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Technical Paper No. 1, Series B

Air Bubble Resorption

by

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August, 1949

Minneapolis, Minnesota

See Later Edition

A I R B U B B L E R E S O R P T I O N

SYNOPSIS

This paper describes an analysis and experiment directed at determining the laws governing the rate of solution of a gas bubble in turbulent liquid. The object of the research was to determine methods for resorbing air bubbles which have been freed from the water in a water tunnel. A basic equation governing the resorption process which has been developed and partially verified in the work is presented as Eq. (13) in the text. Useful approximate forms of this equation are given as Eqs. (14b) and (14d) in the text.

The basic equation has led to several suggested methods for accomplishing resorption in a water tunnel. These include: (1) a resorber method already developed at the California Institute of Technology [1]*; (2) a method in which air in solution in water would be completely replaced in the closed water tunnel circuit by another gas such as carbon dioxide; and (3) a method in which a lengthened return circuit would be combined with a fine-scale turbulence, introduced in the return circuit to hasten air bubble resorption while keeping the bubbles from rising. The time required for resorption by any of these methods may be estimated from the basic equation.

INTRODUCTION

When a closed or open jet water tunnel is used to study cavitation on a body, dissolved air is freed in the cavitation bubbles and continues to recirculate as small bubbles until the test section may be so clouded as to preclude further observation. In a free jet water tunnel, air or gas may be entrained at the free surface and recirculated as small bubbles through the test section without cavitation. Furthermore, in cavitation tests of propellers and other airfoil shapes, the dissolved air content of the water is important in that it determines the pressure or speed at which cavitation will begin. If the dissolved air content changes during tests, results will not be consistent. These difficulties pose the problem of finding a method to free

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*Numbers in brackets refer to the bibliography on page 30.

the water tunnel test section of small bubbles being recirculated and at the same time to maintain a constant dissolved gas content.

The problem for a closed jet water tunnel has been solved at the California Institute of Technology by the design of a resorber for putting air bubbles back into solution [1]. This resorber is not adequate for a free jet tunnel, of course, since the water will approach saturation rapidly. It is such a large structure that even for a closed or open jet structure it was thought that some simpler method might be devised if a more thorough study of the matter were made. The research described in this paper is directed at such a study.

Not until the present research was finished, was it known that the California Institute of Technology had made a much more complete investigation of the resorption problem than was indicated in reference 1. The present paper, therefore, covers much of the same ground as a second California Institute of Technology paper, "Air Resorption in Water Tunnels" [2]. The present research gives a somewhat different viewpoint from the analysis in reference 2, adds confirming experimental data, and discusses other factors in resorption besides those mentioned in reference 2. It is interesting to note that on the basis of the present research alone, it was concluded that the existing California Institute of Technology resorber was of adequate design.

Two basic methods of solving the problem posed in the first paragraph have been considered:

(1) A method in which air bubbles would be physically removed from the water. In one particular method that was investigated experimentally, bubbles would be bled off by external suction applied at slots on the convex surfaces of the turning vanes downstream from the diffuser. For a closed or open jet tunnel, the water drawn off would be replaced by water previously saturated with air at high pressure, to maintain uniform air content in the test section. Replacement would be made through slots on the concave surfaces of the turning vanes at the bottom of the tunnel where the pressure is high. For a free jet tunnel, replacement would be made in a similar manner except that the air content of the new water might be slightly less than that in the test section. Preliminary tests indicated that about 5 per cent of the flow would have to be sucked off to withdraw

a very large part (but not all) of the bubbles. Details of this method have not been worked out, and the idea was dormant pending the completion of the present studies.

(2) A resorption method in which gas bubbles would be dissolved in the tunnel fluid before appearing at the test section. This method cannot be applied very well to a free jet tunnel. The California Institute of Technology resorber is one adaptation of this method. Another possible adaptation, mentioned later in this paper, is the substitution for air of another gas whose bubbles will dissolve more rapidly than air. A preliminary test in a model water tunnel using carbon dioxide failed, however, because of the difficulty of removing enough of the air from the water.

The present paper presents an analysis of the resorption problem in turbulent fluid and some experimental results obtained for air bubbles in a special apparatus designed for the purpose. The results are put in a form useful for tunnel design. The experimental studies were considered necessary since the available literature [3, 4, 5] did not give information on the rate of solution of gas bubbles in fluid as turbulent as that in a normal water tunnel circuit.

TABLE OF SYMBOLS

A - Surface area of gas-liquid interface (ft^2)

a - Characteristic linear dimension of water tunnel or other apparatus used in forming Reynolds number (ft)

C - Concentration of gas dissolved in liquid (lbs/ft^3)

C_G - Value of C if the liquid were saturated at the pressure p and temperature θ (lbs/ft^3)

C_L - Actual value of C in the interior of a liquid at the pressure p and temperature $\bar{\theta}$ (lbs/ft^3)

C_q - Difference between the value of C at the edge of the molecular sublayer and C_L (lbs/ft^3)

D - Diffusivity or specific coefficient of diffusion (ft^2/sec)

K_1, K_2, K_3, K_4 - Coefficients

K_L - Liquid film coefficient (ft/sec)

L - Linear dimension representing scale of turbulence (ft)

n - Dimensionless exponent

p - Absolute pressure at which resorption is taking place (lbs/ft^2)

p_o - Absolute pressure at which the concentration $\underline{C_L}$ would saturate the liquid at the temperature $\underline{\theta}$ (lbs/ft^2)

R - Radius of gas bubble at any time (ft)

R_1 - Initial radius of gas bubble at beginning of resorption (ft)

r - Radial distance from origin of coordinates (ft)

$Re = \frac{Ua}{\nu}$ - Reynolds number

T - Time required for complete resorption of a gas bubble of initial radius $\underline{R_1}$ (sec)

t - Time (sec)

U - Relative velocity of liquid and fixed boundary (ft/sec)

W - Weight of gas in a bubble (lbs)

$\alpha = \frac{C_L}{C_G}$ - Relative saturation of the liquid

$\beta = \frac{C_G}{\gamma}$ - Solubility in gas volumes per unit volume of liquid at the pressure $\underline{p}$ and the temperature θ.

γ - Unit weight of gas at the pressure $\underline{p}$ and the temperature $\underline{\theta}$ (lbs/ft^3)

δ - Thickness of diffusing boundary layer (ft)

δ' - Thickness of molecular diffusing sublayer (ft)

θ - Absolute temperature at which resorption takes place (units are not specified since $\underline{\theta}$ is taken as constant in this paper)

κ - Thermometric conductivity (ft^2/sec)

ω - Angular velocity of rotating disk ($\frac{1}{sec}$)

ν - Kinematic viscosity (ft^2/sec)

ANALYSIS

In order to make the analysis rigorous and to be able to remove the proposed experiments from the water tunnel and place them in an apparatus where conditions can be more carefully controlled and observed, it is necessary to place certain conditions on the work. The first of these is that the relative saturation of the liquid with gas is unaffected by the formation and resorption of bubbles, even for heavy cavitation, as long as resorption is completed in one tunnel cycle. This appears to describe the actual conditions in a water tunnel since only a very small fraction of the total water volume enters a cavitating region in one cycle. (The same conclusion was reached in reference 2 following some preliminary equations.) The second condition is that resorption take place in a constant pressure--constant temperature region. This requirement is also satisfied in a water tunnel circuit if the circuit is divided into reaches of approximately constant average pressure. On this basis, it is permissable to study the resorption of a single gas bubble in a liquid of constant pressure and of such volume that its saturation is relatively unaffected by solution of the bubble.

The study of solution of a gas bubble in liquid may be attacked from two somewhat different viewpoints. One is the Lewis-Whitman concept of gas and liquid films at the bubble interface with the liquid [6]. The gas film is supposed to be composed of gasses that are less readily dissolved than those forming the bulk of the bubble. In the cavitating process, however, the only gasses to be dissolved are those that came out of the liquid in the first place, oxygen and nitrogen (or possibly some other pure gas if it is substituted for these); and, since these have approximately equal diffusivity, there can be no gas film in the problem being considered here.

When the gas film is ignored, the Lewis-Whitman equations for rate of gas transfer per unit area at the interface become [6]:

$$\frac{1}{A}\frac{dW}{dt} = -K_L\,(C_G - C_L) \tag{1}$$

where

W is the weight of gas in the bubble (lbs)

A is the interface area (ft^2)

t is the time (sec)

C_G is the concentration of the gas in the liquid if the liquid were saturated at the pressure and temperature of resorption and is the concentration at the gas-liquid interface (lbs/ft^3)

C_L is the actual concentration of the gas in the liquid at a distance from the bubble (lbs/ft^3)

(C_G-C_L) is then the difference in concentration between the liquid touching the bubble and the uniform liquid some distance away

K_L is an empirical liquid film coefficient (ft/sec).

The negative sign has been added so that $\frac{dW}{dt}$ will be negative when $C_G > C_L$, all other quantities being positive. The experiments previously referred to [3, 4, 5], as well as many others, have been directed toward evaluating $\underline{K_L}$ for different conditions. In spite of this, its value is not known for all conditions, and the factors which influence its value are poorly understood.

Equation (1) does not contain explicitly all of the factors which must be measured in an experimental study of solution of gas bubbles. The rate of gas transfer per unit area at the bubble surface may also be written:

$$\frac{1}{A}\frac{dW}{dt} = D\left.\frac{dC}{dr}\right|_{r=R} \tag{2}$$

where

$\left.\frac{dC}{dr}\right|_{r=R}$ is the rate of change of concentration at the surface (lbs/ft^4)

D is the diffusivity (ft^2/sec)

and the other terms are as previously defined.

For comparative purposes, Eqs. (1) and (2) may be combined to give:

$$K_L = -\frac{D\left.\frac{dC}{dr}\right|_{r=R}}{C_G - C_L} \tag{3}$$

D is a constant fluid property for a given gas and liquid, and it corresponds to the kinematic viscosity (ν) in surface shear computations in fluid flow and to the thermometric conductivity (κ) in heat transfer computations. At room temperatures, D is of the order of 2×10^{-8} ft^2/sec for air in water (ν is of the order 1×10^{-5} and κ of the order 1.5×10^{-6} for water, both in units of ft^2/sec). Values of D are not too well fixed as yet, but, experimental values for several gasses have been collected by Arnold [7] who also gives an empirical formula for D for other gasses.

The rate of change of concentration at the surface, $\left.\frac{dC}{dr}\right|_{r=R}$, must be evaluated in Eq. (2). Consider that the diffusion takes place in a layer of thickness δ surrounding the bubble (see Fig. 1). Several cases can be recognized: (1) there is little or no relative motion between the gas and liquid, and the liquid is not turbulent; (2) there is considerable relative motion, and the liquid is not turbulent; and (3) the liquid is turbulent. It is believed that all recorded experiments to date have been conducted as one or the other of the first two cases. This paper is concerned with the third case.

In the first case, δ may be considered uninfluenced by the relative motion so that $\left.\frac{dC}{dr}\right|_{r=R}$ is some constant function of ($C_G - C_L$). From Eq. (3), it is seen that this implies a constant K_L over a range of conditions from no motion to small relative motion of gas and liquid. That K_L is constant in this range is borne out by experiment, numerous examples being cited in references 3, 4, and 5. In the second case, δ comes under the influence of the fluid motion boundary layer between the gas and liquid. Since the boundary

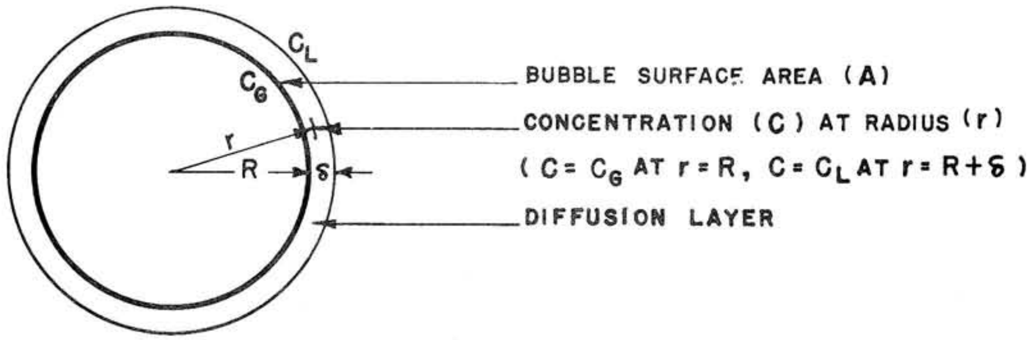

Fig. 1 Gas Bubble in Turbulent Flow

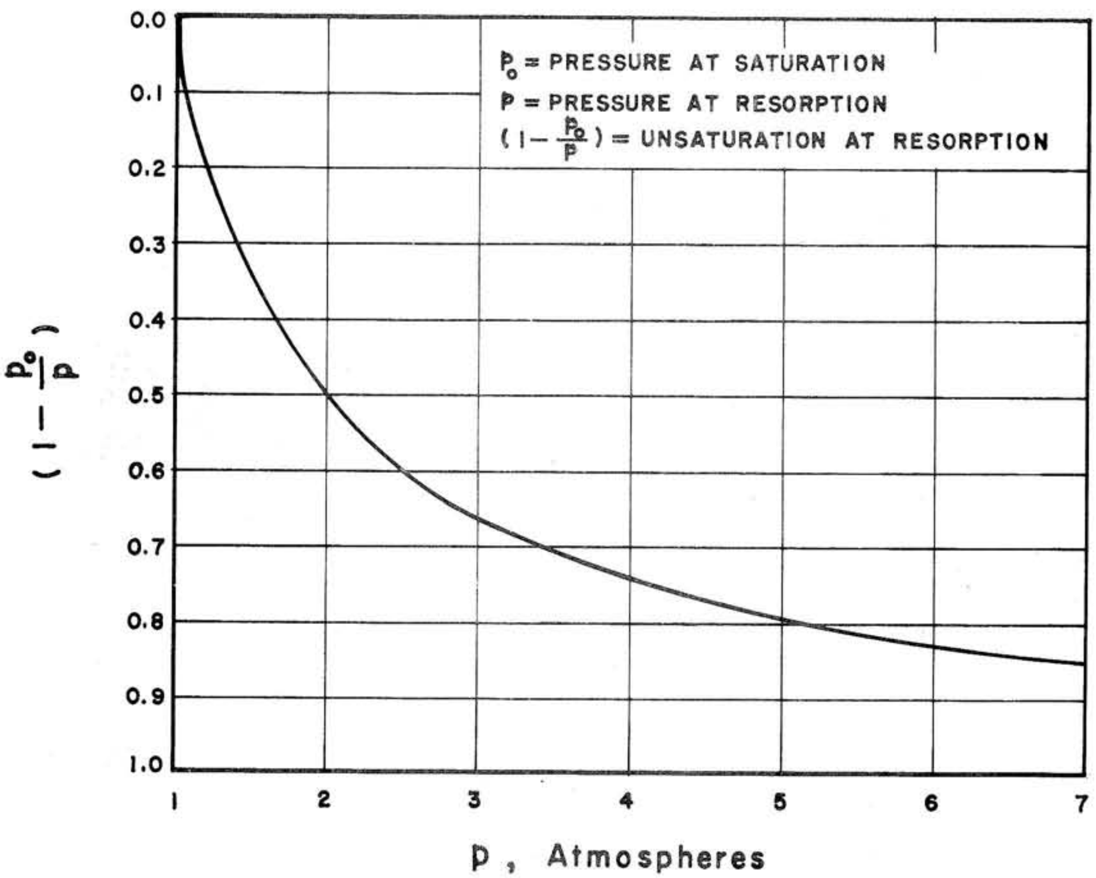

Fig. 2 Variation of Unsaturation with Superimposed Pressure (p_o = 1 Atmos.)

layer decreases in thickness with increasing Reynolds numbers, δ may also be expected to decrease with increasing Reynolds numbers (increasing relative motion, increasing linear dimensions, or decreasing kinematic viscosity). Thus $\left.\frac{dC}{dr}\right|_{r = R}$ must increase (K_L must increase) with increases in Reynolds number in the second case. Again there is much experimental evidence to corroborate this conclusion. Hutchinson and Sherwood [4], for instance, found that K_L increases with about the 0.6 power of the speed (or Reynolds number) in this region. (Hutchinson and Sherwood used a free surface rather than a bubble, but the reasoning follows the same lines.) Becker, quoted by Lewis and Whitman [6], found K_L to vary with the 0.8 power of the speed.

In the third case, δ is also influenced by the liquid motion. By mixture length theories of turbulence, an equation of the form of Eq. (2) still applies to turbulent diffusion, with a turbulent diffusivity substituted for D. The turbulent diffusivity for fully developed turbulence is of a so much higher order of magnitude than D that the diffusing layer may be considered limited to a very small region around the bubble where the turbulence is not fully developed. This small region may be subdivided into two parts: an inner sublayer where all relative motion has been almost completely damped by viscosity, in which diffusion is wholly molecular and governed by the coefficient D; and, a surrounding layer where diffusion becomes more and more a turbulent phenomenon and less and less a molecular phenomenon until full turbulence exists at the edge of the layer. (There is no sharp line, only a nominal line marking the limits of the diffusing layer and sublayer.) These subdivisions correspond to Kármán's laminar and transitional boundary layers in fluid and heat flow analysis. This method of subdividing the diffusing layer should be valid whether the bubble is large and moving mainly with the mean velocity of the liquid, or small and moving with the turbulent fluctuations as well as with the mean velocity. However, the thickness of the layers would be materially different in the two cases. (The bubbles move with the turbulent fluctuations in the apparatus used in these experiments.)

The molecular sublayer must be very thin so that the variations of concentration within it may be taken as approximately linear. That is:

$$\frac{dC}{dr} = -\frac{(C_G - C_L) - C_q}{\delta'} = -\frac{C_G}{\delta'}\left(1 - \alpha - \frac{C_q}{C_G}\right) \tag{4}$$

throughout the sublayer,

where

δ' is the thickness of the sublayer (ft)

C_q is the small difference in concentration between the uniform liquid at the edge of the diffusing layer and the liquid at the edge of the sublayer (lbs/ft^3)

$\alpha = \frac{C_L}{C_G}$ is the relative saturation of the liquid, and $(1-\alpha)$ may be called the unsaturation.

In Eq. (4), C_G and α are determined by the nature of the gas and liquid, the pressure, and the temperature. The quantities δ' and C_q/C_G, however, are dependent on a number of less readily defined variables. Important among these are probably the turbulence and viscosity of the liquid. It is reasonable to assume that the influence of these variables is measured by the Reynolds number, Re, of the main flow, defined as:

$$Re = \frac{Ua}{\nu} \tag{5}$$

where

U is the mean velocity of the liquid (ft/sec)

a is a linear dimension of the fixed boundaries (ft)

ν is the kinematic viscosity (ft^2/sec).

If δ' is taken as a function of Re, it is necessary to introduce an arbitrary length, L, to maintain dimensional homogeneity. That is, the substitution,

$$\frac{\delta'}{L} = f_1 \text{ (Re)} \tag{6}$$

will make Eq. (4) dimensionally correct. Since δ' probably decreases with an increase in Reynolds number, it is convenient to represent Eq. (6) by the approximation:

$$\frac{\delta'}{L} = \frac{1}{K_1 \text{ Re}^n} \tag{6a}$$

where K_1 and n are numerical constants. The length, L, must be related to some physical dimension of either the bubble or the external system boundaries, but its significance will not be investigated further at this point.

In addition to the variables already mentioned, C_q/C_G is probably dependent to an important degree on the diffusivity. The diffusivity may be introduced in the ratio $\frac{\nu}{D}$, corresponding to the Prandtl number ($\frac{\nu}{\kappa}$) in heat flow. Then:

$$\frac{C_q}{C_G} = f_2 \ (Re, \frac{\nu}{D}). \tag{7}$$

The quantity C_q/C_G will probably decrease with increasing Re and with decreasing $\frac{\nu}{D}$.

It is desirable to assign a shape to the gas surface. Observations in the water tunnel and in the experimental apparatus to be described later, show that the bubbles are essentially spherical most of the time. Then:

$$W = \frac{4}{3}\pi R^3 \gamma \tag{8a}$$

and

$$A = 4\pi R^2 \tag{8b}$$

where R is the bubble radius at any time in feet, and γ is the unit weight of the gas in the bubble at the pressure and temperature of resorption in pounds per cubic foot.

Combining Eqs. (2), (4), (6a), and (8) yields the equation:

$$\frac{L}{\beta D} \frac{dr}{dt} = -K_1 \ (1 - \alpha - \frac{C_q}{C_G}) \ Re^n \tag{9}$$

where $\beta = \frac{C_G}{\gamma}$ is the solubility in gas volumes per unit volume of liquid at the pressure and temperature of resorption. Both sides of Eq. (9) are dimensionless.

Equation (9) is the basic equation containing all of the important variables influencing the rate of gas bubble solution in turbulent liquid. K_1 and n must be determined by experiment. β and D are fluid properties determined by the type of gas and liquid, the pressure, and the temperature. Actually, β may be taken as independent of the pressure, for, by Henry's law and the perfect gas law which apply almost exactly to the gasses likely to be used:

$$\beta = \frac{C_G}{\gamma} = \frac{HpR\theta}{p} = HR\theta \tag{10}$$

where

p is the pressure at which resorption is taking place

H is Henry's constant which is independent of p but varies with θ

R is the gas constant which is a universal constant

θ is the absolute temperature.

Values of β at atmospheric pressure for various temperatures may be found in standard handbooks, and Arnold[7] gives values for D. Re and α are determined by operating conditions in the tunnel. The former is given by Eq. (5) and α may be determined by direct sampling or indirectly by making use of Henry's law:

$$\alpha = \frac{C_L}{C_G} = \frac{Hp_o}{Hp} = \frac{p_o}{p} \tag{11}$$

where

p is the pressure at which resorption is being accomplished

p_o is the pressure at which the liquid would be saturated at the concentration C_L. p_o must be measured in the same units as p, and the liquid must be at the same temperature for both measurements.

The vapor pressure is omitted from Eq. (11) but may be included by subtracting it from both p_o and p.

Only L in Eq. (9) remains to be defined physically and C_q/C_G must be evaluated. The quantity C_q/C_G cannot be evaluated theoretically at present but an experimental value will be obtained in another part of the paper. For sufficiently effective turbulence, or as a first approximation in any event, $C_q/C_G \rightarrow 0$ ($\delta \approx \delta'$) while dR/dt remains finite, and Eq. (9) may be written in the limiting form:

$$\frac{L}{\beta D}\frac{dR}{dt} = -K_1\ (1-\alpha)\ Re^n. \tag{9a}$$

Data obtained in these experiments and referred to on page 20, following, and in Fig. 6 show conclusively that for a given bubble in a turbulent flow with constant Reynolds number, pressure and temperature, dR/dt is independent of R (the gas bubble radius) and t. Assuming that this independence will extend to the limiting case expressed by Eq. (9a), L must also be independent of R. On the basis of these assumptions, L must then be determined by the system boundaries; it is probably a measure of the scale of the turbulence. L may be considered proportional to a, a characteristic linear dimension of the fixed boundaries, unless artificial turbulence devices such as screens are installed in the water tunnel circuit. The inability to define L exactly is the greatest drawback toward studying the resorption of air bubbles outside the water tunnel under consideration.

It might be noted that following the above development, K_L in Eq. (3) becomes:

$$K_L = \frac{DK_1 Re^n}{L}\left(1 - \frac{C_q/C_G}{1-\alpha}\right) = \frac{D}{\delta'}\left(1 - \frac{C_q/C_G}{1-\alpha}\right) \tag{12}$$

and in the limiting case:

$$K_L = \frac{DK_1 Re^n}{L} = \frac{D}{\delta'} \tag{12a}$$

Hutchinson and Sherwood [4] showed in their experiments that, although there was some slight tendency for K_L to increase with D, there was no direct proportionality between the two. It should be noted, however, that their measurements were made in an entirely laminar regime, or at best in only very slight turbulence, and consequently are not applicable to Eqs. (12) and (12a).

Equation (9) may be integrated quite simply since dR/dt is known to be independent of both R and t. Letting $t = 0$ when $R = R_1$, ~~some initial~~ value, and $t = T$ when $R = 0$, Eq. (9) integrates to:

$$T = \frac{L}{K_1 \beta D \left(1 - \alpha - \frac{C_q}{C_G}\right) Re^n} R_1 \tag{13}$$

For application to tunnel design, the limiting case obtained from Eq. (9a) is probably sufficiently accurate:

$$T = \frac{L}{K_1 \beta D (1-\alpha) Re^n} R_1. \tag{14}$$

It must be emphasized that Eq. (14) applies only for intense turbulence and is otherwise only a convenient approximation. Equation (14) may be written in the following alternative forms:

$$T = \frac{K_2 a}{D\beta (1-\alpha) Re^n} \tag{14a}$$

where $K_2 = \frac{L}{K_1 a}$ includes the proportionality between L and a and may vary from one installation to another,

$$T = \frac{\delta'}{D\beta\ (1-\alpha)} R_1 \tag{14b}$$

$$T = \frac{1}{K_L \beta\ (1-\alpha)} R_1 \tag{14c}$$

$$T = \frac{K_3}{(1-\alpha)} R_1 \tag{14d}$$

where $K_3 = \frac{1}{K_L \beta}$

and

$$T = K_4 R_1 \tag{14e}$$

where $K_4 = \frac{L}{K_1 \beta D\ (1-\alpha)\ Re^n}$

Equation (14c) corresponds to Eq. (28) of reference 2. It should be noted that this equation is approximate to the extent that $1-\alpha$ in the denominator should be corrected by the factor C_q/C_G.

In designing a water tunnel, the velocity and length of the return circuit determine the time available for resorption. The factors in Eqs. (14) (where α, Re, and L apply to the return circuit and R_1 is the maximum bubble radius converted to return circuit pressure) determine the time required for resorption. The former must equal or exceed the latter for complete resorption. Assuming the initial bubble size, R_1, and the air concentration, C_L, are fixed by the problems to be studied in a water tunnel, Eq. (14) indicates the measures that may be taken to resorb the cavitation bubbles before they return to the test section. These methods are:

(1) Lengthen the tunnel return circuit and reduce the velocity so that the time a bubble remains in the return circuit will be equal to or greater than T. This method involves the risk of having the small bubbles rise and coalesce at the top of the return circuit duct, forming larger bubbles which take still longer to dissolve. If tunnel operation would permit, this defect could be overcome by agitation or rotation of the fluid in the return circuit. It is best overcome by using long vertical legs in the return circuit in connection with method (2) following.

(2) Reduce $\underline{T}$ by increasing the unsaturation, $(1-\underline{\alpha})$. This is done by increasing p, the pressure at resorption, in accordance with Eq. (11). Plotting $(1-\underline{\alpha}) = (1-\frac{p_0}{p})$ against $\underline{p}$ for a given p_0, as in Fig. 2, shows that the effectiveness of this method rapidly decreases as p is increased, and a pressure four or five times the saturation pressure approaches the economical limit. A combination of this method with method (1) results in the California Institute of Technology resorber design adequately described in references 1 and 2.

(3) Reduce $\underline{T}$ by increasing $\underline{Re}$. This method is limited practically by the difficulty of producing a very large increase in $\underline{Re}$.

(4) If the supposition that $\underline{L}$ is a measure of the scale turbulence is correct, reduce $\underline{T}$ by reducing $\underline{L}$; that is, introduce a small scale turbulence without changing the tunnel dimensions. $\underline{L}$ would be reduced by introducing a number of fine screens in the tunnel return circuit at some cost in energy consumption. Some reduction in $\underline{T}$ should be obtainable by this method, and if used in connection with method (1), it might prove adequate for complete resorption. Furthermore, production of a finer scale turbulence would tend to retard coalescence of small bubbles at the top of a horizontal pipe, and it would break up large bubbles.

(5) Reduce $\underline{T}$ by increasing the product, $\underline{\beta D}$. This would be done by completely substituting another gas for air. Carbon dioxide, for instance, offers a product $\underline{\beta D}$ approximately 20 times that for air and would be resorbed in 1/20 the time for air if $C_q/C_G = 0$, other conditions being the same. Because $C_q/C_G \neq 0$ and depends on $\underline{D}$, the factor 20 might not be achieved in tunnel operation but with sufficient turbulence it should be approached closely. This method presents the difficult task of removing all of the air from the water and requires that similarity in the cavitation process be examined.

Of the methods described above, the California Institute of Technology resorber is probably the most practical and has been proved by use. Method (5) warrants some research, and method (4) should not be entirely disregarded. All of the above methods will be most effective in intense turbulence.

EXPERIMENTAL METHOD

An experimental apparatus was designed in which the unknowns in Eq. (9) could be determined. A photograph of the apparatus is contained in Fig. 3, and the apparatus with auxiliary equipment is shown in Fig. 4. Figure 5 is a drawing giving specifications of the apparatus.

Essentially, the apparatus consisted of a small lucite cylinder capped at both ends. Just within one end, a brass rotor was connected to a motor and could be made to rotate at all speeds from 1200 to several thousand RPM by changing the voltage on the motor. The other end cap was drilled for pressure control tubes, a bubble feed tube, and a thermocouple. Pressure or vaccuum could be applied to the entire system (through a rubber diaphragm to prevent contamination) from the laboratory compressed air and vacuum lines. Pressures from a little above zero to 11 atmospheres absolute were available.

The gas bubble feed tube was not used for the air studies but was intended for possible future use with other gasses. Air bubbles were generated by a cavitation process from the liquid under test. This was done by opening the valve at the periphery of the cylinder to the atmosphere (or to a lower pressure, if required) and turning the rotor at high speed until many very small cavitation bubbles were formed. The valve was then closed and the speed reduced so that a number of these small bubbles would coalesce on the cylinder axis, forming a larger bubble. The remaining small bubbles were dissolved by an increase in pressure, leaving the larger bubble for study. It was attempted to make this bubble initially of from 0.01- to 0.02-in. radius at the pressure under study.

The bubbles as described above were not true air bubbles but contained approximately 2/3 nitrogen and 1/3 oxygen, since that is the proportion in which these gasses dissolve in water. Water tunnel bubbles probably contain the same proportions of nitrogen and oxygen.

Turbulence was produced in the apparatus by the liquid rotating inside the cylindrical boundary. The turbulence varied quite noticeably with the rotational speed; using water, there was practically no turbulence at 1200 RPM, but at 3600 RPM and at higher temperatures, the turbulence was so great that a 0.02-in. radius bubble would frequently be sheared in two. After the apparatus was used for a time, the rotor developed a barely perceptible wobble, and it was evident that a more intense turbulence was being obtained

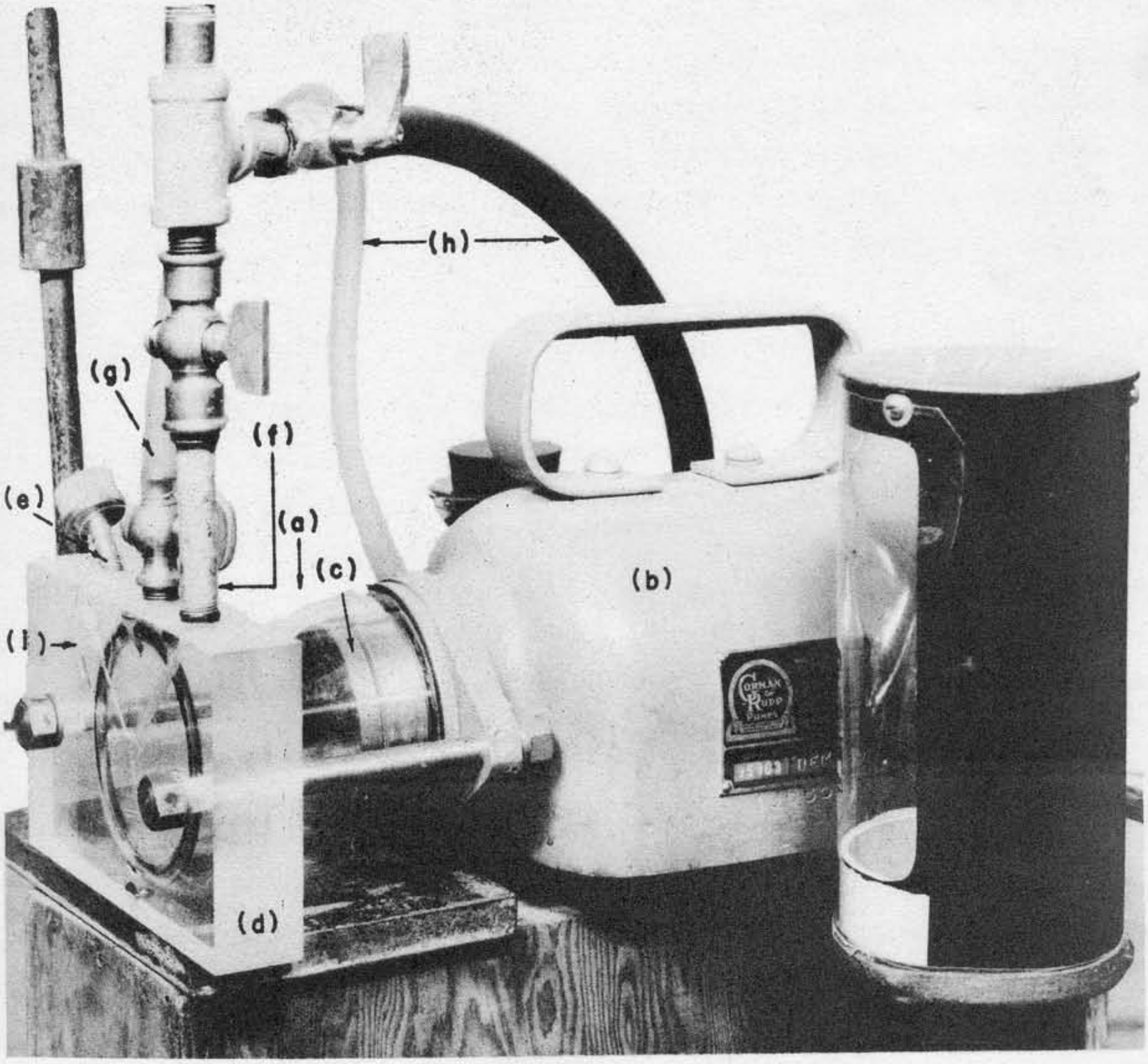

Fig. 3 Test Cylinder and Motor

(a) Lucite Cylinder
(b) Motor
(c) Rotor
(d) Lucite end cap
(e) Bubble feed tube
(f) Tube open to center of cylinder
(g) Tube open to periphery of cylinder
(h) Pressure or vacumn lines
(i) Thermocouple leads

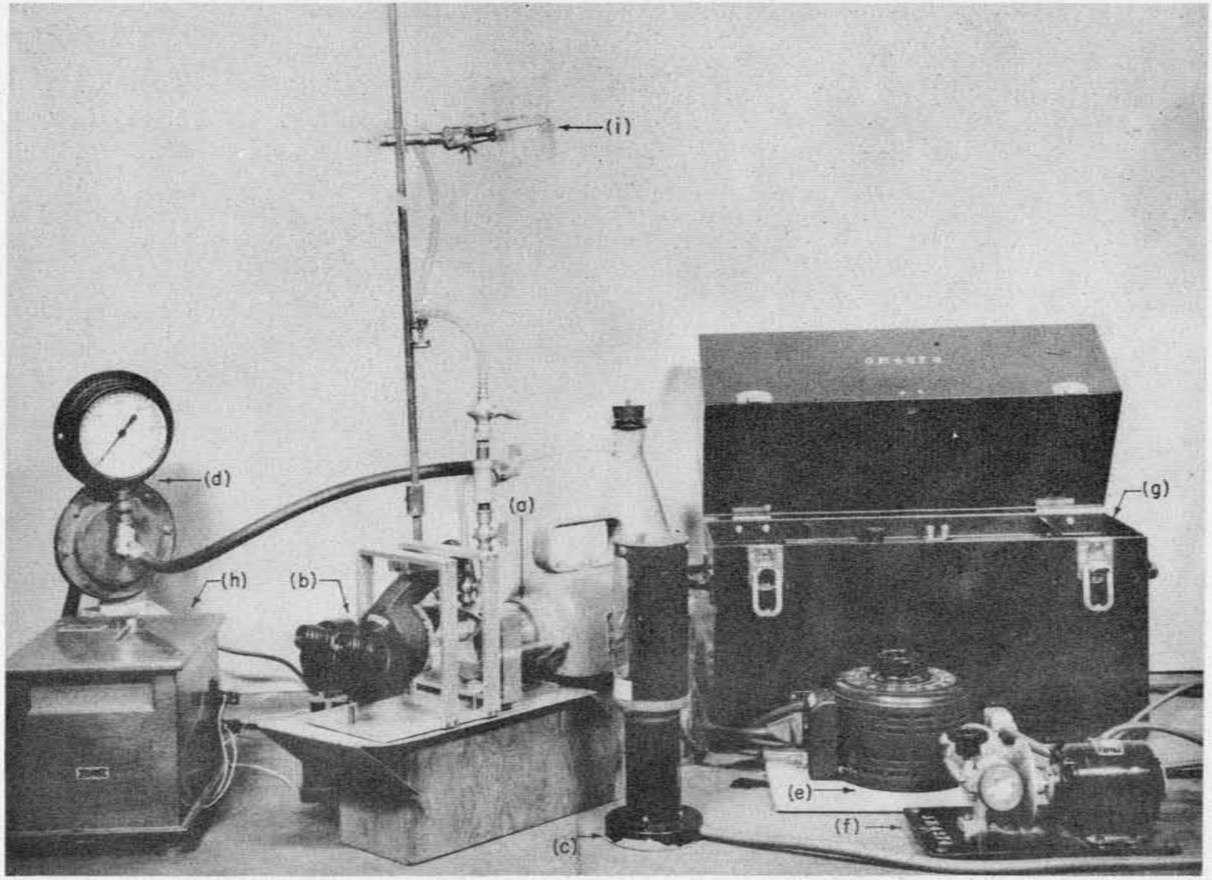

Fig. 4 Test Cylinder and Auxillary Equipment

(a) Test cylinder and motor
(b) Microscope
(c) Stroboscope light
(d) Pressure diaphragm and gauge
(e) Motor speed control
(f) Stroboscope flash control
(g) Stroboscope
(h) Galvanometer attached to thermocouple
(i) Water reservoir and reference pressure control

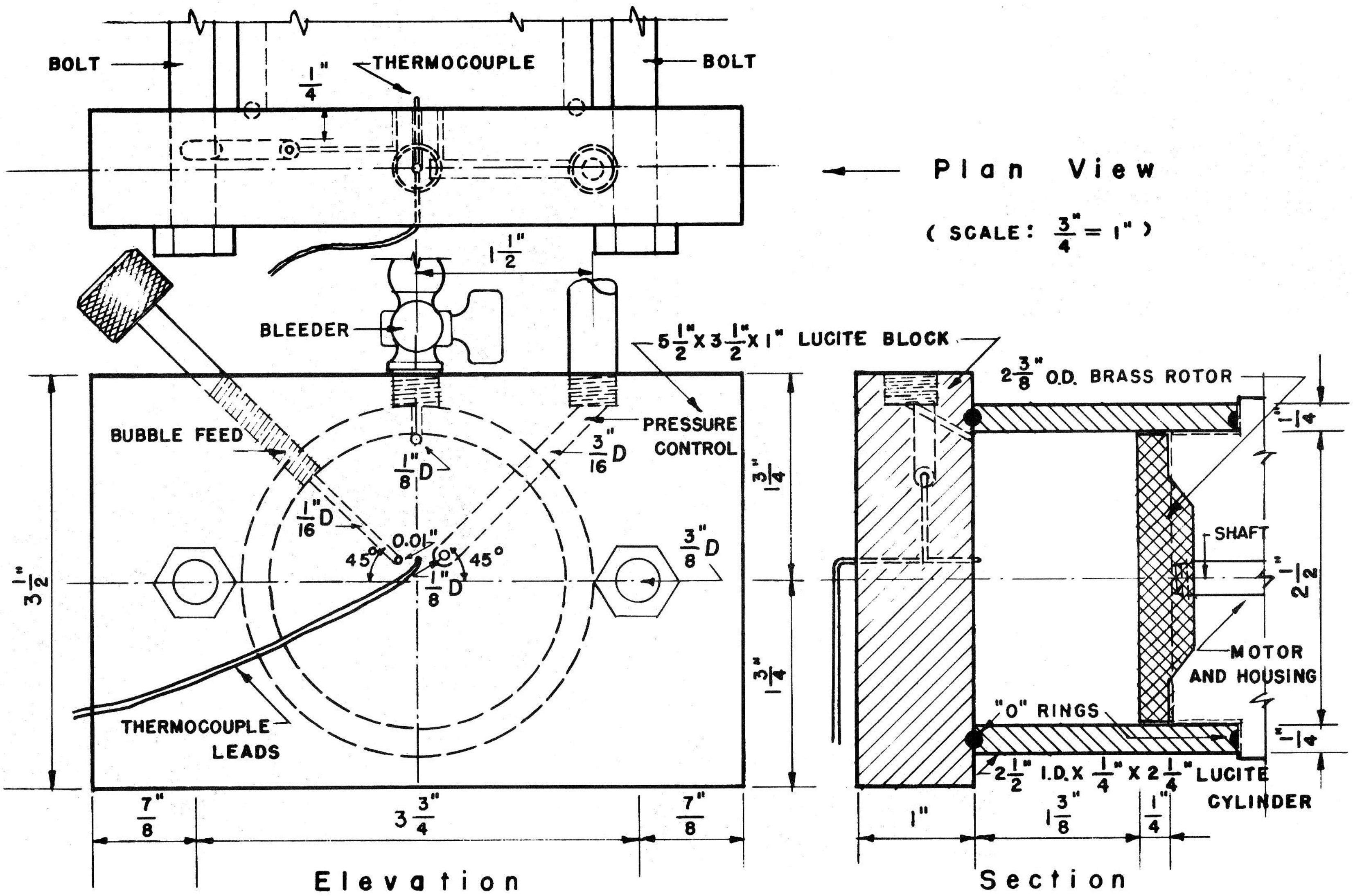

Fig. 5 Details of Test Cylinder

for a given rotational speed. (Turbulence was observed by noting the path of a small bubble in the water.) During the experiments, the bubbles followed a more or less spiral orbit around the cylindrical axis and about 1/8 to 3/8 in. from it. They were moved back and forth in the longitudinal direction very rapidly by the turbulence, remaining near the center of the cylinder, longitudinally. Bubbles of about 0.001-in. radius and smaller did not remain on the spiral path so near the axis but were circulated throughout the liquid.

All experiments described in this paper began with distilled water saturated at 1.08 atmospheres and at a temperature determined by the room temperature and by the rotational speed of 3600 RPM. (1.08 atmospheres corresponds to the pressure due to the water level in the open cup at the top of the photograph in Fig. 4. This cup was ordinarily kept at the top of the support rod shown in the photograph.) The value of $\underline{\alpha}$ at any time was determined from Eq. (11) using $p_o = 1.08$.

The requirement that the test volume should be large compared to the air bubble volume was satisfied in this apparatus. When saturated at 1.08 atmospheres at 95° F, the water in the test cylinder contains about 0.105 cu in. of air. A 0.02-in. radius bubble at 8 atmospheres (the highest pressure used in these experiments) and 95° F contains about 2.5×10^{-4} cu in. of air measured at 1.08 atmospheres. This is less than 1/4 of 1 per cent of the air content of the cylinder and may be considered negligible.

Bubble size was determined using a compound microscope at the lucite end of the cylinder. The microscope was aligned parallel to and about 1/8 in. above the axis of the cylinder and focused on the longitudinal center of the cylinder. A magnification of 18 diameters was used. The depth of focus was about 1/4 in. and the field of view about 1/2 inch in diameter. A reticule in one of the eyepieces was calibrated by placing a scale in the cylinder prior to the runs with bubbles. For obtaining diameter readings, the bubbles were stopped with an Edgerton stroboscope in such a position that they would be superimposed on the calibrated scale in the reticule. This was a very difficult procedure at first, but after some practice, it proved possible to obtain a diameter reading after only two or three seconds of observation. (A photographic method of making diameter readings was tried, but with the magnification required, the light source became so cumbersome that this method was discarded in favor of the simpler direct readings.)

The general plan of the experiments involved measurements of dR/dt in Eq. (9) while two of the variables were changed independently. These variables were $(1-\alpha)$ and Reynolds number. The product βD could have been varied by changing the gas in this apparatus and the length L could have been varied by inserting fine screens on a diametral plane near the cylindrical wall, but the experiments were discontinued before these phases could be undertaken. Variations in $(1-\alpha)$ were produced by changing the pressure, p, in the range from 2 to 8 atmospheres absolute, while holding constant the rotational speed of the disk at 3600 RPM. (Both Re and β varied, however, since the temperature changed from day to day with room temperature and from run to run as the frictional resistance of the shaft seal changed with applied pressure. The water content of the cylinder was so small that its temperature was influenced by all frictional changes.) Variations in Reynolds numbers were produced mainly by changes in rotational speed from 1200 to 3600 RPM but also by uncontrollable variations in temperature from day to day between 95° F and 125° F, while the pressure was held constant at 3 atmospheres absolute. And, as already noted, the turbulence changed near the end of the experiments because of a wobble in the rotor.

DISCUSSION OF EXPERIMENTAL RESULTS

Figure 6 contains typical plots from which dR/dt measurements were obtained. The plotted points are microscope readings of bubble diameter in inches plotted against the time, read on a stop watch, in seconds. All of the data, without exception, plotted as straight lines when plotted in the form of Fig. 6. These straight lines give K_4 in Eq. (14e). It should be noted that the zero diameter point was not obtained by the microscope since, as stated previously, small bubbles did not remain in the microscope field for observation. Instead, the streak of the bubble in a strong, continuous light was observed until it vanished, and this time was taken as the time of zero diameter. The zero point is not very accurate since occasionally the small bubbles were dashed against a wall and broken prematurely; or again they contained minute dirt particles and failed to dissolve in the normal time. Also, near the end, the streak appeared and disappeared for a time, and if there were any other minute particles in the water, it was difficult to distinguish between them and the air bubbles. For this reason, the zero point is given little weight in drawing the lines of Fig. 6.

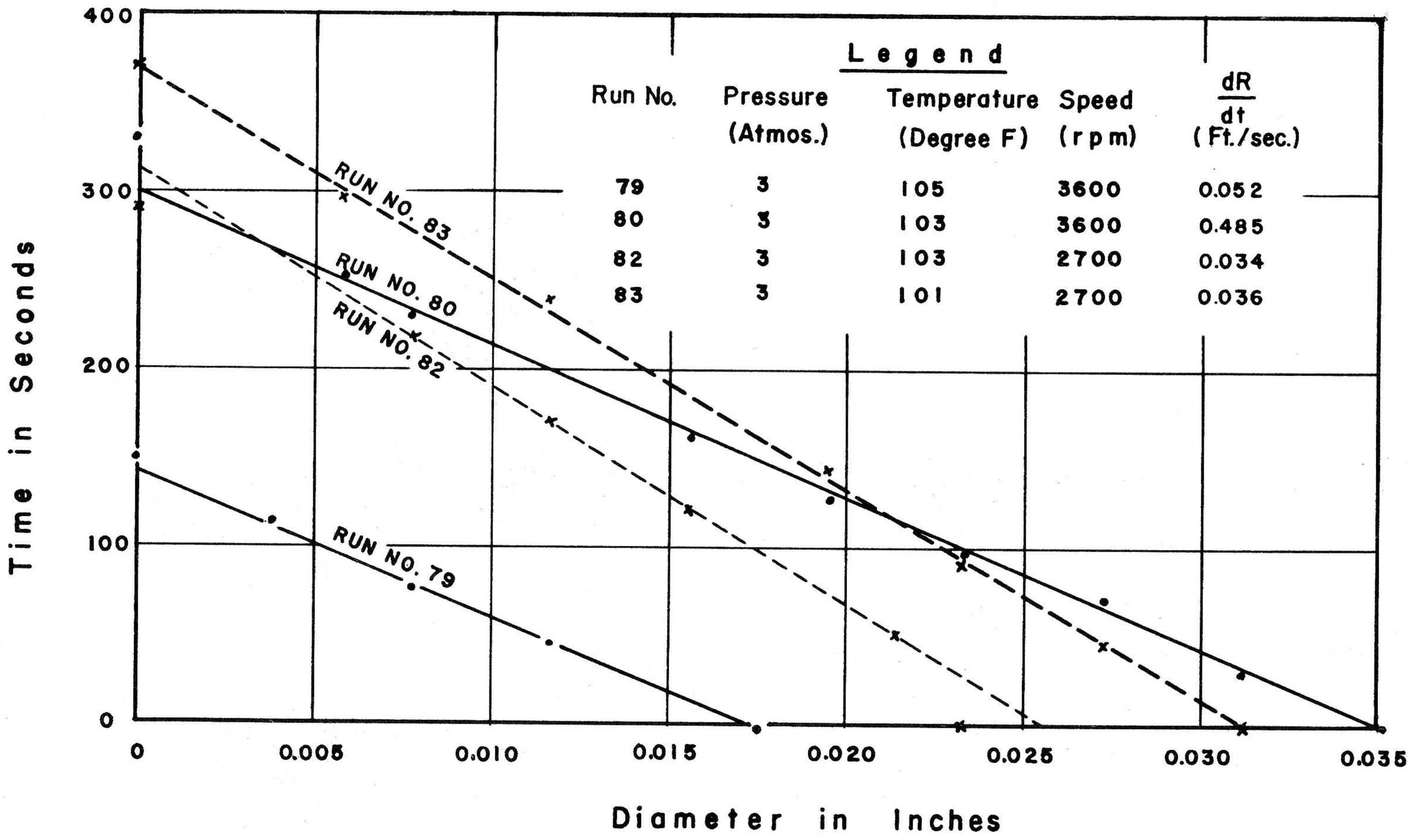

Fig.6 Variation of Bubble Size with Time

There was some latitude in drawing the straight lines through the points in plots like Fig. 6, enough to permit a possible variation of 5 or 6 per cent in dR/dt simply by shifting the line. Other sources of error in applying the measured data to Eq. (9) are:

(1) The value of $(1-\underline{\alpha})$ may be in error by as much as 2 per cent because of the difficulty of determining when the water was saturated at the pressure p_o. In spite of the fact that distilled water was used and that the system was generally closed, dirt seemed to get into the water so that frequent changes were necessary. (The experiments were run only two days a week, on the average, and the equipment stood idle at other times.) After each change, saturation was accomplished by mixing air with the water until a given bubble size would be maintained for at least 15 minutes at the pressure p_o. It was found, however, that even after this point was reached, a 1- or 2-per cent change in air content would still leave dR/dt equal to zero over the 15-minute period. Since $\underline{\alpha}$ approaches 50 per cent in some runs, but is generally smaller, the error in $(1-\underline{\alpha})$ should not exceed about 2 per cent.

(2) A more serious error lies in determining the exact speed of the rotor. A universal motor was used to drive the rotor, and its speed varied by as much as 5 per cent over a very short period of time and perhaps by as much as 10 per cent, during the life of a bubble. A synchronous motor should have been used. These changes of speed changed the turbulence and rate of solution. In the experiments, the nominal speed of the rotor as first set by the stroboscope has been taken as the average speed.

The temperature measuring errors were probably very small. Furthermore, temperature errors tend to be counterbalancing through their effects on $\underline{\beta}$ and $\underline{\nu}$; errors in measuring temperature, therefore, may be entirely disregarded.

The constant $\underline{n}$ has been evaluated from the data first. To make the evaluation, the data for the runs at constant pressure (variable Reynolds numbers) have been inserted in Eq. (9a), wherein C_q/C_G is assumed zero. In this equation, Reynolds number has been taken as:

$$\mathrm{Re} = \frac{\omega a^2}{\nu} \tag{15}$$

where

$\underline{\omega}$ is the angular velocity of the rotor in revolutions per second

a is the radius of the rotor in ft (0.1 ft)

$\underline{\nu}$ is the kinematic viscosity in ft^2 sec.

The actual angular velocity of the liquid near the axis was slightly greater than ω as determined by observing the bubble speed with the stroboscope. Perhaps this value should have been used in defining Reynolds number since it measures the relative speed of liquid and fixed surface, but the only result would be a change in the constant K_1 in Eqs. (9), (13), and (14). Since the data was first computed using Eq. (15), it has been evaluated with that definition for Reynolds number.

Of the remaining factors in Eq. (9a), $\underline{\alpha}$ was obtained from Eq. (11) as explained previously, $\underline{L}$ was placed equal to the rotor radius (0.1 ft) so that K_2, Eq. (14a), rather than K_1 is measured by the experiments, $\underline{D}$ was taken as 1.95×10^{-8} ft^2/sec from Huffner's determination cited by Pekeris [5] and Arnold [7] (this being the value for air and only approximately correct for the gas in the bubble), and $\underline{\beta}$ and $\underline{\nu}$ were taken from handbooks, the values at atmospheric pressure being used (the value for air again being used for $\underline{\beta}$). Some extrapolation for $\underline{\beta}$ was required at the high temperatures. The above factors in Eq. (9a) have been plotted in Fig. 7, on logarithmic scales, in the grouping $\frac{1}{1-\alpha}\frac{a}{\beta D}\frac{dR}{dt}$ versus $\underline{Re}$. All data for the same Reynolds number taken on the same date have been averaged together to reduce the scatter of the points in Fig. 7. The scatter in the averaged data may be attributed at least partially to the change in turbulence as the apparatus was used. The group of points in Fig. 7 with slashes through them, for instance, were obtained from the last runs made in the apparatus when the turbulence was quite large because of the wobbling rotor. Three straight lines representing three values of $\underline{n}$ have been drawn through the points in the figure. Probably $n \approx 1.25$ best represents the data at the higher Reynolds numbers where full turbulence was being approached, but a higher value would be possible.

Using $n = 1.25$, the data obtained at the constant rotational speed of 3600 RPM and variable pressure have been plotted on arithmetical scales in Fig. 8 in the grouping $\frac{1}{Re^{1.25}}\frac{a}{\beta D}\frac{dR}{dt}$ versus $(1-\underline{\alpha})$. Again averaged data have been used. The intercept on the $(1-\underline{\alpha})$ axis of the straight line drawn through the points in the figure is the average value of C_q/C_G for the turbulence

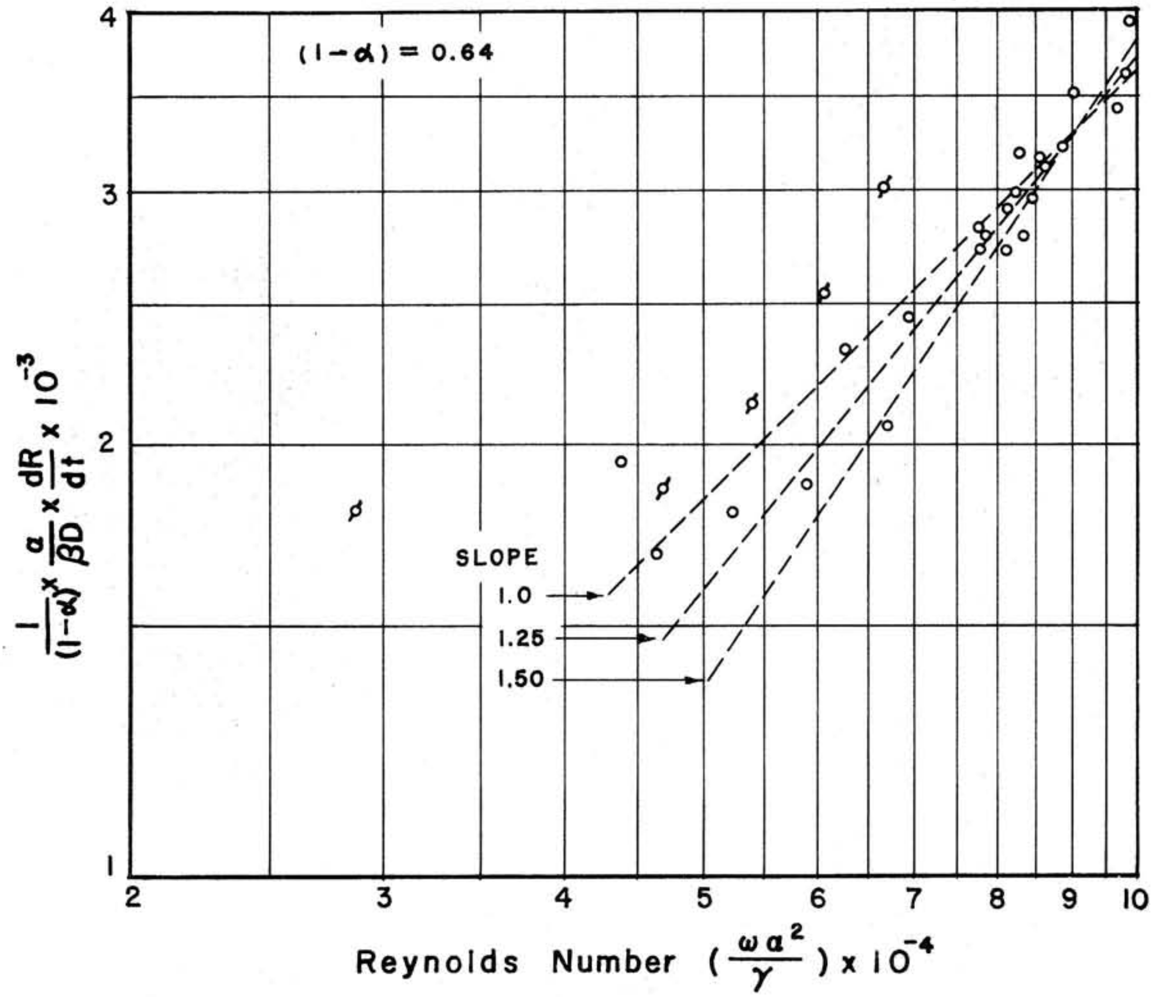

Fig. 7 Effect of Reynolds Number on Rate of Solution

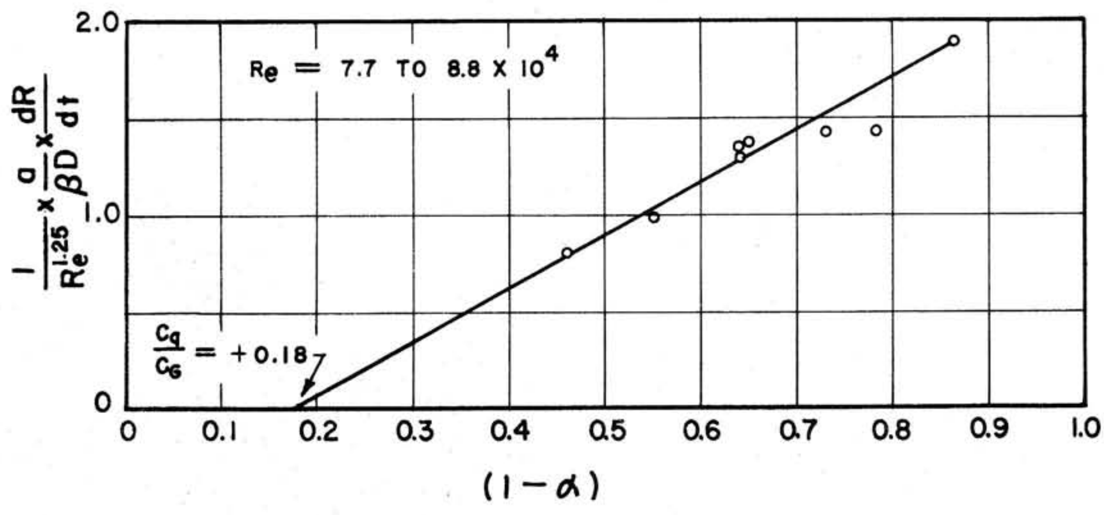

Fig. 8 Effect of Saturation on Rate of Solution

existing at the speed of 3600 RPM. The value of C_q/C_G is 0.18. (In Fig. 8, the straightness of the line depends on the validity of Henry's law and on the proper choice of parameters in Eq. (9). The line appears reasonably straight.) Had n = 1.5 been used, C_q/C_G would be less than 0.18 since the higher Reynolds numbers occurred at the larger values of $(1-\underline{\alpha})$, because of temperature increases.

Data from all of the experimental runs have been plotted on arithmetic scales in Fig. 9 in the form of Eq. (9), with n = 1.25 and $C_q/C_G = 0.18$. Actually, the value of C_q/C_G should probably be increased as the Reynolds number decreases, but it has been taken as constant in plotting the points for the figure. The points obtained at 3600 RPM and 3 atmospheres pressure have been distinguished from those obtained with variable speed and those obtained with variable pressure by different symbols as shown on the legend on the figure. The equation:

$$\frac{a}{\underline{\beta} D} \frac{dR}{dt} = 2.85 \times 10^{-3} (0.82 - \underline{\alpha}) \, Re^{1.25} \tag{16}$$

appears to fit the data reasonably well except in the lower turbulence region where an increase in C_q/C_G would have made an improvement and except for the last runs in which the change in turbulence was not accounted for in the parameter. From Eq. (16), $\underline{K_1}$ in Eqs. (9) and (13) is 2.85×10^{-3} and includes the proportionality between $\underline{L}$ and $\underline{a}$. Equation (13) becomes:

$$T = \frac{a}{2.85 \, \underline{\beta} D \, (0.82 - \underline{\alpha}) \, Re^{1.25}} R_1 \times 10^3. \tag{17}$$

Equation (17) would be applicable to a water tunnel whose turbulence characteristics are similar to those of the apparatus used in these experiments. At a still higher Reynolds number, the above constants should be applicable to Eq. (14a), giving the approximate formula:

$$T = \frac{a}{2.85 \, \underline{\beta} D \, (1-\underline{\alpha}) \, Re^{1.25}} R_1 \times 10^3. \tag{18}$$

At a rotational speed of 3600 RPM and a temperature of 100° F, $\underline{\delta'}$ in Eq. (6a) becomes:

$$\delta' = \frac{a}{K_1 Re^n} \; 2.5 \times 10^{-5} \text{ ft} \approx 0.0003 \text{ in.}$$

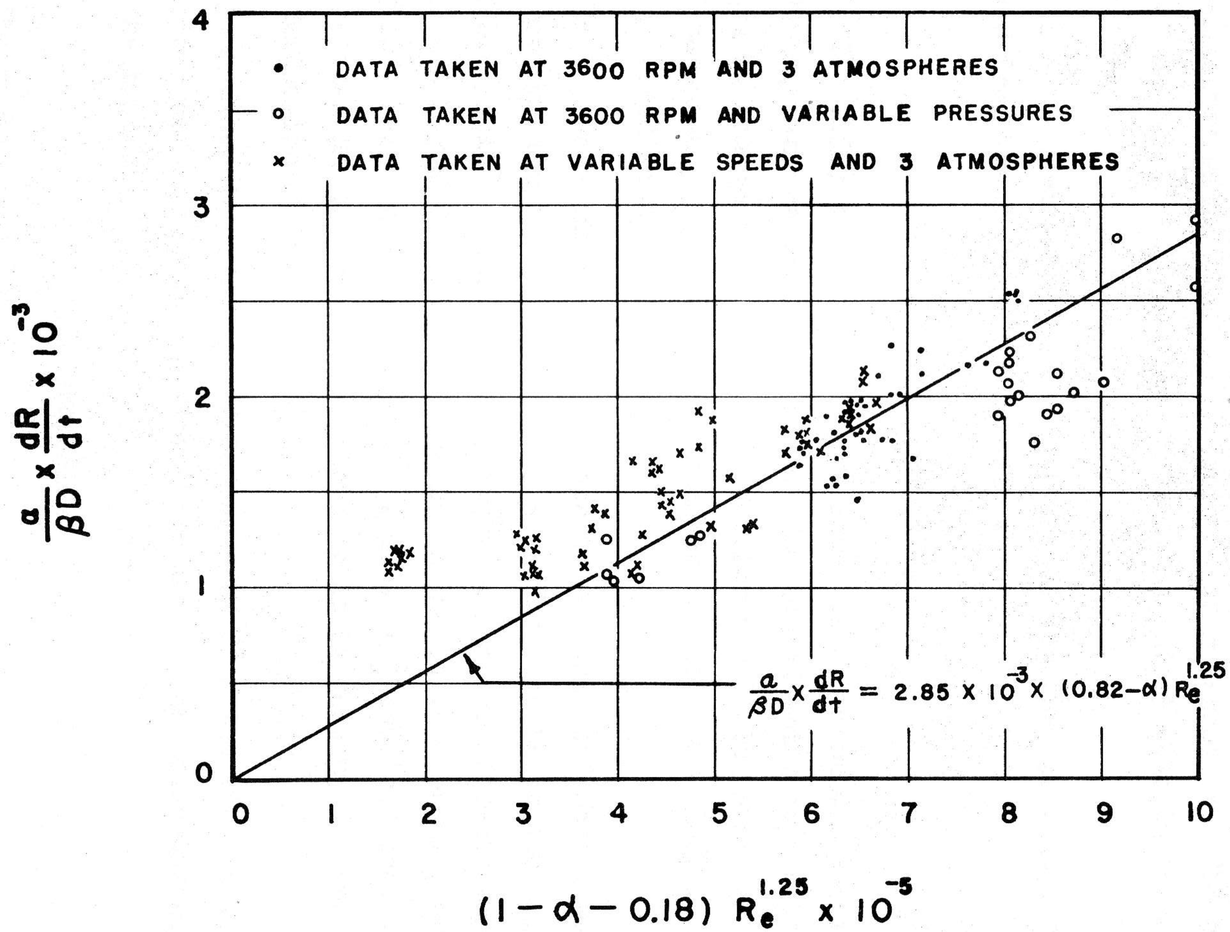

Fig. 9 Effect of Reynolds Number and Relative Saturation on Rate of Solution

This compares well in order of magnitude with the thickness of the laminar sublayer from fluid and heat flow results at high Reynolds numbers. Under the same conditions and with $\alpha = 0$, K_L in Eq. (12) becomes:

$$K_L = \frac{D}{\delta'}\left(1 - \frac{C_q/C_G}{1-\alpha}\right) = 6.4 \times 10^{-4} \text{ ft/sec (70 cm/hr).}$$

This compares with a value of K_L of 70 cm/hr deduced for the California Institute of Technology tunnel without a resorber, in reference 2. The agreement between the data for that tunnel and the most turbulent case in these experiments is remarkable.

If it is desired to use an equation of the form of (14d), the equation derived from these experiments would be:

$$T = \frac{1.17 \times 10^5}{(0.82 - \alpha)} R_1 \tag{17a}$$

When $\alpha = 0$, Eq. (17a) determines the minimum time for bubble resorption in a system having the characteristics of the experimental apparatus. This equation may be compared with Eq. (29) of reference 2, in which the constant 1.70×10^5 based on a value of K_L of 32 cm/hr was used and $(1-\alpha)$ rather than $(1-\alpha-\frac{C_q}{C_G})$ appeared in the denominator.

If the Reynolds number expressed by Eq. (15) had been taken as $\frac{\omega a^2}{\nu}$ in accordance with the previous argument, K_2 would have the value 16.2×10^{-3} instead of 2.85×10^{-3}. (The values of Reynolds numbers in these experiments would have been 2.3×10^5 and lower, based on the latter formula, while they approached 10^6, based on Eq. (15).) The values of δ' and the succeeding constants calculated in the paragraphs above would be unchanged, however, by a change in definition of Reynolds number.

Some other interesting points garnered from the air bubble experiments described herein, but not pertinent to the present discussion, are mentioned in the appendix.

CONCLUSIONS

The equation:

$$T = \frac{L}{K_1 \beta D \left(1-\alpha-\frac{C_q}{C_G}\right) Re^n} R_1 \tag{13}$$

has been proposed and partially verified as the basic equation governing gas bubble resorption in turbulent liquid. The constants, K_1 and n, have been evaluated as 0.0162 and 1.25, respectively, and $\frac{C_q}{C_G} = 0.18$ at the highest turbulence level for the apparatus used in these experiments. T is the time of resorption in seconds, R_1 is the initial bubble radius in feet, D is the diffusivity in square feet per second (about 1.95×10^{-8} for air in water), β is the solubility in gas volumes per volume of liquid at the temperature of resorption (to be taken from handbooks), α is the relative saturation of the liquid at resorption (obtainable from Eq. (11), page 9), Re is the Reynolds number based on the relative motion of the liquid and the fixed boundaries, and L is a linear dimension measuring the scale of the turbulence in the liquid in feet and is taken as equal to the boundary dimension in Re in obtaining the above value of K_1. The constant K_1 will change with the definition of Re and L, but it is believed that $n \approx 1.25$ is a universal constant for high turbulence. The value of C_q/C_G will also change with the turbulence and with the ratio $\frac{\nu}{D}$, but should approach zero for intense turbulence.

For turbulence corresponding to that used in the experimental apparatus, the doubt regarding the definition of Re and L may be resolved by using the equation in the approximate form:

$$T = \frac{\delta'}{\beta D (1-\alpha)} R_1 \tag{14b}$$

where δ' (the thickness of the laminar sublayer) was found equal to 2.5×10^{-5} ft and C_q/C_G has been taken as zero for intense turbulence. Furthermore, if the bubbles are air in water at a temperature of 100° F, the equation may be simplified to:

$$T = \frac{K_3}{1-\alpha} R_1 \tag{14d}$$

where $K_3 = 1.17 \times 10^5$ sec per ft, and the relative saturation is the only parameter to be determined.

Equation (14d) and the associated constant K_3 confirm the resorption analysis made at the California Institute of Technology [2] in connection with the design of a resorber.

Equation (13) led to several suggested methods of accomplishing resorption within a single tunnel circuit. The most promising of these were:

(1) the resorber already introduced at the California Institute of Technology and based on increasing ($1-\underline{\alpha}$) in Eq. (13) while providing more time for resorption; (2) a method in which the product $\underline{\beta}$ D in Eq. (13) would be increased many fold by substituting another gas such as carbon dioxide for air in the water; and (3) a method in which the scale of turbulence would be decreased by providing fine screens in combination with an increased travel time through the tunnel circuit. The second method, in particular, should be given some attention.

ACKNOWLEDGEMENT

The work on air bubble resorption was performed in connection with the study of a pilot model of a free jet water tunnel sponsored by the Office of Naval Research under Contract N8Onr - 66202 and in connection with a model study of an open jet water tunnel sponsored by the David Taylor Model Basin, U. S. Navy, under Contract NObs - 34208. The experimental data were taken by Dr. C. S. Chao and the paper, particularly the analysis, was reviewed critically by James S. Holdhusen.

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APPENDIX

Some Observations from the Air Bubble Resorption Apparatus

In addition to the direct measurement of dR/dt in bubble solution described in the body of this paper, some qualitative observations of other phenomenon were also obtained. One of these was a conclusion that the rate of growth of an air bubble once formed in supersaturated liquid is governed by the same equation and constants as the rate of solution. This is in accordance with similar previous conclusions cited in the absorption symposium [3] listed in the bibliography. The principal difference between solution and formation of a bubble is that in solution Eq. (13) is followed to zero radius as nearly as can be determined, whereas in formation the bubble appears to form nearly instantaneously with some finite radius.

Other interesting obervations were made regarding the method of formation of bubbles in the cavitation process. As the pressure was reduced and the rotor gradually speeded up to the cavitation point (which, of course, depended on the air content), the liquid which was previously perfectly clear began to generate very minute bubbles. These were first observed in an orbit about 1/2 in. from the cylinder axis throughout the cylinder length. These bubbles were certainly considerably less than 0.001-in. diameter, but how much less was not determined. The bubbles first appeared at about the same distance from the axis for several combinations of pressure, speed, and air content, but there is no certainty that they were generated at that distance. They were definitely generated in the interior of the fluid, however, and not at the walls. It might also be noted that it was impossible to reduce the static pressure in the test cylinder sufficiently to produce cavitation without motion of the rotor.

When the rotor was stopped as soon as a few bubbles were formed, the bubbles floated around in the liquid until they finally attached themselves to one of the surfaces and dissolved when the pressure was restored. If the generation was continued, some of the bubbles coalesced to form larger bubbles on the axis and if it was continued long enough, a hollow cylinder could be formed about 1/4 to 3/8 inch in diameter on the cylindrical axis extending all the way from the brass rotor to the lucite cap. If the pressure in this situation was suddenly increased, the vapor in the hollow space could be seen to condense on the cap and the space rapidly broke down into individual air bubbles. At the lower air contents, the cylindrical space was mostly vapor, even after a long time; but at normal air content, the cylindrical space was mostly air if the cavitation proceeded long enough.

If there were many larger bubbles in the liquid, but not enough to form the hollow cylindrical space first described, the bubbles appeared to form a frothy region about the same size as the cylindrical space would have been. On studying this frothy space with the stroboscope, however, it was seen to be made up of four columns of bubbles extending from the rotor to the cap, each column rotating about the cylindrical axis at about 1/8 in. or more from it. No matter how the speed or pressure of the system were altered, there were always four columns of bubbles under these conditions and there were no bubbles outside the four columns.

The turbulence in this apparatus was apparently non-isotropic. At least the paths of the small bubbles which were moved around by the turbulence were long in the longitudinal direction of the cylinder and practically indistinguishable in the radial direction. Under a strong light, a small bubble moving under the influence of turbulence when seen from the side of the cylinder looked almost exactly like a particle under the influence of Brownian movement.

The bubble shape in the apparatus was almost always spherical, but occasionally a distorted bubble was seen. Ocassionally, a speck of dirt would be trapped on a bubble surface and appeared to be held rigidly in the surfsce. Advantage was taken of such dirt specks to observe that the bubble apparently did not rotate about any axix in the bubble, but only about the cylindrical axis. Dirt specks definitely reduced the rate of solution and runs where dirt was picked up were discontinued. On one run, a bubble surface was completely covered with a dirt crust after it had shrunk to a certain size, and it failed to shrink further regardless of increases in pressure or speed, being entirely insulated by the crust.

If the apparatus was turned on end with the cylinder at the top and the rotor horizontal, a bubble in high turbulence would tend to remain in the center of the cylinder, longitudinally. That is, the bubble appeared to resist buoyancy and to be held entirely by the turbulent forces.