{it}^{obs}, might be different from the allocation under perfect risk-sharing, $c_{it}^*(s)$.

Now we assume two additional assumptions: (i) an identical homothetic preference for every household, and (ii) the sum of the observed consumption over households is equal to the aggregate resource and hence the sum of the equilibrium consumption under complete markets. The first assumption enables us to directly use the Euler equation (2.9) to impute the prices from the aggregate resource in the economy. The second assumption enables us to impute the aggregate resource as the sum of the observed consumption.

Specifically, we assume that

$$\beta_i = \beta \qquad \text{and} \qquad u_i(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}, \qquad \forall i \in I. \tag{2.10}$$

Note that if $\gamma = 1$, $u_i(c) = \log(c)$ will be assumed. Given this identical homothetic preference, individual equilibrium consumption is a fixed fraction of the aggregate resource. Combining this fact with (2.9) gives

$$\beta^{t-1} \left(\frac{\sum_{i \in I} y_{it}(s)}{\sum_{i \in I} y_{i1}(s)} \right)^{-\gamma} = \frac{P_t(s)}{P_1(s)}. \tag{2.11}$$

This implies that prices do not depend on the distribution of wealth, but only the total amount of wealth in the whole economy. That is, the equation (2.11) depends neither on the specific history of shocks s nor its associated probability. It means that one unit of consumption is more expensive when the aggregate resource is scarce, and vice versa.

By assumption (ii), the aggregate resource must be equal to the aggregate consumption. Hence we obtain

$$\beta^{t-1} \left(\frac{\sum_{i \in I} c_{it}^{obs}}{\sum_{i \in I} c_{1t}^{obs}} \right)^{-\gamma} = \frac{P_t(s)}{P_1(s)}, \tag{2.12}$$

which is a formula for the state prices on the observed history. We normalize $P_1(s)$ to be one.

In this way, we can impute $P_t(s)$ and calculate the risk-sharing measures given a panel data set of consumption. Next section summarizes the data we use to study how efficiently households share their risk in the United States.

2.2 Data

This section describes the Consumer Expenditure Survey (CEX) data, which we use for the analysis. It also explains how we construct a synthetic panel data set from the CEX data and estimate cohort-level consumption and income, as well as our definition of consumption measure.

2.2.1 Consumer Expenditure Survey (CEX)

The CEX is issued by Bureau of Labor Statistics and provides a continuous flow of information on the buying habits of American consumers. It collects data on major items of expense, household characteristics, and income. In each quarter, 20 percent of the sample are new consumer units (CUs) introduced for the first time. Each CU in the sample is interviewed every three months over a 15-month period.

They use uniform questionnaires to collect expenditure information from the previous three months, while income information are collected in the second and fifth interviews only and they contain annual values for the 12 months prior to the interview month. Financial information is interviewed in the fifth quarter only. It is estimated that the CEX expenditure data covers about 90 to 95 percent of expenditures. For this broad coverage of expenditure, we decided to use the CEX.[8]

2.2.2 Non-Durable Consumption and Income

Our definition of consumption measure is household expenditures on non-durable goods and services, following Kocherlakota and Pistaferri (2008). It includes food (at home and away from home), alcoholic beverages and tobacco, heating fuels and utilities, transports (including gasoline), personal care, clothing and footwear, entertainments, other services (including domestic services). It excludes expenditure on various durables, housing (furniture, appliances, etc.), education and health.

Since our analysis focuses on nondurable consumption, we define household's income by consistently excluding the effect of the other durable expenditure. First, we calculate durable consumption by subtracting nondurable consumption from the total expenditure, which is aggregated according to the CEX definition. Next, we calculate the total income for a household by adding after-tax income to lump sum income, that is, (FINCATAX + NONINCMX). Finally, we subtract durable consumption from the total income, and then obtain household's income for nondurables. Our definition of household's income will be used to purchase nondurable goods or for financial purposes including insurance purchases, savings, or loan payments.

Since CEX households join the survey in different month, we collect the expenditure data in the MTAB file based on calendar periods, that is, the month when expenditures were made. We collect the data of income and sample weights in FMLY file and merge them with the expenditure data. Then, we aggregate them up to the annual data after

[8] Although the Panel Study of Income Dynamics (PSID) provides panel data, its coverage of expenditure is far small. It used to collect only food expenditure, although it starts asking other expenditure categories as well in the recent years.

forming synthetic cohort panel data. We deflate consumption expenditures and income using consumer price index for urban consumers (CPI-U) with the year 2000 as the basis year.[9]

We adjust consumption and income by dividing them by equivalence scales to control for household size. Equivalence scales take into account the fact that the needs of a household is not proportional to the number of persons in the household due to economies of scale in consumption.[10] Among many definitions of equivalence scales, we use "square root scale", which is equal to the square root of the number of persons in a household. Hence, our definition of consumption measure is constant price expenditures on nondurables and services over the square root of household size. The use of equivalence scales has a mechanical consequence on our risk-sharing measures; it decreases welfare gain and income risk measures, and increases partial insurance measure. This is because variations in consumption and income will decrease by being divided by equivalence scales.

2.2.3 Synthetic Cohorts

In order to measure the degree of risk-sharing, we need a panel data, although the CEX is designed as repeated cross-section data. Therefore, we construct a synthetic panel, as in Attanasio and Davis (1996). We form synthetic cohorts based on the birth year and educational attainment of the household head. That is, we construct a panel data set of each representative consumer unit with one of the combination of these characteristics. Therefore, in this empirical study, risk-sharing means insurance to cohort-specific risk, and not household-specific risk.

The birth cohorts are defined by 5-year band. The oldest cohort consists of people who were born between January 1910 and December 1914. We focus on household heads of age 25 to 75. The educational attainment is categorized into three levels: less than

[9] We do not use price index for each type of goods. According to Attanasio and Davis (1996), variations in price indexes are small in their sample from 1980 to 1990. Since we use the data from 1980 to 2006, it might be interesting to use appropriate price index for each type of expenditure because the increase in imports from China in the 2000s might decrease the price of cheaper goods, keeping the price of luxury goods unchanged. This effect may be important in consumption smoothing for low income households.

[10] In an extreme case, a couple living in one bedroom apartment only pay a half of the rent per person compared to a single person living in the same apartment.

high-school degree, high-school degree, and collage degree or more.

The number of CUs in a synthetic cohort varies across cohorts. This variation in the number of CUs in synthetic cohorts can be problematic. The time-series data of synthetic cohorts with few CUs tend to be much volatile than that of synthetic cohorts with many CUs, because household-specific risk are not averaged out. This leads to high standard errors for synthetic cohorts with few CUs. Also, if the time-series of consumption and income are too short, the risk-sharing measures suffer from a small sample bias. In an extreme case where the data is observed for only one year, the both measures of the welfare cost and the income risk take zero by construction.

We restrict the data as follows. First, we exclude the samples with incomplete response on income variables since it may be a signal of large measurement errors in the response. Next, we drop the synthetic cohorts whose number of CUs is strictly less than 5. Finally, we exclude the synthetic cohorts whose observations are strictly less than 15 years. We end up with 41 synthetic cohorts in our sample.

2.2.4 Estimation of Cohort-level Variables

Given the definition of synthetic cohorts, we estimate consumption and income paths for each cohort. Since we use the same way of estimation for both consumption and income, we explain how to construct the estimates of consumption paths for each cohort.

We consider a reduced form relationship between cohort-level consumption and individual household-level consumption in the cohort as follows:

$$\log(c_{ict}) = \log(c_{ct}) + \varepsilon_{ict}, \qquad \varepsilon_{ict} \sim i.i.d.(0, \sigma_{ct}^2) \tag{2.13}$$

where c_{ict} is consumption level of household i in cohort c at time t, c_{ct} is cohort-level consumption for cohort c at time t, and ε_{ict} is a household-specific idiosyncratic shock at time t, which has mean zero and variance σ_{ct}^2. That is, we model log of individual consumption as a random draw from a distribution with mean $\log(c_{ct})$ and variance σ_{ct}^2.

In this reduced form model, the simple average of $\log(c_{ict})$ over households in cohort c at time t is a consistent estimate of $\log(c_{ct})$ by the law of large numbers. Since the CEX is a random sample from U.S. population, we use the CEX sample weights in taking the average. We interpret the CEX sample weights as the number of off-sample households who are represented by the consumer unit in the sample. Namely, we consider that there are ω_{ict} households who are similar to household i, and hence whose consumptions are equal to c_{ict}. Therefore, our estimate of cohort-level logged consumption is the weighted average of logged consumption expenditures over households in the cohort, using the CEX sample weights. That is,

$$\widehat{\log(c_{ct})} := \frac{1}{\omega_{ct}} \sum_{i \in I_{ct}} \omega_{ict} \log(c_{ict}), \tag{2.14}$$

where I_{ct} is the set of households in cohort c at time t, ω_{ict} is the CEX sample weights, and $\omega_{ct} := \sum_{i \in I_{ct}} \omega_{ict}$.[11]

We construct annual estimates by aggregating monthly cohort-level consumptions from the MTAB file in the CEX. We choose the annual frequency, rather than quarterly frequency, for our analysis because risk-sharing can be tested more accurately in the low frequency data as discussed by Hayashi et al. (1996).

We estimate cohort-level income in the same way as cohort-level consumption. That is, our estimate of cohort-level logged income is the weighted average of logged income over households in the cohort, using the CEX sample weights.

We obtain the standard errors of the risk-sharing measures using a bootstrap method. For each bootstrap replication, we resample consumer units within a cohort for every period from the empirical distribution weighted by the CEX sample weights. Thus, each consumer unit is chosen with probability ω_{ict}/ω_{ct} at time t. This resampling procedure mechanically results in re-weighting the sample differently from the CEX sample weights, keeping the total sample weights in the cohort unchanged.[12] The number of replications is shown below tables.

We apply our theory of risk-sharing measures to the estimated cohort-level consumption and income paths. The results with standard errors are summarized in the next section.

[11] In order to obtain $\widehat{c_{ct}}$ from $\widehat{\log(c_{ct})}$, we need an adjustment based on asymptotic normality under the Central Limit Theorem. Specifically, we use the following formula:

$$\widehat{c_{ct}} = \exp(\widehat{\log(c_{ct})}) \times \exp(-\hat{\sigma}_{ct}^2/2N_{ct})$$

where N_{ct} is the number of sample for cohort c at time t and $\hat{\sigma}_{ct}^2$ is the estimated variance.

[12] We take a shortcut in this resampling procedure by setting a unit of resample at 50,000. That is, we resample 50,000 consumer units at once in terms of the CEX sample weights. Since the total CEX sample weights is about 6,000,000 for each month, we need about 120 random draws for each month, in making one bootstrap replication.

2.3 Results

We calculate our measures of risk-sharing using the constructed CEX synthetic panel data. The first subsection documents the main results on risk-sharing in the U.S. We find that households in the U.S. achieved the degree of partial insurance of 64% (out of 100%), and the welfare cost from the uninsured income risk was 0.9% of their annual expenditure on non-durables and services. In the second subsection, we show a positive correlation between the magnitude of income risk and the degree of partial insurance. This positive correlation between risk and risk-sharing has implications on the nature of the income risk and the risk-sharing behavior of households with different income risks. The third subsection documents that the degree of partial insurance is higher for a household group with a higher educational attainment. We also find that the amount of income risk also tends to be increasing with educational attainments, which imply the positive correlation with the degree of partial insurance. In the fourth subsection, we examine the role played by taxes and subsidies and find that they contribute to risk-sharing by reducing income risk. The final subsection shows the results under various values of parameters to investigate the sensitivity of the measures.

All the results in the first to fourth subsections are estimated under one specific set of parameters of the utility function and the baseline estimates of the state prices. The discount factor, β, is set at 0.98 and the coefficient of relative risk aversion, γ, is set at 2. the state prices are estimated from the CEX aggregate estimates of the consumption measure that we define in the data section. The age-specific taste shifters are estimated from the average age-consumption profile. Specifically, b_j is the average consumption, which is adjusted by the square root equivalence scale, for households whose head is in the age-group j.

2.3.1 Risk-Sharing Measures

Table 1 shows the economy-wide average of the risk-sharing measures: the welfare cost from the lack of full-insurance ($\delta(\mathbf{c}^i, \mathbf{s})$), the amount of income risk ($\delta(\mathbf{y}^i, \mathbf{s})$), and the effectiveness of partial insurance ($\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$). $\delta(\mathbf{c}^i, \mathbf{s})$ shows that U.S. households would gain 0.9% of their actual consumption expenditure on average if they could fully insure their cohort-specific income risk on the realized history $\mathbf{s}$. In other words, households

Table 2.1: Measures of Efficiency in Risk Sharing (Weighted Average)

Welfare Cost $\delta(\mathbf{c}^i, \mathbf{s})$	**Income Risk** $\delta(\mathbf{y}^i)$	**Partial Insurance** $\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$
0.9%	3.6%	64%
(0.03)	(0.09)	(1.04)

The risk-sharing measures are estimated at synthetic cohort level and weighted by the CEX sample weights in obtaining the average. The data covers from 1980 to 2009. The parameter values used for this table are $\beta = 0.98$, $\gamma = 2$, and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$. The age-specific taste shifters are considered. The bootstrapped standard errors are reported in parentheses. The number of bootstrap replications is 50.

whose annual expenditure on nondurables and services was \$10,000 are willing to give up \$90 per year if they can get full-insurance on the realized history.

$\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$ indicates that U.S. household insured 64% of their income risk on average by risk-sharing mechanisms available in the actual economy. Recall that $\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$ takes 0% if the consumption path is equal to the income path, and takes 100% if the consumption path satisfies the Euler equation under full insurance every period. Therefore, 64% implies that the degree of partial insurance achieved in the actual economy is high, but does not achieve full-insurance. This is consistent with the historical rejections of the full-insurance hypothesis in the risk-sharing literature, such as Cochrane (1991), Mace (1991), and Nelson (1994).

Another measure of the degree of partial insurance $\hat{\phi}(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$, which has a monetary value implication, was 2.7% (= 3.6%-0.9%). This means that the risk-sharing mechanisms available in the U.S. economy reduced the welfare cost from the income risk by 2.7% of the annual expenditure on non-durables and services. In other words, the welfare gain from the partial insurance available in the U.S. economy was 2.7% of the annual expenditure on non-durables and services.

Recall that all of our risk-sharing measures are measured at synthetic-cohort level, although Table 1 only shows the average values. In Appendix 2, we show the histograms of each measures.

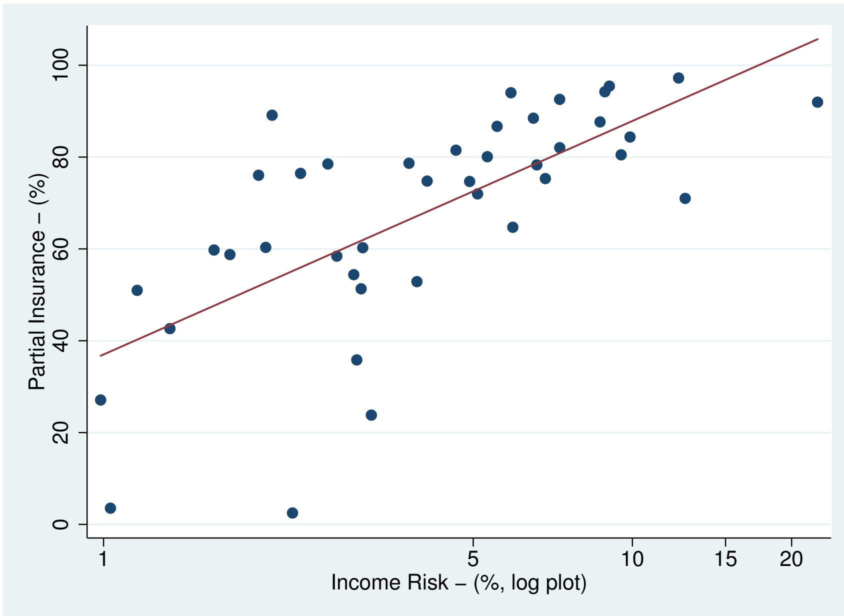

Figure 2.1: Partial Insurance v.s. Income Risk

This figure is the scatter plot of the estimates of $\delta(\mathbf{y}^i, \mathbf{s})$ and $\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$ for 42 synthetic cohorts. Use a log scale (base 2) for x-axis. Use $\beta = 0.98$, $\gamma = 2$, and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$. The fitted line is

$$\begin{array}{cclcll} \phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s}) & = & 37.0 & + & 15.3\ \log_2(\delta(\mathbf{y}^i, \mathbf{s})), & \qquad R^2 = 0.47, \\ & & (7.1) & & (2.6) & \end{array}$$

with the standard errors shown in parentheses. The coefficient on $\log_2(\delta(\mathbf{y}^i, \mathbf{s}))$ is statistically positive with t-value = 5.8. The correlation is 0.7.

2.3.2 Risk and Risk-Sharing

Interestingly, we find a positive correlation between $\delta(\mathbf{y}^i, \mathbf{s})$ and $\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$: the amount of income risk that households face and the degree of partial insurance.[13] Figure 2 is a scatter plot of $\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$ and log of $\delta(\mathbf{y}^i, \mathbf{s})$ in base 2. The degree of partial insurance and the size of income risk were positively correlated with coefficient 0.7. The solid line in Figure 1 is the fitted line of the regression model shown at the bottom of Figure 2. The regression result implies that doubling $\log_2(\delta(\mathbf{y}^i, \mathbf{s}))$ is associated with 15.3% increase in the degree of partial insurance $\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$.

[13] This is why the relationship (2.7) does not hold for the weighted averages reported in Table 1.

Table 2.2: Risk-Sharing Measures by Education Group

	Welfare Cost $\delta(\mathbf{c}^i)$	**Income Risk** $\delta(\mathbf{y}^i)$	**Partial Insurance** $\phi(\mathbf{c}^i, \mathbf{y}^i)$
No Degree	1.7%	4.1%	**54%**
High School	0.7%	1.8%	**57%**
College	1.0%	4.8%	**76%**
More	0.9%	9.7%	**90%**
All	0.9%	3.6%	**64%**

The risk-sharing measures are estimated at synthetic cohort level and weighted by the CEX sample weights in obtaining the average. The data covers from 1980 to 2009. The parameter values used for this table are $\beta = 0.98$, $\gamma = 2$, and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$. The age-specific taste shifters are considered.

This result implies either or both of the following. The first implication is that people with a considerable amount of income risk may tend to make more of an effort in hedging risk. This is a prediction of models with limited enforcement. In particular, Kocherlakota (1996) and Krueger and Perri (2006) argue that households facing higher income risk have a smaller incentive to deviate from the contract and hence the constrained optimal contract of those households can attain better risk-sharing. The second implication is that the variation in the amount of income risk mainly comes from transitory income risk. The degree of partial insurance may be higher for them reflecting the amount of transitory income risk, which are easily insured, even if households with a high income risk make the same amount of effort to hedge their risk.

2.3.3 Education and Risk-Sharing

Table 2 reports the estimates of the risk-sharing measures for four education groups. The four categories of educational attainments are (1) no degree, (2) high school degree, (3) college degree, and (4) more (graduate degree). Table 2 shows a clear pattern of the degree of partial insurance $\phi(\mathbf{c}^i, \mathbf{y}^i)$; households with a higher educational attainment achieve a high degree of partial insurance. $\phi(\mathbf{c}^i, \mathbf{y}^i)$ is 54% for households with no degree, while it is 90% for households with graduate degrees. Blundell et al. (2008) also examined the degree of partial insurance for different education groups and found that a higher degree of partial insurance for households with a higher educational attainment.

Table 2 also indicates that the amount of income risk $\delta(\mathbf{y}^i)$ is also increasing with educational attainment. Although the household group with no degree is an exception, this also implies that households with a higher educational attainment hedged a larger portion of their income risk because of their higher income risk.

Also, it is interesting that the welfare cost takes similar values for three education groups: 0.7% for high school educated households and 1.0% for college or more educated households. Households with no degree have the largest welfare cost from the lack of full insurance (1.7%). It is mainly because the degree of partial insurance is low (54%). Their income risk is moderate (4.1%) in a sense that it is the second lowest among those for four education groups.

2.3.4 Role of Taxes and Transfers

In order to investigate the role of taxes and transfers as a risk-sharing mechanism, we estimate the welfare cost in the autarky economy with taxes and transfers. So far, we use the before-tax household income data $\mathbf{y}^{B,i}$ in measuring the welfare cost in the autarky economy. Now, we use the after-tax income data $\mathbf{y}^{A,i}$ in measuring the welfare cost from the lack of full-insurance in the autarky economy with taxes and transfers. In the economy, households consume their income net of taxes and transfers.

Table 3 shows the results. The first and second columns of Table 3 shows that the welfare cost from the lack of full-insurance is lower in the autarky economy with taxes and transfers (3.6% v.s. 3.5%). This implies that taxes and transfers play a role as risk-sharing mechanism in the U.S., consistent with the results provided by Blundell et al. (2008).

From this result, we also find the magnitude of the reduction in the welfare cost is quantitatively small. Although the welfare cost was eventually reduced by 2.7% (= 3.6% − 0.9%), this result means that taxes and transfers reduced only 0.1% (= 3.6% − 3.5%). However, we noticed that the tax data in the CEX is very sparse and its quality is low. Roughly speaking, there are only a half of the households provided logically consistent responses to the federal tax questions. Therefore, in order to assess the quantitative importance of taxes and transfers, we need to re-estimate $\delta(\mathbf{y}^{A,i}, \mathbf{s})$ using the after-tax income data with a better quality.

Table 2.3: Role of Taxes and Transfers

Autarky Economy $\delta(\mathbf{y}^{B,i},\mathbf{s})$	**Autarky+Tax** $\delta(\mathbf{y}^{A,i},\mathbf{s})$	**Actual Economy** $\delta(\mathbf{c}^{i},\mathbf{s})$
3.6%	3.5%	0.9%
(0.09)	(0.09)	(0.03)

y^B: Before-tax income y^A: After-tax income

The risk-sharing measures are estimated at synthetic cohort level and weighted by the CEX sample weights in obtaining the average. The data covers from 1980 to 2009. The parameter values used for this table are $\beta = 0.98$, $\gamma = 2$, and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$. The age-specific taste shifters are considered.

2.3.5 Sensitivity Analysis

In this subsection, we show the results under different parameter values of the utility function. Tables 2.4 - 2.6 respectively show $\delta(\mathbf{c}^i,\mathbf{s})$, $\delta(\mathbf{y}^i,\mathbf{s})$, and $\phi(\mathbf{c}^i,\mathbf{y}^i,\mathbf{s})$ for different levels of the discount factor, β, and the risk aversion coefficient, γ. The risk-sharing measures continuously respond to changes in the discount factor and the risk aversion coefficient.

As the risk aversion coefficient γ increases, $\delta(\mathbf{c}^i,\mathbf{s})$ and $\delta(\mathbf{y}^i,\mathbf{s})$ increase. This result is intuitive. As households become risk averse, they evaluate more the utility cost from variation in consumption. Therefore, they would anticipate more cost from income variation and thus gain more from full-insurance. Since both $\delta(\mathbf{c}^i,\mathbf{s})$ and $\delta(\mathbf{y}^i,\mathbf{s})$ increase with the risk aversion coefficient γ, the degree of partial insurance $\phi(\mathbf{c}^i,\mathbf{y}^i,\mathbf{s})$ can go up and down, depending on the relative increase of $\delta(\mathbf{c}^i,\mathbf{s})$ to $\delta(\mathbf{y}^i,\mathbf{s})$. Tables 2.6 shows that $\phi(\mathbf{c}^i,\mathbf{y}^i,\mathbf{s})$ actually exhibits a hump-shaped pattern in responding to γ when the discount factor β is less than 1.1. When the discount factor β is above 1.2, $\phi(\mathbf{c}^i,\mathbf{y}^i,\mathbf{s})$ is increasing with γ.

In response to changes in the discount factor β, $\delta(\mathbf{c}^i,\mathbf{s})$ and $\delta(\mathbf{y}^i,\mathbf{s})$ show an increasing or hump-shaped pattern. Both $\delta(\mathbf{c}^i,\mathbf{s})$ and $\delta(\mathbf{y}^i,\mathbf{s})$ initially increase with β but start decreasing at around $\beta = 1.1$. Again, the degree of partial insurance $\phi(\mathbf{c}^i,\mathbf{y}^i,\mathbf{s})$ can go up and down, depending on the relative change of $\delta(\mathbf{c}^i,\mathbf{s})$ to $\delta(\mathbf{y}^i,\mathbf{s})$. These patterns depend on the timing of observed fluctuations in consumption and income.

2.4 Conclusion

We propose a framework for assessing the realized performance of partial insurance in the actual economy, compared to hypothetical full-insurance. This framework consists of three measures: the welfare cost of uninsured risk $\delta(\mathbf{c}^i, \mathbf{s})$, the size of household's income risk $\delta(\mathbf{y}^i, \mathbf{s})$, and the percentage of risk that households insure under partial insurance $\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$. We apply the framework to the U.S. economy by constructing a synthetic panel data set from the Consumer Expenditure Survey (CEX) for the period of 1980-2009. We find that, on average, 64% of cohort-specific income risk was insured, and the remaining uninsured risk cost 1% of the annual expenditure on nondurables and services.

This paper has two contributions to the literature. The first contribution is to provide new empirical evidence on the relationship between the magnitude of income risk and the degree of partial insurance. There was a positive correlation between them for U.S. households. This implies either or both of the following: (1) households bearing a higher income risk tended to hedge a larger portion of their risk, or (2) households with a higer income risk insured a larger portion of their income risk because most of their income risk was transitory and easily insurable. The first implication is a prediction of the risk-sharing theories under limited enforcement. The second implication is about the type of income risk, rather than the risk-hedging effort. Further research to fully reconcile this positive correlation using a model will be insightful to households' risk-sharing behaviors.

The second contribution is to provide a comprehensive framework for characterizing the welfare cost from the lack of full-insurance as a composite of the magnitude of income risk and the degree of partial insurance. This decomposition gives an insight to risk-sharing behaviors. For example, it seems puzzling that, the stockholder's consumption path typically fluctuates more than the non-stockholder's one (Mankiw and Zeldes (1991), although stockholders have more options to smooth their consumption path by holding a diversified portfolio. The framework implies that this is possibly because stockholders face more income risk and still bear a larger amount of uninsured risk than non-stockholders.

Although it is important to consider heterogeneous preferences in assessing risk-sharing performances, it is beyond the scope of this paper. It requires a way to estimate the state prices in an economy with different types of agents. The estimation in this paper assumes the identical homothetic preference.

Since a synthetic cohort approach is employed, household's idiosyncratic risk is not examined in this paper. Although the analysis on cohort-specific risk has several advantages (Attanasio and Davis (1996)), it is also interesting to investigate how important the idiosyncratic risk is. Such research can be done by applying the framework to a panel data on consumption and income. In the case of the U.S., the PSID panel data can be used, after it is combined with the CEX to obtain an estimate of expenditure on nondurables and services, as conducted by Blundell, Pistaferri, and Preston (2008).

Table 2.4: Welfare Cost $\delta(\mathbf{c}^i, \mathbf{s})$ under various parameter values

	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
$\beta = 0.9$	0.36	0.63	0.93	1.55	3.17	6.50
$\beta = 0.95$	0.45	0.79	1.18	1.99	4.12	8.08
$\beta = 0.98$	0.49	$\boxed{0.88}$	1.33	2.27	4.71	9.01
$\beta = 1$	0.52	0.94	1.42	2.43	5.08	9.56
$\beta = 1.1$	0.55	1.07	1.64	2.85	5.86	10.75
$\beta = 1.2$	0.54	1.07	1.63	2.75	5.31	9.18
$\beta = 1.5$	0.51	1.00	1.46	2.29	3.76	5.08

Table 2.5: Income Risk $\delta(\mathbf{y}^i, \mathbf{s})$ under various parameter values

	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
$\beta = 0.9$	1.19	2.67	4.11	6.66	11.28	16.79
$\beta = 0.95$	1.43	3.29	5.12	8.43	14.26	19.88
$\beta = 0.98$	1.57	$\boxed{3.64}$	5.73	9.57	16.39	22.14
$\beta = 1$	1.66	3.84	6.09	10.30	17.88	23.84
$\beta = 1.1$	1.87	4.20	6.76	12.13	24.02	34.31
$\beta = 1.2$	1.90	4.05	6.36	11.31	24.39	42.55
$\beta = 1.5$	1.91	3.81	5.66	9.13	17.10	33.79

Table 2.6: Degree of Partial Insurance $\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$ under various parameter values

	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
$\beta = 0.9$	49.93	65.06	67.47	67.51	62.21	46.86
$\beta = 0.95$	49.71	64.58	67.01	67.18	61.76	44.45
$\beta = 0.98$	50.59	$\boxed{64.37}$	66.73	67.07	62.31	45.75
$\beta = 1$	51.43	64.33	66.62	67.10	63.01	47.70
$\beta = 1.1$	55.15	64.36	66.43	67.78	67.79	62.41
$\beta = 1.2$	53.87	61.23	63.51	66.13	70.33	72.99
$\beta = 1.5$	40.89	47.65	51.19	56.20	65.72	76.60

The risk-sharing measures are estimated at synthetic cohort level and weighted by the CEX sample weights in obtaining the average. The data covers from 1980 to 2009. The parameter values used for this table are $\beta = 0.98$, $\gamma = 2$, and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$. The age-specific taste shifters are considered. The bootstrapped standard errors are reported in parentheses. The number of bootstrap replications is 50.

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Appendix A

Appendix to Chapter 1

A.1 Decomposition of the Change in the Homeownership Rate

Let h_t be the homeownership rate in the year t. Then, as is discussed in Chambers et al. (2009), it can be expressed as the weighted sum of the group-specific homeownership rates;

$$h_t = \sum_j \sum_e \mu_{jt} s^e_{jt} h^e_{jt}$$

where h^e_{jt} is the homeownership rate among households whose householder is at age j and has the level of education e, s^e_{jt} is the share of e-households among the j-households, and μ_{jt} is the share of j-households in the whole population.

Then, the change in the homeownership rate h_t from the year t_0 to t can be written as follows;

$$\begin{aligned} h_t - h_{t_0} &= \sum_j \mu_{jt_0} s^{nc}_{jt_0} \boldsymbol{\Delta} \mathbf{h^{nc}_{jt}} && \text{(Change in ownership among noncollege)} \\ &+ \sum_j \mu_{jt_0} s^{c}_{jt_0} \boldsymbol{\Delta} \mathbf{h^{c}_{jt}} && \text{(Change in ownership among college)} \\ &+ \sum_j \sum_e \mu_{jt_0} \boldsymbol{\Delta} \mathbf{s^{e}_{jt}} h^e_{jt_0} && \text{(Change in the share of college educated)} \\ &+ \sum_j \sum_e \boldsymbol{\Delta} \boldsymbol{\mu}_{\mathbf{jt}} s^e_{jt_0} h^e_{jt_0} && \text{(Change in the age structure of population)} \end{aligned}$$

$$+ \quad \text{(Covariance Terms)}$$

where $\Delta x_t := x_t - x_{t_0}$. Dividing both sides by $h_t - h_{t_0}$ gives the contribution of each factor. Table 1 shows the results for the periods of 1995-2000, 2000-2005, and 2005-2010.

A.2 Homeownership Rate by Age and Education

Table A.1: Homeownership Rate by Age and Education of Householder

					Percentage Change	
		2000	2005	2010	'00-'05	'05-'10
18 - 29						
	Non College	29.4	30.4	27.2	3.4	-10.6
	College	35.4	43.7	37.6	23.2	-13.8
30 - 39						
	Non College	55.6	54.6	48.5	-1.8	-11.1
	College	67.5	71.9	64.1	6.5	-10.8
40 - 49						
	Non College	68.1	68.3	62.2	0.3	-8.9
	College	80.9	82.2	80.4	1.6	-2.2
50 - 59						
	Non College	75.8	75.2	72.0	-0.8	-4.2
	College	85.4	86.4	84.3	1.2	-2.5
60 -						
	Non College	79.3	80.3	78.7	1.2	-2.0
	College	86.5	86.1	86.2	-0.4	0.1
All						
	Non College	65.6	66.2	63.6	0.9	-3.9
	College	73.5	77.0	74.4	4.7	-3.4

Note: The data source is the March CPS supplement. Age and education level of a household are defined by those of the householder.

A.3 Decline in the Average Labor Earnings

Table A.2: Decline in the Average Labor Earnings by Age and Education

	Non-College	College
20-22	-1.3	-10.0
23-25	-8.3	3.6
26-28	-4.6	-1.3
29-31	-7.3	-3.0
32-34	-12.3	-1.3
35-37	-7.3	-4.2
38-40	-6.5	-5.9
41-43	-11.1	1.3
44-46	-4.8	-3.6
47-49	-13.3	-8.4
50-52	-6.7	-1.1
53-55	-2.4	-3.4
56-58	-1.4	6.3
59-61	-1.2	-11.1
62-64	0.9	14.2

Note: The table reports the decline in the average real labor earnings in each age and education group from 2005 to 2009. The data sources are the CPI and the March CPS supplement in 2006 and 2010. Age and education level of a household are defined by those of the householder.

Appendix B

Appendix to Chapter 2

B.1 Theory behind the Risk-Sharing Measures

Households perfectly share their risk in a competitive equilibrium in a complete market, where they can trade securities contingent on any state of the economy. Thus, we characterize the consumption allocation with full risk insurance as an equilibrium outcome in a complete market where all households trade Arrow-Debreu securities at time 0.

Consider a pure exchange economy with a set of households, I, in a complete market. Every household $i \in I$ has an instantaneous utility function, $u_i(\cdot)$, has a discount factor, β_i, and lives until time T. $u_i(\cdot)$ only depends on consumption. Each household receives income $y_{it}(s)$ as an endowment at time t, which depends on $s \in S \subseteq R^T$, a history of the states of the economy until time T. Let $\pi(s)$ be the probability that s occurs.

We define a competitive equilibrium as a sequence of prices and an allocation of quantities such that (i) the allocation solves household's utility maximization problem given the prices for every i in I, and that (ii) the consumption goods market clears with the allocation.

In a competitive equilibrium, every household $i \in I$ solves the following problem given the state price, $P_t(s)$.[1]

$$\text{(HH)} \qquad \max_{\{\{c_{it}(s)\}_{t=1}^T\}_{s\in S}} \sum_{s\in S}\pi(s)\left[\sum_{t=1}^T \beta_i^{t-1}u_i(c_{it}(s))\right]$$
$$s.t. \quad \sum_{s\in S}\pi(s)\left[\sum_{t=1}^T P_t(s)c_{it}(s)\right] \le \sum_{s\in S}\pi(s)\left[\sum_{t=1}^T P_t(s)y_{it}(s)\right].$$

By assuming that $u(\cdot)$ is continuous and locally non-satiated, the solution to this problem, $\{c_{it}^*(s)\}_{t=1}^T$, is also the solution to the following cost minimization problem.

$$\text{(CM0)} \qquad \min_{\{\{c_{it}(s)\}_{t=1}^T\}_{s\in S}} \sum_{s\in S}\pi(s)\left[\sum_{t=1}^T P_t(s)c_{it}(s)\right]$$
$$s.t. \quad \sum_{s\in S}\pi(s)\left[\sum_{t=1}^T \beta_i^{t-1}u_i(c_{it}(s))\right] \ge \sum_{s\in S}\pi(s)\left[\sum_{t=1}^T \beta_i^{t-1}u_i(c_{it}^*(s))\right].$$

That is, each household minimizes their *expected* expenditure, keeping the *expected* utility level equal to or greater than the level they achieve in a complete market with the state prices, $P_t(s)$.

Note that $P_t(s)$ is not the Arrow-Debreu price because it does not take the probability $\pi(s)$ into account. We can show that $P_t^*(s) = \pi(s)P_t(s)$ for all s, where $P_t^*(s)$ is the Arrow-Debreu price, by solving for an equilibrium.

We need to remove the *expected* expenditure and the *expected* utility level from our formulation because they are not directly observed in data. Data contain the information on a particular realized history, s. To remove the *expected* expressions, we change (CM0) into history-by-history minimization problems, where each household minimizes their expenditure *for each history*, s, in equilibrium. The proof of the equivalence between such history-by-history minimization problems and (CM0) requires household's preferences to be separable for each history of the states, in addition to the conditions of the duality theorem.

The cost minimization problem, (CM0), can be solved by two steps. First, we solve for the expenditure function for each history, $\rho_i(U_i(s), s)$, given prices.

$$\text{(CM1)} \qquad \rho_i(U_i(s), s) := \min_{\{c_{it}(s)\}_{t=1}^T} \sum_{t=1}^T P_t(s)c_{it}(s)$$

[1] $P_t(s)$ is not the Arrow-Debreu price because it does not take the probability $\pi(s)$ into account. We can show that $P_t^*(s) = \pi(s)P_t(s)$ for all s, where $P_t^*(s)$ is the Arrow-Debreu price, by solving for an equilibrium.

$$s.t. \quad \sum_{t=1}^{T} \beta_i^{t-1} u(c_{it}(s)) \geq U_i(s).$$

Second, we minimize the expected expenditure using the expenditure function for each history. That is, given the minimized expenditure function, we optimally allocate the utility level for each history, keeping the expected utility level the same as or greater than the level achieved in equilibrium.

$$(\text{CM2}) \qquad \min_{\{U_i(s)\}_{s\in S}} \sum_{s\in S} \pi(s)\rho_i(U_i(s), s)$$

$$s.t. \quad \sum_{s\in S} \pi(s)U_i(s) \geq \sum_{s\in S} \pi(s) \left[\sum_{t=1}^{T} \beta_i^{t-1} u_i(c_{it}^*(s)) \right].$$

The following proposition formally establishes the equivalence between the original cost minimization problem and the two-step cost minimization problem.

Proposition 1 The two-step cost minimization problem formulated as (CM1) and (CM2) is equivalent to the original cost minimization problem, (CM0).

(Proof) First, the objective functions of both (CM0) and the two-step cost minimization problem are the same, after we plug the solution to (CM2) in the required utility of (CM1). Next, the solution to (CM0) is affordable in the two-step cost minimization problem, (CM1) and (CM2). On the other hand, the solution to the two-step cost minimization problem is affordable in (CM0). Hence, the two cost minimization problems are equivalent. ■

Proposition 1 implies that the solution to (CM0) solves (CM1) for each history given $U^*(s)$, which is the solution to (CM2) that satisfies

$$U_i^*(s) = \sum_{t=1}^{T} \beta_i^{t-1} u_i(c_{it}^*(s)).$$

Hence, combining these two facts, $c_{it}^*(s)$ solves the following cost minimization problem for each history, $s \in S$, given Arrow-Debreu prices, $P_t(s)$.

$$(\text{CM3}) \qquad \min_{\{c_{it}(s)\}_{t=1}^{T}} \sum_{t=1}^{T} P_t(s) c_{it}(s)$$

$$s.t. \quad \sum_{t=1}^{T} \beta_i^{t-1} u_i(c_{it}(s)) \geq \sum_{t=1}^{T} \beta_i^{t-1} u_i(c_{it}^*(s)).$$

This means that the equilibrium consumption path for each history minimizes the expenditure for each history, given the utility level it attains. Since there is no expected expression, (CM3) has a direct implication to the data, which include the information on only one realized history.

Suppose that all households efficiently share their risk in a complete market. That is, suppose that the observed consumption path in data is an equilibrium consumption path for a certain history. Then, the observed consumption path must solve (CM3), which means that the observed consumption path must minimize the expenditure for the observed history, given the utility level from the observed consumption path. In other words, in the case of perfect risk-sharing, the minimized expenditure in (CM3) must equal to the expenditure for the observed consumption path.

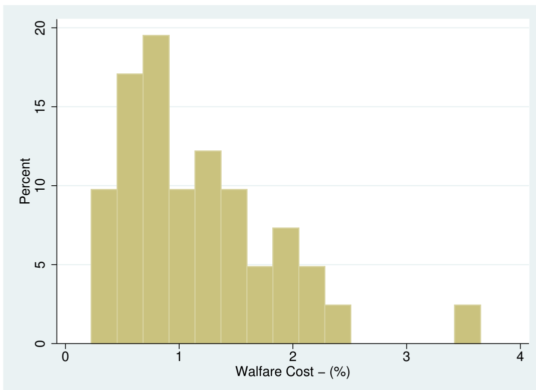
This motivates us to see how different the observed expenditure is from the minimized expenditure in (CM3). The difference will be large if the observations are far from an equilibrium outcome in a complete market. Conversely, the difference will be small if households efficiently share their risk with others. Therefore, this difference can be interpreted as a measure of efficiency in risk-sharing. Specifically, it measures the cost of being away from complete market. In section 2, we provide a formal definition of the welfare cost.

B.2 Distribution of the Risk-Sharing Measures

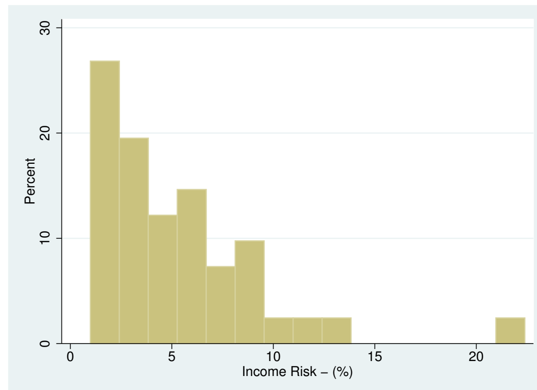
Our risk-sharing measures are estimated at synthetic-cohort level, or household-group level. Therefore, we can examine the risk-sharing performance of households in the form of distributions. Figure B.1 show the histograms of the welfare cost from the lack of full-insurance $\delta(\mathbf{c}^i, \mathbf{s})$, the amount of income risk $\delta(\mathbf{y}^i, \mathbf{s})$, and the degree of partial insurance $\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$. The amount of income risk $\delta(\mathbf{y}^i, \mathbf{s})$ is measured as the welfare cost from the lack of full-insurance in the autarky economy.

The distribution of the welfare cost $\delta(\mathbf{c}^i, \mathbf{s})$ are right-skewed (Figure B.1, Panel (a)). This means that most households have a small welfare cost from the lack of full-insurance. More than 90 % of households would have less than 3% of their actual expenditure on non-durable goods and services, as the cost from the lack of full-insurance to cohort-specific income shocks.

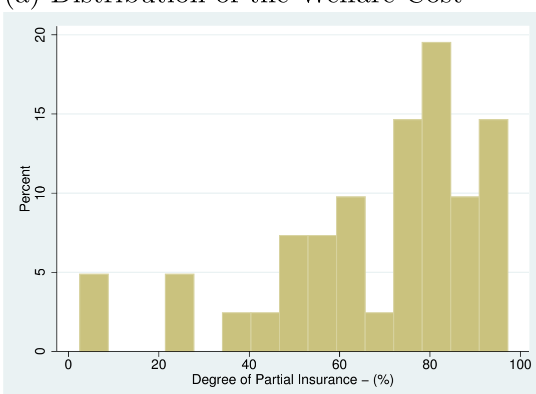
The welfare cost in the autarky economy $\delta(\mathbf{y}^i, \mathbf{s})$ also has a right skewed distribution, although the support of the distribution is much wider than that of $\delta(\mathbf{c}^i, \mathbf{s})$ (Figure B.1, Panel (b)). That is, most households faced to a small amount of income risk. The wider dispersion of the welfare cost in the autarky economy $\delta(\mathbf{y}^i, \mathbf{s})$ implies that only a portion of income variations is transmitted to consumption variations. It also indicates that households with large income fluctuations insure most of them and have small consumption fluctuations. Actually, The distribution of $\phi(\mathbf{c}^i, \mathbf{y}^i, \mathbf{s})$ is skewed to the left, reflecting this fact (Figure B.1, Panel (c)). Households typically hedged a sizable portion of their income risk.



(a) Distribution of the Welfare Cost



(b) Distribution of the Income Risk



(c) Distribution of the Partial Insurance

Figure B.1: Distributions of Risk Sharing Measures

The risk-sharing measures are estimated at synthetic cohort level and weighted by the CEX sample weights in obtaining the average. The data covers from 1980 to 2009. The parameter values used for this table are $\beta = 0.98$, $\gamma = 2$, and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$. The age-specific taste shifters are considered. The bootstrapped standard errors are reported in parentheses. The number of bootstrap replications is 50.