Application of the Bifactor Model to Computerized Adaptive Testing

A DISSERTATION
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA
BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

David J. Weiss, Adviser

January 2011

Acknowledgements

I would like to acknowledge my thesis adviser, Dr. David J. Weiss who has provided me with endless support, help, and direction throughout the implementation of this study. This thesis could not have been completed without his teaching and encouragement. He showed me the way of a true scholar, and encouraged me to overcome many difficult situations in the preceding years of graduate work.

I am thankful to my committee members, Dr. Niels G. Waller, Dr. Mark L. Davison, and Dr. Michael C. Rodriguez for their valuable time and input. Their comments and suggestions have significantly improved the quality of this thesis.

I would like to express my thanks to my former advisors, Dr. Soonmook Lee, and Dr. Jwa K. Kim who guided me to study psychometric method in the beginning of graduate student. I would like to express my sincere appreciation to Dr. Soonmook Lee who encouraged and guided me to keep psychometrics with constant concern.

My family deserves special recognition. I would like to express special thanks to my father, Sangwon Seo, and my mother, Hansam Choi for their support and love. I express my deep thanks and respect to my wife, Jaehyun Hong for her understanding, sacrifice, and patience. Finally, thanks to my son, Hongjin Seo for making me happy during the conduct of the study.

Abstract

Most computerized adaptive tests (CAT) have been studied under the framework of unidimensional item response theory. However, many psychological variables are multidimensional and might benefit from using a multidimensional approach to CAT. In addition, a number of psychological variables (e.g., quality of life, depression) can be conceptualized as being consistent with a bifactor model (Holzinger & Swineford, 1937) in which there is a general dimension and some number of subdomains with each item loading on only one of those domains. The present study extended the work on bifactor CAT of Weiss & Gibbons (2007) in comparison to a fully multidimensional bifactor method using multidimensional maximum likelihood θ estimation and Bayesian θ estimation for the bifactor model (MBICAT algorithm). Although Weiss and Gibbons applied the bifactor model to CAT (BICAT algorithm), their methods for item selection and scoring were based on unidimensional IRT methods. Therefore, this study investigated a fully multidimensional bifactor CAT algorithm using simulated data. The MBICAT algorithm was compared to two variations of the BICAT algorithm under three different conditions: different numbers of group factors, variations in the group factor discriminations, and trait (θ) estimation method. A fixed test length was used as the termination criterion for the CATs for Study 1. The accuracy of $\hat{\theta}$ using the BICAT algorithm and the MBICAT algorithm was evaluated with the correlation between θs and $\hat{\theta}$s, the root mean square error (RMSE), and the observed standard error (OSE). Two termination criteria (OSE = .50 and .55) were used to investigate efficiency of the MBICAT for Study 2. This study demonstrated that the MBICAT algorithm worked well when latent scores on the secondary dimensions were estimated properly.

Although the MBICAT algorithm did not improve the accuracy and efficiency of the general factor scores compared to two the BICAT algorithms, MBICAT showed an improvement in the accuracy and efficiency for the group factors. In the two BICAT algorithms, the use of differential entry on the group factors, as in Weiss and Gibbons (2007), did not make a difference compared to initial item at θ of 0 for both the general factor and group factor scales (Gibbons, et al., 2008) in terms of accuracy and efficiency.

Table of Contents

List of Tables

List of Figures

CHAPTER 1: INTRODUCTION

Measurement in psychology is the process of quantifying observations to represent latent traits. Establishing precision of measurement is a main issue in test theories and applications. In order to measure latent traits accurately, a variety of models and theories have been developed. In line with these developments, the objective of this research was to apply a bifactor model to computerized adaptive testing (CAT), and develop an operational technique to measure multidimensional as well as unidimensional latent traits under CAT.

There was a great change in theories of psychological testing during the 1960s. Although classical test theory (CTT; Spearman, 1907) had made meaningful contributions to test development for a long time, it has many shortcomings. One of the strong assumptions in CTT is the parallelism of tests (true score variances and error variances are the same across two parallel tests). However, an ideal test model requires invariance properties such as test-independence of true scores and sample-independence of item parameters. Satisfying these requirements, item response theory (IRT) has a strong emphasis on, and has been successful in providing procedures for, putting item difficulty and person trait on a common scale.

The invariance property of IRT model parameters makes it theoretically possible to solve some important measurement problems that have been difficult to handle with CTT (Petersen, Cook, & Stocking, 1983). These problems have included issues of test linking/equating and CAT (e.g., Kingsbury &Weiss, 1983). Since persons and items are placed on an identical scale in IRT, equating can be performed without assumptions of score distributions. This makes it possible to compare examinees on a common scale despite the fact that they are measured in different groups and with different items. Item parameters (difficulty and discrimination) are linearly transformable across different samples from a population. This makes it possible to create large banks of items that are linked onto a common scale for CAT (Embretson & Reise, 2000).

In designing and constructing measurement instruments, the IRT test information function (TIF) provides explicit procedures for varying characteristics to meet specific measurement objectives. Since the TIF can be obtained by summing the values of item information functions conditional on the latent trait, θ, TIFs show how well a test measures examinees at each value of θ. Therefore, a TIF is an index of local precision at the test level and useful for assuring desirable measurement characteristics and for developing a test instrument satisfying a target information function.

In order to provide each examinee with a precise θ estimate, a target information function should be high and constant across θ. However, a conventional test with a fixed set of items will have either a peaked or rectangular information function, or a TIF in between these two major types. A peaked conventional test is designed to provide a large amount of information in a narrow interval of θ, while a rectangular conventional test is designed to have flat TIF and, therefore, equal precision across all θ values (Weiss, 1982). Consequently, a conventional test would require a very large number of items in order to have high and equal precision across all θ values. In contrast to using a conventional test, CAT can yield a high and equal degree of precision for all examinees, and requires fewer items than a conventional test to reach a given degree of precision (e.g., Weiss & Kingsbury, 1984).

However, in light of the benefits of CAT in the unidimensional context, there has been little research that is instrumental for implementing multidimensional CAT effectively in psychological data. Recently, some research has become available that provides ideas how CAT based on the bifactor model can be applied to implement multidimensional CAT. Thus, the ultimate purpose of this study was to provide the underlying theory, a concrete algorithm, and operational procedures for implementing CAT with the bifactor model, and to evaluate the performance of the model.

The Current Research Issues

For the past several decades, it has been repeatedly demonstrated that a CAT version of a test can be at least 50% shorter, on average, than using a conventional test with equal or better precision of measurement (e.g., Kingsbury & Weiss, 1980; Kingsbury & Weiss, 1983; Weiss, 1985). However, most CAT has been studied under the framework of unidimensional IRT (UIRT). UIRT has been so influential as to bring about an increased use of CAT. However, there are still many unanswered questions in the use of UIRT for CAT. The questions are in regard to two strong assumptions in UIRT: local independence and unidimensionality.

Hambleton and Cook (2005) stated that the local independence assumption would be satisfied if all of the test items measure a single latent trait. Specifically, they argued that local independence would hold when pairs of test items are uncorrelated in data of examinees that are at the same θ level, which is called the weak form of local independence. However, since local item dependence often results from an additional dimension of a test (Wainer & Thissen, 1996), it goes beyond unidimensionality and can be evaluated, at least conceptually, for multidimensional tests as well as unidimensional tests. Thus, the plausibility of the local independence assumption can be tested by factor analytic techniques which are used to detect dimensionality. Consequently, the dimensionality test should be performed before examining local item independence and determining IRT models for data.

Although most IRT models currently in use assume that test items measure a single dominant latent trait, it is not always practical to assume that a test measures only a single latent trait (Reise, Morizot, & Hays, 2007). Most personality inventories in psychology are designed to measure multidimensional latent traits rather than a single latent trait (e.g, the NEO personality inventory; Costa & McCrae, 1992). Therefore, it is compelling to introduce a multidimensional latent space, where multidimensional IRT (MIRT) modeling would be adopted beyond the framework of unidimensionality in search of a more generalizable model to fit real data.

Notwithstanding that the use of UIRT in CAT has become popular in practice, thereby bypassing the strong proposal of MIRT modeling, a few researchers have applied MIRT models to CAT (Luecht, 1996; Segall, 1996). In fact, several procedures already have been proposed to implement CAT for multidimensional data. A multiple scale procedure proposed earlier by Brown and Weiss (1977) is based on a multi-factor model. However, it assumes that each test is unidimensional. In a multiple-scale procedure, one item loads on only one dimension. Even though MIRT models allow each item to load on more than one latent trait and that traits are allowed to correlate with each other, most MIRT models are limited to exploratory analyses. Li and Schafer (2005) mentioned some major problems in exploratory MIRT in that unique solutions for estimated item loadings are not obtained, and interpretation of each dimension, along with the determination of the number of dimensions, is subjective. However, confirmatory item factor analysis in a context of MIRT could help resolve these practical problems (McLeod, Swygert, & Thissen, 2001). Initially, the confirmatory multi-UIRT model (Segall, 1996) was applied to CAT in order to consider correlations among factors. However, these models confine each item to measuring a single latent trait similar to the multiple scale procedure, which is not realistic for multidimensional constructs in psychology.

In order to relax this constraint, bifactor modeling was applied to CAT that (1) measures more than one latent trait, (2) yields readily interpretable latent traits, and (3) estimates directly item and person parameters jointly (a confirmatory MIRT model). Initially, Weiss and Gibbons (2007) implemented a CAT with the bifactor model (BICAT) for mental health data. However, their algorithm of implementing item selection and scoring is not multidimensional in an authentic sense. The algorithm for selecting items and estimating trait levels is still based on unidimensional testing. Therefore, the next step is to develop an algorithm of item selection and scoring that is truly multidimensional for CAT with a bifactor model.

Objective of the Study

Weiss and Gibbons (2007) showed that CAT with a bifactor model (BICAT) could be an implementation of CAT for measuring individuals on an instrument that has scales that could be fit by a dichotomous bifactor model. However, there are no studies comparing the precision and efficiency of their work with other multidimensional CATs (MCAT; Segall, 1996; Luecht, 1996). The present study had three objectives to advance MCAT over and beyond the previous work of BICAT. First, a monte-carlo study was conducted to implement the BICAT method of Weiss and Gibbons, and Gibbons et al. (2008). The second objective of the present study was to apply to the bifactor model item selection and θ estimation methods that are truly multidimensional using multidimensional maximum likelihood estimation (MLE) and Bayesian estimation (MBICAT: the multidimensional bifactor algorithm for CAT). Although Weiss and Gibbons applied the bifactor model to CAT, their methods for item selection and θ estimation were based on unidimensional scales. Therefore, the third and final objective was to compare the newly developed MBICAT algorithm with the BICAT algorithm.

An overarching goal encompassing the above three objectives was to investigate the efficiency and precision that CAT with the bifactor model allows in the measurement of multidimensional latent traits underlying psychological data when the data structure is consistent with the bifactor model. In order to answer this overarching question, a series of specific questions were addressed:

1) How would the bifactor model in CAT work when latent scores on the secondary dimension are estimated properly (multidimensional latent traits are estimated)?
2) In the BICAT algorithm, does the use of differential entry, as in Weiss and Gibbons (2007), make a difference compared to entry item at θ of 0 for both the general factor and group factor scales (Gibbons, et al., 2008)?
3) How does the BICAT algorithm compare to the MBICAT algorithm applying Segall's multidimensional item selection and scoring methods?

4) How are all the above questions related to the CAT with bifactor model affected by the number of group factors, the factor types (general or group factor), different bifactor loadings (group factor discrimination condition), and θ estimation methods?
5) Is it possible to develop a multidimensional bifactor algorithm that could allow multidimensional item selection and θ estimation methods for CAT?

Unidimensional IRT-Based CAT (UCAT)

UCAT Algorithm

Advancements in IRT over the past several decades have opened ways of powerful data analysis in the testing area, such as differential item functioning and test score linking/equating, and CAT. In settings of CAT, items are tailored to an individual examinee while he/she is taking a test. Specifically, CAT allows a test administrator to control measurement precision and to maximize the efficiency of the testing process. Characteristics of CAT include a pre-calibrated item bank, use of differential entry points into the CAT, a procedure of item selection, an estimation method, and a criterion for terminating the test. Since the 1970s research has shown that these characteristics of CAT are most readily achieved by adopting UIRT (Kingsbury & Weiss, 1980; Kingsbury & Weiss, 1983; McBride & Martin, 1983).

Pre-calibrated item bank. The implementation of CAT requires developing a large bank of test items. A bank might contain thousands of items, and all items are assumed to measure an identical latent trait on the same scale. However, it is unreasonable and impractical to gather a single group of thousands of examinees to develop a good item bank of so many items. Therefore, it is frequently necessary to link subsets of items administered to different groups onto a target matrix using a reference group to create a large item bank (e.g., Hambleton, Swaminathan, & Rogers, 1991; Kolen & Brennan, 2004). UIRT offers pre-calibrated item parameters and a reasonable method for linking subsets of test items, owing to the invariance properties of parameters in items and examinees. Consequenctly, linking procedures in UIRT modeling enable item developers to have many hundreds of pre-calibrated items prior to implementing CAT.

Entry item. An entry item should be determined before implementing CAT. Usually, an entry item in CAT is assigned based on a θ of 0 because it is difficult to obtain valid prior information about the θ level of an examinee. If the difficulty level of an entry item is selected to be close to the examinee's θ level, it will improve the efficiency of CAT (Weiss, 1985). In practice, since CAT begins with an item of median difficulty level, the item would be readily overexposed. Therefore, several possible methods are proposed to reduce the item exposure rate. One possible method is to use random selection of the first few items from a subset of the item bank. One specific procedure for determining an entry item in CAT is to combine IRT and Bayesian statistical methods (e.g., Baker, 1992; Weiss & McBride, 1984).

Item selection rule. One of the key factors characterizing CAT is the item selection rule, which is essential to continue the adaptive testing process after an entry item is given to an examinee. Item selection rules in CAT are based on concepts of item information in IRT (Weiss, 1985). Given the current estimate of an examinee's θ, the most informative item among the remaining items should be chosen for the next item. Given that appropriate computer software is available, two reasonable choices are conceivable for CAT implementation (Weiss, 1985): the maximum information procedure and Bayesian selection procedures. The former allows CAT to select an item with the maximum information at the examinee's most recently estimated θ level. The latter (Owen, 1975) is to select the item minimizing the expected posterior variance of the θ estimates.

Scoring procedure. Continuous estimation and updating of an examinee's θ level can be performed after each item is administered in an adaptive test, and the next item to be administered can be selected based on the θ level as estimated from all previous item responses of an examinee. Individual θ levels can be estimated by maximum likelihood or Bayesian methods (Weiss & McBride, 1984). If item parameters are assumed known, examinees' θ level can be estimated from the likelihood function, which is the product of all IRFs (probabilities of answering an item correctly or incorrectly). Usually, the local maximum value of the likelihood function given a theta value can be obtained by setting the first derivative of the natural log of the likelihood function at zero. However, maximum likelihood methods can be used only when there is a mixed response pattern (not all 0 or 1 responses). On the other hand, Bayesian methods can be used for any response pattern because they are based on Bayes' rule that is proportional to the product of the likelihood and prior probability. Usually, a prior probability distribution of θ assumes a standard normal distribution. In Bayesian estimation methods, the Bayesian modal estimator is to find the maximum value of a posterior distribution of θ (MAP). The expected a posteriori (EAP) method is to find the mean of the posterior distribution of θ. Owen's (1975) method is to use the posterior distribution with the normal ogive model replacing the logistic model in the calculation of the likelihood.

Termination criterion. One of the characteristics of CAT is that test administration can be terminated when each examinee is measured with a pre-specified degree of precision, which will guarantee equiprecision in measuring θ levels of all examinees. The selection of a termination criterion varies over different objectives of CAT. One criterion is the standard error of θ estimates, in maximum likelihood methods, which allows CAT to terminate when the standard error of θ estimates reaches the prespecified value. The other is the variance of the posterior distribution in Bayesian θ estimation methods, which allows CAT to terminate when the variance of the posterior distribution becomes smaller than a pre-specified value. In many applied test settings, CAT is terminated when a pre-determined number of items is reached. However, using a fixed number of items as the termination criterion is inappropriate for CAT because it does not provide all examinees with equal precision in measuring θ (Weiss, 2004).

Multiple Scales CAT

Much of the research and development in CAT has been done in the context of achievement testing. Achievement domains are mostly considered both unidimensional and relatively homogenous. Some achievement tests might measure a single variable without substantial variation in content. On the other hand, others might measure a relatively unidimensional domain with two or more content domains underlying the primary dimension. However, UCAT does not consider the varied content categories of the items within an item bank due to the maximum information item selection procedure.

In a CAT setting with several homogeneous scales, several procedures were proposed to achieve “content balance” among examinees in several domains (e.g., Green, Bock, Humphreys, Linn, & Reckase, 1984; Kingsbury & Zara, 1991; Weiss & Kingsbury, 1984), because a content balancing procedure is one alternative to solve a practical issue in UCAT.

Kingsbury and Zara (1991) proposed an algorithm to control content on an item-by-item basis as items are administered. A content-balanced adaptive test provides users with a test that adequately represents each of the content domains included. For example, each content-balanced CAT would administer items according to a pre-specified ratio of content such as 50% from math content and 50% from verbal content.

However, by modifying the maximum information item selection procedure, a content balancing procedure could do harm to the efficiency of CATs, which would in turn result in longer tests than an unrestricted CAT to reach the test objective (assuming that test length is allowed to vary). In addition, in order to balance content domains, the target percentage of items being administered should be calculated for all content areas before implementing CAT. Furthermore, a content balancing procedure does not provide an examinee with an estimate of trait level in each content domain, but rather provides a single estimate of the general trait given a test, which means that it has operated within the context of the UIRT model. In the present study a procedure of content balancing will be designed to improve measurement efficiency in a context of multiple dimensions.

An issue closely related to content balancing procedures is an application of CAT to measure an examinee on multiple scales. Lord and Novick (1968) and McDonald (1981) considered scale homogeneity as unidimensionality. If researchers consider each homogeneous scale as a unidimensional scale, a CAT with multiple scales cannot only achieve content balancing but can also take into account multidimensionality in latent traits. However, since a CAT with multiple scales proceeds separately for each scale to measure an individual examinee, it ignores information from other traits and proceeds as if they were uncorrelated. Even though a CAT with multiple scales provides each examinee with a trait score for each content scale, it is not a practical procedure because most test batteries or instruments with multiple scales result in scores that are intercorrelated to some degree.

Brown and Weiss (1977) indicated that trait scores are usually correlated across different scales, ranging from $r = .30$ to $r = .50$. Therefore, they considered the trait score correlations across different scales in CAT. Different starting values were generated using inter-scale correlations that were obtained using trait estimates of a test development group.

First, a pair of scales with the highest correlation was chosen. Then, each of the remaining scales in turn, was regressed on the chosen two scales. Of these remaining scales, the one with the highest multiple correlation with the first two scales was added as the third scale of choice. This process was then repeated to select the fourth scale, computing multiple correlations with the first three scales as the predictors of each of the remaining scales, in turn, as the criterion variable. One scale is added to the predictor set at each subsequent stage. Also the subscales can be ordered by how well they can predict the last remaining scale. Finally, the multiple regression equations (the subscales ordered are the predictors and the last remaining scale is the criterion) can be used to predict an examinee's initial θ estimate on each new CAT in the battery as a starting value for that CAT. This algorithm was called the "inter-subtest" branching procedure (Brown & Weiss, 1977), which further enhances the efficiency of the CAT with separate multiple scales by providing more accurate starting values for each subsequent scale in the battery.

The inter-subtest branching procedure would reduce the number of items that are needed to measure an examinee on multiple correlated traits. In the context of an achievement test battery, several researchers (Brown & Weiss, 1977; Gialluca & Weiss, 1979; Maurelli & Weiss, 1981) demonstrated that test lengths in a battery were reduced by 80% or more of their full test lengths with no reduction in measurement precision. Specifically, Gialluca and Weiss showed that total test length was reduced another 1% to 5% with multiple scales by using an inter-subtest branching strategy.

However, Brown and Weiss (1977) argued that it is necessary to determine an optimal and generalizable procedure for ordering a set of subtests, and questioned whether a unidimensional model can be applied across subtests for adaptive testing using achievement test batteries. Although the inter-subtest branching approach is appealing in terms of its simple structure, it might bring an undesirable bias due to using number-correct scores from each subtest to determine an order of subscales. Therefore, it is compelling to consider multidimensional item parameter estimates while the trait correlations across scales are preserved.

Furthermore, Weiss and Suhadolnik (1982) pointed out two factors that can affect the performance of CAT. The first is the nature of the item pool from which items are drawn.

The second is an effect of multidimensionality on the performance of IRT-based CAT. Weiss and Suhadolnik argued that the number of items of an adaptive test should be at least two times as many as UIRT to overcome the effects of multidimensionality. This would suggest that CAT with multidimensional IRT should be able to be implemented for multidimensional data without loss of efficiency in CAT.

Multidimensional CAT (MCAT)

UIRT has contributed to the measurement of not only ability/achievement variables but also personality and psychological variables. Notwithstanding the enormous utility of CAT in educational achievement data, few studies have applied CAT to personality and psychiatric data. In terms of psychological measurement, research has demonstrated that UCAT can be meaningfully applied to the measurement of attitudes and personality variables (Baek, 1997; Dodd, De Ayala, & Koch, 1995; Reise & Waller, 1991). However, most of CAT research with personality and psychiatric data has assumed unidimensionality in modeling latent traits.

Several studies have examined the effects that UIRT modeling applied to multidimensional data might have on item parameter estimation (e.g., Ackerman, 1989; Ansley & Forsyth, 1985; Reckase, 1974). A general finding from these studies is that if there is a predominant general factor in the data, the presence of multidimensionality has little effect on estimation of item and trait parameters. On the other hand, if the data have strong secondary factors beyond the primary factor, application of a UIRT model will result in a serious distortion of the measurement characteristics. Indeed, the validity of UIRT applications (linking, model-fit, parameter estimation, scoring, and adaptive testing) would also be questioned in a situation where it is reasonable to hypothesize a multidimensional latent space. Therefore, CAT might not guarantee the optimal test for individual examinees unless IRT parameter estimates are accurately pre-specified, given an appropriate model for the data.

Through both adaptive and conventional testing, Folk and Green (1989) examined the effects that unidimensional modeling applied to two-dimensional data might have on item parameters. Folk and Green's study demonstrated that when non-dominant factors do not affect scale scores seriously, θ can be estimated with an assumption of a unidimensional latent trait underlying the data in conventional testing. However, if the two-dimensional latent traits are relatively uncorrelated and dominant in data, using one or the other trait makes a great difference in θ estimates. Folk and Green concluded that the difference between θ estimates in CAT was greater than in conventional testing due to the fact that UIRT item discrimination parameter estimates were used for both the item selection and θ estimation procedures in CAT.

Since Bock and Aitkin (1981) extended the IRT model to a multidimensional case, some researchers have studied CAT using a pool of items calibrated under MIRT models. Initially, Bloxom and Vale (1987) developed multidimensional adaptive estimation procedures. They extended Owen's sequential Bayesian adaptive updating algorithm (Owen, 1975) to the multidimensional case. Later, Tam (1992) evaluated a multidimensional adaptive estimation procedure through precision, test information, and computational time. However, these studies (Bloxom & Vale, 1987; Tam, 1992) considered the implementation of CAT with respect to only θ estimation methods. They did not address the procedure for multidimensional adaptive item selection that considers prior knowledge of a multivariate distribution of θ.

Bloxom and Vale (1987), and Tam (1992) initially developed MCAT, but their MCAT failed to demonstrate advantages over unidimensional adaptive testing (Segall, 1996). Therefore, Segall (1996) developed multidimensional Bayesian item selection and θ estimation procedures, and demonstrated that his MCAT was more efficient than a UCAT in terms of test length and precision. In addition to gaining efficiency, his MCAT could be used as an instrument for measuring different content traits for examinees from nine different subtests of the Armed Services Vocational Aptitude Battery (ASVAB; Moreno & Segall, 1992). Luecht (1996) also demonstrated the efficiency of MCAT. He observed that MCAT with content constraints could achieve approximately the same precision with 25% to 40% fewer items than were required in UCAT in regard to the measurement of latent traits.

Furthermore, Li and Schafer (2005) showed that UCAT and MCAT conditional on the constraints of item exposure rate were quite capable of producing accurate estimates of reading and math abilities. Specifically, compared with UCAT, MCAT slightly increased the accuracy of θ estimates for examinees at the low- and high-end of an ability scale in both reading and math tests. Therefore, MCAT appears to be an efficient method for ensuring adequate coverage of content in adaptive testing, and provides a separate multidimensional vector of estimated abilities for each examinee.

Multidimensional IRT (MIRT)

A pattern of dichotomous item responses (0 or 1) in MIRT is represented by a set of probabilities for all composites of responses (0, 1) corresponding to two or more underlying traits. Therefore, each item has multiple discriminations and possibly multiple difficulties with a single guessing parameter for the 3PLM. In the late 1970s and early 1980s, researchers began to develop practical MIRT models. The compensatory models in MIRT have an underlying assumption that a high level in one dimension might compensate for deficiency in another. Therefore, the functions of θ are additive. The multidimensional compensatory 3PLM (MC3PL) (Reckase, 1985) can be described as

$$P(u_{ij}=1\,|\,\mathbf{a}_i,d_i,c_i,\boldsymbol{\theta}_j)=c_i+\frac{(1-c_i)}{1+\exp\left[-1.7\left(\sum_{k=1}^{m}a_{ik}\theta_{jk}+d_i\right)\right]}, \tag{1}$$

where $\mathbf{a}_i$ is the row vector of discriminations of item *i* (a_{ik} is the discrimination of item *i* for dimension *k*), d_i is an intercept parameter representing the difficulty of item *i*, c_i is the guessing parameter of item i, and $\boldsymbol{\theta}_j$ is a vector of latent traits for examinee *j* (θ_{jk} is the latent trait of examinee *j* on dimension *k*). d_i can be seen to be a function of both $\mathbf{a}_i$ and $\mathbf{b}_i$ ($d_i=-\mathbf{a}_i\mathbf{b}_i'$).

In the vein of UIRT models, when *c* is set at zero, the MC3PL model becomes a multidimensional compensatory two-parameter logistic model (MC2PL). The MC2PL model becomes a multidimensional Rasch model when all of the discrimination parameters are set at 1. The interpretation of item parameters is the same across all compensatory models except that difficulty is reversed, and is equivalent to UIRT models. In MIRT models, an overall discrimination index is defined as the multidimensional discrimination index (MDISC; Reckase, 1985); item difficulty, b_{ik} on dimension *k*, is defined with d_i.

$$\mathrm{MDISC}_i=\left(\sum_{k=1}^{m}a_{ik}^2\right)^{1/2}, \tag{2}$$

and

$$b_{ik} = \frac{-d_i}{a_{ik}}. \tag{3}$$

However, MIRT models have not incorporated the potential correlations among latent traits. The models have an assumption of orthogonality of latent traits, as shown in Equation 1. Therefore, MIRT models have been developed through the connection between the normal ogive model and item factor analysis (IFA) to accommodate correlations among latent traits. When the classical linear factor model is applied to binary items, it is called item factor analysis (Bock, Gibbons, & Muraki, 1988).

Full-Information Item Factor Analysis

Bock, Gibbons, and Muraki (1988) introduced an IRT-based item factor analysis called "full-information item factor analysis (FIIFA)" that does not require calculation of inter-item correlation coefficients and is not strongly limited by the number of items. In addition, full-information methods are no longer limited in applications due to the number of factors and the total number of response patterns increasing exponentially.

In the FIIFA model, the conditional probability of an item score u_{ij} = 1, a correct response to item *i* by examinee *j* with trait vector $\boldsymbol{\theta}_j$, can be described as

$$P(u_{ij} = 1 \mid \boldsymbol{\theta}_j,\ \boldsymbol{\lambda}_i,\ \tau_i) = \frac{1}{\sqrt{2\pi}\sigma_i}\int_{\tau_i}^{\infty} \exp\left(-\frac{1}{2}\left(\frac{X_i - \boldsymbol{\lambda}_i\boldsymbol{\theta}_j}{\sigma_i}\right)^2\right) dX_i\,, \tag{4}$$

where $\boldsymbol{\lambda}_i$ is factor loading vector of item *i* and τ_i is the threshold of item *i*. The latent variable X_i is assumed to follow N(0,1), and the data are assumed to be sampled from a population of people whose θs follow a particular multivariate distribution. Provisionally, the underlying trait vector $\boldsymbol{\theta}_j$ is assumed to follow MVN(**0, I**); however, this assumption could be relaxed to allow for correlated traits and non-normal distributions (Bock, Gibbons, & Muraki, 1988). If the traits are orthogonal, error variances, σ_i^2 are given by $\mathbf{I}-\boldsymbol{\lambda}\boldsymbol{\lambda}'$. If the traits are correlated, σ_i^2 can be presented by $\mathbf{I}-\boldsymbol{\lambda}\boldsymbol{\Phi}\boldsymbol{\lambda}'$, where $\boldsymbol{\Phi}$ is the trait intercorrelation matrix. Consequently, the trait correlations in FIIFA can affect the model by interposing $\boldsymbol{\Phi}$ between $\boldsymbol{\lambda}$ and $\boldsymbol{\lambda}'$.

Estimation of item parameters. Estimates of factor loadings and thresholds are obtained from slope and intercept values estimated in the framework of IRT modeling. Since Takane and De Leeuw (1987) showed the formal equivalence of the marginal likelihood of the multidimensional two-parameter normal ogive model and the marginal likelihood of the IFA for dichotomous variables, the parameters of the MIRT model can be linearly transformed to those of the FIIFA model. The parameters of MIRT—$\mathbf{a}_i$ (a vector of slopes) and d_i (the intercept parameter)—can be expressed by the parameters of the FIIFA model, $\boldsymbol{\lambda}_i$, τ_i, and σ_i^2 and vice-versa. Given factor loadings and thresholds, the item slope and intercept of the *k*th dimension can be obtained by

$$a_{ik}=\frac{\lambda_{ik}}{\sigma_i} \text{ and } d_i=-\frac{\tau_i}{\sigma_i}, \tag{5}$$

where $\sigma_i=\sqrt{1-\boldsymbol{\lambda}_i\boldsymbol{\Phi}\boldsymbol{\lambda}_i'}$. Factor loadings and the thresholds of *k* dimensions can be transformed from item slopes and intercepts as

$$\lambda_{ik}=\frac{a_{ik}}{\sqrt{1+\mathbf{a}_i\boldsymbol{\Phi}\mathbf{a}_i'}} \quad \text{and } \tau_i=\frac{-d_i}{\sqrt{1+\mathbf{a}_i\boldsymbol{\Phi}\mathbf{a}_i'}}. \tag{6}$$

Since the MIRT model is relatively simpler than the FIIFA model, item intercepts and slopes are estimated to obtain factor loadings and slopes.

The marginal maximum likelihood (MML) method can be adopted to estimate slopes and intercepts of the MIRT model given a multivariate distribution representing an examinee's $\boldsymbol{\theta}$. By adopting the multidimensional extension of the conditional independence assumption (i.e., the items are uncorrelated conditional on the underlying trait vector, $\boldsymbol{\theta}$), the probability of an observed response pattern conditional on the trait vector $\boldsymbol{\theta}_j$ can be represented as

$$P(\mathbf{u}_j \mid \boldsymbol{\theta}_j, \mathbf{a}_i, d_i) = \prod_{i=1}^{n} \left[P(u_{ij} = 1 \mid \boldsymbol{\theta}_j, \mathbf{a}_i, d_i) \right]^{u_{ij}} \left[1 - P(u_{ij} \mid \boldsymbol{\theta}_j, \mathbf{a}_i, d_i) \right]^{1-u_{ij}}, \tag{7}$$

where n equals the number of items in the test. This conditional probability is called a likelihood function conditional on the trait vector $\boldsymbol{\theta}_j$. For a random subject sampled from a population with a continuous $\boldsymbol{\theta}$ distribution g($\boldsymbol{\theta}$), the unconditional likelihood function of $\mathbf{u}_j$ for a k-dimensional trait vector is

$$P(\mathbf{u}_j) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} P(\mathbf{u}_j \mid \boldsymbol{\theta}_j)\, g(\boldsymbol{\theta})\, d\theta_1 d\theta_2 \cdots d\theta_k, \tag{8}$$

where g($\boldsymbol{\theta}$) is the population multivariate density of trait vector $\boldsymbol{\theta}$ with mean $\mathbf{0}$ and covariance **I**, Equation 8 can be approximated by a k-dimensional Gauss-Hermite quadrature (e.g., Bock, Gibbons, & Muraki, 1988). Through substitution of the quadrature point, X_k, and quadrature weight, A(X_k), for the values of the multidimensional trait $\boldsymbol{\theta}$, the unconditional probability is approximated by the weighted sum,

$$P(\mathbf{u}_j) \cong \sum_{qk}^{Q} \cdots \sum_{q2}^{Q} \sum_{q1}^{Q} P(\mathbf{u}_j \mid X_{q1}, X_{q2}, \cdots, X_{qk}) A(X_{q1}) A(X_{q2}) \cdots A(X_{qk}), \tag{9}$$

where X_k is the quadrature point for the k -dimensional trait vector and A(X_k) is the corresponding weight of X_k. Given the frequency r_s of a particular response pattern $\mathbf{u}_s$ for n items in a sample of N examinees, the number of distinct pattern is $s < \min(2^n, N)$, and the joint likelihood function of items and examinees for the sample is represented as a multinomial function with parameters N and $P(\mathbf{u}_s)$,

$$M = \frac{N!}{r_1!r_2!\cdots r_s!} P(\mathbf{u}_1)^{r_1} P(\mathbf{u}_2)^{r_2} \cdots P(\mathbf{u}_s)^{r_s}, \tag{10}$$

where $P(\mathbf{u}_s)$ is the unconditional probability for unique responses. Equation 10 is often referred to as an unconditional likelihood function, of which the log-likelihood form is more commonly used in MML estimation method,

$$\log M = r_1 \log P_1 + r_2 \log P_2 + \cdots + r_s \log P_s. \tag{11}$$

In Equation 11, the MIRT model parameters of the slopes and intercepts can be obtained by maximizing it, and then item parameters are converted to factor loadings and threshold parameters of the FIIFA model by using Equation 6.

In order to estimate item parameters, Bock and Aitkin (1981) developed an expectation/maximization (EM) algorithm to maximize the log likelihood presented as Equation 11. The EM algorithm for MML is an iterative procedure consisting of an expectation (E) stage and a maximization (M) stage. The iterative procedure successively improves the expected frequencies for correct responses and θ levels. In the expectation stage, the expected number of examinees at each quadrature point representing pre-determined θ levels is computed. At the same time, the expected number of examinees correctly answering each particular item is also computed. Then, using these expected frequencies, item parameter estimates are found to maximize the likelihood in the maximization stage. Final item parameter estimates and standard errors are obtained through the Newton-Raphson iteration procedure when a criterion value converges to a small value.

Implementation of the FIIFA model. The TESTFACT program (Wood et al., 2003) was developed for the full-information exploratory item factor analysis model with an IRT perspective. The thresholds and factor loadings in FIIFA can be estimated by all distinct response patterns instead of joint proportions of item pairs, using the marginal maximum likelihood estimation method. Typically, since EM solutions converge so slowly, it is important to begin with an accurate starting value to estimate item parameters. The TESTFACT program performs a principal axis factor analysis on the tetrachoric correlation matrix of items to obtain starting values. Because the factors of the principal axis factor analysis are orthogonal, their loadings can be used to obtain starting values after transforming into item slopes and intercepts. Then, item slopes and intercepts estimated by the MML method are linearly transformed again into factor loadings and thresholds. Finally, the resulting full-information factor pattern can be rotated orthogonally or obliquely. The TESTFACT program provides factor solutions with orthogonal (VARIMAX) or oblique (PROMAX) rotations. TESTFACT does not provide a root mean square residual (RMSR) index, but does provide a residual matrix. The dimensionality decision is based on a test of the difference between chi-square statistics.

Many studies have compared full-information with full-responses and bivariate-information with inter-item correlation procedures using simulated and real data. Most studies have compared the method in terms of parameter recovery. Finger (2002)

conducted a simulation study using unidimensional data to compare two procedures, and found that the bivariate-information procedure using ULS (unweighted least squares) estimation method was more accurate than the full-information procedure.
Using simulated multidimensional data sets, Gosz and Walker (2002) found that both approaches performed well in terms of parameter recovery, depending on conditions related to the amount of multidimensionality in the data. For more information on the comparison between full-information and bivariate-information procedures, see Bolt (2005).

The MCAT Algorithm

Initially, Segall (1996) used a confirmatory simple structure to implement MCAT, where items within one scale were assumed to measure the same latent trait and each item was loaded on only one latent trait. For that reason, this confirmatory simple structure MIRT model is called a "multi-unidimensional" IRT (multi-UIRT) model or a "between-item" MIRT model (Wang & Chen, 2004).

The CAT algorithm used with the multi-UIRT model specifies two more things than UCAT. One is a θ estimation procedure to obtain a provisional θ estimate after each administration of an item. Multidimensional θ estimation procedures estimate multiple latent traits simultaneously. The other is an item selection algorithm that provides an efficient choice of items based on the examinees' provisional θ estimates. Two commonly used procedures of θ estimation in UIRT (maximum likelihood and Bayesian estimation) also can be used with a MCAT.

Multidimensional θ estimation. The likelihood of a vector of observed responses $\mathbf{u}$ given $\boldsymbol{\theta}$ is expressed in Equation 7, where i is the adaptively administered items. The maximum likelihood (ML) estimates on p-dimensional traits can be obtained by setting the first partial derivative of the log likelihood function of the p simultaneous equations at zero as follows

$$\frac{\partial}{\partial \theta} \ln L(\mathbf{u} \mid \boldsymbol{\theta}) = \mathbf{0}, \tag{12}$$

where

$$\frac{\partial}{\partial \theta} \ln L(\mathbf{u} \mid \boldsymbol{\theta}) = \begin{bmatrix} \frac{\partial}{\partial \theta_1} \ln L(\mathbf{u} \mid \boldsymbol{\theta}) \\ \frac{\partial}{\partial \theta_2} \ln L(\mathbf{u} \mid \boldsymbol{\theta}) \\ \vdots \\ \frac{\partial}{\partial \theta_p} \ln L(\mathbf{u} \mid \boldsymbol{\theta}) \end{bmatrix}. \tag{13}$$

The partial derivative of the log likelihood with respect to θ_k (for k = 1, 2, …, p) takes a similar form to the unidimensional 2PLM (Baker, 1992) as shown below,

$$\frac{\partial}{\partial \theta_k} \ln L(\mathbf{u} \mid \boldsymbol{\theta}) = 1.7 \sum_{i=1}^{v} \frac{a_{ik} P_i(\boldsymbol{\theta}) \left[\mathrm{u}_i - P_i(\boldsymbol{\theta}) \right]}{P_i(\boldsymbol{\theta})}, \tag{14}$$

where k is a dimension of $\boldsymbol{\theta}$, and v is the number of adaptively administered items. Since the likelihood in Equation 7 has no closed form solution, the Newton-Raphson procedure can be used to approximate it. Let $\theta^{(n)}$ be the nth approximation to the value of $\boldsymbol{\theta}$ that maximizes $\log L(\mathbf{u} \mid \boldsymbol{\theta})$. Then the maximum likelihood estimate can be approximated by an iterative procedure:

$$\hat{\boldsymbol{\theta}}^{(n+1)} = \hat{\boldsymbol{\theta}}^{(n)} - \boldsymbol{\delta}^{(n)}, \tag{15}$$

where the $\hat{\boldsymbol{\theta}}^{(n+1)}$ is the vector for updated estimates of $\hat{\boldsymbol{\theta}}^{(n)}$, and the convergence criterion at the nth iteration is

$$\boldsymbol{\delta}^{(n)} = \left[\mathbf{H}\left(\boldsymbol{\theta}^{(n)}\right)\right]^{-1} \times \frac{\partial}{\partial \boldsymbol{\theta}} \ln L\left(\mathbf{u} \mid \boldsymbol{\theta}^{(n)}\right). \tag{16}$$

The matrix $\mathbf{H}\left(\boldsymbol{\theta}^{(n)}\right)$ is the $p \times p$ symmetric matrix of second derivatives or the negative information matrix evaluated at $\boldsymbol{\theta}^{(n)}$. ML estimates can be obtained through successive approximations using Equations 15 and 16. Additional approximations are obtained until the elements of $\boldsymbol{\theta}^{(n)}$ change very little from one iteration to the next.

Multidimensional item selection. In UCAT, items can be selected on the basis of item information. Likewise, in MCAT, the provisional trait estimate vector, $\hat{\boldsymbol{\theta}}^{(n)}$ obtained after responding to the *n*th item, is used to evaluate the item information function (Lord, 1980):

$$I\left(\theta, u_i\right) = \frac{\left[\dfrac{\partial P_i\left(\hat{\theta}^{(n)}\right)}{\partial \theta}\right]^2}{P_i\left(\hat{\theta}^{(n)}\right) Q_i\left(\hat{\theta}^{(n)}\right)}, \tag{17}$$

where u_i is the candidate item response among items in the item bank, $P_i\left(\hat{\theta}^{(n)}\right)$ is the item response function with candidate item *i* at $\hat{\theta}^{(n)}$, and $Q_i\left(\hat{\theta}^{(n)}\right) = 1 - P_i\left(\hat{\theta}^{(n)}\right)$. The only difference from UCAT item selection is that MCAT item selection is based on the volume of the multivariate normal ellipsoid (Anderson, 1984, p. 263).

In a multivariate normal distribution, the volume of the credibility region (Bayesian confidence interval) is given by

$$V_i = \left|\Sigma_i\right|^{1/2} \times g\left(K\right) \times \left[\chi_K^2\left(p\right)\right]^{K/2}, \tag{18}$$

where Σ_i is the covariance matrix of $\boldsymbol{\theta}$ based on the administration of item *i*, *K* is the number of dimensions, g(K) is the surface area of a sphere in k dimensions, and $\chi_K^2(p)$ is the χ^2 value ($df = K$) at the $p \times 100$ percentile. When different candidate items are compared, all terms are constant except the Σ_i in the Equation 18. Therefore, only Σ_i was used to compute the largest decrement in the size of the credibility region. Since Σ_i is related to the observed standard error of estimates, if the $|\Sigma_i|$ is small, the observed standard error of estimates would be small. As a result, the multidimensional item selection strategy is similar to the unidimensional item selection strategy.

In order to select the item that provides the smallest standard error of estimate, decrement in the volume of the credibility region after the administration of item i can be computed as

$$V_i = c\left|\Sigma_{S_{n-1}}\right|^{1/2} - c\left|\Sigma_{i|S_{n-1}}\right|^{1/2}, \qquad (19)$$

where c is a constant defined by $g(K) \times \left[\chi_K^2(p)\right]^{K/2}$, $\Sigma_{S_{n-1}}$ is the covariance matrix of provisional estimates $\hat{\boldsymbol{\theta}}^{n-1}$ after administration of $n-1$ items, and $\Sigma_{i|S_{n-1}}$ is the covariance matrix of provisional estimates $\hat{\boldsymbol{\theta}}^{n-1}$ after administration of $n-1$ items and the candidate item i.

The covariance matrix $\Sigma_{S_{n-1}}$ is equal to the inverse of the information matrix evaluated at $\hat{\boldsymbol{\theta}}^{n-1}$, which is described as

$$\Sigma_{S_{n-1}} = \left[\mathbf{I}_{S_{n-1}}\right]^{-1}, \qquad (20)$$

and the covariance matrix of $\Sigma_{i|S_{n-1}}$ which includes candidate item i can be expressed in terms of the information matrix as

$$\Sigma_{i|S_{n-1}} = \left[\mathbf{I}_{S_{n-1}} + \mathbf{I}_i\right]^{-1}. \tag{21}$$

Consequently, the volume decrement of the credibility region can be illustrated by using Equations 20 and 21 as

$$\left|\Sigma_{S_{n-1}}\right|^{1/2} - \left|\Sigma_{i|S_{n-1}}\right|^{1/2} = \left|\left[\mathbf{I}_{S_{n-1}}\right]^{-1}\right|^{1/2} - \left|\left[\mathbf{I}_{S_{n-1}} + \mathbf{I}_i\right]^{-1}\right|^{1/2}. \tag{22}$$

Since the determinant of the inverse of $\mathbf{I}$ is equal to the inverse of the determinant, the right side of Equation 22 can be redefined as

$$\left|\left[\mathbf{I}_{S_{n-1}}\right]^{-1}\right|^{1/2} - \left|\left[\mathbf{I}_{S_{n-1}} + \mathbf{I}_i\right]^{-1}\right|^{1/2} = \left|\mathbf{I}_{S_{n-1}}\right|^{-1/2} - \left|\mathbf{I}_{S_{n-1}} + \mathbf{I}_i\right|^{-1/2}. \tag{23}$$

Finally, the volume decrement V_i can be maximized by selecting the item *i* that maximizes $\left|\mathbf{I}_{S_{n-1}} + \mathbf{I}_i\right|$ in the right side of Equation 23.

As a result, the item selection method for MCAT is to seek an item maximizing the determinant of information (Segall, 1996). An item with the maximum determinant of information will provide the largest decrement in the volume of the credibility ellipsoid (Segall, 2000, p. 63). This statistical concept has been graphically illustrated in Segall's (2000) article. The information in the *n*th candidate item *i* consisted of two components:

$$\mathbf{I}_{i|S_{n-1}} = \mathbf{I}_{S_{n-1}} + \mathbf{I}_i, \tag{24}$$

where S_{n-1} is the set of administered items $(i_1, i_2, \cdots, i_{n-1})$, and

$$\mathbf{I}_i = (1.702)^2 \mathbf{a}_i \mathbf{a}_i' \left[P_i\left(\hat{\boldsymbol{\theta}}^{(n)}\right) Q_i\left(\hat{\boldsymbol{\theta}}^{(n)}\right)\right]. \tag{25}$$

$\mathbf{I}_i$ is the item information matrix for candidate item i, $\mathbf{a}_i$ is a $k \times 1$ vector of discrimination parameters for item *i*, $P_i\left(\hat{\boldsymbol{\theta}}^{(n)}\right)$ is the item response function with candidate item *i* evaluated at $\hat{\boldsymbol{\theta}}^{(n)}$, $Q_i\left(\hat{\boldsymbol{\theta}}^{(n)}\right)=1-P_i\left(\hat{\boldsymbol{\theta}}^{(n)}\right)$, and the $\mathbf{I}_{S_{n-1}}$ is the sum of $\mathbf{I}_i$ across the previously administered items.

Like UCAT, when $\hat{\boldsymbol{\theta}}^{(n)}$ is obtained from responding to the *n*th item, candidate item *i* is selected to maximize the determinant of information described as

$$\left|\mathbf{I}_{i|S_{n-1}}\left(\hat{\boldsymbol{\theta}}^{(n)},u_i\right)\right|, \tag{26}$$

where $\mathbf{I}_{i|S_{n-1}}$ implies that the information matrix associated with item *i* depends on both the characteristics of the candidate item *i* itself, and the characteristics of the previously administered $n-1$ items. The candidate item *i* which maximizes the determinant of the information matrix $\mathbf{I}_{i|S_{n-1}}$ at $\hat{\boldsymbol{\theta}}^{(n)}$ will provide the largest decrement in the size of the credibility region.

Bayesian θ estimation and item selection. Maximum a posteriori (MAP) is a Bayesian method that incorporates information about the prior θ distribution in order to better approximate the posterior distribution of θ (Bock & Mislevy, 1982). One advantage of using prior information is that an estimate of θ exists when examinees have either all correct or all incorrect responses. MAP was used for CAT for θ estimation and item selection by Segall (1996) as one Bayesian method. According to Bayes' theorem, the posterior density function of $\boldsymbol{\theta}$ is described as

$$f(\boldsymbol{\theta}\,|\,\mathbf{u})=\frac{L(\mathbf{u}\,|\,\boldsymbol{\theta})f(\boldsymbol{\theta})}{f(\mathbf{u})}, \tag{27}$$

where $L(\mathbf{u}\,|\,\boldsymbol{\theta})$ is the likelihood function given by Equation 7; $f(\boldsymbol{\theta})$ is the prior distribution of $\boldsymbol{\theta}$, MVN($\mathbf{0},\boldsymbol{\Phi}$); and $f(\mathbf{u})$ is the marginal probability of **u**. The MAP estimates of $\boldsymbol{\theta}$ can be approximated by setting the partial derivative of the log of the posterior distribution in

Equation 27 at zero.

The Bayesian item selection method adjusts the ML item selection method like the Bayesian θ estimation method for selecting candidate item i by maximizing the determinant of the posterior information matrix as

$$\left| \mathbf{I}_{i|S_{n-1}} \left(\hat{\boldsymbol{\theta}}^{(n)}, u_i \right) + \boldsymbol{\Phi}^{-1} \right|, \tag{28}$$

where $\boldsymbol{\Phi}^{-1}$ is the inverse of the covariance matrix of the prior distribution of trait vector $\boldsymbol{\theta}$. The only difference from ML item selection is that the Bayesian method considers the prior distribution of the trait vector $\boldsymbol{\theta}$.

Issues of the MCAT algorithm. In light of the advances of MCAT, most previous studies have applied the multi-UIRT model to implement MCAT in educational achievement data. However, MCATs with the multi-UIRT model have several drawbacks. First, the assumption of simple structure might lead to some poorly specified loadings in some data (Segall, 1996). In practice, the multi-UIRT model is probably more common in large-scale testing situations, and might not be appropriate for data in which each item loads on multiple traits, as in psychological data. Second, the derivation of disattenuated correlations among trait scores, $\rho_{kk'}$, is not built on a theoretical base, but they are estimated from the disattenuated correlations among subtests.

However, if a certain confirmatory approach to multidimensional item parameter estimates offers a better fit to the data than the simple structure obtained by the multi-UIRT, the above problems will be solved. Along this line, the confirmatory bifactor model can be conceived as an alternative solution that would yield readily interpretable latent traits, which is suitable for items measuring more than one latent trait and takes account of the correlations among latent traits. In the bifactor model, the inter-factor correlations $\rho_{kk'}$ would be identified by the general factor without the need of estimating correlations. Consequently, a confirmatory bifactor model eliminates the arbitrary part of estimating inter-factor correlations following FIIFA and multi-UIRT models. Recent empirical research has demonstrated that the bifactor model is suitable for large items pool that span many correlated dimensions (e.g., Gibbons & Hedeker, 1992 ; Reise, Morizot, & Hays, 2007).

A CAT With the Bifactor Model

In practice, bifactor modeling has been considered as an approach to resolving the limitation in detecting the number of dimensions in MIRT models. A bifactor model can circumvent the problem by having only two types of factors: general and group factors. Gibbons and Hedeker (1992) presented a full-information item bifactor model extending the UIRT model to multidimensional data, where each item is related to only a general factor and a single group factor. Since bifactor analysis constrains each item to have a nonzero loading on the primary dimension and on only one secondary dimension among multiple group factors, the computational requirements for item parameter estimation of the bifactor model are relatively lighter than other MIRT models. Above all, the bifactor model explicates specific content structures containing a general factor and multiple group factors, which is the primary motivator of using the bifactor model. Several researchers have demonstrated the benefits of using the bifactor model to conceptualize psychological constructs (Chen, West, & Sousa, 2006; Gibbons & Hedeker, 1992; Reise, Morizot, & Hays, 2007).

The Bifactor Model

As mentioned above, Bock and Aitkin (1981) extended the IRT model to the multidimensional case, where each item is related to one or more underlying latent dimensions, traits, or constructs of interest. If inter-factor correlations show that the factors are substantially correlated, researchers might wish to estimate a general level of performance over all dimensions, while simultaneously taking into account the redundant information within the item subsets that would reduce the precision in estimating the general factor. In that case, the bifactor model (Holzinger & Swineford, 1937), consisting of a general factor and independent group factors, can be fitted well to data.

The bifactor model not only furnishes simple structure on orthogonal reference axes (the integral of the product of any two different functions is zero), but also provides a more complete rationale of structuring psychological traits than that given by conventional oblique solutions. The simple structure on orthogonal reference axes provides simplicity of analysis and easy interpretation of the factor structure. Holzinger and Swineford (1937) originally applied the term "bifactor" to a test measuring psychological traits. They defined the bifactor pattern as a theoretical framework in which all variables are explained by a general factor and group factors, both as the first-order factors. This bifactor pattern assumes that uncorrelated group factors are independent of the general factor. The bifactor model allows only one of the $k = 2,\ldots, p$ values of λ_{ik} (group factor loadings) to be nonzero including λ_{i1} (general factor loading). For example, the theoretical bifactor pattern in four items can be described as

$$\boldsymbol{\lambda} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & 0 \\ \lambda_{21} & \lambda_{22} & 0 \\ \lambda_{31} & \lambda_{32} & 0 \\ \lambda_{41} & 0 & \lambda_{43} \\ \lambda_{51} & 0 & \lambda_{53} \\ \lambda_{61} & 0 & \lambda_{63} \end{bmatrix} \tag{29}$$

The first column is for the general factor, and the other columns are the group factors in the factor pattern matrix.

The dimensional structure in a bifactor model is pre-determined through prior information. Therefore, the bifactor model is in the confirmatory spirit of fitting a model to data with a hypothesis of a structure, unlike an EFA that estimates the number of common factors, explores a structure, and lets the analysis define the structure. In the perspective of a confirmatory approach, the bifactor model allows each item to have loadings on a single general factor and only one group factor. This initiative by a researcher reduces the number of parameters to be estimated and gives the model more degrees of freedom. In addition, the bifactor model can avoid the problem of estimating inter-factor correlations because the general factor directly contributes to all items, and secondary factors account for the residual information remaining after estimation of the general factor and are independent of each other.

Since the Schmid-Leiman (1957) solution (SLS) is the derivation of a single order of hierarchical factors, the SLS allows building an exploratory bifactor model on the polychoric correlation matrix because the bifactor model is a special case of the hierarchical factor model (Yung, Thissen, & McLeod, 1999). An exploratory bifactor model can be implemented in the R function "schmid" (Schmid & Leiman, 1957), SAS or SPSS macros (Wolff & Preising, 2005). A confirmatory bifactor model can be directly implemented as an R program using the “sem” package, Mplus (Muthén & Muthén, 2004), and LISREL (Jöreskog & Sörbom, 1995). However, those methods are based on a linear factor model using a correlation matrix and do not provide full-information factor analysis. In these applications, the models are employed to obtain the dimensions only, instead of examinees’ scores. Therefore, this research used the confirmatory bifactor pattern represented at the item level (i.e., the IRT perspective) because interest was in examinees’ scores.

Chen, West, and Sousa (2006) compared the bifactor model with a second-order factor model. A bifactor model is potentially applicable when (1) there is a general factor that is hypothesized to account for the communality of the items (2) there are multiple domain group factors, each of which is hypothesized to account for the unique influence of the specific domain over and above the general factor, and (3) a focal interest is in the domain group factors as well as a general factor. On the other hand, a second-order factor model can be specified in which the fit of the second-order structure can be statistically tested, so long as more than two first-order factors are hypothesized. A second-order factor model is potentially applicable when (1) the first-order factors are substantially correlated with each other and (2) there is a second-order factor that takes account of the relationship among the first-order factors. The second-order factor model may be employed to test whether there is a second-order factor explaining the correlation among the first-order factors.

In practice, Reise, Morizot and Hays (2007) demonstrated that 16 items in the consumer assessment of healthcare providers and systems (CAHPS 2.0) were fitted well with both the bifactor model and a second-order factor model (multidimensional solution). However, they showed that the major part of common variance was explained by a general factor in the bifactor model, which is a main conceptual difference with the second-order factor in a second-order factor model. A second-order factor contains a qualitatively different dimension from first-order factors because a second-order factor explains generally the common variance among first-order factors, not observed variables. On the other hand, a general factor in the bifactor model is on the same conceptual level with group factors. As a consequence, Reise et al. (2007) said that although both the bifactor model and second-order factor model can provide the same fit to data, the second-order factor model does not address directly the dimensionality assessment (i.e., how much of the item variance is due to the general construct and how much is due to secondary dimensions), while the bifactor model can do so directly.

Full-Information Item Bifactor Analysis (FIIBFA)

The factor analytic IRT models overcame a restrictive unidimensionality assumption in test structure. However, most factor analytic IRT models are based on EFA. For example, the FIIFA model does not make use of a priori information to determine the number of underlying latent traits. Also the model does not enable us to specify one-to-one relationships between items and factors. In addition, interpretation of factor structure remains subjective.

As an initiative of the confirmatory approach to the FIIFA model, Gibbons and Hedeker (1992) derived a bifactor model for binary response data. Gibbons et al. (2007) extended the work to polytomous data; this method has been called full-information item bifactor analysis (FIIBFA). The FIIBFA model is the first confirmatory approach to IRT modeling to estimate item parameters from multidimensional data, making use of a priori theoretical considerations to investigate relationships between observed variables and latent variables. Through a simplification of the likelihood equations, the FIIBFA assumptions can permit a model with a large number of group factors and conditional dependence among items after conditioning on **θ**. Also, issues of dimensionality and estimation turn out to be relatively easier to handle with the FIIBFA model than the exploratory FIIFA model (e.g., Bock et al., 1988).

Gibbons and Hedeker (1992) specified the FIIBFA model as combining a UIRT model with the multi-UIRT model representing simple structure, meaning that each item is related to a general trait and one group trait only. In the two-dimensional computation in the bifactor model, a primary dimension should be considered first, and then a second dimension would be considered to estimate the probability of a correct response. As a consequence, the conditional probability of correct response in the FIIBFA model can be described as

$$P(u_{ij} = 1 \mid \theta_{j1}, \theta_{jk},\ \lambda_{i1}, \lambda_{ik}\ \ \tau_i) = \Phi_i(\theta_1, \theta_k) = \frac{1}{\sqrt{2\pi}\sigma_i} \int_{\tau_i}^{\infty} \exp\left(-\frac{1}{2} \left(\frac{X_i - \lambda_{i1}\theta_{j1} - \lambda_{ik}\theta_{jk}}{\sigma_i} \right)^2 \right) dX_i, \qquad (30)$$

where $\sigma_i = \sqrt{1 - \lambda_{i1}^2 - \lambda_{ik}^2}$. Unlike FIIFA, the unconditional probability in a FIIBFA model can be justified by evaluating the probability of dimensions 2, … , k, after integrating with respect to the distribution of θ_1 . The bifactor restriction reduces the p-dimensional integral in FIIFA to a two-dimensional integral, one for θ_1 and the other for $\theta_2, \cdots, \theta_k$. In sequence, the integration of secondary dimensions over the probability distribution function of the primary dimension in Equation 30 leads to the unconditional probability of endorsing the full response vector, $\mathbf{u}_j$ as

$$P(\mathbf{u}_j = 1) = \int_{\theta_1} \left\{ \prod_{k=2}^{p} \int_{\theta_k} \left[\prod_{i=1}^{n} \left(\Phi\left[\frac{X_i - \lambda_{i1}\theta_1 - \lambda_{ik}\theta_k}{\sigma_i} \right] \right) \right] g(\theta_k) d\theta_k \right\} g(\theta_1) d\theta_1 , \qquad (31)$$

where p is the number of group factors, and n is the number of items. Because of the bifactor structure, Equation 31 indicates that primary and secondary factors are distributed independently in the test population. Unlike FIIFA, σ_i does not entail any inter-factor correlation due to an assumptions of the bifactor model and can have a unique estimate under the model. As a consequence, the marginal likelihood function of a particular response pattern $\mathbf{u}_j$ can be described as

$$P_l = \int_{\theta_1} \left\{ \prod_{k=2}^{p} \int_{\theta_k} \left[\prod_{i=1}^{n} \left([\Phi_i(\theta_1, \theta_k)]^{u_{ij}} \, [1 - \Phi_i(\theta_1, \theta_k)]^{1-u_{ij}} \right) \right] g(\theta_k) d\theta_k \right\} g(\theta_1) d\theta_1 . \qquad (32)$$

This marginal likelihood function in the FIIBFA model can be approximated by a Gauss-Hermite quadrature point as

$$P_l \cong \sum_{q1}^{Q} \left\{ \prod_{k=2}^{p} \sum_{q_k}^{Q} \left[\prod_{i=1}^{n} \left(\left[\Phi_i(X_{q1}, X_{qk}) \right]^{u_{ij}} \left[1 - \Phi_i(X_{q1}, X_{qk}) \right]^{1-u_{ij}} \right) \right] A(X_{qk}) \right\} A(X_{q1}) , \quad (33)$$

where X_q and $A(X_q)$ are the nodes and corresponding weights of a Gauss-Hermite quadrature. The parameters of the FIIBFA model also can be estimated by MML.

The TESTFACT program implements this confirmatory FIIBFA as well as exploratory FIIFA for dichotomous data. Initially, intercept parameters and slope/discrimination parameters for the general factor and the group factors are estimated and then transformed into factor loadings and thresholds. In order to estimate item parameters, the multinomial function in Equation 10 is also applied to MML for the bifactor model. The item parameters are estimated by setting the derivative of the log multinomial function at zero, which is solved by using a variation of the EM algorithm described by Bock and Aitkin (1981). The one difference from the bifactor model in the EM algorithm is that the conditional probability of the response pattern $\mathbf{u}_j$ in sub-dimension k is weighted by the factor, $E_{lk}(\theta_1)$, which is the expected likelihood for sub-dimension k after marginalizing on the primary θ_1. The starting values of TESTFACT for the bifactor model are fixed as an intercept of zero, and a slope of 1.414 for the general factor and 1.00 for the group factors. Then, item slopes and intercepts estimated by the MML method are linearly transferred into factor loadings and thresholds. Because the factors of the bifactor model are orthogonal, factor rotation methods are not routinely used in TESTFACT for the bifactor model.

Fit of the Bifactor Model

In psychological data, many researchers have tried to hypothesize general constructs that encompass several related domains. For example, Jackson, Ahmed, and Heapy (1976) proposed a general structure comprising several domains of achievement motivation. In practice, the ultimate objective of measurement is to estimate the trait level of an examinee on the primary trait in which the instrument was designed to measure. Along this line, bifactor models have been applied to estimate factor structures of highly related domains that are hypothesized to comprise a general construct (e.g., Gustafsson & Balke, 1993; Mulaik & Quartetti, 1997; Yung, Thissen, & McLeod, 1999).

In the perspective of factor analysis, the bifactor model has been found to be appropriate for explicating psychological constructs on intelligence (Gustafsson & Balke, 1993), personality (Chernyshenko, Stark, & Chan, 2001), and psychiatric symptoms

(Steer, Clark, Beck, & Ranieri, 1995). Gustafsson and Balke (1993) applied the bifactor model to a battery of 16 aptitude tests for the 6^{th} grade and 9^{th} grade ($N = 866$). A confirmatory bifactor model was fitted with a general factor (*g*) along with nine orthogonal group factors.

Chen, West, and Sousa (2006) compared the bifactor model with a second-order factor model using a quality of life data set ($N = 403$). They showed that the bifactor model identified three, rather than the hypothesized four, domain-specific factors beyond the general factor, and the bifactor model fit the data significantly better than the second-order factor model.

For an IRT perspective, Gibbons and Hedeker (1992) applied the FIIBFA model to 20 items selected from an ACT natural science test ($N = 1,000$ examinees). The FIIBFA model showed improvement of fit over a multi-UIRT model (measuring four independent dimensions, without a primary trait dimension): $\chi^2_{difference}(20) = 336$, $P < .0001$. In addition, they also applied the FIIBFA model to the Hamilton Depression Rating Scale (HDRS) for assessment of depression and psychological impairment, which is inherently multidimensional. In their study, they showed that the FIIBFA model fit better than a multi-UIRT model: $\chi^2_{difference}(17) = 75$, $P < .0001$ and suggested that a primary depressive dimension is needed to explain these data. As a result, they argued that since psychological data such as HDRS are multidimensional, it is necessary to consider both a primary dimension and specific sub-dimensions.

In recent years, Gibbons, Immekus, and Bock (in press) applied a FIIBFA model of a primary dimension and 15 symptom domains to dichotomous data of the Psychiatric Diagnostic Screening Questionnaire and compared it to the UIRT model. They showed that the FIIBFA model fit better than the UIRT model: $\chi^2_{difference}(139) = 79{,}624$, $P < .001$. In addition, they fitted the FIIBFA model to polytomous data of the Post Traumatic Growth Inventory (PTGI; Tedeschi & Calhoun, 1996) and the Jenkins Activity Survey (Jenkins, Rosenman, & Zyzansk, 1972), and compared them to the unidimensional IRT model.

Simms, Gros, Watson, and O'Hara (2008) applied the FIIBFA model to the Inventory of Depression and Anxiety Symptoms (IDAS; Watson et al., 2007). They found that the FIIBFA model fit the observed response data significantly better than the UIRT model in all samples, judging from the χ^2 and Bayesian information criterion (Schwartz, 1978).

DeMars (2006) applied the FIIBFA model to testlet-based tests from the Programme for International Student Assessment 2000 (PISA 2000) under violations of the local independence assumption in the IRT model. She found that the FIIBFA model fit better than the UIRT model: $\chi^2_{difference}(5) = 103$, $P < .001$ for the math test and $\chi^2_{difference}(14) = 1{,}182$, $P < .0001$ for reading test.

Rijmen (2010) applied the bifactor model, the restricted testlet, and second-order model to the data stemming from an international English assessment test. According to both AIC (Akaike information criterion) and BIC (Bayesian information criterion), the bifactor model was the preferred model for the data.

Through many empirical studies, FIIBFA for dichotomous data can be expected to fit well under a variety of conditions. Consequently, the FIIBFA model has become one alternative to analyzing data with a general construct in implementing CAT.

The BICAT Algorithm

Weiss and Gibbons (2007) developed an algorithm that implements CAT for the bifactor model with dichotomously scored items. They evaluated and demonstrated the efficiency and precision of the performance of the algorithm in both post-hoc simulation data and live-testing data. The bifactor CAT algorithm by Weiss and Gibbons (2007) works as follows.

Step 1: Fit the bifactor model to an appropriately structured set of item responses. The bifactor model is not useful for all multidimensional psychological data. As mentioned before, it is useful for specific psychological data having a general factor and multiple group factors. Usually, the scales are determined by theory and practical considerations, and it is restricted to item sets in which each item loads on the general factor and only one group factor.

Step 2: Express the factor structure as slope-intercept IRT parameters The FIIBFA model provides factor loadings and threshold parameters from TESTFACT. After estimating the item parameters given a fitted model, the factor loadings and thresholds can be transformed to slope and intercept parameters using Equation 5.

Step 3: Convert the slope-intercept parameter estimates to discrimination and difficulty parameters. Most CAT programs are implemented by the logistic IRT model. Therefore, slope and intercept parameters are converted to discrimination and difficulty parameters using Equation 3.

Step 4: Implement CAT on the general factor for each examinee. According to Reise et al. (2007), the general factor can be interpreted as an essentially unidimensional trait in the IRT model if the general factor loadings are greater than the group factor loadings.

Thus, CAT on the general factor loadings can be implemented using traditional UCAT. Each CAT starts with an initial θ estimate (usually 0.0, but variable starting values might be useful if there is valid prior information). Items with maximum information are selected, and θs are estimated using maximum likelihood. Finally, the CAT is terminated using a fixed standard error of the θ estimate (SEM), allowing the number of items to vary across examinees.

Step 5: For the first group factor, identify those items that were administered to the examinee in Step 4. Items have loadings not only on the general factor but also on group factors in a bifactor model. Therefore, each item has two item discrimination parameters and one item difficulty parameter. Among the items that have loadings on the general factor, those related to the first group factor can be identified. These items will vary across examinees based on their estimated θ level on the general factor.

Step 6: Estimate initial θ for the first group factor using items identified in Step 5 for each examinee. Using the discrimination parameters from the bifactor solution for the first group factor, a θ estimate is computed from these items, and used as the CAT starting θ estimate for the first group factor scale.

Step 7: Implement CAT for the first group factor scale using its discrimination parameters and an appropriate termination criterion. The group factor CATs use the same set of CAT options as the general factor CAT with two exceptions (1) an examinee's CAT on a group factor uses a variable entry θ estimate, based on Steps 5 and 6; and (2) although a fixed SEM was used to terminate the group factor scales, the values of the SEM will likely vary among the scales (other termination criteria can also be used in conjunction with, or independent of, the fixed SEM).

Step 8: Repeat Steps 5 -7 for each additional group factor:

a. Identify administered items from the general factor scale for a given group factor scale.

b. Estimate θ for the group factor scale from those items and their scale discrimination parameters.

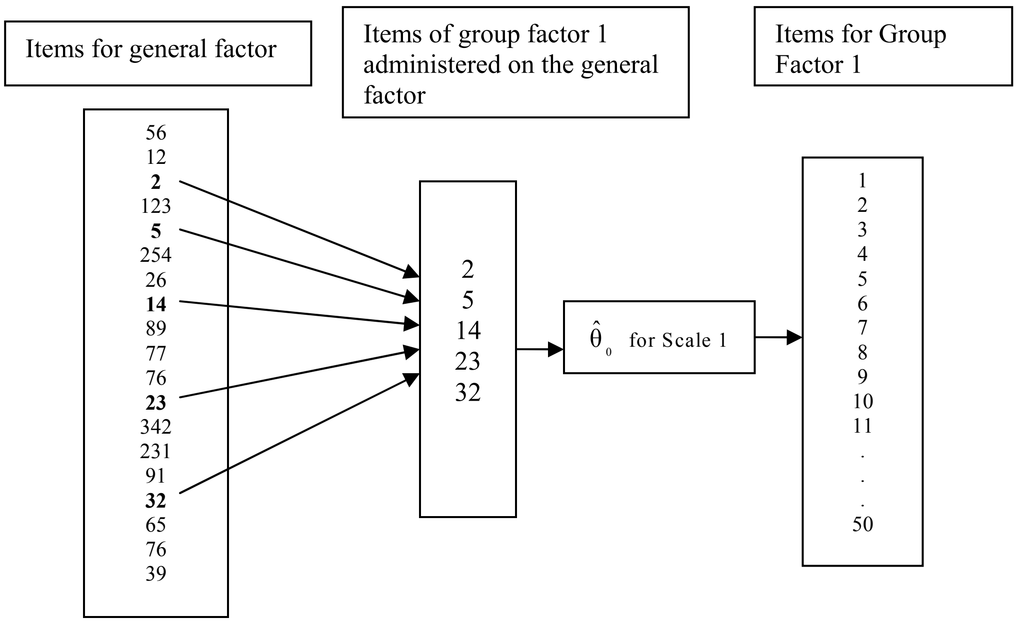
c. Implement CAT for that group factor scale.

Based on the BICAT algorithm, Figure 1 illustrates how items administered to an examinee for scaling the general factor are used in the BICAT algorithm. An examinee's location on the general factor is estimated by the CAT on the general factor, and each examinee receives a subset of items that are potentially different from other examinees. Figure 1 shows that the examinee received 19 items. Of the items answered by the examinee, subsets of items were associated with group factor 1 in the bifactor structure: items 2, 5, 14, 23, and 32. These items were used, in conjunction with their IRT discrimination parameters on group factor scale 1, to derive an initial θ estimate for examinee 1 on group factor scale 1, which was used to select the first item in group factor scale 1. The CAT with group factor scale 1 then proceeded from that starting θ estimate.

The results of using the BICAT algorithm to deliver "The Mood-Anxiety Spectrum Scales" (MASS; Cassano et al., 1997) supported the utility and flexibility of CAT. Weiss and Gibbons (2007) used the completed 616 items of the 626 items in MASS (Mood 155 items, Obsessive-Compulsive 183 items, Panic-Agoraphobia 113 items, and Social Phobia 164 items).

Figure 1

Illustration of Item Usage in the BICAT Algorithm (Weiss & Gibbons, 2007)



They then used the item response data obtained from four groups of examinees (item calibration group, post-hoc CAT calibration group, post-hoc cross-validation group, and live-testing bifactor CAT group).

Weiss and Gibbons (2007) fitted both the UIRT model and FIIBFA model to an identical set of data and demonstrated that the FIIBFA model provided a significant improvement in fit over the UIRT model: $\chi^2_{difference}(616) = 2{,}955$, $P < .0001$. They then applied the bifactor CAT algorithm as described above using EAP θ estimation.

As a termination criterion, they used a pre-specified observed standard error of the θ estimates (General factor = .30, Mood = .35, Obsessive-Compulsive = .475, Panic-Agoraphobia = .40, and Social Phobia = .35). In their analysis, the CAT bifactor algorithm resulted in substantial reduction of items (80% to 90%) in both post-hoc simulation and live bifactor CATs while producing CAT θ estimates that correlated above .90 with θ estimates from the full set of scale items. In addition, the results of the post-hoc simulation provided a good prediction of the results of live testing, confirming the usefulness of post-hoc simulation in the process of developing and implementing bifactor CATs. Results from the live testing, based on a test-retest design, showed high correlations between CAT θ estimates from the post-hoc simulation and retests of live CATs.

Gibbons et al. (2008) administered CAT with the bifactor model to MASS data like Weiss and Gibbons' study (2007) in post-hoc simulation. However, Gibbons et al. selected the initial items corresponding to $\theta = 0$ for a general factor and group factors. The results of this study complemented the previous results implemented by Weiss and Gibbons (2007). They indicated better discriminant validity for CAT by showing both conventionally scored full-scale and CAT effect sizes (difference in scores between two groups of depressed patients) of .63 ($P < .003$) and 1.19 ($P < .001$) respectively.

Furthermore, Immekus, Gibbons, and Rush (2007) studied the BICAT algorithm by conducting a post-hoc simulation administration of the Psychiatric Diagnostic Screening Questionnaire (PDSQ; Zimmerman & Mattia, 2001). For PDSQ full scale, using a fixed standard error termination of .30 and initial item at $\theta = -1, -2, -3, 0, 1, 2, 3$, the correlations between θ estimated by CAT and θ estimated by full test were above .90. For PDSQ subscales, using a fixed standard error termination of .30 and initial item at $\theta = 0$, the correlations between θ estimated by CAT and θ estimated by full test also were above .90.

The previous post-hoc simulation studies (e.g., Immekus, Gibbons, & Rush, 2007; Weiss & Gibbons, 2007) have demonstrated the efficiency of the BICAT algorithm for specific psychological constructs that have both a general factor and several group factors. However, the BICAT algorithm has one limitation. The BICAT algorithm still operates within a unidimensional system of item selection and θ estimation for a general factor and group factors separately. Consequently, the BICAT algorithm might decrease measurement efficiency because it does not consider cross-information gathered from items by both the general factor and group factors in implementing CAT. Therefore, it is necessary to develop truly multidimensional adaptive item selection and θ estimation methods for CAT with the bifactor model. The present study applied multidimensional item selection and θ estimation methods developed by Segall (1996) to a CAT with the bifactor model and developed a unique multidimensional θ estimation method for a CAT with the bifactor model.

The MBICAT Algorithm

The MBICAT algorithm is a special case of the MCAT algorithm by Segall (1996) except that the provisional trait estimate $\hat{\boldsymbol{\theta}}$ is set equal to MVN($\mathbf{0}$, $\mathbf{I}$). Since the bifactor model has an assumption that uncorrelated group factors are independent of the general factor, it is not necessary to consider the correlations of the prior distribution of traits. The item selection methods of the MBICAT algorithm also can use the multidimensional item selection maximizing the determinant of information and the determinant of the posterior information described in Equations 26 and 28. In addition, the MBICAT algorithm implements test termination using the variance of the posterior distribution in three θ estimation methods, as well as a fixed-length method.

EAP estimation method. EAP θ estimates are relatively easy to compute because it does not require the first and second partial derivatives of the likelihood function that are employed to compute a solution as in MAP and MLE methods. In addition, the EAP method is non-iterative, which permits fast calculations of the provisional θ estimate in CAT (Bock & Aitkin, 1981). Although EAP has benefits in θ estimation, there has been no research to develop EAP methods to estimate the latent traits in MCAT. Thus, the MBICAT algorithm in this study used the EAP estimation method as well as MLE and MAP methods. In this study, the ultimate objective was to estimate the θ level of an examinee on the general factor trait and the group factor traits that the instrument was designed to measure. For the bifactor model, Gibbons et al. (2007) simplified the EAP method for estimating the primary latent variable θ_1 and sub-domain trait scores θ_k, given the observed response vector $\mathbf{u}_j$ for an examinee j. The EAP estimate of θ_1 for an examinee j is

$$\hat{\theta}_{j1} = E(\theta_{j1} \mid \mathbf{u}_j, \theta_{j2} \cdots \theta_{jk}) = \frac{1}{P_l} \int_{\theta_1} \theta_{j1} \left\{ \prod_{k=2}^{s} \int_{\theta_k} L_j(\theta_1, \theta_k) g(\theta_k) d\theta_k \right\} g(\theta_1) d\theta_1, \tag{34}$$

where P_l is the unconditional probability of observing response pattern $\mathbf{u}_j$ described in Equation 30, s is the number of group factors, and $L_j(\theta_1, \theta_k)$ is the likelihood function of the bifactor model for examinee j. Similarly, the posterior variance of $\hat{\theta}_{j1}$, which can be used to express the precision of the EAP estimator, is given by

$$V(\theta_{j1} \mid \mathbf{u}_j, \theta_{j2} \cdots \theta_{jk}) = \frac{1}{P_l} \int_{\theta_1} \left(\theta_{j1} - \hat{\theta}_{j1} \right)^2 \left\{ \prod_{k=2}^{s} \int_{\theta_k} L_j(\theta_1, \theta_k) g(\theta_k) d\theta_k \right\} g(\theta_1) d\theta_1. \tag{35}$$

The EAP estimate of θ_k for examinee j is given by

$$\hat{\theta}_{jk} = E(\theta_{jk} \mid \mathbf{u}_j, \theta_{1j}) = \frac{1}{P_l} \int_{\theta_k} \theta_{jk} \int_{\theta_1} \left\{ L_j\left(\theta_1, \theta_k\right) \frac{\Pi_{k=2}^{s} E_{jk}\left(\theta_1, \theta_k\right)}{E_{jk}\left(\theta_1, \theta_k\right)} g\left(\theta_1\right) d\theta_1 \right\} g\left(\theta_k\right) d\theta_k, \quad (36)$$

where $E_{jk}\left(\theta_1, \theta_k\right) = \int_{\theta_k} L_j\left(\theta_1, \theta_k\right) g\left(\theta_k\right) d\theta_k$, (37)

and the corresponding posterior variance of $\hat{\theta}_{jk}$ is

$$V(\theta_{jk} \mid \mathbf{u}_j, \theta_{j1}) = \frac{1}{P_l} \int_{\theta_k} \left(\theta_{jk} - \hat{\theta}_{jk}\right)^2 \int_{\theta_1} \left\{ L_j\left(\theta_1, \theta_k\right) \frac{\Pi_{k=2}^{s} E_{jk}\left(\theta_1, \theta_k\right)}{E_{jk}\left(\theta_1, \theta_k\right)} g\left(\theta_1\right) d\theta_1 \right\} g\left(\theta_k\right) d\theta_k. \quad (38)$$

To evaluate the integrals, Equations 34, 35, 36, 37, and 38 can be reasonably approximated using the Gauss-Hermite quadrature nodes and weights in Equation 9.

Termination criterion in the MBICAT. In Equations 35 and 38, latent traits estimated by the EAP method have posterior variances after administration of each item. Since the observed standard errors (OSE) decrease as the number of items increases, it is possible to employ the OSE as a termination criterion in the MBICAT algorithm through taking square roots of the posterior variances.

Purpose

The objective of this study was to adopt multidimensional item selection and the EAP estimation method to implement a unique MBICAT algorithm in order to estimate a general trait score and group trait scores. The efficiency and precision of the MBICAT algorithm was demonstrated by comparing general and group latent scores estimated by BICAT algorithms.

CHAPTER 2: METHOD

In order to compare the precision of BICAT versus MBICAT, and to investigate the efficiency of MBICAT, two studies were conducted. Monte-carlo simulation studies were conducted to compare true θs with estimated θs using both the BICAT and the MBICAT algorithm. The results from the two studies are summarized and presented separately.

Study 1

Experimental Design

The purpose of Study 1 was to investigate the precision of the BICAT algorithm and the MBICAT algorithm. Four factors that reflect realistic testing situations and could affect the precision of CAT were considered: (1) CAT algorithms (BICATs and MBICAT), (2) different group factor discrimination conditions (low, medium, and high), (3) the number of group factors (two and four), and (4) θ estimation methods (MLE, MAP, and EAP). The comparison was based on three dependent variables, including the correlation between estimated θ and true θ, root mean squared error (RMSE), and observed standard error (OSE).

The second and third independent variables were crossed with the others. Two factors were directly manipulated because item responses were generated independently for six data sets (three different group factor discrimination parameter conditions for the bifactor model with two group factors, and three different group factor discrimination parameter conditions for the bifactor model with four group factors). The other two factors (three CAT algorithms and three θ estimation methods) were manipulated within each of the six response sets.

In order to minimize the sample variance and increase the power to detect the effects of interest, 10 replications were used to compare the differences between $\hat{\theta}$ and θ, for a total of 54 conditions. Thus, average correlation, RMSE, and OSE through 10 replications are presented as dependent variables for Study 1.

Response Generation

In order to approximate the condition of equal measurement precision throughout the θ range, each item bank contained 400 dichotomous items with true item parameters. The item responses were generated according to the bifactor model using an R program (R Development Core Team, 2008). For the monte-carlo simulation study, IRT parameters were specified that could be transformed into factor analytic parameters. The equation for the probability of a correct response for a 2PL bifactor IRT model is

$$P(u_{ij}=1)=\frac{1}{1+\exp\left[-1.702\mathbf{a}_i'\left(\boldsymbol{\theta}_j-b_i\mathbf{1}\right)\right]}, \tag{39}$$

where, $\mathbf{a}$ indicates the vector of discrimination parameters of item i, $\boldsymbol{\theta}_j$ indicates the vector of the general and group factor latent traits of examinee j, and b_i is a multidimensional difficulty parameter that can be transformed to item intercept parameters by Equation 3.

The item responses for this study were generated given the true θs and item parameters using Equation 39. The first step in the data generation process was to generate 400 random numbers from U[0, 1] for each examinee. The probability of a correct response given the 2PL bifactor IRT model was obtained for each item, conditional on θ. These model-based probabilities were compared to the random numbers to obtain the item responses for each item. If the model-based probability was greater than the random number, the response to that item was recorded as correct (1). Likewise, if the model-based probability was less than the random number, the item response was recorded as incorrect (0). This process was repeated for each item to obtain the full item response matrix for the 400 items for each simulated examinee. In order to reduce the variance of the dependent variables, a total of 1,000 examinees were generated within six sets of response matrices (three different group factor discrimination conditions $\times$ two the number of group factors conditions).

Discrimination parameter. Each item was assigned a vector of discrimination parameters $\mathbf{a}_i$ corresponding to each factor (general factor and group factors). Each item was allowed to load on the general factor and a single group factor. Therefore, the vectors of discrimination parameters for items contained in the first group factor took the form $\mathbf{a}_i = \{a_{i1},\ a_{i2},\ 0\}$, where $a_{i1} > 0$ and $a_{i2} > 0$. Similarly, the vectors of discrimination parameters for items that loaded on the second group factor took the form $\mathbf{a}_i = \{a_{i1},\ 0,\ a_{i3}\}$. Given the bifactor model with one general factor and four group factors, the vectors of discrimination parameters for items contained in the first group factor took the form $\mathbf{a}_i = \{a_{i1},\ a_{i2},\ 0,\ 0,\ 0\}$ and those for items loaded on the second group factor took the form $\mathbf{a}_i = \{a_{i1},\ 0,\ a_{i3},\ 0,\ 0\}$, and so forth.

These discriminations for general factor and group factors were obtained from normal distributions for the bifactor models with two group factors or four group factors in order to reflect theoretical characteristics (Harwell, Stone, & Kirisci, 1996). Theoretical characteristics of discrimination parameters suggest that all values would be positive values and follow a normal distribution.

The bifactor model is especially appropriate when researchers have instruments with a dominant general factor (Reise, Morizot, & Hays, 2007). Reise et al. said that when items tend to have small loadings on the general factor and larger loadings on the group factor, the multi-UIRT model should be used, and when the general factor loadings are larger than group factors loadings, the bifactor model should be used to measure traits. Therefore, banks of items were generated that had different magnitudes of factor discrimination parameters to represent typical patterns of bifactor loadings. Figure 2 shows that group factor discrimination conditions A, B, and C were designed to replicate the bifactor loadings with two group factors represented in the CAHPS 2.0 survey (Reise, Morizot, & Hays, 2007). Figure 2 describes three discrimination parameter conditions for the bifactor model with two group factors.

The purpose of the three conditions was to examine the effect of the magnitude of the group factor discrimination parameters on the estimates of group factors. All three conditions were reasonable when translated into factor loadings without any Heywood cases (Heywood, 1931). 200 items were loaded on the first group factor and the second group factor respectively. Item discrimination parameters for the general factor were generated separately from N(1, 0.2) within each group factor discrimination condition. In Condition A, item discrimination parameters for 200 items in each group factor were generated from N(0.5, 0.2), respectively. In Condition B item discriminations for 200 items in each group factor were generated from N(0.6, 0.2), respectively. In condition C, item discrimination parameters for 200 items in both group factors were generated from N(0.7, 0.2) respectively.

Figure 2
Discrimination Parameters Conditions for the Bifactor Model
with Two Group Factors

Condition A — Condition B — Condition C

$$
\begin{bmatrix} \overbrace{\mathrm{N}(1,0.2)}^{a_1} & \overbrace{\mathrm{N}(0.5,0.2)}^{a_2} & \overbrace{\quad 0 \quad}^{a_3} \\ \mathrm{N}(1,0.2) & 0 & \mathrm{N}(0.5,0.2) \end{bmatrix}
\begin{bmatrix} \overbrace{\mathrm{N}(1,0.2)}^{a_1} & \overbrace{\mathrm{N}(0.6,0.2)}^{a_2} & \overbrace{\quad 0 \quad}^{a_3} \\ \mathrm{N}(1,0.2) & 0 & \mathrm{N}(0.6,0.2) \end{bmatrix}
\begin{bmatrix} \overbrace{\mathrm{N}(1,0.2)}^{a_1} & \overbrace{\mathrm{N}(0.7,0.2)}^{a_2} & \overbrace{\quad 0 \quad}^{a_3} \\ \mathrm{N}(1,0.2) & 0 & \mathrm{N}(0.7,0.2) \end{bmatrix}
$$

Figure 3 shows that group factor discrimination conditions D, E and F were designed to replicate the bifactor loadings with four group factors of health-related quality of life (Chen, West, & Sousa, 2006). Figure 3 describes three discrimination parameters conditions for the bifactor model with four group factors.

100 items loaded on each group factor. Item discrimination parameters for 400 items in the general factor were generated from N(1, 0.2) within each condition. In condition D, item discrimination parameters for 100 items in each group factor were generated from N(0.5, 0.2). In condition E, item discriminations for 100 items were drawn from normal distributions with N(0.6, 0.2) for each group factor. In condition F, item discrimination parameters for 100 items in each group factor were drawn from N(0.7, 0.2). In all cases, the item discrimination distributions were separately generated for each of the six response sets. Appendix A shows the item discrimination parameters for each of the six response sets.

Multidimensional difficulty parameter. For an item bank providing equal measurement precision across θs, item difficulty should be evenly and equally distributed throughout the θ continuum of interest (Weiss, 1982). Therefore, for each item bank with 400 items, the item difficulty parameters were randomly generated from a uniform distribution from

–4 to 4 so that they were equally distributed across θs; then they were transformed as item intercept parameters by using Equation 3. Appendix A shows the item difficulty parameters for each of the six response sets.

Figure 3
Discrimination Parameters Conditions for the Bifactor Model
with Four Group Factors

Condition D

$$\begin{array}{c}
\begin{array}{ccccc} \overbrace{\qquad}^{a_1} & \overbrace{\qquad}^{a_2} & \overbrace{\qquad}^{a_3} & \overbrace{\qquad}^{a_4} & \overbrace{\qquad}^{a_5} \end{array} \\
\begin{bmatrix} N(1, 0.2) & N(0.5, 0.2) & 0 & 0 & 0 \\ N(1, 0.2) & 0 & N(0.5, 0.2) & 0 & 0 \\ N(1, 0.2) & 0 & 0 & N(0.5, 0.2) & 0 \\ N(1, 0.2) & 0 & 0 & 0 & N(0.5, 0.2) \end{bmatrix}
\end{array}$$

Condition E

$$\begin{array}{c}
\begin{array}{ccccc} \overbrace{\qquad}^{a_1} & \overbrace{\qquad}^{a_2} & \overbrace{\qquad}^{a_3} & \overbrace{\qquad}^{a_4} & \overbrace{\qquad}^{a_5} \end{array} \\
\begin{bmatrix} N(1, 0.2) & N(0.6, 0.2) & 0 & 0 & 0 \\ N(1, 0.2) & 0 & N(0.6, 0.2) & 0 & 0 \\ N(1, 0.2) & 0 & 0 & N(0.6, 0.2) & 0 \\ N(1, 0.2) & 0 & 0 & 0 & N(0.6, 0.2) \end{bmatrix}
\end{array}$$

Condition F

$$\begin{array}{c}
\begin{array}{ccccc} \overbrace{\qquad}^{a_1} & \overbrace{\qquad}^{a_2} & \overbrace{\qquad}^{a_3} & \overbrace{\qquad}^{a_4} & \overbrace{\qquad}^{a_5} \end{array} \\
\begin{bmatrix} N(1, 0.2) & N(0.7, 0.2) & 0 & 0 & 0 \\ N(1, 0.2) & 0 & N(0.7, 0.2) & 0 & 0 \\ N(1, 0.2) & 0 & 0 & N(0.7, 0.2) & 0 \\ N(1, 0.2) & 0 & 0 & 0 & N(0.7, 0.2) \end{bmatrix}
\end{array}$$

True θ parameter. Since the bifactor model has an assumption that uncorrelated group factors are independent of the general factor, it was not necessary to consider the correlation of the distributions of traits. Therefore, each examinee had latent traits that were orthogonal to each other. Their θs were randomly drawn from a multivariate normal distribution with MVN (**0**, **I**) without any inter-trait correlations. These true θs were generated by the “mvtnorm” function in R for three-and five-dimensional latent traits (one general factor with two group factors and one general factor with four group factors).

The BICAT Algorithm

The procedures of θ estimation and item selection were implemented for the general factor and group factors separately. The first BICAT algorithm followed the Weiss and Gibbon’s method described in Figure 1 [BICAT(S)], and the second BICAT algorithm used $\theta = 0.0$ to select an initial item for both the general and group factors [BICAT(D)].

In order to compare the two BICAT algorithms with the MBICAT algorithm, this study used a fixed-length CAT. Weiss and Gibbons (2007) showed that the mean number of items administered in CAT ranged from about 20 to about 50 items per scale to recover each scale score with a correlation above .90 for all group factor scales. Therefore, the BICAT terminated after 40 items for the general factor and 20 items for each group factor were administered in the bifactor model with two group factors; and 80 items for the general factor and 20 items for each group factor were administered in the bifactor model with four group factors. The response pattern, current θ estimate, and the observed standard errors for each examinee were saved after each item was administered.

Unidimensional θ estimation. MLE, MAP and EAP were used for estimation of θ for each item with respect to the general factor and group factors. A standard normal distribution was used as the prior for EAP and MAP. In the BICAT, the item selection procedure for each factor required an estimate of θ after 1 to n items were administered, except that a prior based on the general factor was used for the group factors. For this reason, an alternative was specified to the MLE θ estimate for response patterns that produce a likelihood that had no maximum. $\hat{\theta}$ was incremented by -1 for each incorrect response, and $+1$ for each correct response, until $\hat{\theta}$ reached 4 in absolute value. This procedure was employed until the response pattern became mixed. However, this was not required for EAP or MAP because these methods can obtain finite θ estimates for non-mixed response patterns.

The Newton-Raphson procedure was used to estimate θ for MLE and MAP. The Newton-Raphson procedure found the maximum of the likelihood using an iterative procedure. The Newton-Raphson iterations continued until the incremental change in $\hat{\theta}$ became less than the criterion of .001. Since EAP does not have a closed form to numerically integrate the θ distribution in the marginalization, a total of 50 quadrature points from –4 to 4 on the standard normal distribution were used to estimate θ.

Unidimensional item selection. The BICAT algorithm used the maximum Fisher information at $\hat{\theta}$ to select next item. The item that provides maximum information at $\hat{\theta}$ provided the greatest increase in test information and the greatest reduction in standard error. The BICAT algorithm was then implemented as summarized in Figure 1.

MBICAT Algorithm

θ estimation and item selection in MBICAT proceeded for all dimensions simultaneously. As a consequence, the MBICAT might administer an unequal number of items from each of the group factor scales, which would result in θ estimates based on different mixtures of group factor scales. Thus, the MBICAT algorithm alternated items that loaded on each group factor, which functioned as content balancing in the MBICAT. In the bifactor model with two group factors, the MBICAT was terminated after 40 items total were administered, with 20 items selected from the item bank measuring the first group factor scale and 20 items selected from the item bank measuring the second group factor scale. In the bifactor model with four group factors, MBICAT was terminated after 80 items total were administered with 20 items loaded from each group factor scale. For selecting the initial item, θ was fixed at 0, which was the midpoint of the scale for all dimensions. This was needed because the MBICAT algorithm did not have prior information about initial θ.

Multidimensional θ estimation. Multidimensional MLE, MAP, and EAP methods were used for estimation of θ for each examinee in the MBICAT. θ estimates using MLE and MAP were computed by Equations 15 and 16, respectively. The Newton-Raphson procedure was used to estimate θ for MLE and MAP. Early in the CAT procedure, it was necessary for MLE to specify an alternative for all correct response patterns or all incorrect response patterns that did not result in a likelihood with a maximum; $\hat{\theta}$ was decreased or increased by 1 for each incorrect response and for each correct response, until $\hat{\theta}$ reached 4 in absolute value. This procedure was employed until the response pattern became mixed.

MAP and EAP can obtain finite θ estimates for non-mixed response patterns because they used a standard normal distribution as the prior. EAP estimates of θ on the general factor and the group factors were directly approximated by Equations 34 and 36 respectively. In the EAP method, since integration of the distribution for an assumed general factor and group factor scores was not in a closed form, 15 quadrature points were used from –4 to 4 on the standard normal distribution for both the general factor and group factors. As a result, a total of 225 quadrature points were used to estimate θ with the EAP method for the bifactor model.

The Newton-Raphson procedure was used to estimate θ with MLE and MAP in the MBICAT, as in UCAT. As shown in Equations 15 and 16, the Newton-Raphson procedure approximated the maximum of the likelihood by using an iterative procedure. The Newton-Raphson iterations were repeated until the incremental change in $\hat{\theta}$ became less than the criterion of .001.

Multidimensional item selection. The MBICAT selected an item that provided maximum posterior information at the current $\hat{\boldsymbol{\theta}}$ using the multidimensional item selection method described in Equation 23. The selected item maximized the determinant of the posterior information matrix in Equation 28.

CAT Simulation Procedures

A program in R (R Core Development Team, 2008) was utilized to implement the two BICAT algorithms and the MBICAT algorithm. Appendix B shows R code for two BICAT algorithms and the MBICAT algorithm. Figure 4 shows the flow chart of both BICAT and MBICAT algorithms. The MBICAT program worked with dichotomously scored items for both MIRT and the bifactor model. The BICAT and MBICAT algorithms implemented the following steps:

Step 1: Generate item parameters and θ parameters based on pre-specified distributions. These were true values of item parameters and θ. All item parameters for each item bank were saved in a parameter file.

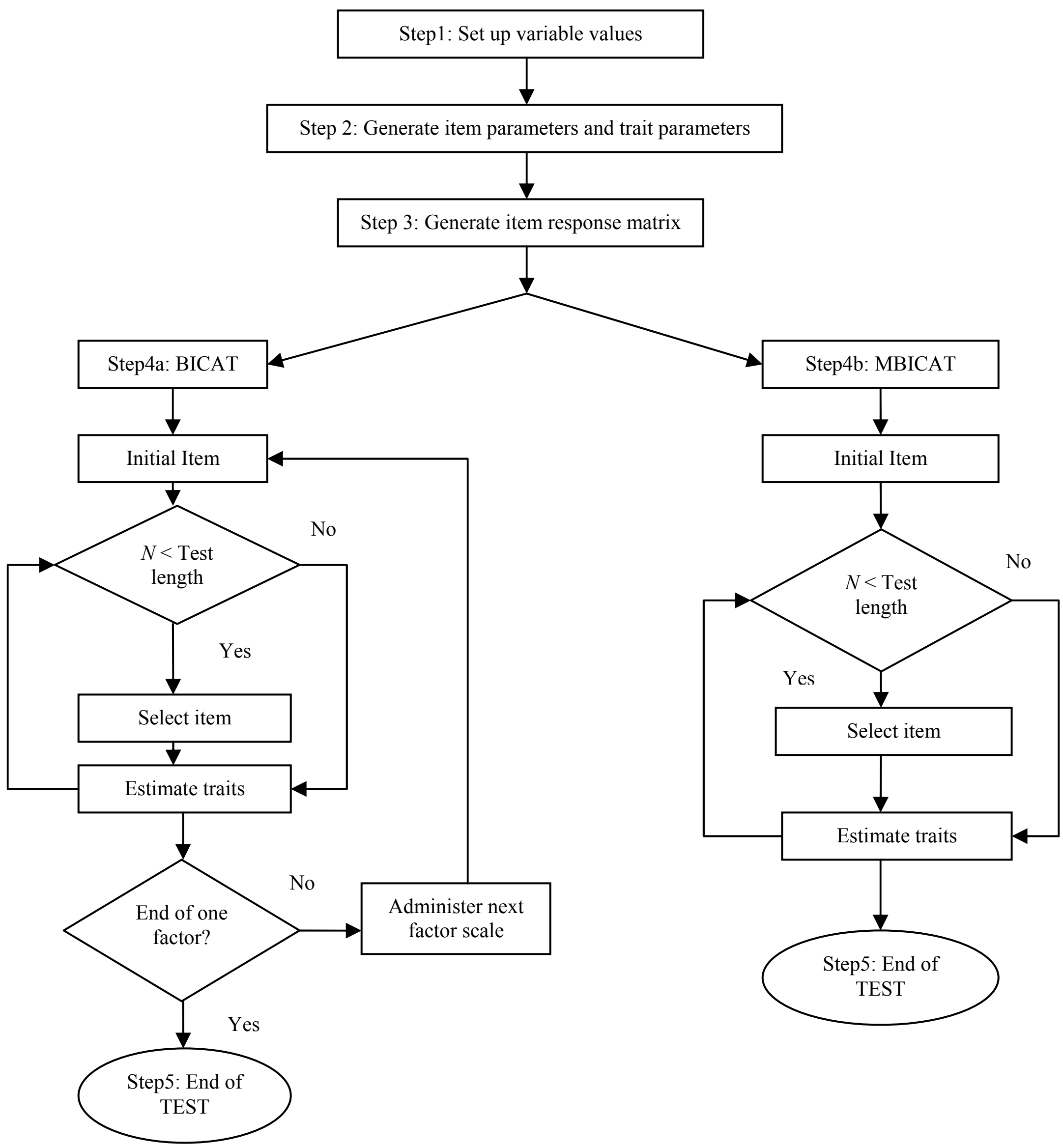
Step 2: Generate item responses based on the true item parameters and θ parameters. Using the bifactor model and true parameters, the probability of each response for an item was calculated from Equation 39. Random numbers from U[0, 1] were generated and compared to the probabilities of responding at each score response.

Step 3: Simulate CAT. This was an iterative process. There were two CAT algorithms.

Step 3a: Follow the BICAT algorithm. The BICAT algorithm proceeded for a general factor and each group factor scale separately. For BICAT(S), θ was set to zero for the first item in both the general factor and group factors. For the BICAT(D), θ was set to zero for the first item on the general factor, and θ or the first item in group factors was dependent on items administered on the general factor. θs on each factor were estimated one at a time.

Figure 4

Flow Chart for BICAT and MBICAT Algorithms



Step 3b: Follow the MBICAT algorithm. The MBICAT algorithm proceeded for a general factor and group factor scales simultaneously. For the first item, $\boldsymbol{\theta}$ for each examinee was set to a vector of zeros. The examinees' $\boldsymbol{\theta}$ on all factors were estimated at the same time by Equations 12, 27, 34, and 36.

Step 4: Obtain final estimated θs . After all "fixed length" items were administered, dependent variables were calculated and saved.

Validation

Generated item responses. In order to validate the response generation program, the TESTFACT program was used to recover the item parameters using the generated responses based on the bifactor model (400 items and 1,000 simulees) using group factor loading Condition A. The correlation, RMSE, and BIAS between true item parameters and estimated item parameters by the TESTFACT program are listed in Table 1. Table 1 shows that item parameters were recovered very well.

Table 1
Correlation, RMSE, and BIAS Between
True Item Parameters and Estimated Item Parameters
by TESTFACT Under the Bifactor Model

Statistic	a_1	a_2	a_3	d
Correlation	0.967	0.960	0.940	0.998
RMSE	0.071	0.059	0.048	0.095
BIAS	-0.027	-0.009	0.009	0.009

BICAT algorithm. The general factor can be interpreted as an essentially unidimensional trait in an IRT model if the general factor loadings are greater than the group factor loadings (Reise, Morizot, & Hays, 2007). Thus, general factor discrimination parameters can be used to implement a traditional UCAT. UCAT using the general factor discrimination parameter was implemented by both the author's CAT program and the FIRESTAR program (Choi, 2007). Both UCATs began with an initial θ = 0. θ estimates from the author's CAT program were compared to those estimated by the FIRESTAR using MLE and EAP estimation methods for the 2PLM. The MLE and EAP θ estimates from the author's program were identical to those estimated by the FIRESTAR program.

MBICAT algorithm. Since there was no available software to implement the MBICAT algorithm, the MBICAT algorithm was developed in the R language by the author. General factor scores, $\hat{\theta}_1$s estimated by the full-length MBICAT algorithm using EAP were compared to those estimated by the TESTFACT program (Wood et al., 2003). Group factor scores and $\hat{\theta}_k$s were not compared because TESTFACT provided only a general factor score for the bifactor model. In group factor discrimination Condition A, examinees' $\hat{\theta}_1$s were estimated by EAP with the same response matrix by using both the author's program and TESTFACT. The two sets of $\hat{\theta}_1$s were identical to each other rounded to two decimal places.

Dependent Variables

The θ estimates obtained from administering a fixed number of items were used for evaluation of the performance of the CAT. Each set of $\hat{\theta}$s provided information about how recovery of θ varied as more items were administered. The precision of $\hat{\theta}$s using the BICAT algorithm and the MBICAT algorithm were evaluated by the correlation between θs and $\hat{\theta}$s, and the root mean square error (RMSE). The efficiency of $\hat{\theta}$s was evaluated by the observed standard error (OSE). These indices provided descriptive information about the recovery of θ for comparisons across the different cells in the research design. The correlation was computed as a Pearson product-moment correlation, $r(\theta_j, \hat{\theta}_j)$. The RMSE and OSE were computed as

$$RMSE\left(\theta_{jk}\right) = \sqrt{\frac{1}{N}\sum_{j=1}^{N}\left(\theta_{jk} - \hat{\theta}_{jk}\right)^2} \tag{40}$$

and

$$OSE\left(\hat{\boldsymbol{\theta}}_j\right) = \frac{1}{\sqrt{-\left(\dfrac{\partial^2 \ln L}{\partial \boldsymbol{\theta}_j^2}\right)}}, \tag{41}$$

where j is an examinee, k is each factor, and N is the number of examinees. RMSE is the square root of the mean of the squared differences between the estimates and true values. It is always positive and reflects the absolute distance from true values. A smaller RMSE means better precision. Since the distributions of correlations, RMSEs, and OSEs are skewed, they were transformed for the ANOVA as (Hays, 1988; Howell, 1992)

$$r_Z = \frac{1}{2} \cdot \ln\left(\frac{1+r}{1-r}\right), \tag{42}$$

$$LRMSE = \log_{10}(RMSE + 1), \text{ and} \tag{43}$$

$$LOSE = \log_{10}(OSE + 1). \tag{44}$$

The ANOVA

Although the average correlation, RMSE, and OSE described the precision and efficiency of the CAT, they needed to be aggregated to detect important effects and estimating the magnitude of effects. Since the independent variables of this study were nominal, a mixed design analysis of variance (ANOVA) was conducted in order to evaluate the effect of the factors in this simulation study (Harwell, Stone, Hsu, & Kirisci, 1996). The number of group factors and group factor discrimination conditions were between-subjects factors, and the three CAT algorithms and three θ estimation methods were within-subjects factors.

ANOVA was conducted separately for the bifactor model with two group factors and with four group factors. Thus, dependent variables for group factors were compared with those for the general factor separately. Each ANOVA was conducted with three dependent variables (r_Z, *LRMSE*, and *LOSE*).

Table 2 shows the 2 (the number of group factors) × 3 (group factor discrimination condition) × 3 (CAT algorithm) × 3 (θ estimation method) mixed design for the general factor. In addition, the effects in the four-way mixed design ANOVA were reported for all group factors separately. Since the monte-carlo study controlled the number of examinees, this study did not perform hypothesis testing. Instead, η^2, the magnitude of the effect of factors in ANOVA, was computed for each of the conditions (Harwell, Stone, Hsu, & Kirisci, 1996). It is defined as the ratio of the sum of squares of a main effect or interaction (SS_{effect}) to total sum of squares (SS_{total}):

$$\eta^2 = \frac{SS_{effect}}{SS_{total}}. \tag{45}$$

Table 2

The Mixed Design ANOVA for the General Factor and Group Factors

Effect	*General Factor*			*Group Factors*		
	df	*SS*	η^2	*df*	*SS*	η^2
Between Subjects						
Group factor discrimination (F)						
Number of group factors (G)						
G × F						
Subjects						
Within Subjects						
Estimation Method (E)						
E × F						
E × G						
E × F × G						
Subjects						
CAT Algorithm (C)						
C × F						
C × G						
C × F × G						
Subjects						
E × C						
E × C × F						
E × C × G						
E × C × F × G						
Subjects						

Study 2

Experimental Design

The aim of the second study was to investigate the efficiency of the MBICAT algorithm for future real data application. This study examined an alternative termination criterion for the MBICAT instead of fixed length. The design of Study 2 was three factorial: 3 (group factor discrimination condition) × 2 (termination criterion: average OSE of all factors were .5 and .55) × 3 (θ estimation method). This study used only the bifactor model with four group factors because psychological instruments contain usually more than three latent traits (Gustafsson & Balke,1993; Reise, Morizot, & Hays, 2007). To ensure stability in the results, 1,000 simulees were generated within each cell. One factor (group factor discrimination condition) was a between-subjects factor. Group factor discrimination conditions D, E and F in Study 1 were applied to Study 2. The other two factors (termination criterion and θ estimation method) were manipulated within each bank defined by group factor discrimination conditions D, E, and F.

The MBICAT algorithm in Study 2 used the multidimensional item selection method maximizing the determinant of the posterior information and multidimensional MLE, MAP, and EAP estimation methods, as in Study 1.

The dependent variables were the correlation between MBICAT $\hat{\theta}$s $\left(\hat{\theta}_C\right)$ and full-scale $\hat{\theta}$s $\left(\hat{\theta}_F\right)$, the root mean square difference between $\hat{\theta}_C$ and $\hat{\theta}_F$ (RMSD), and the average number of items required by MBICAT algorithm to recover full-scale $\hat{\theta}$s with a pre-specified termination criterion.

Termination criterion. The observed standard errors (OSE) of $\hat{\theta}$ were computed for a single examinee from the second derivative of the likelihood function given administered items using the MLE and MAP estimation methods in Equation 41. OSE was computed by the $\mathbf{H}\left(\boldsymbol{\theta}^{(n)}\right)$ in Equation 16 for examinee j at a provisional estimated $\boldsymbol{\theta}$. In the MBICAT algorithm, OSEs of θ_1 and θ_k were calculated by taking the square root of the diagonals of the negative inverse of the second derivative of the likelihood function. Two termination criteria for each examinee were calculated as the average of the diagonal matrix of OSE of .50 or .55 for multidimensional $\boldsymbol{\theta}$ in MLE and MAP. When average OSEs of θ_1 and θ_k met a criterion, the MBICAT was terminated. For the EAP estimation method, Gibbons et al. (2007) simplified the OSE for estimating the primary latent variable θ_1 and sub-domain trait scores θ_k for an examinee j. The posterior variances of $\hat{\theta}_{j1}$ and $\hat{\theta}_{jk}$ described in Equations 35 and 38 were used to express the precision of the EAP estimates. The MBICAT was terminated when both OSEs of θ_1 and θ_k were less than .50 or .55. Note that an ANOVA was not conducted for Study 2.

CHAPTER 3: RESULTS

Estimation Issues

MLE and MAP estimation methods being evaluated required the use of an iterative process in order to maximize a log-likelihood function. To ensure convergence of 1,000 simulees' estimates across each condition, convergence of θ was assessed using an R program (R Core Development Team, 2008). If the θ estimate did not converge for a simulee for MLE and MAP, then the R program would stop for that simulee and estimate again for each simulee using a different set of generated responses.

The first operational problem for MLE and MAP θ estimation methods in the MBICAT was to satisfy the convergence criterion of .001 because it was difficult for the program to satisfy simultaneously the convergence criterion of .001 for all latent traits. Thus, MLE and MAP θ estimation methods in the MBICAT used a maximum of 10 iterations instead of the convergence criterion of .001. To ensure convergence of 1,000 simulees' estimates across each condition, the convergence criterion of each factor was investigated by an R program. Appendix C shows that the average convergence criteria for each latent trait of 1,000 simulees were less than .001 for all latent traits when the maximum number of iterations was set to 10.

The second computational problem was a Hessian or information matrix becoming singular during MLE θ estimation in the MBICAT algorithm. A singular matrix cannot be inverted. Thus, an iteration was immediately terminated because MLE estimation in the MBICAT algorithm involved inverting the Hessian or information matrix. In order to handle a singular information matrix, the MLE procedure took the second derivative of the likelihood function with additional prior multivariate standard normal distributions until a Hessian matrix could be inverted. This treatment would not provide a pure MLE method for the MBICAT algorithm. However, this study included this manipulated MLE method to compare the MLE method within the BICAT algorithms.

Study 1

Independence of Factors

Tables 3 and 4 show average correlations within $\hat{\theta}$s for 10 replications for BICAT(S), BICAT(D), and MBICAT respectively. Table 3 shows all correlations for each combination of the number of group factors, group factor discrimination conditions, and θ estimation methods. In Table 3, $\hat{\theta}$s using the two BICAT algorithms were not independent of each other. $\hat{\theta}$s for group factors were highly correlated with $\hat{\theta}$s from the general factor. For example, in the BICAT(S) algorithm with two group factors using MLE, the correlation between the general factor $\hat{\theta}$s and the first group factor $\hat{\theta}$s was .844, and the correlation between the general factor $\hat{\theta}$s and the second group factor $\hat{\theta}$s was .881. In addition, the correlation between the first group factor $\hat{\theta}$s and the second group factor $\hat{\theta}$s was .614. Table 3 shows that through all conditions, the BICAT algorithms did not provide independence of $\hat{\theta}$s between the general factor and the group factors, and the group factors with each other. However, $\hat{\theta}$s implemented by the MBICAT algorithm relatively satisfied the independence assumption of $\hat{\theta}$s between the general factor and group factors, and the group factors with each other. In Table 3, the correlation between the general factor $\hat{\theta}$s and the first group factor $\hat{\theta}$s was .186, and the correlation

Table 3

Correlations Among $\hat{\theta}$s
for the Bifactor Model With Two Group Factors

Condition, CAT algorithm and θ estimation method	$r(G, g1)$	$r(G, g2)$	$r(g1, g2)$
Condition A: Low group factor discrimination			
BICAT(S)			
MAP	.844	.881	.614
MLE	.844	.880	.613
EAP	.845	.881	.615
BICAT(D)			
MAP	.844	.880	.613
MLE	.844	.880	.613
EAP	.845	.880	.611
MBICAT			
MAP	.186	.144	−.317
MLE	.180	.142	−.312
EAP	.299	.276	−432
Condition B: Medium group factor discrimination			
BICAT(S)			
MAP	.831	.882	.577
MLE	.831	.880	.575
EAP	.832	.880	.575
BICAT(D)			
MAP	.832	.880	.575
MLE	.830	.881	.576
EAP	.832	.880	.575
MBICAT			
MAP	.195	.178	−.326
MLE	.201	.178	−.329
EAP	.328	.321	−.428
Condition C: High group factor discrimination			
BICAT(S)			
MAP	.826	.873	.540
MLE	.827	.872	.544
EAP	.826	.872	.540
BICAT(D)			
MAP	.826	.874	.542
MLE	.826	.872	.544
EAP	.826	.872	.539
MBICAT			
MAP	.225	.197	−.315
MLE	.216	.194	−.316
EAP	.358	.347	−.421

Table 4

Correlations Among $\hat{\theta}$s for the Bifactor Model With Four Group Factors

Condition, CAT algorithm, and θ estimation	*r(G,g1)*	*r(G,g2)*	*r(G,g3)*	*r(G,g4)*	*r(g1,g2)*	*r(g1,g3)*	*r(g1,g4)*	*r(g2,g3)*	*r(g2,g4)*	*r(g3,g4)*
Condition D: Low group factor discrimination										
BICAT(S)										
MAP	.844	.822	.831	.830	.609	.607	.628	.631	.620	.642
MLE	.843	.821	.832	.830	.606	.607	.627	.632	.619	.645
EAP	.843	.822	.831	.831	.608	.607	.628	.631	.620	.643
BICAT(D)										
MAP	.844	.822	.831	.830	.609	.607	.628	.631	.620	.642
MLE	.843	.821	.831	.830	.605	.608	.626	.630	.619	.644
EAP	.843	.821	.831	.830	.608	.607	.628	.631	.620	.642
MBICAT										
MAP	.123	.068	.117	.068	−.169	−.193	−.147	−.137	−.152	−.085
MLE	.121	.071	.117	.067	−.170	−.194	−.145	−.139	−.148	−.087
EAP	.125	.065	.125	.081	−.164	−.194	−.141	−.131	−.145	−.083
Condition E: Medium group factor discrimination										
BICAT(S)										
MAP	.801	.812	.834	.850	.572	.597	.608	.612	.586	.628
MLE	.802	.810	.837	.852	.570	.597	.613	.615	.587	.639
EAP	.801	.811	.833	.849	.570	.595	.608	.611	.583	.626
BICAT(D)										
MAP	.801	.812	.834	.850	.571	.596	.608	.612	.585	.629
MLE	.802	.810	.838	.852	.572	.596	.613	.615	.587	.638
EAP	.801	.812	.834	.850	.571	.596	.608	.612	.585	.629
MBICAT										
MAP	.142	.064	.138	.141	−.162	−.175	−.149	−.113	−.193	−.176
MLE	.144	.066	.140	.141	−.159	−.173	−.155	−.116	−.195	−.175
EAP	.138	.072	.137	.141	−.161	−.174	−.152	−.110	−.191	−.177
Condition F: High group factor discrimination										
BICAT(S)										
MAP	.817	.828	.796	.800	.574	.566	.565	.585	.562	.544
MLE	.816	.829	.796	.801	.575	.565	.566	.586	.565	.548
EAP	.817	.828	.796	.800	.574	.565	.564	.586	.562	.544
BICAT(D)										
MAP	.817	.828	.796	.799	.574	.566	.563	.585	.561	.543
MLE	.815	.829	.796	.800	.575	.563	.562	.586	.565	.546
EAP	.816	.828	.796	.799	.574	.565	.563	.587	.561	.544
MBICAT										
MAP	.150	.158	.107	.132	−.166	−.161	−.135	−.102	−.173	−.163
MLE	.145	.158	.106	.133	−.162	−.163	−.138	−.106	−.174	−.160
EAP	.148	.155	.102	.135	−.168	−.159	−.136	−.106	−.175	−.163

between the general factor $\hat{\theta}$s and the first group factor $\hat{\theta}$s was .144. In addition, the correlation between the first group factor $\hat{\theta}$s and the second group factor $\hat{\theta}$s was −.317. The independence assumption of $\hat{\theta}$s in the MBICAT algorithm was, therefore, relatively supported for each combination of the number of group factors, group factor discrimination conditions, and θ estimation methods.

Correlations of θ with $\hat{\theta}$

Appendix D (Tables D-1 through D-10) shows the results for recovery of true θ for all 10 replications of $r\left(\theta, \hat{\theta}\right)$ using BICAT (S), BICAT(D), and MBICAT algorithms for three group factor discrimination conditions. Table 5 summarizes the average $r\left(\theta, \hat{\theta}\right)$ of 10 simulations within each condition for the general factor and the group factors across all conditions.

Results for the low group factor discrimination conditions (Conditions A and D) show that the average $r\left(\theta, \hat{\theta}\right)$ for the general factor were above .94 for all three CAT algorithms in both bifactor models with two group factors and four group factors. The MBICAT algorithm provided slightly higher correlations than the two BICAT algorithms. Higher correlations were found for the bifactor model with four group factors than for the bifactor model with two group factors because the number of items administered on the general factor in the bifactor model with four group factors was twice the number of items administered on the general factor in the bifactor model with two group factors. There were no large differences among θ estimation methods in each condition.

Results for $r\left(\theta, \hat{\theta}\right)$ for the group factors showed that there were large differences among CAT algorithms in both bifactor models with two group factors and four group factors. The average $r\left(\theta, \hat{\theta}\right)$ using the BICAT algorithms ranged from .488 to .581, while for MBICAT they were ranged from .764 to .815. No consistent differences were observed in terms of the impact of the number of group factors for the two BICAT algorithms. However, all correlations of group factors for the MBICAT algorithm were

Table 5

Average $r\left(\theta,\hat{\theta}\right)$ Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions

Condition, CAT algorithm, and θ estimation method	Number of group factors							
	Two group factors			Four group factors				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	.942	.581	.489	.965	.540	.489	.531	.521
EAP	.942	.581	.489	.965	.540	.489	.531	.520
MLE	.942	.580	.489	.965	.539	.490	.530	.520
BICAT(D)								
MAP	.942	.581	.488	.965	.540	.490	.531	.521
EAP	.942	.581	.488	.965	.540	.489	.531	.520
MLE	.942	.579	.488	.965	.540	.490	.532	.521
MBICAT								
MAP	.950	.790	.765	.972	.809	.795	.814	.810
EAP	.947	.788	.756	.971	.808	.793	.812	.808
MLE	.950	.791	.764	.972	.807	.794	.815	.810
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	.923	.630	.530	.957	.569	.534	.518	.550
EAP	.923	.629	.530	.957	.571	.535	.518	.551
MLE	.923	.630	.529	.956	.569	.536	.515	.546
BICAT(D)								
MAP	.923	.629	.530	.957	.569	.535	.518	.550
EAP	.923	.629	.530	.957	.570	.535	.517	.551
MLE	.923	.630	.527	.956	.568	.535	.514	.547
MBICAT								
MAP	.939	.812	.786	.965	.810	.831	.809	.826
EAP	.934	.810	.774	.965	.808	.830	.810	.827
MLE	.938	.816	.785	.965	.809	.830	.810	.827
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	.904	.666	.566	.945	.590	.601	.600	.632
EAP	.904	.666	.567	.945	.590	.601	.600	.632
MLE	.904	.666	.567	.945	.589	.600	.599	.630
BICAT(D)								
MAP	.904	.667	.566	.945	.591	.601	.599	.632
EAP	.904	.666	.567	.945	.591	.601	.600	.632
MLE	.904	.665	.566	.945	.592	.600	.599	.631
MBICAT								
MAP	.924	.828	.802	.952	.849	.842	.852	.853
EAP	.918	.825	.786	.955	.848	.834	.852	.855
MLE	.923	.829	.798	.952	.848	.841	.852	.855

higher in the bifactor model with four group factors than the bifactor model with two group factors. There were no large differences among θ estimation methods in each condition.

Results for the medium group factor discrimination conditions (Conditions B and E) show that the average $r\left(\theta,\hat{\theta}\right)$ for the general factor were above .923 for all three CAT algorithms in both bifactor models with two group factors and four group factors. The average $r\left(\theta,\hat{\theta}\right)$ using MBICAT were slightly larger than those using the two BICAT algorithms in both bifactor models with two group factors and four group factors. Again, the bifactor model with four group factors showed higher correlations than the bifactor model with two group factors because of the difference in the number of items administered on the general factor. There were also no large differences among θ estimation methods in any condition.

There were large differences among CAT algorithms in $r\left(\theta,\hat{\theta}\right)$ for the group factors for both bifactor models with two group factors and four group factors. Average $r\left(\theta,\hat{\theta}\right)$ using the two BICAT algorithms ranged from .527 to .630, while for MBICAT they ranged from .774 to .830. There were no consistent differences for the correlation between the bifactor model with two group factors and four group factors in both the BICAT and MBICAT algorithms. There were no large differences among θ estimation methods in any condition.

Results for the high group factor discrimination (Conditions C and F) indicate that $r\left(\theta,\hat{\theta}\right)$ for the general factor were above .904 for three CAT algorithms with both two and four group factors. Average $r\left(\theta,\hat{\theta}\right)$ for the general factor using the MBICAT algorithm were slightly higher than those using the two BICAT algorithms in both bifactor models with two group factors and four group factors. The bifactor model with four group factors consistently provided higher correlations for the general factor than those for the bifactor model with two group factors because of the number of items administered on the general factor. There were no large differences among θ estimation methods in any condition.

There were also large differences in average $r\left(\theta,\hat{\theta}\right)$ for the group factors among CAT algorithms in both bifactor models with two group factors and four group factors. The average correlations for the group factors using the two BICAT algorithms ranged from .566 to .667, while average $r\left(\theta,\hat{\theta}\right)$ using MBICAT ranged from .786 to .855. No consistent differences were observed in the correlation between bifactor models with two group factors and four group factors in the two BICAT algorithms. However, all correlations of group factors for the MBICAT algorithm were higher in the bifactor model with four group factors than the bifactor model with two group factors. There was no impact of θ estimation methods in any condition.

The precision of the CAT $\hat{\theta}$ varied according to group factor discrimination conditions. Larger average $r\left(\theta,\hat{\theta}\right)$ was found under high group factor discrimination conditions (C and F) for group factors, while larger average $r\left(\theta,\hat{\theta}\right)$ was found under low group factor discrimination conditions (A and D) for the general factor. These results were more obvious in the two BICAT algorithms compared with the MBICAT algorithm. For example, the correlation for the first group factor increased from .581 (MAP method) to .666 (MAP method), and the correlation on the general factor decreased from .942 (MAP method) to .904 (MAP method) in the bifactor model with two group factors using the BICAT(S) algorithm, as the mean of group factor discrimination parameters increased from .50 (Condition A) to .70 (Condition C). On the other hand, the correlation for the first group factor increased from .790 (MAP method) to .828 (MAP method), and the correlation for the general factor decreased from .950 (MAP method) to .924 (MAP method) in the bifactor model with two group factors using the MBICAT algorithm, as the mean of the group factor discrimination parameters increased from .50 (Condition A) to .70 (Condition C).

Overall, $r\left(\theta,\hat{\theta}\right)$ for group factors tended to increase as the mean of the group factor discrimination parameters increased under the fixed general factor discrimination parameters [mean of 1.0 and standard deviation (SD) of 0.2]. On the other hand, $r\left(\theta,\hat{\theta}\right)$ for the general factor tended to decrease when the mean of the group factor discrimination parameters increased, even if the general factor discrimination parameters had mean of 1.0 and SD of 0.2 across three conditions.

ANOVA

Table 6 shows the results of the mixed design ANOVA with the correlations transformed by Fisher's *z* as the dependent variable for the general factor and the group factors. Appendix E shows the computations of the degrees of freedom and sums of squares of the mixed design using 10 simulation results.

As shown by Table 6 for the general factor, group factor discrimination conditions and the number of group factors accounted for the most variation in the ANOVA models as defined by η^2. The variation accounted for by the group factor discrimination condition was .297, and the variation accounted for by the number of group factors was .499. The remaining factors in the mixed-design ANOVA accounted for a negligible amount of variation in the general factor. However, for group factors, the CAT algorithm factor accounted for the most variation in the ANOVA models as defined by η^2. The variation accounted for by the CAT algorithm was .869.

Table 6

Mixed Design ANOVA Summary for the Fisher's *z* Transformed Correlation

Effect	General Factor			Group Factor		
	df	*SS*	η^2	*df*	*SS*	η^2
Between Subjects						
Group factor discrimination (F)	2	6.676	.297	2	1.165	.037
Number of group factors (G)	1	11.201	.499	1	.017	.001
G × F	2	.249	.011	2	.264	.008
Subjects	54	2.829		54	1.535	
Within Subjects						
Estimation Method (E)	2	.005	<.001	2	.001	<.001
E × F	4	.001	<.001	4	.001	<.001
E × G	2	.003	<.001	2	.001	<.001
E × F × G	4	.001	<.001	4	.001	<.001
Subjects	108	.010		108	.007	
CAT Algorithm (C)	2	1.182	.053	2	27.447	.869
C × F	4	.012	.001	4	.003	<.001
C × G	2	.001	<.001	2	.725	.023
C × F × G	4	.050	<.001	4	.002	<.001
Subjects	108	.212		108	.381	
E × C	4	.009	<.001	4	.002	<.001
E × C × F	8	.001	<.001	8	.001	<.001
E × C × G	4	.004	<.001	4	.002	<.001
E × C × F × G	8	.002	<.001	8	.001	<.001
Subjects	216	.015		216	.012	

The remaining factors in the mixed-design ANOVA accounted for a negligible amount of variation in the group factors. There were no interaction effects among the four factors.

RMSEs

Appendix F shows results for each of the 10 replications of RMSEs using the BICAT(S), BICAT(D), and MBICAT algorithms for three group factor discrimination conditions. Table 7 summarizes the average RMSEs over 10 replications for the general factor and the group factors across all conditions.

Average RMSEs for the low group factor discrimination conditions for the general factor were under .400 for all three CAT algorithms in bifactor models with both two and four group factors. The RMSEs from the MBICAT algorithm were slightly smaller than those using the two BICAT methods in the bifactor models with two group factos and four group factors. Smaller RMSEs for the general factor were observed for the bifactor model with four group factors than those for the bifactor model with two group factors because of the difference in the number of items administered in the general factor. There were no large differences among θ estimation methods across each condition, except for a slight tendency for EAP to have higher RMSE under MBICAT.

There were large differences in the RMSEs for group factors among the CAT algorithms with both the two and four group factors data. The RMSEs from the two BICAT algorithms with low group factor discriminations ranged from .832 to .899, while the RMSEs from the MBICAT algorithm ranged from .575 to .669. No consistent differences were observed across the bifactor models with two or four group factors. Based on Table 7, the number of group factors did not impact RMSE in the two BICAT algorithms, while it had an impact on RMSE in the MBICAT algorithm. Like the general

Table 7

Average RMSEs Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions

Condition, CAT algorithm, and θ estimation method	Number of group factors							
	Two group factors			Four group factors				
	G	g1	g2	G	g1	g2	g3	g4
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	.399	.832	.881	.368	.853	.874	.863	.864
EAP	.399	.832	.880	.368	.854	.874	.864	.867
MLE	.388	.845	.896	.361	.874	.896	.891	.900
BICAT(D)								
MAP	.399	.832	.881	.368	.853	.873	.863	.865
EAP	.399	.832	.880	.368	.854	.874	.864	.867
MLE	.388	.846	.897	.361	.873	.895	.889	.899
MBICAT								
MAP	.311	.622	.638	.239	.589	.597	.578	.575
EAP	.322	.644	.669	.241	.590	.600	.581	.577
MLE	.312	.621	.639	.239	.591	.599	.577	.575
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	.446	.790	.852	.424	.820	.840	.870	.840
EAP	.446	.791	.852	.424	.818	.840	.870	.839
MLE	.437	.796	.863	.417	.828	.860	.884	.860
BICAT(D)								
MAP	.446	.791	.852	.424	.819	.840	.870	.840
EAP	.446	.791	.851	.424	.819	.840	.870	.839
MLE	.437	.796	.864	.417	.828	.860	.884	.859
MBICAT								
MAP	.344	.592	.613	.270	.584	.540	.594	.555
EAP	.358	.611	.644	.271	.587	.541	.593	.554
MLE	.345	.587	.639	.270	.585	.541	.593	.553
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	.484	.757	.823	.440	.795	.819	.805	.772
EAP	.484	.757	.823	.439	.796	.819	.805	.773
MLE	.477	.761	.830	.433	.812	.830	.815	.785
BICAT(D)								
MAP	.484	.757	.824	.440	.795	.819	.805	.771
EAP	.484	.757	.822	.439	.795	.819	.805	.772
MLE	.477	.761	.830	.433	.809	.830	.815	.784
MBICAT								
MAP	.381	.569	.593	.308	.511	.549	.523	.514
EAP	.397	.586	.626	.308	.512	.549	.523	.512
MLE	.383	.568	.598	.307	.512	.549	.523	.512

factor case, the RMSEs of group factors in the MBICAT were smaller in the bifactor model with four group factors than the bifactor model with two group factors. The RMSEs among θ estimation methods were similar to one another across all conditions. Average RMSEs for medium group factor discrimination conditions for the general factor were below .460 for all three CAT algorithms in both bifactor models with two group factors and four group factors. The RMSEs in the MBICAT algorithm were also smaller by .10 to .15 than those in the two BICAT algorithms across both bifactor models with two group factors and four group factors. Smaller RMSEs were also found for the bifactor model with four group factors than those for the bifactor model with two group factors because of the number of items administered for the general factor. The RMSEs were consistent among θ estimation methods across each condition.

There were consistent differences among CAT algorithms for the RMSEs of the group factors in both bifactor models with two group factors and four group factors. The RMSEs for the two BICAT algorithms ranged from .790 to .884, while those for the MBICAT algorithm ranged from .540 to .644. No consistent differences were observed across the two bifactor models with either two or four group factors. Based on Table 7, the number of group factors did not have an impact on RMSE in the two BICAT algorithms, while it had a minor impact on the RMSE in the MBICAT algorithm. Like the general factor case, the RMSEs of the group factors for MBICAT were smaller in the bifactor model with four group factors than the bifactor model with two group factors. There was no impact on the RMSE among θ estimation methods across any condition.

Average RMSEs for high group factor discrimination conditions for the general factor were below .490 for the three CAT algorithms in both bifactor models. The CAT algorithm factor had an impact on the RMSEs. The RMSEs in the MBICAT algorithm were slightly smaller than those in the two BICAT algorithms in both bifactor models. Smaller RMSEs were also found for the bifactor model with four group factors than those for the bifactor model with two group factors because of the number of items administered for the general factor. There was again no impact on the RMSE among θ estimation methods across each condition.

There were large differences in RMSEs for the group factors among CAT algorithms in both bifactor models. The RMSEs in the two BICAT algorithms ranged from .757 to .830, while the RMSE in the MBICAT algorithm ranged from .511 to .626. Based on Table 7, the number of group factors had no impact on RMSE in the two BICAT algorithms, while it had a minor impact on the RMSE in the MBICAT algorithm. Like the general factor case, the RMSEs of group factors in MBICAT were smaller in the bifactor model with four group factors than the bifactor model with two group factors. There was no θ estimation method effect in any condition.

Comparing the values between different group factor discrimination conditions (Table 7), smaller RMSEs were found for group factors under high group factor discrimination conditions (C and F), while the RMSEs were larger for the general factor in those conditions. These results were more obvious in the two BICAT algorithms compared with the MBICAT algorithm. For example, the RMSEs for the first group factor using the BICAT(S) algorithm decreased from .832 (MAP method) to .757 (MAP method) in the bifactor model with two group factors as the mean of group factor discrimination parameters increased from .50 (Condition A) to .70 (Condition C). On the other hand, the RMSEs for the general factor using the BICAT(S) algorithm increased from .399 (MAP method) to .484 (MAP method) in the bifactor model with two group factors as the mean of the group factor discrimination parameters increased from .50 (Condition A) to .70 (Condition C). As another example, the RMSEs for the first group factor using the MBICAT algorithm decreased from .622 (MAP method) to .569 (MAP method) in the bifactor model with two group factors as the mean of the group factor discrimination parameters increased from .50 (Condition A) to .70 (Condition C). On the other hand, the RMSEs on the general factor increased from .311 (MAP method) to .381 (MAP method) in the bifactor model with two group factors. These trends were consistent for the bifactor model with four group factors.

Overall, RMSEs for the group factors tended to decrease as the mean of the group factor discrimination parameters increased when the discrimination parameters of the general factor had mean of 1.0 and SD of .20. On the other hand, the RMSEs for the general factor tended to increase as the mean of discrimination parameters of the group factor increased even if the discrimination parameters of the general factor were set as mean of 1.0 and standard deviation of .20.

ANOVA

Table 8 summarizes the mixed design ANOVA for the general factor and group factors with the transformed RMSE as the dependent variable. Appendix E shows computations of the degrees of freedom and sums of squares of the mixed design ANOVA using 10 replications results.

As shown by Table 8 for the general factor, the group factor discrimination, the number of group factors, and CAT algorithm factors accounted for the most variation in the ANOVA as defined by η^2.

Table 8

Mixed Design ANOVA Summary for the Transformed RMSE

Effect	General factor			Group factors		
	df	*SS*	η^2	*df*	*SS*	η^2
Between Subjects						
Group factor discrimination (F)	2	.041	.149	2	.090	.016
Number of group factors (G)	1	.027	.099	1	.118	.021
G × F	2	.002	.002	2	.037	.007
Subjects	54	.040		54	1.436	
Within Subjects						
Estimation Method (E)	2	.001	.003	2	.004	.001
E × F	4	.001	.003	4	.002	<.001
E × G	2	.001	.003	2	.003	<.001
E × F × G	4	.001	.003	4	.001	<.001
Subjects	108	.002		108	.013	
CAT Algorithm (C)	2	.137	.500	2	3.381	.597
C × F	4	.001	.003	4	.010	.002
C × G	2	.006	.022	2	.079	.014
C × F × G	4	.001	.003	4	.018	.003
Subjects	108	.007		108	.443	
E × C	4	.001	.003	4	.010	.002
E × C × F	8	.001	.003	8	.001	<.001
E × C × G	4	.001	.003	4	.003	<.001
E × C × F × G	8	.001	.003	8	.001	<.001
Subjects	216	.002		216	.014	

The variation accounted for by the group factor discrimination magnitude was .149, the variation accounted for by the number of group factors was .099, and the variation accounted for by the CAT algorithm was .500. The remaining factors in the mixed-design ANOVA accounted for a negligible amount of variation in the general factor. For group factors, only the CAT algorithm factor accounted for a substantial amount of variation in the ANOVA as defined by η^2. The variation accounted for by the CAT algorithm was .597. The other factors in the mixed-design ANOVA accounted for a negligible amount of variation in the group factors. There were no interaction effects among the four factors.

OSEs

Appendix G shows results for all 10 replications of OSEs using the BICAT(S), BICAT(D), and MBICAT algorithms for three group factor discrimination conditions. Table 9 summarizes the average OSEs of 10 replications for the general factor and the group factors across all conditions. The OSEs of estimates were computed by Equation 41.

The values of OSE for the low group factor discrimination conditions for the general factor ranged from .145 to .192. The OSEs from the MBICAT algorithm were larger than those using the two BICAT algorithms in bifactor models with both two and four group factors. Comparing bifactor models with two group and four group factors, smaller OSEs for the general factor were observed for the bifactor model with four group factors because of the larger number of items administered on the general factor. EAP estimation provided the smallest OSEs in the BICAT algorithms, while EAP showed the largest OSEs in the MBICAT algorithm.

There were large differences in OSE for the group factors among CAT algorithms in bifactor models with both two and four group factors. OSEs for the two BICAT algorithms ranged from .349 to .463, while OSEs in the MBICAT algorithm ranged from .413 to .520. The bifactor model with four group factors showed larger OSEs compared with the bifactor model with two group factors. There were OSE differences among the three θ estimation methods across all three CAT algorithms; EAP provided the smallest

Table 9
Average OSEs of Estimates Using BICAT(S), BICAT(D),
and MBICAT Algorithms for Three Group Factor Discrimination Conditions

Condition, CAT algorithm, and θ estimation method	Two group factors: G	g1	g2	Four group factors: G	g1	g2	g3	g4
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	.172	.376	.380	.138	.406	.417	.419	.411
EAP	.145	.349	.353	.112	.378	.389	.392	.383
MLE	.175	.406	.410	.139	.445	.459	.463	.454
BICAT(D)								
MAP	.172	.376	.380	.138	.406	.417	.419	.411
EAP	.145	.349	.353	.112	.378	.389	.392	.383
MLE	.175	.406	.411	.139	.444	.459	.463	.453
MBICAT								
MAP	.188	.448	.467	.153	.483	.517	.495	.499
EAP	.192	.413	.434	.173	.485	.509	.492	.488
MLE	.190	.449	.468	.154	.484	.520	.499	.503
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	.172	.340	.344	.138	.378	.373	.396	.369
EAP	.145	.313	.317	.112	.349	.346	.368	.342
MLE	.175	.362	.367	.139	.407	.404	.430	.397
BICAT(D)								
MAP	.172	.340	.345	.138	.378	.373	.396	.369
EAP	.145	.313	.318	.112	.349	.346	.368	.342
MLE	.175	.362	.367	.139	.407	.404	.430	.397
MBICAT								
MAP	.196	.419	.409	.156	.478	.440	.491	.431
EAP	.214	.393	.413	.201	.481	.455	.510	.461
MLE	.200	.428	.413	.156	.474	.443	.492	.431
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	.171	.310	.314	.131	.337	.343	.336	.320
EAP	.145	.283	.288	.106	.310	.315	.308	.293
MLE	.175	.327	.331	.133	.360	.365	.358	.339
BICAT(D)								
MAP	.171	.309	.314	.131	.337	.343	.336	.319
EAP	.145	.283	.287	.106	.310	.315	.308	.292
MLE	.175	.327	.331	.133	.359	.364	.357	.338
MBICAT								
MAP	.198	.348	.339	.148	.381	.407	.394	.377
EAP	.235	.374	.394	.221	.422	.453	.427	.405
MLE	.197	.348	.357	.149	.384	.406	.395	.380

OSEs in the BICAT algorithms, while EAP showed the largest OSEs in the MBICAT algorithm, probably because of the small number of quadrature points. For the bifactor model with four group factors, MBICAT also had higher OSEs on the group factors than either of the two BICAT algorithms.

OSEs for the medium group factor discrimination conditions (Conditions B and E) on the general factor ranged from .145 to .214 in the two-group factor case and .112 to .201 for the four-group factor case. The OSEs in the MBICAT algorithm were larger than those using the two BICAT algorithms in both two-group and four-group factor conditions. Smaller OSEs for the general factor were observed for the bifactor model with four group factors than two group factors, because of the larger number of items administered on the general factor. EAP had the smallest OSEs in the BICAT algorithms, while EAP had largest OSEs in the MBICAT algorithm, likely because of the small number of quadrature points.

There were large differences on the group factors among CAT algorithms in both bifactor models with two and four group factors. OSEs in the two BICAT algorithms ranged from .313 to .430, while OSEs in the MBICAT algorithm ranged from .393 to .613. Comparing OSEs between the two bifactor models, the bifactor model with four group factors showed larger OSEs compared with the bifactor model with two group factors, except for MAP and MLE in the MBICAT algorithm. There were OSE differences among θ estimation methods in all three CAT algorithms. EAP provided the smallest OSEs in the BICAT algorithms, while EAP showed the largest OSEs in the MBICAT algorithm because of the small number of quadrature points.

OSEs for the high group factor discrimination conditions (Conditions C and F) on the general factor ranged from .171 to .235. The OSEs from the MBICAT algorithm were larger than those using the BICAT algorithms in both bifactor models with two and four group factors. Smaller OSEs for the general factor were observed for the bifactor model with four group factors than with two group factors because of the different number of items administered on the general factor. EAP provided the smallest OSEs in the BICAT algorithms, while EAP showed the largest OSEs in the MBICAT algorithm because of the small number of quadrature points.

There were differences in the OSEs of θ estimates among CAT algorithms in both bifactor models with two and four group factors. The OSEs from the two BICAT algorithms ranged from .283 to .364, while OSEs from the MBICAT algorithm ranged from .339 to .453. The bifactor model with four group factors showed larger OSEs compared with the bifactor model with two group factors. There were OSE differences among θ estimation methods in the three CAT algorithms. EAP also provided the smallest OSEs in the BICAT algorithms, while EAP showed the largest OSEs in the MBICAT algorithm.

Comparing the OSEs among the three group factor discrimination conditions (low, medium, and high), smaller OSEs were found under high group factor discrimination conditions (C and F) for group factors, while the OSEs for the general factor were larger in those conditions in the MBICAT algorithm. The OSEs on the general factor using the two BICAT algorithms were not changed as the mean of group factor discrimination parameters increased from .50 (Conditions A and D) to .70 (Conditions C and F). However, The OSEs on the group factors decreased as the mean of the group factor discrimination parameters increased from .50 to .70. For example, the OSEs on the first group factor using BICAT(S) decreased from .376 (MAP method) to .310 (MAP method), and those using the MBICAT algorithm decreased from .448 (MAP method) to .309 (MAP method) in the bifactor model with two group factors as the mean of the group factor discrimination parameters increased from .50 (Conditions A and D) to .70 (Conditions C and F). On the other hand, the OSEs on the general factor using the MBICAT algorithm increased from .188 (MAP method) to .198 (MAP method) in the bifactor model with two group factors as the mean of group factor discrimination parameters increased from .50 (Conditions A and D) to .70 (Conditions C and F).

Overall, OSEs on the group factors tended to decrease when the mean of group factor discrimination parameters increased as the discrimination parameters of the general factor were set as mean of 1.0 and SD of .20. On the other hand, the OSEs on the general factor tended to increase when the mean of group factor discrimination parameters increased even if the discrimination parameters of the general factor were set as mean of 1.0 and SD of .20 in the MBICAT algorithm.

ANOVA

Table 10 summarizes the mixed design ANOVA for the general factor and the group factors with the transformed OSEs as the dependent variable. Appendix E shows computations of the degrees of freedom and sums of squares of the mixed design ANOVA using 10 replication results.

As shown by Table 10, for the general factor, the number of group factors and CAT algorithm factors accounted for the most variation in the ANOVA models as defined by η^2. The variation accounted for by the number of group factors was .322, and the variation accounted for by the CAT algorithm was .370. The remaining factors in the mixed-design ANOVA accounted for a negligible amount of variation in the general factor. For group factors, the number of group factors and CAT algorithm factors also accounted for the most variation in the ANOVA. The variation accounted for by the group factor discrimination was .226. The variation accounted for by the CAT algorithm was .481. The remaining factors in the mixed-design ANOVA accounted for a negligible amount of variation in the group factors. There were no large interaction effects among the four factors.

Table 10

Mixed Design ANOVA Summary for the Fisher's z Transformed OSEs

Effect	General factor			Group factors		
	df	*SS*	η^2	*df*	*SS*	η^2
Between Subjects						
Group factor discrimination (F)	2	.005	.010	2	.214	.226
Number of group factors (G)	1	.154	.322	1	.053	.061
G × F	2	.008	.017	2	.028	.030
Subjects	54	.001		54	.001	
Within Subjects						
Estimation Method (S)	2	.010	.021	2	.082	.086
E × F	4	.002	.004	4	.004	.004
E × G	2	.006	.013	2	.007	.007
E × F × G	4	.003	.006	4	.005	.005
Subjects	108	.001		108	.001	
CAT Algorithm (C)	2	.177	.370	2	.456	.481
C × F	4	.022	.046	4	.015	.016
C × G	2	.002	.004	2	.004	.004
C × F × G	4	.007	.015	4	.023	.024
Subjects	108	.006		108	.001	
E × C	4	.054	.107	4	.020	.021
E × C × F	8	.004	.008	8	.003	.003
E × C × G	4	.011	.023	4	.016	.017
E × C × F × G	8	.007	.015	8	.010	.011
Subjects	216	.001		216	.001	

Study 2: Variable-Length CAT in MBICAT

Study 2 used an OSE termination criterion for MBICAT instead of fixed length. Two termination criteria for each examinee were calculated as the average of the diagonal matrix of OSE for MLE and MAP and the posterior standard deviations of estimates for each factor for EAP as termination values for 50 and .55.

OSE = 0.50

General factor. Table 11shows that when OSE was set to .50 in Condition D, on average, 36.24 items (range from 27 to 55 items) resulted in a correlation of .974 between $\hat{\theta}_C$ and $\hat{\theta}_F$ by MAP, on average 35.31 items (range from 27 to 55 items) resulted in r = .973 by EAP, and an average of 36.68 items (range from 28 to 55 items) yielded r = .974 by MLE. In Condition E, r = .969 was obtained with an average of 28.80 items (range from 22 to 43 items) by MAP, r = .974 with an average of 32.22 items (range from 26 to 44 items) by EAP, and r = .968 with an average of 28.98 items (range from 22 to 42 items) by MLE. In Condition F, r = .951 was obtained with an average of 20.42 items (range from 15 to 31 items) by MAP, r = .960 with an average of 24.17 items (range from 20 to 35) by EAP, and r = .950 with an average of 20.97 items (range from 16 to 30 items) by MLE. Mean percentage reductions of MAP, EAP, and MLE methods were 90.94%, 91.17%, and 90.83%, respectively, in Condition D, 92.89%, 91.94%, and 92.76% in Condition E, 94.89%, 93.14%, and 94.76% in Condition F. These results indicate that EAP was the least efficient method among the three θ estimation methods in the MBICAT algorithm.

Bias was consistently low for all three group factor discrimination conditions. Bias in Condition D was –. 004 by MAP, –.001 by EAP, and –.004 by MLE.

Table 11

Correlation, Bias and RMSE, Mean and Range of Number of Items Administered, and Percent Mean Reduction in Number of Items for the General Factor Scale (OSE=0.50)

Condition and θ estimation method	$r\left(\hat{\theta}_C, \hat{\theta}_F\right)$	Bias	RMSE	Number of Items		
				Mean	Range	Reduction
Condition D: Low group factor discrimination						
MAP	.974	–.004	.226	36.24	27 - 55	90.94%
EAP	.973	–.001	.229	35.31	27 - 55	91.17%
MLE	.974	–.004	.223	36.68	28 - 55	90.83%
Condition E: Medium group factor discrimination						
MAP	.969	.007	.254	28.80	22 - 43	92.89%
EAP	.974	.009	.228	32.22	26 - 44	91.94%
MLE	.968	.004	.250	28.98	22 - 42	92.76%
Condition F: High group factor discrimination						
MAP	.951	–.006	.299	20.42	15 -31	94.89%
EAP	.960	–.002	.272	24.17	20 - 35	93.14%
MLE	.950	–.004	.296	20.97	16 - 30	94.76%

Bias in Condition E was .007 by MAP, .009 by EAP, and .004 by MLE. Bias in Condition F was –.006 byMAP, –.002 by EAP, and –.004 by MLE. In terms of RMSE, in Condition D, RMSEs by using MAP, EAP, and MLE methods were .226, .229, and .223, respectively. In Condition E, RMSEs by using MAP, EAP, and MLE methods were .254, .228, and 250, respectively. In Condition F, RMSEs by using MAP, EAP, and MLE methods were .299, .272, and .296, respectively. Note that RMSE was higher with higher discrimination on group factors because the number of items administered was smaller due to low OSEs.

Group factors. When OSE was set to .50, results for the group factors for all conditions are shown in Table 12. Note that there are no test length results for the group factors because they are the same as those of the general factor. Results for the four group factor scales were similar to those obtained for the general factor scale. Correlations between $\hat{\theta}_C$ and $\hat{\theta}_F$ in Condition D ranged from .890 to .896. Correlations in Condition E ranged from .857 to .909. Correlations in Condition F ranged from .823 to .888. Bias for all scales and across the three group factor discrimination conditions was negligible, with a slight predominance of small negative and positive values. RMSE patterns of each group factor scale were generally consistent with those of the general factor scale across each estimation method. However, RMSEs of the group factor scales were somewhat larger than for the general factor scale, ranging from about .352 to .386 in Condition D, from about .367 to .430 in Condition E, and from about .404 to .483 in Condition F. Note that RMSE was higher with higher discrimination on group factors because the number of items administered was smaller due to low OSEs.

Table 12

Correlation, Bias and RMSE for Group Factor Scales under OSE=0.50

Group factor and θ estimation method	Condition D			Condition E			Condition F		
	$r(\hat{\theta}_C, \hat{\theta}_F)$	Bias	RMSE	$r(\hat{\theta}_C, \hat{\theta}_F)$	Bias	RMSE	$r(\hat{\theta}_C, \hat{\theta}_F)$	Bias	RMSE
Group Factor 1									
MAP	.896	.007	.382	.881	.003	.402	.823	.006	.483
EAP	.897	.008	.380	.902	–.004	.367	.866	.007	.426
MLE	.896	.011	.381	.878	.004	.408	.831	.009	.461
Group Factor 2									
MAP	.896	.011	.352	.894	–.004	.378	.850	–.008	.461
EAP	.895	.011	.355	.909	–.013	.353	.873	–.001	.427
MLE	.893	.012	.357	.892	–.009	.381	.853	–.010	.471
Group Factor 3									
MAP	.888	.015	.386	.862	–.004	.423	.842	.012	.468
EAP	.890	.011	.386	.888	–.011	.384	.872	.018	425
MLE	.894	.014	.376	.857	–.008	.430	.841	.025	.456
Group Factor 4									
MAP	.890	–.001	.375	.867	–.001	.418	.866	.009	.439
EAP	.890	–.007	.378	.891	–.004	.380	.888	.006	.404
MLE	.892	–.001	.371	.868	.002	.416	.868	.009	.436

OSE = .55

General factor. When OSE was set to .55, results for all scales are shown in Table 13. In Condition D, on average 23.60 items (range from 18 to 37 items) resulted in a correlation of .958 between $\hat{\theta}_C$ and $\hat{\theta}_F$ by MAP, 22.29 items (range from 17 to 33 items) resulted in r = .957 by EAP, and 24.02 items (range from 18 to 35 items) resulted in r = .960 by MLE. In Condition E, r = .951 with an average of 19.60 items (range from 15 to 29 items) by MAP, r = .955 with an average of 20.79 items (range from 17 to 29 items) by EAP, and r = .954 with an average of 20.18 items (range from 16 to 30 items) by MLE. In Condition F, r = .940 with an average of 14.88 items (range from 11 to 25 items) by MAP, r = .945 with an average of 16.48 items (range from 14 to 25) by EAP, and r = .936 with an average of 15.43 items (range from 12 to 24 items) by MLE. Mean percentage reductions in test length for MAP, EAP, and MLE were 94.13%, 94.42%, and 93.99%, respectively, for Condition D; 95.10%, 94.83%, and 94.95% for Condition E; and 96.28%, 95.88%, and 96.14% for Condition F.

Bias of the general factor was consistently low for all three group factor discrimination conditions. Bias in Condition D was −.003 by MAP, −.001 by EAP, and −.004 by MLE. Bias in Condition E was .004 by MAP, and .007 by EAP, and .008 by MLE. Bias in Condition F was –.002 by MAP, –.009 by EAP, and –.012 by MLE. In terms of RMSE, in Condition D, RMSEs by MAP, EAP, and MLE were .282, .288, and .277, respectively. In Condition E, RMSEs by MAP, EAP, and MLE were .307, .294, and .299, respectively; and in Condition F, RMSEs by MAP, EAP, and MLE were .333, .318, and .343, respectively. Note that RMSE was higher with higher discrimination on group factors because the number of items administered was smaller.

Table 13
Correlation, Bias and RMSE, Mean and Range of Number of Items Administered, and Percent Mean Reduction in Number of Items for the General Factor Scale (OSE=0.55)

Condition and θ estimation method	$r\left(\hat{\theta}_C, \hat{\theta}_F\right)$	Bias	RMSE	Number of Items		
				Mean	Range	Reduction
Condition D: Low group factor discrimination						
MAP	.958	−.003	.282	23.60	18 - 37	94.13%
EAP	.957	−.001	.288	22.29	17 - 33	94.42%
MLE	.960	−.004	.277	24.02	18 - 35	93.99%
Condition E: Medium group factor discrimination						
MAP	.951	.004	.307	19.60	15 - 29	95.10%
EAP	.955	.007	.294	20.79	17 - 29	94.83%
MLE	.954	.008	.299	20.18	16 - 30	94.95%
Condition F: High group factor discrimination						
MAP	.940	−.002	.333	14.88	11 - 25	96.28%
EAP	.945	−.009	.318	16.48	14 - 25	95.88%
MLE	.936	−.012	.343	15.43	12 - 24	96.14%

Group factors. When OSE was set to .55, results of group factors for all conditions are shown in Table 14. Again, note that there are no test length results for the group factors because they are the same as those of the general factor. When OSE was set to .55, results for the four group factor scales were similar to those obtained for the general factor scale. Correlations between $\hat{\theta}_C$ and $\hat{\theta}_F$ in Condition D ranged from .825 to .843. Correlations in Condition E ranged from .800 to .853. Correlations in Condition F ranged from .756 to .845. Bias for all scales and across the group factor discrimination conditions was negligible, with a slight predominance of small negative and positive values. RMSE patterns of each group factor scale were generally consistent with those of the general factor scale across each estimation method. However, RMSEs of the group factor scales were somewhat larger than for the general factor scale, ranging from .433 to .475 in Condition D, from .440 to .502 in Condition E, and from .469 to .556 in Condition F. Note again that RMSE was higher with higher discrimination on the group factors because the number of items administered was smaller due to low OSEs.

Table 14

Correlation, Bias and RMSE for Group Factor Scales under OSE=0.55

Group factor and θ estimation method	Condition D			Condition E			Condition F		
	$r(\hat{\theta}_C, \hat{\theta}_F)$	Bias	RMSE	$r(\hat{\theta}_C, \hat{\theta}_F)$	Bias	RMSE	$r(\hat{\theta}_C, \hat{\theta}_F)$	Bias	RMSE
Group Factor 1									
MAP	.843	.003	.462	.832	.008	.472	.758	–.004	.554
EAP	.834	.001	.475	.844	.008	.456	.797	.005	.512
MLE	.840	.004	.467	.824	.021	.483	.756	–.009	.556
Group Factor 2									
MAP	.838	.013	.433	.837	–.001	.462	.784	–.008	.542
EAP	.838	.013	.435	.853	–.008	.440	.817	–.012	.504
MLE	.836	.011	.435	.844	–.008	.453	.788	.004	.540
Group Factor 3									
MAP	.827	–.001	.472	.812	–.002	.488	793	.014	.528
EAP	.828	–.001	.474	.824	–.012	.474	.814	.022	505
MLE	.828	.001	.470	.808	–.007	.492	.799	.010	.522
Group Factor 4									
MAP	.828	.001	.460	.800	–.008	.502	.823	–.001	.498
EAP	.828	.002	.464	.827	–.002	.471	.845	.016	.469
MLE	.825	–.005	.464	.809	–.005	.492	.825	.005	.496

Termination Criterion

The MBICAT algorithm resulted in very substantial reductions in numbers of items (90% to 95%) with both the OSE =.55 and OSE = .50 CAT termination criteria producing θ estimates that correlated with θ estimates from the full sets of scale items well above .90 for the general factor and above .80 for the group factors. Comparisons of the results from the two OSE criteria indicated that OSE of .55 resulted in correlations above .936

for the general factor and above .797 for all group factors, and OSE of .50 resulted in correlations above .951 for the general factor and above .823 for all group factors. The three θ estimation methods provided CAT θ estimates that correlated highly with θ estimates from the total set of items. All three θ estimation methods provided efficient (i.e., shorter) tests across all conditions for the general factor and group factors with both OSE termination criteria.

CHAPTER 4: DISCUSSION AND CONCLUSIONS

This study demonstrated that the MBICAT algorithm worked well when latent scores on the secondary dimension were estimated properly. In the two BICAT algorithms, the use of differential entry on the group factors, as in Weiss and Gibbons (2007), did not make a difference compared to initial item at θ of 0 for both the general factor and group factor scales (Gibbons, et al., 2008) in terms of precision and efficiency. Through examining the correlations among estimated θs, this study showed that the BICAT algorithms resulted in high $\hat{\theta}$ intercorrelations, which means that the general factor $\hat{\theta}s$ were not independent of the group factor $\hat{\theta}$s , and group factor $\hat{\theta}$s were not independent of each other, as is postulated by the bifactor model. On the other hand, the MBICAT algorithm maintained low $\hat{\theta}$ intercorrelations in accordance with the assumption of the bifactor model.

There was no significant impact of CAT algorithm on the correlation and RMSE criteria for the general factor. However, the CAT algorithm had a significant impact on the correlation and the RMSE criteria for the group factors. Therefore, this study demonstrated that the MBICAT algorithm provided more precise estimates than the two BICAT algorithms for the group factors. However, the BICAT(D) algorithm using a differential initial θ for group factors did not improve the precision of θ estimates compared to the BICAT(S) using initial $\theta = 0$ for both the general factor and group factor scales. In addition, the CAT algorithm had an impact on the efficiency of test length. The MBICAT algorithm was less efficient than the two BICAT algorithms because the OSEs of the MBICAT algorithm were higher than those of the two BICAT algorithms.

The number of group factors had an impact on the precision of only group factor θ estimates for the BICAT algorithm, but had an impact on both the general factor and group factors θ estimates for the MBICAT algorithms. For the BICAT algorithm, the correlations between θ and $\hat{\theta}$ for the general factors were higher in the bifactor model with four group factors than the bifactor model with two group factors because of the difference in the number of items administered on the general factor, but there were no consistent differences for the correlation for the group factors between the two different number of group factors conditions. The precision of group factor θ estimates was not affected by the number of items administered on the general factor because θs of the general factor and group factors are estimated separately. However, for the MBICAT algorithm, the correlations between θ and $\hat{\theta}$ were higher in the bifactor model with four group factors than the bifactor model with two group factors for both the general factor and the group factors. The precision of group factor θ estimates was affected by the number of group factors. These results were the same for the RMSE.

OSEs of estimates for the general factors were smaller in the bifactor model with four group factors than the bifactor model with two group factors because of the difference in the number of items administered on the general factor for both the BICAT and MBICAT algorithm. However, OSEs of estimates for the group factors were higher in the bifactor model with four group factors than the bifactor model with two group factors for both the BICAT and MBICAT algorithms. Since multidimensional estimation methods employ the covariance matrix of $\hat{\boldsymbol{\theta}}$ to compute the OSE for each examinee, they would increase the OSE due to more zero group factor discriminations across latent trait estimates for the bifactor model with four group factor than the bifactor model with two group factors.

The magnitude of the group factor discriminations contributed to the precision and efficiency of estimates for both the general factor and group factors across the three CAT algorithms. As the mean of the group factor discrimination parameters increased, the precision of the estimated θs for the group factors increased, while the precision of the estimated θs for the general factor were slightly decreased because the items loading highly on the group factor contributed to the precision of the group factor θ estimates rather than the general factor θ estimates. The precision of group factor estimates was, therefore, dependent on the degree to which the items loaded on group factors. In addition, high group factor discrimination parameters contributed to the efficiency of the CAT for the group factors in both the BICAT and MBICAT algorithms. As the mean of group factor discrimination parameters increased, the OSEs for the general factor slightly increased, while the OSE for the group factors decreased. Thus, the number of items administered for the group factors can be reduced by selecting items highly loaded on the group factors.

The effect of θ estimation method was negligible across the three CAT algorithms. The three θ estimation methods provided similar θ estimates given that more than 20 items were administered for each group factor. The recovery of θs was almost the same across the three θ estimation methods regardless of CAT algorithms. However, EAP had the largest OSEs among the three θ estimation methods for the multidimensional case unlike the unidimensional case, likely because the 15 quadrature points were used for each factor with MBICAT and 50 for the BICAT algorithm. The difference in the OSEs between multidimensional MLE and MAP was negligible across the three group factor discrimination conditions.

Although development of MIRT began many years ago, practical applications of MIRT to CAT are fairly few. This study determined that the MBICAT algorithm provided more accurate estimation of a *k*-dimensional vector of traits for each examinee instead of *k* separate trait estimates using the BICAT algorithm. In addition, this study demonstrated that the MBICAT algorithm satisfied independence of the general factor θ estimates with the group factor θ estimates, and the group factor θ estimates with each other.

Limitations and Implications

Some obvious issues are raised by this study for future research. This study used the bifactor model to implement CAT as one alternative model among many possible multidimensional models for psychological data. By considering the effect of multidimensionality on IRT item parameter estimates, the bifactor model estimates a substantively common trait as well as capturing multiple constructs (Reise, Morizot, & Hays, 2007).

However, the bifactor model is not versatile for all psychological data. Gibbons et al. (2008) mentioned some limitations of the bifactor model. First, the bifactor model specification relies on prior information to indicate the relationships between items and factors. Second, a primary (i.e., general) dimension is assumed to exist. If the test items show a strong simple factor structure, the bifactor model will not be useful. Under these circumstances, it would be appropriate to use a unidimensional model for each factor structure rather than the bifactor model. This is empirically testable by comparing the fit of the bifactor model to corresponding unidimensional models and unrestricted multidimensional models by using the TESTFACT program. Third, the bifactor model requires each item to load on a primary dimension and on no more than one sub-domain.

If items are related to multiple sub-domains, they will not be appropriate for the bifactor model. Therefore, the bifactor model can be adapted to CAT only when the researcher holds certain beliefs or has evidence concerning the actual dimensional structure of a test and after demonstrating that the bifactor model fits better than alternative models.

As empirical issues, the factor structures examined in this study represented only a small subset of the factor structures that might affect the quality of CAT with the bifactor model. In this regard, this study has several limitations that could be of interest in future studies. First, this study generated the data using specific instances of the bifactor structure. This means that the mean and SD of group factor discrimination parameters for each group factor were equally applied to generate group factor discrimination parameters for each item. Second, this study used only two conditions for the number of group factors (two group factors vs. four group factors). Future studies can be undertaken to verify whether the same conclusions can be arrived at when the conditions are further varied. Of particular interest was how much the BICAT(D) improved efficiency of θ recovery over the BICAT(S) when the different bifactor structures were applied to the BICAT algorithm. However, the two BICAT algorithms were identical with each other in terms of precision and efficiency when 20 items were administered for each group factor. Thus, follow-up studies should examine different tests length to compare the two BICAT algorithms.

MCAT is a relatively new area of research. This study was a comparison of CAT algorithms for the bifactor model to evaluate the relative performance of the BICAT and MBICAT algorithms and make recommendations in terms of what θ estimation and termination criterion methods to use when designing a MBICAT. The conclusions of this study were limited to the conditions of item bank, test lengths, and specific factor patterns. Also, in order to improve this study, more multidimensional item selection methods can be considered, such as: (1) minimizing the trace of the inverse of the Fisher information matrix (Mulder & van der Linden, 2008), and (2) maximizing the Kullback-Leibler information (Veldkamp & van der Linden, 2002). Thus, more research needs to be done to compare these new item selection methods with Segall's Bayesian item selection method as used in this study. There are also other potential research issues in MBICAT, such as how to select the first item and how to control item exposure.

Finally, it should be noted that current implementation of the code in R for the MBICAT algorithm was not efficient. For the condition involving 400 items and 1,000 examinees in the bifactor model with two group factors, the average time required to estimate θs using a convergence criterion of .001on a computer with a 3.2 GHz processor and 2 GB of memory was about four hours for the MAP and MLE estimation methods, and was about four days for the EAP estimation method using 15 quadrature points for the general factor and each group factor. Thus, there still remain computation time issues for multidimensional CAT models.

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APPENDIX A

Item Discrimination and Difficulty Parameters for Six Response Sets

Table A1

Item Discrimination and Difficulty Parameters
Under the Low Group Factor Discrimination Condition (Condition A)

Item	a_1	a_2	a_3	b	*Item*	a_1	a_2	a_3	b
1	0.851	0.271	0.000	-2.011	201	1.273	0.000	0.375	1.547
2	1.030	0.602	0.000	2.109	202	0.891	0.000	0.769	3.116
3	0.573	0.389	0.000	1.479	203	1.242	0.000	0.559	2.728
4	1.118	0.824	0.000	-3.863	204	1.230	0.000	0.329	1.795
5	1.024	0.663	0.000	0.859	205	1.233	0.000	0.502	-2.198
6	0.811	0.115	0.000	-1.767	206	0.774	0.000	0.662	-3.266
7	1.033	0.372	0.000	-0.186	207	0.919	0.000	0.715	2.130
8	1.061	0.116	0.000	1.580	208	0.693	0.000	0.816	3.778
9	0.993	0.808	0.000	-2.890	209	0.930	0.000	0.263	-0.840
10	1.113	0.606	0.000	2.283	210	0.869	0.000	0.482	0.582
11	0.748	0.740	0.000	-1.638	211	1.089	0.000	0.615	2.924
12	0.948	0.440	0.000	-2.993	212	1.180	0.000	0.540	2.826
13	1.468	0.589	0.000	1.243	213	1.072	0.000	0.627	-2.554
14	1.187	0.444	0.000	1.300	214	1.030	0.000	0.213	-0.664
15	1.036	0.585	0.000	3.759	215	0.977	0.000	0.527	1.977
16	0.987	0.742	0.000	3.605	216	0.867	0.000	0.358	-3.646
17	1.065	0.628	0.000	-3.545	217	1.026	0.000	0.545	2.877
18	1.198	0.160	0.000	-2.449	218	1.278	0.000	0.628	3.879
19	0.585	0.699	0.000	1.317	219	0.733	0.000	0.298	1.305
20	1.247	0.665	0.000	0.263	220	0.613	0.000	0.767	-3.887
21	1.200	0.207	0.000	1.611	221	1.255	0.000	0.618	1.435
22	0.800	0.009	0.000	-2.543	222	1.116	0.000	0.520	1.104
23	1.060	0.598	0.000	-1.927	223	0.943	0.000	0.688	1.773
24	0.841	0.516	0.000	-0.288	224	0.713	0.000	0.732	2.099
25	1.293	0.351	0.000	2.274	225	1.027	0.000	0.444	-2.722
26	0.998	0.586	0.000	1.344	226	0.880	0.000	0.549	3.964
27	1.073	0.572	0.000	2.854	227	0.472	0.000	0.552	-1.886
28	0.797	0.296	0.000	-3.459	228	1.330	0.000	0.608	0.379
29	0.564	0.412	0.000	0.091	229	1.023	0.000	0.519	-2.858
30	1.111	0.667	0.000	0.282	230	0.980	0.000	0.515	2.582
31	1.226	0.257	0.000	-0.187	231	0.853	0.000	0.392	0.064
32	0.883	0.249	0.000	2.628	232	1.163	0.000	0.499	-2.089
33	0.858	0.269	0.000	-1.550	233	0.907	0.000	0.397	-1.918
34	0.878	0.798	0.000	-0.902	234	1.291	0.000	0.182	2.897
35	0.546	0.150	0.000	-1.957	235	0.747	0.000	0.381	-0.558
36	0.778	0.270	0.000	-1.973	236	1.239	0.000	0.496	-0.984
37	0.767	0.846	0.000	1.169	237	1.244	0.000	0.677	-0.217
38	0.808	0.326	0.000	-2.249	238	0.832	0.000	0.694	-0.288
39	1.355	0.435	0.000	-1.096	239	1.155	0.000	0.412	2.330
40	0.995	0.686	0.000	-3.594	240	1.037	0.000	0.727	-0.855
41	0.870	0.436	0.000	-2.877	241	0.725	0.000	0.547	-2.202
42	0.977	0.252	0.000	-2.684	242	1.078	0.000	0.792	1.199
43	0.805	0.507	0.000	3.398	243	1.063	0.000	0.613	-0.320
44	1.091	0.242	0.000	-0.620	244	1.347	0.000	0.495	-1.871
45	1.109	0.276	0.000	-1.831	245	1.254	0.000	1.040	-2.067
46	1.203	0.381	0.000	2.019	246	0.914	0.000	0.784	2.990
47	0.974	0.574	0.000	-1.991	247	0.804	0.000	0.830	2.537
48	1.259	0.298	0.000	2.476	248	0.866	0.000	0.608	1.879
49	0.891	0.699	0.000	-3.682	249	1.160	0.000	0.316	2.585
50	1.216	0.007	0.000	3.209	250	0.661	0.000	0.355	0.583

Table A1
Item Discrimination and Difficulty Parameters
Under the Low Group Factor Discrimination Condition (Condition A)

Item	a_1	a_2	a_3	b	*Item*	a_1	a_2	a_3	b
51	0.960	0.154	0.000	1.386	251	0.852	0.000	0.245	-1.376
52	0.630	0.682	0.000	2.510	252	1.025	0.000	0.542	3.558
53	1.034	0.750	0.000	1.101	253	0.849	0.000	0.233	0.322
54	0.712	0.749	0.000	1.768	254	0.879	0.000	0.560	0.678
55	1.009	0.630	0.000	-1.963	255	1.221	0.000	0.613	-1.017
56	0.541	0.543	0.000	2.873	256	0.909	0.000	0.603	-3.756
57	1.077	0.484	0.000	-1.381	257	1.148	0.000	0.543	2.821
58	1.000	0.419	0.000	-1.093	258	1.190	0.000	0.773	-3.984
59	0.701	0.717	0.000	2.772	259	1.070	0.000	0.908	-0.964
60	1.066	0.026	0.000	-3.940	260	1.439	0.000	0.835	-2.392
61	1.052	0.486	0.000	-3.928	261	1.168	0.000	0.392	-2.735
62	1.180	0.397	0.000	2.050	262	0.941	0.000	0.363	-1.583
63	1.141	0.487	0.000	1.175	263	1.332	0.000	0.389	-0.473
64	1.283	0.348	0.000	2.845	264	0.876	0.000	0.320	0.769
65	0.974	0.455	0.000	-2.641	265	1.249	0.000	0.871	1.677
66	0.846	0.353	0.000	2.860	266	1.050	0.000	0.323	-0.058
67	0.960	-0.115	0.000	-0.091	267	1.113	0.000	0.763	1.226
68	1.231	0.742	0.000	0.270	268	1.043	0.000	0.472	1.771
69	0.714	0.426	0.000	2.053	269	1.247	0.000	0.628	-3.225
70	0.980	0.419	0.000	3.547	270	1.647	0.000	0.518	-0.558
71	0.896	0.449	0.000	3.093	271	0.693	0.000	0.046	-1.828
72	0.807	0.660	0.000	1.613	272	0.844	0.000	0.903	-1.365
73	0.996	0.508	0.000	-3.620	273	0.918	0.000	0.437	-2.072
74	1.071	0.829	0.000	3.207	274	1.260	0.000	0.936	3.023
75	1.097	0.406	0.000	-0.638	275	0.870	0.000	0.537	1.984
76	1.214	0.735	0.000	2.995	276	0.969	0.000	0.448	3.237
77	0.953	0.028	0.000	-2.713	277	1.332	0.000	0.674	0.959
78	1.168	0.205	0.000	3.329	278	0.686	0.000	0.589	-3.706
79	0.933	0.614	0.000	-0.693	279	0.943	0.000	0.493	-3.635
80	0.652	0.603	0.000	0.899	280	1.193	0.000	0.556	-2.066
81	1.256	0.546	0.000	3.929	281	1.247	0.000	0.837	0.898
82	1.005	0.393	0.000	-3.048	282	1.337	0.000	0.424	-0.103
83	0.796	0.678	0.000	0.501	283	0.949	0.000	0.394	1.750
84	0.871	0.645	0.000	-3.985	284	0.998	0.000	0.823	-1.489
85	0.867	0.505	0.000	-2.794	285	0.937	0.000	0.592	3.432
86	1.003	0.210	0.000	0.857	286	1.248	0.000	0.237	-3.213
87	0.779	0.416	0.000	0.530	287	1.204	0.000	0.677	3.317
88	0.807	0.746	0.000	-2.413	288	0.701	0.000	0.247	-0.316
89	0.497	0.399	0.000	-0.702	289	1.068	0.000	0.256	-1.544
90	0.775	0.450	0.000	-2.587	290	0.892	0.000	0.388	-3.349
91	0.759	0.283	0.000	3.745	291	0.992	0.000	0.382	-3.434
92	0.884	0.440	0.000	-1.745	292	1.007	0.000	0.051	-0.160
93	0.749	0.543	0.000	3.438	293	1.098	0.000	0.459	-1.405
94	1.074	0.502	0.000	-1.176	294	1.192	0.000	0.290	2.529
95	0.951	0.695	0.000	-0.242	295	1.044	0.000	0.622	3.331
96	1.055	0.377	0.000	-1.087	296	0.971	0.000	0.741	1.094
97	0.826	0.504	0.000	-1.911	297	0.853	0.000	0.737	3.496
98	1.189	0.172	0.000	1.737	298	1.097	0.000	0.490	-3.047
99	1.361	0.510	0.000	3.468	299	0.818	0.000	0.376	-3.531
100	1.156	0.432	0.000	-0.753	300	0.832	0.000	0.453	-2.820

Table A1
Item Discrimination and Difficulty Parameters
Under the Low Group Factor Discrimination Condition (Condition A)

Item	a_1	a_2	a_3	b	*Item*	a_1	a_2	a_3	b
101	1.071	0.420	0.000	-3.844	301	0.946	0.000	0.690	-1.026
102	1.194	0.486	0.000	-2.993	302	0.875	0.000	0.598	0.255
103	1.114	0.834	0.000	-3.478	303	0.834	0.000	0.077	2.288
104	1.348	0.594	0.000	2.438	304	1.137	0.000	0.756	3.513
105	0.768	0.376	0.000	1.797	305	1.262	0.000	0.219	-2.800
106	0.723	0.195	0.000	-0.110	306	1.027	0.000	0.334	-0.262
107	0.727	0.575	0.000	2.408	307	0.872	0.000	0.004	1.543
108	0.681	0.503	0.000	-3.817	308	0.932	0.000	0.675	0.113
109	0.927	0.483	0.000	2.286	309	1.359	0.000	0.392	2.349
110	1.137	0.325	0.000	1.482	310	1.154	0.000	0.428	1.655
111	1.043	0.760	0.000	1.953	311	1.463	0.000	0.521	2.281
112	0.973	0.394	0.000	-2.388	312	1.048	0.000	0.228	-2.461
113	0.903	0.515	0.000	2.167	313	1.322	0.000	0.470	1.304
114	1.057	0.386	0.000	-0.769	314	1.143	0.000	0.589	3.297
115	0.915	0.474	0.000	1.177	315	0.866	0.000	0.237	-0.259
116	1.080	0.470	0.000	-1.522	316	0.898	0.000	0.711	2.946
117	1.093	0.289	0.000	-3.159	317	1.008	0.000	0.201	-3.738
118	1.254	0.464	0.000	-0.379	318	0.864	0.000	0.500	-0.579
119	0.847	0.418	0.000	3.225	319	0.800	0.000	0.708	-3.941
120	1.314	0.701	0.000	-2.695	320	1.175	0.000	0.781	-2.815
121	1.317	0.338	0.000	0.637	321	1.369	0.000	0.523	-0.513
122	1.264	0.629	0.000	-2.500	322	1.092	0.000	0.340	-2.997
123	0.788	0.428	0.000	3.702	323	1.307	0.000	1.028	-2.589
124	1.114	0.712	0.000	2.013	324	0.728	0.000	0.235	-0.448
125	1.205	0.744	0.000	-0.147	325	0.728	0.000	0.595	-0.184
126	0.728	0.025	0.000	2.429	326	0.996	0.000	0.475	0.405
127	0.993	0.668	0.000	-2.036	327	1.382	0.000	0.551	-0.597
128	0.983	0.360	0.000	3.471	328	1.405	0.000	0.267	-2.304
129	0.855	0.652	0.000	1.683	329	0.919	0.000	0.256	-0.633
130	1.123	0.355	0.000	3.237	330	0.758	0.000	0.404	-3.945
131	0.844	0.844	0.000	1.281	331	1.294	0.000	0.619	-3.945
132	0.800	0.458	0.000	-3.905	332	0.713	0.000	0.374	0.788
133	0.994	0.439	0.000	-1.858	333	0.916	0.000	0.672	-0.996
134	0.912	0.543	0.000	-3.284	334	1.264	0.000	0.567	1.307
135	1.214	0.357	0.000	0.542	335	1.023	0.000	0.409	3.821
136	0.605	0.450	0.000	-2.732	336	1.134	0.000	0.664	2.485
137	1.149	0.538	0.000	-0.224	337	1.052	0.000	0.541	0.753
138	1.262	0.585	0.000	-0.500	338	1.092	0.000	0.438	2.813
139	0.748	0.614	0.000	-2.133	339	0.436	0.000	0.605	1.762
140	0.829	0.279	0.000	-1.875	340	0.967	0.000	0.714	-3.846
141	1.044	0.789	0.000	-0.271	341	0.863	0.000	0.421	-1.771
142	1.055	0.820	0.000	-2.947	342	1.033	0.000	0.654	-3.057
143	0.932	0.732	0.000	2.863	343	0.962	0.000	0.416	-1.086
144	1.017	0.526	0.000	2.363	344	0.960	0.000	0.495	2.284
145	0.864	0.384	0.000	2.273	345	0.755	0.000	0.457	-1.051
146	0.955	0.292	0.000	2.305	346	1.112	0.000	0.234	2.483
147	1.018	0.768	0.000	-3.100	347	1.000	0.000	0.640	-2.650
148	1.177	0.551	0.000	-3.892	348	0.747	0.000	0.382	2.678
149	0.922	0.682	0.000	1.578	349	0.965	0.000	0.249	-1.672
150	0.973	0.609	0.000	2.258	350	1.096	0.000	0.639	-2.784

Table A1

Item Discrimination and Difficulty Parameters

Under the Low Group Factor Discrimination Condition (Condition A)

Item	a_1	a_2	a_3	b	*Item*	a_1	a_2	a_3	b
151	1.090	0.736	0.000	1.898	351	1.141	0.000	0.528	-2.181
152	1.060	0.408	0.000	-0.040	352	0.845	0.000	0.306	1.278
153	1.118	0.460	0.000	-2.497	353	1.011	0.000	0.104	0.022
154	0.795	0.847	0.000	-2.666	354	1.161	0.000	0.342	3.694
155	1.005	0.631	0.000	0.880	355	0.999	0.000	0.690	1.386
156	1.131	0.537	0.000	2.463	356	0.781	0.000	0.405	3.289
157	1.187	0.612	0.000	1.534	357	0.996	0.000	0.488	0.909
158	0.969	0.603	0.000	-0.483	358	1.102	0.000	0.474	2.343
159	0.978	0.726	0.000	-2.637	359	1.337	0.000	0.648	-0.810
160	1.203	0.283	0.000	-0.632	360	1.006	0.000	0.551	2.736
161	0.723	0.556	0.000	3.201	361	1.125	0.000	0.863	1.354
162	0.952	0.231	0.000	-3.591	362	1.038	0.000	0.573	-3.191
163	0.928	0.585	0.000	-0.699	363	0.766	0.000	0.366	2.677
164	1.104	0.794	0.000	-1.500	364	0.852	0.000	0.794	3.053
165	1.234	0.522	0.000	-1.167	365	1.105	0.000	0.622	0.151
166	0.719	0.526	0.000	-0.011	366	0.776	0.000	0.259	-1.818
167	0.765	0.467	0.000	-2.408	367	0.949	0.000	0.775	3.964
168	1.140	0.475	0.000	-0.094	368	0.730	0.000	0.399	0.648
169	0.751	0.367	0.000	2.586	369	1.052	0.000	0.696	2.748
170	0.830	0.801	0.000	-0.229	370	1.185	0.000	0.537	3.771
171	0.851	0.688	0.000	-1.708	371	0.824	0.000	0.561	-2.963
172	0.759	0.958	0.000	1.989	372	0.754	0.000	0.851	-2.502
173	1.065	0.615	0.000	-2.403	373	1.384	0.000	0.763	0.202
174	0.967	0.375	0.000	0.324	374	1.088	0.000	0.302	3.433
175	0.821	0.198	0.000	-0.728	375	0.817	0.000	0.283	-0.433
176	1.113	0.513	0.000	2.053	376	1.165	0.000	0.279	3.990
177	1.092	0.711	0.000	1.697	377	0.955	0.000	0.385	-0.779
178	1.073	0.624	0.000	0.760	378	0.916	0.000	0.298	2.842
179	0.702	0.661	0.000	-1.247	379	0.990	0.000	0.386	3.080
180	1.331	0.767	0.000	2.416	380	0.996	0.000	0.529	-1.811
181	0.671	0.654	0.000	-2.838	381	1.187	0.000	0.531	-1.914
182	1.139	0.513	0.000	-3.371	382	0.992	0.000	0.418	0.717
183	1.314	0.795	0.000	-3.587	383	1.173	0.000	0.503	-2.489
184	1.060	0.555	0.000	-3.285	384	1.065	0.000	0.760	1.970
185	0.857	0.323	0.000	-1.936	385	1.225	0.000	0.470	0.821
186	1.538	0.450	0.000	2.939	386	0.904	0.000	0.530	3.749
187	1.177	0.452	0.000	0.883	387	1.162	0.000	0.606	-3.244
188	0.909	-0.039	0.000	-1.801	388	0.578	0.000	0.375	3.093
189	1.127	0.656	0.000	-1.204	389	0.800	0.000	0.345	1.166
190	1.176	0.446	0.000	3.142	390	0.890	0.000	0.749	-3.193
191	1.138	0.581	0.000	-1.791	391	0.743	0.000	0.558	-3.830
192	0.892	0.415	0.000	0.523	392	1.240	0.000	0.553	-2.951
193	0.717	0.244	0.000	1.011	393	0.953	0.000	0.642	2.878
194	0.992	0.412	0.000	3.072	394	1.332	0.000	0.334	-3.674
195	0.723	0.540	0.000	-2.546	395	1.305	0.000	0.344	-2.926
196	1.203	0.657	0.000	-3.101	396	1.048	0.000	0.504	-0.856
197	1.080	0.657	0.000	3.771	397	1.211	0.000	0.594	-1.304
198	1.274	0.938	0.000	-3.440	398	1.042	0.000	0.212	0.939
199	0.820	0.706	0.000	-0.129	399	1.118	0.000	0.564	-0.195
200	0.920	0.730	0.000	1.961	400	1.279	0.000	0.383	-1.192

Table A2
Item Discrimination and Difficulty Parameters
Under the Medium Group Factor Discrimination Condition (Condition B)

Item	a_1	a_2	a_3	b	*Item*	a_1	a_2	a_3	b
1	1.129	0.449	0.000	1.041	201	0.798	0.000	0.675	3.234
2	1.178	0.913	0.000	-1.832	202	1.080	0.000	0.621	3.065
3	1.076	0.574	0.000	2.461	203	1.122	0.000	1.169	2.803
4	1.102	0.554	0.000	2.397	204	0.836	0.000	0.655	1.559
5	1.180	0.168	0.000	3.133	205	0.844	0.000	0.472	3.919
6	1.228	0.865	0.000	2.452	206	0.977	0.000	0.607	-1.142
7	1.162	0.348	0.000	2.427	207	0.856	0.000	0.661	-1.601
8	0.992	0.312	0.000	0.986	208	0.827	0.000	0.521	-2.384
9	0.879	0.332	0.000	3.987	209	0.917	0.000	0.928	3.036
10	0.946	0.936	0.000	2.073	210	1.078	0.000	0.382	1.853
11	0.883	0.452	0.000	0.512	211	0.995	0.000	0.602	3.756
12	0.279	0.692	0.000	-2.734	212	0.821	0.000	0.185	-3.819
13	1.245	0.370	0.000	1.901	213	1.233	0.000	0.720	-2.013
14	0.911	0.754	0.000	0.421	214	1.349	0.000	0.585	0.273
15	1.128	0.800	0.000	-3.794	215	0.838	0.000	0.397	-3.420
16	0.978	0.984	0.000	2.744	216	1.009	0.000	0.741	-2.654
17	1.285	0.559	0.000	-3.219	217	0.909	0.000	0.632	-0.510
18	1.168	0.543	0.000	-1.816	218	0.637	0.000	0.557	1.978
19	0.576	0.623	0.000	0.653	219	1.115	0.000	0.412	-3.921
20	1.078	0.496	0.000	-0.413	220	0.725	0.000	0.738	-1.628
21	0.850	0.528	0.000	2.646	221	0.974	0.000	0.717	1.941
22	1.291	0.528	0.000	0.485	222	1.049	0.000	1.024	-2.318
23	0.958	0.167	0.000	-1.522	223	0.882	0.000	0.928	1.797
24	1.231	0.576	0.000	-0.135	224	1.137	0.000	0.752	1.305
25	0.907	0.520	0.000	3.499	225	0.902	0.000	0.338	-2.821
26	0.896	0.607	0.000	-2.618	226	0.763	0.000	0.581	3.375
27	1.251	0.576	0.000	0.202	227	1.167	0.000	0.470	-2.096
28	0.947	0.619	0.000	-0.872	228	1.115	0.000	0.755	-1.336
29	0.958	0.274	0.000	2.801	229	0.833	0.000	0.729	-2.159
30	0.892	0.718	0.000	-3.489	230	1.030	0.000	0.446	0.038
31	1.035	0.489	0.000	-0.489	231	1.359	0.000	0.849	3.682
32	0.829	0.447	0.000	3.059	232	1.199	0.000	0.619	3.262
33	0.734	0.417	0.000	2.496	233	0.826	0.000	0.527	-3.215
34	1.013	0.262	0.000	0.811	234	1.580	0.000	0.699	-3.759
35	1.049	0.466	0.000	-1.138	235	1.201	0.000	1.210	0.298
36	0.919	0.458	0.000	-3.901	236	0.484	0.000	0.605	-3.051
37	0.956	0.196	0.000	1.982	237	0.900	0.000	0.730	-0.278
38	1.015	0.389	0.000	-1.547	238	1.006	0.000	0.473	-3.931
39	1.235	0.647	0.000	0.372	239	0.813	0.000	0.518	0.571
40	0.946	0.617	0.000	-2.754	240	0.874	0.000	0.689	3.592
41	0.910	0.808	0.000	1.270	241	0.925	0.000	0.741	-0.498
42	0.929	0.506	0.000	-3.558	242	0.967	0.000	0.804	1.417
43	0.716	0.639	0.000	-0.892	243	1.259	0.000	0.310	3.486
44	0.822	0.852	0.000	-1.125	244	0.719	0.000	-0.030	2.574
45	1.058	0.374	0.000	2.770	245	1.102	0.000	0.534	1.198
46	0.934	0.793	0.000	-1.213	246	0.884	0.000	0.795	-3.966
47	1.113	0.719	0.000	2.239	247	1.069	0.000	0.713	-1.330
48	0.995	0.474	0.000	1.883	248	0.825	0.000	0.486	1.262
49	1.106	0.524	0.000	-1.168	249	0.830	0.000	0.488	-1.900
50	0.506	1.090	0.000	0.953	250	0.856	0.000	0.461	-3.026

Table A2
Item Discrimination and Difficulty Parameters
Under the Medium Group Factor Discrimination Condition (Condition B)

Item	a_1	a_2	a_3	*b*	*Item*	a_1	a_2	a_3	*b*
51	1.073	0.359	0.000	-0.408	251	0.847	0.000	0.868	-3.671
52	1.224	0.569	0.000	-2.546	252	1.376	0.000	0.692	2.277
53	0.924	0.727	0.000	-0.945	253	1.076	0.000	0.722	2.377
54	1.132	0.780	0.000	3.330	254	1.200	0.000	0.795	-0.834
55	0.907	0.325	0.000	0.133	255	1.274	0.000	0.303	2.466
56	0.964	0.319	0.000	0.694	256	1.232	0.000	0.461	2.108
57	0.983	0.724	0.000	3.113	257	0.930	0.000	0.332	0.470
58	0.607	0.544	0.000	-1.295	258	1.244	0.000	0.467	3.046
59	1.042	0.276	0.000	-0.956	259	0.494	0.000	0.523	1.798
60	1.051	0.771	0.000	-3.770	260	1.474	0.000	0.768	-1.654
61	1.608	0.657	0.000	3.040	261	0.919	0.000	0.608	-1.072
62	0.956	0.766	0.000	-1.303	262	1.222	0.000	0.648	1.324
63	1.024	0.623	0.000	-0.874	263	0.769	0.000	0.415	0.619
64	1.400	0.995	0.000	3.160	264	0.775	0.000	0.763	-1.487
65	0.838	0.734	0.000	3.805	265	1.200	0.000	0.744	3.376
66	1.206	0.264	0.000	-1.577	266	0.943	0.000	0.746	-0.931
67	0.976	0.769	0.000	3.940	267	1.423	0.000	0.726	-0.882
68	0.724	1.007	0.000	-0.406	268	0.683	0.000	1.191	-1.653
69	1.107	0.745	0.000	0.479	269	1.074	0.000	-0.116	-0.703
70	0.984	0.445	0.000	-2.760	270	1.075	0.000	0.653	2.610
71	1.272	0.735	0.000	-0.622	271	0.569	0.000	0.772	2.342
72	0.916	0.621	0.000	3.887	272	1.341	0.000	0.710	3.497
73	0.981	0.506	0.000	-3.281	273	0.822	0.000	1.070	-1.005
74	1.029	0.537	0.000	-0.231	274	1.152	0.000	0.430	-3.209
75	0.980	0.327	0.000	3.922	275	1.410	0.000	0.245	3.237
76	1.212	0.668	0.000	-2.777	276	1.180	0.000	0.886	2.770
77	1.169	0.510	0.000	0.317	277	0.868	0.000	0.122	-0.838
78	1.055	0.826	0.000	3.308	278	1.294	0.000	0.521	2.134
79	0.880	1.018	0.000	-3.713	279	1.016	0.000	0.556	-0.879
80	0.818	0.853	0.000	-2.529	280	0.924	0.000	0.577	0.228
81	1.165	0.559	0.000	0.282	281	1.075	0.000	0.505	-1.835
82	0.649	0.805	0.000	2.070	282	1.096	0.000	0.388	-3.375
83	1.055	0.577	0.000	0.373	283	1.031	0.000	0.820	-2.122
84	0.830	0.718	0.000	-0.854	284	1.273	0.000	0.469	2.015
85	0.976	0.505	0.000	2.884	285	0.907	0.000	0.610	-2.802
86	0.971	0.586	0.000	0.173	286	1.518	0.000	0.606	-0.478
87	1.286	0.658	0.000	1.611	287	0.570	0.000	0.321	-3.968
88	0.951	0.588	0.000	-2.251	288	0.532	0.000	0.808	-1.562
89	0.971	0.668	0.000	2.597	289	0.840	0.000	0.477	-1.561
90	1.036	0.863	0.000	1.170	290	0.959	0.000	0.455	-0.702
91	0.837	0.488	0.000	2.517	291	1.123	0.000	0.554	-3.779
92	0.749	0.482	0.000	-0.096	292	1.146	0.000	0.670	2.804
93	1.238	0.224	0.000	2.284	293	1.154	0.000	0.477	1.169
94	0.996	0.069	0.000	-1.742	294	1.204	0.000	0.935	-0.838
95	0.843	0.459	0.000	1.604	295	1.388	0.000	0.505	-1.593
96	0.825	0.815	0.000	-1.182	296	0.735	0.000	0.623	-1.361
97	1.092	0.543	0.000	-2.786	297	0.772	0.000	0.596	1.561
98	0.915	0.338	0.000	0.668	298	0.937	0.000	0.542	0.514
99	1.332	0.375	0.000	-1.576	299	1.010	0.000	0.786	-1.195
100	0.811	0.811	0.000	-2.842	300	0.797	0.000	0.573	-3.684

Table A2

Item Discrimination and Difficulty Parameters

Under the Medium Group Factor Discrimination Condition (Condition B)

Item	a_1	a_2	a_3	*b*	*Item*	a_1	a_2	a_3	*b*
101	0.945	0.647	0.000	3.625	301	1.074	0.000	0.899	-3.851
102	1.031	0.666	0.000	1.035	302	1.268	0.000	0.820	-2.451
103	1.090	0.681	0.000	-3.624	303	0.913	0.000	0.368	0.145
104	0.944	0.544	0.000	-2.267	304	0.734	0.000	0.843	3.656
105	1.038	0.531	0.000	2.669	305	1.066	0.000	0.946	-2.031
106	1.244	0.501	0.000	-2.421	306	1.002	0.000	0.544	-2.647
107	1.186	0.604	0.000	2.818	307	1.038	0.000	0.043	-1.542
108	0.193	0.401	0.000	-1.792	308	1.503	0.000	0.799	-0.279
109	1.025	0.717	0.000	0.126	309	0.730	0.000	0.211	-2.260
110	1.095	0.684	0.000	-0.505	310	1.242	0.000	0.634	2.843
111	0.832	0.562	0.000	3.360	311	1.289	0.000	0.607	2.112
112	0.945	0.445	0.000	-0.064	312	1.186	0.000	0.818	-1.039
113	1.016	0.600	0.000	-0.553	313	0.863	0.000	0.732	2.405
114	1.103	0.660	0.000	-1.451	314	0.889	0.000	0.700	0.599
115	1.278	0.557	0.000	-2.276	315	0.876	0.000	0.907	-2.679
116	1.213	0.688	0.000	1.252	316	1.067	0.000	0.707	3.856
117	0.764	0.529	0.000	-1.486	317	1.032	0.000	0.814	-0.722
118	1.108	0.352	0.000	-3.573	318	1.132	0.000	0.784	-3.139
119	1.148	0.453	0.000	1.769	319	1.023	0.000	0.548	-2.550
120	0.898	0.610	0.000	-0.862	320	0.840	0.000	0.585	-1.368
121	0.870	0.872	0.000	2.407	321	1.167	0.000	0.601	2.193
122	1.021	0.393	0.000	2.793	322	1.008	0.000	0.649	1.409
123	1.230	0.692	0.000	-2.865	323	1.181	0.000	0.526	3.712
124	1.120	0.237	0.000	-3.629	324	1.268	0.000	0.726	-3.076
125	1.297	0.553	0.000	0.893	325	1.442	0.000	0.386	0.806
126	1.251	0.411	0.000	0.561	326	0.922	0.000	0.449	-3.241
127	1.235	0.491	0.000	3.274	327	1.332	0.000	0.394	-1.066
128	1.143	1.108	0.000	-2.895	328	1.064	0.000	0.581	0.659
129	0.852	0.632	0.000	0.072	329	1.183	0.000	0.530	-3.787
130	1.249	0.622	0.000	1.265	330	0.975	0.000	0.507	-1.579
131	0.967	0.749	0.000	2.765	331	0.846	0.000	0.852	-2.267
132	0.947	0.752	0.000	-2.943	332	1.086	0.000	0.549	0.070
133	0.937	0.807	0.000	2.908	333	1.200	0.000	0.559	1.893
134	0.938	0.410	0.000	2.740	334	0.933	0.000	0.679	2.061
135	0.968	0.399	0.000	2.353	335	0.978	0.000	0.591	-0.090
136	1.182	0.634	0.000	1.676	336	0.977	0.000	0.839	-2.252
137	1.193	0.665	0.000	0.505	337	1.216	0.000	0.569	2.990
138	1.128	0.877	0.000	2.615	338	1.156	0.000	0.725	-0.448
139	1.101	0.531	0.000	3.377	339	0.785	0.000	0.517	-2.565
140	1.116	0.642	0.000	-0.099	340	0.929	0.000	0.503	1.660
141	1.191	0.740	0.000	-3.459	341	1.036	0.000	0.676	-3.396
142	0.833	0.636	0.000	1.753	342	0.836	0.000	0.892	0.051
143	0.949	0.481	0.000	-3.384	343	1.219	0.000	0.766	2.644
144	1.094	0.593	0.000	-2.124	344	1.079	0.000	0.726	-3.131
145	0.941	0.441	0.000	3.391	345	0.744	0.000	0.687	1.988
146	1.224	0.478	0.000	-3.929	346	0.996	0.000	0.885	2.859
147	0.957	0.539	0.000	3.076	347	0.907	0.000	0.614	1.822
148	0.919	0.559	0.000	-1.489	348	1.022	0.000	0.767	3.110
149	0.623	0.417	0.000	-1.966	349	0.986	0.000	0.542	-1.723
150	1.054	0.475	0.000	1.668	350	1.142	0.000	0.172	0.809

Table A2
Item Discrimination and Difficulty Parameters
Under the Medium Group Factor Discrimination Condition (Condition B)

Item	a_1	a_2	a_3	*b*	*Item*	a_1	a_2	a_3	*b*
151	0.773	0.782	0.000	-3.816	351	1.216	0.000	0.773	3.934
152	0.945	0.549	0.000	-2.225	352	1.058	0.000	0.697	-2.929
153	0.945	0.777	0.000	2.930	353	0.992	0.000	0.422	-0.645
154	1.103	0.473	0.000	-1.724	354	1.016	0.000	0.040	-1.588
155	0.925	0.295	0.000	3.438	355	1.067	0.000	0.600	1.622
156	0.670	0.560	0.000	-2.960	356	0.633	0.000	0.639	2.081
157	1.310	0.524	0.000	3.003	357	0.780	0.000	0.706	-2.172
158	1.221	0.248	0.000	3.843	358	1.058	0.000	0.796	0.644
159	1.283	0.882	0.000	0.557	359	1.092	0.000	0.499	-0.390
160	0.889	0.607	0.000	2.279	360	0.834	0.000	0.319	-1.965
161	1.114	0.695	0.000	-3.038	361	0.587	0.000	0.505	-2.556
162	1.193	0.619	0.000	2.083	362	1.147	0.000	0.485	3.755
163	1.041	0.527	0.000	1.928	363	1.624	0.000	0.662	-2.406
164	0.852	0.622	0.000	0.632	364	0.759	0.000	0.338	0.681
165	1.231	0.843	0.000	2.188	365	1.126	0.000	0.357	-1.774
166	1.184	0.650	0.000	-3.839	366	0.704	0.000	0.358	-3.068
167	0.589	0.841	0.000	0.558	367	0.998	0.000	0.176	1.037
168	1.115	0.565	0.000	-0.740	368	0.904	0.000	0.452	1.466
169	0.886	0.729	0.000	2.223	369	0.699	0.000	0.638	-3.470
170	1.198	0.510	0.000	-2.276	370	0.889	0.000	0.760	1.505
171	0.802	0.616	0.000	1.108	371	0.871	0.000	0.587	-2.859
172	0.831	0.414	0.000	1.453	372	1.119	0.000	0.283	1.181
173	1.023	0.761	0.000	3.349	373	1.022	0.000	0.354	1.123
174	1.034	0.468	0.000	-2.043	374	1.055	0.000	0.272	2.560
175	1.080	0.737	0.000	2.881	375	0.733	0.000	0.584	-2.504
176	1.528	0.311	0.000	1.310	376	0.956	0.000	1.085	-2.988
177	0.957	0.675	0.000	0.211	377	1.020	0.000	0.678	-1.663
178	0.797	0.798	0.000	-3.721	378	1.172	0.000	0.889	-1.313
179	1.028	0.605	0.000	-2.680	379	0.703	0.000	0.708	-0.947
180	1.087	0.580	0.000	2.986	380	1.123	0.000	0.596	-2.494
181	1.159	0.349	0.000	2.169	381	1.196	0.000	0.868	2.865
182	1.079	0.267	0.000	0.890	382	0.905	0.000	0.397	-1.148
183	0.765	0.584	0.000	-0.858	383	1.443	0.000	0.361	-3.948
184	1.137	0.593	0.000	-0.574	384	1.248	0.000	0.837	1.106
185	0.664	0.511	0.000	0.920	385	0.964	0.000	0.566	0.800
186	0.881	0.793	0.000	1.273	386	0.347	0.000	1.135	2.685
187	1.208	0.743	0.000	3.374	387	1.093	0.000	0.713	-0.615
188	1.143	0.426	0.000	-2.873	388	0.967	0.000	0.467	-2.186
189	0.638	0.639	0.000	-0.643	389	1.357	0.000	0.760	-0.885
190	1.080	0.679	0.000	1.628	390	1.396	0.000	0.244	3.073
191	1.082	0.696	0.000	-2.978	391	0.978	0.000	0.598	3.379
192	0.928	0.786	0.000	1.816	392	1.174	0.000	0.573	-0.133
193	0.948	0.417	0.000	2.586	393	0.890	0.000	0.393	2.842
194	0.659	0.534	0.000	-1.963	394	0.955	0.000	0.758	0.622
195	0.898	0.317	0.000	-0.654	395	1.259	0.000	0.747	1.689
196	0.743	0.792	0.000	-2.146	396	0.812	0.000	0.438	0.943
197	0.930	0.466	0.000	-2.669	397	0.744	0.000	0.756	2.527
198	0.795	0.437	0.000	3.271	398	0.586	0.000	0.583	-2.182
199	0.933	0.598	0.000	3.952	399	1.453	0.000	1.083	-2.750
200	0.886	0.870	0.000	-2.522	400	0.851	0.000	0.857	0.353

Table A3

Item Discrimination and Difficulty Parameters

Under the High Group Factor Discrimination Condition (Condition C)

Item	a_1	a_2	a_3	b	*Item*	a_1	a_2	a_3	b
1	0.924	1.169	0.000	-0.612	201	0.624	0.000	0.749	0.107
2	0.826	0.521	0.000	0.935	202	0.844	0.000	0.713	2.774
3	1.087	0.502	0.000	-2.640	203	1.030	0.000	0.830	-3.264
4	0.715	0.619	0.000	2.928	204	0.932	0.000	0.554	1.043
5	0.845	0.785	0.000	3.138	205	1.265	0.000	0.790	3.644
6	1.207	0.480	0.000	-2.287	206	1.008	0.000	0.684	1.030
7	1.020	0.477	0.000	-2.760	207	0.906	0.000	0.803	2.091
8	1.025	0.985	0.000	-0.853	208	1.326	0.000	0.775	-1.863
9	0.473	0.714	0.000	-0.587	209	1.015	0.000	0.478	-1.262
10	0.933	0.696	0.000	3.745	210	0.961	0.000	0.547	3.395
11	1.139	0.471	0.000	-1.481	211	1.503	0.000	0.842	2.436
12	1.152	0.448	0.000	-1.435	212	1.040	0.000	1.337	-3.450
13	0.729	0.675	0.000	-1.119	213	1.222	0.000	0.590	0.489
14	0.857	0.861	0.000	-2.553	214	0.961	0.000	0.702	3.373
15	0.975	0.632	0.000	3.836	215	0.871	0.000	0.210	1.770
16	0.692	0.645	0.000	-3.028	216	1.067	0.000	0.630	-3.633
17	1.205	0.627	0.000	1.995	217	1.311	0.000	0.591	0.109
18	1.087	0.390	0.000	2.839	218	0.894	0.000	0.583	-3.551
19	1.181	0.915	0.000	3.671	219	0.717	0.000	0.535	3.147
20	0.698	0.817	0.000	-2.595	220	1.468	0.000	0.581	-3.283
21	0.723	0.549	0.000	0.987	221	0.982	0.000	0.524	3.740
22	0.977	0.665	0.000	3.875	222	0.840	0.000	1.221	1.537
23	1.097	0.478	0.000	-1.369	223	0.852	0.000	0.568	-3.030
24	0.943	0.485	0.000	-0.223	224	0.806	0.000	0.550	1.632
25	1.068	0.534	0.000	1.614	225	0.903	0.000	0.334	-2.465
26	1.198	0.901	0.000	-2.005	226	1.115	0.000	0.725	-1.144
27	0.685	0.563	0.000	-3.752	227	1.084	0.000	0.649	-1.378
28	0.796	1.059	0.000	0.183	228	1.025	0.000	1.096	1.588
29	0.975	0.740	0.000	-2.551	229	0.956	0.000	0.539	3.592
30	1.230	0.784	0.000	3.818	230	1.250	0.000	0.985	-0.954
31	0.715	0.724	0.000	-3.855	231	0.887	0.000	0.788	-1.448
32	0.996	0.614	0.000	-3.041	232	1.177	0.000	0.890	-1.880
33	1.155	0.849	0.000	0.411	233	0.848	0.000	1.249	-0.773
34	0.853	0.842	0.000	-2.602	234	1.242	0.000	1.060	-0.835
35	0.854	0.707	0.000	1.655	235	1.250	0.000	0.743	-3.704
36	0.888	0.644	0.000	-0.979	236	1.264	0.000	0.810	0.879
37	0.459	0.509	0.000	3.209	237	1.032	0.000	0.613	-0.539
38	0.953	0.614	0.000	-3.977	238	1.134	0.000	0.589	-3.640
39	1.010	0.936	0.000	2.663	239	1.128	0.000	0.906	-0.735
40	1.123	0.662	0.000	-0.963	240	1.252	0.000	0.823	0.588
41	1.118	0.761	0.000	1.995	241	1.271	0.000	0.662	2.388
42	0.966	0.807	0.000	-3.170	242	0.837	0.000	0.727	3.598
43	0.680	0.727	0.000	-3.122	243	1.033	0.000	0.426	-0.821
44	1.108	0.536	0.000	-2.607	244	1.132	0.000	0.691	-1.713
45	0.922	0.443	0.000	-0.941	245	0.761	0.000	0.190	-1.774
46	0.744	0.757	0.000	2.866	246	1.141	0.000	0.720	-1.979
47	1.228	0.729	0.000	0.563	247	1.027	0.000	0.639	1.299
48	0.472	1.096	0.000	-3.015	248	0.910	0.000	0.766	0.252
49	1.525	0.465	0.000	0.682	249	1.117	0.000	0.696	-0.218
50	0.989	0.411	0.000	2.161	250	0.978	0.000	0.526	3.578

Table A3
Item Discrimination and Difficulty Parameters
Under the High Group Factor Discrimination Condition (Condition C)

Item	a_1	a_2	a_3	b	*Item*	a_1	a_2	a_3	b
51	0.955	0.764	0.000	-2.326	251	0.987	0.000	0.540	1.892
52	1.074	0.622	0.000	1.812	252	0.755	0.000	0.684	1.441
53	1.086	0.630	0.000	3.288	253	1.327	0.000	0.563	1.908
54	0.800	0.664	0.000	0.102	254	0.858	0.000	0.385	2.109
55	0.862	0.597	0.000	-2.120	255	0.507	0.000	0.885	-2.799
56	0.919	0.884	0.000	1.842	256	1.157	0.000	0.886	-0.402
57	1.086	0.497	0.000	1.518	257	0.786	0.000	1.129	1.807
58	1.242	0.397	0.000	1.654	258	0.997	0.000	0.456	3.374
59	1.057	0.863	0.000	-1.540	259	1.067	0.000	0.516	-1.006
60	0.832	0.585	0.000	0.730	260	1.221	0.000	0.797	0.720
61	0.940	0.781	0.000	3.363	261	1.182	0.000	0.688	2.165
62	0.929	0.535	0.000	1.167	262	1.050	0.000	0.856	-3.546
63	0.955	0.701	0.000	-2.045	263	0.889	0.000	1.016	-2.690
64	0.700	0.658	0.000	-2.477	264	1.403	0.000	0.576	2.607
65	1.113	0.865	0.000	-1.849	265	1.110	0.000	0.614	1.085
66	0.849	0.571	0.000	3.927	266	0.996	0.000	0.304	-1.764
67	0.739	0.643	0.000	0.412	267	1.128	0.000	0.853	-2.963
68	0.788	0.910	0.000	3.108	268	0.715	0.000	0.387	-2.682
69	1.145	1.039	0.000	1.970	269	0.981	0.000	0.898	-1.242
70	0.993	0.824	0.000	0.719	270	0.837	0.000	0.517	1.676
71	1.280	0.760	0.000	-1.955	271	0.776	0.000	1.069	3.479
72	1.010	0.849	0.000	-1.051	272	1.118	0.000	0.735	1.377
73	1.129	0.802	0.000	-3.986	273	1.036	0.000	0.894	0.922
74	0.542	0.742	0.000	-2.664	274	1.055	0.000	0.727	2.666
75	0.814	0.369	0.000	1.496	275	1.104	0.000	0.662	-2.670
76	0.652	0.553	0.000	2.725	276	1.210	0.000	0.840	-0.407
77	1.069	0.895	0.000	-3.124	277	0.768	0.000	0.671	-1.977
78	1.511	0.757	0.000	-2.153	278	0.416	0.000	0.838	0.277
79	1.092	0.689	0.000	-2.262	279	1.151	0.000	0.560	1.963
80	0.716	0.520	0.000	2.128	280	0.864	0.000	1.058	3.046
81	0.763	0.824	0.000	-2.635	281	0.746	0.000	0.745	3.272
82	1.137	0.750	0.000	-1.566	282	1.297	0.000	0.739	2.944
83	0.693	0.589	0.000	0.321	283	0.780	0.000	0.506	-0.648
84	0.811	0.994	0.000	-3.133	284	1.106	0.000	1.069	2.904
85	0.948	0.598	0.000	-0.137	285	1.190	0.000	0.458	2.962
86	1.258	0.538	0.000	0.644	286	0.881	0.000	0.626	-2.115
87	0.961	0.334	0.000	1.411	287	0.979	0.000	0.817	-1.821
88	1.108	0.350	0.000	-1.962	288	0.958	0.000	0.416	-2.402
89	0.790	0.640	0.000	-3.610	289	1.162	0.000	0.849	-3.122
90	1.085	0.733	0.000	0.450	290	1.267	0.000	0.861	1.183
91	0.863	0.777	0.000	-0.516	291	0.684	0.000	0.545	-3.428
92	1.088	0.414	0.000	3.710	292	0.927	0.000	0.649	-1.041
93	0.982	0.803	0.000	-0.756	293	1.221	0.000	0.535	3.642
94	0.935	0.148	0.000	-1.667	294	0.824	0.000	0.801	3.104
95	0.903	1.081	0.000	-1.013	295	0.931	0.000	0.847	-0.867
96	0.969	0.485	0.000	-1.494	296	0.817	0.000	0.501	1.718
97	0.930	0.736	0.000	-0.955	297	0.620	0.000	0.738	0.305
98	0.912	0.577	0.000	3.535	298	1.327	0.000	0.682	2.132
99	0.478	1.174	0.000	-0.096	299	0.849	0.000	0.710	2.479
100	1.171	0.545	0.000	-2.793	300	0.916	0.000	0.578	-2.785

Table A3
Item Discrimination and Difficulty Parameters
Under the High Group Factor Discrimination Condition (Condition C)

Item	a_1	a_2	a_3	b	*Item*	a_1	a_2	a_3	b
101	1.063	0.554	0.000	0.323	301	0.761	0.000	0.480	-2.810
102	0.912	0.769	0.000	0.505	302	1.170	0.000	0.452	-2.652
103	1.077	0.834	0.000	-3.987	303	0.753	0.000	1.052	-3.702
104	0.655	0.726	0.000	-2.483	304	1.642	0.000	0.803	3.301
105	0.885	0.647	0.000	-1.229	305	1.344	0.000	0.504	-3.084
106	0.999	0.738	0.000	1.452	306	0.696	0.000	0.706	1.292
107	0.959	1.039	0.000	-3.095	307	1.289	0.000	0.782	3.816
108	1.023	0.681	0.000	3.747	308	1.041	0.000	0.400	-2.446
109	0.678	1.119	0.000	3.278	309	0.851	0.000	0.684	-3.885
110	1.151	0.531	0.000	-1.956	310	0.699	0.000	0.930	-0.159
111	0.935	0.627	0.000	-1.080	311	1.239	0.000	0.647	3.861
112	0.950	0.685	0.000	-1.339	312	1.320	0.000	0.932	-0.405
113	0.518	0.809	0.000	3.884	313	0.912	0.000	0.891	1.261
114	0.768	1.036	0.000	-0.588	314	1.160	0.000	0.570	1.658
115	1.308	0.628	0.000	-2.146	315	0.978	0.000	0.814	-0.983
116	0.734	0.915	0.000	-3.695	316	1.274	0.000	0.231	3.548
117	1.273	1.076	0.000	3.299	317	1.063	0.000	0.624	-0.004
118	0.984	0.679	0.000	2.610	318	1.108	0.000	0.678	1.322
119	0.928	1.280	0.000	-2.602	319	0.973	0.000	0.615	0.801
120	0.772	1.013	0.000	1.379	320	1.276	0.000	0.718	3.302
121	0.991	0.894	0.000	-3.852	321	1.139	0.000	0.582	-3.310
122	0.910	1.036	0.000	-1.671	322	1.107	0.000	0.761	0.835
123	1.342	0.925	0.000	-0.428	323	1.207	0.000	0.977	0.691
124	1.033	0.923	0.000	3.122	324	1.303	0.000	0.785	2.160
125	0.987	0.706	0.000	-0.252	325	0.864	0.000	0.494	-0.477
126	0.931	0.542	0.000	0.029	326	1.067	0.000	0.901	1.009
127	1.111	0.717	0.000	-2.399	327	0.989	0.000	0.700	0.729
128	1.048	0.590	0.000	0.258	328	1.300	0.000	0.786	2.320
129	0.814	0.525	0.000	1.443	329	0.984	0.000	0.449	3.515
130	0.994	0.792	0.000	-1.285	330	1.006	0.000	0.735	-3.332
131	0.739	0.951	0.000	-1.336	331	0.910	0.000	0.715	1.176
132	0.997	0.475	0.000	1.207	332	1.244	0.000	0.519	-3.959
133	1.370	0.279	0.000	1.859	333	1.123	0.000	0.213	3.114
134	0.706	0.536	0.000	-0.842	334	1.158	0.000	0.923	3.689
135	0.963	0.787	0.000	-2.141	335	1.092	0.000	1.212	-0.193
136	1.039	0.795	0.000	-0.493	336	0.833	0.000	0.927	-0.535
137	1.153	0.599	0.000	-2.642	337	1.107	0.000	1.302	2.007
138	0.964	0.604	0.000	-2.081	338	0.750	0.000	0.481	2.748
139	0.671	0.555	0.000	3.323	339	1.196	0.000	1.037	-3.517
140	1.178	0.727	0.000	-0.117	340	1.357	0.000	0.938	-2.626
141	1.000	0.437	0.000	2.124	341	0.870	0.000	0.587	1.952
142	0.928	0.612	0.000	3.798	342	0.878	0.000	1.057	-0.188
143	1.285	0.756	0.000	-2.873	343	1.078	0.000	0.507	3.024
144	1.212	0.645	0.000	1.359	344	1.092	0.000	0.703	2.296
145	0.950	0.835	0.000	3.828	345	1.381	0.000	0.366	-0.450
146	0.953	0.621	0.000	-2.653	346	1.009	0.000	1.359	0.595
147	0.855	0.694	0.000	-0.732	347	1.198	0.000	1.194	-3.952
148	1.141	0.952	0.000	1.450	348	0.921	0.000	0.605	1.294
149	1.112	0.363	0.000	-3.817	349	0.843	0.000	0.811	-1.337
150	0.917	0.605	0.000	-3.150	350	1.125	0.000	0.791	-1.633

Table A3
Item Discrimination and Difficulty Parameters
Under the High Group Factor Discrimination Condition (Condition C)

Item	a_1	a_2	a_3	b	*Item*	a_1	a_2	a_3	b
151	1.184	1.184	0.000	2.052	351	0.863	0.000	0.807	2.459
152	1.265	0.602	0.000	0.570	352	0.910	0.000	0.586	2.079
153	1.437	0.615	0.000	-0.784	353	1.065	0.000	0.576	0.891
154	1.117	0.830	0.000	-3.343	354	1.157	0.000	0.864	0.676
155	1.110	0.696	0.000	1.189	355	0.728	0.000	0.842	-2.406
156	1.195	0.768	0.000	1.290	356	1.224	0.000	1.163	-1.815
157	1.032	0.839	0.000	3.108	357	0.782	0.000	0.056	-1.284
158	1.045	0.731	0.000	-3.319	358	1.057	0.000	0.710	2.761
159	1.133	0.645	0.000	3.649	359	1.118	0.000	1.057	3.794
160	0.805	0.334	0.000	1.828	360	0.772	0.000	0.894	-0.804
161	0.871	0.603	0.000	-1.777	361	1.614	0.000	0.945	-0.157
162	1.328	0.473	0.000	2.746	362	0.949	0.000	0.826	-3.272
163	1.243	1.031	0.000	1.516	363	1.170	0.000	0.826	1.870
164	1.082	0.920	0.000	-1.767	364	0.722	0.000	0.731	-2.467
165	1.050	0.477	0.000	1.881	365	0.963	0.000	0.706	-0.260
166	0.919	0.788	0.000	0.418	366	1.233	0.000	0.885	1.971
167	1.186	0.428	0.000	-1.836	367	1.051	0.000	0.835	1.581
168	0.862	0.799	0.000	-1.547	368	1.284	0.000	0.686	2.626
169	1.107	0.728	0.000	-1.405	369	1.052	0.000	0.639	-3.927
170	1.281	0.555	0.000	-1.967	370	1.139	0.000	0.656	-0.676
171	1.130	0.204	0.000	2.678	371	1.108	0.000	0.574	-3.791
172	1.057	1.003	0.000	-0.498	372	0.972	0.000	0.712	0.074
173	0.871	0.585	0.000	-1.712	373	0.997	0.000	0.508	-0.316
174	0.752	0.726	0.000	2.946	374	0.927	0.000	1.038	2.347
175	1.053	0.622	0.000	-2.593	375	0.809	0.000	0.588	2.607
176	0.923	1.145	0.000	3.314	376	0.970	0.000	0.568	3.834
177	1.088	0.671	0.000	-3.676	377	1.020	0.000	0.638	2.130
178	1.180	0.671	0.000	0.111	378	1.067	0.000	0.537	-3.706
179	1.368	0.668	0.000	0.411	379	1.004	0.000	0.472	1.009
180	1.390	0.590	0.000	-1.139	380	0.544	0.000	0.523	3.194
181	0.973	0.200	0.000	0.789	381	1.140	0.000	0.664	0.707
182	0.920	0.583	0.000	-0.365	382	1.019	0.000	0.873	3.539
183	0.849	0.892	0.000	-2.203	383	0.211	0.000	0.542	0.387
184	0.877	0.821	0.000	3.930	384	1.146	0.000	0.316	-2.311
185	1.080	0.759	0.000	3.246	385	1.093	0.000	0.667	3.776
186	0.806	0.669	0.000	0.383	386	1.143	0.000	0.655	-3.197
187	0.392	0.646	0.000	-1.240	387	1.292	0.000	0.236	1.266
188	0.987	0.702	0.000	1.274	388	0.832	0.000	0.619	-3.319
189	1.053	0.771	0.000	-3.165	389	1.068	0.000	0.746	-0.015
190	1.023	0.666	0.000	-0.410	390	0.916	0.000	0.662	-2.779
191	0.846	0.448	0.000	0.978	391	1.168	0.000	0.832	0.912
192	0.857	0.885	0.000	-0.822	392	1.137	0.000	0.833	0.579
193	1.006	0.642	0.000	0.368	393	1.029	0.000	0.530	-2.773
194	1.168	0.182	0.000	-1.300	394	0.867	0.000	0.778	-1.755
195	0.927	0.517	0.000	3.748	395	1.195	0.000	0.646	-0.759
196	0.728	0.784	0.000	1.793	396	0.784	0.000	0.443	1.074
197	1.064	0.803	0.000	1.021	397	1.013	0.000	0.545	0.244
198	0.983	0.612	0.000	1.288	398	0.883	0.000	0.899	-2.544
199	1.137	0.824	0.000	0.937	399	1.273	0.000	0.759	0.520
200	1.040	0.594	0.000	2.136	400	1.283	0.000	0.571	-1.477

Table A4
Item Discrimination and Difficulty Parameters
Under the Low Group Factor Discrimination Parameters
(Condition D)

Item	a_1	a_2	a_3	a_4	a_5	b
1	1.192	0.254	0.000	0.000	0.000	-3.895
2	1.076	0.322	0.000	0.000	0.000	-2.271
3	0.869	0.547	0.000	0.000	0.000	-2.480
4	1.368	0.515	0.000	0.000	0.000	-0.760
5	1.133	0.270	0.000	0.000	0.000	-1.379
6	0.754	0.304	0.000	0.000	0.000	0.685
7	0.953	0.706	0.000	0.000	0.000	-1.794
8	1.164	0.062	0.000	0.000	0.000	1.930
9	0.881	0.242	0.000	0.000	0.000	1.590
10	1.021	0.350	0.000	0.000	0.000	0.799
11	0.832	0.680	0.000	0.000	0.000	2.063
12	1.252	0.709	0.000	0.000	0.000	1.856
13	0.987	0.640	0.000	0.000	0.000	-0.839
14	0.519	0.483	0.000	0.000	0.000	1.493
15	1.121	0.187	0.000	0.000	0.000	2.436
16	0.890	0.398	0.000	0.000	0.000	0.962
17	1.170	0.509	0.000	0.000	0.000	-1.786
18	1.340	0.604	0.000	0.000	0.000	-3.812
19	1.302	0.433	0.000	0.000	0.000	3.694
20	0.980	0.176	0.000	0.000	0.000	-1.404
21	1.142	0.736	0.000	0.000	0.000	-2.405
22	0.966	0.377	0.000	0.000	0.000	0.141
23	0.634	0.379	0.000	0.000	0.000	-3.601
24	1.320	0.566	0.000	0.000	0.000	1.900
25	0.432	0.765	0.000	0.000	0.000	-0.833
26	0.756	0.493	0.000	0.000	0.000	0.111
27	1.287	0.271	0.000	0.000	0.000	2.602
28	0.892	0.294	0.000	0.000	0.000	0.171
29	1.168	0.222	0.000	0.000	0.000	1.687
30	0.850	0.653	0.000	0.000	0.000	3.709
31	1.159	0.531	0.000	0.000	0.000	1.071
32	1.110	0.258	0.000	0.000	0.000	3.993
33	0.729	0.583	0.000	0.000	0.000	1.080
34	1.026	0.812	0.000	0.000	0.000	1.290
35	1.009	0.525	0.000	0.000	0.000	2.355
36	0.890	0.571	0.000	0.000	0.000	-3.247
37	1.263	0.532	0.000	0.000	0.000	3.134
38	1.266	0.427	0.000	0.000	0.000	0.079
39	0.912	0.840	0.000	0.000	0.000	-3.193
40	0.941	0.004	0.000	0.000	0.000	-0.386
41	1.145	0.758	0.000	0.000	0.000	1.117
42	1.152	0.653	0.000	0.000	0.000	-2.104
43	0.924	0.579	0.000	0.000	0.000	0.579
44	0.963	0.445	0.000	0.000	0.000	0.408
45	0.922	0.566	0.000	0.000	0.000	-2.387
46	1.174	0.645	0.000	0.000	0.000	-2.678
47	1.009	0.423	0.000	0.000	0.000	-1.208
48	1.090	0.352	0.000	0.000	0.000	2.495
49	1.360	0.461	0.000	0.000	0.000	-3.402

Table A4
Item Discrimination and Difficulty Parameters
Under the Low Group Factor Discrimination Parameters
(Condition D)

Item	a_1	a_2	a_3	a_4	a_5	b
50	0.817	0.782	0.000	0.000	0.000	3.651
51	0.836	0.229	0.000	0.000	0.000	3.310
52	0.859	0.332	0.000	0.000	0.000	-2.715
53	0.871	0.409	0.000	0.000	0.000	3.139
54	1.031	0.265	0.000	0.000	0.000	-2.089
55	0.892	0.526	0.000	0.000	0.000	-0.689
56	1.074	0.262	0.000	0.000	0.000	-0.954
57	1.054	0.136	0.000	0.000	0.000	1.731
58	1.202	0.663	0.000	0.000	0.000	-2.434
59	1.152	0.626	0.000	0.000	0.000	1.851
60	1.197	0.462	0.000	0.000	0.000	-0.790
61	1.157	0.451	0.000	0.000	0.000	-1.886
62	1.082	0.538	0.000	0.000	0.000	1.306
63	0.978	0.532	0.000	0.000	0.000	-1.611
64	0.835	0.436	0.000	0.000	0.000	-3.281
65	0.983	0.467	0.000	0.000	0.000	3.220
66	0.960	0.073	0.000	0.000	0.000	3.454
67	0.669	0.821	0.000	0.000	0.000	-2.633
68	1.193	0.506	0.000	0.000	0.000	3.686
69	0.924	0.743	0.000	0.000	0.000	-3.573
70	0.817	0.303	0.000	0.000	0.000	-2.250
71	1.020	0.474	0.000	0.000	0.000	-3.056
72	1.190	0.868	0.000	0.000	0.000	-0.224
73	1.064	0.751	0.000	0.000	0.000	0.683
74	0.801	0.563	0.000	0.000	0.000	0.005
75	1.295	0.339	0.000	0.000	0.000	-2.095
76	1.289	0.455	0.000	0.000	0.000	-1.352
77	0.903	0.622	0.000	0.000	0.000	0.069
78	0.913	0.533	0.000	0.000	0.000	0.025
79	0.923	0.558	0.000	0.000	0.000	0.255
80	1.240	0.136	0.000	0.000	0.000	3.944
81	0.784	0.573	0.000	0.000	0.000	-3.103
82	1.178	0.590	0.000	0.000	0.000	2.576
83	1.011	0.450	0.000	0.000	0.000	-0.536
84	0.933	0.690	0.000	0.000	0.000	-1.230
85	0.811	0.776	0.000	0.000	0.000	2.364
86	1.169	0.574	0.000	0.000	0.000	1.011
87	1.053	0.240	0.000	0.000	0.000	1.695
88	0.813	0.585	0.000	0.000	0.000	3.426
89	0.847	0.584	0.000	0.000	0.000	-2.118
90	1.116	0.629	0.000	0.000	0.000	-1.618
91	1.155	0.653	0.000	0.000	0.000	1.811
92	0.870	0.250	0.000	0.000	0.000	-3.760
93	1.401	0.286	0.000	0.000	0.000	-2.956
94	1.353	0.591	0.000	0.000	0.000	1.435
95	1.243	0.652	0.000	0.000	0.000	-0.520
96	0.773	0.314	0.000	0.000	0.000	3.770
97	1.045	0.868	0.000	0.000	0.000	-0.463
98	0.947	0.323	0.000	0.000	0.000	1.369

Table A4
Item Discrimination and Difficulty Parameters
Under the Low Group Factor Discrimination Parameters
(Condition D)

Item	a_1	a_2	a_3	a_4	a_5	b
99	1.194	0.278	0.000	0.000	0.000	-3.442
100	0.938	0.787	0.000	0.000	0.000	-2.943
101	1.191	0.000	0.612	0.000	0.000	-0.974
102	0.938	0.000	0.333	0.000	0.000	-2.993
103	1.118	0.000	0.328	0.000	0.000	-0.629
104	0.614	0.000	0.669	0.000	0.000	-0.525
105	0.881	0.000	0.615	0.000	0.000	3.580
106	0.912	0.000	0.169	0.000	0.000	-1.564
107	1.097	0.000	0.435	0.000	0.000	2.089
108	0.597	0.000	0.442	0.000	0.000	-3.897
109	0.824	0.000	0.546	0.000	0.000	-0.193
110	0.997	0.000	0.541	0.000	0.000	-0.512
111	0.923	0.000	0.245	0.000	0.000	2.311
112	1.347	0.000	0.305	0.000	0.000	3.789
113	0.988	0.000	0.785	0.000	0.000	-3.327
114	1.147	0.000	0.412	0.000	0.000	-0.673
115	1.157	0.000	0.800	0.000	0.000	-2.413
116	0.802	0.000	0.521	0.000	0.000	-2.592
117	0.847	0.000	0.686	0.000	0.000	-3.053
118	1.193	0.000	0.394	0.000	0.000	3.511
119	1.139	0.000	0.384	0.000	0.000	-3.553
120	1.140	0.000	0.414	0.000	0.000	3.795
121	1.002	0.000	0.321	0.000	0.000	-0.486
122	1.143	0.000	0.275	0.000	0.000	-1.276
123	1.045	0.000	0.246	0.000	0.000	2.975
124	1.052	0.000	0.420	0.000	0.000	-0.723
125	0.757	0.000	0.257	0.000	0.000	1.088
126	0.912	0.000	0.149	0.000	0.000	0.325
127	1.198	0.000	0.364	0.000	0.000	-1.147
128	1.023	0.000	0.580	0.000	0.000	2.861
129	1.409	0.000	0.862	0.000	0.000	2.196
130	0.835	0.000	0.549	0.000	0.000	-3.075
131	0.667	0.000	0.201	0.000	0.000	0.209
132	0.845	0.000	0.411	0.000	0.000	-0.691
133	1.288	0.000	0.591	0.000	0.000	-2.848
134	0.919	0.000	0.406	0.000	0.000	-0.910
135	0.837	0.000	0.113	0.000	0.000	-1.527
136	0.993	0.000	0.271	0.000	0.000	-3.857
137	1.081	0.000	0.388	0.000	0.000	-3.387
138	1.005	0.000	0.500	0.000	0.000	-2.774
139	0.807	0.000	0.585	0.000	0.000	2.116
140	0.899	0.000	0.717	0.000	0.000	-3.281
141	0.991	0.000	0.089	0.000	0.000	3.633
142	0.865	0.000	0.235	0.000	0.000	1.754
143	0.876	0.000	0.505	0.000	0.000	2.795
144	1.162	0.000	0.441	0.000	0.000	3.605
145	0.831	0.000	0.397	0.000	0.000	-2.305
146	0.914	0.000	0.664	0.000	0.000	1.276
147	0.690	0.000	0.492	0.000	0.000	2.193

Table A4
Item Discrimination and Difficulty Parameters
Under the Low Group Factor Discrimination Parameters
(Condition D)

Item	a_1	a_2	a_3	a_4	a_5	b
148	0.981	0.000	0.162	0.000	0.000	-1.061
149	1.247	0.000	0.453	0.000	0.000	3.662
150	1.030	0.000	0.311	0.000	0.000	-0.756
151	1.094	0.000	0.777	0.000	0.000	0.293
152	1.026	0.000	0.446	0.000	0.000	-1.623
153	1.293	0.000	0.436	0.000	0.000	-1.146
154	0.732	0.000	0.647	0.000	0.000	-2.565
155	1.017	0.000	0.585	0.000	0.000	2.992
156	1.103	0.000	0.674	0.000	0.000	0.668
157	1.333	0.000	0.667	0.000	0.000	1.693
158	1.252	0.000	0.452	0.000	0.000	1.702
159	0.762	0.000	0.717	0.000	0.000	1.839
160	1.552	0.000	0.643	0.000	0.000	-1.511
161	0.692	0.000	0.269	0.000	0.000	3.964
162	0.993	0.000	0.426	0.000	0.000	1.561
163	0.921	0.000	0.309	0.000	0.000	-2.379
164	1.189	0.000	0.482	0.000	0.000	1.856
165	0.922	0.000	0.942	0.000	0.000	-0.579
166	0.523	0.000	0.345	0.000	0.000	-2.073
167	1.186	0.000	0.402	0.000	0.000	-0.991
168	0.818	0.000	0.752	0.000	0.000	0.123
169	1.059	0.000	0.090	0.000	0.000	-3.037
170	0.763	0.000	0.899	0.000	0.000	-3.552
171	0.944	0.000	0.507	0.000	0.000	-3.579
172	1.126	0.000	0.400	0.000	0.000	-2.679
173	1.082	0.000	0.233	0.000	0.000	-1.102
174	1.072	0.000	0.493	0.000	0.000	2.907
175	0.860	0.000	0.801	0.000	0.000	-3.858
176	1.028	0.000	0.749	0.000	0.000	1.000
177	1.242	0.000	0.707	0.000	0.000	-1.461
178	1.346	0.000	0.093	0.000	0.000	-2.718
179	0.973	0.000	0.647	0.000	0.000	1.506
180	1.143	0.000	0.627	0.000	0.000	1.164
181	0.947	0.000	0.485	0.000	0.000	-2.619
182	1.435	0.000	0.687	0.000	0.000	-3.661
183	1.206	0.000	0.400	0.000	0.000	-0.929
184	1.222	0.000	0.221	0.000	0.000	-2.404
185	0.679	0.000	0.436	0.000	0.000	1.594
186	0.948	0.000	0.261	0.000	0.000	1.038
187	0.917	0.000	0.477	0.000	0.000	-2.088
188	1.153	0.000	0.340	0.000	0.000	0.976
189	1.121	0.000	0.737	0.000	0.000	-1.089
190	1.119	0.000	0.291	0.000	0.000	-1.857
191	1.085	0.000	0.329	0.000	0.000	2.789
192	0.951	0.000	0.904	0.000	0.000	3.098
193	0.981	0.000	0.485	0.000	0.000	-0.619
194	1.071	0.000	0.572	0.000	0.000	-1.955
195	0.798	0.000	0.434	0.000	0.000	0.437
196	1.155	0.000	0.503	0.000	0.000	-0.230

Table A4
Item Discrimination and Difficulty Parameters
Under the Low Group Factor Discrimination Parameters
(Condition D)

Item	a_1	a_2	a_3	a_4	a_5	b
197	1.057	0.000	0.746	0.000	0.000	3.650
198	0.851	0.000	0.108	0.000	0.000	2.191
199	0.754	0.000	0.592	0.000	0.000	-2.737
200	1.028	0.000	0.788	0.000	0.000	-1.434
201	1.454	0.000	0.000	0.462	0.000	-3.550
202	0.765	0.000	0.000	0.544	0.000	1.900
203	1.079	0.000	0.000	0.835	0.000	3.100
204	0.813	0.000	0.000	0.352	0.000	-3.794
205	1.212	0.000	0.000	0.756	0.000	-3.402
206	1.091	0.000	0.000	0.038	0.000	2.336
207	0.959	0.000	0.000	0.197	0.000	1.584
208	1.010	0.000	0.000	0.667	0.000	2.572
209	1.232	0.000	0.000	0.452	0.000	-1.224
210	1.012	0.000	0.000	0.570	0.000	-1.656
211	1.207	0.000	0.000	0.515	0.000	2.602
212	1.282	0.000	0.000	0.485	0.000	3.219
213	1.346	0.000	0.000	0.574	0.000	-0.133
214	1.234	0.000	0.000	0.689	0.000	-2.610
215	0.949	0.000	0.000	0.543	0.000	0.729
216	0.892	0.000	0.000	0.324	0.000	-1.406
217	0.886	0.000	0.000	0.043	0.000	-1.307
218	0.570	0.000	0.000	0.618	0.000	1.499
219	0.977	0.000	0.000	0.249	0.000	3.650
220	0.727	0.000	0.000	0.562	0.000	0.195
221	1.085	0.000	0.000	0.560	0.000	-1.673
222	0.825	0.000	0.000	0.073	0.000	3.988
223	0.866	0.000	0.000	0.813	0.000	-0.357
224	0.761	0.000	0.000	0.609	0.000	1.281
225	1.043	0.000	0.000	0.382	0.000	-2.607
226	0.919	0.000	0.000	0.912	0.000	-3.071
227	1.226	0.000	0.000	0.610	0.000	2.494
228	0.724	0.000	0.000	-0.293	0.000	-2.131
229	0.855	0.000	0.000	0.742	0.000	-3.712
230	0.798	0.000	0.000	0.347	0.000	-1.796
231	0.966	0.000	0.000	0.451	0.000	0.467
232	0.613	0.000	0.000	0.515	0.000	0.410
233	1.158	0.000	0.000	0.796	0.000	-1.874
234	1.338	0.000	0.000	0.370	0.000	-1.558
235	0.922	0.000	0.000	0.384	0.000	1.285
236	1.041	0.000	0.000	0.248	0.000	3.067
237	0.970	0.000	0.000	0.342	0.000	2.612
238	0.723	0.000	0.000	0.375	0.000	-0.090
239	1.110	0.000	0.000	0.197	0.000	2.914
240	0.944	0.000	0.000	0.475	0.000	-1.730
241	0.930	0.000	0.000	0.479	0.000	-0.227
242	1.222	0.000	0.000	0.046	0.000	0.496
243	1.081	0.000	0.000	0.530	0.000	-1.388
244	1.053	0.000	0.000	0.448	0.000	0.099
245	1.013	0.000	0.000	0.295	0.000	0.388

Table A4
Item Discrimination and Difficulty Parameters
Under the Low Group Factor Discrimination Parameters
(Condition D)

Item	a_1	a_2	a_3	a_4	a_5	b
246	0.762	0.000	0.000	0.549	0.000	3.457
247	1.319	0.000	0.000	0.424	0.000	-1.363
248	1.215	0.000	0.000	0.473	0.000	-3.814
249	1.209	0.000	0.000	0.622	0.000	0.197
250	0.810	0.000	0.000	0.728	0.000	-1.463
251	1.235	0.000	0.000	0.422	0.000	-1.784
252	0.722	0.000	0.000	0.543	0.000	3.109
253	0.819	0.000	0.000	0.482	0.000	-1.933
254	0.908	0.000	0.000	0.814	0.000	2.991
255	0.632	0.000	0.000	0.253	0.000	-3.202
256	1.068	0.000	0.000	0.338	0.000	-0.513
257	1.024	0.000	0.000	0.433	0.000	-1.307
258	1.196	0.000	0.000	0.546	0.000	-3.438
259	1.182	0.000	0.000	0.266	0.000	-0.225
260	1.236	0.000	0.000	0.405	0.000	3.470
261	1.101	0.000	0.000	0.661	0.000	2.930
262	1.226	0.000	0.000	0.527	0.000	0.783
263	1.339	0.000	0.000	0.549	0.000	0.375
264	1.244	0.000	0.000	0.593	0.000	-3.196
265	1.102	0.000	0.000	0.227	0.000	1.384
266	1.098	0.000	0.000	0.460	0.000	-3.331
267	0.955	0.000	0.000	0.640	0.000	-1.802
268	0.947	0.000	0.000	0.444	0.000	-1.578
269	1.122	0.000	0.000	0.438	0.000	-3.698
270	0.774	0.000	0.000	0.668	0.000	-1.096
271	1.157	0.000	0.000	0.270	0.000	2.115
272	0.835	0.000	0.000	0.943	0.000	0.336
273	1.318	0.000	0.000	0.431	0.000	-0.104
274	1.233	0.000	0.000	0.746	0.000	-3.951
275	1.199	0.000	0.000	0.790	0.000	3.752
276	0.964	0.000	0.000	0.628	0.000	-0.232
277	1.029	0.000	0.000	0.100	0.000	3.607
278	0.897	0.000	0.000	0.285	0.000	-1.572
279	1.100	0.000	0.000	0.229	0.000	-1.591
280	0.745	0.000	0.000	0.389	0.000	-0.401
281	0.934	0.000	0.000	-0.007	0.000	0.481
282	1.017	0.000	0.000	0.678	0.000	0.952
283	1.211	0.000	0.000	0.198	0.000	2.260
284	0.812	0.000	0.000	0.648	0.000	-3.106
285	1.024	0.000	0.000	0.405	0.000	1.787
286	0.927	0.000	0.000	0.598	0.000	1.526
287	0.897	0.000	0.000	0.383	0.000	0.002
288	0.977	0.000	0.000	0.437	0.000	-2.556
289	0.981	0.000	0.000	0.763	0.000	-0.829
290	1.275	0.000	0.000	0.610	0.000	2.203
291	0.967	0.000	0.000	0.440	0.000	-0.724
292	1.323	0.000	0.000	0.435	0.000	3.734
293	0.914	0.000	0.000	0.585	0.000	-1.565
294	1.037	0.000	0.000	0.101	0.000	-3.202

Table A4
Item Discrimination and Difficulty Parameters
Under the Low Group Factor Discrimination Parameters
(Condition D)

Item	a_1	a_2	a_3	a_4	a_5	b
295	1.313	0.000	0.000	0.632	0.000	-3.507
296	0.807	0.000	0.000	0.351	0.000	-2.013
297	1.157	0.000	0.000	0.438	0.000	-3.935
298	0.701	0.000	0.000	0.681	0.000	-1.410
299	0.943	0.000	0.000	0.787	0.000	3.056
300	1.161	0.000	0.000	0.559	0.000	2.779
301	0.970	0.000	0.000	0.000	0.187	1.374
302	0.974	0.000	0.000	0.000	0.238	-3.555
303	0.530	0.000	0.000	0.000	0.512	-1.036
304	1.009	0.000	0.000	0.000	0.419	-1.977
305	1.240	0.000	0.000	0.000	0.785	2.618
306	0.829	0.000	0.000	0.000	0.642	3.388
307	0.713	0.000	0.000	0.000	0.243	1.186
308	1.203	0.000	0.000	0.000	0.569	0.939
309	1.088	0.000	0.000	0.000	0.654	2.534
310	0.781	0.000	0.000	0.000	0.519	3.105
311	0.949	0.000	0.000	0.000	0.380	-0.075
312	0.856	0.000	0.000	0.000	0.475	-3.184
313	1.082	0.000	0.000	0.000	0.274	-1.276
314	1.166	0.000	0.000	0.000	0.437	-0.605
315	1.381	0.000	0.000	0.000	0.886	2.619
316	1.108	0.000	0.000	0.000	0.516	-0.097
317	0.978	0.000	0.000	0.000	0.639	1.225
318	0.938	0.000	0.000	0.000	0.709	3.116
319	1.171	0.000	0.000	0.000	0.771	0.455
320	1.072	0.000	0.000	0.000	0.333	3.166
321	1.134	0.000	0.000	0.000	0.266	2.637
322	0.938	0.000	0.000	0.000	0.211	2.548
323	1.157	0.000	0.000	0.000	0.165	2.756
324	0.960	0.000	0.000	0.000	0.659	2.389
325	0.897	0.000	0.000	0.000	0.377	1.884
326	0.714	0.000	0.000	0.000	0.458	-0.629
327	1.133	0.000	0.000	0.000	0.448	0.933
328	1.353	0.000	0.000	0.000	0.545	-0.232
329	1.106	0.000	0.000	0.000	0.615	3.911
330	0.954	0.000	0.000	0.000	0.811	-3.518
331	0.200	0.000	0.000	0.000	0.477	2.560
332	1.039	0.000	0.000	0.000	0.716	-2.928
333	0.837	0.000	0.000	0.000	0.455	2.046
334	0.934	0.000	0.000	0.000	0.523	-3.253
335	1.198	0.000	0.000	0.000	0.521	-2.055
336	0.961	0.000	0.000	0.000	0.419	-2.904
337	1.057	0.000	0.000	0.000	0.506	1.320
338	1.025	0.000	0.000	0.000	0.801	-0.143
339	0.932	0.000	0.000	0.000	0.830	2.759
340	0.758	0.000	0.000	0.000	0.722	-3.518
341	0.796	0.000	0.000	0.000	0.687	-2.727
342	1.034	0.000	0.000	0.000	0.411	1.614
343	1.244	0.000	0.000	0.000	0.413	-1.410

Table A4
Item Discrimination and Difficulty Parameters
Under the Low Group Factor Discrimination Parameters
(Condition D)

Item	a_1	a_2	a_3	a_4	a_5	b
344	1.120	0.000	0.000	0.000	0.641	-3.585
345	0.933	0.000	0.000	0.000	0.723	-1.445
346	0.929	0.000	0.000	0.000	0.115	2.532
347	1.418	0.000	0.000	0.000	0.511	-1.738
348	0.851	0.000	0.000	0.000	0.027	3.581
349	1.292	0.000	0.000	0.000	0.440	-3.509
350	0.984	0.000	0.000	0.000	0.494	-1.835
351	0.999	0.000	0.000	0.000	0.357	0.808
352	1.030	0.000	0.000	0.000	0.211	-2.398
353	0.842	0.000	0.000	0.000	0.620	3.906
354	0.832	0.000	0.000	0.000	0.700	0.430
355	0.982	0.000	0.000	0.000	0.907	0.517
356	1.243	0.000	0.000	0.000	0.067	-0.207
357	0.769	0.000	0.000	0.000	0.363	3.047
358	1.133	0.000	0.000	0.000	0.486	-1.079
359	0.639	0.000	0.000	0.000	0.308	1.656
360	0.633	0.000	0.000	0.000	0.378	-1.916
361	0.922	0.000	0.000	0.000	0.513	0.232
362	0.771	0.000	0.000	0.000	0.784	1.868
363	1.401	0.000	0.000	0.000	-0.004	-3.678
364	0.729	0.000	0.000	0.000	0.255	3.542
365	0.688	0.000	0.000	0.000	0.896	-2.056
366	0.958	0.000	0.000	0.000	1.056	0.177
367	0.568	0.000	0.000	0.000	0.349	0.441
368	0.998	0.000	0.000	0.000	0.192	1.572
369	1.056	0.000	0.000	0.000	0.467	3.024
370	1.036	0.000	0.000	0.000	0.290	-1.236
371	1.133	0.000	0.000	0.000	0.477	3.595
372	0.934	0.000	0.000	0.000	0.566	-3.554
373	1.102	0.000	0.000	0.000	0.499	3.404
374	0.606	0.000	0.000	0.000	0.538	-0.833
375	0.857	0.000	0.000	0.000	0.677	-0.166
376	0.992	0.000	0.000	0.000	0.444	2.715
377	1.244	0.000	0.000	0.000	0.682	-2.710
378	0.746	0.000	0.000	0.000	0.471	3.042
379	1.198	0.000	0.000	0.000	0.435	-1.912
380	0.696	0.000	0.000	0.000	0.267	-2.760
381	0.854	0.000	0.000	0.000	0.164	3.823
382	0.954	0.000	0.000	0.000	0.221	-2.220
383	1.015	0.000	0.000	0.000	0.604	-1.786
384	1.364	0.000	0.000	0.000	0.167	-2.404
385	0.496	0.000	0.000	0.000	0.459	-2.752
386	1.164	0.000	0.000	0.000	0.620	-2.681
387	1.131	0.000	0.000	0.000	0.417	-2.908
388	1.035	0.000	0.000	0.000	0.304	-2.237
389	1.257	0.000	0.000	0.000	0.475	-1.754
390	0.975	0.000	0.000	0.000	0.364	-3.930
391	1.201	0.000	0.000	0.000	0.536	0.396
392	0.930	0.000	0.000	0.000	0.487	0.110

Table A4
Item Discrimination and Difficulty Parameters
Under the Low Group Factor Discrimination Parameters
(Condition D)

Item	a_1	a_2	a_3	a_4	a_5	b
393	1.238	0.000	0.000	0.000	0.432	3.600
394	1.223	0.000	0.000	0.000	0.254	-2.690
395	1.216	0.000	0.000	0.000	0.401	-0.917
396	0.882	0.000	0.000	0.000	0.266	2.147
397	1.368	0.000	0.000	0.000	0.655	-1.552
398	0.816	0.000	0.000	0.000	0.386	2.409
399	1.114	0.000	0.000	0.000	0.591	3.555
400	0.710	0.000	0.000	0.000	0.448	3.221

Table A5
Item Discrimination and Difficulty Parameters
Under the Medium Group Factor Discrimination Condition
(Condition E)

Item	a_1	a_2	a_3	a_4	a_5	b
1	1.224	0.419	0.000	0.000	0.000	-3.681
2	0.868	0.489	0.000	0.000	0.000	-2.345
3	1.265	0.369	0.000	0.000	0.000	0.113
4	1.112	0.513	0.000	0.000	0.000	-3.339
5	0.997	0.573	0.000	0.000	0.000	-2.291
6	0.925	0.506	0.000	0.000	0.000	-1.800
7	1.209	0.560	0.000	0.000	0.000	-2.544
8	1.155	0.530	0.000	0.000	0.000	-1.826
9	0.993	0.861	0.000	0.000	0.000	2.489
10	0.896	0.361	0.000	0.000	0.000	2.012
11	1.055	0.436	0.000	0.000	0.000	-3.064
12	0.928	0.253	0.000	0.000	0.000	2.368
13	1.396	0.477	0.000	0.000	0.000	-3.787
14	1.271	0.868	0.000	0.000	0.000	-0.148
15	0.709	0.718	0.000	0.000	0.000	-2.023
16	0.645	0.774	0.000	0.000	0.000	-1.617
17	1.272	0.790	0.000	0.000	0.000	-1.372
18	0.904	0.731	0.000	0.000	0.000	-1.590
19	0.989	0.554	0.000	0.000	0.000	1.407
20	1.052	0.685	0.000	0.000	0.000	-2.188
21	0.981	0.664	0.000	0.000	0.000	-1.492
22	0.870	0.470	0.000	0.000	0.000	2.524
23	0.979	0.448	0.000	0.000	0.000	2.447
24	1.100	0.467	0.000	0.000	0.000	-2.850
25	0.737	0.400	0.000	0.000	0.000	3.287
26	1.308	0.794	0.000	0.000	0.000	1.349
27	1.184	0.471	0.000	0.000	0.000	3.390
28	1.147	0.395	0.000	0.000	0.000	-0.016
29	1.217	0.693	0.000	0.000	0.000	1.275
30	1.093	0.908	0.000	0.000	0.000	0.694
31	0.759	0.535	0.000	0.000	0.000	3.459
32	1.417	0.956	0.000	0.000	0.000	-0.976
33	0.797	0.610	0.000	0.000	0.000	2.096
34	0.888	0.696	0.000	0.000	0.000	2.537
35	0.838	0.294	0.000	0.000	0.000	-0.124
36	0.685	0.830	0.000	0.000	0.000	-1.138
37	0.786	0.484	0.000	0.000	0.000	-3.433
38	1.079	0.860	0.000	0.000	0.000	2.909
39	1.255	0.297	0.000	0.000	0.000	-0.337
40	0.870	0.693	0.000	0.000	0.000	1.626
41	1.261	0.777	0.000	0.000	0.000	-1.962
42	1.075	0.792	0.000	0.000	0.000	1.490
43	1.060	0.691	0.000	0.000	0.000	-3.782
44	0.961	0.499	0.000	0.000	0.000	-1.782
45	1.104	0.571	0.000	0.000	0.000	-1.729
46	0.967	0.912	0.000	0.000	0.000	0.410
47	0.933	0.674	0.000	0.000	0.000	-1.751
48	0.847	0.655	0.000	0.000	0.000	-1.004
49	0.981	0.265	0.000	0.000	0.000	1.990

Table A5
Item Discrimination and Difficulty Parameters
Under the Medium Group Factor Discrimination Condition
(Condition E)

Item	a_1	a_2	a_3	a_4	a_5	b
50	1.201	0.409	0.000	0.000	0.000	-0.949
51	1.035	0.900	0.000	0.000	0.000	2.249
52	0.861	0.372	0.000	0.000	0.000	-2.178
53	1.287	0.294	0.000	0.000	0.000	3.574
54	0.999	0.687	0.000	0.000	0.000	2.279
55	0.966	0.711	0.000	0.000	0.000	0.546
56	0.826	0.403	0.000	0.000	0.000	-2.431
57	1.055	0.662	0.000	0.000	0.000	2.443
58	0.943	0.600	0.000	0.000	0.000	2.887
59	0.822	0.247	0.000	0.000	0.000	1.051
60	1.394	0.305	0.000	0.000	0.000	-2.213
61	0.896	0.419	0.000	0.000	0.000	3.516
62	1.118	0.516	0.000	0.000	0.000	1.808
63	0.612	0.710	0.000	0.000	0.000	1.479
64	0.986	0.542	0.000	0.000	0.000	1.081
65	1.231	0.322	0.000	0.000	0.000	-2.559
66	0.703	1.039	0.000	0.000	0.000	1.984
67	1.337	0.621	0.000	0.000	0.000	2.709
68	1.035	0.306	0.000	0.000	0.000	1.144
69	0.820	0.517	0.000	0.000	0.000	1.449
70	0.847	0.596	0.000	0.000	0.000	-1.941
71	0.961	0.610	0.000	0.000	0.000	-0.559
72	1.006	0.941	0.000	0.000	0.000	2.339
73	0.687	0.821	0.000	0.000	0.000	2.026
74	1.118	0.687	0.000	0.000	0.000	-0.403
75	1.115	0.598	0.000	0.000	0.000	3.267
76	1.085	0.460	0.000	0.000	0.000	-1.806
77	0.867	0.626	0.000	0.000	0.000	2.529
78	1.195	0.142	0.000	0.000	0.000	2.689
79	1.229	0.932	0.000	0.000	0.000	-1.176
80	0.766	0.475	0.000	0.000	0.000	3.768
81	0.776	0.723	0.000	0.000	0.000	0.379
82	1.076	0.437	0.000	0.000	0.000	-2.800
83	0.740	0.205	0.000	0.000	0.000	1.236
84	0.739	0.639	0.000	0.000	0.000	-2.902
85	1.147	0.730	0.000	0.000	0.000	1.710
86	0.970	0.512	0.000	0.000	0.000	3.875
87	1.089	0.656	0.000	0.000	0.000	3.898
88	1.040	0.751	0.000	0.000	0.000	-2.300
89	0.940	0.753	0.000	0.000	0.000	2.700
90	1.135	0.772	0.000	0.000	0.000	-2.851
91	0.511	0.551	0.000	0.000	0.000	-0.044
92	1.113	0.489	0.000	0.000	0.000	-1.369
93	1.166	0.251	0.000	0.000	0.000	-2.869
94	0.912	1.128	0.000	0.000	0.000	-3.201
95	0.981	0.464	0.000	0.000	0.000	-3.547
96	0.890	0.630	0.000	0.000	0.000	-2.625
97	0.967	0.574	0.000	0.000	0.000	0.140
98	1.114	0.514	0.000	0.000	0.000	-1.776

Table A5
Item Discrimination and Difficulty Parameters
Under the Medium Group Factor Discrimination Condition
(Condition E)

Item	a_1	a_2	a_3	a_4	a_5	*b*
99	1.186	0.798	0.000	0.000	0.000	2.457
100	0.650	0.650	0.000	0.000	0.000	-2.991
101	1.127	0.000	0.897	0.000	0.000	-1.054
102	0.858	0.000	0.566	0.000	0.000	3.401
103	0.919	0.000	0.387	0.000	0.000	1.817
104	1.039	0.000	0.238	0.000	0.000	-1.179
105	0.920	0.000	0.722	0.000	0.000	-0.984
106	1.037	0.000	0.521	0.000	0.000	3.715
107	1.303	0.000	0.466	0.000	0.000	3.984
108	1.242	0.000	0.612	0.000	0.000	2.978
109	0.698	0.000	0.613	0.000	0.000	2.869
110	1.377	0.000	0.393	0.000	0.000	1.968
111	1.076	0.000	0.602	0.000	0.000	2.145
112	0.888	0.000	0.711	0.000	0.000	-3.319
113	0.600	0.000	0.546	0.000	0.000	-0.986
114	0.964	0.000	0.815	0.000	0.000	2.180
115	1.112	0.000	0.673	0.000	0.000	0.431
116	0.837	0.000	0.803	0.000	0.000	0.250
117	1.347	0.000	0.218	0.000	0.000	-0.307
118	1.283	0.000	0.635	0.000	0.000	-3.552
119	1.154	0.000	0.776	0.000	0.000	0.062
120	0.957	0.000	0.755	0.000	0.000	3.074
121	0.967	0.000	0.921	0.000	0.000	3.884
122	0.892	0.000	0.662	0.000	0.000	0.516
123	1.214	0.000	0.884	0.000	0.000	-2.723
124	1.212	0.000	0.417	0.000	0.000	3.903
125	1.112	0.000	0.322	0.000	0.000	-0.766
126	0.728	0.000	0.314	0.000	0.000	3.367
127	0.740	0.000	0.725	0.000	0.000	2.425
128	1.230	0.000	0.337	0.000	0.000	-3.981
129	0.947	0.000	0.259	0.000	0.000	-0.819
130	1.079	0.000	0.470	0.000	0.000	-1.710
131	1.035	0.000	0.673	0.000	0.000	0.195
132	0.992	0.000	0.653	0.000	0.000	-3.604
133	0.805	0.000	0.541	0.000	0.000	-2.317
134	1.315	0.000	0.875	0.000	0.000	3.275
135	0.977	0.000	0.328	0.000	0.000	-1.644
136	0.803	0.000	0.546	0.000	0.000	-0.060
137	0.804	0.000	0.734	0.000	0.000	2.742
138	1.138	0.000	0.900	0.000	0.000	-2.301
139	1.204	0.000	0.705	0.000	0.000	-2.973
140	0.745	0.000	0.470	0.000	0.000	2.107
141	1.324	0.000	0.856	0.000	0.000	2.129
142	1.248	0.000	0.390	0.000	0.000	1.316
143	0.955	0.000	0.401	0.000	0.000	-3.997
144	1.369	0.000	0.434	0.000	0.000	-0.625
145	0.839	0.000	0.694	0.000	0.000	0.605
146	1.363	0.000	0.588	0.000	0.000	-1.319
147	1.158	0.000	0.303	0.000	0.000	2.295

Table A5
Item Discrimination and Difficulty Parameters
Under the Medium Group Factor Discrimination Condition
(Condition E)

Item	a_1	a_2	a_3	a_4	a_5	b
148	0.941	0.000	0.190	0.000	0.000	1.877
149	1.340	0.000	0.288	0.000	0.000	2.651
150	1.215	0.000	0.651	0.000	0.000	1.772
151	0.795	0.000	0.561	0.000	0.000	0.756
152	0.817	0.000	0.937	0.000	0.000	-2.612
153	1.297	0.000	0.822	0.000	0.000	2.667
154	0.939	0.000	0.625	0.000	0.000	1.009
155	1.154	0.000	0.334	0.000	0.000	-1.183
156	0.953	0.000	0.442	0.000	0.000	-0.581
157	1.259	0.000	0.432	0.000	0.000	-3.417
158	0.913	0.000	0.535	0.000	0.000	2.350
159	0.975	0.000	0.611	0.000	0.000	2.548
160	1.092	0.000	0.494	0.000	0.000	2.288
161	0.511	0.000	0.491	0.000	0.000	2.726
162	0.722	0.000	0.308	0.000	0.000	3.718
163	1.205	0.000	0.647	0.000	0.000	0.177
164	0.791	0.000	0.555	0.000	0.000	-2.065
165	0.897	0.000	0.437	0.000	0.000	-2.807
166	1.241	0.000	0.542	0.000	0.000	-2.060
167	1.284	0.000	0.567	0.000	0.000	2.803
168	0.860	0.000	0.749	0.000	0.000	-0.746
169	0.922	0.000	0.917	0.000	0.000	-1.711
170	0.964	0.000	0.568	0.000	0.000	1.199
171	1.075	0.000	0.703	0.000	0.000	-3.976
172	1.155	0.000	0.866	0.000	0.000	3.769
173	0.909	0.000	0.868	0.000	0.000	-0.552
174	1.181	0.000	0.636	0.000	0.000	-2.139
175	0.428	0.000	0.823	0.000	0.000	-0.124
176	1.166	0.000	0.657	0.000	0.000	-0.112
177	1.242	0.000	0.865	0.000	0.000	-0.053
178	0.968	0.000	0.510	0.000	0.000	-0.367
179	1.126	0.000	0.480	0.000	0.000	3.587
180	1.253	0.000	0.743	0.000	0.000	-2.646
181	1.012	0.000	0.791	0.000	0.000	2.997
182	1.012	0.000	0.866	0.000	0.000	3.482
183	0.877	0.000	0.590	0.000	0.000	1.151
184	1.090	0.000	0.638	0.000	0.000	-2.587
185	1.016	0.000	0.763	0.000	0.000	-3.341
186	1.127	0.000	0.786	0.000	0.000	3.146
187	1.029	0.000	0.733	0.000	0.000	1.237
188	1.101	0.000	0.706	0.000	0.000	3.748
189	0.572	0.000	0.573	0.000	0.000	0.287
190	0.613	0.000	0.545	0.000	0.000	-1.952
191	1.111	0.000	0.296	0.000	0.000	2.143
192	0.910	0.000	0.589	0.000	0.000	3.206
193	0.746	0.000	0.558	0.000	0.000	-3.523
194	1.111	0.000	0.779	0.000	0.000	2.222
195	0.993	0.000	0.825	0.000	0.000	3.927
196	0.770	0.000	0.651	0.000	0.000	-1.669

Table A5
Item Discrimination and Difficulty Parameters
Under the Medium Group Factor Discrimination Condition
(Condition E)

Item	a_1	a_2	a_3	a_4	a_5	b
197	1.042	0.000	0.782	0.000	0.000	2.040
198	1.153	0.000	0.561	0.000	0.000	2.154
199	1.053	0.000	0.732	0.000	0.000	-1.317
200	0.910	0.000	0.645	0.000	0.000	-1.070
201	1.048	0.000	0.000	0.592	0.000	1.513
202	0.894	0.000	0.000	0.809	0.000	3.473
203	0.959	0.000	0.000	0.438	0.000	-3.564
204	1.050	0.000	0.000	0.670	0.000	3.162
205	0.793	0.000	0.000	0.504	0.000	-1.720
206	1.207	0.000	0.000	0.863	0.000	-1.128
207	1.102	0.000	0.000	0.798	0.000	-3.610
208	0.765	0.000	0.000	0.852	0.000	-2.901
209	0.798	0.000	0.000	0.704	0.000	-3.887
210	0.979	0.000	0.000	0.317	0.000	0.772
211	0.774	0.000	0.000	0.476	0.000	0.389
212	0.776	0.000	0.000	0.450	0.000	-1.740
213	0.951	0.000	0.000	0.704	0.000	-0.863
214	1.298	0.000	0.000	0.626	0.000	1.900
215	1.019	0.000	0.000	0.667	0.000	2.689
216	0.988	0.000	0.000	0.757	0.000	-3.355
217	1.362	0.000	0.000	0.328	0.000	1.218
218	0.903	0.000	0.000	0.518	0.000	1.255
219	1.222	0.000	0.000	0.251	0.000	1.001
220	1.226	0.000	0.000	0.665	0.000	2.034
221	1.123	0.000	0.000	0.665	0.000	3.879
222	0.962	0.000	0.000	0.724	0.000	-1.186
223	0.984	0.000	0.000	0.747	0.000	3.189
224	1.229	0.000	0.000	0.600	0.000	-2.751
225	1.052	0.000	0.000	0.377	0.000	-0.126
226	0.972	0.000	0.000	0.605	0.000	2.462
227	1.052	0.000	0.000	0.703	0.000	-2.611
228	1.342	0.000	0.000	0.429	0.000	-2.434
229	1.120	0.000	0.000	0.618	0.000	3.689
230	0.890	0.000	0.000	0.446	0.000	3.999
231	1.236	0.000	0.000	0.521	0.000	3.923
232	0.994	0.000	0.000	0.473	0.000	3.376
233	1.050	0.000	0.000	0.582	0.000	-3.774
234	0.751	0.000	0.000	0.874	0.000	3.509
235	1.004	0.000	0.000	0.563	0.000	-0.599
236	0.955	0.000	0.000	0.911	0.000	2.439
237	1.288	0.000	0.000	0.711	0.000	-2.148
238	1.188	0.000	0.000	0.571	0.000	1.567
239	0.971	0.000	0.000	0.546	0.000	-2.788
240	1.642	0.000	0.000	0.681	0.000	1.554
241	0.991	0.000	0.000	0.444	0.000	1.097
242	1.031	0.000	0.000	0.513	0.000	0.664
243	0.858	0.000	0.000	0.560	0.000	2.304
244	1.111	0.000	0.000	1.025	0.000	2.187
245	0.893	0.000	0.000	0.281	0.000	2.607

Table A5
Item Discrimination and Difficulty Parameters
Under the Medium Group Factor Discrimination Condition
(Condition E)

Item	a_1	a_2	a_3	a_4	a_5	b
246	1.375	0.000	0.000	0.931	0.000	2.138
247	1.123	0.000	0.000	0.982	0.000	1.672
248	0.932	0.000	0.000	1.107	0.000	2.418
249	1.150	0.000	0.000	0.565	0.000	-1.374
250	0.993	0.000	0.000	0.771	0.000	0.473
251	1.213	0.000	0.000	0.436	0.000	-0.664
252	0.886	0.000	0.000	1.089	0.000	3.721
253	0.779	0.000	0.000	0.847	0.000	-3.808
254	1.162	0.000	0.000	0.618	0.000	2.257
255	0.961	0.000	0.000	0.950	0.000	-2.854
256	0.943	0.000	0.000	0.512	0.000	1.406
257	1.001	0.000	0.000	0.768	0.000	-0.377
258	1.077	0.000	0.000	0.978	0.000	-0.223
259	0.930	0.000	0.000	0.549	0.000	-1.310
260	1.010	0.000	0.000	0.604	0.000	3.693
261	1.007	0.000	0.000	0.483	0.000	-3.958
262	0.706	0.000	0.000	0.806	0.000	3.431
263	0.937	0.000	0.000	0.656	0.000	-3.676
264	0.977	0.000	0.000	0.538	0.000	1.696
265	1.239	0.000	0.000	0.681	0.000	-1.385
266	1.028	0.000	0.000	0.805	0.000	1.573
267	0.880	0.000	0.000	0.563	0.000	-0.982
268	1.042	0.000	0.000	0.737	0.000	-2.592
269	1.245	0.000	0.000	0.601	0.000	1.882
270	1.243	0.000	0.000	0.539	0.000	3.581
271	1.142	0.000	0.000	0.746	0.000	2.424
272	1.156	0.000	0.000	0.354	0.000	-0.697
273	0.968	0.000	0.000	0.774	0.000	-2.059
274	1.091	0.000	0.000	0.347	0.000	-1.814
275	0.959	0.000	0.000	0.637	0.000	0.095
276	0.931	0.000	0.000	0.652	0.000	-2.941
277	0.980	0.000	0.000	0.879	0.000	-3.863
278	1.124	0.000	0.000	0.739	0.000	3.268
279	1.139	0.000	0.000	0.457	0.000	3.966
280	1.001	0.000	0.000	0.648	0.000	-3.648
281	0.996	0.000	0.000	0.587	0.000	1.634
282	1.122	0.000	0.000	0.671	0.000	-2.850
283	1.068	0.000	0.000	0.312	0.000	-1.879
284	1.055	0.000	0.000	0.603	0.000	0.492
285	1.232	0.000	0.000	0.826	0.000	-0.109
286	0.936	0.000	0.000	0.639	0.000	2.780
287	0.726	0.000	0.000	0.521	0.000	-1.236
288	1.076	0.000	0.000	0.625	0.000	2.503
289	1.293	0.000	0.000	0.553	0.000	-1.831
290	0.671	0.000	0.000	0.244	0.000	0.708
291	1.176	0.000	0.000	0.675	0.000	-1.218
292	0.928	0.000	0.000	0.301	0.000	3.923
293	0.884	0.000	0.000	0.409	0.000	-1.900
294	1.246	0.000	0.000	0.345	0.000	1.293

Table A5
Item Discrimination and Difficulty Parameters
Under the Medium Group Factor Discrimination Condition
(Condition E)

Item	a_1	a_2	a_3	a_4	a_5	b
295	1.170	0.000	0.000	0.515	0.000	-2.259
296	0.714	0.000	0.000	0.212	0.000	0.393
297	0.876	0.000	0.000	0.673	0.000	-0.954
298	1.236	0.000	0.000	0.511	0.000	-0.438
299	0.900	0.000	0.000	0.533	0.000	-1.637
300	0.994	0.000	0.000	0.703	0.000	1.612
301	0.916	0.000	0.000	0.000	0.543	1.664
302	1.287	0.000	0.000	0.000	0.491	3.385
303	1.188	0.000	0.000	0.000	1.018	1.589
304	0.957	0.000	0.000	0.000	0.288	1.653
305	1.002	0.000	0.000	0.000	0.432	3.844
306	1.027	0.000	0.000	0.000	0.342	1.380
307	1.182	0.000	0.000	0.000	0.486	-2.496
308	0.696	0.000	0.000	0.000	0.775	-3.146
309	1.084	0.000	0.000	0.000	0.433	2.422
310	0.958	0.000	0.000	0.000	0.738	-0.084
311	1.047	0.000	0.000	0.000	0.481	-0.035
312	1.046	0.000	0.000	0.000	0.785	3.664
313	0.792	0.000	0.000	0.000	0.594	-0.581
314	1.171	0.000	0.000	0.000	0.718	-0.438
315	0.922	0.000	0.000	0.000	0.453	2.398
316	0.520	0.000	0.000	0.000	0.367	2.759
317	0.929	0.000	0.000	0.000	0.825	-2.291
318	1.125	0.000	0.000	0.000	0.717	-3.955
319	1.041	0.000	0.000	0.000	0.501	-1.170
320	0.941	0.000	0.000	0.000	0.703	-2.108
321	0.862	0.000	0.000	0.000	0.616	-3.979
322	0.993	0.000	0.000	0.000	0.871	3.186
323	0.790	0.000	0.000	0.000	0.600	3.438
324	0.813	0.000	0.000	0.000	0.193	2.398
325	0.748	0.000	0.000	0.000	0.692	0.880
326	1.013	0.000	0.000	0.000	0.955	1.752
327	0.903	0.000	0.000	0.000	0.921	-1.533
328	0.913	0.000	0.000	0.000	0.744	-3.891
329	1.082	0.000	0.000	0.000	0.577	-3.961
330	1.004	0.000	0.000	0.000	0.875	2.116
331	0.792	0.000	0.000	0.000	0.815	1.956
332	0.749	0.000	0.000	0.000	0.649	-0.705
333	1.031	0.000	0.000	0.000	0.738	-1.168
334	0.952	0.000	0.000	0.000	0.826	2.053
335	1.174	0.000	0.000	0.000	0.973	-2.218
336	0.884	0.000	0.000	0.000	0.608	-0.560
337	1.103	0.000	0.000	0.000	0.472	0.955
338	1.205	0.000	0.000	0.000	0.773	-1.230
339	1.173	0.000	0.000	0.000	0.211	2.252
340	0.823	0.000	0.000	0.000	0.375	-0.391
341	1.200	0.000	0.000	0.000	0.589	3.671
342	0.939	0.000	0.000	0.000	0.298	1.050
343	0.904	0.000	0.000	0.000	0.926	3.515

Table A5

Item Discrimination and Difficulty Parameters
Under the Medium Group Factor Discrimination Condition
(Condition E)

Item	a_1	a_2	a_3	a_4	a_5	b
344	1.034	0.000	0.000	0.000	0.677	0.063
345	0.905	0.000	0.000	0.000	0.535	1.863
346	1.029	0.000	0.000	0.000	0.650	-0.324
347	0.663	0.000	0.000	0.000	0.703	0.454
348	0.733	0.000	0.000	0.000	0.590	-3.996
349	1.039	0.000	0.000	0.000	0.769	-1.785
350	1.181	0.000	0.000	0.000	0.840	-1.097
351	0.737	0.000	0.000	0.000	0.586	1.501
352	1.004	0.000	0.000	0.000	0.756	-1.044
353	1.558	0.000	0.000	0.000	0.622	0.274
354	1.032	0.000	0.000	0.000	0.887	-0.395
355	1.200	0.000	0.000	0.000	0.508	2.208
356	0.991	0.000	0.000	0.000	0.645	-2.473
357	1.334	0.000	0.000	0.000	0.618	3.548
358	0.881	0.000	0.000	0.000	0.411	-3.956
359	1.036	0.000	0.000	0.000	0.691	-2.751
360	1.137	0.000	0.000	0.000	0.423	0.605
361	0.882	0.000	0.000	0.000	0.648	1.102
362	1.270	0.000	0.000	0.000	0.587	3.432
363	1.040	0.000	0.000	0.000	0.522	-0.206
364	1.106	0.000	0.000	0.000	0.730	-1.110
365	0.685	0.000	0.000	0.000	0.105	-2.386
366	1.221	0.000	0.000	0.000	0.842	-0.431
367	0.756	0.000	0.000	0.000	0.394	3.907
368	0.869	0.000	0.000	0.000	0.491	0.354
369	0.613	0.000	0.000	0.000	0.683	-1.232
370	0.876	0.000	0.000	0.000	0.805	-0.665
371	1.059	0.000	0.000	0.000	0.664	0.421
372	0.976	0.000	0.000	0.000	0.728	-3.083
373	1.206	0.000	0.000	0.000	0.743	0.985
374	1.244	0.000	0.000	0.000	0.774	2.012
375	0.835	0.000	0.000	0.000	0.597	2.909
376	1.308	0.000	0.000	0.000	0.677	-2.555
377	1.017	0.000	0.000	0.000	0.415	2.211
378	0.690	0.000	0.000	0.000	0.257	3.185
379	1.330	0.000	0.000	0.000	0.406	-0.701
380	1.171	0.000	0.000	0.000	0.825	1.611
381	1.009	0.000	0.000	0.000	0.757	2.407
382	1.250	0.000	0.000	0.000	0.673	-1.535
383	0.848	0.000	0.000	0.000	0.542	-3.489
384	1.038	0.000	0.000	0.000	0.691	-3.924
385	1.053	0.000	0.000	0.000	0.858	-2.422
386	0.993	0.000	0.000	0.000	0.645	0.893
387	1.334	0.000	0.000	0.000	0.523	-1.623
388	1.115	0.000	0.000	0.000	0.524	0.569
389	0.696	0.000	0.000	0.000	0.690	-3.043
390	0.740	0.000	0.000	0.000	0.716	-3.837
391	0.744	0.000	0.000	0.000	0.619	-2.404
392	0.856	0.000	0.000	0.000	0.670	-2.172

Table A5
Item Discrimination and Difficulty Parameters
Under the Medium Group Factor Discrimination Condition
(Condition E)

Item	a_1	a_2	a_3	a_4	a_5	b
393	1.190	0.000	0.000	0.000	0.896	-3.988
394	1.250	0.000	0.000	0.000	0.188	-2.474
395	1.123	0.000	0.000	0.000	0.544	-3.296
396	1.370	0.000	0.000	0.000	0.880	3.946
397	0.874	0.000	0.000	0.000	1.017	3.077
398	0.794	0.000	0.000	0.000	0.381	-2.029
399	1.288	0.000	0.000	0.000	0.521	0.140
400	0.873	0.000	0.000	0.000	0.447	2.298

Table A6
Item Discrimination and Difficulty Parameters
Under the High Group Factor Discrimination Condition
(Condition F)

Item	a_1	a_2	a_3	a_4	a_5	b
1	0.799	0.319	0.000	0.000	0.000	2.944
2	1.219	0.855	0.000	0.000	0.000	3.158
3	1.362	0.595	0.000	0.000	0.000	3.159
4	0.716	0.471	0.000	0.000	0.000	-1.408
5	1.004	0.746	0.000	0.000	0.000	0.715
6	1.245	0.890	0.000	0.000	0.000	-2.980
7	1.032	0.511	0.000	0.000	0.000	-3.362
8	0.636	0.587	0.000	0.000	0.000	1.431
9	1.319	0.673	0.000	0.000	0.000	-2.298
10	0.906	0.832	0.000	0.000	0.000	-2.302
11	1.321	0.558	0.000	0.000	0.000	-1.721
12	1.258	0.679	0.000	0.000	0.000	-1.257
13	1.259	0.664	0.000	0.000	0.000	-2.446
14	1.057	0.682	0.000	0.000	0.000	-3.444
15	1.062	0.629	0.000	0.000	0.000	1.263
16	1.074	0.686	0.000	0.000	0.000	-3.619
17	1.336	0.929	0.000	0.000	0.000	-0.592
18	1.109	0.805	0.000	0.000	0.000	-3.823
19	0.865	0.772	0.000	0.000	0.000	1.004
20	0.882	0.671	0.000	0.000	0.000	-0.386
21	1.018	0.823	0.000	0.000	0.000	3.284
22	1.102	0.902	0.000	0.000	0.000	3.756
23	0.776	0.509	0.000	0.000	0.000	0.408
24	1.152	0.748	0.000	0.000	0.000	1.358
25	1.113	0.939	0.000	0.000	0.000	2.548
26	1.066	0.560	0.000	0.000	0.000	1.127
27	0.953	0.620	0.000	0.000	0.000	1.563
28	1.337	0.711	0.000	0.000	0.000	3.613
29	0.982	0.750	0.000	0.000	0.000	0.283
30	0.663	0.226	0.000	0.000	0.000	0.763
31	1.157	0.657	0.000	0.000	0.000	3.388
32	1.200	0.661	0.000	0.000	0.000	-0.706
33	1.137	0.944	0.000	0.000	0.000	-1.978
34	1.243	0.754	0.000	0.000	0.000	-0.198
35	0.747	0.826	0.000	0.000	0.000	-3.176
36	1.013	0.782	0.000	0.000	0.000	-0.278
37	0.994	0.824	0.000	0.000	0.000	-3.946
38	1.214	0.612	0.000	0.000	0.000	-0.390
39	0.820	0.636	0.000	0.000	0.000	-0.465
40	0.811	0.433	0.000	0.000	0.000	2.264
41	1.016	0.804	0.000	0.000	0.000	0.149
42	1.308	0.942	0.000	0.000	0.000	3.650
43	0.938	0.330	0.000	0.000	0.000	1.655
44	0.984	0.543	0.000	0.000	0.000	0.076
45	1.057	0.590	0.000	0.000	0.000	-2.880
46	0.948	0.472	0.000	0.000	0.000	-0.688
47	1.249	0.366	0.000	0.000	0.000	1.838
48	0.918	0.337	0.000	0.000	0.000	1.031
49	1.071	0.838	0.000	0.000	0.000	0.002

Table A6
Item Discrimination and Difficulty Parameters
Under the High Group Factor Discrimination Condition
(Condition F)

Item	a_1	a_2	a_3	a_4	a_5	b
50	0.804	0.687	0.000	0.000	0.000	-1.143
51	0.621	0.683	0.000	0.000	0.000	0.863
52	0.715	1.102	0.000	0.000	0.000	-2.417
53	0.850	1.067	0.000	0.000	0.000	3.599
54	0.726	0.954	0.000	0.000	0.000	-2.629
55	0.750	0.311	0.000	0.000	0.000	2.540
56	0.797	0.910	0.000	0.000	0.000	-1.816
57	0.659	0.744	0.000	0.000	0.000	-0.447
58	0.651	0.810	0.000	0.000	0.000	-0.019
59	0.627	0.879	0.000	0.000	0.000	1.598
60	1.030	0.790	0.000	0.000	0.000	0.706
61	0.813	0.693	0.000	0.000	0.000	-3.596
62	0.990	1.058	0.000	0.000	0.000	-2.744
63	0.533	0.645	0.000	0.000	0.000	2.471
64	0.866	0.807	0.000	0.000	0.000	0.936
65	1.400	0.754	0.000	0.000	0.000	-1.116
66	0.947	0.439	0.000	0.000	0.000	1.045
67	1.201	0.628	0.000	0.000	0.000	3.198
68	1.216	0.795	0.000	0.000	0.000	-0.808
69	0.767	0.905	0.000	0.000	0.000	-3.766
70	1.087	0.822	0.000	0.000	0.000	0.405
71	1.354	0.683	0.000	0.000	0.000	-0.375
72	0.790	0.594	0.000	0.000	0.000	0.873
73	1.075	0.521	0.000	0.000	0.000	2.730
74	1.016	0.663	0.000	0.000	0.000	0.645
75	1.446	0.916	0.000	0.000	0.000	0.847
76	0.819	0.796	0.000	0.000	0.000	-1.595
77	0.845	0.848	0.000	0.000	0.000	-0.663
78	1.064	0.991	0.000	0.000	0.000	-2.788
79	0.947	1.174	0.000	0.000	0.000	3.698
80	1.100	0.755	0.000	0.000	0.000	1.001
81	0.866	0.334	0.000	0.000	0.000	3.473
82	1.112	1.166	0.000	0.000	0.000	3.174
83	1.329	0.537	0.000	0.000	0.000	-2.865
84	1.171	0.543	0.000	0.000	0.000	-0.267
85	1.120	0.906	0.000	0.000	0.000	-2.345
86	0.923	0.426	0.000	0.000	0.000	-1.705
87	0.589	0.804	0.000	0.000	0.000	-0.257
88	1.108	0.529	0.000	0.000	0.000	1.984
89	1.256	0.454	0.000	0.000	0.000	-0.031
90	0.928	0.933	0.000	0.000	0.000	3.060
91	1.191	0.777	0.000	0.000	0.000	2.710
92	0.762	0.822	0.000	0.000	0.000	2.373
93	1.360	0.827	0.000	0.000	0.000	3.015
94	1.355	0.720	0.000	0.000	0.000	-2.422
95	1.028	0.625	0.000	0.000	0.000	1.557
96	0.960	0.930	0.000	0.000	0.000	-1.683
97	0.973	0.999	0.000	0.000	0.000	0.501
98	0.864	1.035	0.000	0.000	0.000	2.698

Table A6
Item Discrimination and Difficulty Parameters
Under the High Group Factor Discrimination Condition
(Condition F)

Item	a_1	a_2	a_3	a_4	a_5	b
99	1.089	0.755	0.000	0.000	0.000	2.684
100	0.620	0.331	0.000	0.000	0.000	1.454
101	0.649	0.000	0.911	0.000	0.000	-2.596
102	0.786	0.000	0.803	0.000	0.000	-2.943
103	1.140	0.000	0.760	0.000	0.000	2.998
104	1.015	0.000	0.901	0.000	0.000	0.594
105	1.034	0.000	0.726	0.000	0.000	3.217
106	0.711	0.000	0.424	0.000	0.000	-2.282
107	1.032	0.000	0.876	0.000	0.000	-0.598
108	1.224	0.000	0.977	0.000	0.000	1.191
109	0.970	0.000	0.452	0.000	0.000	3.965
110	0.622	0.000	0.569	0.000	0.000	-3.219
111	0.968	0.000	0.464	0.000	0.000	-2.300
112	1.018	0.000	0.728	0.000	0.000	-0.702
113	0.945	0.000	0.906	0.000	0.000	-0.683
114	0.905	0.000	1.032	0.000	0.000	-3.811
115	1.318	0.000	0.269	0.000	0.000	0.353
116	0.843	0.000	0.023	0.000	0.000	2.380
117	0.698	0.000	1.019	0.000	0.000	1.291
118	0.884	0.000	0.511	0.000	0.000	-1.592
119	0.894	0.000	0.415	0.000	0.000	-1.921
120	1.044	0.000	0.760	0.000	0.000	-1.952
121	1.093	0.000	0.599	0.000	0.000	2.688
122	0.843	0.000	0.533	0.000	0.000	-1.257
123	1.066	0.000	0.480	0.000	0.000	0.346
124	1.179	0.000	1.053	0.000	0.000	1.832
125	1.301	0.000	0.466	0.000	0.000	0.526
126	0.730	0.000	-0.109	0.000	0.000	3.578
127	0.944	0.000	0.669	0.000	0.000	-0.428
128	1.054	0.000	0.521	0.000	0.000	0.784
129	1.143	0.000	0.974	0.000	0.000	2.311
130	0.941	0.000	0.640	0.000	0.000	3.411
131	0.967	0.000	0.444	0.000	0.000	-3.110
132	0.756	0.000	0.489	0.000	0.000	1.893
133	0.662	0.000	0.713	0.000	0.000	-3.430
134	1.214	0.000	0.701	0.000	0.000	-0.331
135	1.010	0.000	0.926	0.000	0.000	0.865
136	0.949	0.000	0.706	0.000	0.000	-2.776
137	1.095	0.000	0.676	0.000	0.000	0.325
138	0.904	0.000	0.549	0.000	0.000	-0.129
139	0.956	0.000	0.730	0.000	0.000	-1.310
140	0.897	0.000	0.493	0.000	0.000	1.205
141	0.901	0.000	0.634	0.000	0.000	-1.434
142	1.211	0.000	0.433	0.000	0.000	1.463
143	0.951	0.000	0.460	0.000	0.000	-2.775
144	0.908	0.000	0.470	0.000	0.000	-1.521
145	1.214	0.000	0.739	0.000	0.000	-0.971
146	0.699	0.000	0.926	0.000	0.000	-1.815
147	0.738	0.000	0.618	0.000	0.000	-2.379

Table A6
Item Discrimination and Difficulty Parameters
Under the High Group Factor Discrimination Condition
(Condition F)

Item	a_1	a_2	a_3	a_4	a_5	b
148	0.887	0.000	0.654	0.000	0.000	-3.149
149	0.779	0.000	0.927	0.000	0.000	3.993
150	1.467	0.000	0.638	0.000	0.000	0.112
151	1.100	0.000	0.892	0.000	0.000	2.411
152	0.685	0.000	1.072	0.000	0.000	2.445
153	0.915	0.000	0.497	0.000	0.000	0.914
154	0.983	0.000	0.445	0.000	0.000	3.626
155	1.154	0.000	0.754	0.000	0.000	-1.801
156	1.097	0.000	0.982	0.000	0.000	0.497
157	0.922	0.000	0.548	0.000	0.000	-1.221
158	0.970	0.000	0.781	0.000	0.000	-0.658
159	0.767	0.000	0.346	0.000	0.000	-0.413
160	0.921	0.000	0.754	0.000	0.000	2.174
161	1.228	0.000	0.845	0.000	0.000	3.910
162	1.642	0.000	0.543	0.000	0.000	-0.493
163	0.992	0.000	0.868	0.000	0.000	-1.113
164	0.927	0.000	1.012	0.000	0.000	1.453
165	0.844	0.000	0.824	0.000	0.000	0.214
166	0.767	0.000	0.362	0.000	0.000	-0.797
167	1.050	0.000	0.699	0.000	0.000	2.963
168	0.844	0.000	0.836	0.000	0.000	-1.735
169	1.010	0.000	0.612	0.000	0.000	-0.067
170	0.790	0.000	0.639	0.000	0.000	-3.475
171	1.239	0.000	0.811	0.000	0.000	0.839
172	1.083	0.000	0.388	0.000	0.000	-0.886
173	0.764	0.000	0.263	0.000	0.000	-1.967
174	0.953	0.000	0.817	0.000	0.000	-3.766
175	1.060	0.000	0.551	0.000	0.000	1.798
176	0.779	0.000	0.498	0.000	0.000	-3.163
177	0.747	0.000	0.782	0.000	0.000	0.549
178	1.142	0.000	0.467	0.000	0.000	-2.790
179	0.985	0.000	0.604	0.000	0.000	2.532
180	1.153	0.000	0.634	0.000	0.000	0.653
181	0.931	0.000	0.624	0.000	0.000	-1.199
182	0.867	0.000	0.374	0.000	0.000	-3.934
183	1.368	0.000	0.940	0.000	0.000	-0.653
184	1.215	0.000	0.703	0.000	0.000	-2.939
185	1.128	0.000	0.788	0.000	0.000	2.945
186	1.162	0.000	0.515	0.000	0.000	3.982
187	1.208	0.000	0.773	0.000	0.000	3.643
188	0.802	0.000	0.419	0.000	0.000	-2.790
189	1.134	0.000	0.975	0.000	0.000	-3.136
190	0.805	0.000	0.252	0.000	0.000	0.239
191	0.958	0.000	0.452	0.000	0.000	2.476
192	1.420	0.000	0.865	0.000	0.000	-2.872
193	1.434	0.000	0.569	0.000	0.000	1.908
194	1.058	0.000	0.572	0.000	0.000	-0.184
195	1.147	0.000	0.442	0.000	0.000	1.098
196	1.241	0.000	0.593	0.000	0.000	3.446

Table A6
Item Discrimination and Difficulty Parameters
Under the High Group Factor Discrimination Condition
(Condition F)

Item	a_1	a_2	a_3	a_4	a_5	*b*
197	0.978	0.000	0.622	0.000	0.000	-0.108
198	0.873	0.000	0.727	0.000	0.000	-1.386
199	0.723	0.000	0.906	0.000	0.000	3.944
200	1.455	0.000	0.743	0.000	0.000	1.555
201	0.977	0.000	0.000	0.781	0.000	0.933
202	0.669	0.000	0.000	0.707	0.000	-2.617
203	1.111	0.000	0.000	1.196	0.000	-2.415
204	0.525	0.000	0.000	0.944	0.000	-2.798
205	0.896	0.000	0.000	0.443	0.000	-0.364
206	1.172	0.000	0.000	1.032	0.000	0.199
207	0.893	0.000	0.000	0.468	0.000	-2.088
208	0.646	0.000	0.000	0.767	0.000	-1.456
209	1.541	0.000	0.000	0.660	0.000	-0.237
210	0.724	0.000	0.000	0.399	0.000	0.931
211	0.534	0.000	0.000	0.450	0.000	2.993
212	0.830	0.000	0.000	0.547	0.000	0.000
213	1.324	0.000	0.000	0.472	0.000	-1.198
214	0.982	0.000	0.000	0.856	0.000	3.520
215	0.994	0.000	0.000	0.753	0.000	-1.264
216	0.771	0.000	0.000	0.752	0.000	-1.486
217	0.962	0.000	0.000	0.960	0.000	3.136
218	0.971	0.000	0.000	0.571	0.000	0.699
219	0.681	0.000	0.000	1.226	0.000	2.641
220	0.949	0.000	0.000	1.283	0.000	1.046
221	0.940	0.000	0.000	0.563	0.000	-0.979
222	1.190	0.000	0.000	0.784	0.000	-3.340
223	1.042	0.000	0.000	0.813	0.000	-1.684
224	1.022	0.000	0.000	0.783	0.000	-2.390
225	0.987	0.000	0.000	0.910	0.000	0.317
226	0.747	0.000	0.000	0.982	0.000	-2.692
227	1.163	0.000	0.000	0.880	0.000	-0.019
228	1.047	0.000	0.000	0.967	0.000	2.094
229	0.878	0.000	0.000	0.596	0.000	0.494
230	1.112	0.000	0.000	1.043	0.000	3.866
231	0.801	0.000	0.000	0.500	0.000	-2.922
232	1.171	0.000	0.000	0.503	0.000	3.913
233	1.002	0.000	0.000	0.605	0.000	-1.899
234	0.977	0.000	0.000	0.588	0.000	-1.563
235	0.857	0.000	0.000	1.101	0.000	2.118
236	1.219	0.000	0.000	0.858	0.000	-1.299
237	1.097	0.000	0.000	1.095	0.000	2.815
238	0.802	0.000	0.000	1.021	0.000	3.937
239	0.953	0.000	0.000	0.733	0.000	3.738
240	1.077	0.000	0.000	0.583	0.000	-1.127
241	0.741	0.000	0.000	0.577	0.000	0.603
242	0.860	0.000	0.000	0.723	0.000	3.751
243	0.844	0.000	0.000	0.357	0.000	-1.657
244	1.090	0.000	0.000	0.624	0.000	1.093
245	1.011	0.000	0.000	0.903	0.000	-2.728

Table A6
Item Discrimination and Difficulty Parameters
Under the High Group Factor Discrimination Condition
(Condition F)

Item	a_1	a_2	a_3	a_4	a_5	b
246	1.090	0.000	0.000	0.532	0.000	2.542
247	1.565	0.000	0.000	0.315	0.000	1.918
248	0.919	0.000	0.000	0.653	0.000	3.506
249	0.928	0.000	0.000	0.843	0.000	1.437
250	0.862	0.000	0.000	0.733	0.000	-0.222
251	1.035	0.000	0.000	0.791	0.000	3.358
252	0.573	0.000	0.000	0.564	0.000	-2.493
253	1.263	0.000	0.000	0.581	0.000	-1.437
254	0.765	0.000	0.000	0.672	0.000	3.037
255	1.252	0.000	0.000	0.937	0.000	-2.071
256	0.973	0.000	0.000	0.821	0.000	-1.575
257	0.928	0.000	0.000	0.656	0.000	-0.006
258	1.285	0.000	0.000	0.697	0.000	-0.996
259	0.745	0.000	0.000	0.621	0.000	3.493
260	1.191	0.000	0.000	0.867	0.000	-3.329
261	1.359	0.000	0.000	0.965	0.000	-0.376
262	1.283	0.000	0.000	0.754	0.000	2.265
263	0.920	0.000	0.000	0.780	0.000	2.680
264	1.078	0.000	0.000	0.952	0.000	0.960
265	0.904	0.000	0.000	0.317	0.000	0.869
266	0.869	0.000	0.000	0.529	0.000	3.736
267	0.701	0.000	0.000	0.596	0.000	-0.916
268	0.828	0.000	0.000	0.505	0.000	-0.604
269	0.818	0.000	0.000	0.760	0.000	-0.828
270	0.946	0.000	0.000	0.683	0.000	-0.959
271	1.010	0.000	0.000	0.947	0.000	2.010
272	0.819	0.000	0.000	0.865	0.000	3.283
273	0.807	0.000	0.000	0.892	0.000	3.562
274	0.740	0.000	0.000	0.432	0.000	-0.107
275	1.199	0.000	0.000	0.751	0.000	0.877
276	0.802	0.000	0.000	0.724	0.000	0.763
277	0.978	0.000	0.000	0.458	0.000	2.267
278	1.228	0.000	0.000	0.707	0.000	2.919
279	0.985	0.000	0.000	0.798	0.000	3.908
280	1.163	0.000	0.000	0.802	0.000	2.721
281	0.801	0.000	0.000	0.747	0.000	0.335
282	1.073	0.000	0.000	0.379	0.000	-3.457
283	1.113	0.000	0.000	0.568	0.000	-1.202
284	0.868	0.000	0.000	0.925	0.000	-3.933
285	0.787	0.000	0.000	0.489	0.000	-0.111
286	1.100	0.000	0.000	0.881	0.000	-3.335
287	1.187	0.000	0.000	0.858	0.000	0.740
288	1.039	0.000	0.000	0.705	0.000	-0.769
289	0.743	0.000	0.000	0.774	0.000	-3.854
290	1.389	0.000	0.000	0.703	0.000	-0.746
291	1.040	0.000	0.000	0.568	0.000	3.807
292	1.350	0.000	0.000	0.534	0.000	3.462
293	0.768	0.000	0.000	0.396	0.000	2.415
294	0.782	0.000	0.000	0.909	0.000	1.887

Table A6
Item Discrimination and Difficulty Parameters
Under the High Group Factor Discrimination Condition
(Condition F)

Item	a_1	a_2	a_3	a_4	a_5	b
295	1.202	0.000	0.000	0.729	0.000	1.664
296	0.869	0.000	0.000	0.591	0.000	-1.487
297	0.837	0.000	0.000	0.541	0.000	2.972
298	0.593	0.000	0.000	0.889	0.000	0.175
299	1.367	0.000	0.000	0.540	0.000	-0.944
300	1.055	0.000	0.000	0.392	0.000	-3.279
301	0.748	0.000	0.000	0.000	0.807	-3.944
302	0.851	0.000	0.000	0.000	0.988	2.044
303	1.314	0.000	0.000	0.000	0.593	-1.012
304	1.320	0.000	0.000	0.000	0.726	1.271
305	1.073	0.000	0.000	0.000	0.836	-1.249
306	1.073	0.000	0.000	0.000	0.388	-3.065
307	0.728	0.000	0.000	0.000	0.341	-2.024
308	0.765	0.000	0.000	0.000	0.886	-3.802
309	1.029	0.000	0.000	0.000	0.833	-2.276
310	1.319	0.000	0.000	0.000	0.501	1.965
311	0.926	0.000	0.000	0.000	0.687	-0.615
312	1.039	0.000	0.000	0.000	0.720	-3.824
313	1.088	0.000	0.000	0.000	0.908	1.552
314	1.502	0.000	0.000	0.000	0.719	2.697
315	0.917	0.000	0.000	0.000	0.477	-2.289
316	1.012	0.000	0.000	0.000	1.001	-1.107
317	0.718	0.000	0.000	0.000	0.690	3.219
318	1.123	0.000	0.000	0.000	0.671	3.013
319	0.954	0.000	0.000	0.000	0.765	-0.644
320	1.273	0.000	0.000	0.000	0.462	0.552
321	1.280	0.000	0.000	0.000	0.650	3.429
322	1.594	0.000	0.000	0.000	0.766	0.072
323	0.961	0.000	0.000	0.000	0.684	-0.024
324	1.088	0.000	0.000	0.000	0.452	-3.371
325	1.061	0.000	0.000	0.000	1.023	0.311
326	0.999	0.000	0.000	0.000	0.895	-0.349
327	0.839	0.000	0.000	0.000	0.682	-2.010
328	1.092	0.000	0.000	0.000	0.643	-2.266
329	0.647	0.000	0.000	0.000	0.554	-1.345
330	1.171	0.000	0.000	0.000	0.491	0.301
331	1.089	0.000	0.000	0.000	0.511	-3.444
332	1.255	0.000	0.000	0.000	0.462	1.926
333	0.739	0.000	0.000	0.000	0.790	-1.931
334	1.072	0.000	0.000	0.000	0.684	-3.238
335	0.971	0.000	0.000	0.000	0.673	-2.272
336	1.234	0.000	0.000	0.000	0.534	-2.846
337	1.237	0.000	0.000	0.000	0.953	2.671
338	0.943	0.000	0.000	0.000	0.576	-3.564
339	0.853	0.000	0.000	0.000	0.439	1.708
340	0.892	0.000	0.000	0.000	0.678	-3.731
341	1.297	0.000	0.000	0.000	0.425	2.046
342	1.186	0.000	0.000	0.000	0.528	-3.609
343	0.709	0.000	0.000	0.000	0.829	2.907

Table A6
Item Discrimination and Difficulty Parameters
Under the High Group Factor Discrimination Condition
(Condition F)

Item	a_1	a_2	a_3	a_4	a_5	b
344	0.957	0.000	0.000	0.000	1.166	0.449
345	0.803	0.000	0.000	0.000	0.894	0.383
346	0.734	0.000	0.000	0.000	0.655	3.933
347	1.122	0.000	0.000	0.000	0.983	2.183
348	0.863	0.000	0.000	0.000	0.557	-1.908
349	0.790	0.000	0.000	0.000	0.539	1.403
350	1.074	0.000	0.000	0.000	0.814	3.022
351	1.022	0.000	0.000	0.000	0.681	-0.947
352	0.749	0.000	0.000	0.000	1.012	0.500
353	0.850	0.000	0.000	0.000	0.876	0.915
354	0.899	0.000	0.000	0.000	0.737	2.561
355	1.070	0.000	0.000	0.000	0.662	0.383
356	1.182	0.000	0.000	0.000	0.861	-3.185
357	1.309	0.000	0.000	0.000	1.000	1.739
358	1.181	0.000	0.000	0.000	0.714	2.399
359	0.951	0.000	0.000	0.000	0.778	1.075
360	1.073	0.000	0.000	0.000	0.447	-2.169
361	1.027	0.000	0.000	0.000	0.521	1.713
362	1.071	0.000	0.000	0.000	0.715	-1.120
363	0.996	0.000	0.000	0.000	0.926	-3.195
364	1.243	0.000	0.000	0.000	0.757	3.521
365	1.137	0.000	0.000	0.000	0.826	-0.988
366	0.918	0.000	0.000	0.000	0.538	0.179
367	0.728	0.000	0.000	0.000	0.932	-0.004
368	1.096	0.000	0.000	0.000	0.797	0.397
369	0.957	0.000	0.000	0.000	0.552	-2.403
370	0.642	0.000	0.000	0.000	0.719	-0.945
371	0.818	0.000	0.000	0.000	0.361	0.908
372	1.021	0.000	0.000	0.000	0.824	-2.788
373	1.070	0.000	0.000	0.000	0.555	2.575
374	0.854	0.000	0.000	0.000	0.424	-0.167
375	1.226	0.000	0.000	0.000	0.900	1.367
376	0.872	0.000	0.000	0.000	0.575	-2.464
377	1.164	0.000	0.000	0.000	0.687	-0.888
378	0.780	0.000	0.000	0.000	0.443	-0.851
379	0.774	0.000	0.000	0.000	0.837	1.830
380	1.184	0.000	0.000	0.000	0.855	0.405
381	1.140	0.000	0.000	0.000	0.725	0.063
382	0.876	0.000	0.000	0.000	0.795	1.212
383	1.307	0.000	0.000	0.000	0.773	1.387
384	0.784	0.000	0.000	0.000	0.944	-0.743
385	1.007	0.000	0.000	0.000	0.757	-2.382
386	1.051	0.000	0.000	0.000	0.902	-3.729
387	1.290	0.000	0.000	0.000	0.568	1.509
388	1.152	0.000	0.000	0.000	0.956	2.273
389	1.001	0.000	0.000	0.000	0.768	-2.993
390	1.017	0.000	0.000	0.000	0.811	-0.877
391	0.819	0.000	0.000	0.000	0.554	-3.165
392	1.125	0.000	0.000	0.000	0.989	-0.594

Table A6
Item Discrimination and Difficulty Parameters
Under the High Group Factor Discrimination Condition
(Condition F)

Item	a_1	a_2	a_3	a_4	a_5	b
393	0.828	0.000	0.000	0.000	0.418	-2.054
394	1.261	0.000	0.000	0.000	0.398	1.011
395	1.153	0.000	0.000	0.000	1.002	-3.408
396	1.206	0.000	0.000	0.000	0.537	-3.716
397	0.873	0.000	0.000	0.000	0.933	3.143
398	1.110	0.000	0.000	0.000	0.694	2.720
399	1.172	0.000	0.000	0.000	0.704	-3.876
400	0.940	0.000	0.000	0.000	0.606	2.611

APPENDIX B

R Codes for Two BICAT Algorithms and MBICAT Algorithm

APPENDIX B1

BICAT(S) Algorithms with Two Group Factors

```
N<-1000
LH<-40
LH1<-20
LH1<-20
items<-400
examinees <- c(1:N)
theta.gen<-0
dd<-1.702
I<-c(1,1,1)
# Reading data
response<-as.matrix(read.table("G://BICAT3a//f2res.txt",header=FALSE,
nrow=N,comment.char= "",colClasses="numeric"))
responses<-matrix(response, nrow = length(examinees), ncol=items)
pattern<-read.table(file=paste("G://BICAT3a//pattern.txt",sep=""),header=FALSE,sep=" ")
ability<-read.table(file=paste("G://BICAT3a//ability.txt",sep=""),header=FALSE,sep=" ")
difficulty<-read.table(file=paste("G://BICAT3a//difficulty.txt",sep=""),header=FALSE,sep=" ")
General<-pattern[,1]
G1<-pattern[,2]
G2<-pattern[,3]
item.b<-difficulty[,1]
item.c<-rep(0,items)
# MAP Estimation
MAP<-matrix(c(NA),N,3)
MAPSE<-matrix(c(NA),N,3)
# EAP Estimation
EAP<-matrix(c(NA),N,3)
MAPSE<-matrix(c(NA),N,3)
#MLE Estimation
MLE<-matrix(c(NA),N,3)
MLESE<-matrix(c(NA),N,3)
for (e in 1:length(examinees)) {
res.General<- responses[e,]
res.G1<-responses[e,1:200]
res.G2<-responses[e,201:400]
General.mat<-matrix(c(NA),nrow=3,ncol=LH)
G1.mat<-matrix(c(NA),nrow=3,ncol=LH1)
G2.mat<-matrix(c(NA),nrow=3,ncol=LH2)
General.a<-matrix(c(NA),LH,1)
General.b<-matrix(c(NA),LH,1)
General.c<-matrix(c(NA),LH,1)
G1.a<-matrix(c(NA),LH1,1)
G1.b<-matrix(c(NA),LH1,1)
G1.c<-matrix(c(NA),LH1,1)
G2.a<-matrix(c(NA),LH2,1)
G2.b<-matrix(c(NA),LH2,1)
G2.c<-matrix(c(NA),LH2,1)
a1<-General
b1<-item.b
c1<-item.c
a2<-G1[1:200]
b2<-item.b[1:200]
c2<-item.c[1:200]
```

```
a3<-G2[201:400]
b3<-item.b[201:400]
c3<-item.c[201:400]
################################################
#    General Factor                                        #
################################################
#~~~~~~~~~~~~~ MLE Estimation ~~~~~~~~~~~~~~~~#
theta.General<-0
for (k in 1:LH) {
ITEM<-0
INFO<-matrix(c(NA),length(res.General),1)
or (h in 1:length(res.General)) {
NFO[h]<-(dd^2)*(a1[h]^2) *
(1-(c1[h] + ((1 - c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General -b1[h]))))))) /
c1[h] + ((1 -c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General - b1[h]))))))) *
(((c1[h] + ((1 - c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General -b1[h]))))))-c1[h]) /
1-c1[h]))^2)}
Z<-round(matrix(max(INFO)),digits=7)
or (j in 1:length(res.General))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
General.mat[1,k]<-res.General[j]
es.General<-res.General[-j]
General.a[k,1]<-a1[j]
General.b[k,1]<-b1[j]
General.c[k,1]<-c1[j]
a1<-a1[-j]
b1<-b1[-j]
c1<-c1[-j]
break}}
TEM<-0+k
DLL<- function (theta) { L<-0
for (h in 1:ITEM) {
P<-General.c[h]+(1-General.c[h])*(1/(1+exp((-dd)*General.a[h]*(theta-General.b[h]))))
Q<-1-P
L<-L+General.a[h]*((P-General.c[h])/(P*(1-General.c[h])))*(General.mat[1,h]-P)
LL<-(1.702)*L-theta
}}
DLL2<-function(theta) { L2<-0
for (i in 1:ITEM){
P<-General.c[i]+(1-General.c[i])*(1/(1+exp((-dd)*General.a[i]*(theta-General.b[i]))))
Q<-1-P
L2<-L2+(General.a[i])^2*((P-General.c[i])/(1-General.c[i]))*(Q/P)*((General.mat[1,i]*General.c[i]-
P^2)/P)
LL2<-(1.702)^2*L2-1
}}
Niter <- 10
for (iter in 1:Niter){
theta.General <- theta.General - (solve(DLL2(theta.General))) %*% (DLL(theta.General))
SE.General<-sqrt(-solve(DLL2(theta.General)))
}
General.mat[2,k]<-theta.General
General.mat[3,k]<-SE.General
}
MAP[e,1]<-General.mat[2,k]
```

```
MAPSE[e,1]<-General.mat[3,k]
#~~~~~~~~~~~~~~~~~~~~~~ EAP Estimation ~~~~~~~~~~~~~~~~~ #
theta.General<-0
for (k in 1:LH) {
ITEM<-0
INFO<-matrix(c(NA),length(res.General),1)
for (h in 1:length(res.General)) {
INFO[h]<-(dd^2)*(a1[h]^2) *
((1-(c1[h] + ((1 - c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General -b1[h]))))))) /
(c1[h] + ((1 -c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General - b1[h]))))))) *
((((c1[h] + ((1 - c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General -b1[h]))))))-c1[h]) /
(1-c1[h]))^2)}
Z<-round(matrix(max(INFO)),digits=7)
for (j in 1:length(res.General))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
General.mat[1,k]<-res.General[j]
res.General<-res.General[-j]
General.a[k,1]<-a1[j]
General.b[k,1]<-b1[j]
General.c[k,1]<-c1[j]
a1<-a1[-j]
b1<-b1[-j]
c1<-c1[-j]
break
}}
ITEM<-0+k
LLM<- function (theta) { L<-1
for (h in 1:ITEM) {
L <- L * (((General.c[h] + ((1 - General.c[h]) *
(1/(1 + exp((-dd) * General.a[h] * (theta -General.b[h]))))))^General.mat[1,h])*(1-(General.c[h] + ((1 -
General.c[h]) * (1/(1 + exp((-dd) * General.a[h] * (theta -General.b[h])))))))^(1-General.mat[1,h]))
}}
NDIST<- function (theta) { (1/sqrt(2*pi*1^2))*exp((-(theta-0)^2)/2*1^2) }
POST<-function(theta) { NDIST(theta)*LLM(theta)}
nQ<-50
node<-seq(from=-4,to=4,by=((4--4)/nQ))
weight <- exp(-0.5*(node^2))
weight <- weight / sum(weight)
NUM<-0
VAR<-0
DENOM<-0
for (z in 1:nQ) {
NUM<-NUM+node[z]*LLM(node[z])*weight[z]
DENOM<-DENOM+LLM(node[z])*weight[z]
theta.General<-NUM/DENOM
VAR<-VAR+(node[z]-theta.General)^2*LLM(node[z])*weight[z]}
SE.General<-sqrt(VAR/DENOM)
General.mat[2,k]<-theta.General
General.mat[3,k]<-SE.General
}
EAP[e,1]<-General.mat[2,k]
EAPSE[e,1]<-General.mat[3,k]
#~~~~~~~~~~~~~~~~~~~~ MLE Estimation ~~~~~~~~~~~~~~~~~~~#
theta.General<-0
```

```
for (k in 1:LH) {
ITEM<-0
INFO<-matrix(c(NA),length(res.General),1)
for (h in 1:length(res.General)) {
INFO[h]<-(dd^2)*(a1[h]^2) *
((1-(c1[h] + ((1 - c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General -b1[h]))))))) /
(c1[h] + ((1 -c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General - b1[h]))))))) *
((((c1[h] + ((1 - c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General -b1[h]))))))-c1[h]) /
(1-c1[h]))^2)}
Z<-round(matrix(max(INFO)),digits=7)
for (j in 1:length(res.General))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
General.mat[1,k]<-res.General[j]
res.General<-res.General[-j]
General.a[k,1]<-a1[j]
General.b[k,1]<-b1[j]
General.c[k,1]<-c1[j]
a1<-a1[-j]
b1<-b1[-j]
c1<-c1[-j]
break}}
ITEM<-0+k
DLL <- function (theta) { LL <-0
for (v in 1:ITEM) {  LL <-  LL + ((General.mat[1,v] - (General.c[v] + ((1 - General.c[v]) *
(1/(1 + exp((-1.702) * General.a[v] * (theta - General.b[v]))))))) * ((1-General.c[v]) *
((exp(General.a[v]*1.702*(theta-General.b[v]))*General.a[v]*1.702)/
(1+exp(General.a[v]*1.702*(theta-General.b[v])))^2))/((General.c[v] + ((1 - General.c[v]) *
(1/(1 + exp((-1.702) * General.a[v] * (theta - General.b[v]))))))*(1-(General.c[v] + ((1 - General.c[v]) *
(1/(1 + exp((-1.702) * General.a[v] * (theta - General.b[v]))))))))) }}
DLL2<- function (theta) { (DLL(theta+eps) - DLL(theta))/eps }
eps<-1/10^8
delta<-1
if (ITEM == sum(General.mat[1,1:ITEM]) | sum(General.mat[1,1:ITEM]) == 0)      {
if (sum(General.mat[1,1:ITEM])==0) theta.General<-theta.General - 1
if (sum(General.mat[1,1:ITEM])==ITEM) theta.General<-theta.General + 1
SEMLE<-1}
else
{ while (abs(delta) > .001) {
if (DLL2(theta.General) > 0) delta<- -DLL(theta.General)/-DLL2(theta.General) else delta <- -
DLL(theta.General)/DLL2(theta.General)
if (delta < -1) { delta <--1 }
if (delta > 1) { delta <- 1 }
theta.General <- theta.General + delta
if (abs (theta.General) > 6) { theta.General <- theta.gen
SEMLE<-NA
break
}
SEMLE<-sqrt(1/(-DLL2(theta.General)))
} }
General.mat[2,k]<-theta.General
General.mat[3,k]<-SEMLE
}
MLE[e,1]<-General.mat[2,k]
MLESE[e,1]<-General.mat[3,k]
```

```
################################################
# GROUP FACTOR 1                                               #
################################################
#~~~~~~~~~~~~~~~~~ MAP Estimation ~~~~~~~~~~~~~~#
theta.G1<-0
for (k in 1:LH1) {
INFO<-matrix(c(NA),length(res.G1),1)
for (h in 1:length(res.G1)) {
INFO[h]<-(dd^2)*(a2[h]^2) * ((1-(c2[h] + ((1 - c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h]))))))) /
(c2[h] + ((1 -c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h]))))))) *
((((c2[h] + ((1 - c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h]))))))-c2[h]) /
(1-c2[h]))^2)}
Z<-round(matrix(max(INFO)),digits=7)
for (j in 1:length(res.G1))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
G1.mat[1,k]<-res.G1[j]
res.G1<-res.G1[-j]
G1.a[k,1]<-a2[j]
G1.b[k,1]<-b2[j]
G1.c[k,1]<-c2[j]
a2<-a2[-j]
b2<-b2[-j]
c2<-c2[-j]
break}}
ITEM<-0+k
DLL<- function (theta) { L<-0
for (h in 1:ITEM) {
P<-G1.c[h]+(1-G1.c[h])*(1/(1+exp((-dd)*G1.a[h]*(theta-G1.b[h]))))
Q<-1-P
L<-L+G1.a[h]*((P-G1.c[h])/(P*(1-G1.c[h])))*(G1.mat[1,h]-P)
LL<-(1.702)*L-theta
}}
DLL2<-function(theta) { L2<-0
for (i in 1:ITEM){
P<-G1.c[i]+(1-G1.c[i])*(1/(1+exp((-dd)*G1.a[i]*(theta-G1.b[i]))))
Q<-1-P
L2<-L2+(G1.a[i])^2*((P-G1.c[i])/(1-G1.c[i]))*(Q/P)*((G1.mat[1,i]*G1.c[i]-P^2)/P)
LL2<-(1.702)^2*L2-1
}}
Niter <- 10
for (iter in 1:Niter){
theta.G1 <- theta.G1- (solve(DLL2(theta.G1))) %*% (DLL(theta.G1))
SE.G1<-sqrt(-solve(DLL2(theta.G1)))
}
G1.mat[2,k]<-theta.G1
G1.mat[3,k]<-SE.G1
}
MAP[e,2]<-G1.mat[2,k]
MAPSE[e,2]<-G1.mat[3,k]
#~~~~~~~~~~~~~~~~~ EAP Estimation ~~~~~~~~~~~~~~~~#
theta.G1<-0
for (k in 1:LH1) {
INFO<-matrix(c(NA),length(res.G1),1)
for (h in 1:length(res.G1)) {
```

```
INFO[h]<-(dd^2)*(a2[h]^2) *
((1-(c2[h] + ((1 - c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h]))))))) /
(c2[h] + ((1 -c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h]))))))) *
((((c2[h] + ((1 - c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h]))))))-c2[h]) /
(1-c2[h]))^2)}
Z<-round(matrix(max(INFO)),digits=7)
for (j in 1:length(res.G1))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
G1.mat[1,k]<-res.G1[j]
res.G1<-res.G1[-j]
G1.a[k,1]<-a2[j]
G1.b[k,1]<-b2[j]
G1.c[k,1]<-c2[j]
a2<-a2[-j]
b2<-b2[-j]
c2<-c2[-j]
break}}
ITEMs<-0+k
LLM<- function (theta) { L<-1
for (h in 1:ITEMs) {
L <- L * (((G1.c[h] + ((1 - G1.c[h]) *
(1/(1 + exp((-dd) * G1.a[h] * (theta -G1.b[h]))))))^G1.mat[1,h])*(1-(G1.c[h] + ((1 - G1.c[h]) *
(1/(1 + exp((-dd) * G1.a[h] * (theta -G1.b[h])))))))^(1-G1.mat[1,h]))
}}
NDIST<- function (theta) { (1/sqrt(2*pi*1^2))*exp((-(theta-0)^2)/2*1^2) }
POST<-function(theta) { NDIST(theta)*LLM(theta)}
nQ<-50
node<-seq(from=-4,to=4,by=((4--4)/nQ))
weight <- exp(-0.5*(node^2))
weight <- weight / sum(weight)
NUM<-0
VAR<-0
DENOM<-0
for (z in 1:nQ) {
NUM<-NUM+node[z]*LLM(node[z])*weight[z]
DENOM<-DENOM+LLM(node[z])*weight[z]
theta.G1<-NUM/DENOM
VAR<-VAR+(node[z]-theta.G1)^2*LLM(node[z])*weight[z]}
SE.G1<-sqrt(VAR/DENOM)
G1.mat[2,k]<-theta.G1
G1.mat[3,k]<-SE.G1
}
EAP[e,2]<-G1.mat[2,k]
EAPSE[e,2]<-G1.mat[3,k]
#~~~~~~~~~~~~~~~~~~~ MLE Estimation ~~~~~~~~~~~~~~~~~~#
theta.G1<-0
for (k in 1:LH1) {
INFO<-matrix(c(NA),length(res.G1),1)
for (h in 1:length(res.G1)) {
INFO[h]<-(dd^2)*(a2[h]^2) *
((1-(c2[h] + ((1 - c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h]))))))) /
(c2[h] + ((1 -c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h]))))))) *
((((c2[h] + ((1 - c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h]))))))-c2[h]) /
(1-c2[h]))^2)}
```

```
Z<-round(matrix(max(INFO)),digits=7)
for (j in 1:length(res.G1))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
G1.mat[1,k]<-res.G1[j]
res.G1<-res.G1[-j]
G1.a[k,1]<-a2[j]
G1.b[k,1]<-b2[j]
G1.c[k,1]<-c2[j]
a2<-a2[-j]
b2<-b2[-j]
c2<-c2[-j]
break}}
ITEM<-0+k
DLL <- function (theta) { LL <-0
for (v in 1:ITEM) {  LL <-  LL + ((G1.mat[1,v] - (G1.c[v] + ((1 - G1.c[v]) *
(1/(1 + exp((-1.702) * G1.a[v] * (theta - G1.b[v]))))))) * ((1-G1.c[v]) *
((exp(G1.a[v]*1.702*(theta-G1.b[v]))*G1.a[v]*1.702)/
(1+exp(G1.a[v]*1.702*(theta-G1.b[v])))^2))/((G1.c[v] + ((1 - G1.c[v]) *
(1/(1 + exp((-1.702) * G1.a[v] * (theta - G1.b[v]))))))*(1-(G1.c[v] + ((1 - G1.c[v]) *
(1/(1 + exp((-1.702) * G1.a[v] * (theta - G1.b[v]))))))))) }}
DLL2<- function (theta) { (DLL(theta+eps) - DLL(theta))/eps }
eps<-1/10^8
delta<-1
if (ITEM == sum(G1.mat[1,1:ITEM]) | sum(G1.mat[1,1:ITEM]) == 0)
{    if (sum(G1.mat[1,1:ITEM])==0) theta.G1<-theta.G1 - 1
if (sum(G1.mat[1,1:ITEM])==ITEM) theta.G1<-theta.G1 + 1
SEMLE<-1}
else
{ while (abs(delta) > .001) {
if (DLL2(theta.G1) > 0) delta<- -DLL(theta.G1)/-DLL2(theta.G1) else delta <- -
DLL(theta.G1)/DLL2(theta.G1)
if (delta < -1) { delta <--1 }
if (delta > 1) { delta <- 1 }
theta.G1 <- theta.G1 + delta
if (abs (theta.G1) > 6) { theta.G1 <- theta.gen
SEMLE<-NA
break
}
SEMLE<-sqrt(1/(-DLL2(theta.G1)))
} }
G1.mat[2,k]<-theta.G1
G1.mat[3,k]<-SEMLE
}
MLE[e,2]<-G1.mat[2,k]
MLESE[e,2]<-G1.mat[3,k]
##################################################
#   GROUP FACTOR 2                                                           #
##################################################
#~~~~~~~~~~~~~~~ MAP Estimation ~~~~~~~~~~~~~~ #
theta.G2<-0
for (k in 1:LH2) {
INFO<-matrix(c(NA),length(res.G2),1)
for (h in 1:length(res.G2)) {
INFO[h]<-(dd^2)*(a3[h]^2) *
```

```
((1-(c3[h] + ((1 - c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))) /
(c3[h] + ((1 -c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))) *
((((c3[h] + ((1 - c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))-c3[h]) /
(1-c3[h]))^2)}
Z<-round(matrix(max(INFO)),digits=7)
for (j in 1:length(res.G2))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
G2.mat[1,k]<-res.G2[j]
res.G2<-res.G2[-j]
G2.a[k,1]<-a3[j]
G2.b[k,1]<-b3[j]
G2.c[k,1]<-c3[j]
a3<-a3[-j]
b3<-b3[-j]
c3<-c3[-j]
break
}}
ITEM<-0+k
DLL<- function (theta) { L<-0
for (h in 1:ITEM) {
P<-G2.c[h]+(1-G2.c[h])*(1/(1+exp((-dd)*G2.a[h]*(theta-G2.b[h]))))
Q<-1-P
L<-L+G2.a[h]*((P-G2.c[h])/(P*(1-G2.c[h])))*(G2.mat[1,h]-P)
LL<-(1.702)*L-theta
}}
DLL2<-function(theta) { L2<-0
for (i in 1:ITEM){
P<-G2.c[i]+(1-G2.c[i])*(1/(1+exp((-dd)*G2.a[i]*(theta-G2.b[i]))))
Q<-1-P
L2<-L2+(G2.a[i])^2*((P-G2.c[i])/(1-G2.c[i]))*(Q/P)*((G2.mat[1,i]*G2.c[i]-P^2)/P)
LL2<-(1.702)^2*L2-1
}}
Niter <- 10
for (iter in 1:Niter){
theta.G2 <- theta.G2 - (solve(DLL2(theta.G2))) %*% (DLL(theta.G2))
SE.G2<-sqrt(-solve(DLL2(theta.G2)))
}
G2.mat[2,k]<-theta.G2
G2.mat[3,k]<-SE.G2
}
MAP[e,3]<-G2.mat[2,k]
MAPSE[e,3]<-G2.mat[3,k]
}
#~~~~~~~~~~~~~~~~ EAP Estimation Method ~~~~~~~~~~~~~~~~~~#
theta.G2<-0
for (k in 1:LH2) {
INFO<-matrix(c(NA),length(res.G2),1)
for (h in 1:length(res.G2)) {
INFO[h]<-(dd^2)*(a3[h]^2) *
((1-(c3[h] + ((1 - c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))) /
(c3[h] + ((1 -c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))) *
((((c3[h] + ((1 - c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))-c3[h]) /
(1-c3[h]))^2)}
Z<-round(matrix(max(INFO)),digits=7)
```

```
for (j in 1:length(res.G2))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
G2.mat[1,k]<-res.G2[j]
res.G2<-res.G2[-j]
G2.a[k,1]<-a3[j]
G2.b[k,1]<-b3[j]
G2.c[k,1]<-c3[j]
a3<-a3[-j]
b3<-b3[-j]
c3<-c3[-j]
break}}
ITEM1<-0+k
LLM<- function (theta) { L<-1
for (h in 1:ITEM1) {
L <- L * (((G2.c[h] + ((1 - G2.c[h]) *
(1/(1 + exp((-dd) * G2.a[h] * (theta -G2.b[h]))))))^G2.mat[1,h])*(1-(G2.c[h] + ((1 - G2.c[h]) *
(1/(1 + exp((-dd) * G2.a[h] * (theta -G2.b[h])))))))^(1-G2.mat[1,h]))
}}
NDIST<- function (theta) { (1/sqrt(2*pi*1^2))*exp((-(theta-0)^2)/2*1^2) }
POST<-function(theta) { NDIST(theta)*LLM(theta)}
nQ<-50
node<-seq(from=-4,to=4,by=((4--4)/nQ))
weight <- exp(-0.5*(node^2))
weight <- weight / sum(weight)
NUM<-0
VAR<-0
DENOM<-0
for (z in 1:nQ) {
NUM<-NUM+node[z]*LLM(node[z])*weight[z]
DENOM<-DENOM+LLM(node[z])*weight[z]
theta.G2<-NUM/DENOM
VAR<-VAR+(node[z]-theta.G2)^2*LLM(node[z])*weight[z]}
SE.G2<-sqrt(VAR/DENOM)
G2.mat[2,k]<-theta.G2
G2.mat[3,k]<-SE.G2
}
EAP[e,3]<-G2.mat[2,k]
EAPSE[e,3]<-G2.mat[3,k]
}
#~~~~~~~~~~~~~~~~~~~~~ MLE Estimation ~~~~~~~~~~~~~~~~~~~~#
theta.G2<-0
for (k in 1:LH2) {
INFO<-matrix(c(NA),length(res.G2),1)
for (h in 1:length(res.G2)) {
INFO[h]<-(dd^2)*(a3[h]^2) *
((1-(c3[h] + ((1 - c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))) /
(c3[h] + ((1 -c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))) *
((((c3[h] + ((1 - c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))-c3[h]) /
(1-c3[h]))^2)}
Z<-round(matrix(max(INFO)),digits=7)
for (j in 1:length(res.G2))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
G2.mat[1,k]<-res.G2[j]
```

```
res.G2<-res.G2[-j]
G2.a[k,1]<-a3[j]
G2.b[k,1]<-b3[j]
G2.c[k,1]<-c3[j]
a3<-a3[-j]
b3<-b3[-j]
c3<-c3[-j]
break}}
ITEM<-0+k
DLL <- function (theta) { LL <-0
for (v in 1:ITEM) {  LL <-  LL + ((G2.mat[1,v] - (G2.c[v] + ((1 - G2.c[v]) *
(1/(1 + exp((-1.702) * G2.a[v] * (theta - G2.b[v]))))))) * ((1-G2.c[v]) *
((exp(G2.a[v]*1.702*(theta-G2.b[v]))*G2.a[v]*1.702)/
(1+exp(G2.a[v]*1.702*(theta-G2.b[v])))^2))/((G2.c[v] + ((1 - G2.c[v]) *
(1/(1 + exp((-1.702) * G2.a[v] * (theta - G2.b[v]))))))*(1-(G2.c[v] + ((1 - G2.c[v]) *
(1/(1 + exp((-1.702) * G2.a[v] * (theta - G2.b[v])))))))))) }}
DLL2<- function (theta) { (DLL(theta+eps) - DLL(theta))/eps }
eps<-1/10^8
delta<-1
if (ITEM == sum(G2.mat[1,1:ITEM]) | sum(G2.mat[1,1:ITEM]) == 0)
{    if (sum(G2.mat[1,1:ITEM])==0) theta.G2<-theta.G2 - 1
if (sum(G2.mat[1,1:ITEM])==ITEM) theta.G2<-theta.G2+ 1
SEMLE<-1}
else
{ while (abs(delta) > .001) {
if (DLL2(theta.G2) > 0) delta<- -DLL(theta.G2)/-DLL2(theta.G2) else delta <- -
DLL(theta.G2)/DLL2(theta.G2)
if (delta < -1) { delta <--1 }
if (delta > 1) { delta <- 1 }
theta.G2 <- theta.G2 + delta
if (abs (theta.G2) > 6) { theta.G2 <- theta.gen
SEMLE<-NA
break
}
SEMLE<-sqrt(1/(-DLL2(theta.G2)))
} }
G2.mat[2,k]<-theta.G2
 G2.mat[3,k]<-SEMLE
}
MLE[e,3]<-G2.mat[2,k]
MLESE[e,3]<-G2.mat[3,k]
}
```

APPENDIX B2

BICAT(D) Algorithm with Two Group Factors

```
N<-1000
LH<-40
LH1<-20
LH1<-20
items<-400
examinees <- c(1:N)
theta.gen<-0
dd<-1.702
I<-c(1,1,1)
# Reading data
response<-as.matrix(read.table("G://BICAT3a//f2res.txt",header=FALSE,
nrow=N,comment.char= "",colClasses="numeric"))
responses<-matrix(response, nrow = length(examinees), ncol=items)
pattern<-read.table(file=paste("G://BICAT3a//pattern.txt",sep=""),header=FALSE,sep=" ")
ability<-read.table(file=paste("G://BICAT3a//ability.txt",sep=""),header=FALSE,sep=" ")
difficulty<-read.table(file=paste("G://BICAT3a//difficulty.txt",sep=""),header=FALSE,sep=" ")
General<-pattern[,1]
G1<-pattern[,2]
G2<-pattern[,3]
item.b<-difficulty[,1]
item.c<-rep(0,items)
# MAP Estimation
WMAP<-matrix(c(NA),N,3)
WMAPSE<-matrix(c(NA),N,3)
# EAP Estimation
WEAP<-matrix(c(NA),N,3)
WMAPSE<-matrix(c(NA),N,3)
#MLE Estimation
WMLE<-matrix(c(NA),N,3)
WMLESE<-matrix(c(NA),N,3)
for (e in 1:length(examinees)) {
res.General<- responses[e,]
res.G1<-responses[e,1:200]
res.G2<-responses[e,201:400]
General.mat<-matrix(c(NA),nrow=3,ncol=LH)
G1.mat<-matrix(c(NA),nrow=3,ncol=LH1)
G2.mat<-matrix(c(NA),nrow=3,ncol=LH2)
General.a<-matrix(c(NA),LH,1)
General.b<-matrix(c(NA),LH,1)
General.c<-matrix(c(NA),LH,1)
G1.a<-matrix(c(NA),LH1,1)
G1.b<-matrix(c(NA),LH1,1)
G1.c<-matrix(c(NA),LH1,1)
G2.a<-matrix(c(NA),LH2,1)
G2.b<-matrix(c(NA),LH2,1)
G2.c<-matrix(c(NA),LH2,1)
a1<-General
b1<-item.b
c1<-item.c
a2<-G1[1:200]
b2<-item.b[1:200]
```

```
c2<-item.c[1:200]
a3<-G2[201:400]
b3<-item.b[201:400]
 c3<-item.c[201:400]
################################################
#           General Factor                              #
################################################
#~~~~~~~~~~~~~ MLE Estimation ~~~~~~~~~~~~~~#
theta.General<-0
for (k in 1:LH) {
ITEM<-0
INFO<-matrix(c(NA),length(res.General),1)
for (h in 1:length(res.General)) {
INFO[h]<-(dd^2)*(a1[h]^2) *
((1-(c1[h] + ((1 - c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General -b1[h]))))))) /
(c1[h] + ((1 -c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General - b1[h]))))))) *
((((c1[h] + ((1 - c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General -b1[h]))))))-c1[h]) /
(1-c1[h]))^2)}
Z<-round(matrix(max(INFO)),digits=7)
for (j in 1:length(res.General))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
General.mat[1,k]<-res.General[j]
res.General<-res.General[-j]
General.a[k,1]<-a1[j]
General.b[k,1]<-b1[j]
General.c[k,1]<-c1[j]
a1<-a1[-j]
b1<-b1[-j]
c1<-c1[-j]
break}}
ITEM<-0+k
DLL<- function (theta) { L<-0
for (h in 1:ITEM) {
P<-General.c[h]+(1-General.c[h])*(1/(1+exp((-dd)*General.a[h]*(theta-General.b[h]))))
Q<-1-P
L<-L+General.a[h]*((P-General.c[h])/(P*(1-General.c[h])))*(General.mat[1,h]-P)
LL<-(1.702)*L-theta
}}
DLL2<-function(theta) { L2<-0
for (i in 1:ITEM){
P<-General.c[i]+(1-General.c[i])*(1/(1+exp((-dd)*General.a[i]*(theta-General.b[i]))))
Q<-1-P
L2<-L2+(General.a[i])^2*((P-General.c[i])/(1-General.c[i]))*(Q/P)*((General.mat[1,i]*General.c[i]-
P^2)/P)
LL2<-(1.702)^2*L2-1
}}
Niter <- 10
for (iter in 1:Niter){
theta.General <- theta.General - (solve(DLL2(theta.General))) %*% (DLL(theta.General))
SE.General<-sqrt(-solve(DLL2(theta.General)))
}
General.mat[2,k]<-theta.General
General.mat[3,k]<-SE.General
}
```

```
WMAP[e,1]<-General.mat[2,k]
WMAPSE[e,1]<-General.mat[3,k]
#~~~~~~~~~~~~~~~~~~~ EAP Estimation ~~~~~~~~~~~~~~~~#
theta.General<-0
for (k in 1:LH) {
ITEM<-0
INFO<-matrix(c(NA),length(res.General),1)
for (h in 1:length(res.General)) {
INFO[h]<-(dd^2)*(a1[h]^2) *
((1-(c1[h] + ((1 - c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General -b1[h]))))))) /
(c1[h] + ((1 -c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General - b1[h])))))) *
((((c1[h] + ((1 - c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General -b1[h]))))))-c1[h]) /
(1-c1[h]))^2)}
Z<-round(matrix(max(INFO)),digits=7)
for (j in 1:length(res.General))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
General.mat[1,k]<-res.General[j]
res.General<-res.General[-j]
General.a[k,1]<-a1[j]
General.b[k,1]<-b1[j]
General.c[k,1]<-c1[j]
a1<-a1[-j]
b1<-b1[-j]
c1<-c1[-j]
break
}}
ITEM<-0+k
LLM<- function (theta) { L<-1
for (h in 1:ITEM) {
L <- L * (((General.c[h] + ((1 - General.c[h]) *
(1/(1 + exp((-dd) * General.a[h] * (theta -General.b[h]))))))^General.mat[1,h])*(1-(General.c[h] + ((1 -
General.c[h]) * (1/(1 + exp((-dd) * General.a[h] * (theta -General.b[h])))))))^(1-General.mat[1,h]))
}}
NDIST<- function (theta) { (1/sqrt(2*pi*1^2))*exp((-(theta-0)^2)/2*1^2) }
POST<-function(theta) { NDIST(theta)*LLM(theta)}
nQ<-50
node<-seq(from=-4,to=4,by=((4--4)/nQ))
weight <- exp(-0.5*(node^2))
weight <- weight / sum(weight)
NUM<-0
VAR<-0
DENOM<-0
for (z in 1:nQ) {
NUM<-NUM+node[z]*LLM(node[z])*weight[z]
DENOM<-DENOM+LLM(node[z])*weight[z]
theta.General<-NUM/DENOM
VAR<-VAR+(node[z]-theta.General)^2*LLM(node[z])*weight[z]}
SE.General<-sqrt(VAR/DENOM)
General.mat[2,k]<-theta.General
General.mat[3,k]<-SE.General
}
WEAP[e,1]<-General.mat[2,k]
WEAPSE[e,1]<-General.mat[3,k]
#~~~~~~~~~~~~~~~~~~~ MLE Estimation ~~~~~~~~~~~~~~~~#
```

```
theta.General<-0
for (k in 1:LH) {
ITEM<-0
INFO<-matrix(c(NA),length(res.General),1)
for (h in 1:length(res.General)) {
INFO[h]<-(dd^2)*(a1[h]^2) *
((1-(c1[h] + ((1 - c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General -b1[h]))))))) /
(c1[h] + ((1 -c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General - b1[h]))))))) *
((((c1[h] + ((1 - c1[h]) * (1/(1 + exp((-dd)*a1[h]*(theta.General -b1[h]))))))-c1[h]) /
(1-c1[h]))^2)}
Z<-round(matrix(max(INFO)),digits=7)
for (j in 1:length(res.General))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
General.mat[1,k]<-res.General[j]
res.General<-res.General[-j]
General.a[k,1]<-a1[j]
General.b[k,1]<-b1[j]
General.c[k,1]<-c1[j]
a1<-a1[-j]
b1<-b1[-j]
c1<-c1[-j]
break}}
ITEM<-0+k
DLL <- function (theta) { LL <-0
for (v in 1:ITEM) {  LL <-  LL + ((General.mat[1,v] - (General.c[v] + ((1 - General.c[v]) *
(1/(1 + exp((-1.702) * General.a[v] * (theta - General.b[v]))))))) * ((1-General.c[v]) *
((exp(General.a[v]*1.702*(theta-General.b[v]))*General.a[v]*1.702)/
(1+exp(General.a[v]*1.702*(theta-General.b[v])))^2))/((General.c[v] + ((1 - General.c[v]) *
(1/(1 + exp((-1.702) * General.a[v] * (theta - General.b[v]))))))*(1-(General.c[v] + ((1 - General.c[v]) *
(1/(1 + exp((-1.702) * General.a[v] * (theta - General.b[v])))))))))) }}
DLL2<- function (theta) { (DLL(theta+eps) - DLL(theta))/eps }
eps<-1/10^8
delta<-1
if (ITEM == sum(General.mat[1,1:ITEM]) | sum(General.mat[1,1:ITEM]) == 0)      {
if (sum(General.mat[1,1:ITEM])==0) theta.General<-theta.General - 1
if (sum(General.mat[1,1:ITEM])==ITEM) theta.General<-theta.General + 1
SEMLE<-1}
else
{ while (abs(delta) > .001) {
if (DLL2(theta.General) > 0) delta<- -DLL(theta.General)/-DLL2(theta.General) else delta <- -
DLL(theta.General)/DLL2(theta.General)
if (delta < -1) { delta <--1 }
if (delta > 1) { delta <- 1 }
theta.General <- theta.General + delta
if (abs (theta.General) > 6) { theta.General <- theta.gen
SEMLE<-NA
break
}
SEMLE<-sqrt(1/(-DLL2(theta.General)))
} }
General.mat[2,k]<-theta.General
General.mat[3,k]<-SEMLE
}
WMLE[e,1]<-General.mat[2,k]
```

```
WMLESE[e,1]<-General.mat[3,k]
################################################
#     GROUP FACTOR 1                                                    #
################################################
#~~~~~~~~~~~~~~MAP Estimation~~~~~~~~~~~~~~~~#
WG<-cbind(t(General.mat),General.a,General.b,General.c )
newdata <- WG[ which(WG[,3]<= 200), ]
group1.mat<-t(newdata)
sec<-WG[ which(WG[,3]>200), ]
group2.mat<-t(sec)
#Initial Theta for group factor 1
theta.H1<-0
ITEMS<-length(group1.mat[1,])
WDLL <- function (theta) { LL <-0
for (v in 1:ITEMS) {  LL <-  LL + ((group1.mat[1,v] - (group1.mat[6,v] + ((1 - group1.mat[6,v]) *
(1/(1 + exp((-1.702) * group1.mat[4,v] * (theta - group1.mat[5,v]))))))) * ((1-group1.mat[6,v]) *
((exp(group1.mat[4,v]*1.702*(theta-group1.mat[5,v]))*group1.mat[4,v]*1.702)/
(1+exp(group1.mat[4,v]*1.702*(theta-group1.mat[5,v])))^2))/((group1.mat[6,v] + ((1 - group1.mat[6,v])
*(1/(1 + exp((-1.702) * group1.mat[4,v] * (theta - group1.mat[5,v]))))))*(1-(group1.mat[6,v] + ((1 -
group1.mat[6,v]) *  (1/(1 + exp((-1.702) * group1.mat[4,v] * (theta - group1.mat[5,v]))))))))) }}
WDLL2<- function (theta) { (WDLL(theta+eps) - WDLL(theta))/eps }
eps<-1/10^8
delta<-1
if (ITEMS == sum(group1.mat[1,1:ITEMS]) | sum(group1.mat[1,1:ITEMS]) == 0)
{    if (sum(group1.mat[1,1:ITEMS])==0) theta.H1<-theta.H1 - 1
if (sum(group1.mat[1,1:ITEMS])==ITEMS) theta.H1<-theta.H1+ 1
SEMLE<-1}
else
{while (abs(delta) > .001) {
if (WDLL2(theta.H1) > 0) delta<- -WDLL(theta.H1)/-WDLL2(theta.H1) else delta <- -
WDLL(theta.H1)/WDLL2(theta.H1)
if (delta < -1) { delta <--1 }
if (delta > 1) { delta <- 1 }
theta.H1 <- theta.H1 + delta
if (abs (theta.H1) > 6) { theta.H1 <- theta.gen
SEMLE<-NA
break
}}}
theta.G1<-theta.H1
for (k in 1:LH1) {
INFO<-matrix(c(NA),length(res.G1),1)
for (h in 1:length(res.G1)) {
INFO[h]<-(dd^2)*(a2[h]^2) * ((1-(c2[h] + ((1 - c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h]))))))) /
(c2[h] + ((1 -c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h])))))) *
((((c2[h] + ((1 - c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h]))))))-c2[h]) /
(1-c2[h]))^2)}
Z<-round(matrix(max(INFO)),digits=7)
for (j in 1:length(res.G1))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
G1.mat[1,k]<-res.G1[j]
res.G1<-res.G1[-j]
G1.a[k,1]<-a2[j]
G1.b[k,1]<-b2[j]
G1.c[k,1]<-c2[j]
```

```
a2<-a2[-j]
b2<-b2[-j]
c2<-c2[-j]
break}}
ITEM<-0+k
DLL<- function (theta) { L<-0
for (h in 1:ITEM) {
P<-G1.c[h]+(1-G1.c[h])*(1/(1+exp((-dd)*G1.a[h]*(theta-G1.b[h]))))
Q<-1-P
L<-L+G1.a[h]*((P-G1.c[h])/(P*(1-G1.c[h])))*(G1.mat[1,h]-P)
LL<-(1.702)*L-theta
}}
DLL2<-function(theta) { L2<-0
for (i in 1:ITEM){
P<-G1.c[i]+(1-G1.c[i])*(1/(1+exp((-dd)*G1.a[i]*(theta-G1.b[i]))))
Q<-1-P
L2<-L2+(G1.a[i])^2*((P-G1.c[i])/(1-G1.c[i]))*(Q/P)*((G1.mat[1,i]*G1.c[i]-P^2)/P)
LL2<-(1.702)^2*L2-1
}}
Niter <- 10
for (iter in 1:Niter){
theta.G1 <- theta.G1- (solve(DLL2(theta.G1))) %*% (DLL(theta.G1))
SE.G1<-sqrt(-solve(DLL2(theta.G1)))
}
G1.mat[2,k]<-theta.G1
G1.mat[3,k]<-SE.G1
}
WMAP[e,2]<-G1.mat[2,k]
WMAPSE[e,2]<-G1.mat[3,k]
#~~~~~~~~~~~~~~~~~~~ EAP Estimation ~~~~~~~~~~~~~~~~~~#
theta.G1<-theta.H1
for (k in 1:LH1) {
INFO<-matrix(c(NA),length(res.G1),1)
for (h in 1:length(res.G1)) {
INFO[h]<-(dd^2)*(a2[h]^2) *
((1-(c2[h] + ((1 - c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h]))))))) /
(c2[h] + ((1 -c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h])))))) *
((((c2[h] + ((1 - c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h]))))))-c2[h]) /
(1-c2[h]))^2)}
Z<-round(matrix(max(INFO)),digits=7)
for (j in 1:length(res.G1))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
G1.mat[1,k]<-res.G1[j]
res.G1<-res.G1[-j]
G1.a[k,1]<-a2[j]
G1.b[k,1]<-b2[j]
G1.c[k,1]<-c2[j]
a2<-a2[-j]
b2<-b2[-j]
c2<-c2[-j]
break}}
ITEMs<-0+k
LLM<- function (theta) { L<-1
for (h in 1:ITEMs) {
```

```
L <- L * (((G1.c[h] + ((1 - G1.c[h]) *
(1/(1 + exp((-dd) * G1.a[h] * (theta -G1.b[h]))))))^G1.mat[1,h])*(1-(G1.c[h] + ((1 - G1.c[h]) *
(1/(1 + exp((-dd) * G1.a[h] * (theta -G1.b[h])))))))^(1-G1.mat[1,h]))
}}
NDIST<- function (theta) { (1/sqrt(2*pi*1^2))*exp((-(theta-0)^2)/2*1^2) }
POST<-function(theta) { NDIST(theta)*LLM(theta)}
nQ<-50
node<-seq(from=-4,to=4,by=((4--4)/nQ))
weight <- exp(-0.5*(node^2))
weight <- weight / sum(weight)
NUM<-0
VAR<-0
DENOM<-0
for (z in 1:nQ) {
NUM<-NUM+node[z]*LLM(node[z])*weight[z]
DENOM<-DENOM+LLM(node[z])*weight[z]
theta.G1<-NUM/DENOM
VAR<-VAR+(node[z]-theta.G1)^2*LLM(node[z])*weight[z]}
SE.G1<-sqrt(VAR/DENOM)
G1.mat[2,k]<-theta.G1
G1.mat[3,k]<-SE.G1
}
WEAP[e,2]<-G1.mat[2,k]
WEAPSE[e,2]<-G1.mat[3,k]
#~~~~~~~~~~~~~~~~~~~ MLE Estimation ~~~~~~~~~~~~~~~#
theta.G1<-theta.H1
for (k in 1:LH1) {
INFO<-matrix(c(NA),length(res.G1),1)
for (h in 1:length(res.G1)) {
INFO[h]<-(dd^2)*(a2[h]^2) *
((1-(c2[h] + ((1 - c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h]))))))) /
(c2[h] + ((1 -c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h]))))))) *
((((c2[h] + ((1 - c2[h]) * (1/(1 + exp((-dd)*a2[h]*(theta.G1 -b2[h]))))))-c2[h]) /
(1-c2[h]))^2)}
Z<-round(matrix(max(INFO)),digits=7)
for (j in 1:length(res.G1))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
G1.mat[1,k]<-res.G1[j]
res.G1<-res.G1[-j]
G1.a[k,1]<-a2[j]
G1.b[k,1]<-b2[j]
G1.c[k,1]<-c2[j]
a2<-a2[-j]
b2<-b2[-j]
c2<-c2[-j]
break}}
ITEM<-0+k
DLL <- function (theta) { LL <-0
for (v in 1:ITEM) {  LL <-  LL + ((G1.mat[1,v] - (G1.c[v] + ((1 - G1.c[v]) *
(1/(1 + exp((-1.702) * G1.a[v] * (theta - G1.b[v]))))))) * ((1-G1.c[v]) *
((exp(G1.a[v]*1.702*(theta-G1.b[v]))*G1.a[v]*1.702)/
(1+exp(G1.a[v]*1.702*(theta-G1.b[v])))^2))/((G1.c[v] + ((1 - G1.c[v]) *
(1/(1 + exp((-1.702) * G1.a[v] * (theta - G1.b[v]))))))*(1-(G1.c[v] + ((1 - G1.c[v]) *
(1/(1 + exp((-1.702) * G1.a[v] * (theta - G1.b[v])))))))))) }}
```

```
DLL2<- function (theta) { (DLL(theta+eps) - DLL(theta))/eps }
eps<-1/10^8
delta<-1
if (ITEM == sum(G1.mat[1,1:ITEM]) | sum(G1.mat[1,1:ITEM]) == 0)
{    if (sum(G1.mat[1,1:ITEM])==0) theta.G1<-theta.G1 - 1
if (sum(G1.mat[1,1:ITEM])==ITEM) theta.G1<-theta.G1 + 1
SEMLE<-1}
else
{ while (abs(delta) > .001) {
if (DLL2(theta.G1) > 0) delta<- -DLL(theta.G1)/-DLL2(theta.G1) else delta <- -
DLL(theta.G1)/DLL2(theta.G1)
if (delta < -1) { delta <--1 }
if (delta > 1) { delta <- 1 }
theta.G1 <- theta.G1 + delta
if (abs (theta.G1) > 6) { theta.G1 <- theta.gen
SEMLE<-NA
break
}
SEMLE<-sqrt(1/(-DLL2(theta.G1)))
} }
G1.mat[2,k]<-theta.G1
G1.mat[3,k]<-SEMLE
}
WMLE[e,2]<-G1.mat[2,k]
WMLESE[e,2]<-G1.mat[3,k]
##################################################
#        GROUP FACTOR 2                                          #
##################################################
#~~~~~~~~~~~~~~ MAP Estimation ~~~~~~~~~~~~~~~#
theta.H2<-0
ITEMs<-length(group2.mat[1,])
GDLL <- function (theta) { LL <-0
for (v in 1:ITEMs) {  LL <-  LL + ((group2.mat[1,v] - (group2.mat[6,v] + ((1 - group2.mat[6,v]) *
(1/(1 + exp((-1.702) * group2.mat[4,v] * (theta - group2.mat[5,v]))))))) * ((1-group2.mat[6,v]) *
((exp(group2.mat[4,v]*1.702*(theta-group2.mat[5,v]))*group2.mat[4,v]*1.702)/
(1+exp(group2.mat[4,v]*1.702*(theta-group2.mat[5,v])))^2))/((group2.mat[6,v] + ((1 - group2.mat[6,v]) *
(1/(1 + exp((-1.702) * group2.mat[4,v] * (theta - group2.mat[5,v]))))))*(1-(group2.mat[6,v] + ((1 -
group2.mat[6,v]) *  (1/(1 + exp((-1.702) * group2.mat[4,v] * (theta - group2.mat[5,v])))))))) }}
GDLL2<- function (theta) { (GDLL(theta+eps) - GDLL(theta))/eps }
eps<-1/10^8
delta<-1
if (ITEMs == sum(group2.mat[1,1:ITEMs]) | sum(group2.mat[1,1:ITEMs]) == 0)
{    if (sum(group2.mat[1,1:ITEMs])==0) theta.H2<-theta.H2 – 1
if (sum(group2.mat[1,1:ITEMs])==ITEMs) theta.H2<-theta.H2+ 1
SEMLE<-1}
else
{ while (abs(delta) > .001) {
if (GDLL2(theta.H2) > 0)
delta<- -GDLL(theta.H2)/-GDLL2(theta.H2) else delta <- - GDLL(theta.H2)/GDLL2(theta.H2)
if (delta < -1) { delta <--1 }
if (delta > 1) { delta <- 1 }
theta.H2 <- theta.H2 + delta
if (abs (theta.H2) > 6) { theta.H2 <- theta.gen
SEMLE<-NA
break
```

```
}}}
theta.G2<-theta.H2
for (k in 1:LH2) {
INFO<-matrix(c(NA),length(res.G2),1)
for (h in 1:length(res.G2)) {
INFO[h]<-(dd^2)*(a3[h]^2) *
((1-(c3[h] + ((1 - c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))) /
(c3[h] + ((1 -c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))) *
((((c3[h] + ((1 - c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))-c3[h]) /
(1-c3[h]))^2)}
Z<-round(matrix(max(INFO)),digits=7)
for (j in 1:length(res.G2))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
G2.mat[1,k]<-res.G2[j]
res.G2<-res.G2[-j]
G2.a[k,1]<-a3[j]
G2.b[k,1]<-b3[j]
G2.c[k,1]<-c3[j]
a3<-a3[-j]
b3<-b3[-j]
c3<-c3[-j]
break
}}
ITEM<-0+k
DLL<- function (theta) { L<-0
for (h in 1:ITEM) {
P<-G2.c[h]+(1-G2.c[h])*(1/(1+exp((-dd)*G2.a[h]*(theta-G2.b[h]))))
Q<-1-P
L<-L+G2.a[h]*((P-G2.c[h])/(P*(1-G2.c[h])))*(G2.mat[1,h]-P)
LL<-(1.702)*L-theta
}}
DLL2<-function(theta) { L2<-0
for (i in 1:ITEM){
P<-G2.c[i]+(1-G2.c[i])*(1/(1+exp((-dd)*G2.a[i]*(theta-G2.b[i]))))
Q<-1-P
L2<-L2+(G2.a[i])^2*((P-G2.c[i])/(1-G2.c[i]))*(Q/P)*((G2.mat[1,i]*G2.c[i]-P^2)/P)
LL2<-(1.702)^2*L2-1
}}
Niter <- 10
for (iter in 1:Niter){
theta.G2 <- theta.G2 - (solve(DLL2(theta.G2))) %*% (DLL(theta.G2))
SE.G2<-sqrt(-solve(DLL2(theta.G2)))
}
G2.mat[2,k]<-theta.G2
G2.mat[3,k]<-SE.G2
}
W MAP[e,3]<-G2.mat[2,k]
WMAPSE[e,3]<-G2.mat[3,k]
}
#~~~~~~~~~~~~~~~~~~~ EAP Estimation ~~~~~~~~~~~~~ #
theta.G2<-theta.H2
for (k in 1:LH2) {
INFO<-matrix(c(NA),length(res.G2),1)
for (h in 1:length(res.G2)) {
```

```
INFO[h]<-(dd^2)*(a3[h]^2) *
((1-(c3[h] + ((1 - c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))) /
(c3[h] + ((1 -c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))) *
((((c3[h] + ((1 - c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))-c3[h]) /
(1-c3[h]))^2)}
Z<-round(matrix(max(INFO)),digits=7)
for (j in 1:length(res.G2))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
G2.mat[1,k]<-res.G2[j]
res.G2<-res.G2[-j]
G2.a[k,1]<-a3[j]
G2.b[k,1]<-b3[j]
G2.c[k,1]<-c3[j]
a3<-a3[-j]
b3<-b3[-j]
c3<-c3[-j]
break}}
ITEM1<-0+k
LLM<- function (theta) { L<-1
for (h in 1:ITEM1) {
L <- L * (((G2.c[h] + ((1 - G2.c[h]) *
(1/(1 + exp((-dd) * G2.a[h] * (theta -G2.b[h]))))))^G2.mat[1,h])*(1-(G2.c[h] + ((1 - G2.c[h]) *
(1/(1 + exp((-dd) * G2.a[h] * (theta -G2.b[h])))))))^(1-G2.mat[1,h]))
}}
NDIST<- function (theta) { (1/sqrt(2*pi*1^2))*exp((-(theta-0)^2)/2*1^2) }
POST<-function(theta) { NDIST(theta)*LLM(theta)}
nQ<-50
node<-seq(from=-4,to=4,by=((4--4)/nQ))
weight <- exp(-0.5*(node^2))
weight <- weight / sum(weight)
NUM<-0
VAR<-0
DENOM<-0
for (z in 1:nQ) {
NUM<-NUM+node[z]*LLM(node[z])*weight[z]
DENOM<-DENOM+LLM(node[z])*weight[z]
theta.G2<-NUM/DENOM
VAR<-VAR+(node[z]-theta.G2)^2*LLM(node[z])*weight[z]}
SE.G2<-sqrt(VAR/DENOM)
G2.mat[2,k]<-theta.G2
G2.mat[3,k]<-SE.G2
}
WEAP[e,3]<-G2.mat[2,k]
WEAPSE[e,3]<-G2.mat[3,k]
}
#~~~~~~~~~~~~~~~~~~MLE Estimation ~~~~~~~~~~~~~~#
theta.G2<-theta.H2
for (k in 1:LH2) {
INFO<-matrix(c(NA),length(res.G2),1)
for (h in 1:length(res.G2)) {
INFO[h]<-(dd^2)*(a3[h]^2) *
((1-(c3[h] + ((1 - c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))) /
(c3[h] + ((1 -c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))) *
((((c3[h] + ((1 - c3[h]) * (1/(1 + exp((-dd)*a3[h]*(theta.G2 -b3[h]))))))-c3[h]) /
```

```
(1-c3[h]))^2)}
Z<-round(matrix(max(INFO)),digits=7)
for (j in 1:length(res.G2))
{ if (Z == round(INFO[j],digits=7))
{ INFO<-INFO[-j]
G2.mat[1,k]<-res.G2[j]
res.G2<-res.G2[-j]
G2.a[k,1]<-a3[j]
G2.b[k,1]<-b3[j]
G2.c[k,1]<-c3[j]
a3<-a3[-j]
b3<-b3[-j]
c3<-c3[-j]
break}}
ITEM<-0+k
DLL <- function (theta) { LL <-0
for (v in 1:ITEM) {  LL <-  LL + ((G2.mat[1,v] - (G2.c[v] + ((1 - G2.c[v]) *
(1/(1 + exp((-1.702) * G2.a[v] * (theta - G2.b[v]))))))) * ((1-G2.c[v]) *
((exp(G2.a[v]*1.702*(theta-G2.b[v]))*G2.a[v]*1.702)/
(1+exp(G2.a[v]*1.702*(theta-G2.b[v])))^2))/((G2.c[v] + ((1 - G2.c[v]) *
(1/(1 + exp((-1.702) * G2.a[v] * (theta - G2.b[v]))))))*(1-(G2.c[v] + ((1 - G2.c[v]) *
(1/(1 + exp((-1.702) * G2.a[v] * (theta - G2.b[v])))))))) }}
DLL2<- function (theta) { (DLL(theta+eps) - DLL(theta))/eps }
eps<-1/10^8
delta<-1
if (ITEM == sum(G2.mat[1,1:ITEM]) | sum(G2.mat[1,1:ITEM]) == 0)
{    if (sum(G2.mat[1,1:ITEM])==0) theta.G2<-theta.G2 - 1
if (sum(G2.mat[1,1:ITEM])==ITEM) theta.G2<-theta.G2+ 1
SEMLE<-1}
else
{ while (abs(delta) > .001) {
if (DLL2(theta.G2) > 0) delta<- -DLL(theta.G2)/-DLL2(theta.G2) else delta <- -
DLL(theta.G2)/DLL2(theta.G2)
if (delta < -1) { delta <--1 }
if (delta > 1) { delta <- 1 }
theta.G2 <- theta.G2 + delta
if (abs (theta.G2) > 6) { theta.G2 <- theta.gen
SEMLE<-NA
break
}
SEMLE<-sqrt(1/(-DLL2(theta.G2)))
} }
G2.mat[2,k]<-theta.G2
G2.mat[3,k]<-SEMLE
}
WMLE[e,3]<-G2.mat[2,k]
WMLESE[e,3]<-G2.mat[3,k]
}
```

APPENDIX B3

MBICAT Algorithm with Two Group Factors

```
N<-1000
LH<-40
items<-400
examinees <- c(1:N)
MAP<-matrix(c(NA),N,3)
EAP<-matrix(c(NA),N,3)
MLE<-matrix(c(NA),N,3)
mat<-matrix(c(NA),nrow=1,ncol=LH)
IM<-matrix(c(NA),LH,1)
aM<-matrix(c(NA),LH,3)
bM<-matrix(c(NA),LH,1)
cM<-matrix(c(NA),LH,1)
EST<-matrix(c(NA),LH,3)
matE<-matrix(c(NA),nrow=1,ncol=LH)
IE<-matrix(c(NA),LH,1)
aE<-matrix(c(NA),LH,3)
bE<-matrix(c(NA),LH,1)
cE<-matrix(c(NA),LH,1)
ESTE<-matrix(c(NA),LH,3)
matW<-matrix(c(NA),nrow=1,ncol=LH)
IW<-matrix(c(NA),LH,1)
aW<-matrix(c(NA),LH,3)
bW<-matrix(c(NA),LH,1)
cW<-matrix(c(NA),LH,1)
ESTW<-matrix(c(NA),LH,3)
##################################################
#      MAP Estimation                                #
##################################################
for (e in 1:length(examinees)) {
u<- responses[e,]   # Responses for MAP
u1<-responses[e,]   #Responses for MLE
u2<-responses[e,]   #Responses for EAP
a1<-patterns
b1<-item.b
c1<-item.c
a2<-patterns
b2<-item.b
c2<-item.c
a3<-patterns
b3<-item.b
c3<-item.c
thetaH<-c(0,0,0)
theta.General<-c(0,0,0)
thetaW<-c(0,0,0)
Ni<-0
I<-c(1,1,1)
Wis<-0
WisE<-0
WisW<-0
for (k in 1:LH) {
sigma<-matrix(c(1,0,0,0,1,0,0,0,1),nrow=3,ncol=3,byrow=T)
```

```
Ni<-0+k
Wi<-array(NA,dim=c(length(a1[1,]),length(a1[1,]),length(u)))
for (i in 1:length(u)) {
P<-c1[i]+(1-c1[i])*(1/(1+exp((-1.702)*t(a1[i,])%*%(thetaH-b1[i]*I))))
Q<-1-P
Wi[,,i]<-(1.702)^2*a1[i,]%*%t(a1[i,])*as.numeric((Q/P)*((P-c1[i])/(1-c1[i]))^2)
}
Ii<-matrix(NA,nrow=length(u),ncol=1,byrow=T)
for ( i in 1:length(u)) {
Ii[i]<-det(solve(sigma)+Wis+Wi[,,i])
}
Z<-round(matrix(max(Ii,na.rm=TRUE)),digits=7)
for (i in 1:length(u))
{ if (Z == round(Ii[i],digits=7))
{ IM[k,1]<-Ii[i]
Wis<-Wis+Wi[,,i]
Ii<-Ii[-i]
mat[1,k]<-u[i]
u<-u[-i]
aM[k,]<-a1[i,]
bM[k,1]<-b1[i]
cM[k,1]<-c1[i]
a1<-a1[-i,]
b1<-b1[-i]
c1<-c1[-i]
break }}
DLL<-0
DLL2<-0
DLL <- function (theta) {
Vi <-0
for (i in 1:Ni) {
P<-cM[i]+(1-cM[i])*(1/(1+exp((-1.702)*t(aM[i,])%*%(theta-bM[i]*I))))
Q<-1-P
Vi<-Vi+ ((P-cM[i])*(mat[1,i]-P))/ ((1-cM[i])*P)  * aM[i,]
LL <- (1.702)*Vi - solve(sigma)%*%(theta-0)
}}
DLL2<- function (theta) {
SSi<-0
for (i in 1:Ni){
P<-cM[i]+(1-cM[i])*(1/(1+exp((-1.702)*t(aM[i,])%*%(theta-bM[i]*I))))
Q<-1– P
SSi<-SSi+( aM[i,]%*%t(aM[i,])) * as.numeric( (Q*(P-cM[i])*(cM[i]*mat[1,i]-P^2))/ (P^2*(1-cM[i])^2) )
LL2<-(1.702)^2*SSi -solve(sigma)
}}
Niter <- 10
for (iter in 1:Niter){
thetaH <- thetaH - (solve(DLL2(thetaH))) %*% (DLL(thetaH))
SEMLE<- – solve(DLL2(thetaH))
}
EST[k,]<-thetaH
MAP[e,]<-EST[k,]
}
###################################################
#           EAP Estimation                        #
###################################################
```

```
for (p in 1:LH) {
Ni<-p
WiE<-array(NA,dim=c(length(a2[1,]),length(a2[1,]),length(u1)))
for (i in 1:length(u1)) {
P<-c2[i]+(1-c2[i])*(1/(1+exp((-1.702)*t(a2[i,])%*%(theta.General-b2[i]*I))))
Q<-1-P
WiE[,,i]<-(1.702)^2*a2[i,]%*%t(a2[i,])*as.numeric((Q/P)*((P-c2[i])/(1-c2[i]))^2)
}
IiE<-matrix(NA,nrow=length(u1),ncol=1,byrow=T)
for ( i in 1:length(u1)) {
IiE[i]<-det(solve(sigma)+WisE+WiE[,,i])
}
Z<-round(matrix(max(IiE,na.rm=TRUE)),digits=7)
for (i in 1:length(u1))
{ if (Z == round(IiE[i],digits=7))
{ IE[p,1]<-IiE[i]
WisE<-WisE+WiE[,,i]
IiE<-IiE[-i]
matE[1,p]<-u1[i]
u1<-u1[-i]
aE[p,]<-a2[i,]
bE[p,1]<-b2[i]
cE[p,1]<-c2[i]
a2<-a2[-i,]
b2<-b2[-i]
c2<-c2[-i]
break }}
LF1<- function (theta) { L<-1
for (h in 1:Ni) {
if (aE[h,3]==0) { L <- L * (((cE[h] + ((1 - cE[h]) *
(1/(1+exp((-1.702)*t(aE[h,-3])%*%(theta[-3]-bE[h,-3]*I[-3]))))))^matE[1,h])*(1-(cE[h]+((1-cE[h])*
(1/(1+exp((-1.702)*t(aE[h,-3])%*%(theta[-3]-bE[h,-3]*I[-3])))))))^(1-matE[1,h]))}
else          {L <- L*1}
}   }
LF2<- function (theta) { L<-1
for (h in 1:Ni) {
if (aE[h,2]==0) {L <- L * (((cE[h] + ((1 - cE[h]) *
(1/(1+exp((-1.702)*t(aE[h,-2])%*%(theta[-3]-bE[h,-3]*I[-3]))))))^matE[1,h])*(1-(cE[h]+((1-cE[h])*
(1/(1+exp((-1.702)*t(aE[h,-2])%*%(theta[-3]-bE[h,-3]*I[-3])))))))^(1-matE[1,h]))}
else          {L <- L*1}
}  }
nQ<-15
node<-seq(from=-4,to=4,by=((4--4)/nQ))
weight <- exp(-0.5*(node^2))
weight <- weight / sum(weight)
NUM.GEN<-0
DENOM.GEN<-0
NUM.GRO3<-0
NUM.GRO4<-0
DENOM.GRO3<-0
DENOM.GRO4<-0
VAR<-0
or (z in 0:nQ+1)   {
NUM.GEN1<-0
NUM.GEN2<-0
```

```
or (w in 0:nQ+1)  {
NUM.GEN1<-NUM.GEN1+(LF1(c(node[z],node[w],node[w]))*weight[w])
NUM.GEN2<-NUM.GEN2+(LF2(c(node[z],node[w],node[w]))*weight[w])
}
NUM.GEN<-NUM.GEN+node[z]*NUM.GEN1*NUM.GEN2*weight[z]
DENOM.GEN<-DENOM.GEN+NUM.GEN1*NUM.GEN2*weight[z]
}
theta.GE<-NUM.GEN/DENOM.GEN
for (w in 0:nQ+1)          {
NUM.GRO1<-0
NUM.GRO2<-0
for (z in 0:nQ+1) {
NUM.GEN1<-0
NUM.GEN2<-0
for(k in 0:nQ+1) {
NUM.GEN1<-NUM.GEN1+(LF1(c(node[z],node[k],node[k]))*weight[k])
NUM.GEN2<-NUM.GEN2+(LF2(c(node[z],node[k],node[k]))*weight[k])
}
NUM.GRO1<-NUM.GRO1+(LF1(c(node[z],node[w],node[w]))*weight[z]) *NUM.GEN2
NUM.GRO2<-NUM.GRO2+(LF2(c(node[z],node[w],node[w]))*weight[z])  *NUM.GEN1
}
NUM.GRO3<-NUM.GRO3+node[w]*NUM.GRO1*weight[w]
NUM.GRO4<-NUM.GRO4+node[w]*NUM.GRO2*weight[w]
DENOM.GRO3<-DENOM.GRO3+    NUM.GRO1*weight[w]
DENOM.GRO4<-DENOM.GRO4+    NUM.GRO2*weight[w]
}
theta.G1<-NUM.GRO3/DENOM.GRO3
theta.G2<-NUM.GRO4/DENOM.GRO4
theta.Gen<-c(theta.GE,theta.G1,theta.G2)
theta.General<-as.matrix(theta.Gen)
VAR<-VAR+(node[z]-theta.General)^2*LLM(node[z])*weight[z]}
SE.General<-sqrt(VAR/DENOM)    }
General.mat[k,]<-theta.General
General.mat[3,k]<-SE.General
ESTE[p,]<-theta.General
EAP[e,]<-ESTE[p,]
}
##########################################
 #   MLE Estimation                                          #
##########################################
for (k in 1:LH) {
Ni<-0+k
WiW<-array(NA,dim=c(length(a3[1,]),length(a3[1,]),length(u2)))
for (i in 1:length(u1)) {
P<-c3[i]+(1-c3[i])*(1/(1+exp((-1.702)*t(a3[i,])%*%(thetaW-b3[i]*I))))
Q<-1-P
WiW[,,i]<-(1.702)^2*a3[i,]%*%t(a3[i,])*as.numeric((Q/P)*((P-c3[i])/(1-c3[i]))^2)
}
IiW<-matrix(NA,nrow=length(u2),ncol=1,byrow=T)
for ( i in 1:length(u2)) {
IiW[i]<-det(solve(sigma)+WisW+WiW[,,i])
}
Z<-round(matrix(max(IiW,na.rm=TRUE)),digits=7)
for (i in 1:length(u2))
{ if (Z == round(IiW[i],digits=7))
```

```
{ IW[k,1]<-IiW[i]
WisW<-WisW+WiW[,,i]
IiW<-IiW[-i]
matW[1,k]<-u2[i]
u2<-u2[-i]
aW[k,]<-a3[i,]
bW[k,1]<-b3[i]
cW[k,1]<-c3[i]
a3<-a3[-i,]
b3<-b3[-i]
c3<-c3[-i]
break }}
DLLE<-0
DLLE2<-0
DLLE <- function (theta) {
Vi <-0
for (i in 1:Ni) {
P<-cW[i]+(1-cW[i])*(1/(1+exp((-1.702)*t(aW[i,])%*%(theta-bW[i]*I))))
Q<-1-P
Vi<-Vi+aW[i,] * ((P-cW[i])*(matW[1,i]-P))/ ((1-cW[i])*P)
LL <- 1.702*Vi
}}
DLLE2<- function (theta) {
SSi<-0
for (i in 1:Ni){
P<-cW[i]+(1-cW[i])*(1/(1+exp((-1.702)*t(aW[i,])%*%(theta-bW[i]*I))))
Q<-1-P
SSi<-SSi+( aW[i,]%*%t(aW[i,])) * as.numeric( (Q*(P-cW[i])*(cW[i]*P-P^2))/ (P^2*(1-cW[i])^2) )
LL2<- – (1.702)^2*SSi
}}
if ( Ni == sum(matW[1,1:Ni]) | sum(matW[1,1:Ni]) == 0)
{  if (sum(matW[1,1:Ni])==0) thetaW<- thetaW – I
if (sum(matW[1,1:Ni])==Ni) thetaW<-thetaW+ I
SEMLE<-1}
else
{
Niter <- 10
for (iter in 1:Niter){
thetaW <- thetaW -(solve(-DLLE2(thetaW))) %*% (DLLE(thetaW))
SEMLE<- –solve(DLLE2(thetaW))
} }
ESTW[k,]<-thetaW
MLE[e,]<-ESTW[k,]
}  }
```

APPENDIX C

Mean Convergence Criteria of Each Latent Trait for 1,000 Simulees With Ten Iterations

Table C1

Mean Convergence Criteria of MBICAT
For the Bifactor Model with Two Group Factors

Conditions and θ estimation method		*G*	*g1*	*g2*
Condition A	MAP	<.0001	<.0001	<.0001
	MLE	<.0001	<.0001	<.0001
Condition B	MAP	<.0001	<.0001	<.0001
	MLE	<.0001	<.0001	<.0001
Condition C	MAP	<.0001	<.0001	<.0001
	MLE	<.0001	<.0001	<.0001

Table C2
Mean Convergence Criteria of MBICAT
For the Bifactor Model With Four Group Factors

Conditions and θ estimation method		*G*	*g1*	*g2*	*g3*	*g4*
Condition D	MAP	<.0001	<.0001	<.0001	<.0001	<.0001
	MLE	<.0001	<.0001	<.0001	<.0001	<.0001
Condition E	MAP	<.0001	<.0001	<.001	<.0001	<.0001
	MLE	<.0001	<.0001	<.0001	<.0001	<.0001
Condition F	MAP	<.0001	<.0001	<.0001	<.0001	<.0001
	MLE	<.0001	<.0001	<.0001	<.0001	<.0001

APPENDIX D

$r\left(\theta,\hat{\theta}\right)$ Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions

Table D1

$r\left(\theta,\hat{\theta}\right)$ Using BICAT(S), BICAT(D) and MBICAT Algorithms

for Three Group Factor Discrimination Conditions (Replication 1)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.948	0.662	0.475	0.969	0.488	0.378	0.453	0.489
EAP	0.948	0.664	0.476	0.968	0.488	0.380	0.454	0.487
MLE	0.947	0.663	0.469	0.968	0.503	0.380	0.443	0.499
BICAT(D)								
MAP	0.948	0.662	0.477	0.969	0.487	0.378	0.454	0.489
EAP	0.948	0.665	0.476	0.968	0.487	0.380	0.454	0.487
MLE	0.947	0.666	0.472	0.968	0.504	0.377	0.443	0.498
MBICAT								
MAP	0.951	0.814	0.799	0.973	0.839	0.795	0.738	0.809
EAP	0.949	0.818	0.780	0.972	0.830	0.791	0.740	0.807
MLE	0.953	0.797	0.811	0.973	0.836	0.792	0.739	0.806
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.926	0.718	0.503	0.944	0.641	0.552	0.618	0.452
EAP	0.926	0.716	0.506	0.945	0.647	0.550	0.616	0.451
MLE	0.924	0.718	0.499	0.944	0.646	0.564	0.614	0.433
BICAT(D)								
MAP	0.926	0.717	0.504	0.944	0.643	0.554	0.619	0.450
EAP	0.926	0.714	0.508	0.945	0.642	0.552	0.616	0.450
MLE	0.924	0.716	0.498	0.944	0.649	0.564	0.612	0.433
MBICAT								
MAP	0.949	0.850	0.835	0.947	0.856	0.807	0.834	0.797
EAP	0.943	0.847	0.833	0.948	0.857	0.806	0.834	0.796
MLE	0.952	0.839	0.845	0.946	0.860	0.805	0.833	0.799
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.905	0.759	0.572	0.945	0.621	0.599	0.649	0.582
EAP	0.904	0.759	0.570	0.945	0.621	0.600	0.650	0.584
MLE	0.901	0.749	0.576	0.945	0.614	0.598	0.651	0.584
BICAT(D)								
MAP	0.905	0.758	0.575	0.945	0.620	0.598	0.648	0.581
EAP	0.904	0.759	0.581	0.945	0.621	0.598	0.650	0.585
MLE	0.901	0.749	0.581	0.945	0.616	0.597	0.648	0.584
MBICAT								
MAP	0.937	0.863	0.870	0.955	0.880	0.850	0.839	0.822
EAP	0.924	0.863	0.859	0.956	0.880	0.849	0.840	0.823
MLE	0.938	0.860	0.872	0.954	0.879	0.848	0.838	0.821

Table D2

$r\left(\theta,\hat{\theta}\right)$ Using BICAT(S), BICAT(D) and MBICAT Algorithms

for Three Group Factor Discrimination Condition (Replication 2)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.949	0.628	0.458	0.973	0.434	0.438	0.323	0.522
EAP	0.949	0.622	0.459	0.973	0.433	0.437	0.323	0.522
MLE	0.950	0.621	0.460	0.973	0.429	0.451	0.322	0.528
BICAT(D)								
MAP	0.949	0.627	0.457	0.973	0.434	0.438	0.324	0.523
EAP	0.949	0.624	0.456	0.973	0.434	0.437	0.325	0.522
MLE	0.950	0.622	0.461	0.973	0.430	0.456	0.327	0.532
MBICAT								
MAP	0.956	0.831	0.760	0.973	0.802	0.818	0.802	0.856
EAP	0.955	0.831	0.756	0.973	0.801	0.817	0.797	0.847
MLE	0.956	0.834	0.757	0.974	0.799	0.814	0.802	0.854
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.925	0.663	0.495	0.965	0.543	0.519	0.485	0.578
EAP	0.925	0.662	0.495	0.965	0.544	0.518	0.483	0.578
MLE	0.928	0.662	0.497	0.965	0.537	0.521	0.478	0.580
BICAT(D)								
MAP	0.925	0.659	0.494	0.965	0.543	0.519	0.485	0.579
EAP	0.925	0.662	0.494	0.965	0.544	0.518	0.483	0.578
MLE	0.928	0.663	0.499	0.965	0.533	0.522	0.475	0.579
MBICAT								
MAP	0.939	0.841	0.796	0.972	0.867	0.800	0.859	0.841
EAP	0.933	0.848	0.762	0.971	0.864	0.801	0.861	0.839
MLE	0.940	0.850	0.797	0.971	0.866	0.797	0.857	0.842
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.912	0.684	0.539	0.951	0.619	0.583	0.610	0.618
EAP	0.912	0.686	0.544	0.951	0.617	0.583	0.611	0.622
MLE	0.916	0.688	0.534	0.952	0.622	0.588	0.606	0.621
BICAT(D)								
MAP	0.912	0.684	0.538	0.951	0.619	0.586	0.611	0.621
EAP	0.912	0.684	0.542	0.951	0.618	0.587	0.612	0.623
MLE	0.916	0.688	0.537	0.952	0.624	0.586	0.605	0.623
MBICAT								
MAP	0.930	0.847	0.815	0.958	0.835	0.812	0.868	0.842
EAP	0.924	0.849	0.791	0.958	0.836	0.810	0.869	0.847
MLE	0.924	0.849	0.800	0.958	0.837	0.814	0.869	0.844

Table D3

$r\left(\theta,\hat{\theta}\right)$ Using BICAT(S), BICAT(D) and MBICAT Algorithms

for Three Group Factor Discrimination Condition (Replication 3)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.958	0.510	0.455	0.966	0.605	0.413	0.637	0.530
EAP	0.958	0.508	0.458	0.966	0.606	0.412	0.635	0.527
MLE	0.959	0.501	0.457	0.966	0.606	0.409	0.629	0.528
BICAT(D)								
MAP	0.958	0.510	0.451	0.966	0.605	0.413	0.637	0.529
EAP	0.958	0.509	0.455	0.966	0.606	0.412	0.635	0.527
MLE	0.959	0.505	0.458	0.966	0.607	0.409	0.632	0.532
MBICAT								
MAP	0.966	0.822	0.784	0.971	0.813	0.715	0.808	0.805
EAP	0.965	0.829	0.755	0.969	0.810	0.716	0.808	0.804
MLE	0.967	0.824	0.784	0.972	0.809	0.719	0.815	0.807
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.943	0.587	0.497	0.958	0.499	0.554	0.425	0.573
EAP	0.944	0.585	0.494	0.958	0.501	0.554	0.423	0.577
MLE	0.945	0.603	0.492	0.958	0.497	0.556	0.419	0.578
BICAT(D)								
MAP	0.943	0.586	0.499	0.958	0.499	0.554	0.424	0.573
EAP	0.944	0.583	0.498	0.958	0.501	0.554	0.422	0.576
MLE	0.945	0.604	0.488	0.958	0.494	0.558	0.416	0.578
MBICAT								
MAP	0.958	0.860	0.801	0.964	0.801	0.820	0.790	0.820
EAP	0.955	0.851	0.770	0.964	0.802	0.818	0.789	0.822
MLE	0.958	0.860	0.794	0.964	0.808	0.819	0.792	0.823
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.935	0.604	0.518	0.949	0.492	0.546	0.620	0.624
EAP	0.936	0.605	0.519	0.950	0.493	0.547	0.621	0.624
MLE	0.934	0.610	0.517	0.949	0.488	0.552	0.620	0.635
BICAT(D)								
MAP	0.935	0.604	0.518	0.949	0.491	0.544	0.620	0.622
EAP	0.936	0.604	0.516	0.950	0.494	0.545	0.621	0.622
MLE	0.934	0.608	0.518	0.949	0.490	0.552	0.620	0.631
MBICAT								
MAP	0.949	0.859	0.806	0.955	0.840	0.835	0.825	0.843
EAP	0.943	0.846	0.772	0.954	0.838	0.836	0.823	0.844
MLE	0.951	0.861	0.796	0.955	0.841	0.836	0.823	0.847

Table D4

$r\left(\theta,\hat{\theta}\right)$ Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Condition (Replication 4)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.958	0.630	0.480	0.961	0.557	0.476	0.519	0.528
EAP	0.958	0.630	0.478	0.961	0.557	0.475	0.516	0.526
MLE	0.959	0.635	0.487	0.961	0.553	0.474	0.516	0.521
BICAT(D)								
MAP	0.958	0.629	0.479	0.961	0.557	0.476	0.518	0.528
EAP	0.958	0.629	0.475	0.961	0.557	0.475	0.516	0.526
MLE	0.959	0.634	0.486	0.961	0.552	0.479	0.516	0.523
MBICAT								
MAP	0.966	0.775	0.733	0.973	0.828	0.727	0.793	0.802
EAP	0.965	0.773	0.731	0.974	0.826	0.729	0.789	0.807
MLE	0.967	0.774	0.726	0.973	0.823	0.714	0.792	0.800
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.923	0.668	0.547	0.950	0.571	0.567	0.514	0.600
EAP	0.923	0.669	0.546	0.950	0.567	0.570	0.516	0.600
MLE	0.922	0.664	0.549	0.950	0.568	0.567	0.507	0.595
BICAT(D)								
MAP	0.923	0.664	0.547	0.950	0.570	0.567	0.514	0.601
EAP	0.923	0.665	0.546	0.950	0.567	0.569	0.516	0.599
MLE	0.922	0.667	0.542	0.950	0.568	0.570	0.507	0.592
MBICAT								
MAP	0.938	0.800	0.751	0.964	0.764	0.817	0.855	0.850
EAP	0.938	0.799	0.759	0.963	0.760	0.816	0.856	0.852
MLE	0.936	0.798	0.758	0.962	0.770	0.820	0.855	0.849
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.905	0.712	0.578	0.936	0.538	0.726	0.615	0.613
EAP	0.905	0.712	0.581	0.937	0.539	0.727	0.615	0.610
MLE	0.906	0.713	0.573	0.937	0.531	0.725	0.613	0.613
BICAT(D)								
MAP	0.905	0.712	0.577	0.936	0.537	0.726	0.614	0.613
EAP	0.905	0.712	0.580	0.937	0.539	0.727	0.613	0.609
MLE	0.906	0.710	0.571	0.937	0.535	0.725	0.615	0.615
MBICAT								
MAP	0.933	0.823	0.811	0.952	0.876	0.858	0.899	0.836
EAP	0.930	0.821	0.802	0.953	0.876	0.859	0.898	0.838
MLE	0.932	0.810	0.809	0.952	0.872	0.859	0.900	0.839

Table D5

$r\left(\theta,\hat{\theta}\right)$ Using BICAT(S), BICAT(D) and MBICAT Algorithms

for Three Group Factor Discrimination Condition (Replication 5)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.944	0.590	0.555	0.976	0.375	0.509	0.424	0.477
EAP	0.944	0.593	0.558	0.976	0.374	0.509	0.423	0.476
MLE	0.946	0.588	0.552	0.975	0.378	0.508	0.414	0.461
BICAT(D)								
MAP	0.944	0.592	0.554	0.976	0.375	0.510	0.423	0.477
EAP	0.944	0.595	0.558	0.976	0.374	0.510	0.423	0.476
MLE	0.946	0.581	0.550	0.975	0.378	0.510	0.415	0.462
MBICAT								
MAP	0.957	0.753	0.765	0.979	0.737	0.794	0.777	0.781
EAP	0.953	0.770	0.774	0.977	0.739	0.795	0.775	0.770
MLE	0.959	0.776	0.771	0.979	0.740	0.798	0.783	0.779
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.931	0.645	0.586	0.964	0.651	0.518	0.477	0.674
EAP	0.930	0.645	0.586	0.964	0.651	0.516	0.479	0.674
MLE	0.931	0.643	0.587	0.964	0.655	0.507	0.462	0.673
BICAT(D)								
MAP	0.931	0.643	0.589	0.964	0.649	0.518	0.477	0.675
EAP	0.930	0.640	0.589	0.964	0.649	0.516	0.479	0.674
MLE	0.931	0.640	0.583	0.964	0.654	0.508	0.463	0.673
MBICAT								
MAP	0.942	0.790	0.786	0.968	0.881	0.822	0.776	0.854
EAP	0.936	0.792	0.788	0.968	0.878	0.821	0.777	0.852
MLE	0.939	0.784	0.783	0.969	0.882	0.815	0.778	0.854
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.908	0.683	0.617	0.958	0.658	0.655	0.635	0.584
EAP	0.907	0.683	0.615	0.958	0.657	0.655	0.636	0.585
MLE	0.908	0.682	0.621	0.958	0.654	0.653	0.630	0.587
BICAT(D)								
MAP	0.908	0.684	0.616	0.958	0.658	0.651	0.635	0.585
EAP	0.907	0.684	0.616	0.958	0.660	0.651	0.636	0.586
MLE	0.908	0.679	0.627	0.958	0.662	0.654	0.633	0.581
MBICAT								
MAP	0.917	0.803	0.798	0.966	0.891	0.873	0.880	0.839
EAP	0.912	0.800	0.788	0.966	0.888	0.874	0.883	0.838
MLE	0.916	0.810	0.801	0.965	0.889	0.870	0.878	0.831

Table D6

$r\left(\theta,\hat{\theta}\right)$ Using BICAT(S), BICAT(D) and MBICAT Algorithms

for Three Group Factor Discrimination Condition (Replication 6)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.926	0.403	0.480	0.961	0.579	0.513	0.495	0.476
EAP	0.926	0.403	0.479	0.960	0.578	0.514	0.494	0.471
MLE	0.926	0.394	0.473	0.960	0.572	0.520	0.502	0.478
BICAT(D)								
MAP	0.926	0.402	0.479	0.961	0.578	0.513	0.496	0.476
EAP	0.926	0.402	0.476	0.960	0.578	0.514	0.494	0.471
MLE	0.926	0.383	0.470	0.960	0.574	0.519	0.509	0.477
MBICAT								
MAP	0.945	0.743	0.753	0.966	0.762	0.825	0.794	0.778
EAP	0.935	0.715	0.742	0.965	0.768	0.824	0.794	0.777
MLE	0.944	0.737	0.751	0.965	0.761	0.829	0.792	0.780
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.900	0.456	0.517	0.957	0.570	0.555	0.580	0.535
EAP	0.900	0.453	0.516	0.957	0.571	0.557	0.582	0.539
MLE	0.899	0.447	0.517	0.956	0.575	0.549	0.582	0.537
BICAT(D)								
MAP	0.900	0.458	0.513	0.957	0.570	0.555	0.580	0.537
EAP	0.900	0.457	0.511	0.957	0.572	0.557	0.581	0.540
MLE	0.899	0.449	0.517	0.956	0.571	0.547	0.581	0.534
MBICAT								
MAP	0.928	0.761	0.761	0.964	0.784	0.843	0.836	0.822
EAP	0.918	0.740	0.747	0.963	0.781	0.846	0.836	0.824
MLE	0.926	0.780	0.763	0.965	0.782	0.843	0.842	0.823
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.862	0.516	0.563	0.955	0.576	0.514	0.607	0.530
EAP	0.862	0.515	0.555	0.955	0.576	0.513	0.607	0.528
MLE	0.861	0.516	0.566	0.955	0.578	0.513	0.602	0.528
BICAT(D)								
MAP	0.862	0.518	0.562	0.955	0.578	0.515	0.607	0.529
EAP	0.862	0.519	0.556	0.955	0.575	0.515	0.607	0.532
MLE	0.861	0.516	0.562	0.955	0.585	0.512	0.597	0.531
MBICAT								
MAP	0.908	0.766	0.778	0.960	0.848	0.871	0.846	0.863
EAP	0.898	0.757	0.755	0.960	0.848	0.874	0.842	0.860
MLE	0.907	0.776	0.773	0.960	0.854	0.875	0.848	0.860

Table D7

$r\left(\theta,\hat{\theta}\right)$ Using BICAT(S), BICAT(D) and MBICAT Algorithms

for Three Group Factor Discrimination Condition (Replication 7)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.932	0.679	0.409	0.953	0.511	0.588	0.636	0.652
EAP	0.932	0.677	0.413	0.953	0.509	0.587	0.638	0.651
MLE	0.934	0.670	0.410	0.953	0.522	0.587	0.639	0.650
BICAT(D)								
MAP	0.932	0.677	0.403	0.953	0.511	0.588	0.635	0.652
EAP	0.932	0.678	0.408	0.953	0.509	0.587	0.637	0.650
MLE	0.934	0.665	0.406	0.953	0.523	0.585	0.639	0.651
MBICAT								
MAP	0.939	0.787	0.750	0.967	0.794	0.848	0.864	0.835
EAP	0.934	0.795	0.743	0.967	0.797	0.841	0.865	0.834
MLE	0.935	0.781	0.752	0.966	0.791	0.846	0.865	0.835
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.913	0.722	0.469	0.959	0.507	0.547	0.545	0.560
EAP	0.913	0.722	0.470	0.959	0.510	0.546	0.538	0.561
MLE	0.913	0.719	0.476	0.958	0.509	0.560	0.540	0.549
BICAT(D)								
MAP	0.913	0.721	0.470	0.959	0.507	0.547	0.543	0.561
EAP	0.913	0.725	0.472	0.959	0.512	0.546	0.538	0.560
MLE	0.913	0.719	0.469	0.958	0.507	0.556	0.541	0.551
MBICAT								
MAP	0.928	0.815	0.783	0.972	0.787	0.886	0.809	0.811
EAP	0.925	0.814	0.775	0.972	0.785	0.887	0.810	0.812
MLE	0.924	0.817	0.768	0.972	0.778	0.886	0.811	0.814
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.895	0.746	0.488	0.958	0.587	0.540	0.566	0.767
EAP	0.897	0.745	0.493	0.958	0.586	0.541	0.566	0.765
MLE	0.894	0.739	0.497	0.958	0.591	0.546	0.569	0.760
BICAT(D)								
MAP	0.895	0.745	0.485	0.958	0.591	0.541	0.566	0.767
EAP	0.897	0.745	0.493	0.958	0.590	0.541	0.566	0.767
MLE	0.894	0.742	0.493	0.958	0.596	0.544	0.570	0.765
MBICAT								
MAP	0.917	0.842	0.809	0.950	0.858	0.835	0.845	0.884
EAP	0.909	0.839	0.789	0.951	0.858	0.832	0.849	0.887
MLE	0.912	0.837	0.796	0.951	0.857	0.836	0.851	0.883

Table D8

$r\left(\theta,\hat{\theta}\right)$ Using BICAT(S), BICAT(D) and MBICAT Algorithms

for Three Group Factor Discrimination Condition (Replication 2)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.947	0.481	0.536	0.958	0.587	0.519	0.597	0.535
EAP	0.946	0.478	0.535	0.958	0.587	0.519	0.597	0.534
MLE	0.949	0.481	0.533	0.958	0.592	0.508	0.590	0.542
BICAT(D)								
MAP	0.947	0.481	0.536	0.958	0.586	0.519	0.597	0.534
EAP	0.946	0.481	0.538	0.958	0.587	0.519	0.597	0.533
MLE	0.949	0.489	0.531	0.958	0.590	0.508	0.591	0.544
MBICAT								
MAP	0.957	0.811	0.776	0.965	0.785	0.824	0.827	0.822
EAP	0.956	0.799	0.787	0.965	0.789	0.821	0.825	0.827
MLE	0.956	0.815	0.777	0.964	0.788	0.823	0.822	0.818
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.937	0.543	0.583	0.952	0.494	0.496	0.585	0.444
EAP	0.937	0.543	0.587	0.952	0.496	0.499	0.583	0.444
MLE	0.936	0.532	0.570	0.952	0.489	0.498	0.584	0.435
BICAT(D)								
MAP	0.937	0.542	0.583	0.952	0.492	0.497	0.585	0.444
EAP	0.937	0.544	0.587	0.952	0.495	0.500	0.584	0.444
MLE	0.936	0.538	0.568	0.952	0.486	0.495	0.583	0.435
MBICAT								
MAP	0.946	0.826	0.804	0.959	0.763	0.843	0.807	0.800
EAP	0.944	0.821	0.800	0.960	0.764	0.841	0.810	0.800
MLE	0.950	0.836	0.812	0.960	0.752	0.844	0.807	0.801
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.923	0.581	0.619	0.915	0.564	0.609	0.577	0.709
EAP	0.923	0.582	0.620	0.915	0.565	0.606	0.576	0.709
MLE	0.922	0.583	0.614	0.915	0.559	0.608	0.584	0.704
BICAT(D)								
MAP	0.923	0.579	0.619	0.915	0.563	0.609	0.578	0.710
EAP	0.923	0.581	0.619	0.915	0.565	0.607	0.577	0.710
MLE	0.922	0.587	0.611	0.915	0.554	0.610	0.582	0.705
MBICAT								
MAP	0.924	0.826	0.799	0.937	0.827	0.816	0.855	0.887
EAP	0.926	0.828	0.796	0.936	0.827	0.814	0.857	0.887
MLE	0.925	0.833	0.797	0.939	0.825	0.815	0.852	0.889

Table D9

$r\left(\theta,\hat{\theta}\right)$ Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Condition (Replication 9)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.941	0.567	0.465	0.965	0.670	0.499	0.579	0.449
EAP	0.941	0.570	0.465	0.965	0.669	0.498	0.582	0.447
MLE	0.942	0.573	0.471	0.965	0.656	0.494	0.583	0.438
BICAT(D)								
MAP	0.941	0.566	0.466	0.965	0.670	0.499	0.579	0.449
EAP	0.941	0.567	0.463	0.965	0.669	0.498	0.582	0.447
MLE	0.942	0.570	0.466	0.965	0.657	0.492	0.582	0.438
MBICAT								
MAP	0.946	0.764	0.683	0.974	0.863	0.784	0.849	0.813
EAP	0.944	0.752	0.675	0.975	0.861	0.775	0.849	0.813
MLE	0.946	0.756	0.673	0.974	0.863	0.783	0.850	0.818
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.922	0.608	0.488	0.960	0.611	0.553	0.516	0.510
EAP	0.924	0.610	0.485	0.959	0.613	0.555	0.518	0.516
MLE	0.925	0.614	0.492	0.959	0.604	0.554	0.519	0.512
BICAT(D)								
MAP	0.922	0.611	0.489	0.960	0.612	0.553	0.512	0.510
EAP	0.924	0.609	0.485	0.959	0.614	0.555	0.515	0.516
MLE	0.925	0.610	0.490	0.959	0.610	0.550	0.521	0.514
MBICAT								
MAP	0.936	0.774	0.682	0.966	0.817	0.840	0.759	0.811
EAP	0.933	0.772	0.667	0.965	0.814	0.838	0.757	0.810
MLE	0.932	0.769	0.672	0.967	0.809	0.841	0.760	0.811
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.913	0.648	0.519	0.932	0.603	0.683	0.549	0.546
EAP	0.913	0.645	0.516	0.932	0.601	0.683	0.549	0.548
MLE	0.912	0.645	0.521	0.932	0.592	0.677	0.544	0.539
BICAT(D)								
MAP	0.913	0.648	0.518	0.932	0.604	0.682	0.550	0.547
EAP	0.913	0.649	0.515	0.932	0.602	0.682	0.552	0.549
MLE	0.912	0.646	0.518	0.932	0.594	0.681	0.547	0.547
MBICAT								
MAP	0.920	0.804	0.688	0.936	0.778	0.841	0.830	0.827
EAP	0.919	0.805	0.684	0.937	0.784	0.845	0.829	0.834
MLE	0.921	0.800	0.681	0.938	0.779	0.842	0.837	0.835

Table D10

$r\left(\theta,\hat{\theta}\right)$ Using BICAT(S), BICAT(D) and MBICAT Algorithms

for Three Group Factor Discrimination Condition (Replication 10)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.925	0.607	0.606	0.971	0.540	0.557	0.584	0.520
EAP	0.924	0.605	0.610	0.971	0.538	0.557	0.582	0.520
MLE	0.923	0.615	0.613	0.971	0.534	0.563	0.587	0.515
BICAT(D)								
MAP	0.925	0.604	0.610	0.971	0.541	0.557	0.583	0.520
EAP	0.924	0.604	0.614	0.971	0.539	0.557	0.579	0.521
MLE	0.923	0.617	0.613	0.971	0.533	0.562	0.591	0.517
MBICAT								
MAP	0.937	0.805	0.820	0.979	0.873	0.803	0.856	0.806
EAP	0.930	0.796	0.802	0.978	0.871	0.798	0.853	0.802
MLE	0.938	0.812	0.815	0.979	0.870	0.802	0.857	0.804
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.905	0.644	0.637	0.959	0.580	0.502	0.407	0.514
EAP	0.906	0.641	0.637	0.959	0.586	0.502	0.409	0.513
MLE	0.906	0.649	0.641	0.958	0.584	0.506	0.410	0.507
BICAT(D)								
MAP	0.905	0.645	0.637	0.959	0.585	0.502	0.408	0.512
EAP	0.906	0.643	0.635	0.959	0.585	0.502	0.410	0.515
MLE	0.906	0.652	0.641	0.958	0.587	0.506	0.413	0.513
MBICAT								
MAP	0.921	0.808	0.838	0.969	0.775	0.830	0.766	0.846
EAP	0.915	0.804	0.818	0.970	0.767	0.830	0.771	0.848
MLE	0.921	0.816	0.837	0.969	0.777	0.828	0.766	0.847
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.877	0.696	0.675	0.943	0.615	0.527	0.579	0.687
EAP	0.877	0.695	0.676	0.943	0.612	0.526	0.579	0.688
MLE	0.880	0.692	0.674	0.942	0.623	0.519	0.584	0.681
BICAT(D)								
MAP	0.877	0.694	0.673	0.943	0.615	0.528	0.578	0.690
EAP	0.877	0.694	0.675	0.943	0.615	0.527	0.576	0.689
MLE	0.880	0.693	0.675	0.942	0.621	0.517	0.584	0.682
MBICAT								
MAP	0.902	0.849	0.828	0.943	0.829	0.823	0.838	0.865
EAP	0.892	0.838	0.806	0.943	0.827	0.824	0.839	0.865
MLE	0.902	0.846	0.832	0.943	0.825	0.821	0.835	0.869

APPENDIX E

Formulas for the ANOVA Sums of Squares and Degrees of Freedom

Sums of Squares

There were 60 response sets (10 replications × 6 between subjects) and 9 within subject. Thus, a total of 540 unique cells were modeled by the mixed-design ANOVA. For purposes of the formulas to be presented below, the following notation was used:

F = Group factor discrimination condition,
G = Number of group factors,
E = Estimation method,
C = CAT algorithm,
Bet = total between subjects variability in the model,
W_E = within subjects variability of estimation method,
W_C = within subjects variability of CAT algorithm,
W_{EC} = within subjects variability of estimation and CAT algorithm, and
With = total within subjects variability in the model,
$\overline{X}$ = mean of all observations.

$$SS_{total} = \sum_{j=1}^{540} (X_j - \overline{X})^2 \tag{E1}$$

$$SS_{Bet} = \sum_{i=1}^{60} (\overline{X}_i - \overline{X})^2 \tag{E2}$$

$$SS_{With} = SS_{total} - SS_{Bet} \tag{E3}$$

$$SS_F = 180 \sum_{c=1}^{3} (\overline{X}_F - \overline{X})^2 \tag{E4}$$

$$SS_G = 270 \sum_{c=1}^{2} (\overline{X}_G - \overline{X})^2 \tag{E5}$$

$$SS_E = 180 \sum_{c=1}^{3} (\overline{X}_E - \overline{X})^2 \tag{E6}$$

$$SS_C = 180 \sum_{c=1}^{3} (\overline{X}_C - \overline{X})^2 \tag{E7}$$

$$SS_{Cells(FG)} = 90 \sum_{c=1}^{6} (\overline{X}_{FG} - \overline{X})^2 \tag{E8}$$

$$SS_{FG} = SS_{Cells(FG)} - SS_F - SS_G \tag{E9}$$

$$SS_{Cells(EF)} = 60 \sum_{c=1}^{9} (\overline{X}_{EF} - \overline{X})^2 \tag{E10}$$

$$SS_{EF} = SS_{Cells(EF)} - SS_E - SS_F \tag{E11}$$

$$SS_{Cells(CF)} = 60 \sum_{c=1}^{9} (\overline{X}_{CF} - \overline{X})^2 \tag{E12}$$

$$SS_{CF} = SS_{Cells(CF)} - SS_C - SS_F \tag{E13}$$

$$SS_{Cells(EG)} = 60\sum_{c=1}^{9}(\bar{X}_{EG} - \bar{X})^2 \quad \text{(E14)}$$

$$SS_{EG} = SS_{Cells(EG)} - SS_E - SS_G \quad \text{(E15)}$$

$$SS_{Cells(CG)} = 90\sum_{c=1}^{6}(\bar{X}_{CG} - \bar{X})^2 \quad \text{(E16)}$$

$$SS_{CG} = SS_{Cells(CG)} - SS_C - SS_G \quad \text{(E17)}$$

$$SS_{Cells(CE)} = 60\sum_{c=1}^{9}(\bar{X}_{CE} - \bar{X})^2 \quad \text{(E18)}$$

$$SS_{CE} = SS_{Cells(CE)} - SS_C - SS_E \quad \text{(E19)}$$

$$SS_{Cells(EFG)} = 30\sum_{c=1}^{18}(\bar{X}_{EFG} - \bar{X})^2 \quad \text{(E20)}$$

$$SS_{EFG} = SS_{Cells(EFG)} - SS_F - SS_G - SS_E - SS_{FG} - SS_{EF} - SS_{EG} \quad \text{(E21)}$$

$$SS_{Cells(CFG)} = 30\sum_{c=1}^{18}(\bar{X}_{CFG} - \bar{X})^2 \quad \text{(E22)}$$

$$SS_{CFG} = SS_{Cells(CFG)} - SS_F - SS_G - SS_C - SS_{FG} - SS_{CF} - SSC_G \quad \text{(E23)}$$

$$SS_{Cells(ECF)} = 20\sum_{c=1}^{27}(\bar{X}_{ECF} - \bar{X})^2 \quad \text{(E24)}$$

$$SS_{ECF} = SS_{Cells(ECF)} - SS_F - SS_E - SS_C - SS_{EF} - SS_{CF} - SS_{EC} \quad \text{(E25)}$$

$$SS_{Cells(ECG)} = 30\sum_{c=1}^{18}(\bar{X}_{ECG} - \bar{X})^2 \quad \text{(E26)}$$

$$SS_{ECG} = SS_{Cells(ECG)} - SS_G - SS_E - SS_C - SS_{EG} - SS_{CG} - SS_{EC} \quad \text{(E27)}$$

$$SS_{Cells(ECFG)} = 10\sum_{c=1}^{54}(\bar{X}_{ECFG} - \bar{X})^2 \quad \text{(E28)}$$

$$\begin{aligned} SS_{ECFG} &= SS_{Cells(ECFG)} - SS_F - SS_E - SS_C - SS_G - SS_{FG} - SS_{EF} - SS_{CF} \\ &- SS_{EG} - SS_{CG} - SS_{EC} - SS_{EFG} - SS_{CFG} - SS_{ECF} - SS_{ECG} \end{aligned} \quad \text{(E29)}$$

$$SS_{subject(Bet)} = SS_{Bet} - SS_F - SS_G - SS_{FG} \quad \text{(E30)}$$

$$SS_{Subject(W_E)} = SS_{Within} - SS_E - SS_{EF} - SS_{EG} - SS_{EFG} \quad \text{(E31)}$$

$$SS_{Subject(W_C)} = SS_{Within} - SS_C - SS_{CF} - SS_{CG} - SS_{CFG} \quad \text{(E32)}$$

$$\begin{aligned} SS_{Subject(W_{EC})} &= SS_{Within} - SS_E - SS_C - SS_{EF} - SS_{EG} - SS_{CF} - SS_{CG} - SS_{EC} \\ &- SS_{EFG} - SS_{CFG} - SS_{ECF} - SS_{ECG} - SS_{ECFG} \end{aligned} \quad \text{(E33)}$$

Degrees of Freedom

The following terms are defined for the *df* reported below:

f = Group factor discrimination condition (3),
g= Number of group factors (2),
e= Estimation method (3),
c = CAT algorithm (3),
n= number of replications (10),

$$df_F = (f-1) = 2 \quad \text{(E34)}$$

$$df_G = (g-1) = 1 \quad \text{(E35)}$$

$$df_E = (e-1) = 2 \quad \text{(E36)}$$

$$df_C = (c-1) = 2 \quad \text{(E37)}$$

$$df_{FG} = (f-1)(g-1) = 2 \quad \text{(E38)}$$

$$df_{FE} = (f-1)(e-1) = 4 \quad \text{(E39)}$$

$$df_{FC} = (f-1)(c-1) = 4 \quad \text{(E40)}$$

$$df_{GE} = (g-1)(e-1) = 2 \quad \text{(E41)}$$

$$df_{GC} = (g-1)(c-1) = 2 \quad \text{(E42)}$$

$$df_{EC} = (e-1)(c-1) = 4 \quad \text{(E43)}$$

$$df_{FGE} = (f-1)(g-1)(e-1) = 4 \quad \text{(E44)}$$

$$df_{FGC} = (f-1)(g-1)(c-1) = 4 \quad \text{(E45)}$$

$$df_{FEC} = (f-1)(e-1)(c-1) = 8 \quad \text{(E46)}$$

$$df_{GEC} = (g-1)(e-1)(c-1) = 4 \quad \text{(E47)}$$

$$df_{FECG} = (f-1)(e-1)(c-1)(g-1) = 8 \quad \text{(E48)}$$

$$df_{Subject(Bet)} = f \times g \times (n-1) = 54 \quad \text{(E49)}$$

$$df_{Subject(W_E)} = f \times g \times (e-1) \times (n-1) = 108 \quad \text{(E50)}$$

$$df_{Subject(W_C)} = f \times g \times (c-1) \times (n-1) = 108 \quad \text{(E51)}$$

$$df_{Subject(W_{EC})} = f \times g \times (e-1) \times (c-1) \times (n-1) = 216 = 108 \quad \text{(E52)}$$

APPENDIX F

RMSEs Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions

Table F1

RMSEs Using BICAT(S), BICAT(D) and MBICAT Algorithms for
Three Group Factor Discrimination Conditions (Replication 1)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Three group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.357	0.764	0.894	0.371	0.842	0.887	0.825	0.905
EAP	0.355	0.763	0.893	0.372	0.842	0.887	0.825	0.906
MLE	0.346	0.769	0.906	0.365	0.857	0.917	0.876	0.933
BICAT(D)								
MAP	0.357	0.763	0.892	0.371	0.842	0.887	0.823	0.905
EAP	0.355	0.762	0.893	0.372	0.842	0.887	0.824	0.906
MLE	0.346	0.765	0.904	0.365	0.856	0.919	0.875	0.933
MBICAT								
MAP	0.284	0.597	0.612	0.234	0.512	0.556	0.588	0.592
EAP	0.292	0.611	0.662	0.241	0.525	0.562	0.586	0.593
MLE	0.282	0.616	0.599	0.235	0.516	0.560	0.586	0.595
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.402	0.710	0.876	0.379	0.748	0.795	0.819	0.871
EAP	0.400	0.712	0.874	0.379	0.744	0.798	0.821	0.870
MLE	0.394	0.710	0.886	0.373	0.748	0.801	0.824	0.900
BICAT(D)								
MAP	0.402	0.711	0.876	0.379	0.747	0.794	0.818	0.871
EAP	0.400	0.714	0.873	0.379	0.748	0.796	0.821	0.870
MLE	0.394	0.713	0.885	0.373	0.745	0.800	0.825	0.899
MBICAT								
MAP	0.290	0.540	0.565	0.291	0.503	0.556	0.580	0.573
EAP	0.307	0.562	0.609	0.289	0.501	0.558	0.579	0.576
MLE	0.282	0.556	0.552	0.293	0.497	0.559	0.581	0.571
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.441	0.666	0.828	0.455	0.762	0.804	0.783	0.758
EAP	0.442	0.666	0.830	0.454	0.762	0.803	0.782	0.757
MLE	0.438	0.675	0.826	0.448	0.780	0.815	0.789	0.772
BICAT(D)								
MAP	0.441	0.667	0.825	0.455	0.762	0.805	0.783	0.760
EAP	0.442	0.666	0.821	0.454	0.762	0.805	0.783	0.757
MLE	0.438	0.674	0.823	0.448	0.777	0.817	0.790	0.771
MBICAT								
MAP	0.327	0.518	0.515	0.302	0.448	0.526	0.557	0.525
EAP	0.351	0.532	0.560	0.301	0.451	0.528	0.557	0.524
MLE	0.325	0.522	0.514	0.305	0.451	0.529	0.558	0.526

Table F2

RMSEs Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions (Replication 2)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Three group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.413	0.858	0.849	0.385	0.932	0.946	0.985	0.875
EAP	0.410	0.863	0.848	0.385	0.932	0.947	0.987	0.877
MLE	0.397	0.870	0.871	0.376	0.967	0.961	1.019	0.922
BICAT(D)								
MAP	0.413	0.859	0.850	0.385	0.931	0.946	0.983	0.876
EAP	0.410	0.862	0.850	0.385	0.931	0.947	0.985	0.877
MLE	0.397	0.869	0.872	0.376	0.966	0.954	1.016	0.919
MBICAT								
MAP	0.307	0.615	0.597	0.245	0.601	0.600	0.582	0.501
EAP	0.322	0.654	0.611	0.245	0.604	0.600	0.589	0.517
MLE	0.310	0.610	0.600	0.239	0.606	0.603	0.581	0.503
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.467	0.825	0.829	0.440	0.882	0.822	0.990	0.804
EAP	0.468	0.825	0.830	0.441	0.882	0.825	0.991	0.803
MLE	0.454	0.829	0.846	0.431	0.895	0.851	1.007	0.818
BICAT(D)								
MAP	0.467	0.829	0.830	0.440	0.882	0.822	0.990	0.803
EAP	0.468	0.826	0.830	0.441	0.881	0.824	0.991	0.803
MLE	0.454	0.827	0.843	0.431	0.898	0.850	1.010	0.818
MBICAT								
MAP	0.361	0.597	0.557	0.260	0.528	0.553	0.585	0.526
EAP	0.380	0.615	0.596	0.265	0.533	0.554	0.583	0.530
MLE	0.358	0.581	0.558	0.264	0.529	0.557	0.589	0.526
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.507	0.803	0.795	0.451	0.788	0.805	0.834	0.783
EAP	0.507	0.801	0.790	0.452	0.790	0.806	0.835	0.780
MLE	0.489	0.800	0.811	0.440	0.803	0.818	0.851	0.798
BICAT(D)								
MAP	0.507	0.803	0.795	0.451	0.788	0.801	0.833	0.779
EAP	0.507	0.804	0.792	0.452	0.789	0.801	0.833	0.778
MLE	0.489	0.801	0.810	0.440	0.801	0.820	0.853	0.797
MBICAT								
MAP	0.384	0.585	0.534	0.312	0.539	0.566	0.522	0.523
EAP	0.406	0.601	0.564	0.311	0.538	0.569	0.521	0.517
MLE	0.401	0.582	0.552	0.311	0.535	0.564	0.521	0.521

Table F3

RMSEs Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions (Replication 3)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Three group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.407	0.880	0.884	0.343	0.884	0.934	0.747	0.836
EAP	0.408	0.881	0.884	0.343	0.883	0.935	0.750	0.840
MLE	0.392	0.906	0.907	0.337	0.889	0.965	0.778	0.871
BICAT(D)								
MAP	0.407	0.880	0.889	0.343	0.884	0.934	0.747	0.836
EAP	0.408	0.881	0.886	0.343	0.883	0.935	0.749	0.840
MLE	0.392	0.901	0.907	0.337	0.888	0.966	0.774	0.868
MBICAT								
MAP	0.282	0.576	0.583	0.239	0.654	0.700	0.570	0.574
EAP	0.291	0.604	0.629	0.243	0.657	0.698	0.571	0.574
MLE	0.281	0.572	0.583	0.235	0.660	0.696	0.561	0.571
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.469	0.819	0.853	0.419	0.858	0.781	0.855	0.791
EAP	0.468	0.821	0.854	0.419	0.857	0.783	0.855	0.788
MLE	0.451	0.814	0.876	0.412	0.867	0.807	0.874	0.802
BICAT(D)								
MAP	0.469	0.820	0.852	0.419	0.858	0.781	0.855	0.791
EAP	0.468	0.822	0.851	0.419	0.857	0.782	0.856	0.788
MLE	0.451	0.812	0.876	0.412	0.869	0.803	0.877	0.803
MBICAT								
MAP	0.315	0.515	0.563	0.269	0.593	0.524	0.573	0.545
EAP	0.331	0.558	0.609	0.270	0.593	0.527	0.575	0.543
MLE	0.313	0.515	0.571	0.270	0.585	0.525	0.569	0.542
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.499	0.807	0.835	0.464	0.839	0.890	0.797	0.765
EAP	0.499	0.807	0.833	0.464	0.838	0.890	0.797	0.767
MLE	0.488	0.809	0.847	0.457	0.864	0.896	0.807	0.771
BICAT(D)								
MAP	0.499	0.807	0.835	0.464	0.838	0.892	0.798	0.767
EAP	0.499	0.806	0.835	0.464	0.836	0.892	0.798	0.769
MLE	0.488	0.810	0.846	0.457	0.863	0.894	0.807	0.773
MBICAT								
MAP	0.345	0.518	0.556	0.311	0.497	0.585	0.567	0.513
EAP	0.374	0.558	0.604	0.311	0.501	0.583	0.569	0.513
MLE	0.339	0.514	0.568	0.311	0.495	0.582	0.571	0.508

Table F4

RMSEs Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions (Replication 4)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Three group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.378	0.756	0.940	0.373	0.743	0.760	0.896	0.887
EAP	0.378	0.756	0.941	0.374	0.743	0.761	0.900	0.889
MLE	0.372	0.766	0.951	0.366	0.769	0.788	0.916	0.920
BICAT(D)								
MAP	0.378	0.756	0.941	0.373	0.743	0.760	0.897	0.887
EAP	0.378	0.756	0.943	0.374	0.743	0.761	0.900	0.889
MLE	0.372	0.765	0.949	0.366	0.769	0.783	0.917	0.918
MBICAT								
MAP	0.325	0.609	0.719	0.230	0.494	0.582	0.632	0.616
EAP	0.320	0.617	0.747	0.228	0.495	0.581	0.638	0.609
MLE	0.330	0.609	0.728	0.233	0.500	0.596	0.634	0.619
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.433	0.719	0.889	0.419	0.770	0.732	0.872	0.741
EAP	0.433	0.718	0.890	0.419	0.772	0.730	0.871	0.741
MLE	0.428	0.729	0.897	0.412	0.773	0.760	0.884	0.756
BICAT(D)								
MAP	0.433	0.722	0.890	0.419	0.770	0.732	0.872	0.741
EAP	0.433	0.721	0.890	0.419	0.772	0.730	0.871	0.742
MLE	0.428	0.726	0.903	0.412	0.773	0.756	0.883	0.759
MBICAT								
MAP	0.346	0.579	0.698	0.259	0.605	0.496	0.551	0.485
EAP	0.343	0.585	0.709	0.261	0.609	0.498	0.548	0.482
MLE	0.353	0.582	0.690	0.263	0.598	0.493	0.551	0.486
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.469	0.675	0.864	0.430	0.856	0.757	0.813	0.727
EAP	0.469	0.675	0.862	0.429	0.855	0.755	0.813	0.730
MLE	0.460	0.677	0.875	0.424	0.872	0.757	0.822	0.740
BICAT(D)								
MAP	0.469	0.675	0.865	0.430	0.855	0.757	0.814	0.726
EAP	0.469	0.675	0.862	0.429	0.854	0.756	0.814	0.731
MLE	0.460	0.681	0.877	0.424	0.868	0.757	0.820	0.738
MBICAT								
MAP	0.358	0.545	0.627	0.295	0.491	0.564	0.456	0.507
EAP	0.364	0.551	0.659	0.293	0.493	0.562	0.457	0.505
MLE	0.362	0.567	0.629	0.294	0.498	0.562	0.456	0.503

Table F5

RMSEs Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions (Replication 5)

CAT Algorithm and θ estimation Method	*Two group factors*			*Three group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.356	0.816	0.790	0.389	0.881	0.903	0.870	0.832
EAP	0.357	0.815	0.788	0.389	0.881	0.903	0.872	0.835
MLE	0.347	0.830	0.811	0.380	0.914	0.931	0.914	0.900
BICAT(D)								
MAP	0.356	0.815	0.791	0.389	0.881	0.902	0.870	0.832
EAP	0.357	0.813	0.788	0.389	0.881	0.902	0.872	0.835
MLE	0.347	0.836	0.813	0.380	0.914	0.930	0.912	0.899
MBICAT								
MAP	0.266	0.657	0.603	0.224	0.605	0.623	0.571	0.541
EAP	0.279	0.658	0.602	0.230	0.603	0.622	0.575	0.555
MLE	0.261	0.630	0.597	0.224	0.602	0.619	0.564	0.543
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.392	0.768	0.766	0.393	0.725	0.794	0.847	0.824
EAP	0.394	0.768	0.766	0.393	0.725	0.797	0.845	0.824
MLE	0.386	0.777	0.776	0.386	0.731	0.831	0.874	0.834
BICAT(D)								
MAP	0.392	0.771	0.764	0.393	0.726	0.794	0.847	0.822
EAP	0.394	0.773	0.763	0.393	0.726	0.797	0.845	0.823
MLE	0.386	0.780	0.778	0.386	0.733	0.831	0.872	0.833
MBICAT								
MAP	0.310	0.612	0.578	0.254	0.455	0.505	0.600	0.577
EAP	0.327	0.624	0.585	0.256	0.458	0.506	0.598	0.580
MLE	0.318	0.619	0.583	0.253	0.453	0.514	0.597	0.576
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.431	0.731	0.741	0.426	0.808	0.778	0.840	0.802
EAP	0.433	0.732	0.743	0.424	0.809	0.779	0.839	0.804
MLE	0.426	0.735	0.744	0.419	0.829	0.796	0.854	0.824
BICAT(D)								
MAP	0.431	0.730	0.743	0.426	0.808	0.783	0.839	0.801
EAP	0.433	0.731	0.742	0.424	0.808	0.784	0.840	0.802
MLE	0.426	0.738	0.739	0.419	0.819	0.794	0.851	0.828
MBICAT								
MAP	0.368	0.595	0.565	0.280	0.485	0.496	0.521	0.516
EAP	0.378	0.608	0.584	0.281	0.490	0.494	0.515	0.518
MLE	0.370	0.585	0.562	0.286	0.489	0.501	0.527	0.528

Table F6
RMSEs Using BICAT(S), BICAT(D) and MBICAT Algorithms for
Three Group Factor Discrimination Conditions (Replication 6)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Three group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.453	0.961	0.934	0.387	0.868	0.965	0.910	0.892
EAP	0.453	0.961	0.935	0.388	0.869	0.965	0.912	0.898
MLE	0.446	0.983	0.954	0.380	0.903	0.975	0.933	0.926
BICAT(D)								
MAP	0.453	0.961	0.935	0.387	0.869	0.965	0.909	0.892
EAP	0.453	0.961	0.936	0.388	0.870	0.965	0.912	0.898
MLE	0.446	0.990	0.958	0.380	0.901	0.975	0.926	0.928
MBICAT								
MAP	0.338	0.685	0.675	0.275	0.680	0.626	0.627	0.617
EAP	0.371	0.727	0.722	0.277	0.672	0.628	0.628	0.620
MLE	0.343	0.692	0.675	0.276	0.682	0.621	0.630	0.615
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.506	0.926	0.905	0.455	0.852	0.971	0.885	0.853
EAP	0.507	0.928	0.906	0.455	0.852	0.969	0.884	0.849
MLE	0.502	0.945	0.914	0.448	0.859	0.990	0.890	0.872
BICAT(D)								
MAP	0.506	0.925	0.908	0.455	0.852	0.971	0.886	0.851
EAP	0.507	0.925	0.910	0.455	0.850	0.970	0.884	0.847
MLE	0.502	0.943	0.916	0.448	0.864	0.992	0.891	0.873
MBICAT								
MAP	0.385	0.664	0.666	0.294	0.639	0.626	0.599	0.565
EAP	0.415	0.703	0.710	0.296	0.643	0.621	0.599	0.563
MLE	0.388	0.642	0.664	0.291	0.641	0.626	0.589	0.565
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.567	0.886	0.867	0.458	0.872	0.871	0.754	0.841
EAP	0.568	0.886	0.872	0.457	0.872	0.872	0.753	0.844
MLE	0.564	0.893	0.871	0.446	0.880	0.891	0.780	0.858
BICAT(D)								
MAP	0.567	0.885	0.868	0.458	0.869	0.870	0.754	0.843
EAP	0.568	0.884	0.871	0.457	0.872	0.870	0.754	0.841
MLE	0.564	0.894	0.875	0.446	0.873	0.891	0.783	0.854
MBICAT								
MAP	0.430	0.658	0.648	0.295	0.562	0.486	0.493	0.493
EAP	0.457	0.681	0.696	0.295	0.561	0.481	0.500	0.497
MLE	0.433	0.646	0.652	0.294	0.550	0.480	0.492	0.497

Table F7

RMSEs Using BICAT(S), BICAT(D) and MBICAT Algorithms for
Three Group Factor Discrimination Conditions (Replication 7)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Three group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.409	0.772	0.946	0.351	0.858	0.802	0.781	0.849
EAP	0.409	0.773	0.943	0.351	0.860	0.803	0.779	0.849
MLE	0.398	0.789	0.961	0.344	0.861	0.811	0.799	0.860
BICAT(D)								
MAP	0.409	0.775	0.950	0.351	0.858	0.802	0.782	0.849
EAP	0.409	0.773	0.946	0.351	0.860	0.803	0.780	0.851
MLE	0.398	0.794	0.962	0.344	0.860	0.813	0.797	0.859
MBICAT								
MAP	0.345	0.655	0.674	0.235	0.604	0.525	0.505	0.618
EAP	0.359	0.669	0.712	0.236	0.601	0.536	0.503	0.620
MLE	0.353	0.661	0.672	0.238	0.608	0.528	0.503	0.618
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.453	0.728	0.909	0.395	0.808	0.842	0.815	0.830
EAP	0.452	0.728	0.908	0.396	0.805	0.844	0.820	0.829
MLE	0.448	0.732	0.913	0.390	0.822	0.848	0.831	0.861
BICAT(D)								
MAP	0.453	0.729	0.908	0.395	0.807	0.842	0.816	0.829
EAP	0.452	0.725	0.906	0.396	0.803	0.843	0.820	0.829
MLE	0.448	0.732	0.917	0.390	0.823	0.851	0.831	0.858
MBICAT								
MAP	0.371	0.612	0.637	0.239	0.581	0.465	0.569	0.574
EAP	0.381	0.635	0.675	0.238	0.581	0.462	0.568	0.572
MLE	0.382	0.607	0.655	0.239	0.588	0.465	0.566	0.570
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.481	0.703	0.896	0.418	0.816	0.821	0.793	0.621
EAP	0.479	0.705	0.893	0.419	0.817	0.821	0.793	0.624
MLE	0.477	0.710	0.898	0.412	0.828	0.832	0.800	0.640
BICAT(D)								
MAP	0.481	0.704	0.898	0.418	0.813	0.820	0.793	0.620
EAP	0.479	0.704	0.893	0.419	0.814	0.820	0.793	0.622
MLE	0.477	0.705	0.901	0.412	0.825	0.834	0.800	0.634
MBICAT								
MAP	0.397	0.568	0.603	0.319	0.512	0.528	0.509	0.446
EAP	0.416	0.592	0.655	0.316	0.511	0.532	0.504	0.440
MLE	0.408	0.577	0.620	0.315	0.514	0.527	0.501	0.448

Table F8

RMSEs Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions (Replication 8)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Three group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.397	0.856	0.843	0.380	0.892	0.828	0.798	0.825
EAP	0.401	0.857	0.844	0.380	0.892	0.828	0.799	0.828
MLE	0.383	0.876	0.864	0.373	0.898	0.859	0.824	0.849
BICAT(D)								
MAP	0.397	0.855	0.842	0.380	0.892	0.828	0.797	0.825
EAP	0.401	0.855	0.842	0.380	0.892	0.828	0.798	0.828
MLE	0.383	0.869	0.866	0.373	0.900	0.859	0.824	0.847
MBICAT								
MAP	0.300	0.565	0.609	0.263	0.684	0.540	0.555	0.545
EAP	0.304	0.588	0.612	0.262	0.677	0.544	0.557	0.536
MLE	0.302	0.559	0.607	0.265	0.679	0.542	0.561	0.549
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.432	0.808	0.804	0.439	0.873	0.863	0.810	0.920
EAP	0.432	0.808	0.801	0.440	0.871	0.861	0.812	0.921
MLE	0.424	0.829	0.832	0.434	0.880	0.878	0.822	0.940
BICAT(D)								
MAP	0.432	0.810	0.803	0.439	0.873	0.863	0.810	0.920
EAP	0.432	0.807	0.800	0.440	0.872	0.862	0.811	0.921
MLE	0.424	0.824	0.834	0.434	0.882	0.880	0.822	0.940
MBICAT								
MAP	0.336	0.545	0.574	0.283	0.647	0.531	0.589	0.600
EAP	0.341	0.554	0.588	0.280	0.646	0.536	0.585	0.600
MLE	0.321	0.529	0.565	0.281	0.660	0.529	0.588	0.597
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.472	0.778	0.772	0.438	0.719	0.834	0.763	0.786
EAP	0.471	0.777	0.771	0.438	0.719	0.836	0.764	0.786
MLE	0.464	0.782	0.788	0.433	0.736	0.836	0.762	0.789
BICAT(D)								
MAP	0.472	0.779	0.772	0.438	0.720	0.833	0.763	0.786
EAP	0.471	0.777	0.772	0.438	0.719	0.835	0.764	0.785
MLE	0.464	0.778	0.792	0.433	0.741	0.835	0.763	0.788
MBICAT								
MAP	0.395	0.546	0.582	0.307	0.483	0.609	0.484	0.513
EAP	0.394	0.541	0.587	0.310	0.484	0.612	0.481	0.513
MLE	0.391	0.534	0.584	0.303	0.486	0.610	0.488	0.507

Table F9
RMSEs Using BICAT(S), BICAT(D) and MBICAT Algorithms for
Three Group Factor Discrimination Conditions (Replication 9)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Three group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.403	0.793	0.847	0.338	0.813	0.795	0.890	0.858
EAP	0.402	0.791	0.846	0.338	0.814	0.795	0.888	0.861
MLE	0.389	0.801	0.858	0.331	0.829	0.830	0.904	0.895
BICAT(D)								
MAP	0.403	0.793	0.848	0.338	0.813	0.793	0.890	0.858
EAP	0.402	0.793	0.848	0.338	0.814	0.794	0.888	0.861
MLE	0.389	0.803	0.861	0.331	0.827	0.828	0.904	0.895
MBICAT								
MAP	0.324	0.618	0.671	0.222	0.555	0.553	0.568	0.545
EAP	0.331	0.632	0.692	0.217	0.559	0.563	0.569	0.547
MLE	0.324	0.628	0.680	0.221	0.556	0.554	0.566	0.538
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.453	0.761	0.834	0.408	0.831	0.883	0.837	0.820
EAP	0.451	0.760	0.835	0.409	0.829	0.882	0.836	0.816
MLE	0.442	0.766	0.843	0.401	0.840	0.896	0.844	0.835
BICAT(D)								
MAP	0.453	0.758	0.835	0.408	0.830	0.883	0.840	0.820
EAP	0.451	0.759	0.837	0.409	0.828	0.882	0.837	0.815
MLE	0.442	0.770	0.843	0.401	0.834	0.899	0.843	0.834
MBICAT								
MAP	0.351	0.610	0.671	0.261	0.605	0.575	0.637	0.531
EAP	0.365	0.610	0.690	0.264	0.610	0.579	0.639	0.532
MLE	0.361	0.615	0.680	0.259	0.617	0.573	0.636	0.530
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.477	0.728	0.813	0.428	0.697	0.705	0.812	0.828
EAP	0.478	0.730	0.815	0.428	0.699	0.704	0.810	0.826
MLE	0.472	0.738	0.819	0.423	0.729	0.721	0.822	0.844
BICAT(D)								
MAP	0.477	0.727	0.814	0.428	0.696	0.705	0.811	0.826
EAP	0.478	0.727	0.815	0.428	0.698	0.705	0.809	0.826
MLE	0.472	0.737	0.822	0.423	0.724	0.717	0.819	0.836
MBICAT								
MAP	0.390	0.573	0.665	0.332	0.536	0.522	0.536	0.551
EAP	0.401	0.579	0.675	0.330	0.531	0.517	0.538	0.540
MLE	0.387	0.578	0.672	0.326	0.536	0.522	0.526	0.539

Table F10
RMSEs Using BICAT(S), BICAT(D) and MBICAT Algorithms for
Three Group Factor Discrimination Conditions (Replication 10)

CAT Algorithm and θ estimation Method	Number of group factors							
	Two group factors			Three group factors				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.409	0.843	0.866	0.359	0.808	0.894	0.902	0.881
EAP	0.409	0.844	0.863	0.360	0.810	0.894	0.904	0.884
MLE	0.403	0.839	0.863	0.352	0.838	0.900	0.920	0.926
BICAT(D)								
MAP	0.409	0.845	0.863	0.359	0.807	0.894	0.903	0.881
EAP	0.409	0.845	0.859	0.360	0.808	0.894	0.906	0.883
MLE	0.403	0.837	0.862	0.352	0.837	0.901	0.915	0.924
MBICAT								
MAP	0.326	0.632	0.626	0.217	0.456	0.646	0.573	0.585
EAP	0.345	0.669	0.683	0.223	0.459	0.652	0.579	0.590
MLE	0.324	0.623	0.634	0.217	0.460	0.648	0.571	0.588
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.443	0.812	0.839	0.480	0.838	0.890	0.949	0.930
EAP	0.443	0.814	0.840	0.482	0.832	0.890	0.948	0.931
MLE	0.435	0.806	0.837	0.473	0.850	0.918	0.966	0.958
BICAT(D)								
MAP	0.443	0.811	0.840	0.480	0.833	0.890	0.948	0.932
EAP	0.443	0.812	0.842	0.482	0.832	0.890	0.947	0.929
MLE	0.435	0.804	0.837	0.473	0.846	0.918	0.965	0.950
MBICAT								
MAP	0.363	0.627	0.600	0.286	0.650	0.548	0.654	0.561
EAP	0.377	0.651	0.652	0.283	0.660	0.548	0.648	0.557
MLE	0.363	0.615	0.600	0.287	0.648	0.551	0.654	0.559
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.485	0.765	0.805	0.428	0.780	0.899	0.854	0.789
EAP	0.485	0.765	0.805	0.427	0.783	0.900	0.854	0.788
MLE	0.476	0.766	0.805	0.422	0.782	0.916	0.858	0.798
BICAT(D)								
MAP	0.485	0.767	0.808	0.428	0.780	0.899	0.854	0.787
EAP	0.485	0.767	0.806	0.427	0.780	0.900	0.856	0.788
MLE	0.476	0.765	0.805	0.422	0.783	0.919	0.858	0.796
MBICAT								
MAP	0.403	0.567	0.612	0.321	0.550	0.596	0.574	0.545
EAP	0.421	0.602	0.662	0.323	0.552	0.596	0.574	0.546
MLE	0.403	0.569	0.607	0.322	0.556	0.600	0.578	0.539

APPENDIX G

OSEs Using BICAT(S), BICAT(D) and MBICAT Algorithms For Three Group Factor Discrimination Conditions

Table G1
OSEs of Estimates Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions (Replication 1)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.171	0.376	0.381	0.138	0.406	0.416	0.419	0.411
EAP	0.145	0.349	0.353	0.112	0.378	0.388	0.392	0.383
MLE	0.174	0.406	0.411	0.139	0.445	0.459	0.464	0.453
BICAT(D)								
MAP	0.171	0.376	0.381	0.138	0.406	0.416	0.419	0.411
EAP	0.145	0.349	0.354	0.112	0.378	0.388	0.392	0.383
MLE	0.174	0.405	0.412	0.139	0.444	0.459	0.463	0.453
MBICAT								
MAP	0.181	0.448	0.466	0.153	0.485	0.517	0.496	0.498
EAP	0.192	0.413	0.436	0.170	0.484	0.508	0.487	0.484
MLE	0.191	0.449	0.459	0.155	0.486	0.521	0.498	0.505
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.171	0.340	0.345	0.138	0.378	0.373	0.395	0.370
EAP	0.145	0.313	0.318	0.112	0.350	0.346	0.367	0.342
MLE	0.174	0.362	0.368	0.139	0.407	0.404	0.430	0.398
BICAT(D)								
MAP	0.171	0.340	0.345	0.138	0.378	0.373	0.395	0.369
EAP	0.145	0.313	0.318	0.112	0.350	0.346	0.367	0.342
MLE	0.174	0.361	0.368	0.139	0.407	0.404	0.430	0.398
MBICAT								
MAP	0.197	0.420	0.410	0.155	0.479	0.440	0.491	0.429
EAP	0.214	0.391	0.415	0.201	0.482	0.455	0.510	0.463
MLE	0.200	0.428	0.413	0.156	0.481	0.439	0.492	0.430
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.171	0.310	0.315	0.131	0.337	0.343	0.335	0.319
EAP	0.145	0.283	0.288	0.106	0.311	0.315	0.308	0.292
MLE	0.175	0.326	0.332	0.132	0.360	0.365	0.357	0.339
BICAT(D)								
MAP	0.171	0.310	0.315	0.131	0.337	0.343	0.335	0.319
EAP	0.145	0.283	0.288	0.106	0.311	0.315	0.308	0.292
MLE	0.175	0.326	0.332	0.132	0.360	0.365	0.357	0.338
MBICAT								
MAP	0.198	0.318	0.339	0.149	0.380	0.409	0.391	0.376
EAP	0.235	0.373	0.394	0.221	0.422	0.454	0.426	0.404
MLE	0.199	0.348	0.354	0.150	0.382	0.408	0.393	0.377

Table G2
OSEs of Estimates Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions (Replication 2)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.171	0.377	0.380	0.138	0.406	0.417	0.419	0.411
EAP	0.145	0.350	0.353	0.113	0.378	0.389	0.392	0.383
MLE	0.174	0.408	0.411	0.140	0.445	0.459	0.463	0.455
BICAT(D)								
MAP	0.171	0.377	0.381	0.138	0.406	0.417	0.419	0.411
EAP	0.145	0.350	0.353	0.113	0.378	0.389	0.392	0.383
MLE	0.174	0.408	0.411	0.140	0.445	0.459	0.464	0.454
MBICAT								
MAP	0.189	0.448	0.466	0.153	0.484	0.517	0.494	0.499
EAP	0.192	0.414	0.433	0.176	0.488	0.511	0.493	0.491
MLE	0.191	0.450	0.469	0.155	0.489	0.521	0.497	0.504
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.171	0.341	0.345	0.138	0.377	0.374	0.396	0.369
EAP	0.145	0.314	0.317	0.112	0.349	0.347	0.368	0.342
MLE	0.174	0.364	0.367	0.139	0.406	0.404	0.431	0.397
BICAT(D)								
MAP	0.171	0.341	0.345	0.138	0.377	0.374	0.396	0.369
EAP	0.145	0.314	0.318	0.112	0.349	0.347	0.368	0.342
MLE	0.174	0.364	0.367	0.139	0.406	0.404	0.431	0.397
MBICAT								
MAP	0.197	0.420	0.410	0.157	0.480	0.439	0.493	0.431
EAP	0.215	0.393	0.412	0.201	0.479	0.456	0.511	0.461
MLE	0.201	0.427	0.414	0.158	0.482	0.441	0.494	0.432
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.171	0.311	0.315	0.131	0.338	0.343	0.336	0.320
EAP	0.145	0.284	0.288	0.106	0.311	0.315	0.309	0.293
MLE	0.174	0.329	0.332	0.133	0.360	0.365	0.358	0.339
BICAT(D)								
MAP	0.171	0.311	0.314	0.131	0.337	0.342	0.336	0.319
EAP	0.145	0.284	0.288	0.106	0.311	0.315	0.309	0.292
MLE	0.174	0.328	0.331	0.133	0.359	0.365	0.357	0.339
MBICAT								
MAP	0.201	0.319	0.340	0.149	0.359	0.407	0.394	0.377
EAP	0.235	0.375	0.394	0.221	0.422	0.452	0.427	0.404
MLE	0.197	0.348	0.354	0.150	0.384	0.406	0.395	0.380

Table G3

OSEs of Estimates Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions (Replication 3)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.172	0.374	0.380	0.137	0.406	0.416	0.418	0.411
EAP	0.146	0.347	0.352	0.112	0.379	0.388	0.391	0.383
MLE	0.176	0.404	0.410	0.139	0.445	0.458	0.462	0.455
BICAT(D)								
MAP	0.172	0.374	0.380	0.137	0.406	0.416	0.418	0.411
EAP	0.146	0.347	0.352	0.112	0.379	0.388	0.391	0.383
MLE	0.176	0.404	0.410	0.139	0.445	0.458	0.462	0.454
MBICAT								
MAP	0.189	0.449	0.468	0.151	0.482	0.516	0.494	0.496
EAP	0.193	0.413	0.435	0.171	0.484	0.508	0.490	0.487
MLE	0.189	0.450	0.469	0.153	0.483	0.519	0.496	0.502
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.172	0.339	0.344	0.138	0.378	0.373	0.396	0.369
EAP	0.146	0.312	0.317	0.112	0.350	0.346	0.368	0.342
MLE	0.176	0.360	0.367	0.139	0.407	0.403	0.431	0.398
BICAT(D)								
MAP	0.172	0.339	0.345	0.138	0.378	0.373	0.396	0.369
EAP	0.146	0.312	0.317	0.112	0.350	0.346	0.368	0.342
MLE	0.176	0.360	0.367	0.139	0.408	0.403	0.431	0.397
MBICAT								
MAP	0.197	0.420	0.409	0.157	0.480	0.441	0.493	0.430
EAP	0.214	0.392	0.414	0.199	0.482	0.454	0.511	0.461
MLE	0.200	0.423	0.414	0.156	0.481	0.440	0.494	0.431
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.172	0.309	0.315	0.131	0.337	0.342	0.336	0.320
EAP	0.146	0.283	0.288	0.106	0.310	0.315	0.308	0.293
MLE	0.175	0.325	0.332	0.132	0.360	0.365	0.358	0.340
BICAT(D)								
MAP	0.172	0.309	0.315	0.131	0.337	0.342	0.336	0.320
EAP	0.146	0.282	0.288	0.106	0.310	0.315	0.308	0.293
MLE	0.175	0.325	0.332	0.132	0.359	0.364	0.358	0.339
MBICAT								
MAP	0.201	0.319	0.339	0.148	0.381	0.408	0.392	0.377
EAP	0.236	0.374	0.395	0.220	0.421	0.453	0.427	0.405
MLE	0.197	0.348	0.354	0.149	0.383	0.383	0.409	0.377

Table G4

OSEs of Estimates Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions (Replication 4)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.172	0.376	0.379	0.138	0.405	0.416	0.419	0.411
EAP	0.145	0.349	0.352	0.112	0.378	0.388	0.391	0.383
MLE	0.174	0.406	0.410	0.139	0.444	0.458	0.463	0.454
BICAT(D)								
MAP	0.172	0.375	0.380	0.138	0.405	0.416	0.419	0.411
EAP	0.145	0.349	0.352	0.112	0.378	0.388	0.391	0.383
MLE	0.174	0.406	0.410	0.139	0.444	0.458	0.463	0.454
MBICAT								
MAP	0.189	0.449	0.468	0.151	0.482	0.516	0.494	0.496
EAP	0.193	0.413	0.435	0.171	0.484	0.508	0.490	0.487
MLE	0.189	0.450	0.469	0.153	0.483	0.519	0.496	0.502
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.171	0.340	0.344	0.138	0.379	0.372	0.396	0.369
EAP	0.145	0.313	0.317	0.112	0.350	0.346	0.369	0.342
MLE	0.174	0.362	0.366	0.139	0.408	0.402	0.431	0.398
BICAT(D)								
MAP	0.171	0.340	0.344	0.138	0.379	0.372	0.396	0.369
EAP	0.145	0.313	0.317	0.112	0.350	0.346	0.369	0.342
MLE	0.174	0.362	0.366	0.139	0.408	0.402	0.431	0.397
MBICAT								
MAP	0.197	0.420	0.410	0.157	0.480	0.439	0.493	0.431
EAP	0.215	0.393	0.412	0.201	0.479	0.456	0.511	0.461
MLE	0.201	0.427	0.414	0.158	0.482	0.441	0.494	0.432
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.171	0.310	0.314	0.131	0.337	0.342	0.335	0.319
EAP	0.145	0.283	0.287	0.106	0.310	0.315	0.308	0.292
MLE	0.174	0.327	0.331	0.133	0.360	0.364	0.357	0.339
BICAT(D)								
MAP	0.171	0.309	0.314	0.131	0.337	0.342	0.335	0.319
EAP	0.145	0.283	0.287	0.106	0.310	0.315	0.308	0.292
MLE	0.174	0.327	0.331	0.133	0.359	0.364	0.357	0.338
MBICAT								
MAP	0.198	0.318	0.339	0.149	0.380	0.409	0.391	0.376
EAP	0.235	0.373	0.394	0.221	0.422	0.454	0.426	0.404
MLE	0.199	0.348	0.354	0.150	0.382	0.408	0.393	0.377

Table G5

OSEs of Estimates Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions (Replication 5)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.171	0.376	0.380	0.138	0.405	0.417	0.420	0.412
EAP	0.145	0.349	0.353	0.112	0.378	0.389	0.392	0.384
MLE	0.175	0.406	0.410	0.139	0.444	0.459	0.464	0.455
BICAT(D)								
MAP	0.171	0.376	0.380	0.138	0.405	0.417	0.420	0.412
EAP	0.145	0.348	0.353	0.112	0.378	0.389	0.392	0.384
MLE	0.175	0.406	0.410	0.139	0.444	0.459	0.463	0.455
MBICAT								
MAP	0.189	0.449	0.468	0.151	0.482	0.516	0.494	0.496
EAP	0.193	0.413	0.435	0.171	0.484	0.508	0.490	0.487
MLE	0.189	0.450	0.469	0.153	0.483	0.519	0.496	0.502
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.172	0.340	0.345	0.138	0.378	0.374	0.397	0.370
EAP	0.145	0.313	0.317	0.113	0.349	0.347	0.368	0.342
MLE	0.175	0.362	0.367	0.140	0.407	0.405	0.432	0.398
BICAT(D)								
MAP	0.172	0.340	0.345	0.138	0.377	0.374	0.397	0.370
EAP	0.145	0.313	0.318	0.113	0.349	0.347	0.368	0.342
MLE	0.175	0.362	0.367	0.140	0.407	0.405	0.432	0.397
MBICAT								
MAP	0.197	0.420	0.410	0.157	0.480	0.439	0.493	0.431
EAP	0.215	0.393	0.412	0.201	0.479	0.456	0.511	0.461
MLE	0.201	0.427	0.414	0.158	0.482	0.441	0.494	0.432
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.172	0.310	0.314	0.132	0.339	0.344	0.336	0.320
EAP	0.145	0.283	0.288	0.106	0.312	0.316	0.309	0.293
MLE	0.175	0.327	0.331	0.133	0.363	0.366	0.358	0.340
BICAT(D)								
MAP	0.172	0.310	0.314	0.132	0.339	0.343	0.336	0.320
EAP	0.145	0.283	0.288	0.106	0.312	0.316	0.309	0.293
MLE	0.175	0.327	0.331	0.133	0.361	0.366	0.358	0.339
MBICAT								
MAP	0.198	0.318	0.339	0.149	0.380	0.409	0.391	0.376
EAP	0.235	0.373	0.394	0.221	0.422	0.454	0.426	0.404
MLE	0.199	0.348	0.354	0.150	0.382	0.408	0.393	0.377

Table G6

OSEs of Estimates Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions (Replication 6)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.172	0.377	0.380	0.138	0.406	0.417	0.420	0.411
EAP	0.146	0.350	0.353	0.113	0.378	0.389	0.393	0.383
MLE	0.175	0.408	0.410	0.140	0.445	0.460	0.464	0.453
BICAT(D)								
MAP	0.172	0.377	0.380	0.138	0.406	0.417	0.420	0.411
EAP	0.146	0.350	0.353	0.113	0.378	0.389	0.393	0.383
MLE	0.175	0.408	0.410	0.140	0.445	0.459	0.464	0.453
MBICAT								
MAP	0.189	0.449	0.468	0.151	0.482	0.516	0.494	0.496
EAP	0.193	0.413	0.435	0.171	0.484	0.508	0.490	0.487
MLE	0.189	0.450	0.469	0.153	0.483	0.519	0.496	0.502
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.172	0.341	0.344	0.138	0.376	0.374	0.395	0.367
EAP	0.146	0.314	0.317	0.112	0.348	0.347	0.367	0.340
MLE	0.175	0.363	0.367	0.139	0.405	0.405	0.430	0.395
BICAT(D)								
MAP	0.172	0.341	0.344	0.138	0.376	0.374	0.395	0.367
EAP	0.146	0.314	0.317	0.112	0.348	0.347	0.367	0.340
MLE	0.175	0.363	0.367	0.139	0.405	0.405	0.430	0.395
MBICAT								
MAP	0.197	0.420	0.410	0.157	0.480	0.439	0.493	0.431
EAP	0.215	0.393	0.412	0.201	0.479	0.456	0.511	0.461
MLE	0.201	0.427	0.414	0.158	0.482	0.441	0.494	0.432
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.172	0.311	0.314	0.131	0.336	0.343	0.336	0.320
EAP	0.145	0.284	0.287	0.106	0.310	0.315	0.308	0.293
MLE	0.175	0.328	0.331	0.133	0.359	0.365	0.358	0.340
BICAT(D)								
MAP	0.172	0.311	0.314	0.131	0.336	0.343	0.336	0.320
EAP	0.145	0.284	0.287	0.106	0.309	0.315	0.308	0.293
MLE	0.175	0.328	0.331	0.133	0.357	0.365	0.357	0.339
MBICAT								
MAP	0.198	0.318	0.339	0.149	0.380	0.409	0.391	0.376
EAP	0.235	0.373	0.394	0.221	0.422	0.454	0.426	0.404
MLE	0.199	0.348	0.354	0.150	0.382	0.408	0.393	0.377

Table G7

OSEs of Estimates Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions (Replication 7)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.171	0.376	0.380	0.138	0.405	0.417	0.419	0.412
EAP	0.145	0.349	0.353	0.112	0.378	0.389	0.392	0.384
MLE	0.174	0.407	0.411	0.139	0.444	0.459	0.464	0.454
BICAT(D)								
MAP	0.171	0.376	0.381	0.138	0.405	0.417	0.419	0.412
EAP	0.145	0.349	0.353	0.112	0.378	0.389	0.392	0.384
MLE	0.174	0.407	0.411	0.139	0.444	0.459	0.464	0.454
MBICAT								
MAP	0.189	0.449	0.468	0.151	0.482	0.516	0.494	0.496
EAP	0.193	0.413	0.435	0.171	0.484	0.508	0.490	0.487
MLE	0.189	0.450	0.469	0.153	0.483	0.519	0.496	0.502
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.171	0.340	0.344	0.138	0.378	0.372	0.396	0.368
EAP	0.144	0.313	0.317	0.112	0.349	0.346	0.368	0.341
MLE	0.174	0.363	0.367	0.139	0.407	0.403	0.432	0.396
BICAT(D)								
MAP	0.171	0.340	0.345	0.138	0.378	0.372	0.396	0.368
EAP	0.144	0.314	0.318	0.112	0.349	0.346	0.368	0.341
MLE	0.174	0.363	0.367	0.139	0.407	0.403	0.432	0.396
MBICAT								
MAP	0.197	0.420	0.410	0.157	0.480	0.439	0.493	0.431
EAP	0.215	0.393	0.412	0.201	0.479	0.456	0.511	0.461
MLE	0.201	0.427	0.414	0.158	0.482	0.441	0.494	0.432
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.171	0.310	0.314	0.131	0.338	0.343	0.336	0.320
EAP	0.145	0.284	0.288	0.106	0.311	0.315	0.309	0.293
MLE	0.174	0.328	0.331	0.133	0.360	0.365	0.359	0.339
BICAT(D)								
MAP	0.171	0.310	0.314	0.131	0.337	0.343	0.336	0.320
EAP	0.145	0.284	0.288	0.106	0.311	0.315	0.309	0.293
MLE	0.174	0.327	0.331	0.133	0.360	0.365	0.358	0.339
MBICAT								
MAP	0.198	0.318	0.339	0.149	0.380	0.409	0.391	0.376
EAP	0.235	0.373	0.394	0.221	0.422	0.454	0.426	0.404
MLE	0.199	0.348	0.354	0.150	0.382	0.408	0.393	0.377

Table G8
OSEs of Estimates Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions (Replication 8)

CAT Algorithm and θ estimation Method	Number of group factors							
	Two group factors			Four group factors				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.171	0.377	0.380	0.138	0.406	0.417	0.419	0.410
EAP	0.145	0.350	0.352	0.112	0.379	0.389	0.392	0.383
MLE	0.175	0.407	0.410	0.139	0.445	0.459	0.462	0.452
BICAT(D)								
MAP	0.171	0.377	0.380	0.138	0.406	0.417	0.419	0.410
EAP	0.145	0.350	0.352	0.112	0.379	0.389	0.392	0.383
MLE	0.175	0.407	0.410	0.139	0.445	0.459	0.462	0.452
MBICAT								
MAP	0.189	0.449	0.468	0.151	0.482	0.516	0.494	0.496
EAP	0.193	0.413	0.435	0.171	0.484	0.508	0.490	0.487
MLE	0.189	0.450	0.469	0.153	0.483	0.519	0.496	0.502
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.172	0.341	0.344	0.138	0.378	0.372	0.394	0.370
EAP	0.145	0.314	0.317	0.112	0.350	0.345	0.366	0.343
MLE	0.175	0.363	0.366	0.139	0.407	0.402	0.429	0.398
BICAT(D)								
MAP	0.172	0.341	0.344	0.138	0.378	0.372	0.394	0.370
EAP	0.145	0.314	0.317	0.112	0.350	0.345	0.366	0.343
MLE	0.175	0.363	0.366	0.139	0.407	0.403	0.428	0.399
MBICAT								
MAP	0.197	0.420	0.410	0.157	0.480	0.439	0.493	0.431
EAP	0.215	0.393	0.412	0.201	0.479	0.456	0.511	0.461
MLE	0.201	0.427	0.414	0.158	0.482	0.441	0.494	0.432
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.171	0.310	0.314	0.131	0.336	0.343	0.335	0.319
EAP	0.145	0.284	0.287	0.106	0.310	0.315	0.307	0.292
MLE	0.175	0.327	0.331	0.132	0.359	0.365	0.356	0.339
BICAT(D)								
MAP	0.171	0.310	0.314	0.131	0.336	0.343	0.335	0.319
EAP	0.145	0.284	0.287	0.106	0.310	0.315	0.307	0.292
MLE	0.175	0.327	0.331	0.132	0.359	0.365	0.356	0.338
MBICAT								
MAP	0.198	0.318	0.339	0.149	0.380	0.409	0.391	0.376
EAP	0.235	0.373	0.394	0.221	0.422	0.454	0.426	0.404
MLE	0.199	0.348	0.354	0.150	0.382	0.408	0.393	0.377

Table G9

OSEs of Estimates Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions (Replication 9)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.172	0.375	0.381	0.138	0.406	0.417	0.419	0.408
EAP	0.145	0.348	0.353	0.113	0.378	0.389	0.392	0.381
MLE	0.175	0.406	0.411	0.140	0.445	0.459	0.462	0.450
BICAT(D)								
MAP	0.172	0.376	0.381	0.138	0.406	0.417	0.419	0.408
EAP	0.145	0.348	0.353	0.113	0.378	0.389	0.392	0.381
MLE	0.175	0.406	0.411	0.140	0.444	0.459	0.462	0.450
MBICAT								
MAP	0.189	0.449	0.468	0.151	0.482	0.516	0.494	0.496
EAP	0.193	0.413	0.435	0.171	0.484	0.508	0.490	0.487
MLE	0.189	0.450	0.469	0.153	0.483	0.519	0.496	0.502
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.172	0.340	0.345	0.138	0.378	0.373	0.395	0.370
EAP	0.145	0.313	0.318	0.113	0.350	0.346	0.367	0.343
MLE	0.175	0.362	0.367	0.140	0.407	0.403	0.430	0.399
BICAT(D)								
MAP	0.172	0.340	0.345	0.138	0.378	0.373	0.395	0.370
EAP	0.145	0.313	0.318	0.113	0.349	0.346	0.367	0.343
MLE	0.175	0.362	0.367	0.140	0.407	0.403	0.430	0.399
MBICAT								
MAP	0.197	0.420	0.410	0.157	0.480	0.439	0.493	0.431
EAP	0.215	0.393	0.412	0.201	0.479	0.456	0.511	0.461
MLE	0.201	0.427	0.414	0.158	0.482	0.441	0.494	0.432
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.172	0.310	0.315	0.131	0.337	0.342	0.336	0.319
EAP	0.145	0.283	0.288	0.106	0.310	0.315	0.308	0.292
MLE	0.175	0.326	0.332	0.132	0.360	0.365	0.358	0.338
BICAT(D)								
MAP	0.172	0.310	0.315	0.131	0.337	0.342	0.336	0.318
EAP	0.145	0.283	0.288	0.106	0.310	0.315	0.308	0.292
MLE	0.175	0.326	0.332	0.132	0.359	0.364	0.357	0.337
MBICAT								
MAP	0.198	0.318	0.339	0.149	0.380	0.409	0.391	0.376
EAP	0.235	0.373	0.394	0.221	0.422	0.454	0.426	0.404
MLE	0.199	0.348	0.354	0.150	0.382	0.408	0.393	0.377

Table G10

OSEs of Estimates Using BICAT(S), BICAT(D) and MBICAT Algorithms for Three Group Factor Discrimination Conditions (Replication 10)

CAT Algorithm and θ estimation Method	*Number of group factors*							
	Two group factors			*Four group factors*				
	G	*g1*	*g2*	*G*	*g1*	*g2*	*g3*	*g4*
Conditions A and D: Low group factor discrimination								
BICAT(S)								
MAP	0.172	0.375	0.380	0.138	0.406	0.416	0.420	0.411
EAP	0.146	0.348	0.352	0.113	0.378	0.388	0.393	0.383
MLE	0.175	0.406	0.410	0.140	0.445	0.458	0.464	0.453
BICAT(D)								
MAP	0.172	0.376	0.380	0.138	0.406	0.416	0.420	0.411
EAP	0.146	0.348	0.353	0.113	0.378	0.388	0.393	0.383
MLE	0.175	0.405	0.411	0.140	0.444	0.458	0.464	0.453
MBICAT								
MAP	0.189	0.449	0.468	0.151	0.482	0.516	0.494	0.496
EAP	0.193	0.413	0.435	0.171	0.484	0.508	0.490	0.487
MLE	0.189	0.450	0.469	0.153	0.483	0.519	0.496	0.502
Conditions B and E: Medium group factor discrimination								
BICAT(S)								
MAP	0.172	0.340	0.344	0.138	0.376	0.374	0.395	0.367
EAP	0.146	0.313	0.317	0.112	0.348	0.347	0.367	0.340
MLE	0.175	0.362	0.367	0.139	0.405	0.405	0.430	0.395
BICAT(D)								
MAP	0.172	0.340	0.344	0.138	0.376	0.374	0.395	0.367
EAP	0.146	0.313	0.317	0.112	0.348	0.347	0.367	0.340
MLE	0.175	0.362	0.367	0.139	0.405	0.405	0.430	0.395
MBICAT								
MAP	0.197	0.420	0.410	0.157	0.480	0.439	0.493	0.431
EAP	0.215	0.393	0.412	0.201	0.479	0.456	0.511	0.461
MLE	0.201	0.427	0.414	0.158	0.482	0.441	0.494	0.432
Conditions C and F: High group factor discrimination								
BICAT(S)								
MAP	0.172	0.310	0.314	0.131	0.336	0.342	0.336	0.320
EAP	0.145	0.283	0.287	0.106	0.309	0.315	0.309	0.293
MLE	0.175	0.327	0.331	0.133	0.359	0.365	0.359	0.340
BICAT(D)								
MAP	0.172	0.310	0.314	0.131	0.336	0.342	0.336	0.320
EAP	0.145	0.283	0.287	0.106	0.309	0.315	0.309	0.293
MLE	0.175	0.327	0.331	0.133	0.358	0.365	0.358	0.339
MBICAT								
MAP	0.198	0.318	0.339	0.149	0.380	0.409	0.391	0.376
EAP	0.235	0.373	0.394	0.221	0.422	0.454	0.426	0.404
MLE	0.199	0.348	0.354	0.150	0.382	0.408	0.393	0.377