

Designing Optimal Strategies for Surveillance and Control of Invasive Forest Pests

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Dedication

This thesis is dedicated to my parents, Hideo and Hiroko Horie, for their love, endless support, and encouragement.

Abstract

This thesis focuses on the theme of detecting and managing invasive forest pests. Chapter 1 opens the dissertation with a review of relevant literature. In Chapter 2, we model optimal detection of sub-populations of invasive species that establish ahead of an advancing front. For many invaders, eradication of the main population is an untenable goal. However, it may be possible to treat and eradicate smaller sub-populations upon detection. Increasing detection effort is costly, yet leads to earlier detection of smaller populations. The level of detection effort that optimally balances this tradeoff depends, in part, on the distance from the main front: locations closer to the front with shorter management horizons enjoy lower reductions in overall cost from intervention. We find that the uninfested landscape is divided into two zones, characterized by different dynamically optimal management plans: a suppression zone and an eradication zone. In the suppression zone, optimal detection effort increases with distance from the front. At the distance where the suppression zone yields to the eradication zone, optimal detection effort plateaus at its maximum level.

In Chapter 3, we develop a model of optimal surveillance and control of forest pathogens and apply it to the case of oak wilt in a region within Anoka County, Minnesota. Managers allocate limited budgets between surveillance and control activities. Furthermore, they try to determine the locations where these efforts will have the most benefit. Our model allows for a heterogeneous landscape, where grid cells are differentiated by the number of trees and the number of infected trees. We develop a cost curve associated with the expected fraction of healthy trees saved from becoming

infected. We find that the cost curve of saving healthy trees from infection is upward sloping, and that marginal cost increases along with the increase in the desired fraction of healthy trees. We also explore characteristics of sites selected for surveillance. In particular, we examine the characteristics of sites that make them high-priority sites for surveillance when the budget level is relatively low. We find that the best surveillance strategy is to prioritize sites with relatively low expected unit surveillance cost per tree saved from infection. Our results offer practical guidance to managers in charge of deciding how and where to spend limited public dollars when the goal is to reduce the number of trees newly infected by oak wilt.

Invasive species are major factors damaging forests on private lands. In the fourth chapter, we consider a situation where private lands are at risk of being infested by an invasive species. In Chapter 4, we model a private landowners' forest protection problem, in which each landowner decides among three possible strategies: prevention, monitoring and treatment, and no treatment. Landowners are differentiated by how much they value the forest on their property. Landowners do not know whether or not their forest will become infested, nor do they know whether monitoring will find an infestation if present: these events are probabilistic. We examine the change in the proportions of the population choosing these three treatments in both the landowners' equilibrium and the full information social optimum as the accuracy of monitoring changes. We find that the proportion of landowners taking preventive and no action increases as the accuracy of monitoring decreases; monitoring ceases to be chosen when monitoring accuracy declines below a threshold value. We also investigate the possible effects of a policy that raises

the accuracy of monitoring on social welfare in both the landowners' equilibrium and the full information social optimum. We find that the policy closes the gap in social welfare between the landowners' equilibrium and the full information social optimum. However, it decreases social welfare in the full information social optimum. This finding casts doubt on whether improving the accuracy of monitoring is worth pursuing. Chapter 5 closes the thesis by summarizing the studies and discussing future extensions.

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Chapter 1

Introduction

Invasive species are alien species which cause economic or environmental harm or harm to human health. One important category of environmental harm is the damage caused by invasive pests and pathogens to forests: damaged caused by invasive diseases and pests including Dutch elm disease, oak wilt, sudden oak death, gypsy moth, emerald ash borer, and Asian longhorn beetle have been valued at an estimated \$120 billion per year (Pimentel, Zuniga, & Morrison, 2000). Much attention has been paid to reducing these costs by stopping or slowing the spread of invasive forest pests and pathogens. This thesis investigates cost effective strategies to manage invasive forest pests, emphasizing the tradeoff which arises when an ecosystem manager determines both surveillance intensities to obtain information about the location of pest populations and pest control intensities. The tradeoff associated with these two activities arises because of various constraints such as time and budget constraints which an ecosystem manager faces when she aims to minimize the sum of damages caused by pests and costs to manage them.

In Chapter 2, we consider a situation where the time available for management is limited. This situation takes place when it is impossible to stop the advance of the main front of an invasive pest, yet it is beneficial to detect and control sub-populations of the pests, which are cast ahead of the front, till the main front reaches a given location. The management of gypsy moth provides us with a prominent example. Gypsy moth was

introduced in 1868 from France to Boston in Massachusetts, U.S., and escaped in 1882. Since then, it has been spreading in the northern eastern part of the U.S. Despite of the public and private intense efforts to control its spread, the spread has not been stopped. Now it is considered that eradication of gypsy moth or even stopping or pushing back the front of infested area is infeasible. Instead, the national Slow the Spread program run by United States Department of Agriculture Animal and Plant Health Inspection Service (USDA-APHIS) aims to slow the spread of gypsy moth (Sharov, Leonard, Liebhold, Roberts, & Dickerson, 2002).

When an ecosystem manager has to detect and control invasive pests within a given length of time, she faces a tradeoff. Assume that an ecosystem manager aims to minimize costs associated with managing invasive species: the direct cost of surveillance effort, the cost of damage caused by the pest before detection occurs, and the costs of both the pest control and the pest damage that occur after detection. Increasing surveillance effort shortens the time to detect the pest. This reduces the time that the invasive population grows before control starts, thereby reducing the costs of pest damage both before and after detection. Pest control costs also decrease when the pest is found earlier. However, increases in surveillance efforts also result in expensive surveillance costs.

This central tradeoff involved in determining the optimal intensity of surveillance was investigated by Mehta, Haight, Homans, Polasky, & Venette (2007). Before this paper, the economic literature focused on questions related to pest control (e.g., Olson & Roy, 2004; Olson & Roy, 2005; Olson & Roy, 2008; Sharov & Liebhold, 1998). Mehta et al. (2007) introduced the importance of timing as it relates to invasive

species management: pest control can be conducted only after pest populations are found through surveillance activities. They incorporated the surveillance stage into a model of invasive pest management.

However, Mehta et al. (2007) assumed an *ad hoc* increasing function relating the cost of the pest to the size of the stock upon detection. This assumption could be supported by the idea that larger populations are more costly to eradicate, and they cause greater damage. However, the management problem is itself an optimization problem and we introduce and solve this optimization problem in Chapter 2. This provides us with a richer understanding of the relationship between stock size upon detection and post-detection damage and management costs. In particular, we allow an ecosystem manager to choose between eradication and suppression strategies after deciding the surveillance intensities. In our model, the surveillance intensities also endogenously determine the timing to switch the management activities from pest surveillance to pest control.

We formulate the problem divided into two parts: the surveillance phase and the pest control phase. In the pest control phase, we use optimal control theory to derive a value function which represents the discounted cost of an optimally managed sub-population of an invasive pest, where costs comprise both damage costs and management costs. In the surveillance phase, by controlling the surveillance intensities, we minimize the sum of the discounted costs of damage and the surveillance during the detection phase and the discounted cost of an optimally managed sub-population of an invasive pest (post-detection) represented by a value function mentioned above. We assume that surveillance intensities are kept constant throughout the detection phase. The value function obtained in the pest control phase depends on both the size of the population

upon detection and the upper limit on the amount of time remaining. We impose an inequality constraint via a transversality condition on the ending stock so that the ending stock must be non-negative. Using a transversality condition for the ending date, the ending date of the management horizon is constrained to be less than or equal to the date at which the main front arrives.

By solving this problem, we investigate the optimal timing to switch from surveillance to pest control when the management time is limited. This investigation provides us with information about the relationship between the optimal surveillance effort levels and the optimal post-detection pest control strategy--eradication or suppression--for each given overall management time horizon.

In Chapter 3, we shed light on the problem of managing a pest over space when budgets are limited. We consider a situation where, within a given budget level, an ecosystem manager has to determine where to conduct surveillance to detect invasive pathogen, and also has to determine where to control the detected pathogen. Moreover, we consider a case where the distribution of the source of a new infestation and susceptible species are heterogeneous over space. We explore the allocation of a limited budget between surveillance and control and the optimal allocation of these activities over a landscape. This study addresses two important issues in invasive pest management: the effect of heterogeneity associated with uncertainty about the degree of infestation among sites on the optimal selection of sites managed and allocation of a limited budget to different activities –surveillance and control.

The setting we examine applies to the case of invasive fungal pathogen management. Invasive fungal pathogens such as oak wilt, chestnut blight, and Dutch elm

disease have had widespread effects on urban forests and have increasingly threatened tree species nationwide. For fungal pathogens, the most effective ways of controlling spread is the detection and removal of infected trees. In the case of oak wilt, for example, trees killed by oak wilt produce spores in the following spring, and these spores are a source of pathogen for new infections. However, budgets available for the surveillance and removal are typically very limited and governments must decide how to allocate those limited funds among sites with different features. For example, should locations with low numbers of infected trees be given priority since these locations can more easily become disease-free, or should locations with low inspection costs be given precedence because more diseased trees can be identified with the same budget? In addition, governments face a tradeoff between search and control activities. Greater spending for surveillance enables governments to identify more infected trees, but increased surveillance limits the budget remaining for removal of infected trees.

While Chapter 2 focused on the timing issue raised by the order of management activities, the study in Chapter 3 focuses on location: the eradication or suppression of the species is possible only on the sites where surveillance is conducted. For pathogens, the detection process of pathogen management needs to exactly identify an infected tree in each location. Pathogens control is not broadcast (such as aerial spraying), but instead is applied to particular trees. Forest managers must know which trees are infected before removing them. This implies a set of important tradeoffs that are considered when sites are selected for surveillance and treatment. When more sites are selected for surveillance, more sites are eligible for treatment; however, when a greater proportion of the budget is spent on surveillance, fewer resources are available for suppression and containment.

To explore this, we build a two stage mixed integer programming model in which the objective is to minimize the expected number of newly infected trees. In the first stage (surveillance), the manager chooses sites to be surveyed from all the sites in the management area. In the second stage (control), the manager determines the proportions of infected tree populations removed in each site within the sites surveyed in the first stage. We have two constraints. First, the budget constraint ensures that the total costs of surveillance and treatment do not exceed the budget level. Second, we ensure that trees cannot be removed in a given site unless the site has been surveyed: the number of trees removed in each site is bounded above by the number of trees that have been identified as infected. This model is adapted from Snyder, Haight, & ReVelle (2004) who develop a similar model for the purpose of maximizing the expected number of species represented in sites selected for preservation. Using this model, we provide information about cost-efficient tradeoffs between detection and control activities.

Chapters 2 and 3 focused on large scale government-funded management programs. This approach lends itself to pests for which individual property owners can do little to shield themselves from damage because the pests disperse over long distances or local control measures are non-existent or ineffective. Chapter 4 focuses on another approach which is suited to slow-growing and locally controllable pathogens. This approach encourages private landowners to undertake management efforts on their own properties to reduce the spread of the disease to neighboring properties.

In Chapter 4, we change our perspective of management from a sole ecosystem manager to multiple landowners' management of invasive pests. Here we examine effects

of information on landowners' behavior in the equilibrium and effects of information on social welfare in the equilibrium. In Chapter 4, we consider situations where landowners can manage the invader effectively by either preventing the invader from establishing on their property or by mitigating damage to trees once the invader arrives and becomes established. Prevention has the added benefit of reducing the overall probability that neighboring properties become infested. Because some of the benefits of preventive action are external and accrue to neighbors, it is likely that private action will yield less than the socially optimal level of prevention. We model this situation, and then introduce a government policy to subsidize prevention. We examine whether, and under what conditions, this policy can achieve the socially optimal outcome.

In its basic structure, our model shares elements with models found in the literature on optimal immunization to protect against disease and models of optimal taxation (e.g., Brito, Sheshinski, & Intriligator, 1991; Francis, 1997; Gersovitz & Hammer, 2004). In our model, we allow for a richer set of possible behaviors on the part of affected landowners. In particular, we include the possibility of monitoring and detection, where the relationship between monitoring and detection is probabilistic.

In our model, the probability of the infestation is composed of two parts. One is the probability of the introduction of the invasive species to the area, which is determined exogenously. The second is the probability of the spread of the species once it is introduced. This probability is determined endogenously by the outcome of the actions of landowners; the larger the fraction of landowners who do not take preventive action, the higher the probability of spread and thus the higher the overall probability of infestation.

We envision a situation where property owners have three possible options regarding the invader. One is to apply a costly preventive treatment, such as a pesticide injection, in advance which protects trees from any damage and also prevents the parcel of land from being a source of pests to neighboring parcels. A second option is to forego prevention but monitor trees and apply a curative treatment to mitigate damage in case trees do become affected and monitoring is successful in detecting the invader. Landowners choosing a monitoring strategy may be fortunate and escape infestation altogether. However, if the species establishes on these properties, damage can be mitigated at a cost, but trees can still be a source of infestation to neighboring properties, adding to the overall probability of infestation. Monitoring is not guaranteed to find the pest if it occurs on the property: detection occurs with some probability between zero and one. A third option is to take no action and leave trees fully vulnerable to the invasive species. No mitigation occurs, and these parcels are also a source of spread and increased probability of infestation in neighboring properties.

We build a model of a continuum of landowners where each landowner is differentiated by her valuation of the trees on her property. Each landowner is endowed with an income and a piece of forested land. We assume that she has separable utility over consumption good and the forest amenity. Except for the valuation of the forest amenity, landowners are identical. That is, they have identical income endowments, forest amenity endowments, and utilities from the consumption good.

The key outcome of the model is the probability of infestation. Landowners choose their strategy based on the probability of infestation, but this infestation is affected by how many landowners choose a preventive strategy. A higher number of

landowners choosing prevention leads to a lower probability of infestation for the remaining landowners. This reduction in probability leads to a lower incentive to follow a preventive strategy. Thus, the probability of infestation is endogenous to the model. This model feature is similar to models found in models of immunization to disease where it becomes tempting for individuals not to incur the cost of immunization when the prevalence of disease is low.

The regulator does not have information about which landowner has which characteristic, but he knows the distribution of the characteristics. The regulator is assumed to try to maximize social welfare by choosing a subsidy for prevention that is financed by taxing all landowners. This subsidy increases the fraction of landowners choosing to prevent, thus reducing the probability of spread. In addition to reducing the probability of infestation, this policy redistributes income. We consider the sensitivity of the optimal policy and social welfare to economic parameters: the probability that the pest will be detected given a monitoring strategy and costs of management alternatives.

Chapter 2

Optimal Detection Strategies for an Established Invasive Pest

2.1 Introduction

Concern about the invasion of non-indigenous species into the U. S. landscape dates back at least to the early 1900s with the spread of the gypsy moth (*Lymantria dispar*), an insect that was introduced from France to Massachusetts and which soon began to defoliate large swaths of hardwood trees (Liebhold, Halversen, & Elmes, 1992). The gypsy moth is still causing damage, and its territory has spread—and continues to spread—thousands of miles outward from its origin. Recently, attention has broadened from concern about specific invaders like the gypsy moth to a wide array of terrestrial and aquatic non-native species. The issue was given national prominence with Executive Order 13112 signed by President Clinton in 1999 (Clinton, 1999). With the increased severity of the problem has come increased attention by economists into finding efficient ways to manage the growing threat.

At least two broad themes have emerged in the literature on the economics of invasive species. One has to do with the efficient use of resources in the prevention of potential invasions, the control of existing populations, and the balance of effort between prevention and control (e.g., Costello & McAusland, 2003; Eiswerth & Johnson, 2002; Olson, 2006). Some attention has also been paid to the intermediate step—detection of

established populations so that management can commence (Mehta et al., 2007). Another theme is the central role that spatial dynamics play in both species dispersal and management (Burnett, Kaiser, & Roumasset, 2007; Wilen 2007). Established populations spread throughout the landscape wherever there is suitable habitat, and the management of these species often takes the form of containment through the establishment and maintenance of a barrier zone. Sharov and Liebhold (1998) view this problem through an economic lens to find an optimal containment strategy in time and space.

In this paper, we draw upon both themes to develop a model of detection where the optimal detection strategy may depend upon the location of a population. In particular, we consider the optimal spatial allocation of detection effort when it is possible only to slow, not stop, the advance of the main front of an invasive species. However, it may well be possible to detect and control sub-populations of the species that erupt ahead of the front. Long-range dispersal of forest pests, weed varieties, and aquatic species can be attributed to human causes (transport of eggs or larvae on vehicles, seeds hitchhiking in horticultural material, vessel movements from infested to uninfested waters). Examples in the forestry setting include gypsy moth, emerald ash borer (*Agrilus planipennis* Fairmaire) and oak wilt (caused by *Ceratocystis fagacearum*). Gypsy moth is the subject of a national “slow-the-spread” program because eradication is no longer deemed possible (Sharov et al., 2002). Emerald ash borer, introduced in the 1990s, has resisted eradication efforts and continues to spread outward from its origin in Michigan and Ontario (Poland & McCullough, 2006). Vigorous detection programs are underway to find and rout satellite populations. Oak wilt continues its natural spread north, and long-distance transport through human means has carried it in a leapfrog fashion into the

Upper Peninsula of Michigan where it is being aggressively managed (Forest, Mineral, and Fire Management Division, Michigan Department of Natural Resource, 2008).

We assume that the natural dispersal of the species is inevitable and that the rate of spread is exogenous. This assumption allows us to focus on the problem of detecting sub-populations far from the main front. In reality, this is a key simplification because--through vigilance and aggressive control--it is in fact possible to slow or reverse the spread of the advancing front. Thus, the rate of spread may itself be a variable of choice. It is in the context of managing the rate of spread that we see one example of optimal spatial allocation of detection effort in the literature. Sharov, Liebhold, & Roberts (1998) have studied how to distribute detection intensity in the transition zone between infested and uninfested areas so as to retard or reverse the spread of the main front in the most cost-effective manner. They find that that it is best to have the highest intensity of traps closest to the front, with diminishing intensity as the distance from the front increases.

In our paper, we look at the problem of optimal detection over space from a different perspective. In particular, we focus on transport of species beyond the transition zone modeled by Sharov et al. (1998). Our approach recognizes that the duration of management of sub-populations is constrained by the amount of time remaining before the main front arrives. Locations close to the front have less time remaining than locations that are more distant. These differences imply different levels of potential cost savings from early detection; in particular, shorter management horizons translate into lower cost savings from intervention. The optimal intensity of detection effort varies over space along with this variation in cost savings due to management.

In the next section of this paper, we lay out our model of optimal detection. Section 3 discusses our simulation results and Section 4 concludes.

2.2 Model Development

We envision a situation where an invader is spreading throughout a landscape; this landscape is divided into an infested and an uninfested zone. The boundary separating the infested and uninfested zones, known as the population front, advances at a constant rate of speed which could either be the natural rate of spread or some controlled rate. Human dispersal can carry sub-populations from the infested zone into the uninfested zone through, for example, unintentional transport of egg masses or larvae on vehicles or in firewood. As in Mehta et al. (2007) we assume that we know the population size of these sub-populations and that control of the sub-populations is only possible once they are detected.

Our modeling strategy is to consider the management problem at an arbitrary distance from the main front, d . With the rate of advance of the main front equal to v , the time remaining before the main front engulfs the arbitrary point in the landscape, T_{\max} , is equal to d/v . See Figure 2.1. Each point in the uninfested zone is characterized by a different ending date to the management problem. After we develop the model for a single distance, we examine how the problem changes when distance from the front increases.

The management problem is to choose detection effort (E) to minimize total costs that accrue before the main front arrives and management is no longer possible. There are three components to total costs: the direct cost of detection effort ($C_1(E)$), the cost of

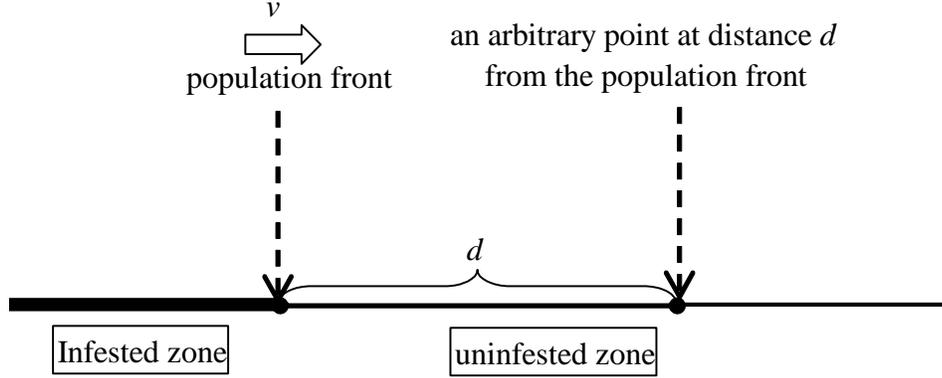


Figure 2.1: Distance From the Front and Time Available for Management

Note: At an arbitrary point in the uninfested zone at distance d from the population front, with the rate of advance of the main front equal to v , the time remaining before the main front engulfs the arbitrary point in the landscape, T_{\max} , is equal to d/v .

damage caused by the pest before detection occurs ($C_2(E)$), and the costs of both pest control and pest damage that occur after detection ($C_3(E)$). We assume that detection effort (E) reduces the date of detection (τ) in a deterministic manner.

2.2.1 Cost of Detection Effort

The first cost component is the direct cost of detection effort. We choose a quadratic specification for the instantaneous cost of direct detection effort (bE^2) to reflect factors such as overtime pay, etc., for more intense detection programs. The present value of the discounted stream of detection effort costs until detection occurs is

$$C_1(E) \equiv \int_0^{\tau(E)} e^{-rt} bE^2 dt = \frac{bE^2}{r} (1 - e^{-r\tau(E)}) \quad (2.1)$$

where r is the discount rate. The marginal direct cost of detection effort is the derivative of this expression:

$$MC_1 = \frac{dC_1}{dE} = \underbrace{\frac{2bE}{r} (1 - e^{-r\tau(E)})}_{+} + \underbrace{bE^2 e^{-r\tau(E)} \frac{d\tau}{dE}}_{-}$$

The first term of this expression is positive, reflecting the additional cost of instantaneous effort throughout the detection period. However, increased detection effort reduces the detection period (so that $d\tau/dE < 0$), making the second term negative and conferring a cost savings. The strength of this effect depends on the function relating detection effort to the date of detection.

2.2.2 Cost of Pest Damage

The second cost term is the present value of the discounted stream of damages caused by the pest before detection occurs:

$$C_2(E) = \int_0^{\tau(E)} e^{-rt} p x_0 e^{at} dt = \frac{p x_0}{(a-r)} (e^{(a-r)\tau(E)} - 1) \quad (2.2)$$

where x_0 is the stock level at the beginning of the time horizon, p is the cost per unit of the pest, and a is the exponential growth rate of the pest. The marginal cost of pre-detection pest damage with an increase in detection effort is the derivative of this expression:

$$MC_2 = \frac{dC_2}{dE} = \underbrace{p x_0 e^{(a-r)\tau(E)} \frac{d\tau}{dE}}_{-}$$

This term is negative, representing a cost savings, because an increase in E reduces the date of detection, shrinking the period of time in which an uncontrolled pest causes damage.

2.2.3 Post-Detection Costs

Once detection occurs, management can begin. Intuition would suggest that earlier detection would lead to lower costs in the management phase because the pest is discovered at a smaller, more manageable stage. One approach to modeling this idea is to embed a simple increasing function of the stock size upon detection. This is the approach followed by Mehta et al. (2007). Here, we follow an alternative approach: we solve for the dynamically optimal path of pest control within the time constraint imposed by the advancing front of the pest. This allows us to determine the explicit relationship between the starting stock and the optimized cost. In addition, we can examine the relationship between the duration of the management horizon and optimized cost. Earlier detection leads to a longer possible management horizon which may, in fact, lead to higher costs in the management phase. And, since the duration of the management horizon is determined by location relative to the advancing front, this modeling approach permits us to see how location affects the determination of optimal detection effort.

Starting at the date of detection τ , the management problem is to minimize the present value of the stream of discounted damage costs and control costs given the size of the stock when it is detected:

$$\min \int_{\tau}^T e^{-r(t-\tau)}(px(t) + cR(t)^2)dt \quad (2.3)$$

subject to $\dot{x}(t) = ax(t) - R(t)$

$$x(\tau) = x_\tau$$

$$T - \tau \leq T_{\max} - \tau$$

$$x(T) \geq 0$$

The exponential pest growth function is now modified to include removals, $R(t)$. The cost of removals is quadratic so that the instantaneous cost of pest removal is $cR(t)^2$. The time horizon is constrained be less than or equal to $T_{\max} - \tau$, where T_{\max} is the date at which the main front arrives and the sub-population becomes subsumed in the main population.

We solve this problem using optimal control theory to find the optimal paths of removals ($R^*(t)$) and stock ($x^*(t)$). We accommodate the inequality restrictions on the terminal stock and terminal time and find out which of the terminal conditions are binding. We solve for the optimal ending time and ending state, subject to the inequality constraints, and the corresponding optimal paths of removal and stock. We then insert the optimal paths of the stock and removal into the integral (2.3) to get an expression for the present value of the discounted optimized stream of damage and removal costs. This value function is a function of the starting stock level ($x(\tau)$) and the amount of time in the horizon ($T_{\max} - \tau$):

$$V(x(\tau), T_{\max} - \tau) = \int_{\tau}^{T^*} e^{-r(t-\tau)}(px^*(t) + cR^*(t)^2)dt. \quad (2.4)$$

Note that the terminal time, T^* , is optimally chosen and may be different than T_{\max} .

Details of our derivation can be found in Appendix 1.

We are now in a position to write down our third cost term. The post-detection cost is the cost as summarized by the value function, discounted back from the date of detection τ to the beginning of the time horizon, 0:

$$C_3(E) \equiv e^{-r\tau(E)} V(x(\tau(E)), T_{\max} - \tau(E)). \quad (2.5)$$

The marginal effect of detection effort on post-detection costs is the derivative of this expression:

$$MC_3 = \frac{dC_3}{dE} = \underbrace{-e^{-r\tau} rV \frac{d\tau}{dE}}_{+} + e^{-r\tau} \frac{\partial V}{\partial x(\tau)} \underbrace{\frac{dx(\tau)}{d\tau} \frac{d\tau}{dE}}_{-} + e^{-r\tau} \frac{\partial V}{\partial (T_{\max} - \tau)} \underbrace{\frac{d(T_{\max} - \tau)}{d\tau} \frac{d\tau}{dE}}_{+}.$$

An increase in detection effort leads to an earlier detection date ($\frac{d\tau}{dE} < 0$), and this earlier detection date has three distinct effects. The first effect is that a greater level of detection effort increases post-detection costs simply because the costs, V , must be borne sooner and will therefore be discounted less. The second effect is that the population is smaller upon detection with increased detection effort. A smaller population size affects costs through the value function. If, as intuition suggests, $\frac{\partial V}{\partial x(\tau)} > 0$, this second effect is negative so that an increase in detection effort decreases costs. The third effect is that the management horizon ($T_{\max} - \tau$) is longer with increased detection effort. Depending on the solution to the dynamic optimization problem for a particular detection date, this change in the management horizon may also affect the value function. If, for example, $\frac{\partial V}{\partial (T_{\max} - \tau)} > 0$, an increase in detection effort would increase costs through this third effect. If, on the other hand, $\frac{\partial V}{\partial (T_{\max} - \tau)} = 0$, changes in the time horizon would have no marginal effect on cost.

2.2.4 Characterizing Optimal Detection Effort

The optimal detection effort level (E^*) is the solution to the following cost minimization problem:

$$\min_E C_1(E) + C_2(E) + C_3(E). \quad (2.6)$$

Equivalently, unless there is a corner solution so that optimal detection effort is equal to zero, E^* is the level at which marginal costs from all sources are equal to zero (i.e., where $MC_1(E) + MC_2(E) + MC_3(E) = 0$). This optimality condition summarizes the economic tradeoff in choosing a detection effort level and can be useful for interpretation. However, we cannot solve the problem with a direct application of the optimality condition. Instead, we evaluate costs for a range of detection effort levels along the lines of the following example. Figure 2.2 is a graphical depiction of the population of the pest at a given distance from the front from the perspective of time 0. Different detection effort levels correspond with different dates of detection, so that earlier dates of detection like τ_1 are associated with high effort levels and late dates of detection like τ_3 are associated with low effort levels. The dynamically optimal paths of the pest once detection occurs are represented by the paths that depart from the no-management stock path at these alternative dates of detection. We summarize the cost of these paths in our value function that depends on detection effort through the date of detection, $\tau(E)$, and the stock level at the time of detection, $x(\tau(E))$.

We observe that, with little detection effort, a late date of detection, and a high stock level at the start of management (Path A), the terminal time constraint ($T - \tau \leq T_{max} - \tau$) is binding. In this case, application of the free ending state transversality condition ($\lambda(T) = 0$) reveals that it is optimal to have a positive stock at the end of the

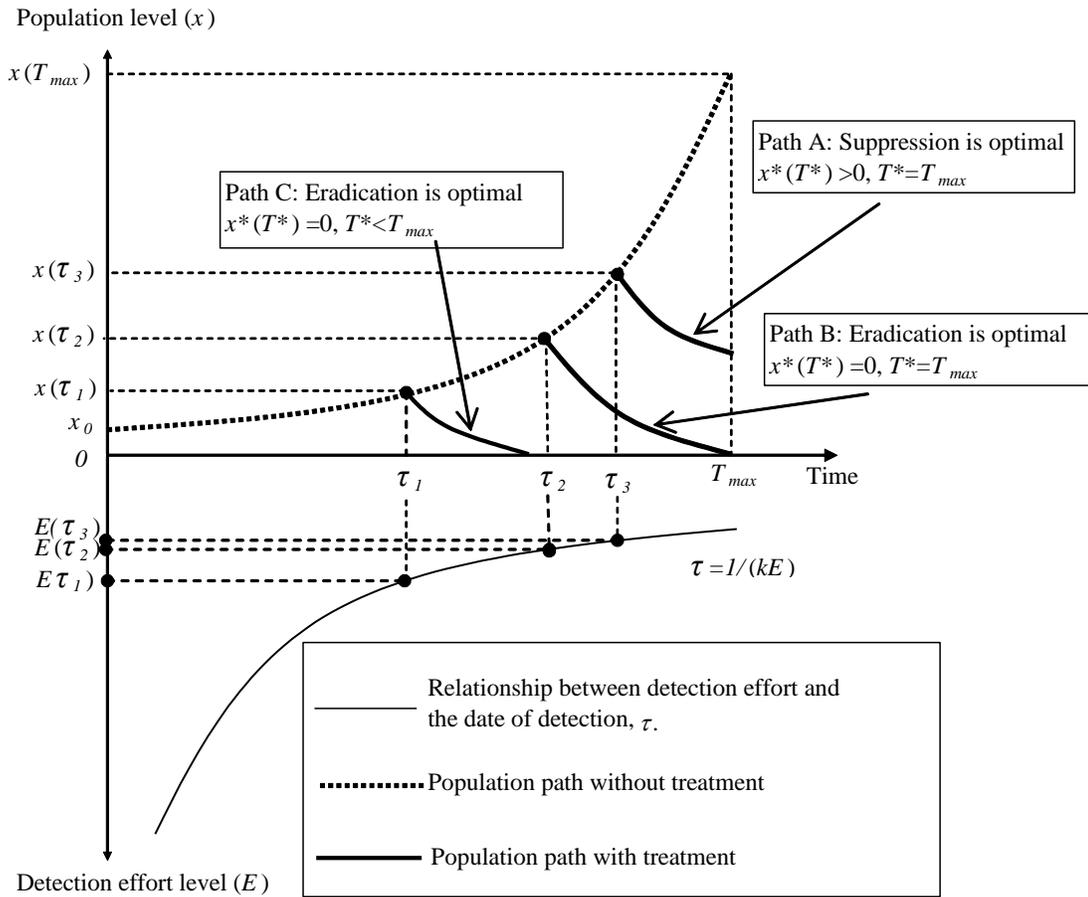


Figure 2.2: Pest Population Paths with Alternative Detection Effort Levels

horizon. This path represents a “suppression” strategy. On the other hand, with a high level of detection effort, an early detection date, and a low stock level at the start of management (Path C), the optimal ending date of the management horizon may occur before the front arrives at T_{\max} . The application of the transversality condition for a free ending time ($H(T^*) = 0$) indicates that the population be driven to zero before T_{\max} . Path C represents an “eradication” strategy where the inequality constraint on the ending stock ($x(T) \geq 0$) is binding. The intermediate case is one in which there is moderate detection

effort, an intermediate date of detection, and a medium size population level (Path B). Here, the sub-population is driven to zero just as the main front arrives so that both constraints are just binding.

The solution procedure requires that the dynamic optimization problem be solved for each possible starting time—and this involves evaluating the present value of the stream of costs with alternative terminal conditions to find out which terminal constraints are binding. After the value function for each starting time is calculated, total costs for a range of possible detection times can be computed. From these computations, the detection effort level associated with the minimum cost can be determined.

2.2.5 Multiple Distances from the Front

We now turn to determining the effects of having a longer potential period of time in which the species can be managed: how does optimal detection effort change further from the front, where the pest is due to arrive at a later date? In particular, what can we say about the effect of changes in $T_{\max} - \tau$ on the value function? While a direct approach—taking the derivative of the value function (Equation (A1.11)) with respect to the time horizon—yields an expression that is too complicated to sign, it is fruitful to consider effects of increasing detection effort on the starting stock level, $x(\tau)$. In Appendix 1, we show that $\frac{\partial V}{\partial x(\tau)} > 0$ in the suppression zone. Furthermore, an increase in the available time augments the value of starting with a reduced stock level ($\frac{\partial^2 V}{\partial x(\tau) \partial (T_{\max} - \tau)} > 0$ (A1.9)). This suggests that the cost savings from having a lower starting stock level due to increased detection effort are magnified with a longer time

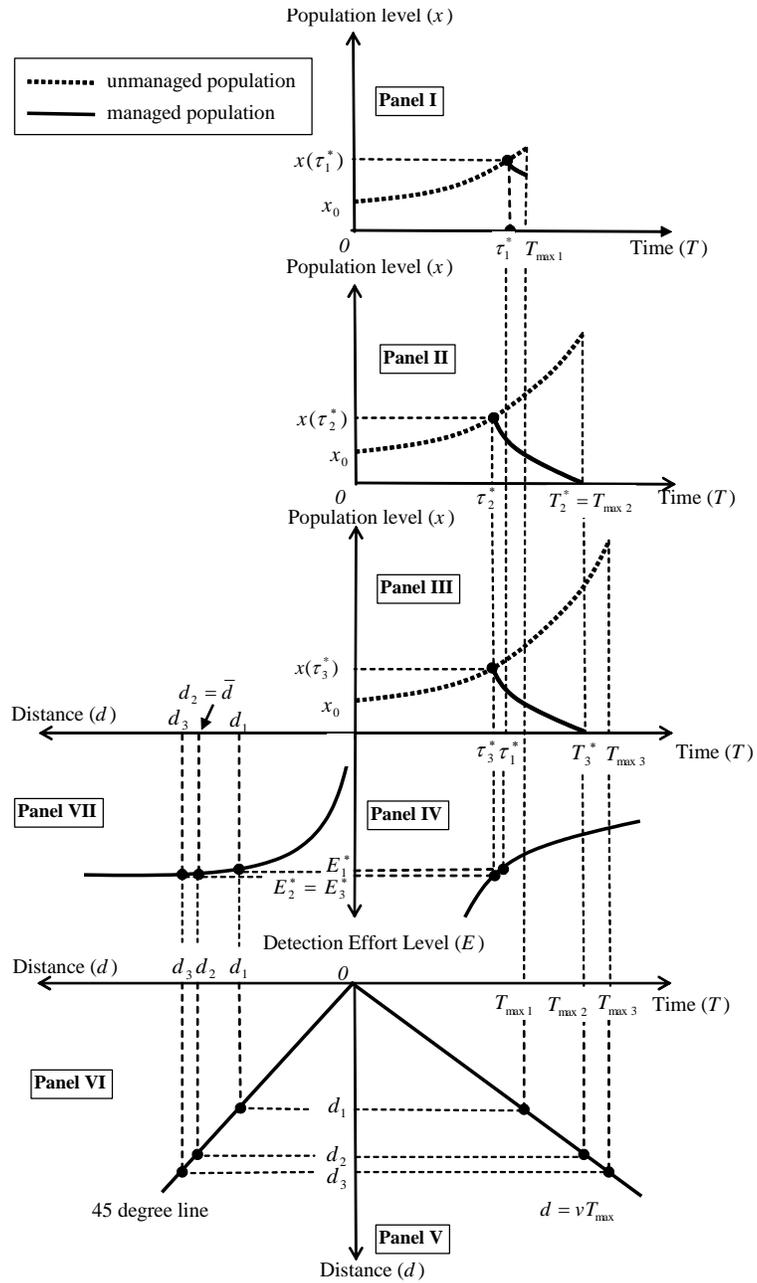


Figure 2.3: Relationship between the Management Horizon (T_{\max}) and Optimal Detection Effort

horizon. If this effect is strong, optimal detection effort should *increase* with distance from the front.

When eradication is optimal, the stock is driven to zero before the end of the horizon and the terminal time constraint (that $T^* - \tau < T_{\max} - \tau$) is not binding. Therefore, increases in the time horizon ($T_{\max} - \tau$) have no effect on the value function (either directly through $\frac{\partial V}{\partial (T_{\max} - \tau)}$ or indirectly through a stock effect) and consequently no effect on the optimal detection effort level. Figure 2.3 illustrates how increasing the distance from the front affects optimal detection effort.

Unlike Figure 2.2, where we show the dynamically optimal path from a variety of detection dates, we now show—for each ending date—the paths that correspond to the results of our cost minimization problem (2.6). So, for example, Panel I shows a short time horizon where the ending time is $T_{\max 1}$. In this case, it is optimal to detect the pest in τ_1^* and pursue a suppression strategy. Panel II shows that, for a longer time horizon ($T_{\max 2}$), it is optimal to eradicate the species at the very moment that the horizon comes to an end. In Panel III, we observe that the optimal strategy for an even longer time horizon is to eradicate the species well before the end of the horizon. Note that the ending dates ($T_{\max 1}, T_{\max 2}, T_{\max 3}$) correspond to different distances from the front (d_1, d_2, d_3), via the equation $T_{\max} = \frac{d}{v}$ shown in Panel V, and translated into distances on the horizontal axis in Panel VI via the 45 degree line in Panel V. Also, the optimal dates of detection ($\tau_1^*, \tau_2^*, \tau_3^*$) are associated with their corresponding optimal detection effort levels (E_1^*, E_2^*, E_3^*) through a function relating the date of detection to the level of detection effort in Panel IV.

Table 2.1: Benchmark Parameter Values

Parameter	Meaning	Value
a	Exponential growth rate	4×0.1^2
b	Detection cost parameter	5×0.1^3
c	Treatment cost parameter	0.1^7
r	Discount rate	0.1
p	Damage cost parameter	2×0.1^4
k	Detectability coefficient	10^4
x_0	Starting population size	10^5

Note: See Appendix 2 for sources.

Panel VI shows a plausible pattern relating E^* to distance. Optimal detection effort increases with distance from the front when suppression is an optimal strategy. In this zone, the dominant effect of changes in the potential time horizon is to encourage more spending on detection because it becomes more valuable to find the pest at a manageable size when more time remains. When eradication becomes the optimal strategy (here, at distance d_2), further increases in distance have no effect on the optimal detection effort level. This is because further increases in distance increase the time horizon, but it is not optimal to take advantage of additional time: because the time constraint is not binding, loosening it has no effect on the optimal decision--it is always optimal to find the species at time τ_2^* and eradicate. The reductions in marginal cost from finding the pest even sooner are outweighed by increases in marginal cost from expending further detection effort.

2.3 Simulations

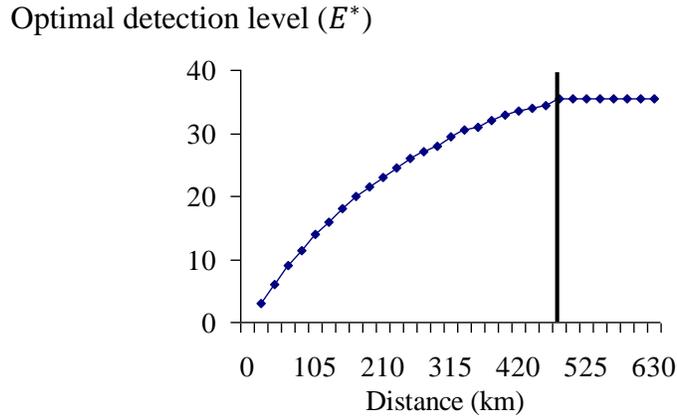


Figure 2.4: Relationship between the Distance from the Population Front and Optimal Detection Effort Levels with Base Case Parameter Values

For our simulations, we adapt the probabilistic model of Mehta et al. (2007) by using the expected date of detection so that $\tau = \frac{1}{kE}$, where $k > 0$ is the “detectability” coefficient. A higher k reduces the date of detection, as does a higher E . In our simulations, we use parameters drawn from the gypsy moth case as described in Appendix 2 and summarized in Table 2.1. We evaluate a range of possible detection effort levels for a range of possible ending dates to determine the relationship we report in this section. We then convert the ending date to a distance measure using a rate of 21 km/year. This is the maximum rate of gypsy moth spread found in the literature (Liebhold et al., 1992). Alternative rates of spread would only affect the conversion between the maximum ending date and distance through the equation $d = vT_{\max}$, changing the scale of the horizontal axis in the figures.

Table 2.2: Sensitivity Analysis Parameters

Case	Parameter	x_0	a	p	C	B	k	r
1	Benchmark	10^5	4×0.1^2	1.5×0.1^4	0.1^7	5×0.1^3	10^4	0.1
2	Starting population size(x_0)	1.2×10^5	4×0.1^2	1.5×0.1^4	0.1^7	5×0.1^3	10^4	0.1
3	Exponential growth rate (a)	10^5	1.2×0.1	1.5×0.1^4	0.1^7	5×0.1^3	10^4	0.1
4	Damage cost parameter (p)	10^5	4×0.1^2	3×0.1^4	0.1^7	5×0.1^3	10^4	0.1
5	Treatment cost parameter (c)	10^5	4×0.1^2	1.5×0.1^4	1.25×0.1^7	5×0.1^3	10^4	0.1
6	Detectability coefficient (k)	10^5	4×0.1^2	1.5×0.1^4	0.1^7	3×0.1^2	10^4	0.1
7	Discount rate (r)	10^5	4×0.1^2	1.5×0.1^4	0.1^7	5×0.1^3	10^4	0.15

Figure 2.4 shows our results for the base case: optimal detection effort increases with the distance from the front until a plateau is reached. This pattern echoes the pattern in Panel VII of Figure 2.3. For these parameters, the plateau is reached at approximately 500 km. from the front.

2.3.1 Sensitivity Analysis

Our sensitivity analysis reveals how changes in each parameter affect our benchmark case results. One by one, we increase each parameter to the values indicated in Table 2.2 to find out how and whether: (1) the distance from the front at which eradication becomes the optimal policy (\bar{d}) is affected by a change in the parameter value and (2) the optimal detection effort level is affected at any given distance from the front. We are also interested in learning whether the pattern identified in the base case—that detection effort increases until a plateau is reached—is repeated when parameters are changed. These results are presented in Figure 2.6 and in Table 2.3.

Observe first that the pattern of the base case is indeed repeated: optimal detection effort does increase with distance from the front, and then levels off, in all of these

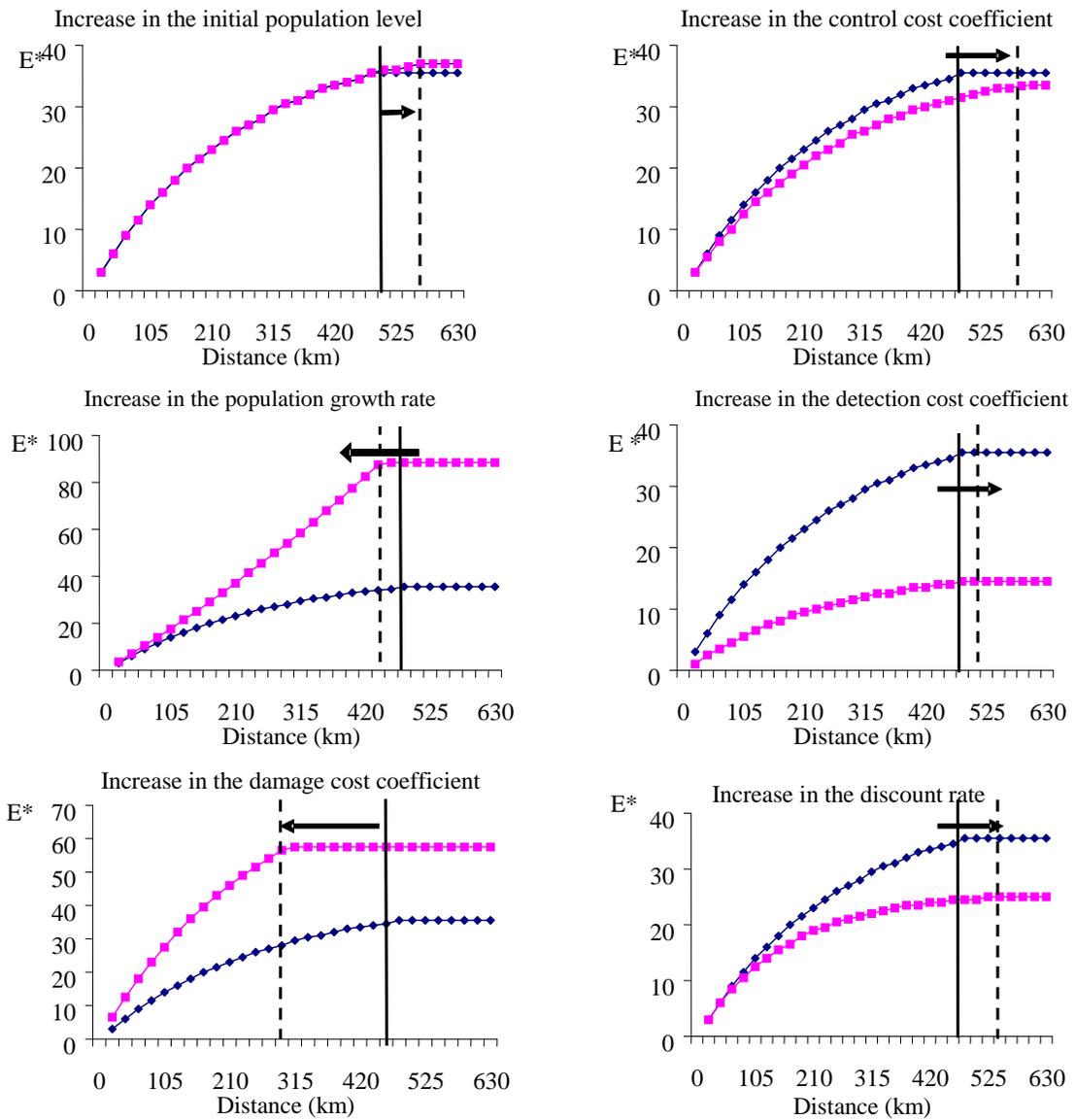


Figure 2.5: Results of the Sensitivity Analysis

alternative scenarios. While this is not proof that this pattern will be repeated with *every* possible parameter configuration, it does demonstrate the robustness of this general result

Table 2.3: Summary of the Sensitivity Analysis

Parameter	Optimal Detection Effort in suppression zone	Optimal Detection Effort in eradication zone	Distance from the front at which the eradication zone begins
Starting stock level (x_0)	0	+	+
Pest growth rate (a)	+	+	-
Damage coefficient (p)	+	+	-
Control cost (c)	-	-	+
Detection cost (b)	-	-	+
Discount rate (r)	-	-	+

for a healthy range of parameter values. We now turn to the effects of changing individual parameters.

Case 1: Starting stock level An increase in the starting stock level increases \bar{d} , the distance separating the suppression zone from the eradication zone. The detection effort level remains the same in the suppression zone. This surprising result is confirmed by applying the implicit function theorem to the marginal cost condition to find the derivative of E^* with respect to x_0 . In the suppression zone, this derivative is equal to zero (See Appendix 1, equation (A1.11)). Optimal detection effort does increase with x_0 in the eradication zone.

Case 2: Pest growth rate An increase in the pest growth rate reduces \bar{d} , making eradication an optimal strategy in more locations. An increase in this parameter also increases the optimal detection effort level for any given distance from the front, indicating the importance of finding the population at a small size.

Case 3: *Pest damage coefficient* As with an increase in the pest growth rate, an increase in the pest damage coefficient reduces \bar{d} . Optimal detection effort also increases for every distance from the front.

Case 4: *Control cost coefficient* An increase in the control cost coefficient increases \bar{d} due to the high cost of reducing the species' population size. It also reduces optimal detection effort for any given distance from the front.

Case 5: *Detection cost coefficient* As one might expect, an increase in the detection effort cost coefficient reduces the optimal detection effort level for any distance from the front. The borderline between the suppression zone and the eradication zone occurs further from the front.

Case 6: *Discount rate* An increase in the discount rate increases \bar{d} , the distance separating the suppression zone from the eradication zone. An increase in the discount rate also reduces optimal detection effort everywhere.

2.4. Conclusions

In this paper, we show that the distance from an advancing front has an influence over how much effort should be devoted to finding sub-populations of a pest that emerge ahead of that front. A greater distance from the front confers a longer span of time within which to enjoy the cost savings that can be derived from a pest management program and so may justify a more aggressive approach to finding and treating the pest. Because we embed a dynamically optimal pest management strategy into the problem of determining the optimal level of detection effort, we get the unexpected result that, at some point, it is no longer optimal to increase detection effort. This is because eradication becomes the

optimal strategy and the time constraint imposed by the date of arrival of the advancing front no longer has an effect on the optimization problem.

Our paper provides a foundation for future work that might consider a more complex setting. Here, we make the simplifying assumption that the starting population size is at the same (known) level at all distances from the front. It would be more realistic to introduce a continuous stochastic arrival process in which the arrival rate depends on the factors guiding human dispersal. We could also imagine endogenizing the rate at which the front advances, combining the two purposes of detection effort—controlling the rate of advance and detecting and managing sub-populations beyond the transition zone.

Chapter 3

Optimal Strategies for the Surveillance and Control of Forest Pathogens

3.1 Introduction

Invasive fungal pathogens have had widespread effects on forests and have increasingly threatened tree species nationwide. Chestnut blight virtually eliminated native chestnuts, and now Dutch elm disease and oak wilt are spreading throughout forested areas (Hansen, 2008; Loo, 2009). Because mature urban trees are particularly valuable to urban residents, protection of these trees from threats posed by invasive pests and pathogens is a high priority for urban foresters (Holmes, Aukema, Holle, Liebhold, & Sills, 2009). For fungal pathogens, the most effective means of controlling spread is the detection and removal of infected trees. In the case of oak wilt, for example, trees killed by oak wilt produce spores in the following spring, and these spores are a source of pathogen for new infections. However, budgets available for detection and removal are typically very limited and governments must decide how to allocate those limited funds among sites with different features. For example, should locations with low numbers of infected trees be given priority since these locations can more easily become disease-free, or should locations with low inspection costs be given precedence because more diseased trees can be identified with the same budget? In addition, governments face a tradeoff

between search and control activities. Greater spending for surveillance enables governments to identify more infected trees, but increased surveillance limits the budget remaining for removal of infected trees.

In this paper, we focus on the relationship between optimal detection and control strategies when budgets are limited. We use models to inform decisions about sites to select for surveillance and removal of infected trees. These models provide information about cost-efficient tradeoffs between detection and control activities. Moreover, they describe how and to what extent optimal site selection and the budget allocation between the surveillance and control of forest pathogens are sensitive to chosen parameter values. In building our model, we draw from the site selection literature, which is generally aimed at choosing sites to preserve for the purpose of maintaining biodiversity (e.g., Margules, Nicholls, & Pressey, 1988; Church, Stoms, & Davis, 1996; Stokland, 1997; Ando, Camm, Polasky, & Solow, 1998; Polasky, Camm, & Garber-Yonts, 2001; Snyder et al., 2004). In our case, we are choosing sites on which to focus inspection activities in order to remove trees found to be infected. We formulate a two-stage site selection model with a budget constraint. A mixed integer formulation allows us to solve the model by determining the level of treatment intensities as well as selecting sites for surveillance and treatment.

In recent years, economists have begun to be interested in how best to manage invasive pests. The literature generated by this interest focuses on two main policy questions. The first question considers optimal trade policy between importing and exporting countries for the prevention of new species introductions. The central tradeoff

is between the damage foregone that would otherwise have been caused by an invader and the welfare lost by implementing two policy tools: a tariff added to reduce trade flows and the inspection of goods at ports of entry (e.g., McAusland & Costello, 2003; Costello & McAusland, 2004; Olson & Roy, 2005). The second set of questions considers optimal eradication or suppression policies to control established invasive species. Olson and Roy (2002) and Olson and Roy (2008) set up and solve a stochastic dynamic optimization problem and show the conditions under which an eradication strategy becomes dominated by a suppression strategy. The central tradeoff here is between costs of control and benefits in terms of foregone damage. Several authors have examined these questions in a spatially explicit setting, including Sharov and Liebhold (1998) who formulate a dynamic optimization approach to solving for an optimal barrier zone.

Recently Mehta et al. (2007) incorporated the surveillance stage into the model of the invasive species management. They point out the importance of timing: eradication or suppression of invasive species populations can be conducted only after populations are found through surveillance activities. This implies that damage caused by invasive species continues to occur during the surveillance phase. Higher intensities of surveillance enable earlier population control and less environmental damage but cost more. This is the central tradeoff involved in determining the optimal intensity of surveillance. While Mehta et al. (2007) focused on the timing issue raised by the order of management activities, this study focuses on location: the eradication or suppression of the species is possible only on the sites where surveillance is conducted. This implies a

set of important tradeoffs that are considered when sites are selected for surveillance and treatment. When more sites are selected for surveillance, more sites are eligible for treatment; however, when a greater proportion of the budget is spent on surveillance, fewer resources are available for suppression and containment. To explore this, we build a two stage mixed integer programming model in which the objective is to minimize the expected number of newly infected trees. In the first stage, the manager chooses sites to be surveyed from all the sites in the management area. In the second stage, the manager determines the number of infested hosts to be removed within the sites surveyed in the first stage. This model is adapted from Snyder et al. (2004) who develop a similar model for the purpose of maximizing the expected number of species represented in sites selected for preservation.

In Section 3.2, we formulate our mixed integer programming model. In Section 3.3, we characterize the solution of the problem for a simple two site case. This case provides intuition for the more complex model in our application. In Section 3.4, we apply our model to the case of oak wilt management in Anoka County Minnesota. Section 3.5 concludes with a discussion of the model and its implications.

3.2 Model Development

We develop a model of a single forest management area composed of a number of distinct sites. The manager's goal is to control the invasive pathogen in the management area, and the overarching objective is to minimize the number of newly infected trees in a

given year. The choice variables are (1) yes-no variables for each site indicating whether surveillance is undertaken in the inspection stage, and (2) the number of infected trees removed in each site in the treatment stage. We have two constraints. First, the budget constraint ensures that the total costs of surveillance and treatment do not exceed the budget level. Second, we ensure that treatment cannot occur in a given site unless the site has been inspected: the number of trees removed in each site is bounded above by the number of trees that have been identified as infected. We introduce this constraint because, for pathogens, the detection process of pathogen management needs to exactly identify an infected tree in each location. Treatment for pathogens is not broadcast (such as aerial spraying), but instead is applied to particular trees. Forest managers must know which trees are infected before removing them.

Forest managers must conduct detailed surveillance before treatment can occur, but they may have some idea of the proportion of infected trees on each site. We consider two characterizations of their information set: one, that the proportion of infected trees on each site is known exactly, even though infected trees are not specifically identified; and two, that the manager does not know the proportion of infected trees, but has some sense of the proportion based on a small sample of trees in each site. We first develop the model with a known proportion, and then generalize by incorporating a distribution of possible infection proportions for each site that reflects the manager's uncertain knowledge about this key piece of information.

3.2.1 A Forest Pathogen Growth Model

Our forest landscape is composed of distinct sites, each containing N_j host trees. A proportion, θ_j , of host trees in any given site j are infected by an invasive pathogen. The number of infected trees (I_j) and healthy (H_j) trees at the beginning of the period can be written:

$$I_j \equiv \theta_j N_j$$

$$H_j \equiv (1 - \theta_j) N_j$$

In the absence of management, new infections, Q_j , are an increasing function of the number of infected trees up to the point where all healthy trees are infected:

$$Q_j = \min[H_j, g_j I_j].$$

Growth rates for pathogens (g_j) may be site-specific because growth may depend on characteristics such as tree density or soil type that vary over the landscape.

Managers can slow or stop infections by removing infected trees. The number of newly infected trees, Q_j , on site j is a function of how many infected trees remain after the manager removes R_j trees from the site¹:

$$Q_j = \min[H_j, g_j (I_j - R_j)] \tag{3.1}$$

¹ With the oak wilt pathogen, infective spore mats are produced under the bark of dead trees in the spring after trees are killed by the pathogen. Only dead trees left standing are a source of pathogen for new infections. (Juzwik, Harrington, MacDonald, & Appel, 2008; Juzwik, 2009)

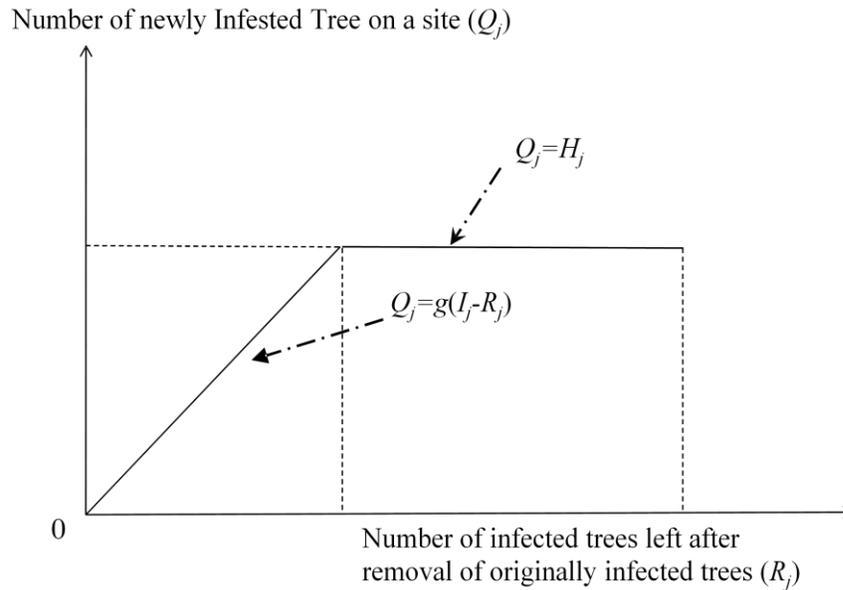


Figure 3.1: Spread of Infection

Figure 3.1 illustrates the relationship between new infections and the number of trees remaining after removal of infected trees. The number of new infections, Q_j , is on the vertical axis and the number of infected trees left after removal, $I_j - R_j$, is on the horizontal axis. At the origin, no infective trees remain after removal and so no new infections occur. The number of newly infected trees increases linearly as the number infected trees remaining after removal ($I_j - R_j$) increases (and the number of trees removed is reduced). For some combinations of g_j , H_j , and I_j , a plateau is reached where all the healthy trees on the site (H_j) become infected. Along this plateau, the number of infected trees remaining after removal is high enough to infect all the remaining healthy trees. Further increases in infective trees can cause no additional damage because all

healthy trees are already infected. We define M_j as the number of trees removed at the kink in the function; $I_j - M_j$ is the number of infective trees that are sufficient to infect all the trees on a site. We solve for the location of the start of the plateau by setting $H_j = g_j(I_j - M_j)$ and solving for M_j to get:

$$M_j = I_j - \frac{H_j}{g_j}. \quad (3.2)$$

From a management perspective, then, M_j represents the minimum number of trees that must be removed to preserve at least one healthy tree. If removal is less than this minimum, removal will be ineffective in saving any trees from infection: the number of newly infected trees Q_j remains H_j for $R_j \leq M_j$. Only when $R_j > M_j$ will the number of newly infected trees be reduced. It is possible that no plateau exists on a site; there will be no plateau when $I_j - \frac{H_j}{g_j} < 0$. This combination of parameters occurs with a low growth rate, a low number of infected trees, and abundant numbers of healthy trees. In cases without a plateau, any amount of removal will be effective in lowering the number of newly infected trees.

3.2.2 A Management Model with Known Proportions of Infected Trees

The management area consists of J sites with a known number of possible host trees, N_j , on each site, $j = 1, \dots, J$. The manager's objective is to minimize the number of newly infected trees in the entire management area:

$$\min \sum_{j=1}^J Q_j = \min \sum_{j=1}^J \min[H_j, g_j(I_j - R_j)]. \quad (3.3)$$

The manager has two sets of choice variables. The first is a set of binary variables (X_j) indicating whether a site is inspected ($X_j = 1$) or not ($X_j = 0$). The second is a set of continuous variables, R_j : the number of infected trees removed in each site. The minimization is subject to the constraint that tree removal can only occur in sites that have been inspected by the manager:

$$0 \leq R_j \leq I_j X_j \quad (3.4)$$

and the budget constraint:

$$\sum_{j=1}^J c_1 N_j X_j + \sum_{j=1}^J c_2 R_j \leq B \quad (3.5)$$

where c_1 is the per-tree cost of inspection in site j , c_2 is the per-tree cost of removal in site j , and B is the size of the budget. This budget constraint reflects the notion that, once a site is selected for inspection, all trees in that site must be examined and evaluated for the presence of the disease. The assumption of comprehensive inspection creates a fixed cost of site inspection that is an increasing function of the total number of trees on the site.

3.2.3. A Management Model When the Proportions of Infected Trees are Unknown

3.2.3.1 Forming Beliefs About the Number of Infected Trees on Each Site

If the true proportion of infected trees in the population of trees in a given site, θ_j , is known, the number of infected trees could be calculated as the product of θ_j and the number of trees on that site, N_j . If this proportion is not known, a forest manager can estimate θ_j by inspecting a small random sample of trees on each site and finding the sample proportion of infected trees, θ'_j . While the sample proportion provides information about the true proportion, the true proportion remains unknown. However, the forest manager can formulate a belief about the true proportion of infected trees based on the sample. We envision that the manager formulates a belief—in the Bayesian sense—about the distribution of the true parameter θ_j . This distribution translates into a belief about the distribution of the number of infected trees in each site when the parameter is multiplied by the number of trees on the site.

Bernoulli trials result in successes (1) or failures (0), according to some underlying probability. In this context, a “success” is finding an infected tree and a “failure” is finding a healthy tree. The beta distribution characterizes the distribution of the true probability of success (here, the true proportion of infected trees) based on the number of successes and failures in a sample. In particular, the probability density function of θ_j , defined over an compact interval [0,1] is characterized by two site specific parameters α_j and β_j :

$$f(\theta_j; \alpha_j, \beta_j) = \frac{\Gamma(\alpha_j + \beta_j)}{\Gamma(\alpha_j) + \Gamma(\beta_j)} \theta_j^{\alpha_j-1} (1 - \theta_j)^{\beta_j-1}$$

where $(\alpha_j, \beta_j) = (n\theta'_j, n(1 - \theta'_j))$ and $\Gamma(\cdot)$ is the probability density function of gamma distribution. The number of infected trees in a sample is α_j , and the number of healthy trees in a sample is β_j . In this conception of the problem as a one period model, the

manager formulates a single belief about the distribution based on a single set of samples in each site. While this structure is amenable to a Bayesian updating approach in which new information can be incorporated and new beliefs formed, we confine ourselves to a single episode of belief formation.

3.2.3.2 Approximating Continuous Distributions with Sets of Discrete Scenarios

After formulating beliefs about distribution of the true proportion of infected trees on each site j , a forest manager can approximate the true distribution by generating a set of scenarios of infection states in the management area. A scenario of infection in the management area is a vector $\theta = (\theta_1, \dots, \theta_j, \dots, \theta_J)$ of proportions of infected trees in all J sites. Each element θ_j of the vector θ is randomly drawn from $[0,1]$ with the belief $f(\theta_j; \alpha_j, \beta_j)$. A manager randomly and independently draws S vectors to generate a set of S scenarios. Let $\theta_j(s)$ denote the s 'th draw of θ_j for site j and $\theta = (\theta_1(s), \dots, \theta_j(s), \dots, \theta_J(s))$ denote the s 'th scenario for all J sites. Together, the set of scenarios $\Theta = (\theta(1), \dots, \theta(s), \dots, \theta(S))$ reflects a range of possible infection proportions in all J sites according to the distribution characterized by site-specific parameters. We assume that the manager considers that each scenario $\theta(s)$ is equally likely: $\theta(s)$ occurs with probability $1/S$.

3.2.3.3 The Management Model

When the proportion of infected trees in each site is unknown, and the manager formulates a belief about the proportion based on a sample, the objective of the manager

is to minimize the expected number of newly infected trees. This expected number of newly infected trees in the management area is the sum over S of the products of the realization of the random variable and their associated probabilities. In this case, since each scenario is equally likely with probability $1/S$, the management problem becomes:

$$\min \frac{1}{S} \sum_{s=1}^S \sum_{j=1}^J Q_j(s).$$

As before, removal can only occur in sites that have been inspected, and this has to be true for every scenario. In addition, the budget constraint must also hold for each scenario. Formally, the manager solves the following problem by selecting sites to inspect, $(X_1, \dots, X_j, \dots, X_J)$, and the number of trees to remove, $(R_1(\theta(s)), \dots, R_j(\theta(s)), \dots, R_J(\theta(s)))_{s=1}^S$, to solve the following problem:

$$\min \frac{1}{S} \sum_{s=1}^S \sum_{j=1}^J Q_j(R_j(\theta(s))) \quad (3.6)$$

subject to

$$0 \leq R_j(\theta(s)) \leq X_j \theta_j(s) N_j, \forall j = 1, \dots, J, \forall s = 1, \dots, S, \quad (3.7)$$

$$\sum_{j=1}^J c_1 X_j N_j + \sum_{j=1}^J c_2 R_j(s) \leq B, \forall s = 1, \dots, S, \quad (3.8)$$

$$Q_j(R_j(\theta(s))) = \min \left[\left((1 - \theta_j(s)) N_j, g(\theta_j(s) N_j - R_j(\theta(s))) \right) \right], \forall j = 1, \dots, J, \forall s = 1, \dots, S, \text{ and} \quad (3.9)$$

$$X_j \in \{0,1\}, \forall j = 1, \dots, J. \quad (3.10)$$

3.3 Characterizing the Solution to a Two Site Problem

We apply this model to a multiple site landscape in the Section 3.4. In order to understand the interactions among the variables of the problem, however, it is helpful to consider a simplified problem involving only two possible locations. For the two site case, the set of all possible combinations of sites $\bar{\Omega}$ is given as

$$\bar{\Omega} = \{\emptyset, \{1\}, \{2\}, \{1,2\}\},$$

where \emptyset is an empty set. In the following discussion, for any combination of sites $\Omega \in \bar{\Omega}$, we call Ω “affordable” if and only if inspection of all the trees on all sites in the set Ω does not exceed the original budget level, B :

$$\sum_{j \in \Omega} c_1 N_j \leq B.$$

Now, we characterize solutions for the tree removal stage, given a set of inspected sites. For each possible combination of inspected sites, Ω , the manager solves the problem defined above in (3.6) through (3.10).

First we consider the case when none of sites are selected for inspection in the surveillance stage: $\Omega = \emptyset$. In this case, by the constraint given by Equation (3.7), cutting no infected trees on both sites is optimal:

$$R_j^*(\Omega) = 0 \text{ for } j = 1,2.$$

Second, we consider cases where only one site is selected for inspection. Sets $\{1\}$ and $\{2\}$ are included in this category. Since only one site is selected for surveillance, it is optimal

to cut as many infected trees on the site as possible within the budget left for infected tree removal, and not to cut any infected trees on sites not surveyed. Thus the solution is given as

$$R_a^*(\Omega) = I_a \text{ for } a \in \Omega \text{ and } R_b^*(\Omega) = 0 \text{ for } b \notin \Omega \text{ if } c_{1a}N_a + c_{2a}I_a \leq B,$$

or

$$R_a^*(\Omega) = \frac{B - c_{1a}N_a}{c_{2a}} \text{ for } a \in \Omega \text{ and } R_b^*(\Omega) = 0 \text{ for } b \notin \Omega \text{ if } c_{1a}N_a + c_{2a}I_a > B.$$

Here, and in subsequent discussions, the higher priority site is labeled site “ a ” and the lower priority site is labeled site “ b .” This notation indicates priority and is distinct from the numerical notation that identifies particular sites (1,2). In this case, since only one site is surveyed and infected tree removal is possible only on the site, the surveyed site becomes automatically the site with the first priority (i.e., assigned the notation a) and the site not surveyed becomes the site with second priority (i.e., assigned the notation b).

Third, it is possible that both sites are surveyed. Here, Ω is the set $\{1,2\}$. If the cost of cutting all the infected trees on all the sites in the set Ω does not exceed the budget left for infected tree removal (so that $\sum_{j \in \Omega} c_{2j}I_j \leq B - \sum_{j \in \Omega} c_{1j}N_j$), then it is optimal to cut all the infected trees in both of the sites in the set Ω . That is,

$$R_j^*(\Omega) = I_j, \forall j \in \Omega$$

If the budget does not allow all trees in inspected sites to be removed, the manager must decide how to allocate the limited budget between the two sites. The higher priority sites

are sites with the higher number of trees saved per dollar spent on tree removal. In the simplest case with many healthy trees, where the first tree removed is effective in reducing new infections (so that no plateau exists and $M_j = 0$), this ratio is given by g_j/c_{2j} . A way to see that this is to note that $g_j R_j$ is equal to the number of trees saved, and $c_{2j} R_j$ is equal to the cost of trees removed so that the quotient, g_j/c_{2j} , is the cost per tree saved. However, when $M_j > 0$, it is possible to incur costs of removal without any resulting reduction in the number of new infections. This will occur when the remaining number of infected trees is sufficient to infect all remaining healthy trees on the site. In this case, the number of healthy trees saved per unit cost of removal is calculated as:

$$\begin{cases} M_j & \text{for } R_j \leq M_j \\ \frac{g_j(R_j - M_j)}{c_{2j}R_j} & \text{for } R_j > M_j. \end{cases}$$

The total cost of tree removal is $c_{2j}R_j$, but tree removal only becomes effective once M_j trees are removed so that the number of trees saved is $g_j(R_j - M_j)$. Therefore, the ratio that determines the highest priority sites depends on the number of infected trees, the number of healthy trees, and the number of trees removed.

Using the principle of choosing the site with the higher number of trees saved per dollar spent on tree removal, we can consider four types of solutions to the problem.

$$R_a^*(\Omega) = I_a \text{ and } R_b^*(\Omega) = \frac{(B - \sum_{j \in \Omega} c_{1j} N_j - c_{2a} I_a)}{c_{2b}} \text{ for } a \text{ and } b \in \Omega$$

In other words, in one site (a), all infected trees will be removed. The rest of the budget will be spent on tree removal in the other site (b). One way in which this type of solution

will occur is when the budget allows for complete removal on either one of the two sites, but not both. The higher priority site (site a) is the one in which complete tree removal will save a higher number of trees. Specifically, this case occurs when:

$$\sum_{j \in \Omega} c_{2j} I_j > B - \sum_{j \in \Omega} c_{1j} N_j,$$

$$c_{2a} I_a \leq B - \sum_{j \in \Omega} c_{1j} N_j, \text{ and}$$

$$\frac{g_a(I_a - M_a)}{c_{2a} I_a} \geq \frac{g_b(I_b - M_b)}{c_{2b} I_b}.$$

Another way in which this solution will occur is when the budget allows for complete removal of infected trees in one site (a) but not the other (b). In this case, the relevant comparison is between the number of trees saved when the entire budget is spent on one site versus the other. Complete tree removal will take place in site a if the number of trees saved when all infected trees are removed from site a exceeds the number of trees saved when the entire budget is spent on incomplete removal of trees from the alternative site (site b). This occurs when these conditions are met:

$$\sum_{j \in \Omega} c_{2j} I_j > B - \sum_{j \in \Omega} c_{1j} N_j,$$

$$c_{2a} I_a \leq B - \sum_{j \in \Omega} c_{1j} N_j,$$

$$c_{2b} I_b > B - \sum_{j \in \Omega} c_{1j} N_j, \text{ and}$$

$$\frac{g_a(I_a - M_a)}{c_{2a} I_a} \geq \frac{g_b\left(\frac{B - \sum_{j \in \Omega} c_{1j} N_j}{c_{2b}} - M_b\right)}{B - \sum_{j \in \Omega} c_{1j} N_j}.$$

It may be possible for incomplete removal in a site to be optimal, even when complete removal in an alternative site is within the budget. In this case, the budget is exhausted in a site (a) where not all trees are removed, but this incomplete removal saves a greater number of trees than when the budget is allocated to an alternative site. This other solution is

$$R_a^*(\Omega) = \frac{B - \sum_{j \in \Omega} c_{1j} N_j}{c_{2a}} \text{ and } R_b^*(\Omega) = 0 \text{ for } a \text{ and } b \in \Omega$$

$$\text{such that } c_{2a} I_a > B - \sum_{j \in \Omega} c_{1j} N_j$$

$$c_{2b} I_b \leq B - \sum_{j \in \Omega} c_{1j} N_j, \text{ and}$$

$$\frac{g_a\left(\frac{B - \sum_{j \in \Omega} c_{1j} N_j}{c_{2a}} - M_a\right)}{B - \sum_{j \in \Omega} c_{1j} N_j} \geq \frac{g_b(I_b - M_b)}{c_{2b} I_b}.$$

A final possibility occurs when the budget allows only incomplete removal in both sites. The higher priority site is again determined as the site where the most trees are saved given the budget available. Mathematically, this possibility happens when

$$c_{2a} I_a > B - \sum_{j \in \Omega} c_{1j} N_j,$$

$$c_{2b} I_b > B - \sum_{j \in \Omega} c_{1j} N_j, \text{ and}$$

$$\frac{g_a\left(\frac{B - \sum_{j \in \Omega} c_{1j} N_j}{c_{2a}} - M_a\right)}{B - \sum_{j \in \Omega} c_{1j} N_j} \geq \frac{g_b\left(\frac{B - \sum_{j \in \Omega} c_{1j} N_j}{c_{2b}} - M_b\right)}{B - \sum_{j \in \Omega} c_{1j} N_j}.$$

To summarize, these solutions imply that, in the infected tree removal stage, budgets left over after surveillance takes place are optimally allocated to sites where more cost

efficient suppression of newly infected trees can take place among the ones where surveillance has been conducted. That is, sites with a positive number of infected trees optimally removed are the ones where more trees can be saved from infection with less cost: these sites are characterized by higher pathogen spread rates g_j , more healthy trees, H_j (so that a plateau, if it exists, is small), and lower per-tree cost of removal, c_{2j} . Note that the combination of a higher spread rate and fewer healthy trees results in a larger plateau.

Now we move to the site selection problem for surveillance. Four types of solutions the surveillance problem can be considered: (i) no sites, (ii) one site, (iii) both sites selected for surveillance.

- (i) *No sites are selected.* ($\Omega^* = \emptyset$): The necessary condition for this solution being optimal is

$$c_{1j}N_j + c_{2j}M_j > B, \forall j \in \Omega, \forall \Omega \in \bar{\Omega}.$$

This condition will hold if the budget is so limited that it does not allow inspection of the least expensive site (i.e., $c_{1j}N_j > B$). This condition could also be met even if inspection is affordable, if the budget allowed for tree removal is not enough to reduce the number of newly infected trees.

- (ii) *Selecting only site a for surveillance is optimal* (i.e., $\Omega^* = \{a\}$): In order to find out whether selecting one site is optimal, we compare the number of infected trees

removed in this one site to all other possibilities. Necessary conditions for the optimality of surveying only site a are the following:

(ii-1) $\{b\}$ is not affordable or $\phi(\{a\}) \leq \phi(\{b\})$ and

(ii-2) $\{a, b\}$ is not affordable or $\phi(\{a\}) \leq \phi(\{a, b\})$.

Condition (ii-1) indicates that preference will be given to a site that (1) is affordable to survey when the alternative site is not affordable or (2) results in fewer newly infected trees than the alternative. Condition (ii-2) compares surveying a single site to surveying both sites. Surveying a single site will be optimal if (1) it is not possible to survey both sites due to the budget constraint or (2) surveying a single site will result in fewer newly infected trees than surveying both sites.

Why might it be optimal to survey a single site when it is within the budget to survey both sites? We relegate a detailed accounting of the budget conditions to Appendix 3, but the essential conclusion is that conditions (ii-1) and (ii-2) above hold when inspection and tree removal on only site a results in saving more healthy trees than if these activities are carried out either only on site b or on both sites a and b .

(iii) *Selecting two sites for surveillance is optimal* $\Omega^* = \{1,2\}$: To have this solution be optimal, the budget level has to satisfy $c_{1a}N_a + c_{1b}N_b + c_{2a}I_a \leq B$. In addition, this strategy has to result in more healthy trees saved than with alternative strategies:

(iii-1) $\phi(\{1,2\}) \leq \phi(\{1\})$, and

$$(iii-2) \phi(\{1,2\}) \leq \phi(\{2\}).$$

3.4 Application

Our application is to the case of oak wilt in Anoka County, Minnesota. Oak wilt is one of the most notorious oak tree-killing fungal pathogens spreading in the eastern and central part of the United States (Juzwik, 2009), and Anoka County has been severely affected by oak wilt. It has been reported that 4,283 pockets have been present in Anoka County from 1992-2007. This number could be an underestimate of actual oak wilt cases in Anoka County because these are only pockets that have been detected by the MN-DNR or by landowners applying for treatment subsidies from the ReLeaf program.

3.4.1 Data Sources

We use three distinct data sources to estimate the total population of trees on each site: the Minnesota Land Cover Classification System (MLCCS), the Forest Inventory Assessment (FIA) database and the oak wilt treatment compliance database. MLCCS is created by Minnesota Department of Natural Resource Central Region (Minnesota Department of Natural Resources Central Region [MN-DNR], 2004). It provides us with detailed geographic information about habitat types (or land use) covering parts of four counties in the Twin Cities metropolitan area in Minnesota: Anoka, Scott, Ramsey, and

Hennepin counties. Among these counties, Anoka is the only county which is completely covered by the MLCCS. The MLCCS defines 252 habitat types.

The oak wilt treatment compliance database provided by the MN-DNR is a statewide inventory of oak wilt pockets. It reports the location and size of oak wilt infection pockets, the year when each infection pocket is reported, the number of infected and healthy trees, the types of treatment applied to both infected and healthy trees, and the status of each infection pocket after treatment. In each reported infection pocket, both infected and healthy may remain after treatment. Some infected trees are removed but some are left and injected with fungicide. If all the infected trees are cut in a given infection pocket, the infection pocket appears “inactive” in the database. If some infected trees are left and treated with fungicide, the infection pocket is reported as “active” in the database. Apparently healthy trees in an infection pocket can be also removed if they are close enough to infected trees to have become infected via root grafts.

This oak wilt compliance database can be used as not only as a good source of information about infection pockets but also to estimate the total number of oak trees in a given habitat of Anoka County. This compliance data was collected through active programs, funded in partnership with the USDA Forest Service and the Minnesota ReLeaf fund, designed to limit further damage. These programs subsidize tree owners for the treatment given to their infected and healthy trees under the condition that the tree owner follows instructions provided by the programs. Instructions require tree owners to treat their infected and healthy trees in a specific way and also to reveal information about their trees: the total numbers of trees, where they stand, the distance between them,

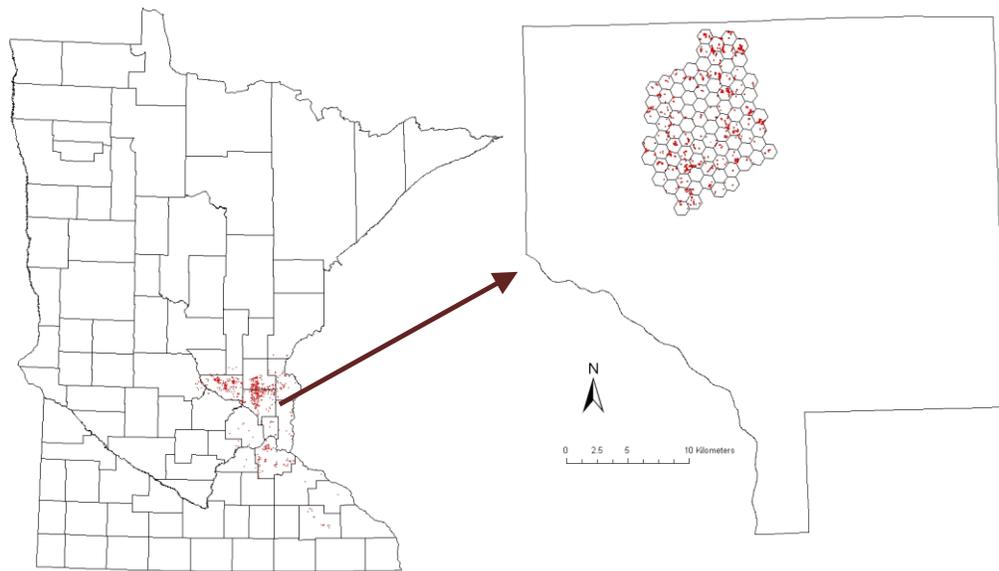


Figure 3.2: Study Area and 90 Grid Cells in Anoka County, Minnesota

Note: Red polygons indicate oak wilt infections.

and so on. Therefore, the total number of oak trees including both infected and healthy trees reported in this compliance data is reliable sample data. Since the data also can identify the location of all the infection pockets, we can identify habitats which they belong to by using the MLCCS database. Thus, overlaying the oak wilt compliance database with MLCCS database enables us to know the representative number of oak trees for each habitat.

3.4.2 Defining Parameters for the Model

We chose to analyze a portion of Anoka County where oak wilt is a particularly severe management problem. The area is in the northwest portion of the county. We divide the area into 90 hexagonal 1 km^2 grid cells. This grid cell surface is the site file. Next we calculate the total population of host trees, N_j , for each site $j = 1, \dots, 90$ by using the data obtained from MLCCS. First, we overlay habitat data obtained from MLCCS database on the site file so that we know the number, size, and types of habitats for each site. From the compliance check data base, we have the density of trees per square kilometers for each habitat type. Summing the sizes (square kilometers) of habitats multiplied with their associated number of trees for each site, we can obtain the total number of trees, N_j , for each site $j = 1, \dots, 90$.

Next, we estimate the proportion of infected trees in the sample from inventory data on site j , θ_j , by using the information on size of active oak wilt infection pocket obtained from the compliance data. The oak wilt compliance data set provides us with the size of infection pockets and whether the oak wilt infection pocket is active or not. We choose only active infection pockets reported in 2005, when the most intense monitoring was conducted. Overlaying this information on the grid cell layer, we can calculate the fraction of each grid cell which is occupied by the infected pockets.

Root grafting between healthy and infected trees and the sap beetle insect vector are the two main factors causing transmission of infection among trees (Juzwik, 2009). In the case of oak wilt, spore mats formed on the surface of the dead infected oak trees

attract sap beetles, and the beetles carry the fungus of oak wilt to healthy trees. However, since sap beetles likely exist uniformly in the range of our study area, it is difficult to reliably estimate different new pocket formation rates for different grid cells. Trees that are grafted together at their roots share biological processes. Therefore, a healthy tree grafted with an infected tree is more likely to be infected than one without root grafts (Epstein 1978). We assume exponential infection growth, where the new population of newly infected trees at the end of the treatment becomes the population of originally infected trees multiplied by constant growth rate (g). This assumption is made to make our optimization problem solvable by using integer programming. We set the growth rate $g = 0.03$.

We calculate the surveillance cost, c_1 , from the wage rate of arborists employed by city and county governments. Arborists employed by governments tend to be ranked in GS6 or 7 in the General Schedule (GS) Locality Pay Table. Taking the average of means of salaries for GS6 and GS7, we calculate the salary for an arborist to be 17.03 dollars per hour. To diagnose whether a tree is diseased or not takes 10 minutes on average. Thus, inspection cost per tree results in 2.84 dollars ($17.03 \text{ dollars} \times 1/6$). We assume that the surveillance cost per tree is uniform over all the sites. The per-tree cost of removal, c_2 , usually depends on both the location and the size of a tree. Larger trees cost more to be removed. On average, trees in Anoka County are 10.475 diameter at breast height (dbh). We use this average value for all trees removed so that the per-tree cost of removal is assumed to be 360 dollars as a benchmark value.

Finally we prepare a set of S scenarios, $\Theta_S = \left(\theta_1(s), \dots, \theta_j(s), \dots, \theta_{90}(s) \right)_{s=1}^S$.

Using a sample size (n) of 20 and the fraction of infected trees $\hat{\theta}_j$ calculated above, we find the parameters for the beta distribution $f(\theta_j; \alpha_j, \beta_j)$ for each site $j = 1, \dots, 90$, where $\alpha_j = n\hat{\theta}_j$ and $\beta_j = n(1 - \hat{\theta}_j)$. Using Matlab (MathWorks 2007), we draw θ_j from an interval $[0,1]$ following $f(\theta_j; \alpha_j, \beta_j)$, S times for each site j . For each draw $s = 1, \dots, S$, we obtain a vector of scenarios $\theta(s) = (\theta_1(s), \dots, \theta_j(s), \dots, \theta_{90}(s))$. Collecting these S scenario vectors, we obtain the set of scenarios Θ .

3.4.3 Results: Performance of the Model

The problem specified in Equations (3.3), (3.4), and (3.5) is solved by using the integrated solution package GAMS (GAMS Development Corporation 1990), which is designed for large and complex linear and mixed integer programming problems. First, we look for the sufficient number of scenarios, \mathcal{S} , which enables the model to yield a robust result. We expect that more scenarios will make it more likely for the model to yield a robust result. However, increasing the number of scenarios is costly. It usually is the case that more scenarios make an integer programming problem more difficult to solve (e.g., more computation time and sometimes no solution (Snyder et al., 2004)). To find an adequate value of \mathcal{S} , we consider three values of S : $S = 100, 1000$, and 2000 . For each value of S , we created 10 sets of scenarios. Thus, for each size, we have 10 different sets of scenarios (Θ_S) with which to run the model. For each size S , we examine which

Table 3.1: Robustness Check

The number of times sites were selected for surveillance in 10 replications	The number of scenarios		
	100	1000	2000
Low budget level ($B=100$ thousand dollars)			
Never	51	62	63
Once	9	1	1
Twice	4	0	0
3 times	1	2	1
4 times	2	1	0
5 times	1	1	2
6 times	1	0	0
7 times	2	1	0
8 times	2	0	1
9 times	2	0	0
10 times	15	22	22
Mean of expected number of trees saved from infection	69.56	69.39	69.46
Standard deviation of expected number of trees saved from infection	0.69	0.16	0.06
The number of times sites were selected for surveillance in 10 replications	The number of scenarios		
	100	1000	2000
High budget level ($B=200$ thousand dollars)			
Never	18	23	23
Once	6	0	2
Twice	0	1	0
3 times	1	1	0
4 times	1	1	0
5 times	1	2	2
6 times	1	0	0
7 times	2	1	0
8 times	4	1	1
9 times	8	1	1
10 times	48	59	61
Mean of expected number of trees saved from infection	126.78	126.14	126.28
Standard deviation of expected number of trees saved from infection	0.93	0.36	0.19

sites were selected for surveillance in each model run and count how many times each site appears as a surveyed site in the 10 runs of the model.

We did this robustness check of the model with two budget constraints: $B = \$100$ thousand and $\$200$ thousand. The upper part of Table 3.2 shows the results when the budget level is relatively low ($B = 100$ thousand dollars), and the lower part shows results when the budget level is relatively high ($B = 200$ thousand dollars). We expect that a perfectly robust model would have sites selected either all 10 times or never. We find that the fraction of sites which are selected as either surveyed or not-surveyed for 10 times consistently increases from 74% to 94% by increasing the number of scenarios from $S = 100$ to 1000 under the relatively low budget level. While we can observe this drastic improvement in the robustness of the model by increasing S from 100 to 1000, the improvement of the robustness is very marginal when S is increased from 1000 to 2000; 95% sites remain selected all 10 times when $S = 2000$ under the relatively low budget level. On the other hand, under the relatively high budget level, the significant improvement in the robustness of the model can still be observed by increasing the number of scenario from $S = 1000$ to 2000; 90 % and 94 % of sites are selected as either surveyed or not- surveyed for 10 times when $S = 1000$ and 2000 respectively. We also reported the average expected number of trees saved from infection. The mean of expected number of trees saved from infection remained approximately constant, and the standard deviation closed tightly around this mean as the number of scenarios increased. We can conclude that the performance of the model greatly increased as the number of

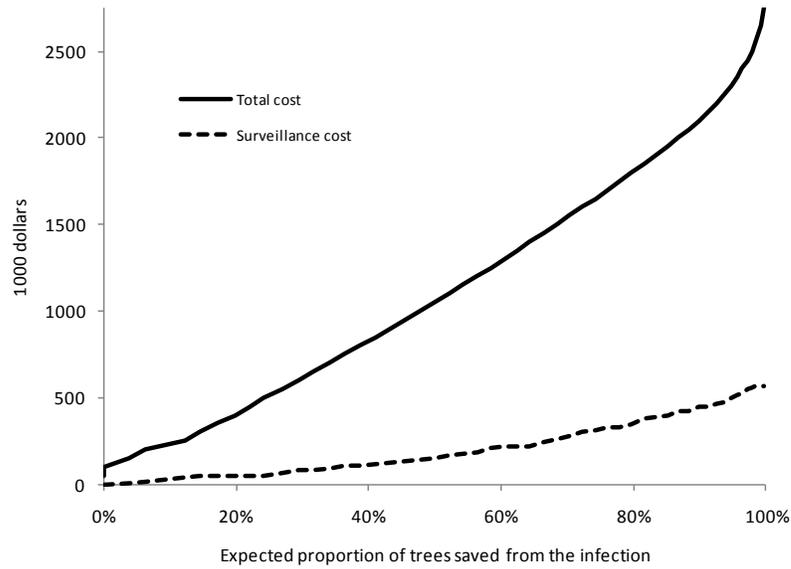


Figure 3.3: Cost Curves for the Targeted Fraction of Healthy Trees Saved

scenarios increased from 100 to 2000 under both budget levels although the model still did not yield perfectly robust results.

3.4.4 Result 1: Total Costs and Surveillance Costs

Having confirmed that a scenario set of 2000 scenarios would give us a robust solution for the problem, we solve the model for 58 different budget levels from 0 to 2,850 thousand dollars with a set of 2000 scenarios. We report the level of budgets required to achieve given expected percentages of healthy trees saved from infection, which is the cost curve, in Figure 3.3. We find that the cost curve has an intercept (100 thousand dollars). This is because saving a positive fraction of healthy trees from infection requires

that a forest manager conducts surveillance on at least one site. Since we assume that a forest manager needs to check the health of all the trees on a given site to conduct surveillance on the site, a manager incurs a fixed cost if she conducts surveillance on a site. We also can find that the slope of the cost curve increases as the fraction of healthy trees saved from infection increases. Note that the curve goes down several times. This is because saving more healthy trees from infection does not necessarily mean surveying more sites. It is optimal for an ecosystem manager not to expand surveyed sites until she removes all the infected trees on sites already surveyed.

3.4.5 Result 2: Characteristics of Sites Selected for Surveillance

We are interested in factors that determine why some sites selected for surveillance and others are not. What makes a site a high priority for limited forest management funds? We address this question by first ranking the sites in order of their selection as budgets increase. Then, we rank sites by other characteristics that might explain the priority ranking to see which ranking matches best. Table 3.2 shows the sites selected for surveillance under different budget levels. Each row represents a site on the landscape, and each column represents a budget level. Cells are shaded when the site is selected for surveillance.

Our first observation is that, once a site is selected for surveillance under a given budget constraint, the site remains selected when the budget constraint is relaxed. We suppose that this is true because of the nature of the scenarios associated with original

Table 3.3: Ranking of Surveyed Sites According to the Expected Number of Trees Saved from Infection

Budget level	Ranking of the sites newly selected for surveillance	Budget level	Ranking of the sites newly selected for surveillance	Budget level	Ranking of the sites newly selected for surveillance
Sites are ranked by expected number of trees saved from infection $\sum_{s=1}^S \theta_j(s) N_j$.					
0	None	1000	None	1950	65,
50	None	1050	19,	2000	55,60,61,66
100	None	1100	20,33	2050	None
150	3,	1150	25,49	2100	62,67,69
200	7,	1200	35,53,	2150	74,
250	1,2,3,5,12	1250	22,28,38	2200	70,72,78
300	13,15	1300	43,	2250	75,
350	6,	1350	None	2300	68,71,73,82
400	None	1400	47,	2350	76,80,83
450	None	1450	37,41,45,52	2400	81,84
500	None	1500	29,44	2450	77,79,85,89
550	8,9,10	1550	39,40,59	2500	86,87,90
600	14,27	1600	23,54,57	2550	88,
650	None	1650	42,58	2600	None
700	4,18	1700	31,56	2650	None
750	24,26	1750	None	2700	None
800	34,	1800	36,64	2750	None
850	16,	1850	11,46,50,51	2800	None
900	21,32	1900	63,	2850	None
950	17,				

Note: Unit of Budget level is 1000 dollars.

fraction of infected trees and the shape of objective function. On any site, the population of infected trees is so small that we never encounter a case where all the originally healthy trees become infected in a site. Therefore, a plateau never appears in the infection growth function on any site; the infection growth function is linear in the number of originally infected trees. Therefore, it is straightforward to rank sites according to the

smallest budget level at which they are included in the set of sites selected for inspection. In ranking sites according to their priority, a ranking of “1” indicates that the site is selected at the lowest budget level where active management occurs. In our application, a budget lower than \$150,000 is insufficient to survey any sites in the study area. When the budget level reaches \$150,000, however, Site 43 is selected for surveillance and treatment. Site 23 is added when the budget reaches \$200,000, so it ranks “2.” A ranking of “90” means that the site is selected only when the budget level reaches a point where all sites are chosen for inspection.

In Table 3.3, we rank sites according to their priority so that, for example, Site 43 is the site corresponding to the \$150,000 budget level, site 43 is the site corresponding to the \$200,000 budget level, and so on. We then find out how these sites rank according to three characteristics. First, we rank the sites by the expected number of trees saved from infection ($N_j \sum_{s=1}^S \theta_j(s)$) in ascending order. This characteristic measures the maximum expected number of trees on a given site by surveying the site. Thus, surveying highly ranked sites according to the expected number of trees saved from infection may lead to maximizing the number of healthy trees saved from infection. Site 43, the first site selected for inspection, ranks “3” according to this characteristic. Site 23, the second site selected (when the budget reaches \$200,000) ranks 7th, and so on.

Next, we rank the sites by the ratio of the expected number of infected trees to the cost of inspecting every tree on the parcel, with the highest ranking corresponding to the highest ratio:

Table 3.4: Ranking of Surveyed Sites by the ratio of the expected number of infected trees to the cost of inspecting every tree on the parcel:

Budget level	Ranking of the sites newly selected for surveillance	Budget level	Ranking of the sites newly selected for surveillance	Budget level	Ranking of the sites newly selected for surveillance
Sites are ranked by the ratio of the expected number of infected trees to the cost of inspecting every tree on the parcel: $N_j \sum_{s=1}^S \theta_j(s) / c_1 N_j$.					
0	None	1000	25,	1950	60,61
50	None	1050	24,	2000	62,63,64,65
100	None	1100	26,27	2050	None
150	1,	1150	30,31,	2100	66,67,68
200	5,	1200	29,33	2150	70,
250	2,3,4,7,	1250	28,32,34	2200	69,71,72
300	6,9,	1300	37,	2250	74,
350	8,	1350	None	2300	73,75,76,77
400	None	1400	36,	2350	78,79,80
450	None	1450	35,38,39,44	2400	81,82
500	None	1500	40,44,48	2450	83,84,85,89
550	10,11,12	1550	42,43,49	2500	86,87,88
600	14,16	1600	41,47,53	2550	90
650	None	1650	50,51	2600	None
700	13,15	1700	45,52	2650	None
750	17,19,	1750	None	2700	None
800	18,	1800	54,57	2750	None
850	20,	1850	46,55,56,58	2800	None
900	21,22	1900	59,	2850	None
950	23,				

Note: Unit of Budget level is 1000 dollars.

$$\frac{N_j \sum_{s=1}^S \theta_j(s)}{c_1 N_j}$$

Results are shown in Table 3.4. In this case, the first site selected (Site 43) has the highest ranking according to this characteristic. The second site selected (Site 23) is ranked 5th according to this characteristic. More roughly, the top 9 sites selected are the top 9 ranked sites. While this characteristic is not a perfect predictor of which sites are selected as the budget increases, there is a rough correspondence that suggests that it has explanatory power.

3.4.6 Result 3: Infected Tree Removal

It is optimal for an ecosystem manager to exhaust her budget left after surveillance by cutting down as many infected trees as possible with the remaining funds. This approach minimizes the number of newly infected trees for any s -th scenario. Also, an ecosystem manager would be indifferent about where to cut infected trees. This is simply because the infection growth rate is uniform over the sites; all healthy trees in sites that have infected trees are vulnerable to infection.

3.5 Discussion

The most effective strategy for limiting the spread of invasive forest pathogens such as oak wilt is to find and remove affected trees. The detection step is an important

one, because trees must be specifically identified as being diseased before being removed; broadcast aerial spraying, for example, is not an option for control of fungi. However, there has generally been little attention paid to the problem of determining how much effort should be devoted to detection of invaders. We use a mixed integer programming approach, inspired by the site selection literature, to choose locations in a grid on which to focus detection and control efforts in a budget-constrained setting. Locations vary by the number of susceptible trees, the number of infected trees, the infection rate, and the cost of tree removal. We characterize the solutions for the infected tree removal stage, and for the surveillance stage. Solutions in the infected tree removal stage imply that, in the infected tree removal stage, budgets left over after surveillance takes place are optimally allocated to sites where more cost efficient suppression of newly infected trees can take place among the ones where surveillance is conducted. That is, sites with positive numbers of infected trees optimally removed are the ones where more trees can be saved from infection with less cost: these sites are characterized by higher pathogen spread rates, more healthy trees, and lower per-tree cost of removal. Necessary conditions for the solutions in the surveillance stage imply that sites on which more healthy trees can be saved from spending a unit of inspection cost tend to be chosen for the inspection. Therefore sites with higher inspection costs due to a high number of trees have to have higher infection spread rates or lower per-tree removal cost if those sites are to be selected for surveillance over sites with fewer trees.

We apply our model to the case of Anoka County, Minnesota, a county with a severe oak wilt problem. We use our model to determine which sites within Anoka

County should be given highest priority for inspection and removal. Our model is adaptable to other regions in which oak wilt is a potential problem. The primary contribution of our study is to offer a tool suggesting practical guidance to managers in charge of deciding how and where to spend limited public dollars when the goal is to reduce the number of trees newly infected by oak wilt. We construct a curve that reflects the cost of protecting healthy trees from infection. This curve provides the manager with information about the level of budgets required to achieve targeted percentages of healthy trees saved from infection. The marginal cost of saving an additional healthy tree from the pathogen increases as the targeted protection level is set higher.

We also explored characteristics of sites selected for the surveillance. We ranked sites by the ratio of the expected number of infected trees to the cost of inspecting every tree on the parcel. We find that this characteristic can predict which sites to be selected for the surveillance as the budget increases, although the prediction is not necessarily perfect. This implies that it is best to conduct surveillance with priority on sites with relatively low expected unit surveillance cost per tree saved from infection. Thus, as the desired fraction of healthy trees saved from infection increases, lower priority sites with higher costs begin to be selected. This drives marginal costs up. This result is consistent with the observation that the fraction of surveillance cost over the total cost is upward sloping in the targeted fraction of healthy trees saved from infection.

Chapter 4

Invasive Species Management on Private Lands

4.1 Introduction

Invasive species are alien species which cause economic or environmental harm or harm to human health. One important category of environmental harm is the damage caused by invasive pests and pathogens to forests: invasive diseases and pests including Dutch elm disease, oak wilt, and gypsy moth have been valued at an estimated \$120 billion per year (Pimentel et al., 1999). There has been great interest in limiting these costs by stopping or slowing the spread of invasive forest pests and pathogens. One approach has been to institute large scale government-funded management programs. This approach lends itself to pests for which individual property owners can do little to shield themselves from damage because the pests disperse over long distances and local control measures are non-existent or ineffective. A prominent example is the national Slow the Spread program to control gypsy moth. In these situations, the public good of protecting forest owners from the advance of an invasive species front justifies public expenditures on landscape-level control. Another approach, suited to slow-growing and locally controllable pathogens such as oak wilt, has been to encourage private landowners

to undertake management efforts on their own properties to reduce the spread of the disease to neighboring properties.

This paper focuses on this second approach--the management of private lands, especially residential areas, at risk of being infested by an invasive species. We consider situations where landowners can manage the invader effectively by either preventing the invader from establishing on their property or by mitigating damage to trees once the invader arrives and becomes established. Prevention has the added benefit of reducing the overall probability that neighboring properties become infested. Because some of the benefits of preventive action are external and accrue to neighbors, it is likely that private action will yield less than the socially optimal level of prevention. We model this situation, and then introduce a government policy to subsidize prevention funded by a lump sum tax on property owners. We examine whether, and under what conditions, this policy can achieve the socially optimal outcome.

In recent years, economists have begun to be interested in how best to manage invasive pests. The literature generated by this interest focuses on two main policy questions. The first question considers optimal trade policy between importing and exporting countries for the prevention of new species introductions. The central tradeoff is between the damage foregone that would otherwise have been caused by an invader and the welfare lost by implementing two policy tools: a tariff added to reduce trade flows and the inspection of goods at ports of entry (e.g., McAusland & Costello, 2003; Costello & McAusland, 2004; Olson & Roy, 2005).

The second set of questions considers optimal eradication or suppression policies to control established invasive species. Olson and Roy (2002) and Olson and Roy (2008)

set up and solve a stochastic dynamic optimization problem and show the conditions under which an eradication strategy becomes dominated by a suppression strategy. The central tradeoff here is between costs of control and benefits in terms of foregone damage. Several authors have examined these questions in a spatially explicit setting, including Sharov and Liebhold (1998) who formulate a dynamic optimization approach to solving for an optimal barrier zone.

Most studies focus in this literature focus on optimal invasive species management in an ecosystem from the perspective of a single manager. However, practically, an ecosystem lies on multiple patches of land. For example, the urban forest exists on land possessed by multiple landowners. Landowners decide on management strategies to protect their own properties from a biological invasion and these private decisions affect the damage caused by the invader on all properties within the forest. This is a traditional externality problem, but surprisingly there has been little work done to analyze how externalities play out in the context of invasive species management.²

Invasive species management on multiple patches of private land shares several common features with the disease control problem (e.g., Brito et al., 1991; Francis, 1995; Gersovitz & Hammer, 2004). In models of disease control, each resident in a community chooses whether to get vaccinated or not. In its basic structure, our model shares elements with models found in the literature on optimal immunization to protect against disease and models of optimal taxation. In particular, Brito et al. (1991) develop a model in

² Exceptions include Wilen (2007), Epanchin-Neill et al. (2009), Epanchin-Neill and Wilen (2010, July a), Epanchin-Neill & Wilen (2010, July b), and Frisvold, Olson, Bean, Betancourt, & Marsh (2007, October) who use cellular automata models to examine decision-making involving multiple landowners.

which disease prevention takes the form of vaccination. A higher number of individuals choosing to vaccinate leads to a lower overall incidence of the disease. Brito et al. (1991) solve for the equilibrium level of vaccination and disease incidence and then examine policies to align the incentives of individuals with the interests of society as a whole. We adapt the approach found in Brito et al. (1991) by recognizing that landowners can prevent infections on their property by treating their trees with fungicides or other chemicals. This is analogous to vaccination both because it prevents individual trees from becoming diseased and it lowers the overall level of disease incidence. However, landowners have an additional option besides prevention and leaving trees fully vulnerable: they can monitor their trees for disease and treat trees once they become infected. Monitoring thus reduces the severity of the disease. We add this behavior to the model.

In Section 4.2, we formulate our model of the private landowner's problem. In Section 4.3, we define and characterize the landowners' equilibrium and the full information social optimum. Next we define the regulator's problem of maximizing social welfare by selecting levels of the subsidy (of prevention activities) and lump sum tax that takes account of the best response of landowners to the subsidy and tax. In the latter part of Section 4.3, we run simulations so that we can derive the solutions of the problems defined in the previous subsections. After running simulations and obtaining intuition for the result of benchmark case, we conduct sensitivity analyses. Lastly, we conclude the discussion in Section 4.4.

4.2 Modeling Landowners' Behavior

4.2.1. Landowner's Utility

We build a model of a continuum of landowners where each landowner is differentiated by her valuation of a forest (or, more generally, an environmental amenity) on her property. Each landowner is endowed with an income and a piece of forest land. Denote by y each landowner's total endowment of income, and by e_1 each owner's endowed size of a forest (i.e., their endowment of healthy trees). We assume that she has separable utility over a consumption good and the forest. Denoting the budgets left for the purchase of consumption goods from the endowed income by b and the size of the forest by e , her utility obtained from the consumption good and her forest, $U(b, e; \theta)$, is

$$U(b, e; \theta) = u(b) + \theta a(e) \quad (4.1)$$

The function $u(b)$ represents utility obtained from income where $u'(b) > 0$ and $u''(b) \leq 0$. The function $\theta a(e)$ represents her utility obtained from consuming environmental amenities, such as shade and a view, supplied by the forest on her property where $a'(e) > 0$ and $a''(e) \leq 0$. We assume that all landowners are distinguished by the valuation parameter θ distributed over a compact interval $[0, \bar{\theta}]$ where $\bar{\theta} > 0$. The total number of individuals is normalized to 1. The cumulative distribution function of θ characterizing the population in terms of its distribution over the valuation of trees is given as $0 \leq F(\theta) \leq 1$, and its corresponding density function is given as $f(\theta)$. Except for the valuation of the amenity, landowners are identical. That is, they have identical income endowments, tree endowments, and preferences for the consumption good.

4.2.2 Infestation and Damage to Trees

In our model we assume that the invasion of species to the community is sure to take place. However, it may or may not spread among properties in the area once it has been introduced. We define the probability of spread among properties as q , where $0 \leq q \leq \bar{q}$. This probability is determined endogenously by the outcome of the actions of landowners: the larger the fraction of landowners who do not take preventive action, the higher the probability of spread and thus the higher the overall probability of infestation. Denoting by $x \geq 0$ the fraction of the population of landowners who do not take preventive action, we can write the probability of the spread of the species among properties, q , as a function of x :

$$q = q(x), \tag{4.2}$$

where $q'(x) \geq 0$, $q''(x) \geq 0$, $q(0) = 0$ and $q(1) = \bar{q}$.

We next introduce the damage of individual forest properties caused by the infestation. We assume that a tree dies for sure once it is infested by the invading species, but not all trees on a property are necessarily affected if an infestation occurs. We assume that, if a forest is not infested at all, then the number of healthy trees remains the endowed number, e_1 . If a forest is infested but its infestation is detected at the early stage and infected trees are removed so that the spread of infestation within the property is stopped, then the number of healthy trees becomes $e_2 > 0$ where $e_2 < e_1$. If a forest is infested and the infestation left untreated, more trees become infested and die than in the previous case so that the number of healthy trees on the property results in $e_3 \geq 0$ where $e_3 < e_2 < e_1$.

4.2.3 Linking Landowners' Behavior and the Probability of Infestation

We envision a situation where property owners have three possible options regarding the invader. One is to apply a costly preventive treatment, such as a pesticide injection, in advance which protects a forest from any damage and also prevents it from being a source of pests to forests on neighboring properties. Denote by $c_1 > 0$ the cost of preventive treatment. By incurring the cost c_1 , a landowner can ensure that all the trees in the forest on her property remain healthy; the total number of healthy trees remains e_1 . When a landowner adopts this option, her utility becomes

$$U(y - c_1, e_1; \theta) = u(y - c_1) + \theta a(e_1), \quad (4.3)$$

which is also her expected utility since she gets this utility level for sure.

A second option is to forego prevention but monitor trees in the forest and apply a curative treatment to mitigate damage in case a forest does become affected and monitoring is successful in detecting the invader. Note that the curative treatment for the forest here specifically means removal of the infected trees in the forest so that the further spread of infection within the forest on the property is stopped. We assume that the monitoring costs $c_2 > 0$. If the species establishes on these properties, damage can be mitigated at a cost $c_3 > 0$, but trees can still be a source of infestation to neighboring properties, adding to the overall probability of infestation. Thus if the infestation of a forest takes place and it is successfully detected by the monitoring so that the curative treatment is conducted, then the landowner obtains the following utility:

$$U(y - c_2 - c_3, e_2; \theta) = u(y - c_2 - c_3) + \theta a(e_2).$$

If the infestation occurs but the infestation is not detected so that the following curative treatment cannot be conducted, the landowner obtains utility

$$U(y - c_2, e_3; \theta) = u(y - c_2) + \theta a(e_3).$$

Alternatively, landowners choosing a monitoring strategy may escape infestation altogether. In this case her utility is given by

$$U(y - c_2, e_1; \theta) = u(y - c_2) + \theta a(e_1).$$

Denoting by $0 \leq r \leq 1$ the probability that monitoring misses the infestation, we can write the expected utility of foregoing prevention but monitoring trees as

$$q(x) \left[\begin{array}{l} (1-r)U(y - c_2 - c_3, e_2; \theta) \\ + rU(y - c_2, e_3; \theta) \end{array} \right] + (1 - q(x))U(y - c_2, e_1; \theta). \quad (4.4)$$

A third option is to take no action and leave trees fully vulnerable to the invasive species. No mitigation occurs, and forests on these properties are also a source of spread and increased probability of infestation in neighboring properties. When a landowner chooses this option and if the infestation of the forest on her property takes place, her utility becomes

$$U(y, e_3; \theta) = u(y) + \theta a(e_3).$$

If infestation does not occur, her utility is given by

$$U(y, e_1; \theta) = u(y) + \theta a(e_1).$$

Thus, the expected utility for a landowner who takes no action and leaves trees fully vulnerable to the invasive species can be written as follows:

$$q(x)U(y, e_3; \theta) + (1 - q(x))U(y, e_1; \theta). \quad (4.5)$$

Now that we have introduced three possible options regarding the invasion of species, we can calculate the fraction of the population who does not prevent the infestation, x . Here, we assume that the preventive action is the most costly option among all the three options. Because of the costliness of the preventive action, landowners with a

low valuation tend not to conduct preventive action. Only landowners with a high valuation tend to take preventive action. Letting θ_1 be the cutoff value of the valuation of a forest such that a landowner with any value of θ larger than θ_1 takes preventive action, the fraction of the population that does not take preventive action, x , is calculated as:

$$x(\theta_1) = \int_0^{\theta_1} f(z)dz = F(\theta_1). \quad (4.6)$$

Finally by (4.2) and (4.6) we can relate the probability of spread of infestation of the invading species among properties not protected by preventive action by its owner, q , and the cutoff value for the valuation parameter, θ_1 :

$$q = q(x(\theta_1)). \quad (4.7)$$

4.3 Analysis

In this section, we derive necessary conditions for the landowners' equilibrium, the full information social optimum, and the equilibrium under a subsidy and tax scheme when a regulator has only incomplete information about landowners' preferences.

4.3.1 Equilibrium of Landowners' Invasive Species Management

The equilibrium outcome of this model is the composition of the population in the area taking the three possible actions in our model: prevention, monitor and cure, and no action. Another key outcome of the model associated with the composition of the population is the probability of the spread of infestation among properties, $q(x(\theta_1))$. Landowners choose their strategy based on the probability of infestation, but this infestation is affected by how many landowners choose a preventive strategy (i.e., the

corresponding value of θ_1). A higher number of landowners choosing prevention (a small value of θ_1^*) leads to a lower probability of infestation of the remaining landowners' properties. This reduction in probability leads to a lower incentive to follow a preventive strategy. Thus, the probability of infestation is endogenous to the model.

The equilibrium of the landowners' invasive species management is defined as follows.

Definition: An equilibrium of the landowners' invasive species management is defined by cutoff points of valuation of forest for the landowners θ_1^* and θ_2^* in the interval of valuation $[0, \bar{\theta}]$ such that

1. For any landowner with valuation θ within the interval $[\theta_1^*, \bar{\theta}]$, selecting the preventive action is the most beneficial given the probability of infestation $q(x(\theta_1^*))$,

$$(i) \quad U(y - c_1, e_1; \theta) \geq q(x(\theta_1^*)) \left[\frac{(1-r)U(y - c_2 - c_3, e_2; \theta) + rU(y - c_2, e_3; \theta)}{rU(y - c_2, e_3; \theta)} \right] + (1 - q(x(\theta_1^*))) U(y - c_2, e_1; \theta_1^*)$$

$$(ii) \quad U(y - c_1, e_1; \theta) \geq q(x(\theta_1^*)) U(y, e_3; \theta) + (1 - q(x(\theta_1^*))) U(y, e_1; \theta)$$

2. For any landowner with valuation θ within the interval $[\theta_2^*, \theta_1^*]$, selecting the monitoring action is the most beneficial given the probability of infestation $q(x(\theta_1^*))$:

$$(i) \quad q(x(\theta_1^*)) [(1-r)U(y - c_2 - c_3, e_2; \theta) + rU(y - c_2, e_3; \theta)] + (1 - q(x(\theta_1^*))) U(y - c_2, e_1; \theta_1^*) \geq U(y - c_1, e_1; \theta)$$

$$\begin{aligned}
\text{(ii)} \quad & q(x(\theta_1^*))[(1-r)U(y-c_2-c_3, e_2; \theta) + rU(y-c_2, e_3; \theta)] \\
& + (1-q(x(\theta_1^*)))U(y-c_2, e_1; \theta_1^*) \\
& \geq q(x(\theta_1^*))U(y, e_3; \theta) + (1-q(x(\theta_1^*)))U(y, e_1; \theta).
\end{aligned}$$

3. For any landowner with valuation θ within the interval $[0, \theta_2^*]$, selecting no action is the most beneficial given the probability of infestation $q(x(\theta_1^*))$:

$$\text{(i)} \quad q(x(\theta_1^*))U(y, e_3; \theta) + (1-q(x(\theta_1^*)))U(y, e_1; \theta) \geq U(y-c_1, e_1; \theta)$$

$$\begin{aligned}
\text{(ii)} \quad & q(x(\theta_1^*))U(y, e_3; \theta) + (1-q(x(\theta_1^*)))U(y, e_1; \theta) \\
& \geq q(x(\theta_1^*))[(1-r)U(y-c_2-c_3, e_2; \theta) + rU(y-c_2, e_3; \theta)] \\
& + (1-q(x(\theta_1^*)))U(y-c_2, e_1; \theta_1^*).
\end{aligned}$$

$$4. \quad 0 \leq \theta_2^* \leq \theta_1^* \leq \bar{\theta}$$

Under different combinations of economic and ecological parameters, the following three different types of equilibrium are possible.

Equilibrium-I: No landowner takes preventive action (i.e., $\theta_1^* = \bar{\theta}$). Any landowner with θ such that $\theta_2^* \leq \theta \leq \bar{\theta}$ takes monitoring action and any landowner with θ such that $0 \leq \theta \leq \theta_2^*$ takes no action, where

$$\theta_2^* = \frac{1}{a(e_2)} \left\{ \frac{u(y) - u(y-c_2)}{\bar{q}(1-r)} + u(y-c_2) - u(y-c_2-c_3) \right\}. \quad (4.8)$$

This occurs if and only if even the landowner evaluating the forest on his property at the highest possible value ($\theta = \bar{\theta}$) would choose monitoring action rather than preventive

action even when the spread of infection is highest (\bar{q}). In other words, the expected damage foregone by taking monitoring and curative actions is less than the expected cost saved at $\theta = \bar{\theta}$, i.e., if and only if

$$\begin{aligned} & \bar{q}\{(1-r)U(y-c_2-c_3, e_2; \theta_2^*) + rU(y-c_2, e_3; \theta_2^*)\} + (1-\bar{q})U(y-c_2, e_1; \theta_2^*) \\ & = \bar{q}U(y, e_3; \theta_2^*) + (1-\bar{q})U(y, e_1; \theta_2^*). \end{aligned} \quad (4.9)$$

Equilibrium-II: Any landowner with θ such that $\theta_1^* \leq \theta \leq \bar{\theta}$ takes preventive action, any landowner with θ such that $\theta_2^* \leq \theta \leq \theta_1^*$ takes monitoring and curative actions, and any landowner with θ such that $0 \leq \theta \leq \theta_2^*$ takes no action, where θ_1^* and θ_2^* satisfy following two equations.

$$\begin{aligned} U(y-c_1, e_1; \theta_1^*) &= q(x(\theta_1^*))\{(1-r)U(y-c_2-c_3, e_2; \theta_1^*) + rU(y-c_2, e_3; \theta_1^*)\} \\ &+ \{1-q(x(\theta_1^*))\}U(y-c_2, e_1; \theta_1^*) \end{aligned} \quad (4.10)$$

$$\begin{aligned} q(x(\theta_1^*))\left\{\begin{array}{l} (1-r)U(y-c_2-c_3, e_2; \theta_2^*) \\ +rU(y-c_2, e_3; \theta_2^*) \end{array}\right\} &+ \{1-q(x(\theta_1^*))\}U(y-c_2, e_1; \theta_2^*) \\ &= q(x(\theta_1^*))U(y, e_3; \theta_2^*) + \{1-q(x(\theta_1^*))\}U(y, e_1; \theta_2^*) \end{aligned} \quad (4.11)$$

Equilibrium-III: Any landowner with θ such that $\theta_1^* \leq \theta \leq \bar{\theta}$ takes preventive action and any landowner with θ such that $0 \leq \theta \leq \theta_1^*$ takes no action, where θ_1^* satisfies

$$U(y-c_1, e_1; \theta_1^*) = q(x(\theta_1^*))U(y, e_3; \theta_1^*) + \{1-q(x(\theta_1^*))\}U(y, e_1; \theta_1^*) \quad (4.12)$$

and $\theta_1^* = \theta_2^*$. In this case, no monitoring occurs.

4.3.2 Full Information Social Optimum

In this section, we consider the problem for the regulator with full information about landowners' valuations for the environmental property on her property, θ . In particular, the regulator knows the level of θ for each landowner. The objective for the regulator is to maximize social welfare $W(\theta_1, \theta_2)$ defined in a utilitarian fashion as the sum of the utilities in the population by determining the level of two cutoff levels for the valuation of trees θ_1 and θ_2 . Then, landowners engage in the management behavior implied by these cutoff levels. The social welfare function is:

$$\begin{aligned}
W(\theta_1, \theta_2) = & \int_0^{\theta_2} f(z) \left(q(\theta_1)U(y, e_3; z) + (1 - q(\theta_1))U(y, e_1; z) \right) dz \\
& + \int_{\theta_2}^{\theta_1} f(z) \left\{ q(\theta_1) \left((1 - r)U(y - c_2 - c_3, e_2; z) + rU(y - c_2, e_3; z) \right) \right. \\
& \quad \left. + (1 - q(\theta_1))U(y - c_2, e_1; z) \right\} dz \\
& \quad + \int_{\theta_1}^{\bar{\theta}} f(z)U(y - c_1, e_1; z) dz
\end{aligned} \tag{4.13}$$

The first term is the welfare of the landowners who take no action, the second term is the welfare of the landowners who choose to monitor, and the last term is the welfare of landowners who take preventive action.

Definition: A full information social optimum of invasive species management is defined by cutoff values of the valuation parameter θ_1^{**} and θ_2^{**} in the interval of valuation $[0, \bar{\theta}]$ such that $W(\theta_1^{**}, \theta_2^{**}) \geq W(\theta_1, \theta_2)$ for any θ_1 and θ_2 .

Just as with the equilibrium mentioned in the previous subsection, letting the socially optimal level of the cutoff levels for the valuation of trees be θ_1^{**} and θ_2^{**} , the following three cases for the solutions of this problem can be considered.

Case 1: $\theta_1^{**} = \bar{\theta}$ and $0 < \theta_2^{**} < \bar{\theta}$ is optimal.

Case 2: $0 < \theta_1^{**} < \bar{\theta}$ and $0 < \theta_2^{**} < \theta_1^{**}$ is optimal.

Case 3: $0 < \theta_1^{**} < \bar{\theta}$ and $\theta_2^{**} = \theta_1^{**}$ is optimal.

Case 1 is a corner solution of this problem such that no landowner engages in prevention, but there are two groups of landowners. One group's valuation θ of their forest is in the interval $[\theta_2^{**}, \bar{\theta}]$. This group monitors their property and applies a cure if necessary. The other group's valuation θ of their forest is in the interval $[0, \theta_2^{**}]$. This group leaves their forests vulnerable to infestation by the invasive species. The necessary condition for this solution is obtained by taking the derivative of the social welfare function with respect to θ_2 and setting $\theta_1^{**} = \bar{\theta}$:

$$f(\theta_2^{**}) \left[+\bar{q}(1-r)\{u(y-c_2) - u(y-c_2-c_3) + \theta_2^{**}(a(e_3) - a(e_2))\} \right] = 0. \quad (4.14)$$

Case 2 is an interior solution of the problem. In this case we have three groups of landowners. The first group consists of landowners with valuation parameter values lying within the interval $[\theta_1^{**}, \bar{\theta}]$, and who take preventive action. The second group has parameter values within the interval $[\theta_2^{**}, \theta_1^{**}]$ and who choose to monitor. The third is the group of landowners characterized by their valuation being in the interval $[0, \theta_2^{**}]$ with $0 < \theta_2^{**} < \theta_1^{**} < \bar{\theta}$ and who take no action. The first order necessary conditions are

$$\begin{aligned} f(\theta_1^{**})q(\theta_1^{**}) & \left[\frac{(1-r)(u(y-c_2-c_3) - u(y-c_2))}{+\theta_1^{**}((1-r)a(e_2) + ra(e_3) - a(e_1))} \right] \\ & + q'(\theta_1^{**}) \int_{\theta_2^{**}}^{\theta_1^{**}} f(z) \left[\frac{(1-r)(u(y-c_2-c_3) - u(y-c_2))}{+z((1-r)a(e_2) + ra(e_3) - a(e_1))} \right] dz \\ & + q'(\theta_1^{**}) \int_{\theta_2^{**}}^{\theta_1^{**}} f(z)z(a(e_3) - a(e_1))dz = 0 \end{aligned} \quad (4.15)$$

and

$$u(y) - u(y - c_2) + q(\theta_1^*)(1 - r) \left\{ \begin{array}{l} u(y - c_2) - u(y - c_2 - c_3) \\ + \theta_2^{**}(a(e_3) - a(e_2)) \end{array} \right\} = 0. \quad (4.16)$$

Case 3 is a special case of Case 2. In this case no landowners choose to conduct monitoring action. Landowners with valuation of the forest $\theta_1^{**} \leq \theta \leq \bar{\theta}$ conduct preventive action and landowners with valuation of the forest $0 \leq \theta \leq \theta_1^{**}$ conduct no action. The necessary condition for this solution is obtained by taking the derivative of the social welfare function with respect to θ_1^* and setting $\theta_2^{**} = \theta_1^{**}$:

$$f(\theta_1^{**}) \{ \bar{q}c(a(e_3) - a(e_1)) + u(y) - u(y - c_1) \} = 0. \quad (4.17)$$

4.3.3 Subsidy-Tax Scheme

If the regulator has full information about the specific preferences of each landowner, he could directly assign responsibilities to prevent and monitor to specific individuals, thereby attaining the socially optimal solution. This is unlikely, however. If the regulator does not have sufficient information about where any individual landowner is on the valuation continuum, but he does know the distribution of the valuation parameter among the population of landowners, he can design strategies that elicit behavior that leads to the social optimum. In particular, the regulator is assumed to try to maximize social welfare defined in (4.13) by choosing a subsidy (s) for prevention that is financed by taxing all landowners (t) given the best responses of landowners to the subsidy and tax. This subsidy increases the fraction of landowners choosing to prevent, thus reducing the probability of spread. The outcome of the best response of landowners is summarized in

the cutoff points $\theta_1^*(s, t)$ and $\theta_2^*(s, t)$. As is discussed in the subsection 3.1, the best response of landowners results in three types of equilibrium: Equilibrium-I, -II, and -III. Thus by plugging $\theta_1^*(s, t)$ and $\theta_2^*(s, t)$ into the necessary conditions (Equation (4.8), (4.9), (4.10), (4.11), and (4.12)) for the equilibria, we can incorporate the best response of landowners to the subsidy and tax into the constraints of our model. That is, we have the following three sets of conditions.

Best Response-I of Landowners (Corresponding to Equilibrium-I): No landowner takes preventive action (i.e., $\theta_1^*(s, t) = \bar{\theta}$). Any landowner with θ such that $\theta_2^*(s, t) \leq \theta \leq \bar{\theta}$ chooses to monitor and any landowner with θ such that $0 \leq \theta \leq \theta_2^*(s, t)$ takes no action, where

$$\theta_2^*(s, t) = \frac{1}{a(e_2)} \left\{ \frac{u(y-t) - u(y-c_2-t)}{\bar{q}(1-r)} + u(y-c_2-t) - u(y-c_2-c_3-t) \right\} \quad (4.18)$$

$$\theta_1^* = \bar{\theta}. \quad (4.19)$$

Best Response-II of Landowners (Corresponding to Equilibrium-II): Any landowner with θ such that $\theta_1^*(s, t) \leq \theta \leq \bar{\theta}$ takes preventive action, any landowner with θ such that $\theta_2^*(s, t) \leq \theta \leq \theta_1^*$ takes monitoring and curative actions, and any landowner with θ such that $0 \leq \theta \leq \theta_2^*(s, t)$ takes no action, where $\theta_1^*(s, t)$ and $\theta_2^*(s, t)$ satisfy following two equations:

$$U(y - c_1 + s, e_1; \theta_1^*(s, t)) = q \left(x(\theta_1^*(s, t)) \right) \left\{ \begin{array}{l} (1-r)U(y - c_2 - c_3 - t, e_2; \theta_1^*(s, t)) \\ + rU(y - c_2 - t, e_3; \theta_1^*(s, t)) \end{array} \right\} \\ + \left\{ 1 - q \left(x(\theta_1^*(s, t)) \right) \right\} U(y - c_2 - t, e_1; \theta_1^*(s, t)) \quad (4.20)$$

and

$$\begin{aligned}
& q\left(x(\theta_1^*(s, t))\right)\left\{(1-r)U\left(y-c_2-c_3-t, e_2; \theta_2^*(s, t)\right)+rU\left(y-c_2-t, e_3; \theta_2^*(s, t)\right)\right\} \\
& \quad +\left\{1-q\left(x(\theta_1^*(s, t))\right)\right\}U\left(y-c_2-t, e_1; \theta_2^*(s, t)\right) \\
& = q\left(x(\theta_1^*(s, t))\right)U\left(y-t, e_3; \theta_2^*(s, t)\right)+\left\{1-q\left(x(\theta_1^*(s, t))\right)\right\}U\left(y-t, e_1; \theta_2^*(s, t)\right).
\end{aligned} \tag{4.21}$$

Best Response-III of Landowners (Corresponding to Equilibrium-III): Any landowner with θ such that $\theta_1^*(s, t) \leq \theta \leq \bar{\theta}$ takes preventive action and any landowner with θ such that $0 \leq \theta \leq \theta_1^*(s, t)$ takes no action, where $\theta_1^*(s, t)$ satisfies

$$\begin{aligned}
U\left(y-c_1+s, e_1; \theta_1^*(s, t)\right) & = q\left(x(\theta_1^*(s, t))\right)U\left(y-t, e_3; \theta_1^*(s, t)\right) \\
& \quad +\left\{1-q\left(x(\theta_1^*(s, t))\right)\right\}U\left(y-t, e_1; \theta_1^*(s, t)\right)
\end{aligned} \tag{4.22}$$

and

$$\theta_2^*(s, t) = \theta_1^*(s, t). \tag{4.23}$$

Moreover, the combined tax/subsidy program is *revenue neutral*, i.e. that tax collections equal subsidy payments. This revenue constraint requires that

$$s\left(1-x\left(\theta_1^*(s, t)\right)\right)-tx\left(\theta_1^*(s, t)\right)=0 \tag{4.24}$$

Thus the regulator maximizes the following social welfare function which takes account of the best response of landowners given by the function (4.25) below subject to the system of best responses of landowners Equations (4.18)-(4.23) and the revenue neutral constraint (4.24).

$$\begin{aligned}
& W(s, t) \\
&= \int_0^{\theta_2^*} f(z)[q(\theta_1^*)U(y - t, e_3; z) + (1 - q(\theta_1^*))U(y - t, e_1; z)]dz \\
&+ \int_{\theta_2^*}^{\theta_1^*} f(z) \left[\begin{aligned} & q(\theta_1^*)\{(1 - r)U(y - c_2 - c_3 - t, e_2; z) + rU(y - c_2 - t, e_3; z)\} \\ & + (1 - q(\theta_1^*))U(y - c_2 - t, e_1; z) \end{aligned} \right] dz \\
&+ \int_{\theta_1^*}^{\bar{\theta}} f(z)U(y - c_1 + s, e_1; z)dz
\end{aligned} \tag{4.25}$$

where $\theta_1^* = \theta_1^*(s, t)$ and $\theta_2^* = \theta_2^*(s, t)$.

In the next section, we derive explicit solutions by solving this problem numerically with simulations, and examine the characteristics of these solutions.

4.4 Simulations

In this section, we specify functional forms for the landowners' utility functions, the probability density function for the valuation parameter, and relationship between the probability of infestation and the fraction of the population taking preventive action. Then, we numerically derive the fractions of the population taking preventive, monitoring and curative, and no actions in the landowner equilibrium. We also solve for social welfare in both the landowners' equilibrium and the full information social optimum. We find the subsidy and tax levels that would lead to the maximum level of social welfare. We explore how these levels change with changes in the accuracy of monitoring. We also examine how changes in costs associated with the management options would change the fractions of the population making alternative choices. Finally, we discuss implications

involved with combining policies to improve the accuracy of monitoring with subsidy and tax policies.

4.4.1 Functional Forms

We choose a linear functional form to describe the relationship between the probability of infestation q and the size of population not taking preventive action, $x(\theta_1)$:

$$q(x(\theta_1)) = \bar{q}x(\theta_1).$$

We assume a uniform distribution of θ so that we have

$$x(\theta) = F(\theta) = \frac{\theta}{\bar{\theta}}.$$

We assume that the utility obtained from consuming goods to be linear in income left over after the cost of treatment is incurred,

$$u(y - c) = \mu(y - c),$$

where c is cost of the chosen management option. We also assume that the utility each landowner derives from the amenity value of her forest is linear in the size of the forest:

$$\theta a(e) = \theta e.$$

These functional forms satisfy the conditions specified above and generate the types of equilibria outlined in the first three subsections (landowners' equilibrium, full information social optimum, equilibrium under subsidy and tax scheme when regulator has only incomplete information) with suitably chosen parameter values. The parameter values for the analysis are summarized in Table 4.1; the first column shows the parameter values of the benchmark case and the remaining columns show parameter values used for the sensitivity analysis (Cases 1 to 3).

Table 4.1: Base Case Parameter Values

Parameter name	Symbol	Parameter value				
		Benchmark	Case1	Case2	Case3	Case4
Size of endowed forest	e_1	100	100	100	100	100
Size of forest infested and partially damaged	e_2	80	<u>60</u>	80	80	80
Size of forest infested and totally damaged	e_3	20	20	20	20	20
Budget for consumption goods	y	100	100	100	100	100
Cost of preventive treatment	c_1	60	60	<u>80</u>	60	60
Cost of monitoring	c_2	10	10	10	<u>40</u>	10
Cost of curative treatment	c_3	5	5	5	5	<u>20</u>
Upper bound for the valuation of the forest	$\bar{\theta}$	100	100	100	100	100
Coefficient for the utility	μ	50	50	50	50	50
Upper bound for the probability of infestation	\bar{q}	1	1	1	1	1

4.4.2 Landowner Equilibrium and Full Information Social Optimum

First we consider the benchmark case of the landowners' equilibrium and full information social optimum. In Figure 4.1, the vertical axis shows the level of valuation of the trees of landowners (θ) where $\bar{\theta} = 100$, and the horizontal axis shows the probabilities that monitoring fails (r); the accuracy of the monitoring is high at the origin and gets lower along the x-axis. The panels of Figure 4.1 show the regions in which landowners choose prevention, monitoring, and no action as a function of their valuation parameter θ and the probability of failing to find an existing infestation given monitoring (r). Panel 1 shows the landowners' equilibrium and Panel 2 shows the full information social optimum. In addition, this figure shows the cutoff values θ_1^* and θ_2^* of the landowners' equilibrium (Panel 1) and θ_1^{**} and θ_2^{**} under the full information optimum (Panel 2) as a function of r .

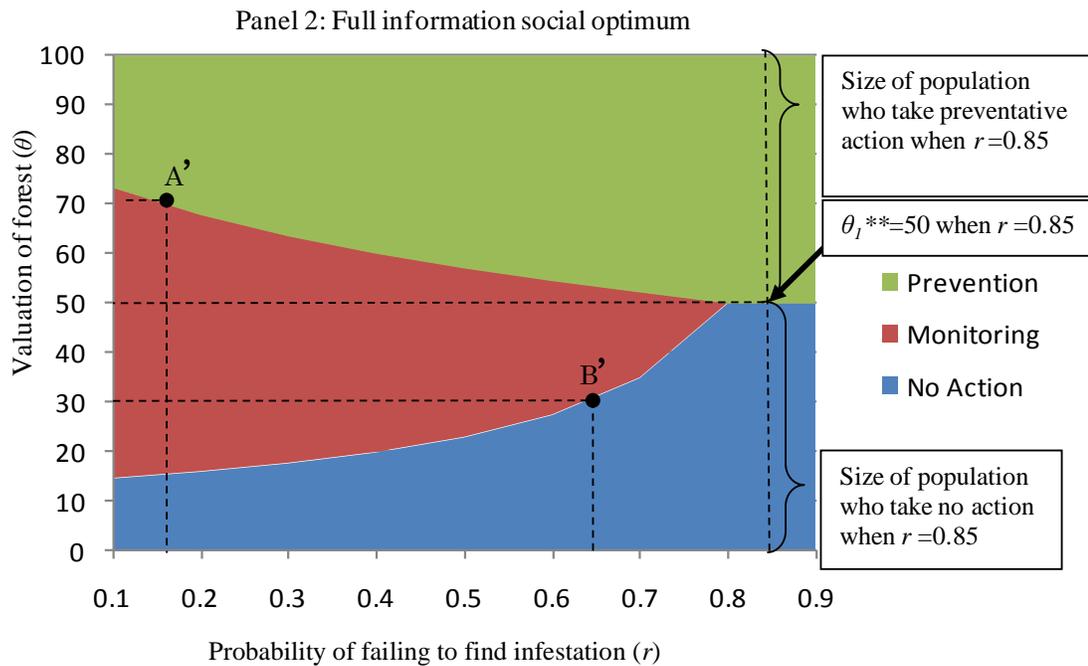
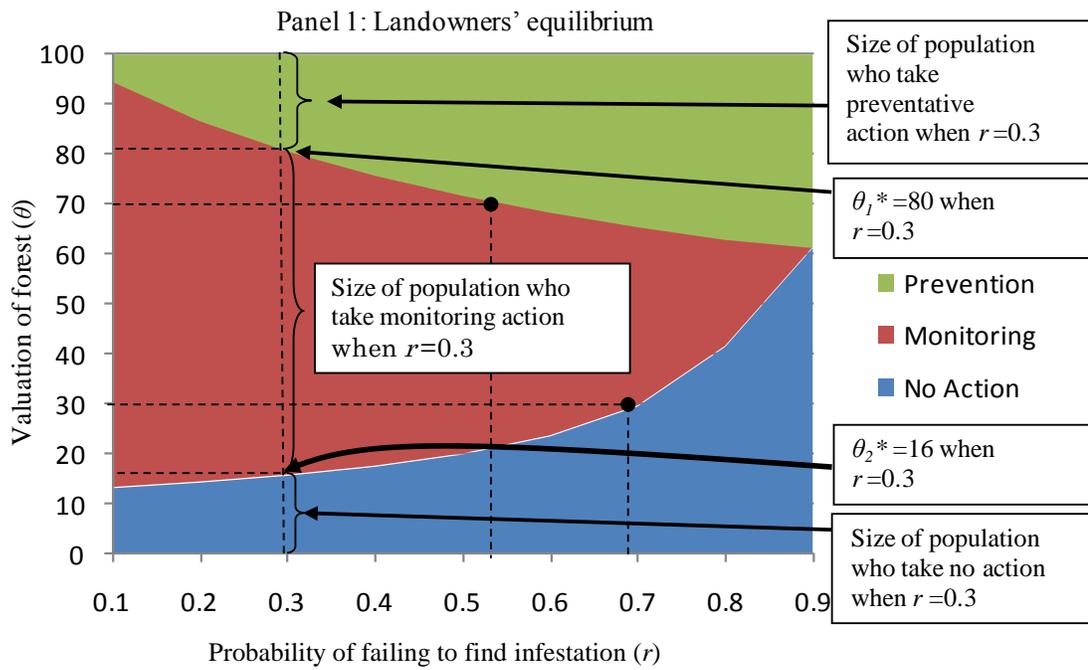


Figure 4.1: Population Structure with Benchmark Parameter Values

For each level of probability of failing to find the infestation, the vertical distance of the green area is the fraction of the population who conducts preventive action, the vertical distance of the red area is the fraction of the population who chooses to monitor, and the vertical distance in the blue area shows the fraction of the population who takes no action. The boundary between the green area and red area or green area and blue area is θ_1^* on Panel 1 and θ_1^{**} on Panel 2. The boundary between the red area and the blue area is θ_2^{**} on Panel 1 and θ_2^* on Panel 2. θ_1^* and θ_2^* on Panel 1 (or θ_1^* and θ_2^* on Panel 2) coincide when the green and blue areas touch each other.

Panel 1 of Figure 4.1 shows that more landowners tend to take preventive action, fewer landowners tend to take monitoring action, and more landowners leave their trees completely vulnerable by not taking either of these actions in the landowner equilibrium as the accuracy of monitoring gets lower. The same tendency is observed in the case of full information optimum. In other words, it is socially optimal for landowners with relatively high forest values to keep preventing the infestation when monitoring gets less accurate, but it is socially optimal for landowners with slightly lower values to switch their action from monitoring to prevention or from monitoring to no-action as the accuracy of the monitoring goes down.

We observe several differences between the landowner equilibrium and the socially optimal solution. First, the monitoring region is smaller in Panel 2 which represents the social optimum. Because the monitor/cure strategy does not reduce the prevalence of the infestation, it does not attenuate the externality caused by landowners taking no action. However, we can find the following difference between Panel 1 and 2: when the regulator knows which landowner has what valuation on her forest, the

regulator forces the landowners with high valuation to switch their actions from monitoring to preventive actions even when the accuracy of monitoring is relatively high. Also, the regulator forces landowners with low valuation to switch their action from monitoring to no action at relatively high accuracy of monitoring. For example the landowner with valuation of her forest $\theta = 70$ (point A in Panel 1 of Figure 4.1) changes her action from monitoring to prevention when the probability of failing to find the infestation becomes less than 0.53. However, the regulator would force the same landowner to switch her action from monitoring to prevention when the probability gets lower than 0.13 (point A' in Panel 2 of Figure 4.1) in order to get social welfare maximized. This gap causes, through the probability of infestation $p(x(\theta_1))$, the landowners with lower valuation on their forests to switch from monitoring to no action at higher accuracy rates in the private optimum than in the social optimum. For example, the landowner with $\theta = 30$ switches her action from monitoring to no action only after the accuracy gets lower than 0.7 (point B in Panel 1). On the other hand, it is socially optimal to change their action when the accuracy gets lower than 0.65 (point B' in Panel 2). This is because, in landowners' equilibrium, fewer landowners with high valuation conduct prevention, and thus more landowners with low valuation need to protect their forests by monitoring instead of leaving their forests vulnerable to the invasive species.

4.4.3 Sensitivity Analysis

Now, we consider the sensitivity of cutoff values and social welfare to biological and economic parameters. We conduct four sensitivity analyses using parameters shown in the last four columns of Table 4.1. Our sensitivity analysis reveals how changes in

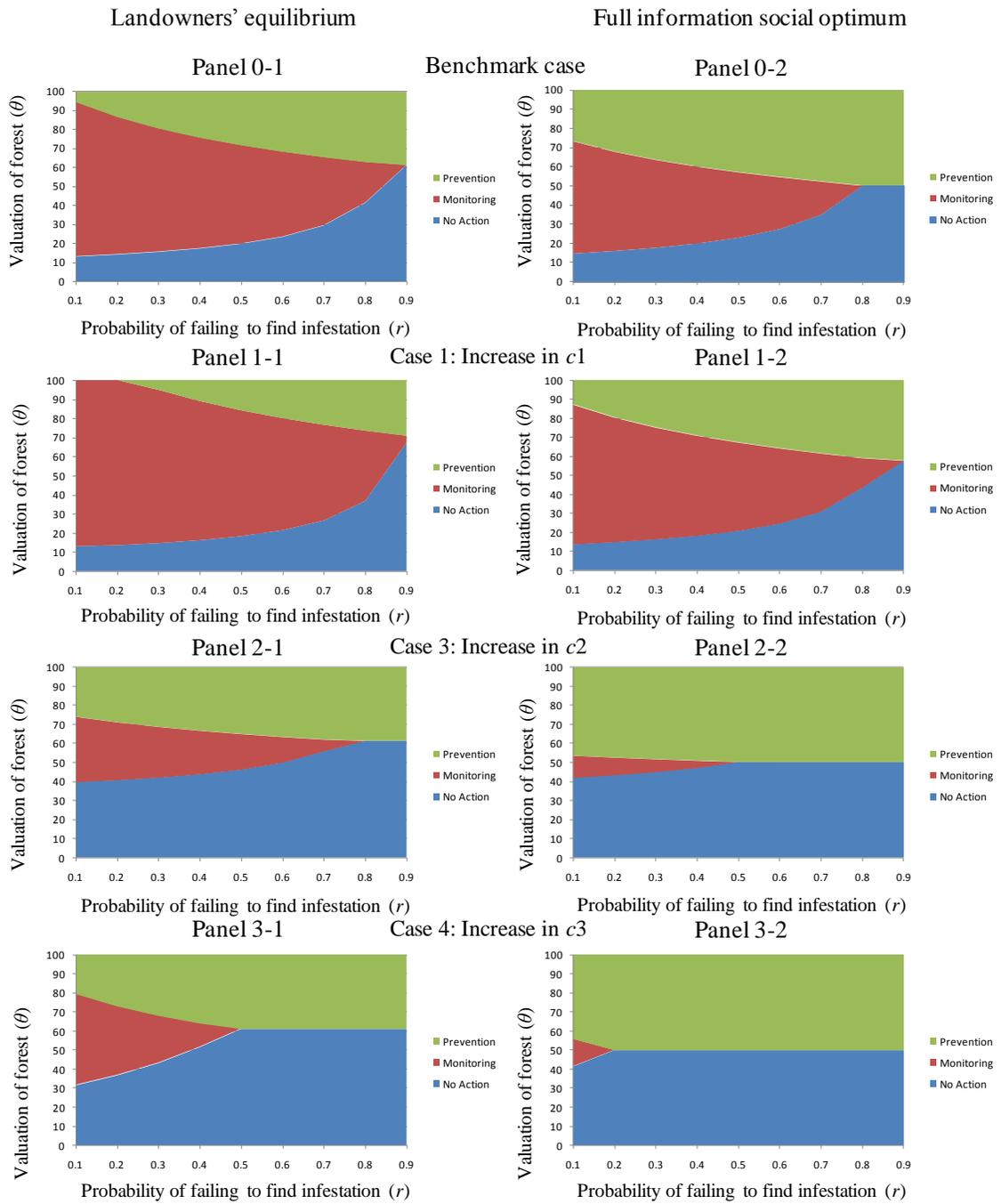


Figure 4.2: Results of the Sensitivity Analysis

each parameter affect our benchmark case results. The results are presented in Figure 4.2. We examine the effect of the change in economic parameters on the fraction of the population conducting preventive, monitoring and no actions. The first column of Figure 4.2 shows the results of sensitivity analyses for the landowners' equilibrium, and the second column reports results for the full information social optimum. The rows indicate the different cases.

Case 1: Increase in cost of preventive treatment, c_1 :

We increase costs of preventive treatment, c_1 , from 60 to 80. The results of landowners' equilibrium are shown in Panel 1-1 in Figure 4.2. Compared to benchmark case, this lowers the incentive of landowners with relatively high valuation of forest to take preventive action and those landowners switch their action from preventive action to monitoring and curative actions. In particular, as is shown in the second row and the first column of the Figure 4.2, when the probability of failing to find infestation is low enough, no landowners takes preventive action. This change in behavior of landowners with higher valuations affects the behavior of landowners with relatively low valuations. A decrease in the fraction of the population taking preventive action increases the probability of infestation. This makes it costlier for landowners with a relatively low valuation of the forest who take no action in the benchmark case to keep taking no action in Case 1. That is, an increase in the probability of infestation causes their expected net benefit obtained from taking monitoring and curative action to exceed their benefit obtained from taking no action. Therefore they now choose a monitoring strategy so that they can protect their forests by finding a more likely infestation in early stage rather than

leaving their forest vulnerable to the infestation. Thus the proportion of population taking monitoring and curative action increases. However, when the accuracy of monitoring becomes very low, landowners with relatively low valuation of forest taking monitoring and curative actions switch their behavior to no action. Thus, the proportion of population taking no action rapidly increases and the proportion choosing to monitor decreases relative to the baseline.

In the full information social optimum, the proportion of population taking preventive action shrinks when the cost of prevention increases. However, compared to the case of landowners' equilibrium, the shrinkage is small. This is because the positive externality of preventive action is taken into account in order to maximize social welfare. Relative to the landowner equilibrium, more landowners with high forest valuations fall into the "prevention" category. This protects the high value forests, and is also the most cost effective way to indirectly protect lower-valued forests through lowering the probability of infestation. As with the landowner equilibrium, landowners with moderately low valuation taking no action in benchmark case will use the monitoring strategy when the cost of prevention increases. The population with very low valuations still takes no action. These landowners allocate their budgets to the consumption good.

Case 2 and 3: Increase in costs of monitoring, c_2 , and curative treatment, c_3 :

Increases in either the cost of monitoring, c_2 , or the cost of curative treatment, c_3 , or both costs together cause monitoring and curative actions to be a less attractive option for landowners. As is shown in Table 4.1, we examined a case where the monitoring cost increases from 10 to 40 (Case 3) and a case where curative treatment cost increases from

5 to 20 (Case 4). Results are reported in Panels 3 and 4 in Figure 4.2. In both cases, landowners with moderately high valuation parameters switch their action from monitoring and curative action to preventive action even when accuracy of monitoring is very high (e.g., $r = 0.9$). This increase in the proportion taking preventive action results in a lower probability of infestation, making it less attractive for landowners with moderately low valuation parameters to choose the monitor/cure strategy. Therefore, they switch from monitor/cure to no action. Thus, the proportion of the population choosing to monitor and cure shrinks compared to benchmark case. Also, the monitor/cure strategy disappears from the population even when the accuracy of monitoring is relatively high.

Relative to the benchmark, a higher fraction of the population engages in prevention when the monitoring and curative costs are higher. Because the monitor/cure strategy is less attractive when it's costlier, a smaller fraction of the population chooses this option and more trees are lost. Reducing the spread rate through more prevention counteracts this effect. The population with low valuation parameters takes no action and enjoys the externality of preventive action taken by landowners with high forest valuations.

4.4.4 Social Welfare

Figure 4.3 shows social welfare obtained in landowners' equilibrium and that obtained in the full information social optimum. First, social welfare in both cases generally increases as the probability of failing to find the infestation increases. As discussed above, in both landowners' equilibrium and full information social optimum, a decrease in accuracy leads to an increase in the proportion of the population taking

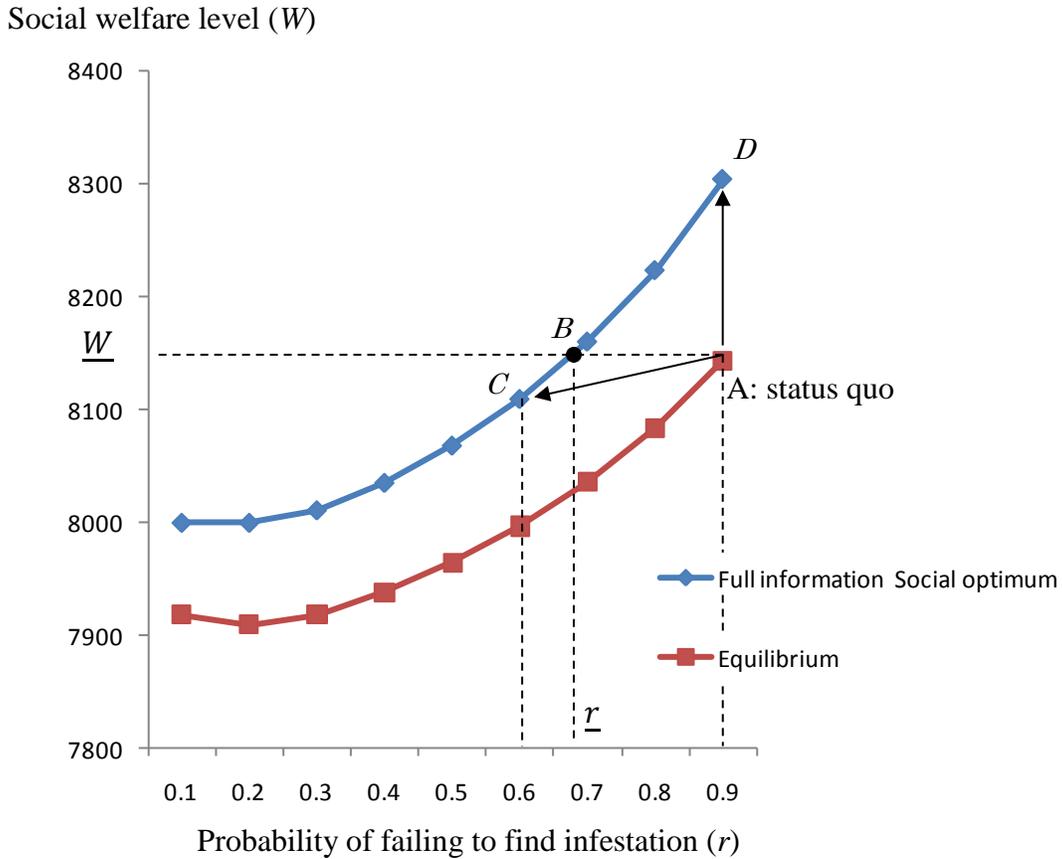


Figure 4.3: Social Welfare and the Accuracy of Monitoring

preventive action, thereby reducing the probability of infestation. This decrease in the probability of infestation increases the welfare of landowners choosing a monitor/cure strategy and those who choose to take no action. It also increases the welfare of those landowners who chose to switch from a monitor/cure strategy to a no action strategy. There are two sources of welfare improvement. First, an increase in the probability of maintaining a healthy forest increases welfare directly. Also, a decrease in the accuracy of monitoring makes it more likely for landowners taking monitoring and curative actions to fail to find the infestation in the early stage. This would tend to negatively to their welfare. However, this negative effect is mitigated by the decrease in probability of

infestation. Also, because of the decrease in the probability of infestation, landowners may choose to switch from monitor/cure to a do-nothing strategy, thereby saving the direct cost— c_2 and c_3 .

It is the differences in the cutoff values of θ that generate the gap in social welfare between the landowners' equilibrium and the full information social optimum. Defining ΔW as the discrepancy between social welfare in landowners' equilibrium and that in full information social optimum,

$$\Delta W \equiv W(\theta_1^{**}, \theta_2^{**}) - W(\theta_1^*, \theta_2^*),$$

we can observe that the discrepancy, ΔW , becomes larger as the accuracy of monitoring decreases in Figure 4.3. This implies that increasing the accuracy of monitoring of landowners may help to close the gap and make the social welfare level in landowner equilibrium close to that in full information social optimum. In Subsection 4.6., we discuss this more fully.

4.4.5 Optimal Subsidy and Tax

Optimal tax and subsidy levels associated with the accuracy of the monitoring of infestation are shown in Figure 4.4. These values are derived using the parameter values of the benchmark case in Table 4.1. When the probability of failing to find infestation is relatively high, the subsidy level starts from a high level ($s^* = 15$), decreases as the probability of failing to find infestation increases, and then remains the same value ($s^* = 10$). At the same time, optimal tax level starts from low level ($t^* = 4.5$), increases as probability of failing to find infestation increases, and stops its increase at the same value ($t^* = 10$). This has an intuitive explanation. As accuracy of monitoring decreases,

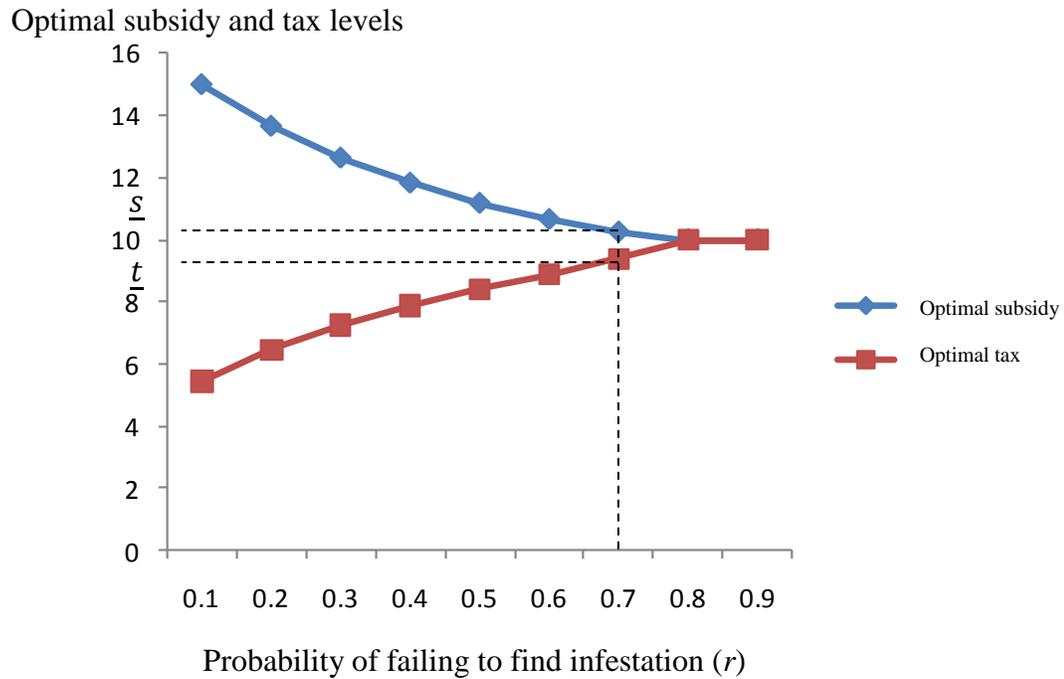


Figure 4.4: Change in Optimal Tax and Subsidy Levels Associated with the Accuracy of Monitoring

the proportion of the population who voluntarily take preventive action increases, thereby reducing the need to subsidize preventive behavior. On the other hand, the proportion of population taking monitoring and curative actions or no action decreases, and therefore by the revenue neutral condition, $tx(\theta_1^*) = s(1 - x(\theta_1^*))$, the tax burden to each landowner increases.

4.4.6 Improving the Accuracy of Monitoring

In this section, we consider mixing a policy of controlling the accuracy of monitoring with a policy inducing landowners to take preventive action. Consider four separate management options to control tree disease. The first two options are related to

making the probability of introduction and spread of disease small. One is establishing barrier zone to shield her community from other communities that have already been infested. The other is increasing the proportion of landowners who take preventive action by using a subsidy and tax scheme. The remaining two options focus on finding trees in the early stages of infestation and curing the trees. To find the infestation in early stage, a manager can conduct surveillance in a community and notify landowners when their trees are affected and give them an opportunity to cure their trees. Finally, the manager can inform landowners about how to identify the early symptoms of the infestation and increase the probability that the landowner will find the disease themselves. In our model, we focus on just two of these four options, omitting the policy of establishing a barrier zone and the comprehensive surveillance option. Also, the accuracy level of monitoring was taken to be exogenous in the model. Our focus was on devising a subsidy and tax scheme to encourage optimal behavior on the part of participants. We derived optimal levels of the subsidy and tax, and we also calculated the optimal level of social welfare achievable using these subsidy and tax rates for each accuracy level of monitoring. We can examine the usefulness of an education campaign that would provide landowners with information about how to identify the early symptom of infestation so that the accuracy level of monitoring can be improved.

As we discussed in Subsection 4.4., we know that, when monitoring is more accurate, social welfare in landowners' equilibrium is closer to that in full information social optimum. However, it is also true that more accurate monitoring leads to less social welfare in full information optimum. Now, the question is how much improvement in the accuracy of monitoring is beneficial. Suppose that current economic and biological

parameters of a given community follow the values of the benchmark case given in Table 4.1. Also, suppose that the current probability of failing to find the infestation is $r = 0.9$ so that the current social welfare level in the landowners' equilibrium is $W = 8140$ which is at point A in Figure 4.3. Moreover, suppose that, without any cost, a policy maker can educate landowners about how to identify early symptoms of infestation so that the probability of successfully finding and curing an infestation increases. The answer is the accuracy of monitoring can be improved to the level where probability of failing to find the infestation becomes $r = \underline{r}$ in Figure 4.3, since $r = \underline{r}$ can guarantee at least the current social welfare level $W = \underline{W} = 8140$ to be achievable in full information social optimum, which is point B by setting subsidy and tax level $s^* = \underline{s}$ and $t^* = \underline{t}$ in Figure 4.4 respectively. If she improves the accuracy of monitoring too much so that probability of failing to find infestation becomes less than $r = \underline{r}$, the maximum achievable social welfare becomes less than $W = \underline{W}$ (e.g., point C in Figure 4.3). However, as is shown in Figure 4.4, setting subsidy and tax as $s^* = 10$ and $t^* = 10$ and leaving the probability of failing to find the infestation to the current level (i.e., $r = 0.9$), a policy maker can achieve social welfare to be 8300 which is point D in Figure 4.3. Therefore, there is no reason for the policy maker to improve the information accuracy level.

4.5 Conclusion

In this paper, we considered a model of landowners' decisions about how to protect their trees from disease. In our model, landowners get utility from their own trees and consumption of a good. Landowners are heterogeneous in terms of their valuation of

trees, and their trees are vulnerable to an invasive disease. Each landowner selects their action from three distinct ways to react to disease; (1) prevention, (2) monitoring and cure, and (3) no action. If a landowner takes preventive action, her trees are protected from infestation for sure. For other landowners, infestation is uncertain. The probability that trees become infested is determined by the proportion of population who does not take preventive action. Damage of trees can be mitigated with a curative treatment. To give such curative treatment to trees, landowners need to monitor the health of trees and find early symptoms of infestation. Both preventive action and monitoring and curative action are associated with costs. Incurring these costs reduces the purchase of the consumption good resulting in less utility gained from consumption.

Following this model, we use specific functional forms and parameter values to derive exact numerical solutions by running simulations. We perform a series of sensitivity analyses by changing the economic parameters to shed light on optimal policies for invasive species management. We explore the change in the proportion of the population choosing these three treatments in both the landowners' equilibrium and the full information social optimum as monitoring accuracy changes. We find that the proportion of landowners choosing preventive and no action dominates the proportion of landowners taking monitoring action as the accuracy of monitoring decreases. We also explore a possible policy which would raise the accuracy of monitoring. We find that, in both the landowners' equilibrium and the full information social optimum, the social welfare level decreases as the accuracy of monitoring increases. Therefore, our analysis suggests that it is not favorable for the regulator to improve the accuracy of monitoring from any level of given accuracy.

While this paper provides useful insights, we acknowledge several caveats. In this paper, analyses are based on a static model. However, using a static model ignores several important aspects of the problem. In a dynamic environment, monitoring might give more benefits to landowners than in a static environment. For example, a landowner may want to leave trees vulnerable to infection in order to learn about the frequency of infestation. After learning this probability, she may decide whether to take preventive action after taking account of the proportion of forests left healthy and the budget available for treatment and for the consumption good. This behavior may enable a landowner to save funds that would otherwise be devoted to prevention. With this behavior, increasing the accuracy of monitoring may not necessarily decrease social welfare. A dynamic model of information gathering may be a useful next step.

Chapter 5

Concluding Remarks

5.1. Summary of the Studies

This thesis investigated optimal management of invasive forest pests. We first explored the optimal invasive pest management from a sole ecosystem manager perspective, which corresponds to the management scheme instituting large scale government-funded management programs. We especially focused on the issues raised by the order of pest management activities; an ecosystem manager needs to detect invasive pests through surveillance before conducting pest control. In Chapter 2, we focused on the timing issue raised by this order of these activities. Using optimal control theory and a numerical simulation, we studied the optimal timing of switching from surveillance to control when the remaining time to manage the pests is limited. Also, we investigated the relationship between optimal pest control strategies--suppression eradication of the sub population--and the remaining time left before the population front reaches a given location of interest. We found that more time available for management may justify a more aggressive approach to finding and controlling the pest. Because we embed a dynamically optimal pest management strategy into the problem of determining the optimal level of detection effort, we get the unexpected result that, at some point, it is no longer optimal to increase detection effort. This is because eradication becomes the

optimal strategy and the time constraint imposed by the date of arrival of the advancing front no longer has an effect on the optimization problem.

In the study of Chapter 3, we took an alternative approach to the issue raised by the order of the surveillance and pest control. Here we focused on the location issue; pest control can be conducted only on the sites where surveillance has been conducted. This location issue is important for the case of managing invasive forest pathogens such as oak wilt, because, for pathogens, the detection process of pathogen management needs to exactly identify an infected tree in each location.

We used a mixed integer programming approach, inspired by the site selection literature, to choose locations in a grid on which to focus detection and control efforts in a budget-constrained setting. Locations vary by the number of susceptible trees and the proportion of infested trees. The primary contribution of our study was to offer a tool suggesting practical guidance to managers in charge of deciding how and where to spend limited public dollars when the goal is to reduce the number of trees newly infected by invasive pathogen. We applied our model to the case of Anoka County, Minnesota, a county which suffers from a severe oak wilt infestation. We used our model to determine which sites within Anoka County should be given highest priority for inspection and removal.

We construct a curve that reflects the cost of protecting healthy trees from infection. This curve provides the manager with information about the level of budgets required to achieve targeted percentages of healthy trees saved from infection. The marginal cost of saving an additional healthy tree from the pathogen increases as the targeted protection level is set higher.

We also investigated characteristics of sites selected for the surveillance. We ranked sites by the ratio of the expected number of infected trees to the cost of inspecting every tree on the parcel. We find that this characteristic can predict which sites to be selected for the surveillance as the budget increases, although the prediction is not necessarily perfect. This implies that it is best to conduct surveillance with priority on sites with relatively low expected unit surveillance cost per tree saved from infection. Thus, as the desired fraction of healthy trees saved from infection increases, lower priority sites with higher costs begin to be selected. This drives marginal costs up. This is consistent with the observation that the fraction of surveillance cost over the total cost is upward sloping in the targeted fraction of healthy trees saved from infection.

In Chapter 4, we changed our perspective of management from a sole system manager to multiple landowners' management of invasive pests. We considered a model analyzing landowners' decisions about how to protect their trees from disease. In our model, landowners get utility from their own trees and consumption of a good. Landowners are heterogeneous in terms of their valuation of trees, and their trees are vulnerable to an invasive disease. Each landowner selects their action from three distinct ways to react to disease; (1) prevention, (2) monitoring and cure, and (3) no action. If a landowner takes preventive action, her trees are protected from infestation for sure. For other landowners, infestation is uncertain. The probability that trees of become infested is determined by the proportion of population who does not take preventive action. Damage of trees can be mitigated with a curative treatment. To give such curative treatment to trees, landowners need to monitor the health of trees and find early symptoms of infestation. Both preventive action and monitoring and curative action are associated with

costs. Incurring these costs reduces the purchase of the consumption good resulting in less utility gained from consumption.

Following this model, we use specific functional forms and parameter values to derive exact numerical solutions by running simulations. We perform a series of sensitivity analyses by changing the economic parameters to shed light on optimal policies for invasive species management. We explore the change in the proportion of the population choosing these three treatments in both the landowners' equilibrium and the full information social optimum as monitoring accuracy changes. We find that the proportion of landowners choosing preventive and no action dominate the proportion of landowners taking monitoring action as the accuracy of monitoring decreases. We also explore a possible policy which would raise the accuracy of monitoring. We find that, in both the landowners' equilibrium and the full information social optimum, the social welfare level decreases as the accuracy of monitoring increases. Therefore, our analysis suggests that it may not be favorable for the regulator to improve the accuracy of monitoring from any given level of accuracy.

5.2 Limitations and Possible Extensions

While the studies in this thesis provide useful insights, we acknowledge several caveats and possible extensions associated with the caveats. We can give more options to an ecosystem manager in the model in Chapter 2 to minimize the damage cost from the sub-population and the pest surveillance and pest control costs. For example, we can let the ecosystem manager to be able to control the spread of the population front. In Chapter 2, we focused only on managing the sub-population. This implies that the remaining time

before the population front arrives at a given location of interest is exogenous, which indeed is consistent with the assumption we set in these model. However, in reality, the ecosystem manager may benefit by controlling the advance of population front. First, slowing the advance of population front would extend the remaining time to manage the sub-population. Second, managing the population front may be able to suppress the dispersal of pests from the population front, which results in suppressing the increase in the size of the sub-population. Thus, it is plausible to consider that an ecosystem manager has an incentive to control the population front in order to manage the sub-population.

Controlling the advancing population front also increases the total management cost, which is the tradeoff for the ecosystem manager to gain the benefits of a slower-moving front. This new tradeoff, which is not presented in the model in Chapter 2, would significantly affect the surveillance intensities to detect sub-population and the following pest control strategies. New predictions of the surveillance intensities and pest control strategies obtained from this improved model would provides us with more useful and detailed information about invasive pest management than the predictions obtained from the current model.

We also can improve the model of Chapter 3 in a meaningful way. Recall that, in the chapter, the site selection model developed for the invasive pathogen management incorporated a set of scenarios of the current proportions of infected trees. The set of scenarios is the approximation of beliefs about distribution of the true proportion of infected trees on each site. This approximation may or may not be a correct way to characterize how managers form beliefs.

To prepare the set of scenarios, we took three steps. In the first step, we estimated proportions of infected trees for each site by using the information on size of active oak wilt infection pocket obtained from compliance data. In the second step, we formulated a beta distribution defined over a compact interval $[0,1]$ for each site where the parameters characterizing the beta distribution are calculated by using the estimated proportions of infected trees. This beta distribution for each site is the belief about distribution of the true proportion of infected trees. In the final step, we randomly drew numbers from an interval $[0,1]$ following the beta distribution for a large number of times (i.e., 2000 times). For each draw, we obtained a vector of scenarios. Collecting these scenario vectors, we obtained the set of scenarios. Note that the compliance data used in the first step to prepare the set of scenarios reflects only the infection pockets reported by tree owners or city foresters. This implies that the number of infection pockets may be under-reported on some sites. If so, there may be errors in the sets of scenarios generated using this method.

In the model of Chapter 3, by not taking account for the errors in the set of scenarios, it is likely, as the budget constraint becomes strict, that the solution of the site selection model may miss the sites which should be selected with priority. It is important to avoid such mistakes, because the aim of our study in Chapter 3 is to establish a model providing us with practical information about the surveillance and the removal of infected trees.

In the static model developed in Chapter 4, each landowner was assumed to have three options to protect trees on her land: preventive action, monitoring and curative action, and leaving trees vulnerable (i.e., doing nothing for trees). By taking monitoring action, a landowner leaves her trees vulnerable to the infestation by invasive pests; her

trees may get infested with some probability. However, at the same time, the monitoring action leaves her the option to protect her trees from further damages by spending extra costs if the trees get actually infested as well as the option to save such costs if the trees do not get infested. These are the benefits of having the options obtained by monitoring in the model.

However, in reality, by monitoring, a landowner can receive more benefits than we assumed in the model. For example, a landowner may want to leave trees vulnerable to infection in order to learn about the frequency of infestation. After learning this probability, she may decide whether to take preventive action after taking account of the proportion of forests left healthy and the budget available for treatment and for the consumption good. This behavior may enable a landowner to save funds that would otherwise be devoted to prevention. With this behavior, increasing the accuracy of monitoring may not necessarily decrease social welfare as is predicted in the model in chapter 4. However, the benefits of “learning by monitoring” can be investigated only in the dynamic model. Thus, a dynamic model of information gathering may be a useful next step.

References

- Ando, A., Camm, J., Polasky, S., & Solow, A. (1998). Species distributions, land values, and efficient conservation. *Science*, 279, 2126-2128.
- Burnett, K. M., Kaiser, B. A., & Roumasset, J. A. (2007). Invasive species control over space and time: *Miconia Calvescens* on Oahu, Hawaii. *Journal of Agricultural and Applied Economics*, 39, 125-132.
- Brito, D. L., Sheshinski, E., & Intriligator, M. (1991). Externalities and compulsory vaccinations. *Journal of Public Economics*, 45, 69-90.
- Church, R. L., Stoms, D. M., & Davis, F. W. (1996). Reserve selection as a maximal covering location problem. *Biological Conservation*, 76, 105-112.
- Clinton, W. J. (1999). Executive Order 13112. *Federal Register*, 64, 6183-6185.
- Costello, C., & McAusland, C. (2003). Protectionism, trade, and measures of damage from exotic species introductions. *American Journal of Agricultural Economics*, 85, 964-975.
- Coulson, R. N., & Witter, J. A. (1984). *Forest entomology: ecology and management*, Wiley-Interscience 2nd Edition.
- Eiswerth, M. E., & Johnson, W. S. (2002). Managing nonindigenous invasive species: insights from dynamic analysis. *Environmental and Resource Economics*, 23, 319-342.
- Epanchin-Neil, R. S., Hufford, M. B., Aslan, C. E., Sexton, J. P., Port, J. D., & Waring, T. M. (2009). Controlling invasive species in complex social landscapes. *Frontiers in Ecology and the Environment*, 8, 210-216.
- Epanchin-Neil, R. S., & Wilen, J. E. (2010, July a). Optimal control of spatial-dynamic processes: the case of biological invasions. Paper presented at the Agricultural & Applied Economics Association's 2010 AAEA, CAES &

- WAEA joint annual meeting, Denver, CO. Retrieved from http://ageconsearch.umn.edu/bitstream/61375/2/Epanchin_Niell%20and%20Wil%20en%2011181.pdf
- Epanchin-Neil, R. S., & Wilen, J. E. (2010, July b). Cooperation, spatial-dynamic externalities, and invasive species management. Paper presented at the Agricultural & Applied Economics Association's 2010 AAEA, CAES & WAEA joint annual meeting, Denver, CO. Retrieved from http://ageconsearch.umn.edu/bitstream/61371/2/Epanchin_Niell%20and%20Wil%20en_11767.pdf
- Forest, Mineral, and Fire Management Division, Michigan Department of Natural Resource. (2008). *Michigan forest health highlights*, 2008.
- Francis, P. J. (1997). Dynamic epidemiology and the market for vaccinations. *Journal of Public Economics*, 63, 383-406.
- Frisvold, G., Olson, A., Bean, T., Betancourt, J., & Marsh, S. (2007, October). Decision model for controlling buffelgrass invasion in an urban-wildland Interface [PowerPoint slides]. Paper presented at Program of Research on the Economics of Invasive Species Management (PREISM) workshop, Economic Research Service, USDA Washington, DC October 18-19. Retrieved from <http://www.farmfoundation.org/news/articlefiles/439-Research%20Presentations%20III-Frisvold.pdf>
- GAMS Development Corporation. (1990). The general algebraic modeling system (GAMS). Retrieved from <http://www.gams.com/>
- Gersovitz, M. & Hammer, J. S. (2004). The economical control of infectious diseases. *Economic Journal*, 114, 1 – 27.
- Hansen, E. M. (2008). Alien forest pathogens: phytophthora species are changing world forests. *Boreal Environment Research*, 13, 33-41.
- Holmes, T. P., Aukema, J. E., Holle, B. V., Liebhold, A. M., & Sills, E. (2009). Economic impacts of invasive species in forests past, present, and future. In R.S. Ostfeld & W.H. Schlesinger (Eds.), *The year in ecology and conservation*

- biology, Annals of the New York Academy of Sciences, 1162, 18-38.*
- Juzwik, J., Harrington, T. C, MacDonald, W. L, & Appel, D. N. (2008). The origin of *Ceratocystis fagacearum*, the oak wilt fungus. *Annual Review of Phytopathology, 46*, 13-26.
- Juzwik, J. (2009). Epidemiology and occurrence of oak wilt in Midwestern, Middle, and South Atlantic states. In Billings, R. F., & Appel, D. N., (Eds.), *Proceedings of the National Oak Wilt Symposium, June 4-7, 2007*. Austin, TX, 55-66.
- Liebhold, A.M., Halversen, J.A., & Elmes, G.A. (1992). Gypsy moth invasion in North America: a quantitative analysis. *Journal of Biogeography, 19*, 513-520.
- Loo, J. A. (2009). Ecological impacts of non-indigenous invasive fungi as forest pathogens. *Biological Invasions, 11*, 81–96.
- Margules, C. R., Nicholls, A. O., & Pressey, R. L. (1988). Selecting networks of reserves to maximize biological diversity. *Biological Conservation, 43*, 63-76.
- Maryland Department of Agriculture. (2007). Plant protection and weed management section, 2007/2008 landscape ash tree removal for emerald ash borer eradication bid announcement and instructions. Retrieved from http://www.mda.state.md.us/plants-pests/eab/bid/arborist_bid_announcement.pdf
- MathWorks. (2007). Programming, Matlab version 7.5, (R2007b). Retrieved from <http://www.mathworks.com/help/techdoc/rn/brb410y-1.html>
- Mayo, J. H., Straka, T. J., & Leonard, D. S. (2003). The cost of slowing the spread of the gypsy moth (Lepidoptera: Lymantriidae). *Journal of Economic Entomology, 96*, 1448-1454.
- McAusland, C., & Costello, C. (2004). Avoiding invasives: trade-related policies for controlling unintentional exotic species introductions. *Journal of Environmental Economics and Management, 48*, 954-977.
- Mehta, S. V., Haight, R. G., Homans, F. R., Polasky, S., & Venette, R. C. (2007). Optimal detection and control strategies for invasive species management.

- Ecological Economics*, 61, 237-245.
- Minnesota Department of Natural Resources Central Region. (2004). Minnesota land cover classification system user manual, version 5.4. Retrieved from http://files.dnr.state.mn.us/assistance/nrplanning/community/mlccs/mlccs_manual_v5_4.pdf
- Olson, L.J. (2006). The economics of terrestrial invasive species: a review of the literature. *Agricultural and Resource Economic Review*, 35, 178-194.
- Olson, L. J., & Roy, S. (2004). The economics of controlling a stochastic biological invasion. *American Journal of Agricultural Economics*, 84, 1311-1316.
- Olson, L. J., & Roy, S. (2005). On prevention and control of an uncertain biological invasion. *Review of Agricultural Economics*, 27, 491-97.
- Olson, L. J., & Roy, S. (2008). Controlling a biological invasion: a non-classical dynamic economic model. *Economic Theory*, 36, 453-69.
- Pimentel, D., Lach, L., Zuniga, R., & Morrison, D. (2000). Environmental and economic costs of nonindigenous species in the United States. *BioScience*, 50, 53-65.
- Poland, T. M., & McCullough, D.G. (2006). Emerald ash borer: invasion of the urban forest and threat to North America's ash resource. *Journal of Forestry*, 104(3), 118-124.
- Polasky, S., Camm, J. D., & Garber-Yonts, B. (2001). Selecting biological reserves cost-effectively: an application to terrestrial vertebrate conservation in Oregon. *Land Economics*, 77, 68-81.
- Sharov, A. A., & Colbert, J. J. (1996). A model for testing hypotheses of gypsy moth, *Lymantria dispar* L., population dynamics. *Ecological Modeling*, 84, 31-51.
- Sharov, A. A., Leonard, D., Liebhold, A. M., Roberts, E. A., & Dickerson, W. (2002). "Slow the Spread": A national program to contain the gypsy moth. *Journal of Forestry*, 100(5) 30-35.

- Sharov, A. A., & Liebhold, A.M. (1998). Bioeconomics of managing the spread of exotic pest species with barrier zones. *Ecological Applications*, 8, 833-845.
- Sharov, A. A., Liebhold, A.M., & Roberts, E.A. (1998). Optimizing the use of barrier zones to slow the spread of gypsy moth (Lepidoptera : Lymantriidae) in North America. *Journal of Economic Entomology*, 91, 165-174.
- Stokland, J. N. (1997). Representativeness and efficiency of bird and insect conservation in Norwegian boreal forest reserves. *Conservation Biology*, 11, 101-111.
- Snyder, S. A., Haight, R. G., & ReVelle, C. S. (2004). A scenario optimization model for dynamic reserve site selection. *Environmental Modeling and Assessment*, 9, 179-187.

Appendix

Appendix 1: Analytical Derivations in Chapter 2

The current value Hamiltonian for Problem (2.3) in the body of the paper is:

$$H = px(t) + cR(t)^2 + \lambda(t)(ax(t) - R(t))$$

and the Pontryagin necessary conditions are:

- i) $0 = 2cR(t) - \lambda(t)$
- ii) $\dot{\lambda}(t) - r\lambda(t) = -p - a\lambda(t)$
- iii) $\dot{x}(t) = ax(t) - R(t)$

Solving equation ii) for the path of the shadow value gives:

$$\lambda(t) = \left(\lambda(T) - \frac{p}{r-a} \right) (e^{-(r-a)(T-t)} - 1) \quad (\text{A1.1})$$

so that the path of removals is, from the first necessary condition,

$$R(t) = \frac{1}{2c} \left(2cR(T) - \frac{p}{r-a} \right) (e^{-(r-a)(T-t)} - 1) = \left(R(T) - \frac{p}{2c(r-a)} \right) (e^{-(r-a)(T-t)} - 1) \quad (\text{A1.2})$$

To derive the complete solution, we determine the ending stock and the ending time, T .

There are three possibilities:

Case A: Eradication Strategy

If the ending stock is to zero and the optimally chosen ending time is less than the maximum possible ending time, $x(T^*) = 0$ and $H(T^*) = 0$.

$$H(T^*) = px(T^*) + cR(T^*)^2 + \lambda(T^*)(ax(T^*) - R(T^*))$$

$$= cR(T^*)^2 + 2cR(T^*)(-R(T^*)) = -cR(T^*)^2 \Rightarrow R(T^*) = 0$$

This derivation implies that removals at the ending date are equal to zero so that the path of removals for an eradication strategy is:

$$R^*(t) = \frac{p}{2c(r-a)} (1 - e^{-(r-a)(T^*-t)}). \quad (\text{A1.3})$$

The path of the stock is determined according to the state equation iii) with this removal strategy so that:

$$x^*(t) = x(\tau)e^{a(t-\tau)} + \frac{p}{2c(r-a)} \left(e^{-(r-a)T^*} \frac{e^{(r-a)t} - e^{at} e^{(r-2a)\tau}}{r-2a} + \frac{1 - e^{a(t-\tau)}}{a} \right), \quad (\text{A1.4})$$

and the terminal time, T^* , is defined as the date at which this path reaches zero.

Case C: Suppression Strategy

If the terminal stock is chosen optimally and the terminal time constraint is binding, the terminal shadow value is equal to zero. Optimal removals are now:

$$R^*(t) = \frac{p}{2c(r-a)} (1 - e^{-(r-a)(T_{\max} - t)}) \quad (\text{A1.5})$$

The only difference in this expression is that the ending time is T_{\max} rather than T^* . The corresponding solution for the path of optimal stock is the same as above, with T_{\max} in place of T^* . The important difference is that the terminal stock is now endogenous and the terminal time is fixed.

Case B: Eradication/Suppression Borderline

In the intermediate case, both the terminal stock constraint and the terminal time constraint are just binding, so that $T = T_{\max}$ and $(T_{\max}) = 0$. The equation that defines this case is:

$$x^*(T_{\max}) = x(\tau)e^{a(T_{\max} - \tau)} + \frac{p}{2c(r-a)} \left(\frac{1 - e^{-(r-2a)(T_{\max} - \tau)}}{r-2a} + \frac{1 - e^{a(T_{\max} - \tau)}}{a} \right) = 0 \quad (\text{A1.6})$$

Solution Procedure

We first solve the problem with the transversality conditions for a free ending time and ending state so that the co-state (λ) and the Hamiltonian ($H(T)$) at the ending time are both equal to zero. If either inequality constraint is violated, we impose the constraints one by one, evaluating the value functions with a free ending state and a constrained ending time ($\lambda(T) = 0$ and $T = T_{\max}$), and with a constrained ending state and a free ending time ($x(T) = 0$ and $H(T) = 0$). If the terminal constraints are satisfied in both cases, we choose the case with the smallest overall cost. If constraints are violated in both cases, we impose both constraints. Once we find the correct terminal conditions, we insert the optimal paths of the stock and removal associated with these terminal conditions into the integral to get an expression for the optimized discounted stream of damage and removal costs.

Analytical Results

The value function, $\int_{\tau}^{T^*} e^{-r(t-\tau)}(px^*(t) + cR^*(t)^2)dt$, can be written explicitly

as:

$$\begin{aligned}
 & V(x(\tau), T^* - \tau) \\
 &= \frac{px(\tau)}{a-r} \left(e^{(a-r)(T^*-\tau)} - 1 \right) + \frac{p^2}{2c(r-a)} \left(\frac{1-e^{r(T^*-\tau)}}{r} + \frac{1-e^{(a-r)(T^*-\tau)}}{a-r} \right) \\
 &+ \frac{p^2}{2c(r-a)(r-2a)} \left(\frac{e^{(a-r)(T^*-\tau)} - e^{-r(T^*-\tau)}}{a} + \frac{e^{2(a-r)(T^*-\tau)} - e^{(a-r)(T^*-\tau)}}{(a-r)} \right) \\
 &+ \frac{p^2}{4c^2(r-a)^2} \left(\frac{1-e^{r(T^*-\tau)}}{r} + \frac{2(e^{-r(T^*-\tau)} - e^{(a-r)(T^*-\tau)})}{a} \right. \\
 &\quad \left. + \frac{e^{-r(T^*-\tau)} - e^{2(a-r)(T^*-\tau)}}{r-2a} \right) \tag{A1.7}
 \end{aligned}$$

where the terminal time T^* is either the upper limit T_{\max} or is endogenously determined. In the suppression zone, where the upper limit is exogenous, the derivative of this expression with respect to the starting stock level is:

$$\frac{\partial V}{\partial x(\tau)} = \frac{p}{a-r} \left(e^{(a-r)(T_{\max} - \tau)} - 1 \right) > 0 \quad (\text{A1.8})$$

and the cross partial is:

$$\frac{\partial^2 V}{\partial x(\tau) \partial (T_{\max} - \tau)} = p e^{(a-r)(T_{\max} - \tau)} > 0. \quad (\text{A1.9})$$

When the optimal ending time becomes a function of the starting stock level (in the eradication zone), the problem of determining the relationship between the value function and the starting stock level becomes intractable.

To find out how optimal detection effort changes with changes in parameters, we apply the implicit function theorem to the first order condition. For example, if we are interested in changes in optimal detection effort with respect to changes in the initial stock level, and if we define $f = MC_1(E^*) + MC_2(E^*) + MC_3(E^*)$, then $\frac{\partial E^*}{\partial x_0} = -\frac{\partial f / \partial x_0}{\partial f / \partial E}$.

Taking the derivative of the first order condition with respect to x_0 , we get:

$$\begin{aligned} \frac{\partial f}{\partial x_0} = & p e^{(a-r)\tau} \frac{\partial \tau}{\partial E} - e^{-r\tau} r \left(\frac{\partial V}{\partial x_\tau} \frac{\partial x_\tau}{\partial x_0} \right) \frac{d\tau}{dE} + e^{-r\tau} \left(\frac{\partial^2 V}{\partial x_\tau^2} \frac{\partial x_\tau}{\partial x_0} \frac{\partial x(\tau)}{\partial \tau} + \frac{\partial V}{\partial x_\tau} \frac{\partial^2 x(\tau)}{\partial \tau \partial x_0} \right) \frac{d\tau}{dE} \\ & + e^{-r\tau} \left(\frac{\partial^2 V}{\partial (T_{\max} - \tau) \partial x_\tau} \frac{\partial x_\tau}{\partial x_0} \frac{\partial (T_{\max} - \tau)}{\partial \tau} \right) \frac{d\tau}{dE}. \end{aligned} \quad (\text{A1.10})$$

In general, this is a complicated expression because the stock level at the beginning of the management horizon can affect the optimally chosen ending date. If, however, we are in the suppression zone, we substitute the relevant derivatives into the equation above, and find:

$$\frac{\partial f}{\partial x_0} = \left\{ \begin{array}{l} pe^{(a-r)\tau} - e^{-r\tau}r(e^{(a-r)(T_{max}-\tau)} - 1)e^{a\tau} \\ +e^{-r\tau}a\frac{p}{a-r}(e^{(a-r)(T_{max}-\tau)} - 1)e^{a\tau} - e^{-r\tau}pe^{(a-r)(T_{max}-\tau)}e^{a\tau} \end{array} \right\} \frac{d\tau}{dE} = 0$$

(A1.11)

Our conclusion is that, in the suppression zone where changes in the starting stock level have no effect on the time horizon, changes in the initial stock level (x_0) have no effect on optimal detection effort.

Appendix 2: Parameter Calculations in Chapter 2

We obtained baseline parameter values for our sensitivity analysis in Chapter 2 from the Gypsy Moth Slow the Spread (STS) program. Sharov and Colbert (2002) predicted that 4 years of treatment was required to eradicate a gypsy moth population with an initial density of 10^5 eggs per hectare. On average, 25×10^3 units of gypsy moth were eradicated per hectare per year. Mayo, Straka, & Leonard (2003) estimated the average treatment costs in the STS pilot project in 1994 to be 69.65 dollars per hectare. Fitting our quadratic removal cost function to these figures implies a control cost coefficient (c) of 0.1^7 dollars per unit of gypsy moth per year in our benchmark case. Damage caused by gypsy moth was estimated as 3.8 dollars per hectare per year in Sharov and Liebhold (1998). This translates into a damage per unit of gypsy moth (p) of 15×0.1^5 dollars per year. Detection effort cost was a cost paid for setting traps to capture male gypsy moths. The cost of traps STS pilot project was estimated as 49.67 dollars per trap, the number of traps placed per hectare was 34,309 units (Mayo et al., 2003), the cost to locate traps per hectare was 123 dollars, and therefore the cost to detect one gypsy moth in a location with a sizable population was calculated as 5×0.1^3 dollars. We use this number as the cost of detection (b) in the benchmark case.

Appendix 3: Derivation of Necessary Conditions for the Optimal Solution in the Surveillance Stage of the Model in Chapter 4

Selecting only site a for surveillance is optimal (i.e., $\Omega^* = \{a\}$) :

First, when the budget level allows the inspection on both sites and complete removal of infected trees only on the site a (i.e., $c_{1a}N_a + c_{1b}N_b + c_{2b}I_a \leq B$) then (ii-1) and (ii-2) are characterized as follows.

$$(ii-1) \Leftrightarrow g_a(I_a - M_a) \geq g_b(I_b - M_b)$$

$$(ii-2) \Leftrightarrow g_b \left(\frac{B - c_{1a}N_a - c_{1b}N_b - c_{2a}I_a}{c_{2b}} - M_b \right) \leq 0$$

A site with more infected trees I_a and an earlier plateau M_a (i.e., more healthy trees, $H_a = N_a - I_a$) and higher spread rate g_a is selected for the surveillance. The second inequality holds if the infection spread rate on the site b is zero ($g_b = 0$) or the amount of budget left for tree removal on site b is not enough to cut more infected trees than M_a , which is the minimum number of infected trees removed to save a positive number of healthy trees on the site.

Second, we consider the level of budget which cannot cover the costs for inspection on both sites and complete infected tree removal on site a but can cover the costs for the inspection and complete infected tree removal on either site a or b (i.e., $c_{1a}N_a + c_{1b}N_b + c_{2a}I_a > B$, $c_{1a}N_a + c_{2a}I_a \leq B$, and $c_{1b}N_b + c_{2b}I_b \leq B$). In this case, the necessary conditions are characterized by

$$(ii-1) \Leftrightarrow g_a(I_a - M_a) \geq g_b(I_b - M_b)$$

$$(ii-2) \Leftrightarrow g_a(I_a - M_a) \geq g_a \left(\frac{B - c_{1a}N_a - c_{1b}N_b}{c_{2a}} - M_a \right).$$

The first inequality is the same as the first case, and the second inequality automatically holds.

Third possible alternative situation is when the budget is not enough to survey and cut all the infected trees on site a but enough to survey and cut all the infected trees on site b (i.e., $c_{1a}N_a + c_{2a}I_a > B$ and $c_{1b}N_b + c_{2b}I_b \leq B$).

$$(ii-1) \Leftrightarrow g_a \left(\frac{B-c_{1a}N_a}{c_{2a}} - M_a \right) \geq g_b (I_b - M_b)$$

$$(ii-2) \Leftrightarrow g_a \left(\frac{B-c_{1a}N_a}{c_{2a}} - M_a \right) \geq g_a \left(\frac{B-c_{1a}N_a-c_{1b}N_b}{c_{2a}} - M_a \right)$$

Again, (ii-2) holds automatically.

The fourth possible situation is that the manager has enough of a budget to survey both sites and remove all the infected trees on site a ($c_{1a}N_a + c_{1b}N_b + c_{2a}I_a \leq B$), but the budget is not enough to survey and remove all the infected trees on only site b ($c_{1b}N_b + c_{2b}I_b > B$). In this case, we have:

$$(ii-1) \Leftrightarrow g_a (I_a - M_a) \geq g_b \left(\frac{B-c_{1b}N_b}{c_{2b}} - M_b \right)$$

$$(ii-2) \Leftrightarrow g_b \left(\frac{B-c_{1a}N_a-c_{1b}N_b-c_{2a}I_a}{c_{2b}} - M_b \right) \leq 0$$

The fifth situation is where budget level is that the budget level is high enough to survey both sites ($c_{1a}N_a + c_{1b}N_b < B$) but it is not enough to cut down all infected trees on site a ($c_{1a}N_a + c_{1b}N_b + c_{2a}I_a > B$). Moreover, if the manager spends her budget only for surveillance on site a it is possible for her to remove all the trees on the site ($c_{1a}N_a + c_{2a}I_a \leq B$), but if she conducts surveillance only on site b complete removal of infected trees on the site b is not possible ($c_{1b}N_b + c_{2b}I_b > B$). Under this situation, the conditions are characterized as follows.

$$(ii-1) \Leftrightarrow g_a(I_a - M_a) \geq g_b \left(\frac{B - c_{1b}N_b}{c_{2b}} - M_b \right)$$

$$(ii-2) \Leftrightarrow g_a(I_a - M_a) \geq g_a \left(\frac{B - c_{1a}N_a - c_{1b}N_b}{c_{2a}} - M_a \right)$$

The last situation is that costs of removing all the infected trees on either site a or b exceed the budget (i.e., $c_{1a}N_a + c_{2a}I_a > B$ and $c_{1b}N_b + c_{2a}I_b > B$). In this case the condition is characterized by

$$(ii-1) \Leftrightarrow g_a \left(\frac{B - c_{1a}N_a}{c_{2a}} - M_a \right) \geq g_b \left(\frac{B - c_{1b}N_b}{c_{2b}} - M_b \right)$$

$$(ii-2) \Leftrightarrow g_a \left(\frac{B - c_{1a}N_a}{c_{2a}} - M_a \right) \geq g_a \left(\frac{B - c_{1a}N_a - c_{1b}N_b}{c_{2b}} - M_a \right)$$

This inequality implies that, under this situation, the site a tends to be selected for inspection if the combinations of relatively high infection, less plateau, and high spread rate of infection appear on the site or the combinations of low infection, more plateau, and low spread rate of infection on the site b appear. In this case, in contrast to the former case, the cost required for the inspection on the site b , $c_{1b}N_b$, involves this condition. Both high per-tree inspection cost c_{1b} and large population N_b on site b leads to a lower budget left for tree removal on site b . This also results in fewer healthy trees saved from infection on the site.

Selecting two sites for surveillance is optimal: $\Omega^ = \{a, b\}$:*

To have this solution be optimal, obviously the budget level has to satisfy $c_{1a}N_a + c_{1b}N_b + c_{2a}I_a \leq B$ where, as was referred in the tree removal stage above, the notation “ a ” indicates the site where the infected trees should be removed with the first

priority among site 1 and 2, and “ b ” indicates the site with second priority. Necessary conditions are the following:

$$(iii-1) \phi(\{a, b\}) \leq \phi(\{a\}), \text{ and}$$

$$(iii-2) \phi(\{a, b\}) \leq \phi(\{b\}).$$

Adding to the condition that surveying both sites and cutting all trees ($c_{1a}N_a + c_{1b}N_b + c_{2a}I_a \leq B$), if the budget level satisfies $c_{1b}N_b + c_{2b}I_b \leq B$, the conditions (iii-1) and (iii-2) are characterized as:

$$\begin{cases} (iii-1) \\ (iii-2) \end{cases} \Leftrightarrow \begin{cases} g_b \left(\frac{B - c_{1a}N_a - c_{1b}N_b - c_{2a}I_a}{c_{2b}} - M_b \right) \geq 0 \\ g_a(I_a - M_a) + g_b \left(\frac{B - c_{1a}N_a - c_{1b}N_b - c_{2a}I_a}{c_{2b}} - M_b \right) \geq g_b(I_b - M_b) \end{cases}$$

If the budget level satisfies $c_{1b}N_b + c_{2b}I_b > B$, the conditions are characterized as follows.

$$\begin{cases} (iii-1) \\ (iii-2) \end{cases} \Leftrightarrow \begin{cases} g_b \left(\frac{B - c_{1a}N_a - c_{1b}N_b - c_{2a}I_a}{c_{2b}} - M_b \right) \geq 0 \\ g_a(I_a - M_a) + g_b \left(\frac{B - c_{1a}N_a - c_{1b}N_b - c_{2a}I_a}{c_{2b}} - M_b \right) \geq g_b \left(\frac{B - c_{1b}N_b}{c_{2b}} - M_b \right) \end{cases}$$