Permanent File Copy
St. Anthony Falls Hydraulic Laboratory

UNIVERSITY OF MINNESOTA
ST. ANTHONY FALLS HYDRAULIC LABORATORY

Technical Paper No. 48, Series B

Studies of the Flow Characteristics of a Compressible, Bubbly Mixture about Supercavitating Bodies and in a Converging-Diverging Nozzle

by
F. R. SCHIEBE, J. M. WETZEL, and K. E. FOERSTER

Prepared for
DAVID TAYLOR MODEL BASIN
Department of the Navy
Washington, D.C.
under
Bureau of Ships General Hydromechanics Research Program
S-R009-01-01
Office of Naval Research Contract Nonr 710(52)

April 1964
Minneapolis, Minnesota

UNIVERSITY OF MINNESOTA
ST. ANTHONY FALLS HYDRAULIC LABORATORY

Technical Paper No. 48, Series B

Studies of the Flow Characteristics of a Compressible, Bubbly Mixture about Supercavitating Bodies and in a Converging-Diverging Nozzle

by

F. R. SCHIEBE, J. M. WETZEL, and K. E. FOERSTER

Prepared for
DAVID TAYLOR MODEL BASIN
Department of the Navy
Washington, D.C.
under
Bureau of Ships General Hydromechanics Research Program
S-R009-01-01
Office of Naval Research Contract Nonr 710(52)

April 1964
Minneapolis, Minnesota

Reproduction in whole or in part is permitted
for any purpose of the United States Government

ABSTRACT

Experimental studies have been made to determine the effect of a compressible, air-water mixture on the drag characteristics of a cavitating body. Data are reported for a series of conical bodies of various slenderness ratios for free stream Mach numbers up to 0.7. Results indicate that the drag coefficient increases with Mach number, although in general not as rapidly as for a non-cavitating body. It was possible to apply Gothert's rule to adequately predict the drag coefficient up to Mach numbers of about 0.6.

A brief study was also conducted to study the flow characteristics of an air-water mixture in a converging-diverging nozzle. Supersonic flow was obtained and shock waves were observed downstream of the throat. Measured throat pressures for choked flow were somewhat higher than those calculated from homogeneous mixture theory.

CONTENTS

LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

A - Area

C_D - Drag coefficient

C_p - Specific heat of gas at constant pressure

C_v - Specific heat of gas at constant volume

C_ℓ - Specific heat of liquid

c - Concentration of gas in mixture by volume

D - Drag force

d - Diameter

E - Bulk modulus of elasticity

h - Height

K - Pitot cylinder coefficient

k - Ratio of specific heats for a gas

M - Mass of gas or liquid

M_∞ - Free stream Mach number

M_c - Cavity Mach number

P - Absolute pressure

P_c - Cavity pressure

P_T - Test section pressure in circular water tunnel

v - Velocity

v_m - Mean mixture velocity

v_s - Sonic velocity

W - Weight flow rate

$\beta - \sqrt{1 - M_\infty^2 (1 + \sigma)}$

β_n - Volume flow fraction = $\frac{cv_g}{cv_g + (1 - c) V_\ell}$, where the subscript n denotes a given point in a converging-diverging nozzle

Γ - $(\mu C_p + C_\ell)/(\mu C_v + C_\ell)$

γ - Specific weight

δ - Volume ratio of gas to liquid

ζ - Slenderness ratio

μ - Mass ratio of gas to liquid

ρ - Density

σ - Cavitation number = $\frac{P - P_c}{\rho_\ell v_\ell^2/2}$

Subscripts not previously defined:

g - Gas

ℓ - Liquid

m - Mixture

o - Stagnation conditions

comp - Compressible flow

incomp - Incompressible flow

1, 2, 3, etc. - Locations in the converging-diverging nozzle

* - Throat conditions

STUDIES OF THE FLOW CHARACTERISTICS OF A COMPRESSIBLE, BUBBLY MIXTURE ABOUT SUPERCAVITATING BODIES AND IN A CONVERGING-DIVERGING NOZZLE

I. INTRODUCTION

The need for higher speed surface ships has accelerated the study of flows about cavitating hydrofoils and other bodies. This work has been considerably stimulated by the discovery of hydrofoil shapes that would provide satisfactory performance characteristics under extreme cavitating conditions. The present thought is to employ these supercavitating sections on hydrofoil craft both as lifting surfaces and as propeller blades, at least until a more satisfactory method of propulsion has been developed.

It has been found that in many applications of these sections which are designed for supercavitation, it would be desirable to ventilate the cavities by introducing air or other gas to the cavity or wake region of a separated flow, thereby increasing the cavity pressure and reducing the cavitation number. This artifice would be especially useful at the lower velocities where under normal operating conditions the cavity would be small or absent altogether. The performance of a hydrofoil section designed for supercavitation under wetted conditions would be poor and the addition of gas would develop the cavity to such a state that performance would be satisfactory.

Added gas leaves the cavity by some entrainment process which depends on many of the parameters of the flow [1]*. The wake region behind the cavity is filled with gas bubbles even at appreciable distances to the rear of the body. As the distance increases, the concentration of gas decreases through gravitation of the bubbles to the free surface.

For stability purposes, at least two supporting foils are required on a hydrofoil craft. The supporting foils may in general be of different size and be arranged in various configurations. However, it is possible that one foil may be operating in the bubbly wake of another foil. The gas-liquid bubbly mixture created by the ventilated forward cavity would be of concern in considering the force characteristics of the downstream foil.

---

*Numbers in brackets refer to the List of References on page 27.

A considerable amount of work has been done on the basic characteristics of gas-liquid mixtures (air bubbles in water), both at this Laboratory and elsewhere [2, 3, 4, 5]. One of the unusual characteristics of a bubbly gas-liquid mixture is the drastic reduction in sound velocity, which can be achieved by the addition of small concentrations of gas to the liquid. As an example, at atmospheric pressure, sonic velocities of the order of 100 fps can be obtained with gas concentration of about 11.5 per cent by volume as compared with a sound speed of approximately 4,800 fps in pure water and 1,100 fps in pure air at normal conditions. The acoustic velocity can be much further reduced by reducing the ambient pressure or increasing the gas concentration.

Hydrofoil craft which employ supercavitating hydrofoil sections are intended to operate at speeds approaching or even exceeding the local sound speed in the bubbly wake of the supporting hydrofoils. It is thus necessary to investigate the problems associated with the determination of the force characteristics of a body operating under such conditions.

In the investigation reported herein, drag coefficients were measured at subsonic mixture velocities on a series of axisymmetric cones under supercavitating conditions in a bubbly two phase flow. The experiments were carried out in the Laboratory 6-in. water tunnel in which air bubbles were injected into the flow just upstream of the test section. The results are presented in Section III.

Concurrent with the above study, a separate program was initiated to investigate the flow properties of an air-water mixture in a converging-diverging nozzle. Of particular interest were the conditions leading to a choked flow and the structure of the mixture in the diverging section of the nozzle for this type of flow. A small facility was constructed for this purpose and the details are described in Section IV.

Some of the results were previously presented in an earlier Memorandum [6] which is superceded by the present report.

II. THEORETICAL CONSIDERATIONS

The velocity of sound in a bubbly two phase gas-liquid medium was calculated by Wood [2]. The sonic velocity, v_s, in a homogeneous medium is given by

$$v_s = (E/\rho)^{1/2} \tag{1}$$

where E = bulk modulus of elasticity and

ρ = density.

If the bubbles are small and evenly distributed, the mixture can be assumed non-resonant to moderate changes in pressure. The mean density and elasticity of such a mixture is

$$\rho_m = c\rho_g + (1 - c)\rho_\ell \tag{2}$$

and

$$E_m = \frac{E_g E_\ell}{cE_\ell + (1 - c)E_g} \tag{3}$$

where c = concentration of gas in the mixture by volume,

ρ_g = gas density,

ρ_ℓ = liquid density,

E_g = gas elasticity, and

E_ℓ = liquid elasticity.

The mean sonic velocity of the homogeneous mixture is then given by

$$v_s = \left\{ \frac{E_g E_\ell}{[cE_\ell + (1 - c)E_g]\,[c\rho_g + (1 - c)\rho_\ell]} \right\}^{1/2} \tag{4}$$

If it is assumed that relatively low frequency sound propagation in a mixture where the gas bubbles are small and uniformly distributed is isothermal rather than adiabatic [7], then the gas elasticity is equal to the absolute pressure P. Evidence will be shown in a later section which tends to validate this assumption for the current group of tests.

Furthermore, if the absolute pressure is of the order of half an atmosphere and the concentration is less than 20 per cent, then the terms $(1 - c)E_g$ and $c\rho_g$ in the denominator of Eq. (4) may safely be neglected and the mean sonic velocity reduces to

$$v_s = \left[\frac{p}{c\,(1 - c)\rho_\ell}\right]^{1/2} \qquad (5)$$

At a pressure of half an atmosphere and a concentration of 20 per cent, the sonic velocity of the mixture becomes 57 fps. In Fig. 1, v_s, is shown as a function of concentration at several pressures in the range of interest for the present investigation.

It is seen that the sonic velocity decreases with decreasing absolute pressure and with increasing concentration. A minimum velocity occurs at a concentration of 50 per cent for each particular pressure. Furthermore, even for rather low concentrations, the sonic velocity is drastically reduced from that in pure water. This reduction is effected primarily by the large change in the elasticity of the mixture while the change in density is rather small. After an initial large decrease, the sonic velocity changes at a much slower rate with increasing concentration. The mixture thus retains the lowered sonic velocity over a rather large range of concentrations.

In a system where the flow of a bubbly mixture is of concern, it is necessary to have some knowledge of the velocity profile in the test section. If a stagnation type device is utilized for this purpose, an equation of motion including the effects of a homogeneous air-water mixture must be obtained.

A review of the literature has revealed that Tangren et al [5] developed the equations of state and motion for the flow of such a mixture with the following assumptions:

(1) The liquid is incompressible; effects due to viscosity, surface tension, and vapor pressure are neglected.

(2) The gas is ideal, insoluble in the liquid, has negligible viscosity, and constant specific heats.

(3) The gas bubbles are small and uniformly distributed.

(4) The gas and liquid are always at the same temperature and the flow is adiabatic.

(5) The flow is one-dimensional and inertial forces are absent.

The equation of state was thus determined as

$$P[1/\rho_m - 1/\rho_\ell(1 + \mu)]^\Gamma = \text{const} \qquad (6)$$

where $\Gamma = (\mu C_p + C_\ell)/(\mu C_v + C_\ell)$,
μ = mass ratio = M_g/M_ℓ;
M_g = mass of gas,
M_ℓ = mass of liquid,
C_p = specific heat of gas at constant pressure,
C_v = specific heat of gas at constant volume, and
C_ℓ = specific heat of liquid.

The equation of motion for the flow of a typical homogeneous mixture to a stagnation region was given to a close approximation, as

$$\rho_\ell v_m^2/2 = -P_o\delta_o \ln \frac{P}{P_o} + P_o - P \qquad (7)$$

where v_m = mixture velocity,
P_o = stagnation pressure,
δ_o = stagnation volume-ratio = $\frac{\rho_\ell}{\rho_{go}}$, and
ρ_{go} = stagnation gas-density.

The first term on the right-hand side of Eq. (7) indicates the effect of the entrained bubbles.

It was shown that the equation of state could be written as

$$\left(\frac{P}{P_o}\right)^{1/\Gamma} = \frac{\delta_o}{\delta} \qquad (8)$$

For a typical mixture in the present experiment Γ is extremely close to unity. Therefore,

$$\delta_o = \delta\frac{P}{P_o} \qquad (9)$$

and δ is related to the gas concentration by

$$\delta = \frac{c}{1 - c} \tag{10}$$

With these substitutions, Eq. (7) now becomes

$$\rho_\ell v_m^{\,2}/2 = P_o - P \left[1 + \frac{c}{1 - c} \ln \frac{P}{P_o}\right] \tag{11}$$

In a simplified approach to the problem of determining the equation of motion, the mixture can be treated as incompressible with the density modified by Eq. (2), in which the term containing the gas density is finally neglected. This approach gives

$$\rho_\ell v_m^{\,2}/2 = \frac{P_o - P}{1 - c} \tag{12}$$

A third analysis has been carried out by Killen [8]. In his analysis assumptions 1, 2, 3, and 5 of Tangren were used. In place of assumption 4, however, the following assumptions were substituted:

(1) The compressibility effects take place entirely within the gas phase.

(2) The process in the gas bubble is reversible adiabatic.

This approach leads to

$$\rho_\ell v_m^{\,2}/2 = \frac{P_o - P}{1 - c} + \frac{c}{1 - c} P \left[\frac{k}{k - 1}\left(\frac{P_o}{P}\right)^{\frac{k-1}{k}} - \frac{1}{k - 1} - \frac{P_o}{P}\right] \tag{13}$$

where k = ratio of specific heats for the gas.

In order to compare these three different equations for the mixture velocity, numerical computations were made for a pressure ratio which was in the range of interest of the experiment, and the results are plotted in Fig. 2. It can be seen that for ambient pressures of the order of one-half an atmosphere and gas concentrations greater than 20 per cent, Eq. (11) and (13) produce almost the same result, while Eq. (12) yields much higher values of velocity. At gas concentrations lower than 20 per cent, the velocity as determined by Eq. (13) deviated from the values obtained from Eq. (11) until at a concentration of 10 per cent, it approached the velocity as determined by Eq. (12). Since the assumptions for the derivation of Eq. (11) seem most reasonable to the authors, it was used in the reduction of data in these experiments. There is still some question remaining as to the effect of slip between the air bubbles and water on a velocity determination of this kind. Slip has been neglected in the calculations.

For reference purposes, it was desirable to have experimental data for various cavitating bodies in an incompressible fluid. It was found that supercavitating flow about axially symmetrical bodies was examined experimentally in great detail by Reichardt [9]. The majority of the bodies studied were cones of height h and base diameter d. The ratios of h/d studied varied from 0 (disk) to 2. In reexamining Reichardt's data, an empirical expression was found as follows relating the drag coefficient, σ, and h/d,

$$C_D = \frac{1.178(1 + \sigma)}{1.49 + 1.18(h/d) + (h/d)^2} \qquad (14)$$

where C_D = drag coefficient,

σ = cavitation number = $\dfrac{P - P_c}{\rho_\ell v_\ell^2/2}$,

P_c = cavity pressure, and

v_ℓ = liquid velocity.

The drag on an axisymmetric body in a compressible subsonic fluid has been studied quite extensively in air [10, 11]. In general, the drag coefficient gradually rises from the incompressible case with increasing free stream Mach number until some local Mach number approaches or exceeds unity. After this point, the drag coefficient rises quite rapidly with the free stream Mach number. This point generally occurs at lower free stream Mach numbers.

The variation of the pressure coefficient with Mach number on a slender, two-dimensional body can be predicted quite well for moderate subsonic Mach numbers by appropriate application of the well-known Gothert and Prandtl-Glauert rules. These rules are fully discussed by Shapiro [10]. In general, for two-dimensional flow about thin bodies, it is possible to apply the Prandtl-Glauert rule to determine the pressure coefficient on a body in a compressible flow from computations made or data taken with the same body in an incompressible flow. In three-dimensional flows, Gothert's more general rule must be used which relates a distorted or affine body in an incompressible flow to the given body in the compressible flow. These rules may be useful also in the prediction of drag coefficients for cases where pressure drag is a large percentage of the total drag force. This is usually the case for most supercavitating flows.

The application of the Prandtl-Glauert rule to a two-dimensional supercavitating wedge in a compressible fluid was discussed by Tulin [12]. Even though the Prandtl-Glauert rule is not applicable to axisymmetric flows, it will be useful to summarize Tulin's result and perform a similar analysis for axisymmetric bodies using Gothert's rule.

The highest local Mach number occurs on the cavity free streamline for a wedge under cavitating conditions. The linearization of the gas dynamic equation then involves the quantity $1 - M_c^2$ rather than $1 - M_\infty^2$. In the case of zero cavitation number the cavity Mach number, M_c, is equal to the free stream Mach number, M_∞. The Prandtl-Glauert rule has been suitably modified by Tulin to give the result

$$\frac{C_{D_{comp}}}{C_{D_{incomp}}} = \frac{1 + \sigma (1 - M_c^2)}{\sqrt{1 - M_c^2 (1 + \sigma)}} \tag{15}$$

where $C_{D_{comp}}$ = drag coefficient at $M_c < 1$,

$C_{D_{incomp}}$ = drag coefficient at $M_c = 0$, and

$M_c = M_\infty \sqrt{1 + \sigma}$.

In the application of Gothert's rule to the present case, assume a slender, axisymmetric body in a compressible flow. Gothert's rule states that the pressure coefficient on this body can be related to the pressure coefficient on a distorted body in a transformed incompressible plane by the following equation

$$C_p (M_\infty, \zeta) = \frac{1}{\beta^2} C_p' (0, \beta\zeta) \tag{16}$$

where $\beta^2 = 1 - M_c^2$ and

ζ = slenderness ratio of body.

The unprimed symbols refer to the compressible flow, and the primed symbols refer to the transformed incompressible flow. If the original body had a length of h and maximum thickness of d, the transformed body will have a length of h/β and the maximum thickness will remain constant. The cavitation number is reduced by a factor of β^2 in the transformation. For a cavitating body where the drag is primarily due to pressure effects, Gothert's rule can also be used in the extrapolation of drag coefficients. Thus,

$$C_D\,(M_\infty, \zeta, \sigma) = \frac{1}{\beta^2}\, C_D{}'\,(0, \beta\zeta, \beta^2\sigma) \tag{17}$$

The calculation of the drag coefficient in the compressible flow thus reduces to the determination of the drag coefficient in the incompressible flow for a distorted body at a different cavitation number. The latter coefficient can be found either analytically or experimentally. In the present case, it was convenient to use the empirical Eq. (14) for conical bodies. Substituting this equation into Eq. (17) and taking into account the change in the x coordinate and cavitation number,

$$C_D\,(M_\infty, \zeta, \sigma) = \frac{1}{\beta^2}\left[\frac{1.178\,(1 + \beta^2\sigma)}{1.49 + 1.18\,\frac{h}{\beta d} + \left(\frac{h}{\beta d}\right)^2}\right] \tag{18}$$

Equation (18) was used for comparison with experimental data and will be discussed in Section III B.

III. DRAG OF SUPERCAVITATING BODIES IN A COMPRESSIBLE, BUBBLY MIXTURE

A. Experimental Apparatus and Procedure

The water tunnel was a closed jet type with a 6-in. diameter test section. When operated in a conventional manner, the test section velocity range is from 15 fps to 55 fps and the test section pressure can be varied independently of the velocity from approximately 1 psia to about 30 psia. This tunnel is conventional in design except for an air separator located upstream of the test section. It is the function of this separator to remove the free-air bubbles from the water and provide the test section with bubble free water. A photograph of the tunnel is shown in Fig. 3 and the facility is further described in Ref. [13]. For the present experiments, a measured amount of air was injected into the region of a screen located immediately upstream of the test section and downstream of the separator in a manner suggested by Campbell and Pitcher [3]. In this way, a bubbly core approximately 4 in. in diameter was formed about the centerline of the test section. This core was surrounded with approximately a 1-in. thick layer of pure water. The injected air was measured with a calibrated orifice meter and controlled by a pressure regulator and a needle valve. The air supply was taken from the Laboratory compressed air system. Figure 4 shows a schematic diagram of the experimental apparatus.

The production of a uniform bubbly mixture was a difficult problem and the method described above was only the best of the methods tried. It would have been more satisfactory to have the entire test section filled with the bubbly flow. Various attempts to accomplish this did not result in satisfactory flows. The main difficulty in achieving this type of flow was the short length available in the tunnel to produce the mixture. It is felt with a longer approach that a better bubble distribution could be achieved. It may be stated, however, that the core type flow does bear some resemblance to the prototype problem as originally stated.

The concentration of the air bubbles in the test section was measured by a concentration meter previously developed at this Laboratory for investigations in air entrained flows in open channels [14]. This meter averages the concentration between two electrodes which are half inch apart. The electrodes were arranged on a traversing rod so that the concentration profile of the test section could be obtained for each test condition.

The velocity profile of the flow through the test section was measured by a pitot cylinder. A system was provided to clear the line of any air bubbles immediately before taking the readings. As explained in the previous section, Eq. (11) was used in combination with the experimentally determined pitot cylinder coefficient. The data were reduced by means of the following equation,

$$v_m = K(2/\rho_\ell \left\{ P_o - P \left[1 + \frac{c}{1 - c} \ln \frac{P}{P_o} \right] \right\})^{1/2} \tag{19}$$

where K = pitot cylinder coefficient, 0.975, from tests in pure water.

Since Reichardt [9] has presented considerable data for a series of cones under supercavitating conditions in pure water, it was decided to use the same series of cones in the present experiments. The ratios of cone height to base diameter of these bodies were 2, 1, 1/2, 1/4, and zero (normal disk). The base diameters were respectively .875, .55, .48, .4, and .375 inches. The diameters of these bodies were selected to give approximately the same total drag as measured by the dynamometer. In addition, a .25-in. diameter disk normal to the flow was tested to determine the existence of any choking effects or other influences of the solid tunnel walls.

The difference between the cavity pressure and the test section pressure was measured with a standard manometer. Pressure taps for this purpose were located on the test section wall and on the strut inside the cavity. In the pure water tests the cavity was artificially ventilated to achieve the desired cavitation number. In the bubbly flows this was not necessary. The cavity was ventilated and a rather constant cavitation number was maintained naturally by the bubbles in the flow. Since the cavitation number has relatively little influence on the drag coefficients compared to the compressibility effects, it was decided to use this naturally obtained cavity and not introduce another complication by trying to vary the cavitation number.

The drag was measured by a single component dynamometer which was constructed for these tests. The drag force was transmitted to the dynamometer mounted on the external wall of the tunnel by a force transmission bar passing through a bellows seal and a hollow strut. The hollow strut was of a symmetrical NACA 16-012 section which protruded into the cavity a short distance downstream of the test body. The test body was sting mounted to the force bar, and the distance from the downstream edge of the test body to the leading edge of the strut varied from 7 diameters for the smaller bodies to about 1/2 diameter for the largest body. The force transmission bar was attached to a linear differential transformer type dynamometer located just outside the test section wall. The dynamometer was calibrated and the output read on a heavily damped, average reading vacuum tube voltmeter. The drag coefficient was computed using the standard expression,

$$C_D = \frac{D}{A\rho {v_m}^2/2}$$

where D = drag, lbs, and
A = base area, ft^2.

The general experimental procedure was as follows:

(1) The tunnel was first operated without the introduction of any air bubbles into the test section. The velocity of flow was approximately that desired for the final test conditions.

(2) A measured rate of air was then introduced into the tunnel. The previously described concentration meter was initially located at the centerline of the tunnel. When the desired concentration was attained, the air flow resulting in this concentration was measured. It was found that three different rates of air flow were sufficient for most of the tests. The pressure difference between the test section and a location downstream on the diffuser was noted and used as a reference for subsequent tests at the desired condition.

(3) A concentration profile was measured across the test section with the concentration probe.

(4) The concentration probe was removed and replaced by the pitot cylinder. A velocity profile was obtained under the desired test section conditions as determined by steps (1) and (2) above.

(5) The appropriate test body, mounted on the dynamometer, was inserted into the center of the tunnel and the drag was measured under the desired test section conditions.

(6) Using centerline values of velocity and concentration, the sound speed, drag coefficient, and free stream Mach number were calculated.

(7) For each flow condition, the resulting data were reduced in the form of a concentration profile, a velocity profile, and drag coefficients for each body. A final composite chart showing the drag coefficient as a function of the free stream Mach number at the test section centerline was then plotted.

B. Discussion of Results

A limited number of drag measurements were taken for each body in pure water. These data were compared with Reichardt's earlier measurements. The agreement with Reichardt's results was nearly perfect. The St. Anthony Falls data are plotted on Fig. 12 at $M_{\infty} = 0$ (to be discussed later). The test section velocity profile for the current tests in pure water is presented in Fig. 5.

As previously mentioned, the air-water mixture in the test section was surrounded by pure water. The average size of the air bubbles in the core was determined as approximately .02 in. in diameter from photographs of the flow. In order to justify the assumption of isothermal flow in the previous development of Eq. (5), some estimation should be made as to the order of magnitude of the time scale involved for the flow to come from free stream conditions to a stagnation condition.

In the current studies if it is assumed that the presence of a 1/4-in. diameter cylinder influences the pressure field to a distance of approximately one diameter upstream of the body, at the highest flow rate used in the tests the approximate time for the flow to come to stagnation is about 5 milliseconds. This rather slow time corresponds to a frequency of less than 200 cps. The computed resonant frequency of the average size bubbles at the test section pressure, on the other hand, is of the order of 6Kcps. This analysis is admittedly rough but sufficient to justify the above mentioned assumption.

Typical measurements of the air concentration profiles across the test section are presented in Figs. 6, 7, and 8. For comparison, they are grouped according to the air flow rate, W_g, into the bubble generator. These air flow rates were respectively 4×10^{-3}, 4.9×10^{-3}, and 5.7×10^{-3} lbs/sec. The Mach number based on conditions at the centerline of the tunnel is indicated on these charts and it can be seen that higher values of air concentration correspond to higher values of Mach number. Even though the air injection rate was constant for each particular plot, the Mach number varied because the test section velocity was increased and therefore, the test section pressure, P_T, was decreased.

Representative plots of velocity profiles for the air-water mixture are shown in Figs. 9, 10, and 11. These profiles were also grouped according to the air flow rate to the bubble generator. It can be seen that the presence of air bubbles in the central portion of the flow increases the mixture velocity in that region over that which would have occurred without bubbles. The Mach number computed from the conditions at the centerline is again indicated on each figure.

In order to obtain some measure of the overall accuracy of the concentration and velocity measurements of the air-water mixture, some of these curves were graphically integrated to obtain the weight rate of flow of the air in the test section. This resulting value was compared with the quantity measured using an orifice meter upstream of the bubble generator. The agreement was better than 3 per cent.

The drag data reduced to the form of a standard drag coefficient versus Mach number plot are shown in Fig. 12. The computed Mach numbers were based on the flow velocity and the air concentration at the centerline of the tunnel. This was a rather arbitrary choice. If average values of the flow velocity and the concentration were used, the Mach numbers would probably have been slightly less than those shown.

It will be noted that two sets of data exist for the disk normal to the flow ($\frac{h}{d} = 0$). The open circles represent data taken with a 0.367-in. diameter disk, and the crossed circles represent data for a 0.25-in. disk. Data were taken with these two bodies to determine if any appreciable effect of the tunnel walls could be detected. The data for both disks apparently scatter without any definite trend being observed.

It should also be mentioned that the visibility of the test body and the cavity in the bubbly flow was poor, particularly for the higher concentrations of air. The presence of the body and the cavity produced a distortion or spreading of the bubbly core in the region of the cavity. A rough estimate of the cavity length resulted in values of about 8 to 10 inches. The cavity length seemed to be about the same for the series of bodies used in the investigation, with the exception of the smaller disk. This probably occurred as the test body diameters were selected to have roughly the same total drag force.

The curves shown in Fig. 12 have been calculated using Gothert's rule as given by Eq. (18). The experimental data generally agree with the computed values up to a Mach number of approximately 0.6 for the blunter bodies and up to approximately 0.7 for the more slender bodies. There seems to be a tendency for the measured drag coefficient to decrease slightly at Mach numbers over 0.6. There is evidence in wind tunnel data that shows for some bodies a similar slight decrease before a sudden sharp increase as the flow somewhere on the body becomes sonic. Since in cavity flows the maximum velocity occurs on the free streamline, the flow in this type of system would be expected to become sonic on the free streamline first.

In general, the experimental data seem to show that in cavity flows the trends are somewhat different from those observed in subsonic air flows [11]. For the latter case, the drag on a blunt body, such as the normal disk, increases very rapidly at much lower Mach numbers than that found in the present experiments.

In the existing facility it was not possible to obtain measurements on a non-cavitating body in an air-water mixture in order to permit direct comparison with the results obtained for subsonic flow in wind tunnels. The low test section pressures necessary to reduce the sonic velocity of the mixture generally caused cavitation to occur.

It will also be noted that computations based on Gothert's rule result in a drag coefficient of infinity at a Mach number of unity. Results from bodies in wind tunnels indicate that the experimental data begin to deviate from the calculated values at Mach numbers of about 0.8. Tulin has discussed the expected behavior of the drag coefficients of two-dimensional supercavitating bodies near Mach 1 [12]. Results show that the drag coefficients did not rise indefinitely as predicted by the subsonic theory, but attain a finite value at Mach 1. It is expected that the axisymmetric case may be somewhat similar.

IV. FLOW CHARACTERISTICS IN A CONVERGING-DIVERGING NOZZLE

A. Description of Converging-Diverging Nozzle

The existing test section of the 6-in. recirculating water tunnel did not permit a study of supersonic flow of an air-water mixture. Rather than modify the test section, it was decided to construct a converging-diverging nozzle on a smaller scale to investigate the characteristics of the bubbly mixture with greater economy of operation.

The general layout of the pilot facility is shown in Fig. 13. The tunnel itself was placed in a vertical position, and the water was supplied to the tunnel from the city water system through a 3-in. line. The water discharge was measured with a calibrated elbow meter located some distance upstream of the tunnel. The water was wasted after passing through the tunnel. A valve located downstream of the diffusor permitted variation of the back pressure. Air was introduced into the mixing section at either of the locations shown, as will be described later. The air was drawn from the Laboratory air supply system, and the flow rate determined by a calibrated orifice meter in the line. Water discharges up to 0.25 cfs and air flow rate of 0.0012 lb/sec were attainable. Since the air had to be forced into the tunnel against the water pressure, resonant conditions developed at small pressure differentials between the water and air systems, which resulted in a fluctuating air supply. By raising the air pressure the pulsation was eliminated, but the minimum air to water volume ratio was then limited to 0.04. The accuracy of low air to water volume ratios **is** in doubt, because of small quantities of gases that are liberated at the valve regulating the water discharge considerably upstream of the tunnel. The heavy throttling necessary for low water discharges produced audible cavitation noises at the valve.

The tunnel itself consisted of a converging section, throat, and diffusor constructed of lucite to facilitate viewing of the flow and bubble structure. The tunnel was two-dimensional, and no transition from the circular mixing section to the rectangular tunnel section was utilized. Sketches of the tunnel longitudinal profile and the mixing section are shown in Fig. 14. The nozzle was shaped from 3/4-in. lucite, and was sandwiched between two side walls made of 1/2-in. lucite. These side walls were bolted to the shaped section and were sealed with rubber "O" ring material extending the entire length of the tunnel. Five pressure taps were provided at the locations shown in Fig. 14 to permit pressure measurements upstream of the contraction, at the throat, and at three locations in the diverging section. Calibrated Bourdon gages were used to measure pressures.

The longitudinal profile of the tunnel shown in Fig. 14 indicates the presence of a second throat. The ordinate is drawn to an expanded scale. It was originally felt that this throat would permit easier observations of the existence of a shock wave. However, first tests indicated that the tunnel would operate better without this throat; therefore, the throat was removed, and a smooth transition made at this location. All measurements reported were taken with the modified section.

Some details of the mixing section are also shown in Fig. 14. The basic system employed a manifold surrounding the 3-in. diameter water inlet pipe. This manifold was attached to an air supply system at four locations. The pipe surrounded by the manifold was drilled with a number of small holes around its circumference, to permit a more or less uniform introduction of air into the conduit. In order to provide more uniform and finer distribution of bubbles in the flow, a close meshed wire screen was installed at the entrance to the rectangular tunnel. Later the 3-in. pipe below the air manifold was filled with wire mesh to help reduce secondary currents present in the water stream.

Tests were also made with a screen of the type used in wells. The wires of this screen were not of circular cross-section, but more of triangular shape with rounded edges. The apex of the triangle was pointed upstream, and the air was introduced directly in the region of the base of the triangle as shown in Fig. 14. It was intended that the wake region of the wires would serve as a mixer to disperse the air throughout the section. This alternate system was found to provide a rather uniform distribution of bubbles in the converging section of the tunnel as will later be seen in photographs.

After the tunnel was fabricated and some preliminary tests conducted, results of other investigators [15] became available. Although the nozzle utilized by Muir and Eichhorn was of different proportions, results similar to those obtained in the present study were reported. Further mention of their work will be made in the following sections.

B. Pressure Measurements

Pressure measurements seemed to indicate that sonic flow was experienced in the throat of the nozzle with air-water mixtures under conditions listed in Table I. For these ranges when the back pressure was increased by

TABLE I

Tabulated Experimental Data for Air-Water Mixtures in Converging-Diverging Nozzle

Run	$W_g \times 10^3$ lb/sec	W_ℓ lb/sec	β_o	β_2^*	β_3	P_o psia	$\frac{P_2^*}{P_o}$	$\frac{P_3}{P_o}$
1	1.20	9.48	.063	.155	.240	23.7	.304	.144
2	2.14	9.48	.096	.212	.300	26.9	.335	.183
3	2.88	9.48	.117	.236	.346	28.9	.370	.194
4	3.66	9.48	.136	.260	.376	31.2	.391	.208
5	4.75	9.48	.160	.290	.414	33.5	.409	.218
6	6.85	9.48	.200	.328	.472	36.7	.458	.232
7	8.41	9.48	.222	.348	.506	39.2	.488	.238
8	10.92	9.48	.258	.383	.548	42.7	.502	.241
9	13.30	9.48	.285	.408	.574	44.9	.526	.256
10	14.58	9.48	.301	.420	.585	46.2	.535	.262
11	15.92	9.48	.310	.431	.599	47.2	.544	.265
12	1.79	11.5	.052	.150	.244	34.9	.264	.121
13	2.48	11.5	.066	.170	.276	37.9	.293	.137
14	3.30	11.5	.082	.196	.310	40.0	.315	.150
15	4.48	11.5	.101	.217	.342	43.0	.354	.172
16	6.00	11.5	.122	.249	.382	46.4	.373	.185
17	7.35	11.5	.138	.262	.403	49.2	.405	.199
18	9.31	11.5	.161	.290	.433	52.2	.423	.212
19	11.00	11.5	.176	.307	.460	54.9	.441	.216
20	12.56	11.5	.192	.322	.478	56.7	.454	.224
21	14.56	11.5	.205	.338	.500	59.7	.469	.228
22	16.10	11.5	.218	.350	.509	61.7	.474	.236
23	17.88	11.5	.232	.362	.529	63.7	.486	.236
24	1.70	14.4	.031	.108	.202	47.7	.220	.086
25	3.25	14.4	.054	.151	.273	50.9	.278	.114
26	4.29	14.4	.065	.166	.296	55.7	.302	.127
27	5.74	14.4	.081	.191	.323	58.5	.327	.147
28	8.13	14.4	.102	.220	.369	63.4	.361	.162
29	10.20	14.4	.118	.242	.384	67.5	.380	.182
30	12.58	14.4	.135	.262	.408	71.7	.398	.195
31	14.78	14.4	.150	.278	.427	74.2	.416	.206

means of an exit valve, the pressure at any given tap in the nozzle remained constant until an apparent shockwave, which was sometimes visible, had passed the tap. The initial large pressure rise was sudden with subsequent pressure rises approximately proportional to the increase in back pressure. The instant a pressure rise was noted at the throat, the pressures also rose in the approach section. These observations are essentially the same as those made for flow of a compressible gas flowing through a converging-diverging nozzle. In this case, the independence of the back pressure on the throat pressure is a result of sonic flow or choked flow in the throat.

It was of interest to compare experimental measurements of the pressure at various positions in the nozzle with those calculated from theory by Tangren et al [5] assuming a homogeneous mixture. In the theory, no consideration was given to the possibility that slip could occur between the air bubbles and water in the region of a strong pressure gradient. With such an assumption it is possible to solve for the flow conditions throughout the nozzle using the momentum and continuity equations and the equation of state for the compressible gas phase.

Muir and Eichhorn [15] in their studies allowed for the existence of a differential velocity between the two phases. The resulting differential equations for air-water flow through a nozzle could not be solved unless some information was available as to the magnitude of the slip velocity or the velocity of one of the phases. Muir and Eichhorn used high-speed synchronized motion picture photography to determine the velocity of the gas bubbles. In the present studies, such equipment was not available, and the facility did not lend itself to the use of electronic methods. As a result, a complete analysis of the flow behavior in the nozzle was not possible. The differential velocity picture was further complicated by the relative magnitude of the buoyancy force on a gas bubble in the long approach section of the nozzle. Since the flow is downward, the gas bubbles would tend to be retarded by the buoyancy force in the absence of an appreciable pressure gradient. In addition, the methods of introducing the air into the water stream may result in a velocity lag of the gas phase. In the manifold system, the air stream enters at right angles to the water stream, and then the mixture passes through a pipe filled with wire mesh where gas bubbles may be retarded in the wake regions. In the alternate air supply system, the air stream enters again at right angles to the water stream directly into a wake region.

Without the knowledge of the slip velocity a local concentration factor cannot be determined. Instead a similar correlation parameter, called the volume flow fraction β_n, is used which is defined as follows:

$$\beta_n = \frac{cv_g}{cv_g + (1 - c)\, v_\ell} \tag{20}$$

Expressions for the velocities may be obtained from the continuity equations:

$$W_g = \gamma_g\, cv_g A = \text{constant} \tag{21}$$

and

$$W_\ell = \gamma_\ell (1 - c)\, v_\ell A = \text{constant} \tag{22}$$

so that the parameter β_n can readily be determined from mass flow and pressure measurements. β_n is equal to c when the phase velocities are the same, and also $\beta_n \lesseqgtr c$ if $v_g \lesseqgtr v_\ell$.

The results of pressure measurements are presented in Fig. 15 and 16 for the tap at the throat and a tap in the diverging section respectively. The local pressures are referred to stagnation pressure conditions. β_o is the corresponding volume flow fraction which is identical to c_o since velocities are zero. β_2 and β_3 are the local volume flow fractions at the throat and in the diverging section, respectively. In Fig. 15, it can be seen that the throat pressure data agree with the experimental data by Muir and Eichhorn. The pressure variations are essentially independent of the liquid mass flow rate. The theoretical pressure variation without slip is shown as a dashed line.

For the pressure variation at the tap in the diverging section, shown in Fig. 16, the mass flow rate has little effect if the correlating parameter is β_o; but the mass flow rate becomes a factor if the pressure variation is plotted against the local β. The variation with mass flow rate may reflect the influence of liberated gas in the water supply for which no allowance was made.

In the absence of liquid and gas velocity measurements at the throat, a mean mixture velocity may be used to describe sonic conditions. The mean mixture velocity is defined as

$$v_m = \frac{W_\ell}{\gamma_\ell A} + \frac{W_g}{\gamma_g A} \qquad (23)$$

It has physical significance only if $v_g = v_\ell$, in which case it is the same as the bulk mixture velocity. A plot of v_m against β_o for all test conditions is shown in Fig. 17. The dashed lines represent cross-plots for constant pressure at the throat. The mean mixture velocities that produce sonic conditions at the throat seem to be low.

C. Visual Observations of Flow

Direct observations of the tunnel in operation revealed very little about the flow behavior of the mixture. The bubble structure of the air-water mixture was discernible in the converging section up to the throat. Beyond the throat, the mixture took on a milky appearance of very uniform consistency, which persisted generally across the shock wave and throughout the remaining portions of the tunnel. The shock wave could be observed only under the following conditions:

(1) low air-water volume ratios,

(2) location of the shock in the rapidly diverging portion of the tunnel,

(3) strong lighting, and

(4) generally an oblique angle of view at the front face of the tunnel.

The location of a shock wave for a given flow condition fluctuated considerably, the fluctuations increasing with increasing air-water ratios. Very high amplitude fluctuations occurred when the shock was located in the narrow portion of the diverging section. Verification of the location of the shock wave could also be obtained by noting the sudden pressure rise on gages located in the diverging section of the tunnel as the back pressure was varied by means of the exit valve.

Density variations across the shock wave were noted only at minimum air-water volume ratios. Along the diverging walls of the tunnel, separation of the flow from the wall was noted below the shock front. At minimum air-water volume ratios, a criss-cross pattern of reflected shock waves seemed to be in evidence in the remaining portion of the tunnel below the initial shock front.

D. Discussion of Photographs of Flow

Efforts to photograph the various flow phenomena in the tunnel proved to be quite difficult, and final results were not as clear as desired. The extremely fine texture of the supersonic air-water mixture caused considerable light diffusion and, consequently, lack of details in the mixture. To make the shock wave more noticeable on a photograph, it was necessary to add red dye to the water in order to bring out density changes. In all the following photographs, the flow is from top to bottom. Figure 18a shows a shock wave for an air-water ratio of $\beta_o = 0.04$. The shock wave is in the vicinity of the arrows in the center of the picture. Figure 18b shows a shock wave in a water and water vapor mixture. Here the transition from a supersonic to a subsonic region brings about a change in phase of the gas-liquid mixture. The supersonic mixture was obtained by operating the tunnel with water only so that water vapor pressures were reached at the throat.

Several series of close-up photographs were taken to examine the details of the bubble structure of the air-water mixtures throughout the nozzle. The close proximity of the camera to the object required that only back lighting be employed. The light source was a high-speed electronic flash unit having an approximate speed of 2 micro-seconds. The low intensity of this light source made it necessary to open the camera lens wide thus reducing the depth of focus. As a result, only the bubbles at the front face of the tunnel are in focus. Each series consists of four photos. Photos a, b, and c of each series show successive sections of the tunnel above, at, and below the throat, respectively. Photo d shows a section further down the tunnel in the rapidly diverging section where the shock wave is normally placed for observation by adjustment of the back pressure. Each photo is approximately a 2:1 enlargement of the actual physical size.

The initial series in Fig. 19 shows a flow condition in which all velocities remain subsonic. Air bubble growth can be noted in the converging section and through the throat; however, not far beyond the throat in the diverging section, the bubbles can be seen to break up very rapidly. Further down in the diverging section, only very small bubbles exist ranging in size from 0.02 in. to 0.1 in. in diameter. Figure 20 is a series of photographs of supersonic flow operation for a low mixture ratio. The break-up of the bubbles is more complete, resulting in an almost homogeneous mixture. The shock wave is not well defined visually, although pressure measurements indicate its presence. A higher rate of discharge and larger air-water mixture ratio is shown in Fig. 21. The break-up of the bubbles takes place more rapidly than in the previous case. The shock wave is not visible except possibly for some separation along the diverging tunnel walls.

V. CONCLUSIONS

On the basis of the tests described in Section III, it appears that the drag coefficient of a supercavitating, axisymmetric body in a stream of a compressible, bubbly mixture was generally higher than that for the same body in pure water. The drag coefficient increased with increasing Mach number up to Mach numbers of approximately 0.6 after which only slight changes were observed. The rate of the drag increase was a function of slenderness ratio of the body, increasing with decreasing slenderness ratio. The drag rise was somewhat less than that experienced for non-cavitating bodies in a subsonic air stream.

Comparisons of the experimental data with calculated values of the drag coefficients based on a modified form of Gothert's rule indicated general agreement at the lower Mach numbers. At Mach numbers greater than 0.6, the theory considerably overestimated the drag coefficient.

Results of the investigation with the converging-diverging nozzle have shown that it is possible to obtain supersonic flow conditions with an air-water mixture. Pressure measurements in the throat of the converging-diverging nozzle agree with those obtained by other investigators [15] with a nozzle of somewhat different proportions and overall layout. Weak shock waves were observed in the diffusor under certain conditions.

Lack of information on the behavior of air-water mixtures in converging-diverging nozzles at the outset of this investigation led to the design of a nozzle which is not ideally suited for detailed measurements of all of the flow characteristics. The data collected for this nozzle are, therefore, limited. It is felt that a more suitable nozzle for the study of air-water mixtures should incorporate the following features:

(1) The converging section of the nozzle should be of such proportions that the accelerative force is large compared to the buoyancy force on a gas bubble, if the nozzle is mounted in a vertical position.

(2) The mixing section and any transition section between the mixing section and the converging section of the nozzle should be designed in such a manner that equal liquid and gas velocities are achieved at the entrance to the converging section; alternatively, features of item (3) should be incorporated.

(3) Provisions for the measurement of phase velocities or the use of electronic equipment to measure the concentration at several points along the nozzle should be made.

(4) If the nozzle is mounted vertically, the overall length should be short enough to eliminate the effect of the elevation head which becomes a complicated variable due to density variations throughout the nozzle.

(5) The nozzle should be proportioned so as to minimize any boundary layer effects.

ACKNOWLEDGEMENTS

Scott Crist and R. Rao were primarily responsible for the collection and reduction of the experimental data. The contributions by W. H. C. Maxwell in various phases of the program are also acknowledged. Preparation of the manuscript for printing was carried out by Mrs. Kathleen Lagerberg and Miss Judy Mike.

LIST OF REFERENCES

[1] Schiebe, F. R. and Wetzel, J. M. Ventilated Cavities on Submerged Three-Dimensional Hydrofoils, University of Minnesota, St. Anthony Falls Hydraulic Laboratory, Technical Paper No. 36, Series B, Minneapolis, Minnesota, December 1961.

[2] Wood, A. B. A Textbook of Sound, Macmillan, New York, 1930.

[3] Campbell, I. J. and Pitcher, A. S. "Flow of Air Water Mixtures," Supplement to the Proceedings of the Joint Admiralty - U. S. Navy Meeting on Hydroballistics, pp. 8 - 28, 1955.

[4] Campbell, I. J. and Pitcher, A. S. "Shock Waves in a Liquid Containing Gas Bubbles," Proceedings of the Royal Society, Vol. 243, 1958.

[5] Tangren, R. F., Dodge, C. H., and Seifert, H. S. "Compressibility Effects in Two-Phase Flow," Journal of Applied Physics, Vol. 20, No. 7, July 1949.

[6] Schiebe, F. R., Wetzel, J. M., and Foerster, K. E. Drag Characteristics of a Supercavitating Body in a Compressible Bubbly Flow, University of Minnesota, St. Anthony Falls Hydraulic Laboratory, Memorandum No. M-99, Minneapolis, Minnesota, April 1964.

[7] Silberman, E. "Sound Velocity and Attenuation in Bubbly Mixtures Measured in Standing Wave Tubes," The Journal of the Acoustical Society of America, Vol. 29, No. 8, August 1957.

[8] Killen, J. M., Personal Communication.

[9] Reichardt, H. The Laws of Cavitation Bubbles at Axially Symmetrical Bodies in a Flow, Ministry of Aircraft Production, Reports and Translations, No. 766, August 15, 1946, (distributed by Office of Naval Research, Washington, D. C.).

[10] Shapiro, A. H. The Dynamics and Thermodynamics of Compressible Fluid Flow, Vol. 1, Ronald Press Company, New York, 1953.

[11] Hoerner, S. F. Fluid Dynamic Drag, Published by the Author, New York, 1958.

[12] Tulin, M. P. "New Developments in the Theory of Supercavitating Flows," Second Symposium on Naval Hydrodynamics, Washington, D. C., August 1958.

[13] Ripken, J. F. and Olson, R. M. A Study of the Influence of Gas Nuclei on Cavitation Scale Effects in Water Tunnel Tests, University of Minnesota, St. Anthony Falls Hydraulic Laboratory, Project Report No. 58, Minneapolis, Minnesota, February 1958.

[14] Lamb, O. P. and Killen, J. M. An Electrical Method for Measuring Air Concentration in Flowing Air-Water Mixtures, University of Minnesota, St. Anthony Falls Hydraulic Laboratory, Technical Paper No. 2, Series B, 1950.

[15] Muir, J. F. and Eichhorn, R. "Compressible Flow of an Air-Water Mixture through a Vertical, Two-Dimensional Converging-Diverging Nozzle," Proc. 1963 Heat Transfer and Fluid Mechanics Institute, Stanford University Press, Stanford, California, 1963.

FIGURES

(1 through 21)

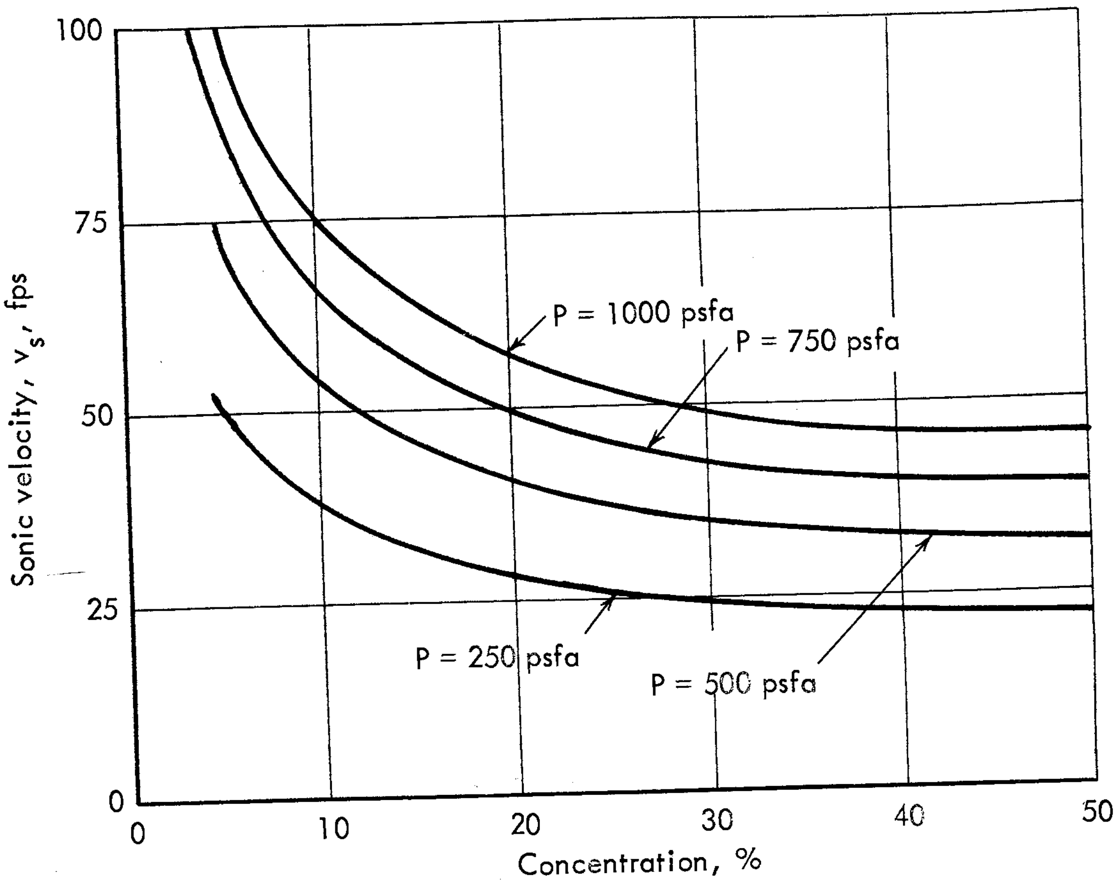


Fig. 1 - Variation of Sonic Velocity with Concentration and Pressure

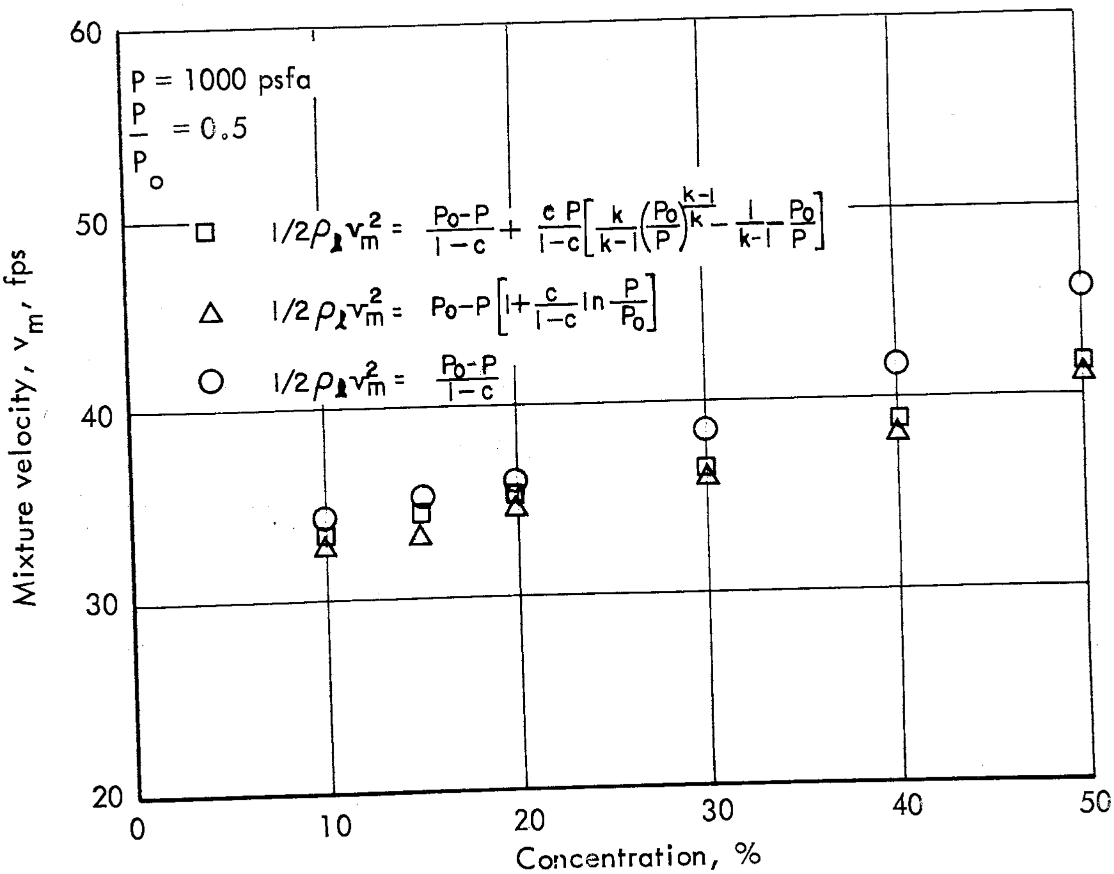


Fig. 2 - Comparison of Various Equations for Calculating Mixture Velocity

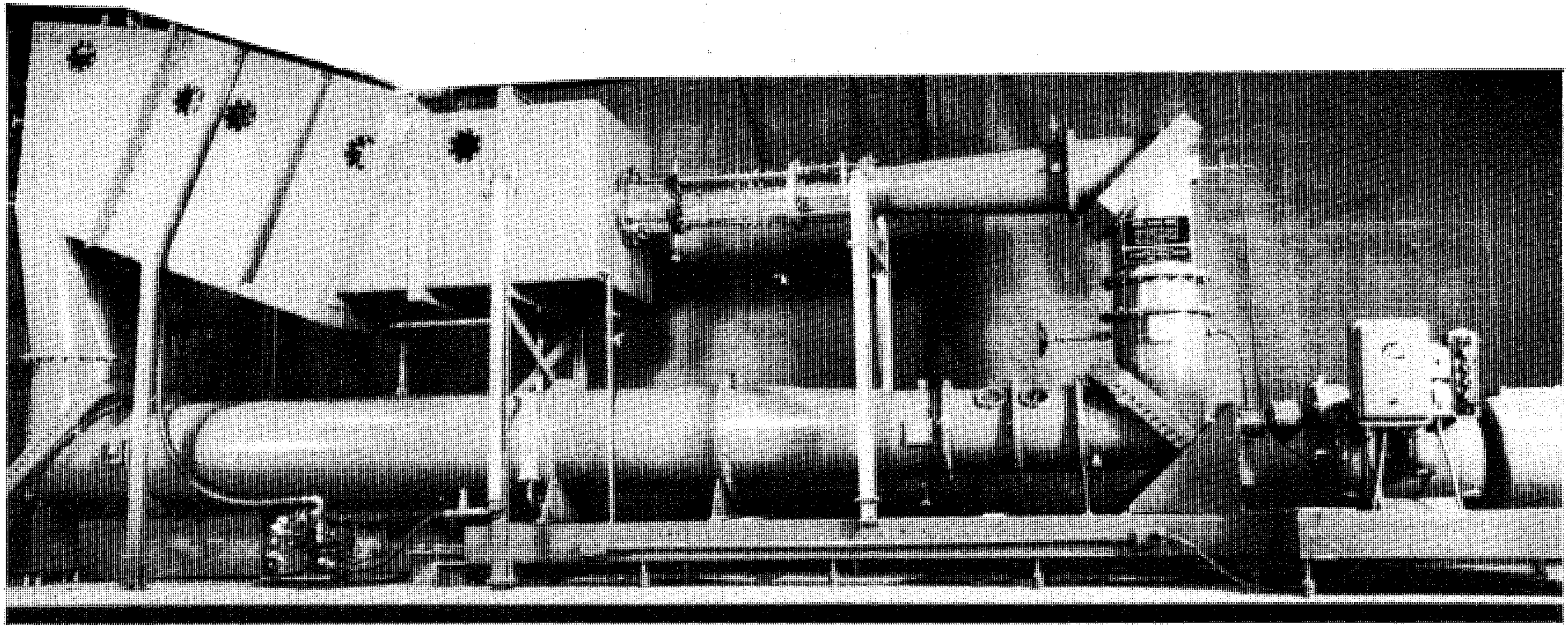


Fig. 3 - Six-Inch Recirculating Water Tunnel with Gas Separator

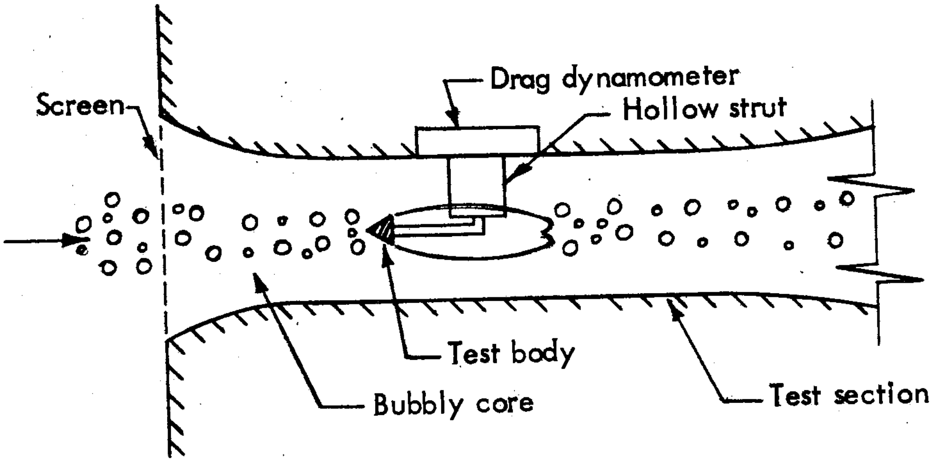


Fig. 4 - Schematic of Flow in Test Section

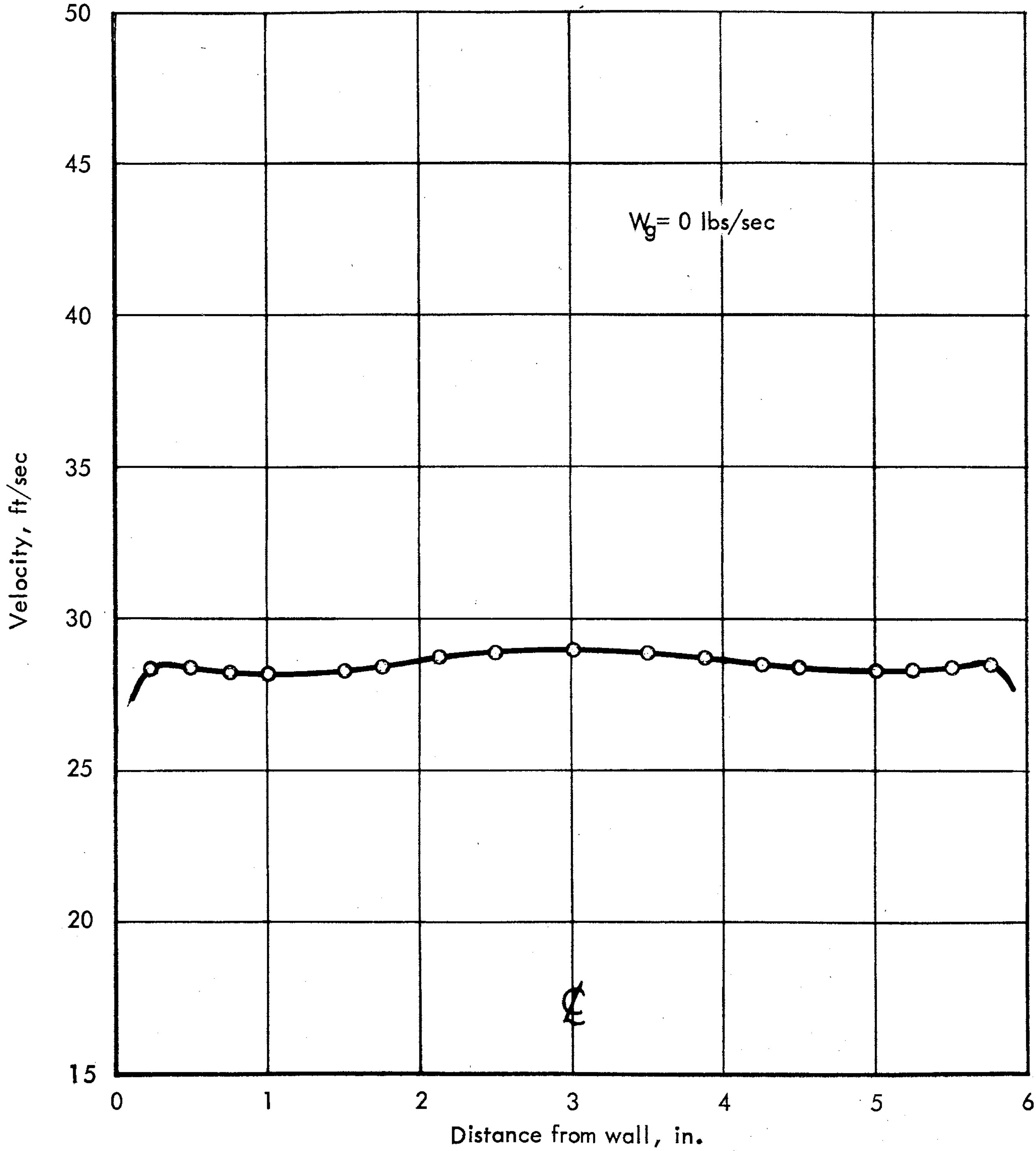


Fig. 5 - Test Section Velocity Profile, Pure Water

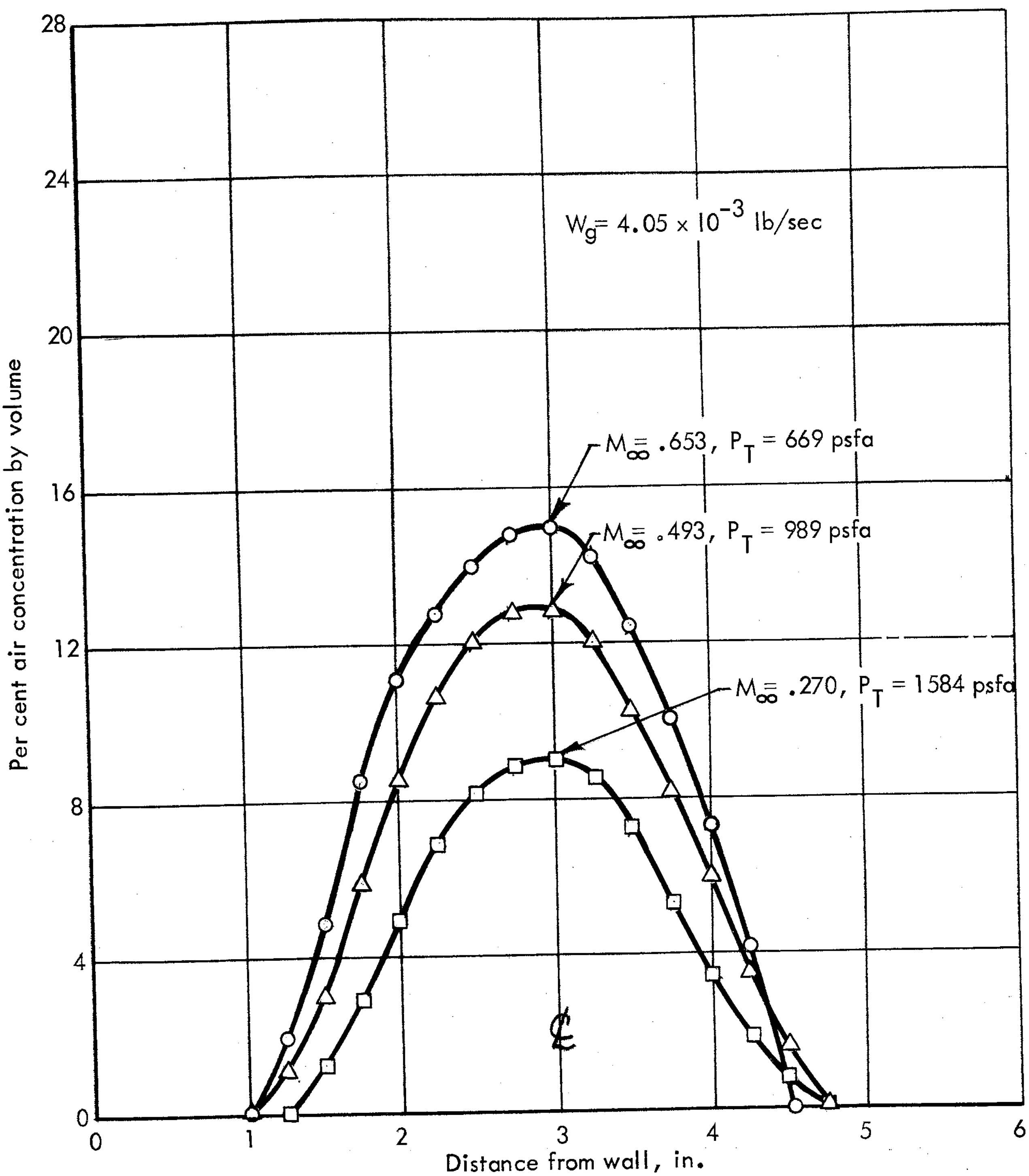


Fig. 6 - Test Section Air Concentration Profiles, $W_g = 4.05 \times 10^{-3}$ lbs/sec

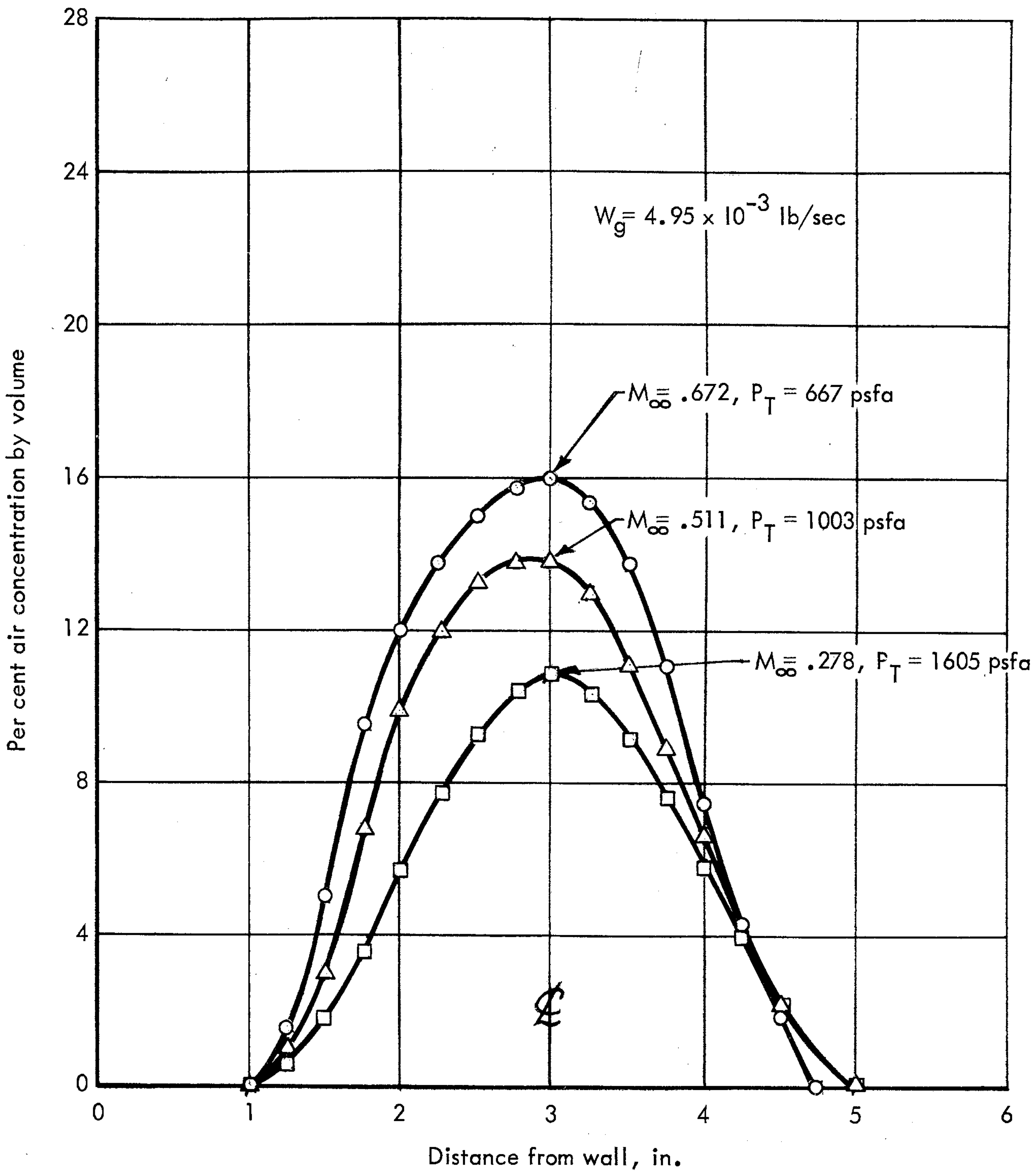


Fig. 7 - Test Section Air Concentration Profiles, $W_g = 4.95 \times 10^{-3}$ lbs/sec

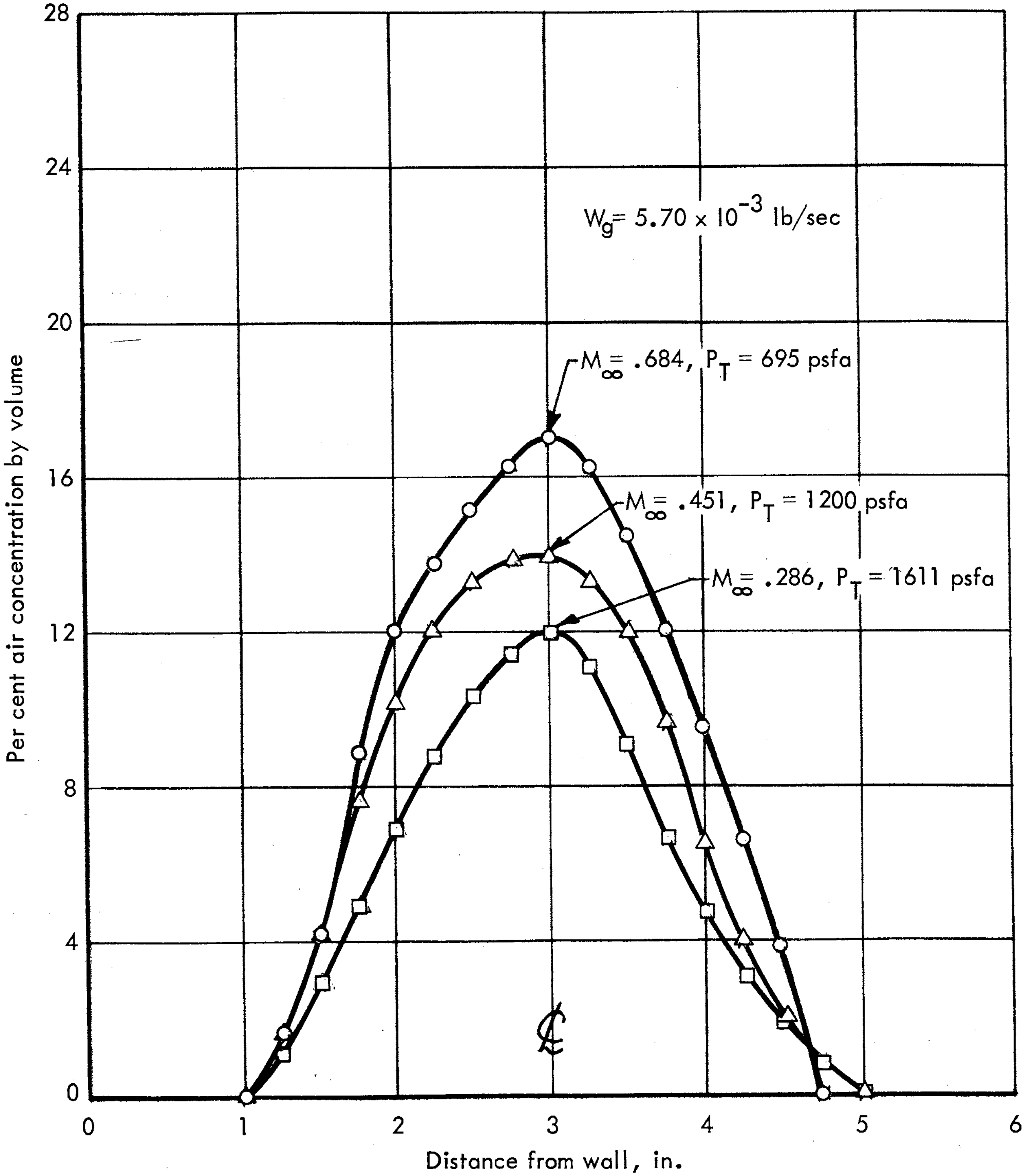


Fig. 8 - Test Section Air Concentration Profiles, $W_g = 5.70 \times 10^{-3}$ lbs/sec

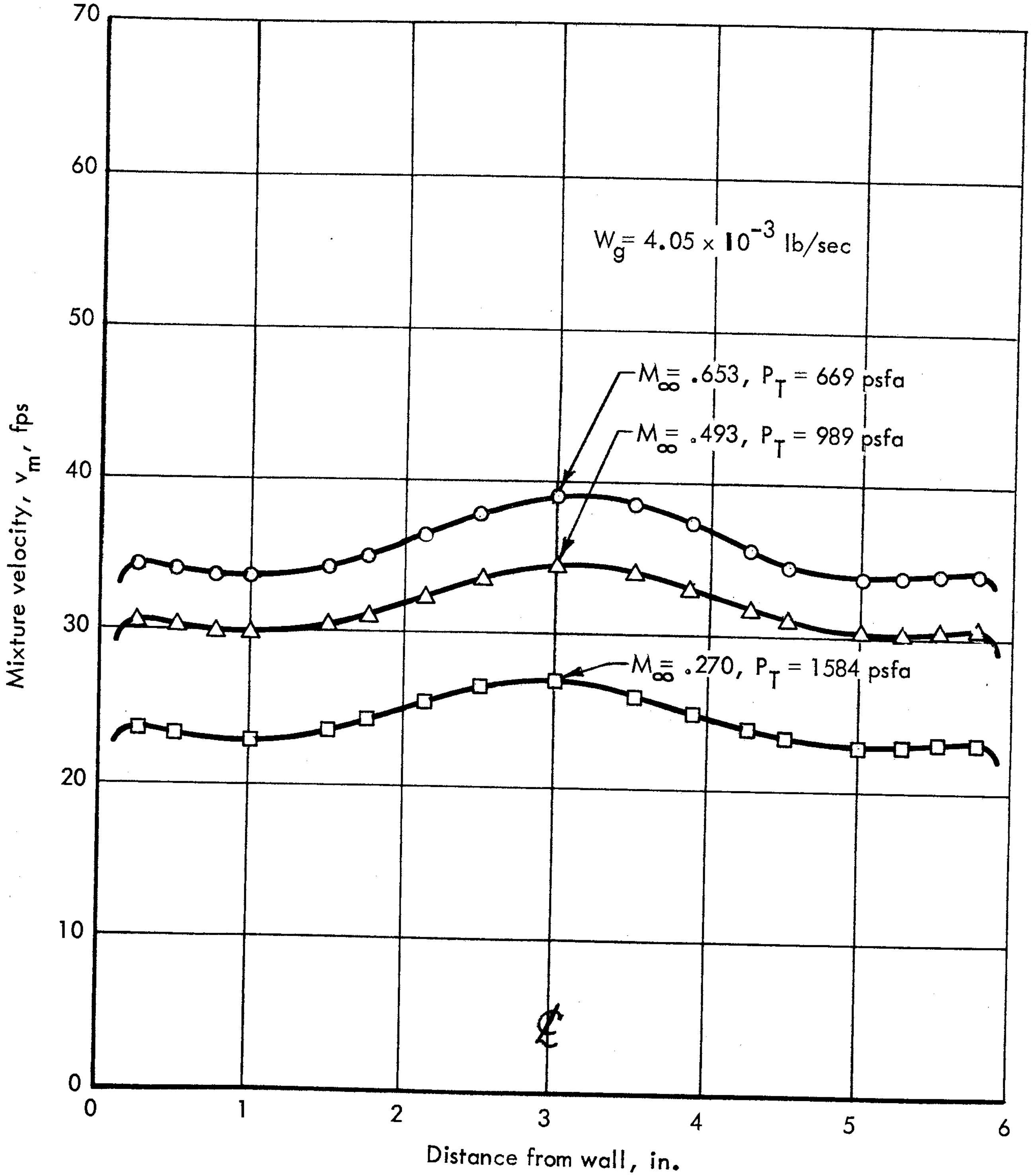


Fig. 9 - Test Section Velocity Profile, Bubbly Mixture, $W_g = 4.05 \times 10^{-3}$ lbs/sec

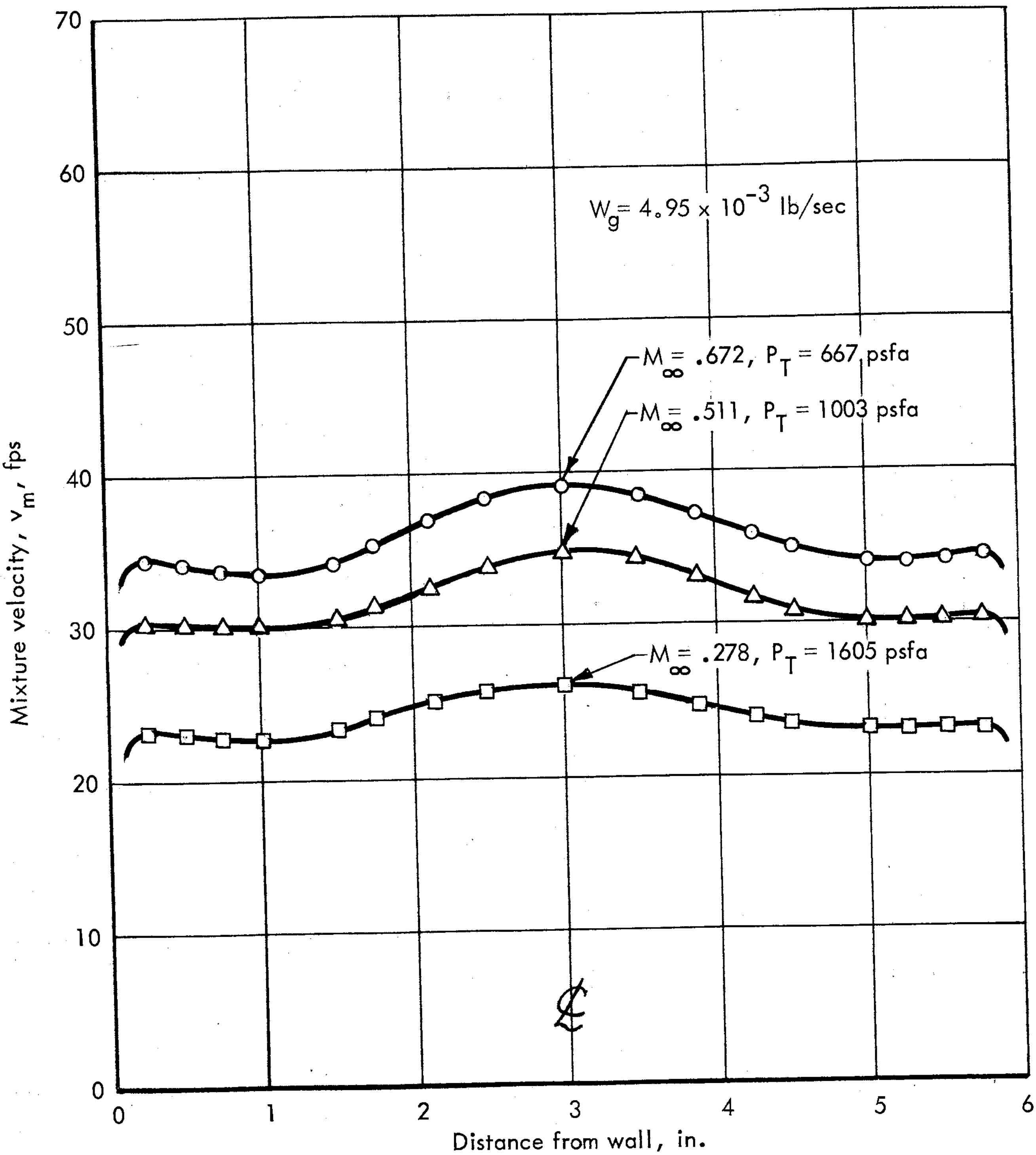


Fig. 10 - Test Section Velocity Profiles, Bubbly Mixture, $W_g = 4.95 \times 10^{-3}$ lbs/sec

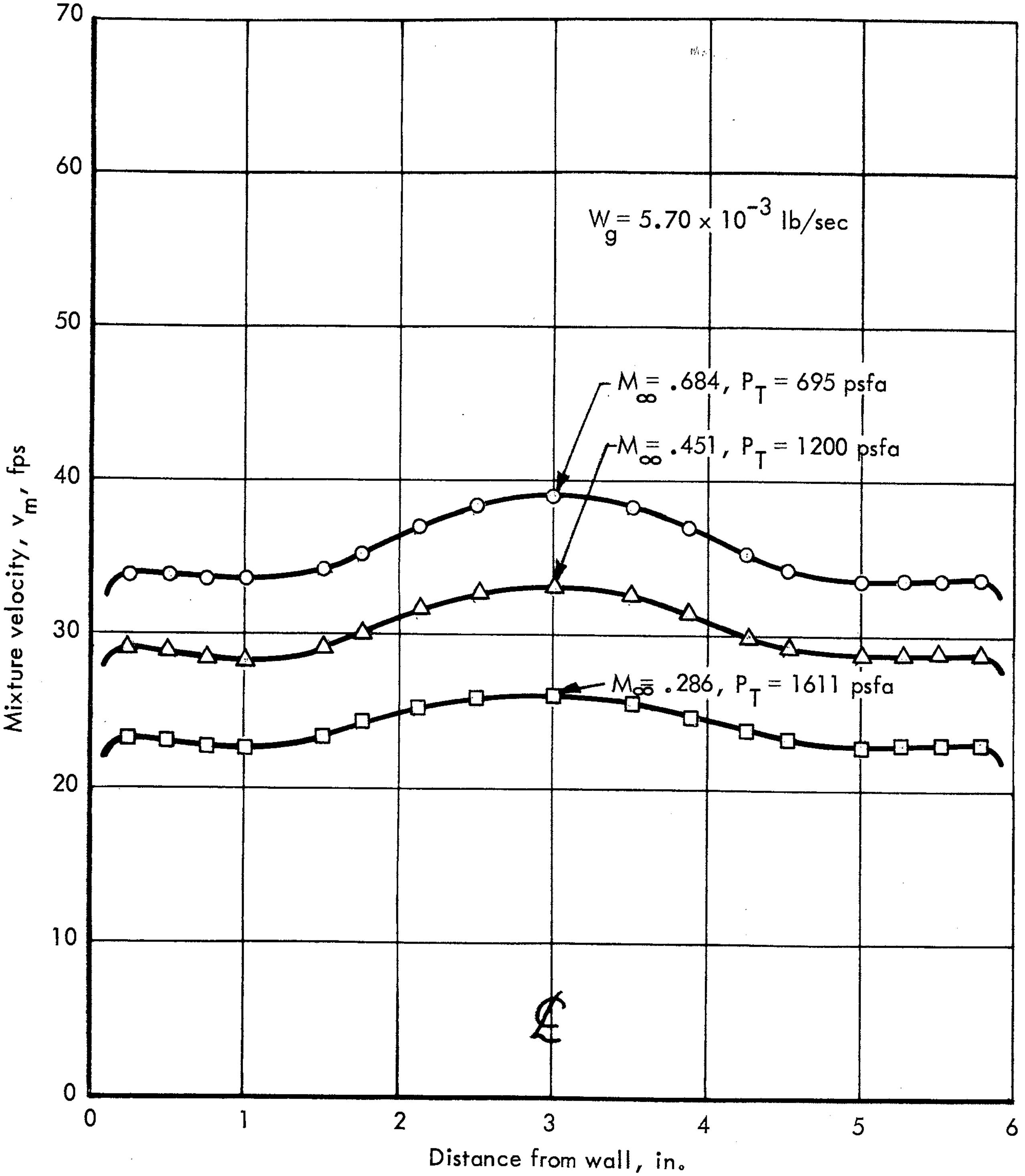


Fig. 11 - Test Section Velocity Profiles, Bubbly Mixture, $W_g = 5.70 \times 10^{-3}$ lbs/sec

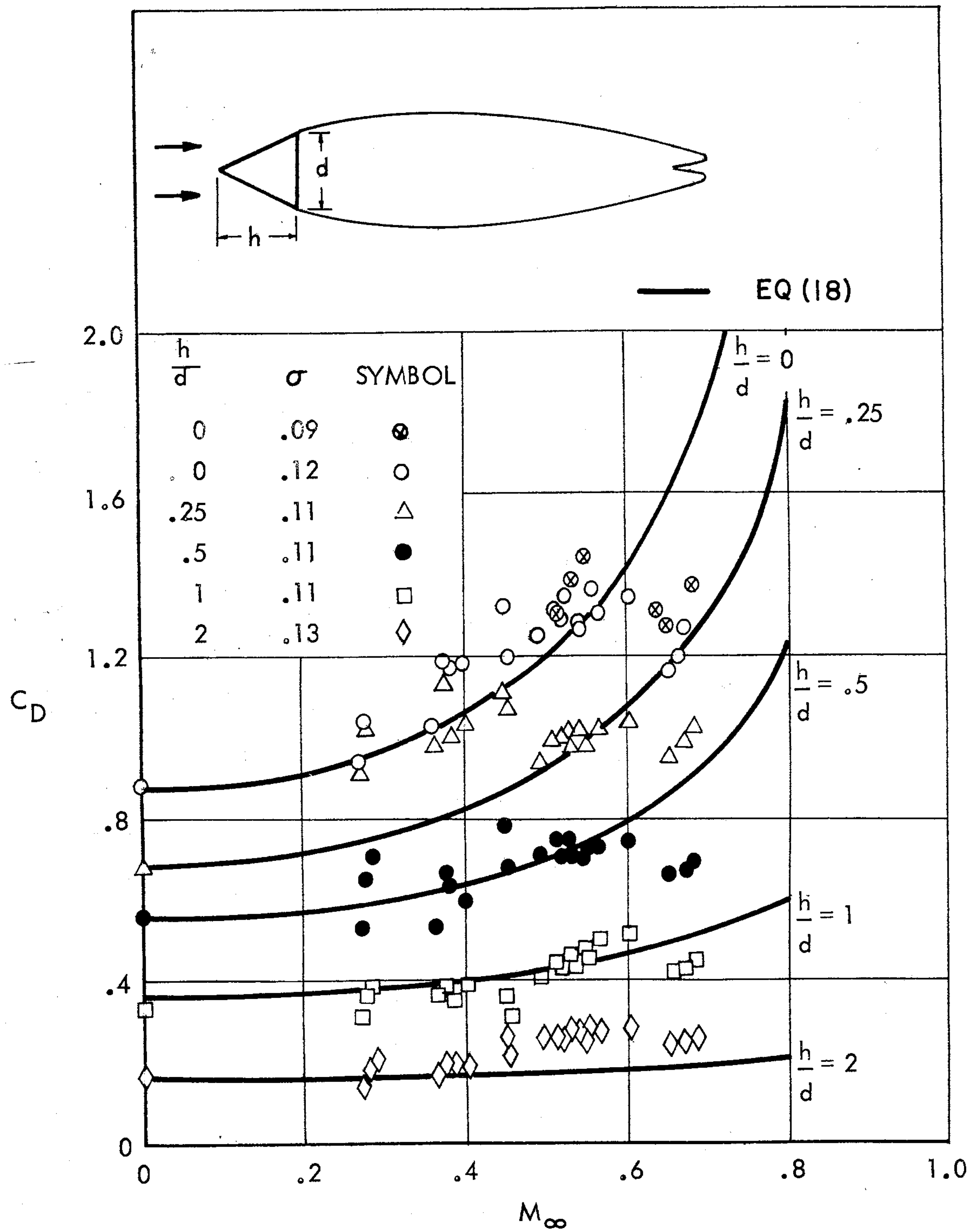


Fig. 12 - Variation of Drag Coefficient with Mach Number for a Series of Supercavitating Cones in a Bubbly Mixture

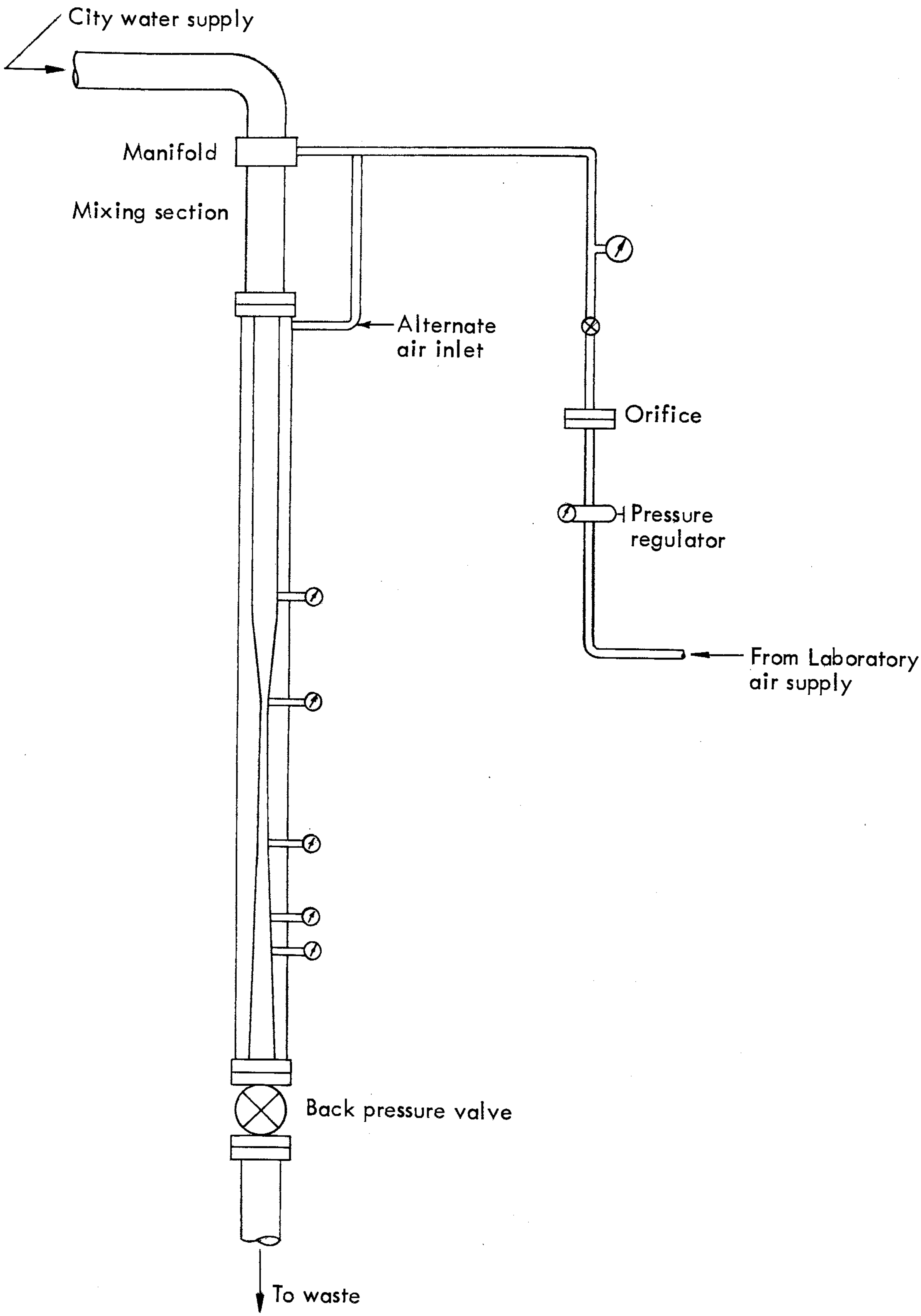
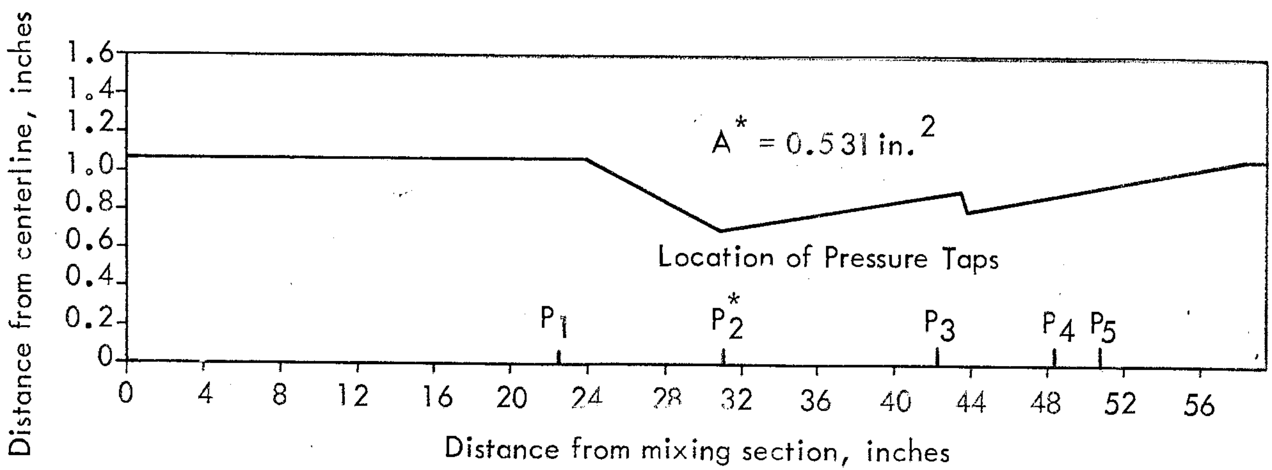
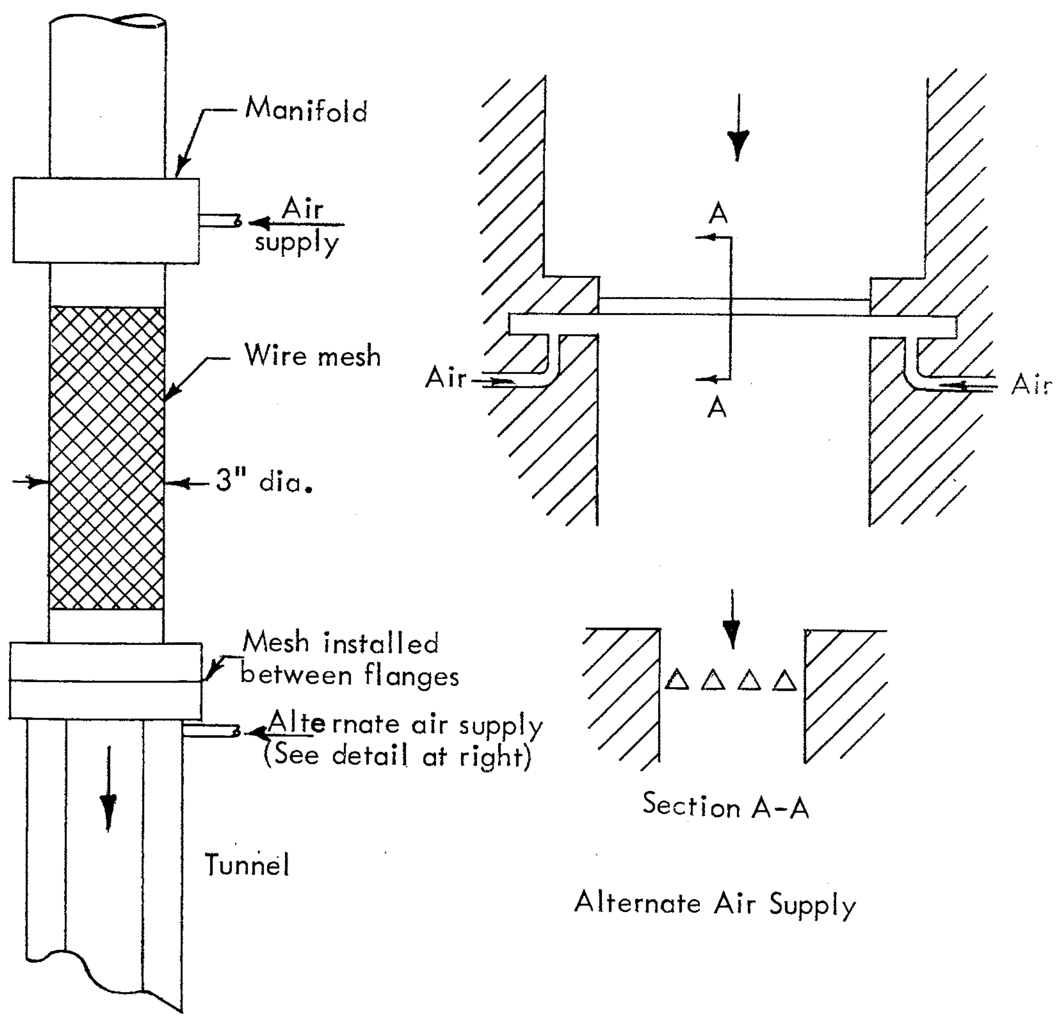


Fig. 13 - General Layout of Converging - Diverging Nozzle

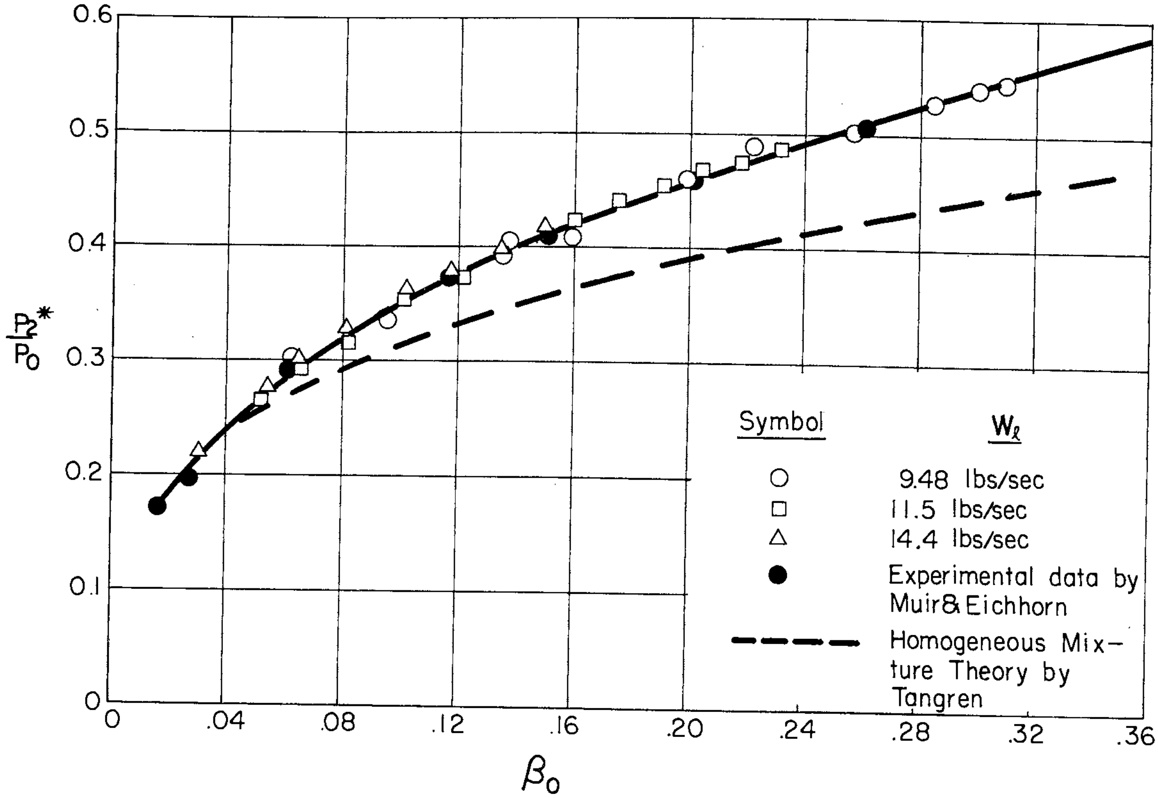


Longitudinal Profile of Nozzle



Alternate Air Supply

Fig. 14 - Details of Converging - Diverging Nozzle



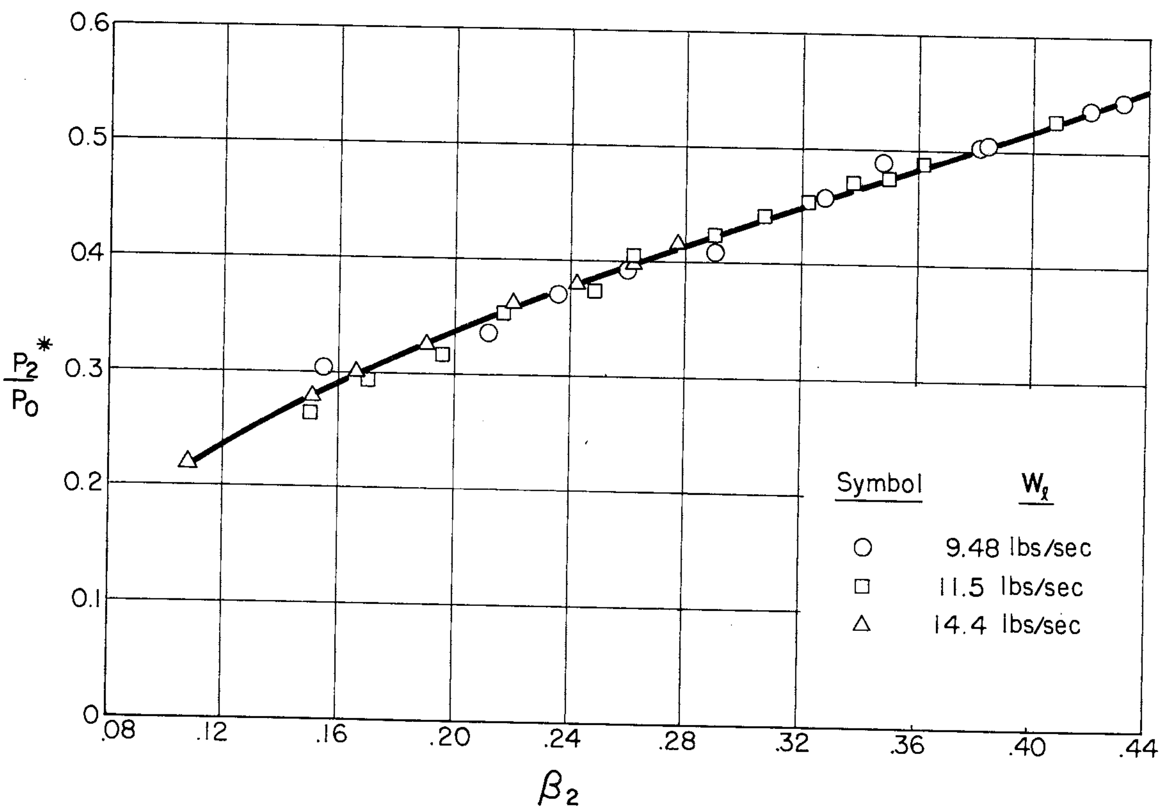


Fig. 15 - Pressure Variations at Throat of Nozzle

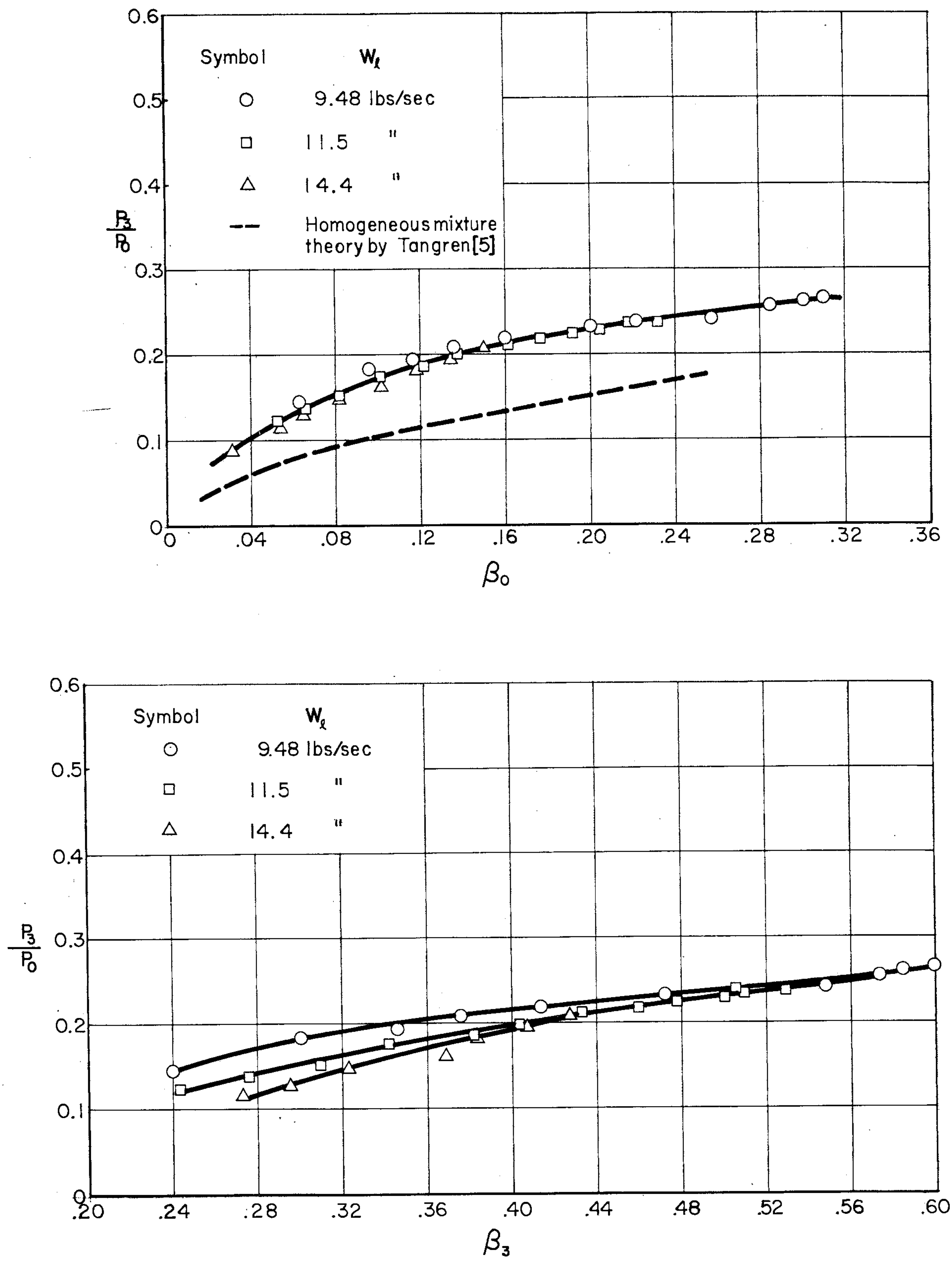


Fig. 16 – Pressure Variations at a Point in the Diverging Section of the Nozzle

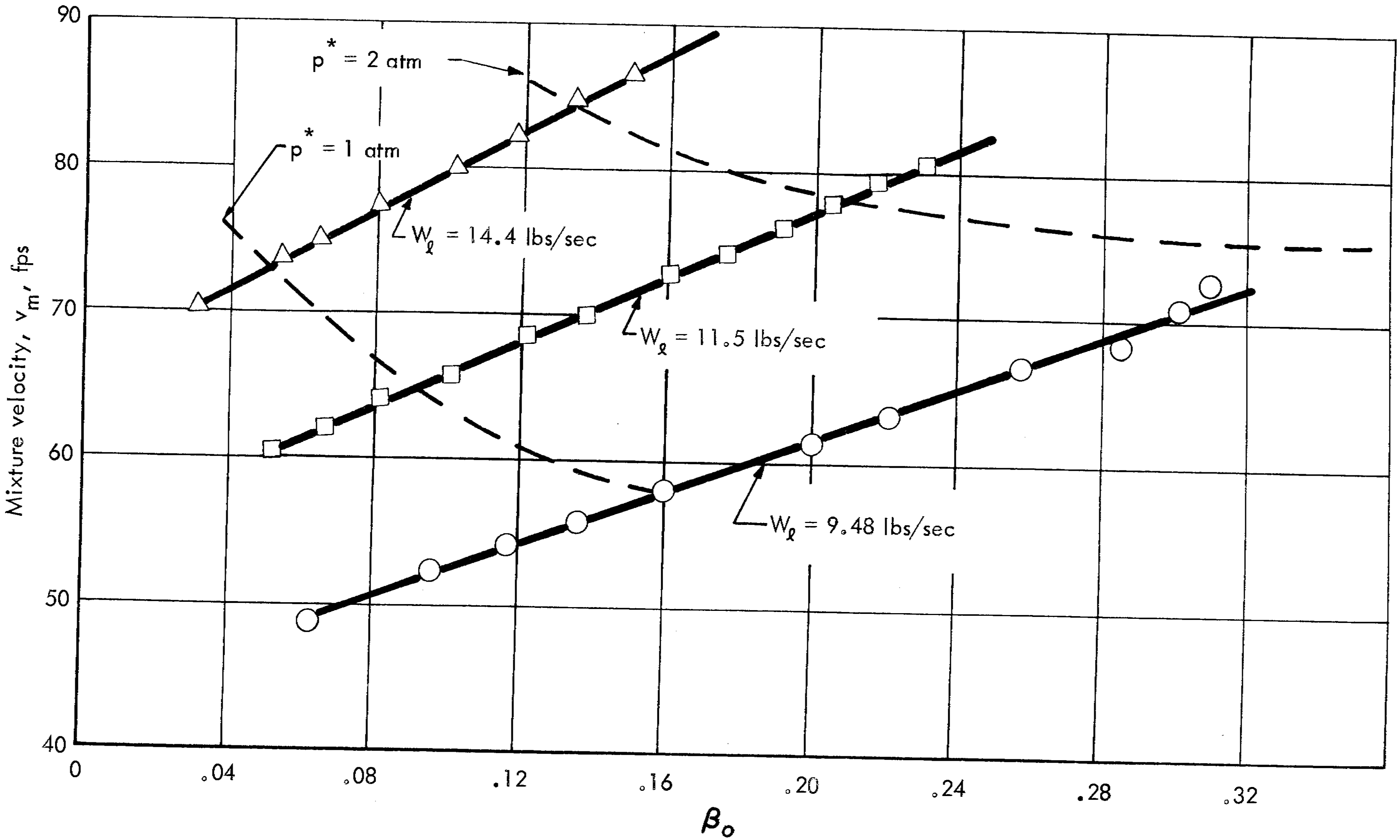
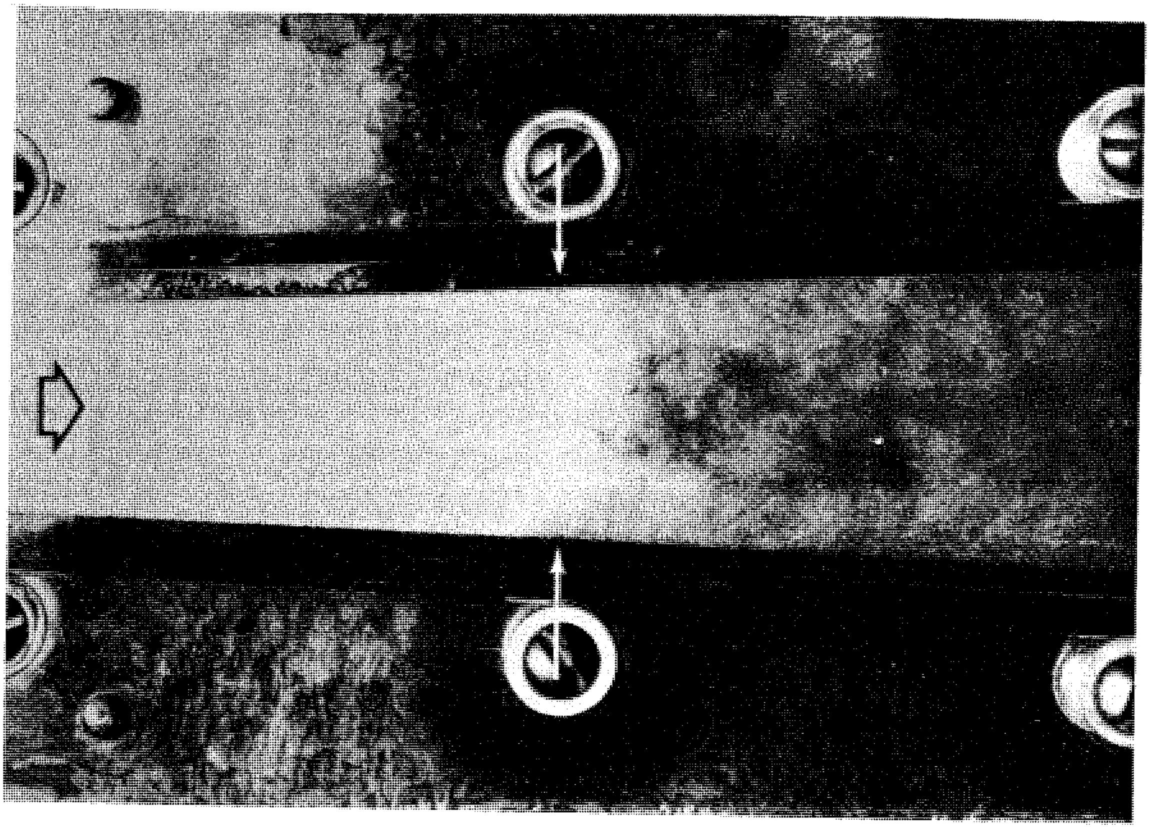


Fig. 17 - Mean Mixture Velocity in the Nozzle Throat

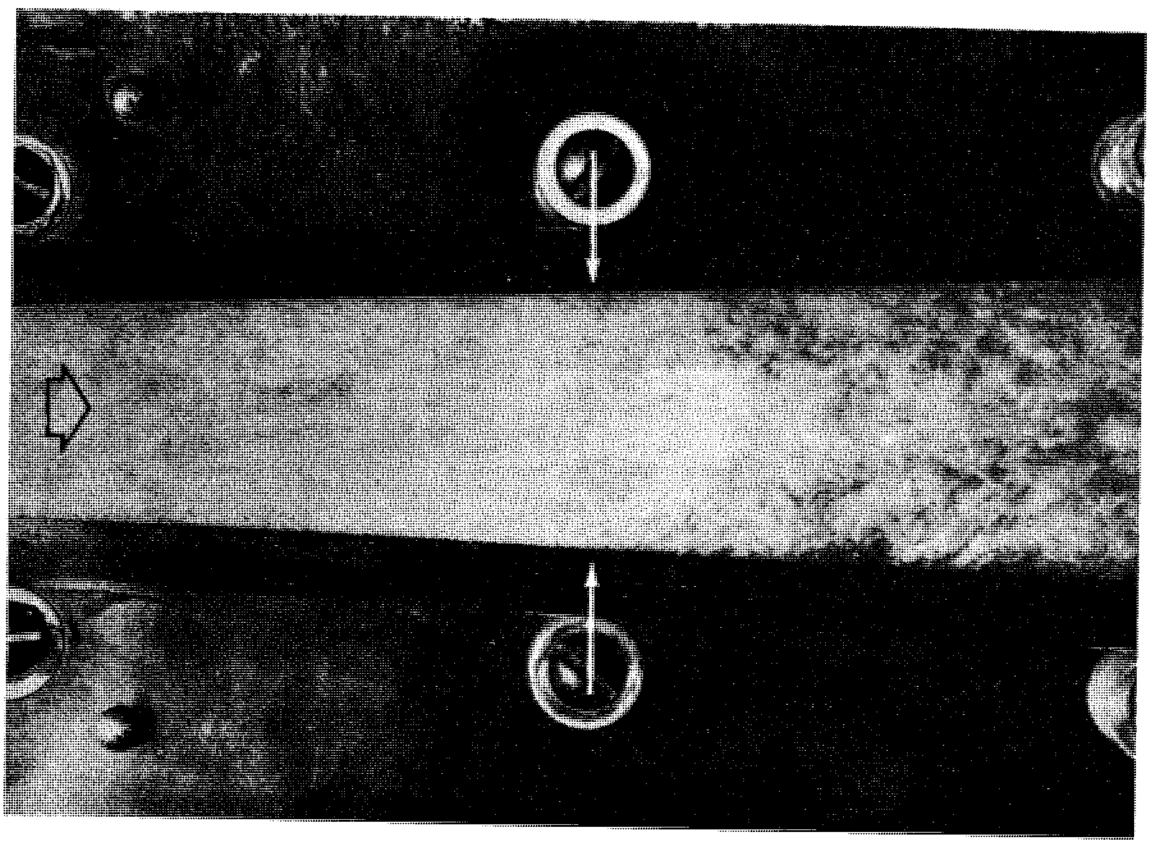
a. Air and Water Mixture, $\beta_o = 0.040$,
$W_g = 3.65 \times 10^{-3}$ lbs/sec, $W_\ell = 17.4$ lbs/sec

b. Water Vapor and Water Mixture

Fig. 18 - Shockwaves in Supersonic Gas-Liquid Mixtures



b.

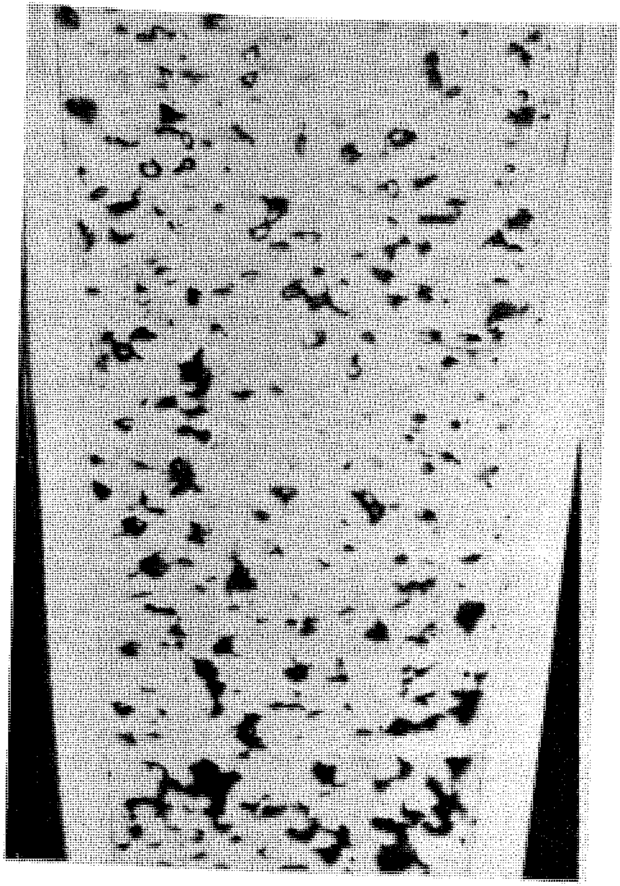


a.

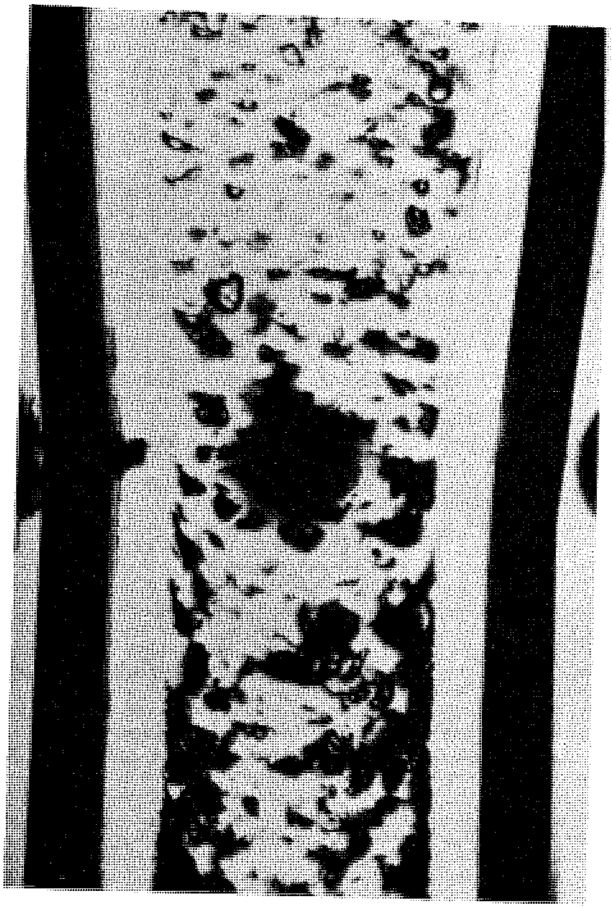
Fig. 19 - Subsonic Air-Water Mixture in a Converging-Diverging Nozzle

$\beta_o = 0.024$, $P_o = 24$ psia, $W_g = 0.48 \times 10^{-3}$ lbs/sec,

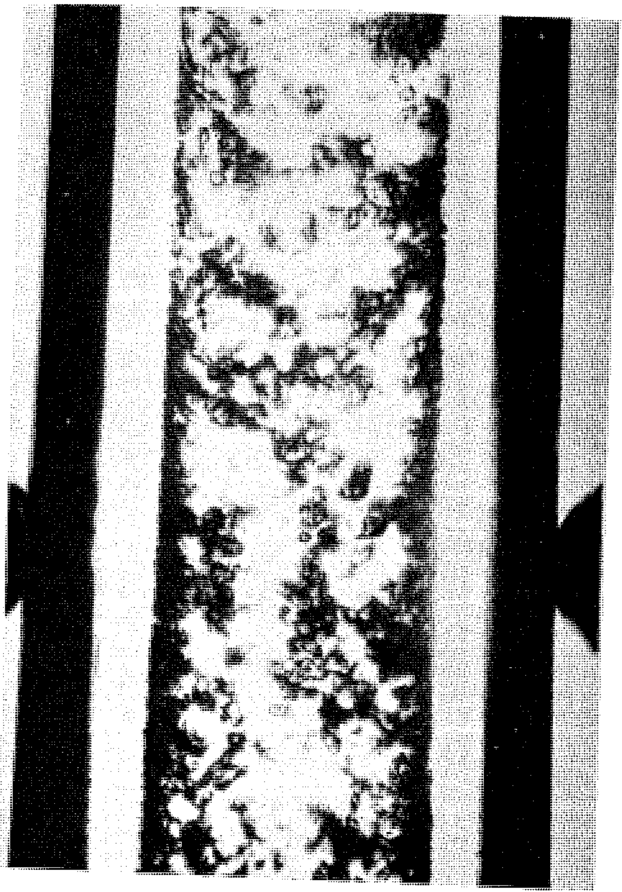
$W_\ell = 10.0$ lbs/sec



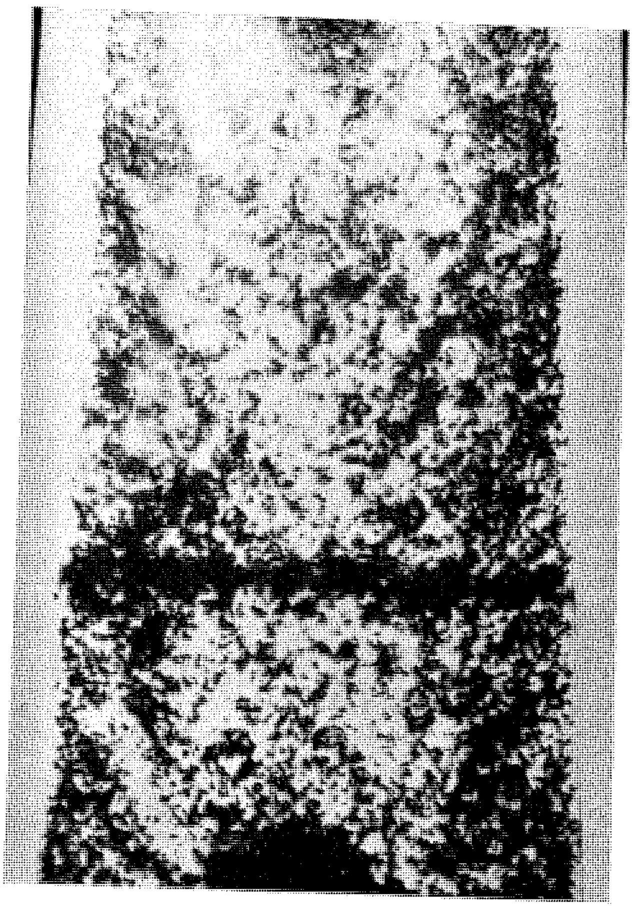
a.



b.



c.

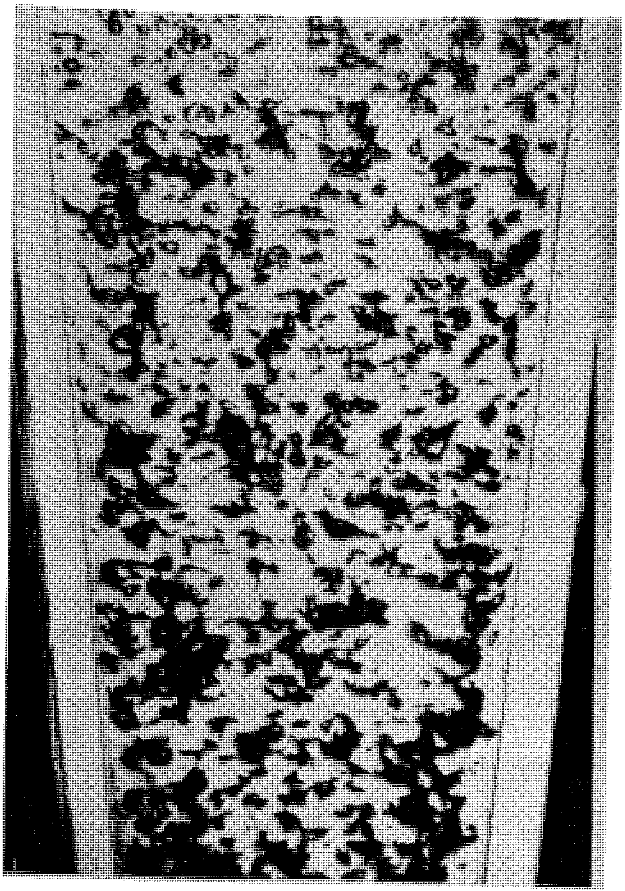


d.

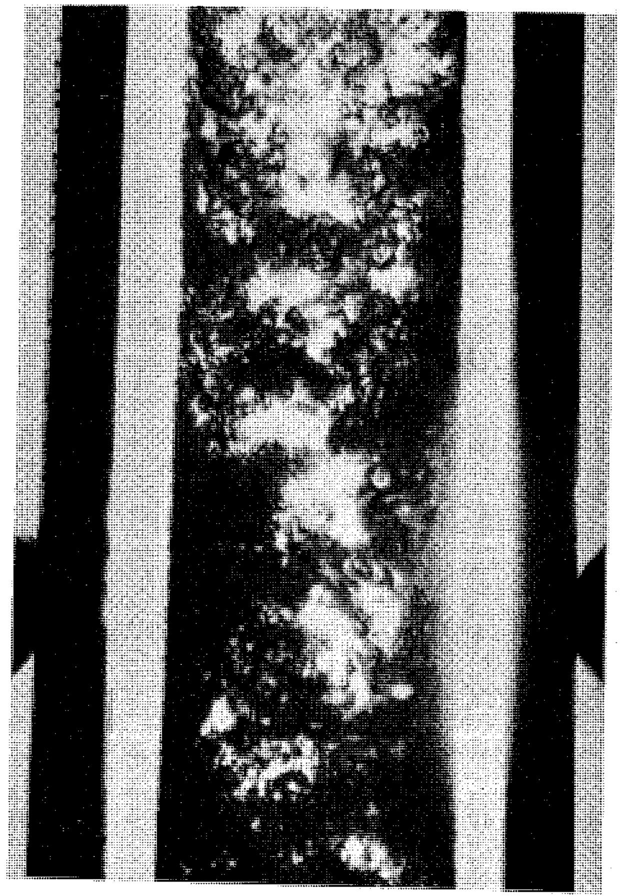
Fig. 20 - Supersonic Air-Water Mixture in a Converging-Diverging Nozzle

$\beta_o = 0.039$, $P_o = 36$ psia, $W_g = 1.47 \times 10^{-3}$ lbs/sec,

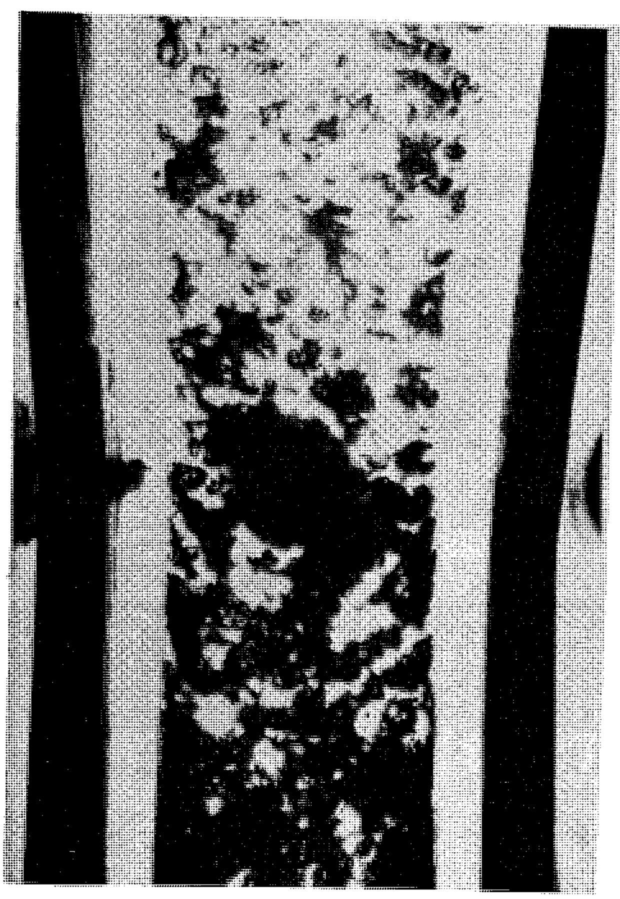
$W_\ell = 12.4$ lbs/sec



a.



c.



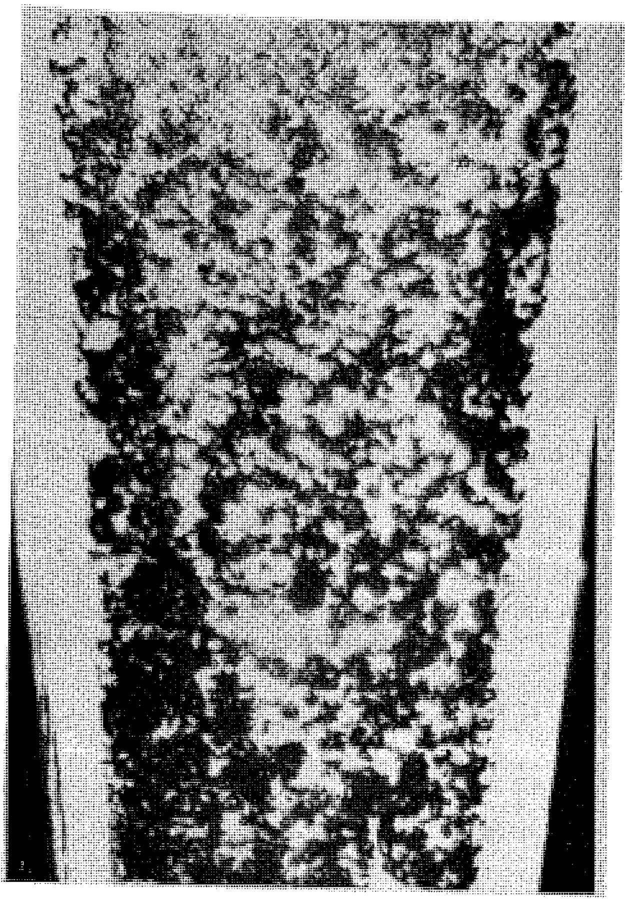
b.

d.

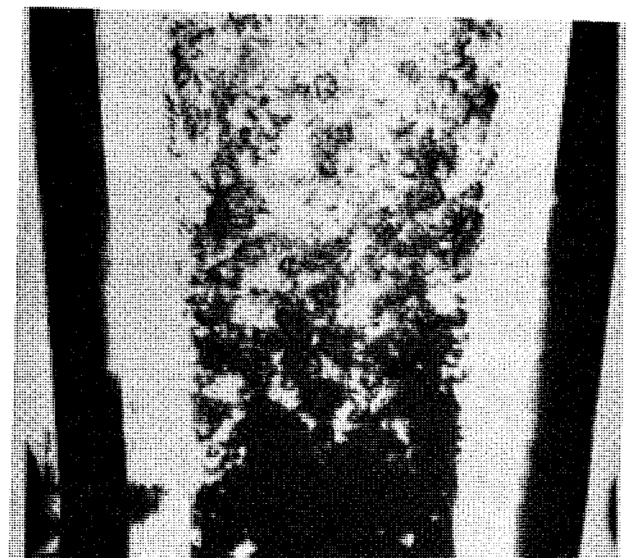
Fig. 21 - Supersonic Air-Water Mixture in a Converging-Diverging Nozzle

$\beta_o = 0.090$, $P_o = 66$ psia, $W_g = 7.95 \times 10^{-3}$ lbs/sec,

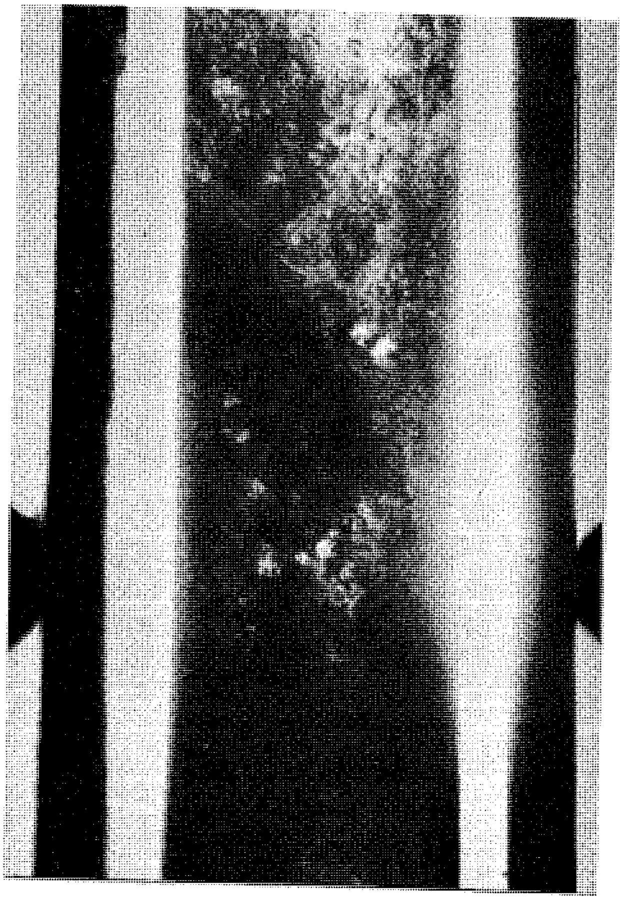
$W_\ell = 15.0$ lbs/sec



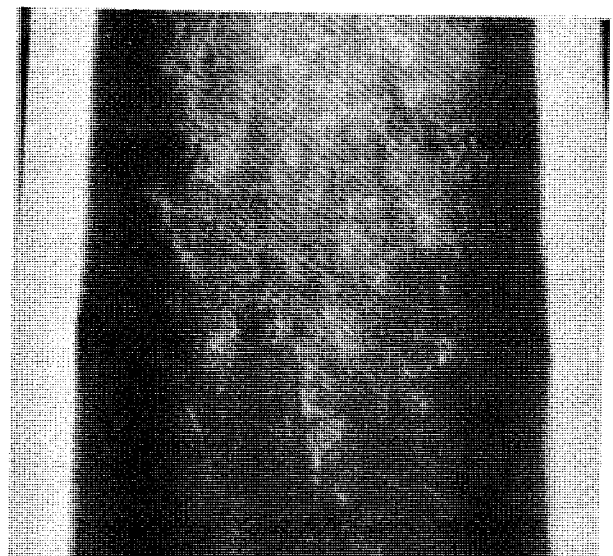
a.



b.



c.



d.

DISTRIBUTION LIST FOR TECHNICAL PAPER NO. 48, SERIES B
of the St. Anthony Falls Hydraulic Laboratory

Copies	Organization
75	Commanding Officer and Director, David Taylor Model Basin, Washington 7, D. C., Attn: Code 513.
11	Chief, Bureau of Ships, Department of the Navy, Washington 25, D. C., Attn: 5 - Technical Library (Code 210L) 1 - Preliminary Design (Code 420) 1 - Hull Design (Code 440) 1 - Research and Division Planning (Code 320) 1 - Code 341B 1 - Code 345
6	Chief, Bureau of Yards and Docks, Department of the Navy, Washington 25, D. C.
4	Chief of Naval Research, Department of the Navy, Washington 25, D. C., Attn: 3 - Mechanics Branch (Code ONR 438) 1 - Acoustic Branch (Code ONR 411)
1	Director, U. S. Naval Research Laboratory, Washington 25, D. C., Attn: Code 2021.
1	Commanding Officer, Office of Naval Research, Branch Office, 230 North Michigan Avenue, Chicago 1, Illinois.
1	Commander, U. S. Naval Ordnance Laboratory, White Oak, Silver Spring, Maryland.
1	Commander, U. S. Naval Ordnance Test Station, 3202 East Foothill Boulevard, Pasadena, California.
1	Commanding Officer and Director, U. S. Navy Underwater Sound Laboratory, Fort Trumbull, New London, Connecticut.
1	Superintendent, U. S. Naval Postgraduate School, Monterey, California, Attn: Librarian.
2	Director, Ordnance Research Laboratory, Pennsylvania State University, University Park, Pennsylvania.
3	Chief, Bureau of Naval Weapons, Department of the Navy, Washington, D. C. 20360.
1	Chief of Research and Development, Department of the Army, Washington 25, D. C.

Copies	Organization
1	U. S. Naval Applied Science Laboratory, Naval Base, Brooklyn, New York, Attn: J. F. Lichtman.
1	Director, U. S. Waterways Experiment Station, Corps of Engineers, P. O. Box 631, Vicksburg, Mississippi.
1	Office of the Chief of Engineers, Department of the Army, Gravelly Point, Washington 25, D. C.
1	Director of Research, National Aeronautics and Space Administration, 1724 F Street, N. W., Washington 25, D. C.
1	Director, Langley Aeronautical Laboratory, National Aeronautics and Space Administration, Langley Field, Virginia.
1	Director, Hydraulic Laboratory, Bureau of Reclamation, Denver Federal Center, Denver, Colorado.
1	National Aeronautics and Space Administration, Lewis Research Center, Cleveland, Ohio, Attn: Mr. R. S. Ruggeri.
20	Defense Documentation Center, Cameron Station, Alexandria, Virginia 22314.
2	Director, National Bureau of Standards, National Hydraulic Laboratory, Washington 25, D. C.
2	Newport News Shipbuilding and Dry Dock Company, Newport News, Virginia, Attn: 1 - Senior Naval Architect 1 - Supervisor, Hydraulic Laboratory
2	California Institute of Technology, Division of Engineering, Pasadena 4, California, Attn: 1 - Dr. M. S. Plesset 1 - Dr. A. T. Ellis
2	Director, Davidson Laboratory, Stevens Institute of Technology, 711 Hudson Street, Hoboken, New Jersey.
1	Director, Hydraulic Laboratory, Worcester Polytechnic Institute, Worcester 2, Massachusetts.
1	Director, Hydrodynamics Laboratory, Massachusetts Institute of Technology, Cambridge 39, Massachusetts.
1	Head, Department of Naval Architecture and Marine Engineering, Massachusetts Institute of Technology, Cambridge 39, Massachusetts.
1	Director, Woods Hole Oceanographic Institute, Woods Hole, Massachusetts.

Copies	Organization
1	Director, Iowa Institute of Hydraulic Research, State University of Iowa, Iowa City, Iowa.
1	University of Michigan, Department of Nuclear Engineering, Ann Arbor, Michigan, Attn: Prof. F. G. Hammitt.
1	S. Logan Kerr and Company, P. O. Box 6, Flourtown, Pennsylvania, Attn: Mr. S. Logan Kerr.
1	Allis Chalmers Mfg. Company, Hydraulic Division, York, Pennsylvania, Attn: Mr. Wm. J. Rheingans.
1	Hydronautics, Incorporated, Pindell School Road, Howard County, Attn: Mr. Phillip Eisenberg.
1	Vitro Laboratories, 14000 Georgia Avenue, Silver Springs, Maryland, Attn: Mr. W. R. Turner.
1	Librarian, Engineering Societies Library, 29 West 39th Street, New York 18, New York.
2	Library, California Institute of Technology, Pasadena, California.
1	Librarian, Massachusetts Institute of Technology, Cambridge 39, Massachusetts.
1	Librarian, Library of Congress, Washington 25, D. C.
1	Librarian, School of Engineering, University of Texas, Austin, Texas.
3	Serials Division, University of Minnesota Library, Minneapolis, Minnesota.
1	Oceanics, Incorporated, Technical Industrial Park, Plainview, Long Island, New York, Attn: Dr. Paul Kaplan, President.
1	Vidya, 1450 Page Mill Road, Palo Alto, California, Attn: Dr. J. Nielsen.
1	Illinois Institute of Technology, Technology Center, Aeronautical Laboratory, Chicago 16, Illinois, Attn: Prof. I. Michelson.

Technical Paper No. 48, Series B
St. Anthony Falls Hydraulic Laboratory

STUDIES OF THE FLOW CHARACTERISTICS OF A COMPRESSIBLE, BUBBLY MIXTURE ABOUT SUPERCAVITATING BODIES AND IN A CONVERGING-DIVERGING NOZZLE, by F. R. Schiebe, J. M. Wetzel, and K. E. Foerster. April 1964. 53 pages incl. 21 illus. Contract Nonr 710(52).

Experimental studies were conducted to determine the effect of a compressible, air-water mixture on the drag of a cavitating body. Data are reported for conical bodies of various slenderness ratios for Mach numbers up to 0.7. The drag coefficient increased with number, although not as rapidly as for a non-cavitating body. A study of an air-water mixture in a de Laval nozzle was also conducted. Supersonic flow was obtained and shocks were observed downstream of the throat. Measured throat pressures for choked flow were slightly higher than calculated values from homogeneous mixture theory.

Available from St. Anthony Falls Hydraulic Laboratory, University of Minnesota, at $1.50 per copy.

1. Supercavitating Flow
2. Two Phase Flow
3. Subsonic Flow
4. Supersonic Flow
5. Converging-Diverging Nozzle

I. Title
II. Schiebe, F. R.
III. Wetzel, J. M.
IV. Foerster, K. E.
V. St. Anthony Falls Hydraulic Laboratory
VI. Contract No. 710(52)

Unclassified

Technical Paper No. 48, Series B
St. Anthony Falls Hydraulic Laboratory

STUDIES OF THE FLOW CHARACTERISTICS OF A COMPRESSIBLE, BUBBLY MIXTURE ABOUT SUPERCAVITATING BODIES AND IN A CONVERGING-DIVERGING NOZZLE, by F. R. Schiebe, J. M. Wetzel, and K. E. Foerster. April 1964. 53 pages incl. 21 illus. Contract Nonr 710(52).

Experimental studies were conducted to determine the effect of a compressible, air-water mixture on the drag of a cavitating body. Data are reported for conical bodies of various slenderness ratios for Mach numbers up to 0.7. The drag coefficient increased with number, although not as rapidly as for a non-cavitating body. A study of an air-water mixture in a de Laval nozzle was also conducted. Supersonic flow was obtained and shocks were observed downstream of the throat. Measured throat pressures for choked flow were slightly higher than calculated values from homogeneous mixture theory.

Available from St. Anthony Falls Hydraulic Laboratory, University of Minnesota, at $1.50 per copy.

1. Supercavitating Flow
2. Two Phase Flow
3. Subsonic Flow
4. Supersonic Flow
5. Converging-Diverging Nozzle

I. Title
II. Schiebe, F. R.
III. Wetzel, J. M.
IV. Foerster, K. E.
V. St. Anthony Falls Hydraulic Laboratory
VI. Contract No. 710(52)

Unclassified

Technical Paper No. 48, Series B
St. Anthony Falls Hydraulic Laboratory

STUDIES OF THE FLOW CHARACTERISTICS OF A COMPRESSIBLE, BUBBLY MIXTURE ABOUT SUPERCAVITATING BODIES AND IN A CONVERGING-DIVERGING NOZZLE, by F. R. Schiebe, J. M. Wetzel, and K. E. Foerster. April 1964. 53 pages incl. 21 illus. Contract Nonr 710(52).

Experimental studies were conducted to determine the effect of a compressible, air-water mixture on the drag of a cavitating body. Data are reported for conical bodies of various slenderness ratios for Mach numbers up to 0.7. The drag coefficient increased with number, although not as rapidly as for a non-cavitating body. A study of an air-water mixture in a de Laval nozzle was also conducted. Supersonic flow was obtained and shocks were observed downstream of the throat. Measured throat pressures for choked flow were slightly higher than calculated values from homogeneous mixture theory.

Available from St. Anthony Falls Hydraulic Laboratory, University of Minnesota, at $1.50 per copy.

1. Supercavitating Flow
2. Two Phase Flow
3. Subsonic Flow
4. Supersonic Flow
5. Converging-Diverging Nozzle

I. Title
II. Schiebe, F. R.
III. Wetzel, J. M.
IV. Foerster, K. E.
V. St. Anthony Falls Hydraulic Laboratory
VI. Contract No. 710(52)

Unclassified

Technical Paper No. 48, Series B
St. Anthony Falls Hydraulic Laboratory

STUDIES OF THE FLOW CHARACTERISTICS OF A COMPRESSIBLE, BUBBLY MIXTURE ABOUT SUPERCAVITATING BODIES AND IN A CONVERGING-DIVERGING NOZZLE, by F. R. Schiebe, J. M. Wetzel, and K. E. Foerster. April 1964. 53 pages incl. 21 illus. Contract Nonr 710(52).

Experimental studies were conducted to determine the effect of a compressible, air-water mixture on the drag of a cavitating body. Data are reported for conical bodies of various slenderness ratios for Mach numbers up to 0.7. The drag coefficient increased with number, although not as rapidly as for a non-cavitating body. A study of an air-water mixture in a de Laval nozzle was also conducted. Supersonic flow was obtained and shocks were observed downstream of the throat. Measured throat pressures for choked flow were slightly higher than calculated values from homogeneous mixture theory.

Available from St. Anthony Falls Hydraulic Laboratory, University of Minnesota, at $1.50 per copy.

1. Supercavitating Flow
2. Two Phase Flow
3. Subsonic Flow
4. Supersonic Flow
5. Converging-Diverging Nozzle

I. Title
II. Schiebe, F. R.
III. Wetzel, J. M.
IV. Foerster, K. E.
V. St. Anthony Falls Hydraulic Laboratory
VI. Contract No. 710(52)

Unclassified

Technical Paper No. 48, Series B
St. Anthony Falls Hydraulic Laboratory

STUDIES OF THE FLOW CHARACTERISTICS OF A COMPRESSIBLE, BUBBLY MIXTURE ABOUT SUPERCAVITATING BODIES AND IN A CONVERGING-DIVERGING NOZZLE, by F. R. Schiebe, J. M. Wetzel, and K. E. Foerster. April 1964. 53 pages incl. 21 illus. Contract Nonr 710(52).

Experimental studies were conducted to determine the effect of a compressible, air-water mixture on the drag of a cavitating body. Data are reported for conical bodies of various slenderness ratios for Mach numbers up to 0.7. The drag coefficient increased with number, although not as rapidly as for a non-cavitating body. A study of an air-water mixture in a de Laval nozzle was also conducted. Supersonic flow was obtained and shocks were observed downstream of the throat. Measured throat pressures for choked flow were slightly higher than calculated values from homogeneous mixture theory.

Available from St. Anthony Falls Hydraulic Laboratory, University of Minnesota, at $1.50 per copy.

1. Supercavitating Flow
2. Two Phase Flow
3. Subsonic Flow
4. Supersonic Flow
5. Converging-Diverging Nozzle

I. Title
II. Schiebe, F. R.
III. Wetzel, J. M.
IV. Foerster, K. E.
V. St. Anthony Falls Hydraulic Laboratory
VI. Contract No. 710(52)

Unclassified

Technical Paper No. 48, Series B
St. Anthony Falls Hydraulic Laboratory

STUDIES OF THE FLOW CHARACTERISTICS OF A COMPRESSIBLE, BUBBLY MIXTURE ABOUT SUPERCAVITATING BODIES AND IN A CONVERGING-DIVERGING NOZZLE, by F. R. Schiebe, J. M. Wetzel, and K. E. Foerster. April 1964. 53 pages incl. 21 illus. Contract Nonr 710(52).

Experimental studies were conducted to determine the effect of a compressible, air-water mixture on the drag of a cavitating body. Data are reported for conical bodies of various slenderness ratios for Mach numbers up to 0.7. The drag coefficient increased with number, although not as rapidly as for a non-cavitating body. A study of an air-water mixture in a de Laval nozzle was also conducted. Supersonic flow was obtained and shocks were observed downstream of the throat. Measured throat pressures for choked flow were slightly higher than calculated values from homogeneous mixture theory.

Available from St. Anthony Falls Hydraulic Laboratory, University of Minnesota, at $1.50 per copy.

1. Supercavitating Flow
2. Two Phase Flow
3. Subsonic Flow
4. Supersonic Flow
5. Converging-Diverging Nozzle

I. Title
II. Schiebe, F. R.
III. Wetzel, J. M.
IV. Foerster, K. E.
V. St. Anthony Falls Hydraulic Laboratory
VI. Contract No. 710(52)

Unclassified

Technical Paper No. 48, Series B
St. Anthony Falls Hydraulic Laboratory

STUDIES OF THE FLOW CHARACTERISTICS OF A COMPRESSIBLE, BUBBLY MIXTURE ABOUT SUPERCAVITATING BODIES AND IN A CONVERGING-DIVERGING NOZZLE, by F. R. Schiebe, J. M. Wetzel, and K. E. Foerster. April 1964. 53 pages incl. 21 illus. Contract Nonr 710(52).

Experimental studies were conducted to determine the effect of a compressible, air-water mixture on the drag of a cavitating body. Data are reported for conical bodies of various slenderness ratios for Mach numbers up to 0.7. The drag coefficient increased with number, although not as rapidly as for a non-cavitating body. A study of an air-water mixture in a de Laval nozzle was also conducted. Supersonic flow was obtained and shocks were observed downstream of the throat. Measured throat pressures for choked flow were slightly higher than calculated values from homogeneous mixture theory.

Available from St. Anthony Falls Hydraulic Laboratory, University of Minnesota, at $1.50 per copy.

1. Supercavitating Flow
2. Two Phase Flow
3. Subsonic Flow
4. Supersonic Flow
5. Converging-Diverging Nozzle

I. Title
II. Schiebe, F. R.
III. Wetzel, J. M.
IV. Foerster, K. E.
V. St. Anthony Falls Hydraulic Laboratory
VI. Contract No. 710(52)

Unclassified

Technical Paper No. 48, Series B
St. Anthony Falls Hydraulic Laboratory

STUDIES OF THE FLOW CHARACTERISTICS OF A COMPRESSIBLE, BUBBLY MIXTURE ABOUT SUPERCAVITATING BODIES AND IN A CONVERGING-DIVERGING NOZZLE, by F. R. Schiebe, J. M. Wetzel, and K. E. Foerster. April 1964. 53 pages incl. 21 illus. Contract Nonr 710(52).

Experimental studies were conducted to determine the effect of a compressible, air-water mixture on the drag of a cavitating body. Data are reported for conical bodies of various slenderness ratios for Mach numbers up to 0.7. The drag coefficient increased with number, although not as rapidly as for a non-cavitating body. A study of an air-water mixture in a de Laval nozzle was also conducted. Supersonic flow was obtained and shocks were observed downstream of the throat. Measured throat pressures for choked flow were slightly higher than calculated values from homogeneous mixture theory.

Available from St. Anthony Falls Hydraulic Laboratory, University of Minnesota, at $1.50 per copy.

1. Supercavitating Flow
2. Two Phase Flow
3. Subsonic Flow
4. Supersonic Flow
5. Converging-Diverging Nozzle

I. Title
II. Schiebe, F. R.
III. Wetzel, J. M.
IV. Foerster, K. E.
V. St. Anthony Falls Hydraulic Laboratory
VI. Contract No. 710(52)

Unclassified

Technical Paper No. 48, Series B
St. Anthony Falls Hydraulic Laboratory

STUDIES OF THE FLOW CHARACTERISTICS OF A COMPRESSIBLE, BUBBLY MIXTURE ABOUT SUPERCAVITATING BODIES AND IN A CONVERGING-DIVERGING NOZZLE, by F. R. Schiebe, J. M. Wetzel, and K. E. Foerster. April 1964. 53 pages incl. 21 illus. Contract Nonr 710(52).

Experimental studies were conducted to determine the effect of a compressible, air-water mixture on the drag of a cavitating body. Data are reported for conical bodies of various slenderness ratios for Mach numbers up to 0.7. The drag coefficient increased with number, although not as rapidly as for a non-cavitating body. A study of an air-water mixture in a de Laval nozzle was also conducted. Supersonic flow was obtained and shocks were observed downstream of the throat. Measured throat pressures for choked flow were slightly higher than calculated values from homogeneous mixture theory.

Available from St. Anthony Falls Hydraulic Laboratory, University of Minnesota, at $1.50 per copy.

1. Supercavitating Flow
2. Two Phase Flow
3. Subsonic Flow
4. Supersonic Flow
5. Converging-Diverging Nozzle

I. Title
II. Schiebe, F. R.
III. Wetzel, J. M.
IV. Foerster, K. E.
V. St. Anthony Falls Hydraulic Laboratory
VI. Contract No. 710(52)

Unclassified

Technical Paper No. 48, Series B
St. Anthony Falls Hydraulic Laboratory

STUDIES OF THE FLOW CHARACTERISTICS OF A COMPRESSIBLE, BUBBLY MIXTURE ABOUT SUPERCAVITATING BODIES AND IN A CONVERGING-DIVERGING NOZZLE, by F. R. Schiebe, J. M. Wetzel, and K. E. Foerster. April 1964. 53 pages incl. 21 illus. Contract Nonr 710(52).

Experimental studies were conducted to determine the effect of a compressible, air-water mixture on the drag of a cavitating body. Data are reported for conical bodies of various slenderness ratios for Mach numbers up to 0.7. The drag coefficient increased with number, although not as rapidly as for a non-cavitating body. A study of an air-water mixture in a de Laval nozzle was also conducted. Supersonic flow was obtained and shocks were observed downstream of the throat. Measured throat pressures for choked flow were slightly higher than calculated values from homogeneous mixture theory.

Available from St. Anthony Falls Hydraulic Laboratory, University of Minnesota, at $1.50 per copy.

1. Supercavitating Flow
2. Two Phase Flow
3. Subsonic Flow
4. Supersonic Flow
5. Converging-Diverging Nozzle

I. Title
II. Schiebe, F. R.
III. Wetzel, J. M.
IV. Foerster, K. E.
V. St. Anthony Falls Hydraulic Laboratory
VI. Contract No. 710(52)

Unclassified

Technical Paper No. 48, Series B
St. Anthony Falls Hydraulic Laboratory

STUDIES OF THE FLOW CHARACTERISTICS OF A COMPRESSIBLE, BUBBLY MIXTURE ABOUT SUPERCAVITATING BODIES AND IN A CONVERGING-DIVERGING NOZZLE, by F. R. Schiebe, J. M. Wetzel, and K. E. Foerster. April 1964. 53 pages incl. 21 illus. Contract Nonr 710(52).

Experimental studies were conducted to determine the effect of a compressible, air-water mixture on the drag of a cavitating body. Data are reported for conical bodies of various slenderness ratios for Mach numbers up to 0.7. The drag coefficient increased with number, although not as rapidly as for a non-cavitating body. A study of an air-water mixture in a de Laval nozzle was also conducted. Supersonic flow was obtained and shocks were observed downstream of the throat. Measured throat pressures for choked flow were slightly higher than calculated values from homogeneous mixture theory.

Available from St. Anthony Falls Hydraulic Laboratory, University of Minnesota, at $1.50 per copy.

1. Supercavitating Flow
2. Two Phase Flow
3. Subsonic Flow
4. Supersonic Flow
5. Converging-Diverging Nozzle

I. Title
II. Schiebe, F. R.
III. Wetzel, J. M.
IV. Foerster, K. E.
V. St. Anthony Falls Hydraulic Laboratory
VI. Contract No. 710(52)

Unclassified

Technical Paper No. 48, Series B
St. Anthony Falls Hydraulic Laboratory

STUDIES OF THE FLOW CHARACTERISTICS OF A COMPRESSIBLE, BUBBLY MIXTURE ABOUT SUPERCAVITATING BODIES AND IN A CONVERGING-DIVERGING NOZZLE, by F. R. Schiebe, J. M. Wetzel, and K. E. Foerster. April 1964. 53 pages incl. 21 illus. Contract Nonr 710(52).

Experimental studies were conducted to determine the effect of a compressible, air-water mixture on the drag of a cavitating body. Data are reported for conical bodies of various slenderness ratios for Mach numbers up to 0.7. The drag coefficient increased with number, although not as rapidly as for a non-cavitating body. A study of an air-water mixture in a de Laval nozzle was also conducted. Supersonic flow was obtained and shocks were observed downstream of the throat. Measured throat pressures for choked flow were slightly higher than calculated values from homogeneous mixture theory.

Available from St. Anthony Falls Hydraulic Laboratory, University of Minnesota, at $1.50 per copy.

1. Supercavitating Flow
2. Two Phase Flow
3. Subsonic Flow
4. Supersonic Flow
5. Converging-Diverging Nozzle

I. Title
II. Schiebe, F. R.
III. Wetzel, J. M.
IV. Foerster, K. E.
V. St. Anthony Falls Hydraulic Laboratory
VI. Contract No. 710(52)

Unclassified