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唯以正直接萬人
至誠依神期勝利
鑑

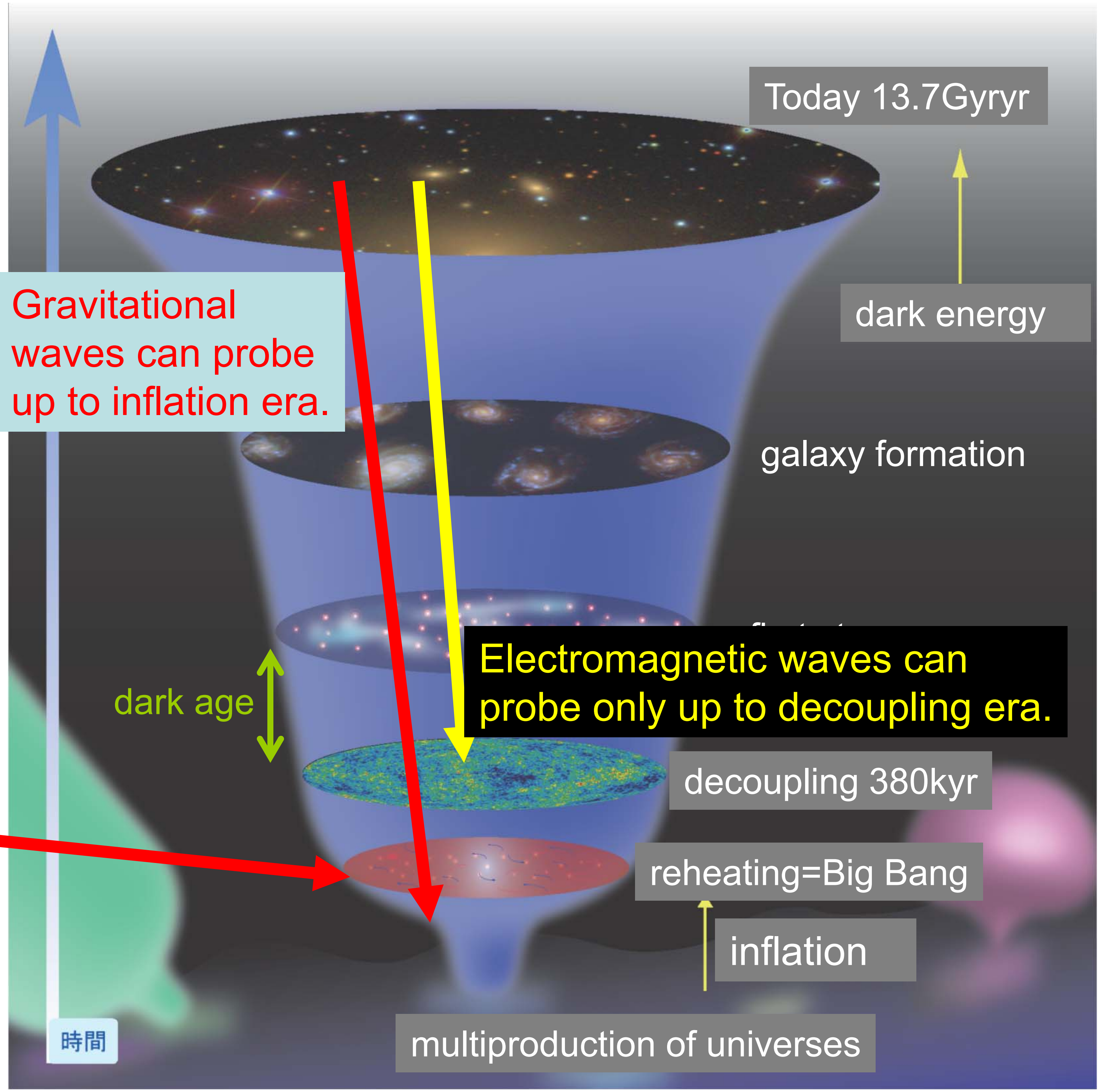
Gravitational Waves as a probe of the early Universe

Jun'ichi Yokoyama (Tokyo)

Gravitational Wave Cosmology

Another tiny dark age between inflation and Big Bang Nucleosynthesis

Shedding new “lignt” on this epoch



Gravitational Radiation as a Probe of the Early Universe

3 separate issues with 2 different origins of gravitational waves

- ☆ **Probe of the Nature of the Dark Matter**

  R. Saito and J. Yokoyama, Phys. Rev. Lett. 102(2009)161101
  R. Saito and J. Yokoyama, Prog. Theor. Phys 123(2010)867.

- ☆ **Probe of the Thermal History after Inflation**

  K. Nakayama, S. Saito, Y. Suwa and J. Yokoyama,
  Phys. Rev. D77(2008)1240001, JCAP 0806(2008)020
  ⇐ N. Seto and J. Yokoyama, J. Phys. Soc. Japan 72(2003)3082

- ☆ **Probe of the Curvaton Scenario**

  K. Nakayama and J. Yokoyama, JCAP 1001(2010)010

Origins of Cosmological Gravitational Waves (Tensor Perturbations)

- ☆ Those generated by d…ity fluctuations as a second-order eff…
  - ⇨ Pro… …imordial Black Holes

What if ?

- ☆ Those generated by … …m effect during inflation (just as densit… …ure fluctautions)
  - ⇨ P… …he Reheat Temperature
  - ⇨ …be of Gravitinos
  - ⇨ Probe of Entropy Productio… …Curvaton

Standard Scenario

What if ?

What constitutes dark matter ?

Particles

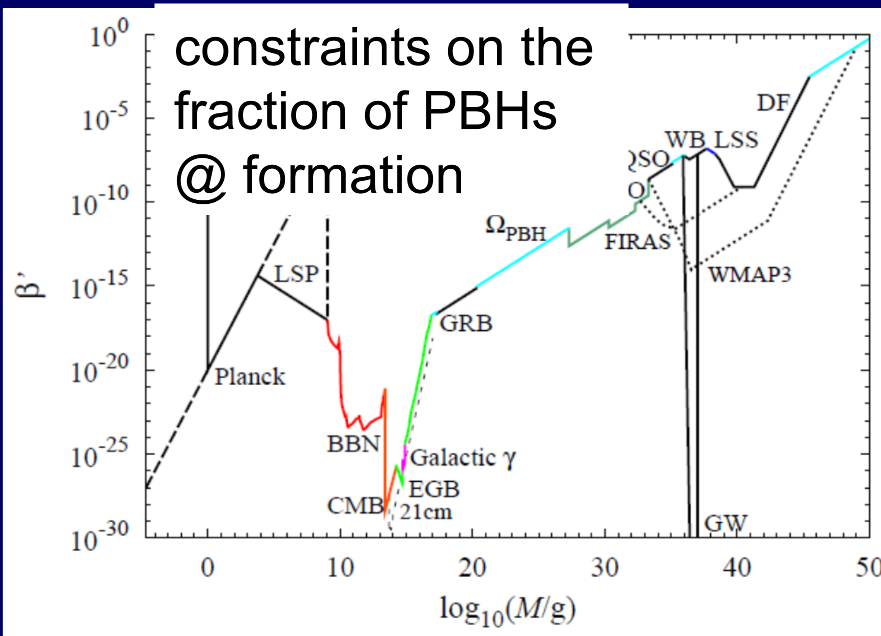
Neutralinos, Axions, etc…

They may be discovered by accelerator experiments or astrophysical observations.

Black Holes

Mini Black Holes with mass 10^{20}—10^{26} g could be dark matter.
Since these light BHs cannot be produced astrophysically, they must be of primordial origin which were created from large fluctuations in the early Universe.
= Primordial Black Holes (PBHs).

How can we confirm/rule out their existence?



Cosmological constraints on PBH abundance
(Carr, Kohri, Sendouda, & JY 10)

Generation of gravitational waves from density fluctuations

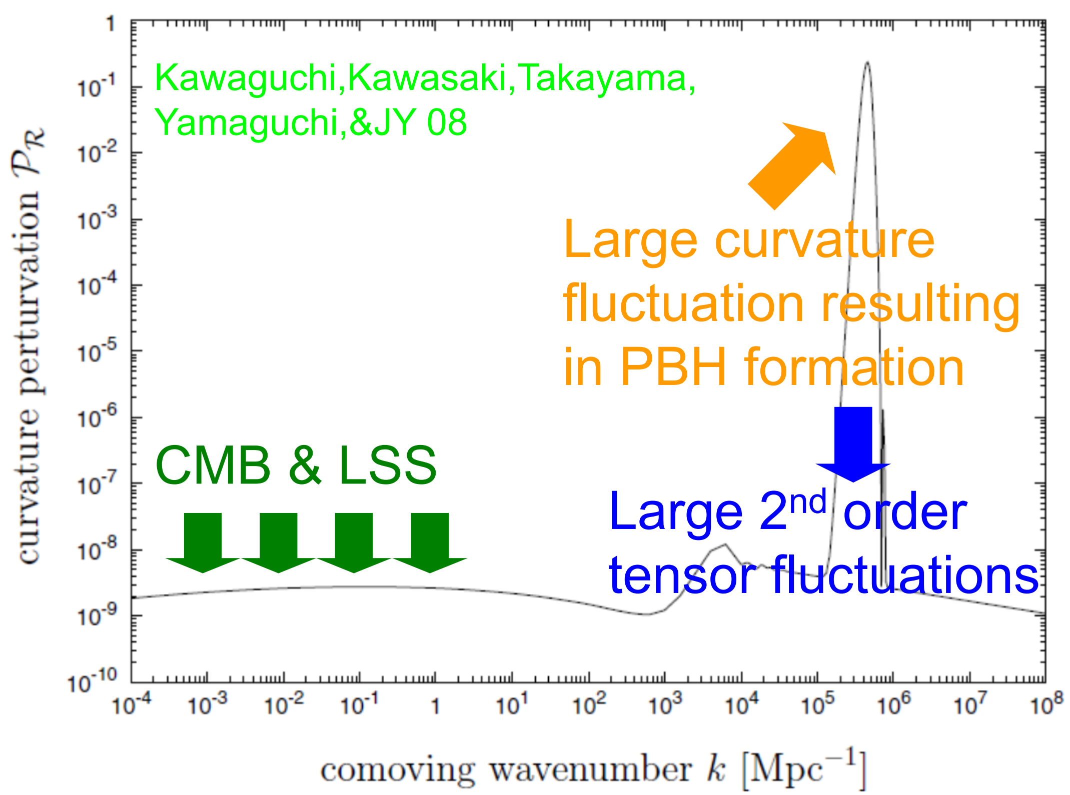
Formation of Primordial Black Holes (PBHs) on a specific mass scale.

Density/curvature fluctuations have a large amplitude on a specific scale.

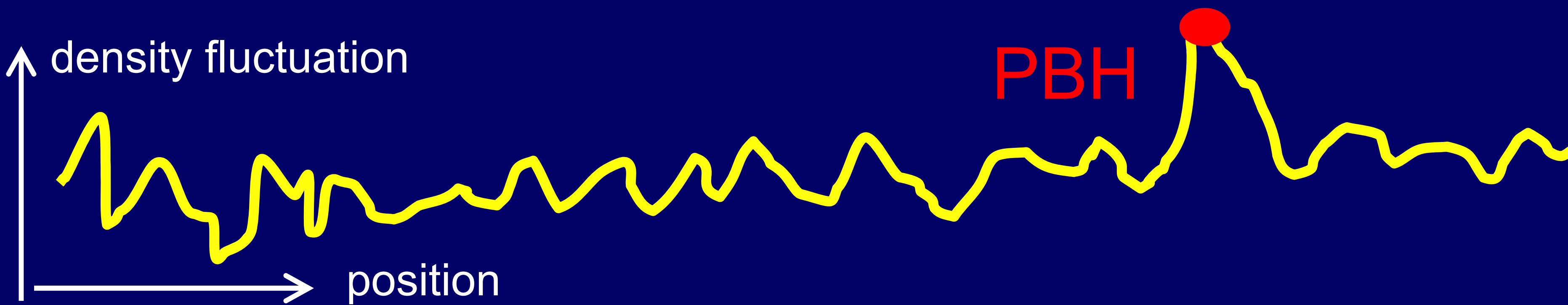
$M_{BH} \approx$ horizon mass when the peak entered the Hubble radius

Gravitational waves (tensor perturbations) generated from second-order density/curvature fluctuations are significant.

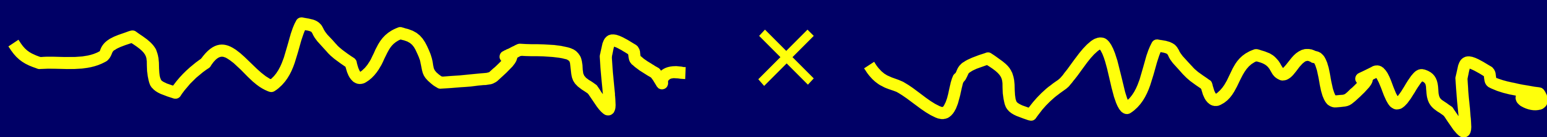
(Mollerach,Harari,& Matarrese 04, Ananda,Clarkson,& Wands 07, Baumann,Steinhardt,Takahashi,& Ichiki 07)



What I am going to discuss is NOT GWs associated with PBH formation which are created only at high σ peaks.



but those created by (density fluctuations) × (density fluctuations) as a second order effect in cosmological perturbation theory.



This effect is much more universal than GWs associated with PBH formation.

Calculation is straight formward...

Perturbed metric: Scalar modes: Φ and Ψ ; Tensor modes h_{ij}.

$$ds^2 = a(\eta)^2 \left[-e^{2\Phi}d\eta^2 + e^{-2\Psi}(\delta_{ij} + 2h_{ij})dx^i dx^j\right]$$

$a(\eta) \propto \eta$ in the radiation dominated regime.

Equation of motion for the transverse-traceless ($\partial_i h^i_j = h^i_i = 0$) tensor mode

$$h^{i\,\prime\prime}_j + 2\mathcal{H}h^{i\,\prime}_j - \partial^2 h^i_j = \mathcal{P}^{is}_{rj} S^r_s \qquad \mathcal{H} \equiv \frac{a'}{a}$$

sourced by the second-order linear scalar perturbations $\Phi = \Psi$:

$$S^r_s = 2\partial^r\Psi\partial_s\Psi - \partial^r(\Psi + \mathcal{H}^{-1}\Psi')\partial_s(\Psi + \mathcal{H}^{-1}\Psi')$$

These equations are obtained by writing the Einstein tensor G^i_j up to second order in Φ and Ψ and first order in h_{ij} and apply the projection operator $\mathcal{P}^{is}_{rj}$ to the transverse-traceless components.

Linear evolution equation suffices for $\Psi_{\boldsymbol{k}}$ in wavenumber space.

$$\Psi''_{\boldsymbol{k}}(\eta) + \frac{4}{\eta}\Psi'_{\boldsymbol{k}}(\eta) + \frac{k^2}{3}\Psi_{\boldsymbol{k}}(\eta) = 0$$

which is easily solved as

$$\Psi_{\boldsymbol{k}}(\eta) = D_k(\eta)\Psi_{\boldsymbol{k}}(0)$$

$$D_k(\eta) = \frac{9}{(k\eta)^2}\left[\frac{\sqrt{3}}{k\eta}\sin\left(\frac{k\eta}{\sqrt{3}}\right) - \cos\left(\frac{k\eta}{\sqrt{3}}\right)\right]$$

Evolution equation for h_{ij} is also easily solved by the Green function method.

$$h^{i\,\prime\prime}_{j} + 2\mathcal{H}h^{i\,\prime}_{j} - \partial^2 h^i_j = \mathcal{P}^{is}_{rj}S^r_s$$

to obtain

$$h_{ij}(\boldsymbol{x},\eta) = \int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\left[h^+_{\boldsymbol{k}}(\eta)\mathrm{e}^+_{ij}(\boldsymbol{k}) + h^\times_{\boldsymbol{k}}(\eta)\mathrm{e}^\times_{ij}(\boldsymbol{k})\right]$$

$$\sum_{i,j}\mathrm{e}^A_{ij}(\boldsymbol{k})\mathrm{e}^B_{ij}(-\boldsymbol{k}) = \delta^{AB}\,\delta^{\alpha\beta}$$

Contribution of GWs with wavenumber □ k to the density parameter

$$\Omega_{\mathrm{GW}}(k,\eta) = \frac{k^3}{6\pi^2\mathcal{H}^2}\left(|h_{\boldsymbol{k}}^{+\prime}|^2 + |h_{\boldsymbol{k}}^{\times\prime}|^2\right)$$

$$= \frac{2}{3}\int^{\eta} \mathrm{d}\eta_1 \int^{\eta} \mathrm{d}\eta_2 \; \eta_1\eta_2 \sin\left[k(\eta-\eta_1)\right] \sin\left[k(\eta-\eta_2)\right] \mathcal{S}_{\boldsymbol{k}}(\eta_1,\eta_2)$$

with

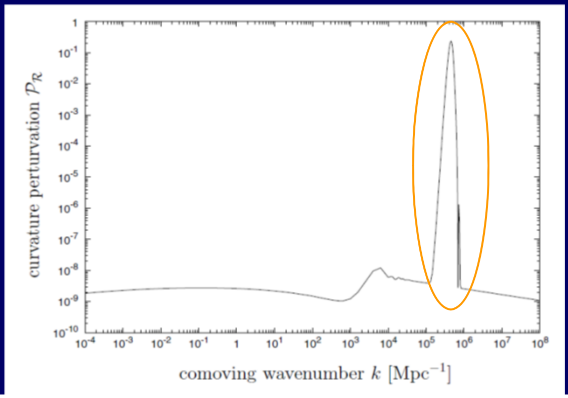
$$\mathcal{S}_{\boldsymbol{k}}(\eta_1,\eta_2) \equiv \int_0^{\infty} \mathrm{d}\tilde{k} \int_{-1}^{1} \mathrm{d}\mu \; \frac{k^3\tilde{k}^3}{|\boldsymbol{k}-\tilde{\boldsymbol{k}}|^3}(1-\mu^2)^2 f(\tilde{k}, |\boldsymbol{k}-\tilde{\boldsymbol{k}}|, \eta_1) f(\tilde{k}, |\boldsymbol{k}-\tilde{\boldsymbol{k}}|, \eta_2)\mathcal{P}_\Psi(\tilde{k})\mathcal{P}_\Psi(|\boldsymbol{k}-\tilde{\boldsymbol{k}}|)$$

where

$$f(k_1,k_2,\eta) \equiv 2D_{k_1}(\eta)D_{k_2}(\eta) - \left[D_{k_1}(\eta) + \mathcal{H}^{-1}D'_{k_1}(\eta)\right]\left[D_{k_2}(\eta) + \mathcal{H}^{-1}D'_{k_2}(\eta)\right]$$

We take a spiky initial scalar power spectrum peaked at $k = k_p$.

$$\mathcal{P}_\Psi(k) \equiv \frac{k^3}{2\pi^2}\langle|\Psi_{\boldsymbol{k}}(0)|^2\rangle = \mathcal{A}^2\delta_D(\ln(k/k_p))$$



Formation of PBHs with mass ≈ horizon mass when $k = k_p$ mode reentered the Hubble radius with the fractional energy density

$$\beta(M_{\rm PBH}) \sim 0.1 \exp\left(-\frac{\Psi_c^2}{2\mathcal{A}^2}\right)$$

we take $\Psi_c = \frac{1}{2}$ for the threshold.

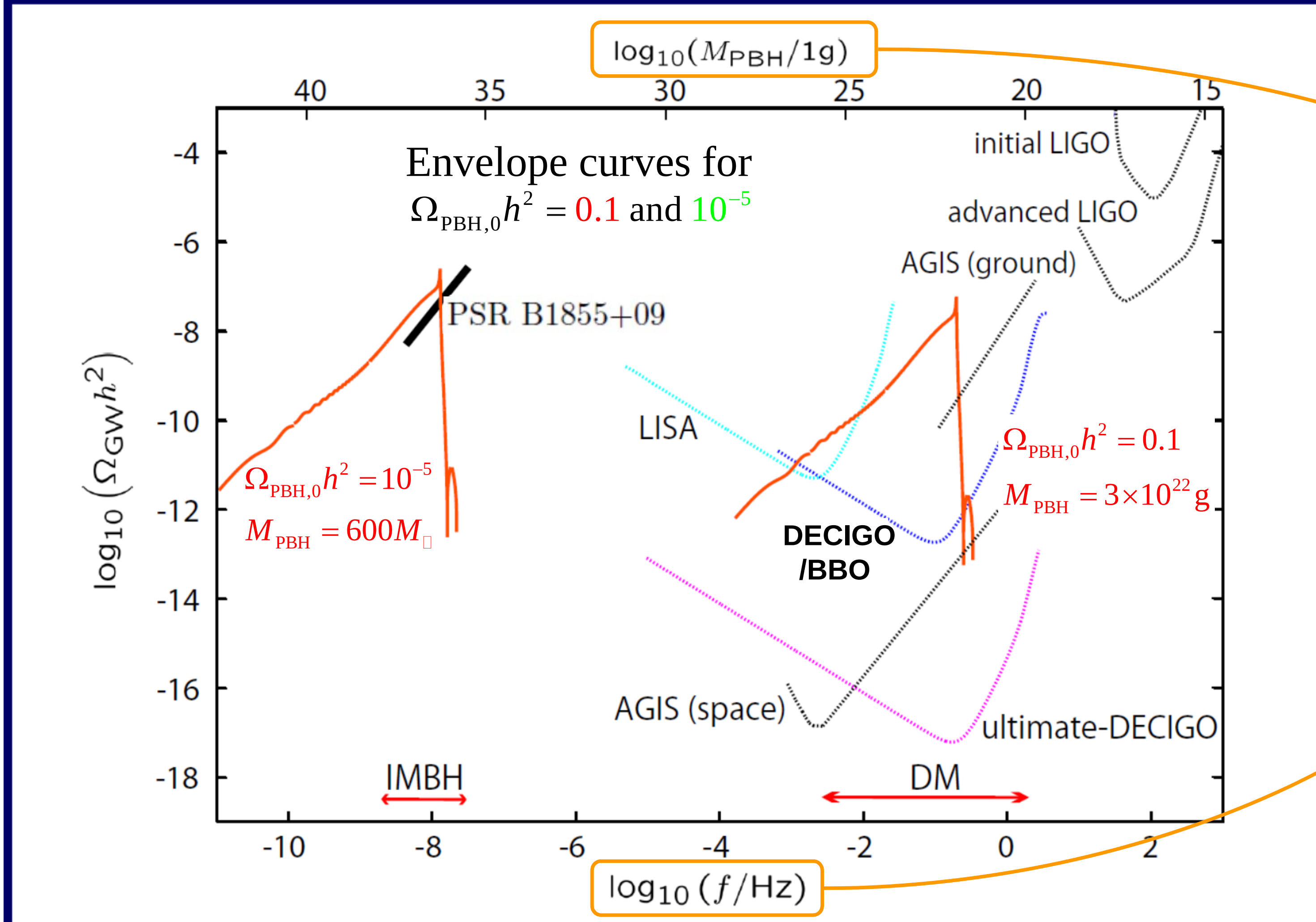
corresponding to the current density parameter

$$\Omega_{\rm PBH,0}h^2 = 2\times 10^6 \beta(M_{\rm PBH}) \left(\frac{M_{\rm PBH}}{10^{36}\,{\rm g}}\right)^{-1/2} \left(\frac{g_{*p}}{10.75}\right)^{-1/3}$$

Gravitational Waves has a peak at the frequency corresponding to k_p namely, at $f_p = 2\pi k_p / a_0$.

$$\Omega_{\rm GW}(f)h^2 \simeq 7\times 10^{-9} \left(\frac{g_{*p}}{10.75}\right)^{-1/3} \left(\frac{\mathcal{A}^2}{10^{-3}}\right)^2 \left(\frac{f}{f_p}\right)^2 \left[1-\left(\frac{f}{2f_p}\right)^2\right]^2 \theta\left(1-\frac{f}{2f_p}\right)$$

$$\equiv A_{\rm GW} \left(\frac{f}{f_p}\right)^2 \left[1-\left(\frac{f}{2f_p}\right)^2\right]^2 \theta\left(1-\frac{f}{2f_p}\right)$$

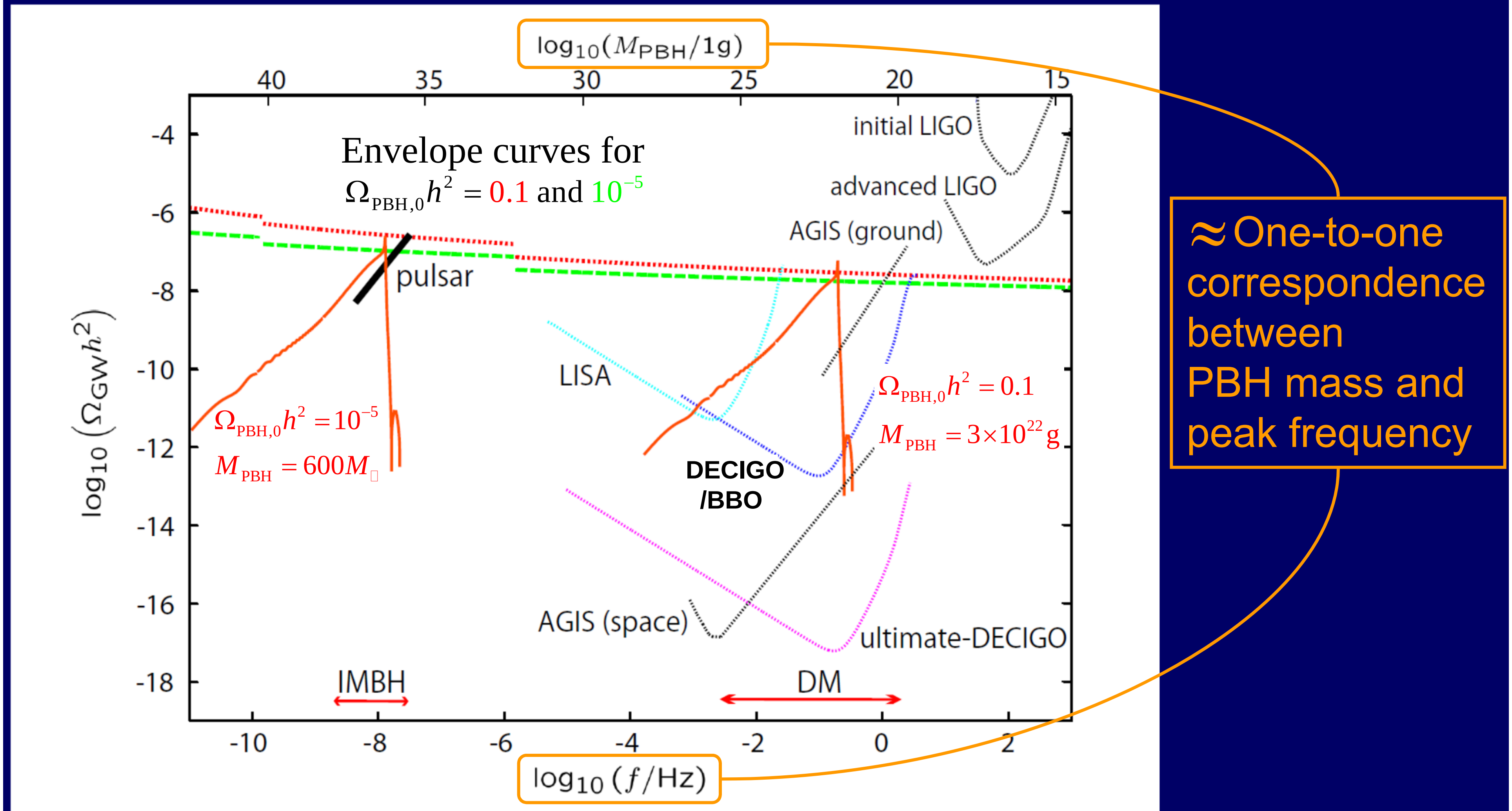


≈One-to-one correspondence between PBH mass and peak frequency

★ Intermediate-mass BHs ($400-5000M_{\odot}$) cannot be PBHs which are supposed to be a source of ultra-luminous X-ray.

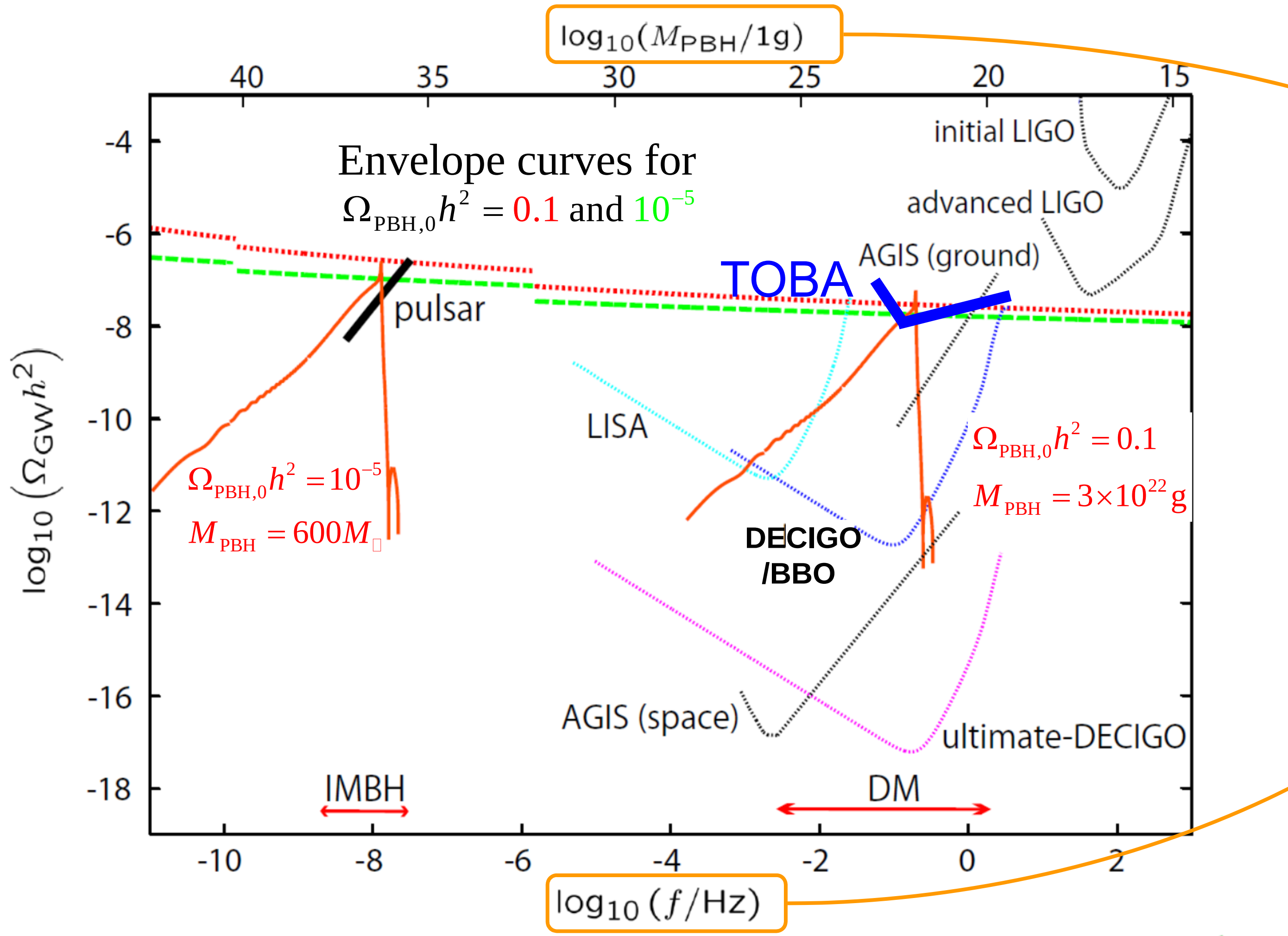
★ Future space-based laser interferometers such as DECIGO or BBO can prove/disprove Black-Hole Dark Matter.

Ando's talk



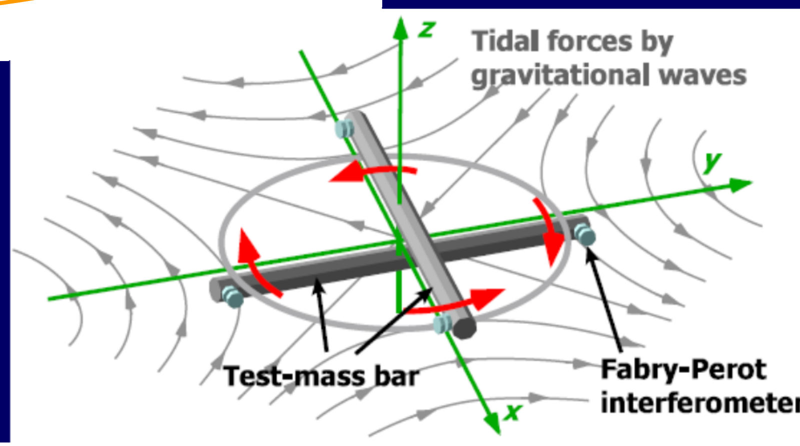
★ Intermediate-mass BHs ($400-5000M_\odot$) cannot be PBHs which are supposed to be a source of ultra-luminous X-ray.

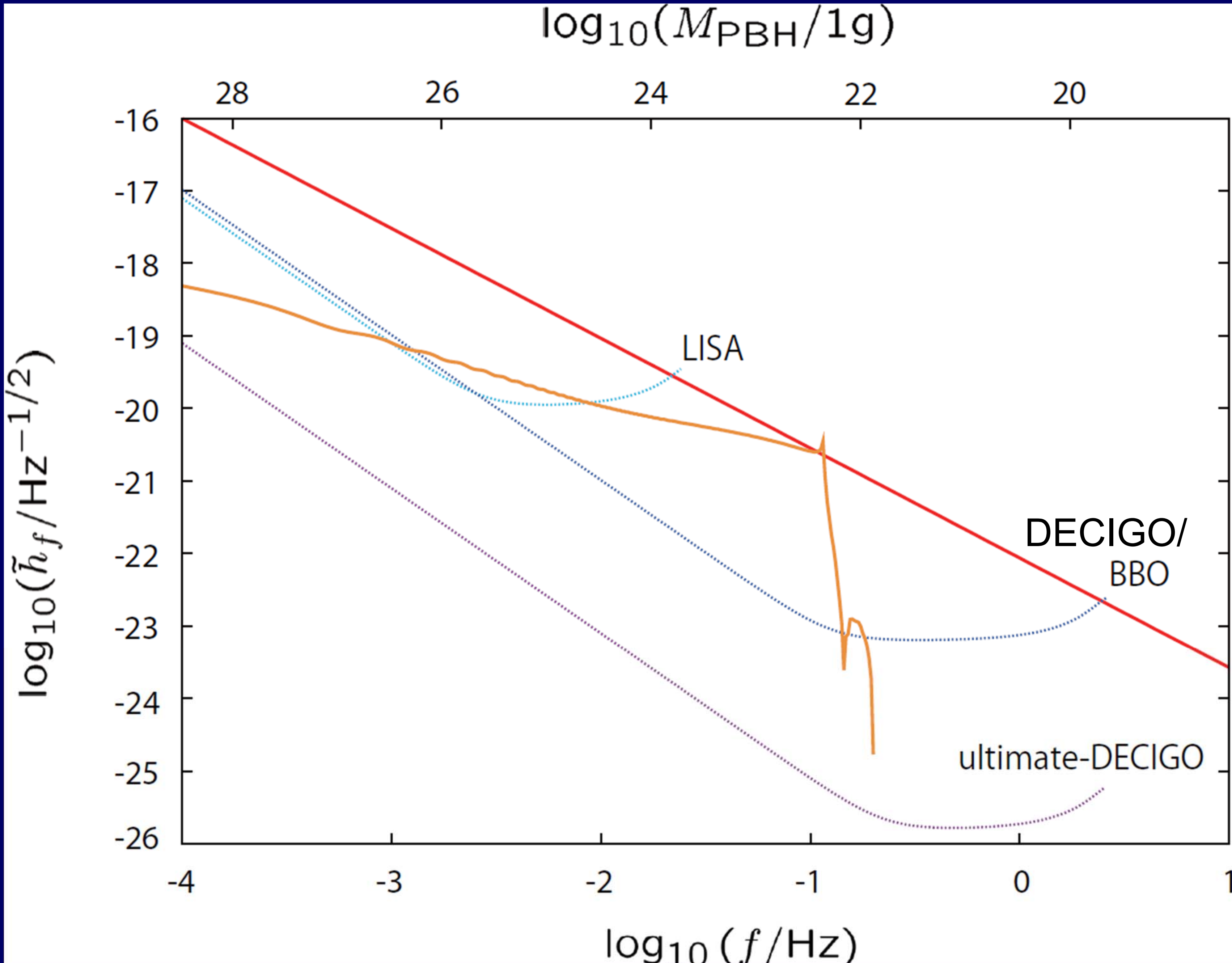
★ AGIS=Atomic Gravitational Interferometric Sensor proposed by Dimopoulos, Graham, Hogan, Kasevich & Rajendran PRD 78(2008)122002



≈One-to-one correspondence between PBH mass and peak frequency

★ TOBA=TOrtion Bar Antenna
Ando et al
Phys Rev Lett 105(2010)161101
10m 7.6ton

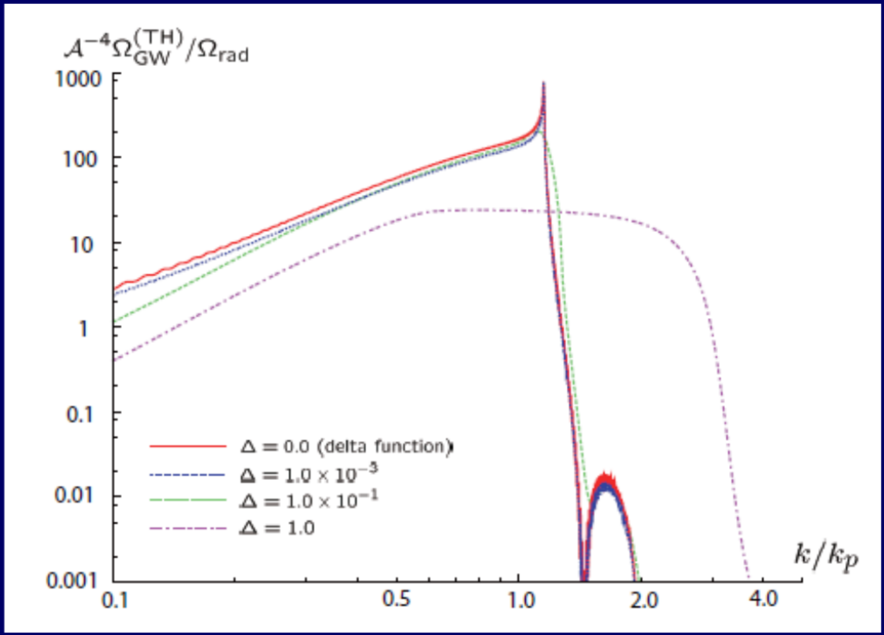


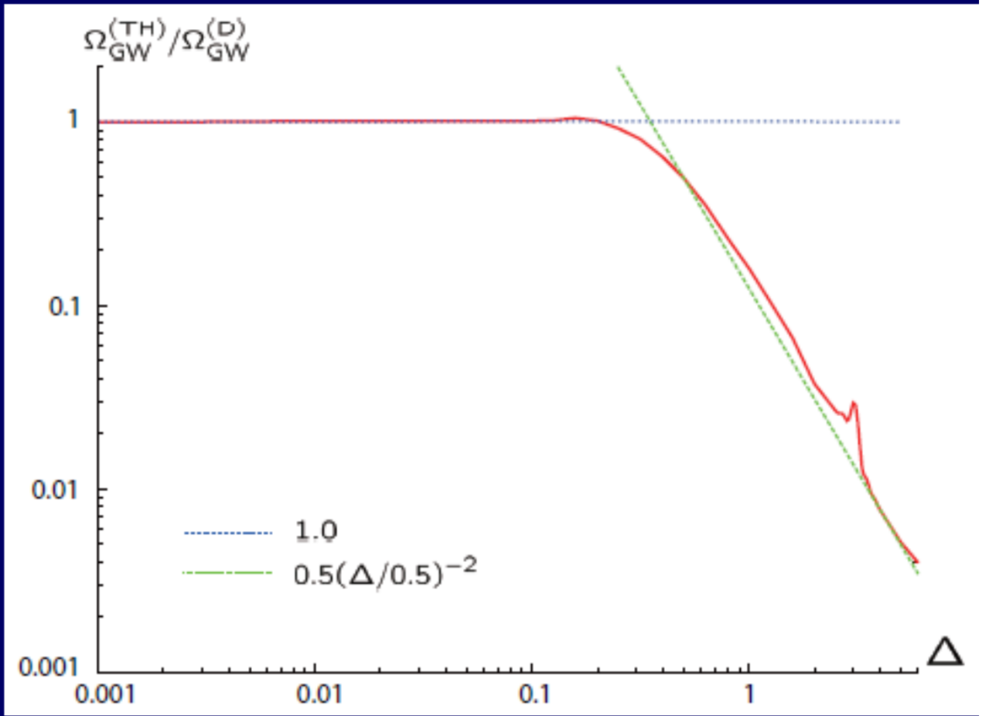


$\log_{10}(M_{\rm PBH}/1{\rm g})$
28
26
24
22
20
-16
-17
-18
-19
-20
-21
-22
-23
-24
-25
-26
$\log_{10}(\tilde{h}_f/{\rm Hz}^{-1/2})$
LISA
DECIGO/
BBO
ultimate-DECIGO
-4
-3
-2
-1
0
1
$\log_{10}(f/{\rm Hz})$

The case the peak of the power spectrum has a finite width Δ with a top-hat shape

$$\mathcal{P}_\Psi^{(\mathrm{TH})} = \begin{cases} \dfrac{\mathcal{A}^2}{2\Delta} & \text{for} \quad |\ln(k/k_p)| < \Delta, \\ 0 & \text{otherwise,} \end{cases}$$





For Δ<0.2 the result is the same as the case with the delta-function peak.

So far:

- ⋆ Non-standard density/curvature fluctuations with a large peak
- ⋆ in the Standard Radiation-dominated Early Universe.

From now on:

- ⋆ Standard nearly scale-invariant spectrum of density/curvature fluctuations
- ⋆ with a non-standard Thermal History in the Early Universe

Second-order GWs are surpassed by the lowest-order quantum GWs.

Tensor Perturbation (Quantum Gravitational Waves)

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + 2h_{ij})dx^i dx^j$$

$$h_{ij} = h_+ e^+_{ij} + h_\times e^\times_{ij}$$

The linearized Einstein action reads

transverse-traceless modes

$$\mathcal{L} = \frac{\mathcal{R}}{16\pi G} = \frac{1}{8\pi G} \times \frac{1}{2}\left[\dot{h}_+^2 - \frac{1}{a^2}(\nabla h_+)^2 + \dot{h}_\times^2 - \frac{1}{a^2}(\nabla h_\times)^2\right] + \cdots$$

It is equivalent with the action of two massless scalar fields $\varphi^A \equiv M_G h_A$.

$$h_{ij} = \sqrt{8\pi G} \sum_{A=+,\times} \int \frac{d^3k}{(2\pi)^{3/2}} \varphi_k^A(t) e^{ikx} e_{ij}^A$$

$$M_G \equiv \frac{1}{\sqrt{8\pi G}}$$

$\varphi_k^A(t)$ satisfies the same eqn as a minimally coupled massless scalar field.

$$\left[\frac{d^2}{dt^2} + 3H\frac{d}{dt} + \frac{k^2}{a^2(t)}\right]\varphi_{\mathbf{k}}^A(t) = 0, \quad a(t) = e^{Ht}$$

Long-wave quantum fluctuation is generated during inflation.

$$\left|\varphi_k^A(t)\right|^2 = \frac{H^2}{2k^3} \quad \text{for } k \ll a(t)H$$

Vacuum fluctuations of 2 Polarization modes of the Graviton

Vacuum fluctuations of 2 massless canonical scalar fields

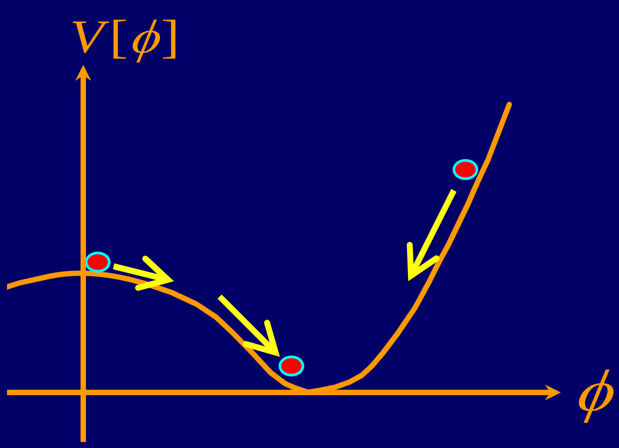
Fluctuation acquired in each logarithmic frequency interval is equal to the Hawking temperature in de Sitter space.

$$|\varphi_{\mathbf{k}}(t)|^2 \frac{4\pi k^3}{(2\pi)^3} = \left(\frac{H}{2\pi}\right)^2$$

Square amplitude per logarithmic frequency (f) interval

$$h_{\rm inf}^2(f) \equiv 2\left\langle h_{ij} h^{ij} \right\rangle = 4\left(\frac{H[\phi(f)]}{2\pi M_G}\right)^2 = \frac{1}{3\pi^2}\frac{V[\phi(f)]}{M_G^2} \equiv \frac{1}{2}\Delta_h^2$$

$$H[\phi(f)] = \frac{V[\phi(f)]^{1/2}}{\sqrt{3}M_G}$$



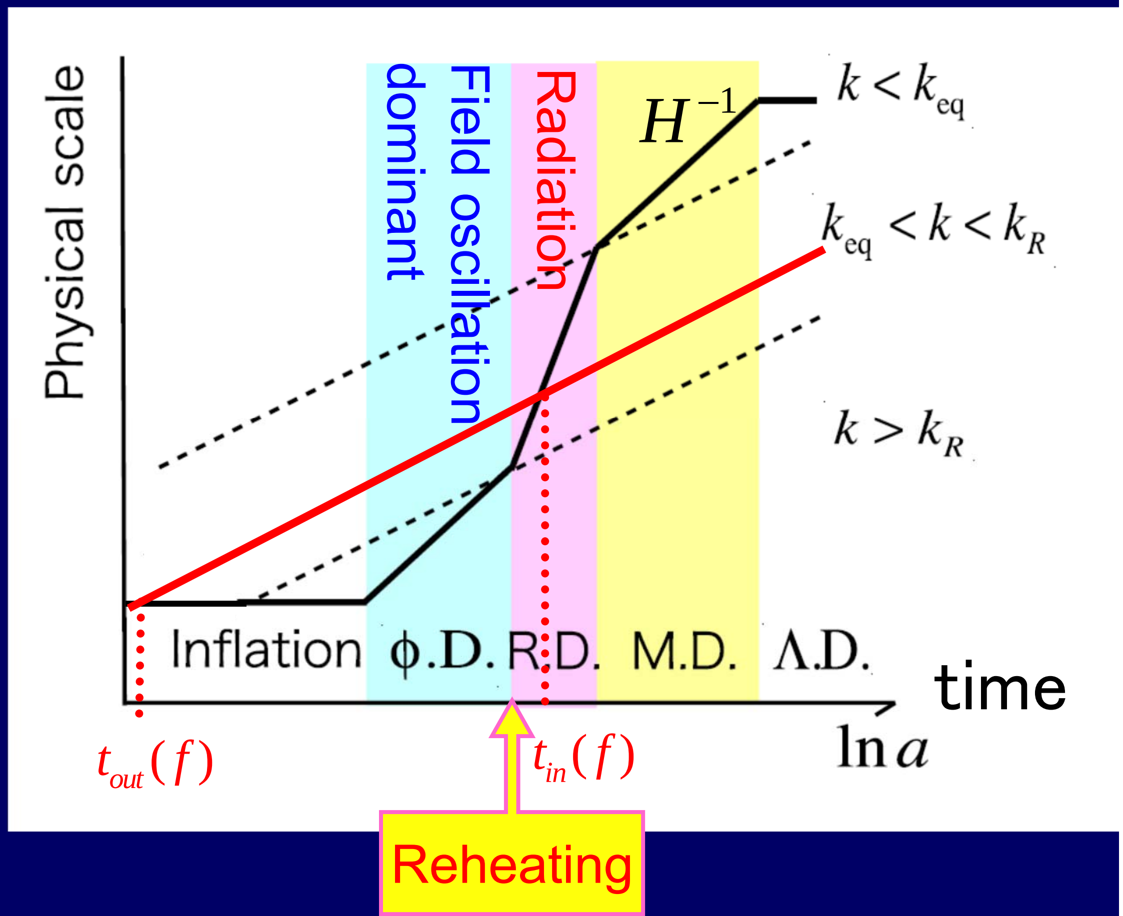
Square amplitude of tensor perturbation

Tensor perturbation has a nearly scale-invariant spectrum at formation.

Evolution of gravitational waves in the inflationary Universe

- Amplitude of GW is constant when its wavelength is longer than the Hubble radius between $t_{out}(f)$ and $t_{in}(f)$.

- After entering the Hubble radius, the amplitude decreases as $\propto a^{-1}(t)$ and the energy density as $\propto a^{-4}(t)$.



When $a(t) \propto t^p \quad (p<1)$, the tensor perturbation evolves as

$$h(f,a) \propto a(t)^{\frac{1-3p}{2p}} J_{\frac{3p-1}{2(1-p)}}\left(\frac{p}{1-p}\frac{k}{a(t)H(t)}\right), \quad k = 2\pi f a(t_0)$$

Density parameter in GW per logarithmic frequency interval

$$\Omega_{GW}(f,t) = \frac{1}{\rho_{cr}(t)} \frac{d\rho_{GW}(f,t)}{d\ln f}$$

When the mode reentered the Hubble horizon at $t \equiv t_{in}(f)$, the angular frequency is equal to $\omega = H\left(t_{in}(f)\right)$, so we find

$$\frac{d\rho_{GW}(f,t_{in}(f))}{d\ln f} = \frac{\omega^2}{32\pi G} h_{\inf}^2(f) = \frac{H^2\left(t_{in}(f)\right)}{32\pi G} h_{\inf}^2(f) = \frac{1}{24}\rho_{cr}(t_{in}(f))\Delta_h^2(f)$$

$$\boxed{\Omega_{GW}(f,t_{in}(f)) = \frac{1}{24}\Delta_h^2(f)}$$

After entering the Hubble horizon,

$$\Omega_{GW}(f,t) = \frac{\rho_{GW,\ln f}(f,t)}{\rho_{tot}(t)} \quad \begin{array}{l} \propto a^{-4}(t) \\ \propto a^{-3(1+w)}(t) \end{array}$$

$$w \equiv p/\rho_{tot}$$

: equation of state
in the early Universe

$$\Omega_{GW}(f,t) \approx \frac{1}{24}\Delta_h^2(f)\left(\frac{a(t_{in}(f))}{a(t)}\right)^{1-3w}$$

Radiation dominated era: constant
Field oscillation dominated era: decreases $\propto a^{-1}(t)$

High frequency modes which entered the Hubble radius in the field oscillation regime acquires a suppression $\propto f^{-2}$.

We may determine the equation of state in the early Universe.

We may determine thermal history of the early Universe.

N. Seto & JY (03), K. Nakayama, S. Saito, Y. Suwa & JY (08)

see also Boyle and Steinhardt (08)

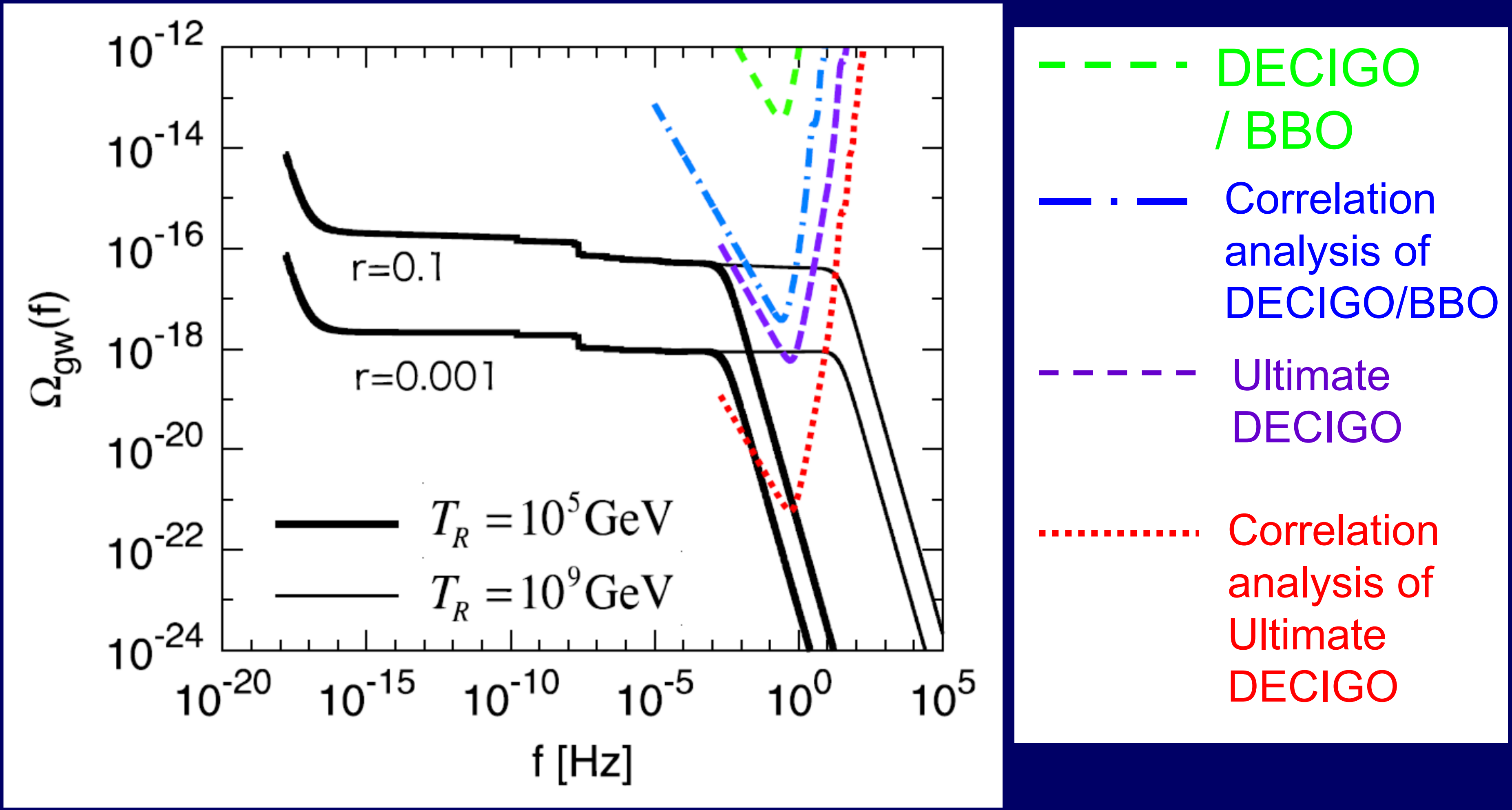
T_R reheat temperature after inflation

Determination of the reheat temperature is important, because,…

- A number of important phenomena took place in the "first dark age".
  Baryo/Leptogenesis, Production/Freezout of CDM, Phase Transitions etc.
  Temperature dependence is important.
  (e.g.) Thermal Leptogenesis requires $T_R > 10^9\,\mathrm{GeV}$ or so.

- Uncertainty in T_R results in uncertainty of the number of e-folds corresponding to the "pivot" scale of CMB observation, say, $k = 0.002\mathrm{Mpc}^{-1}$ up to $\Delta N \approx 12$, which affects precision determination of the inflaton potential.

Thermal History is inprinted on the spectrum of GWs.



$$r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2}$$

Tensor-to-scalar ratio measures the scale of inflation

$$V[\phi] = (3.2\times 10^{16}\,\mathrm{GeV})^4 r = (7.5\times 10^{15}\,\mathrm{GeV})^4 \left(\frac{r}{0.003}\right)$$

Some of the recent particle physics models of inflation

★ Kk … 3)

★ DE … extra dimension

(Silverstein & Tong 04)

… possible.

… ohyama & Kinoshita 07)

★ Ra … , Kallosh, Linde &Quevedo 04)

… uning of parameters

★ Ax …

… vity

★ Higgsflation (Kaloper, Sorbo & JY 08)

★ Ch …

(Kawasaki, Yamaguchi & Yanagida 00)

… etry

★ MS …

… d parameters.

Newer models of “stringy” inflation

★ Monodoromy model

$r = 0.03$ (Silverstein & Westphal 08
McAllister, Silverstein & Westphal 10)

★ Fibre inflation

$r = 0.005$ (Cicoli, Burgers, & Quevedo 09)

★ Warped Wilsonline DBI inflation

$r \sim 0.1$ (Avgoustidis & Zavala 09)

etc

One of the Newest: G-inflation driven by the Galileon

$r = 0.17$ (Kobayashi, Yamaguchi, & JY 10)

The oldest but still viable model of R^2 inflation

$r = 0.003$ (Starobinsky 80)

(may be combined with the origin of dark energy?)

$r \sim 10^{-25}$ It can occur only if previous inflation with a slightly higher energy scale is realized with a sufficiently low reheat temperature, very contrived. (Kamada & JY 10)

- In the standard cosmology, GW with frequency f today reentered the Hubble radius at the temperature,

$$T(f) = 3.8 \times 10^{6} \left(\frac{f}{0.1\mathrm{Hz}} \right) \mathrm{GeV}$$

- Hence by observing GW with $f = 0.1 \sim 10\mathrm{Hz}$, we can probe the thermal history at the epoch $T = 10^{6-8}\mathrm{GeV}$ and later.

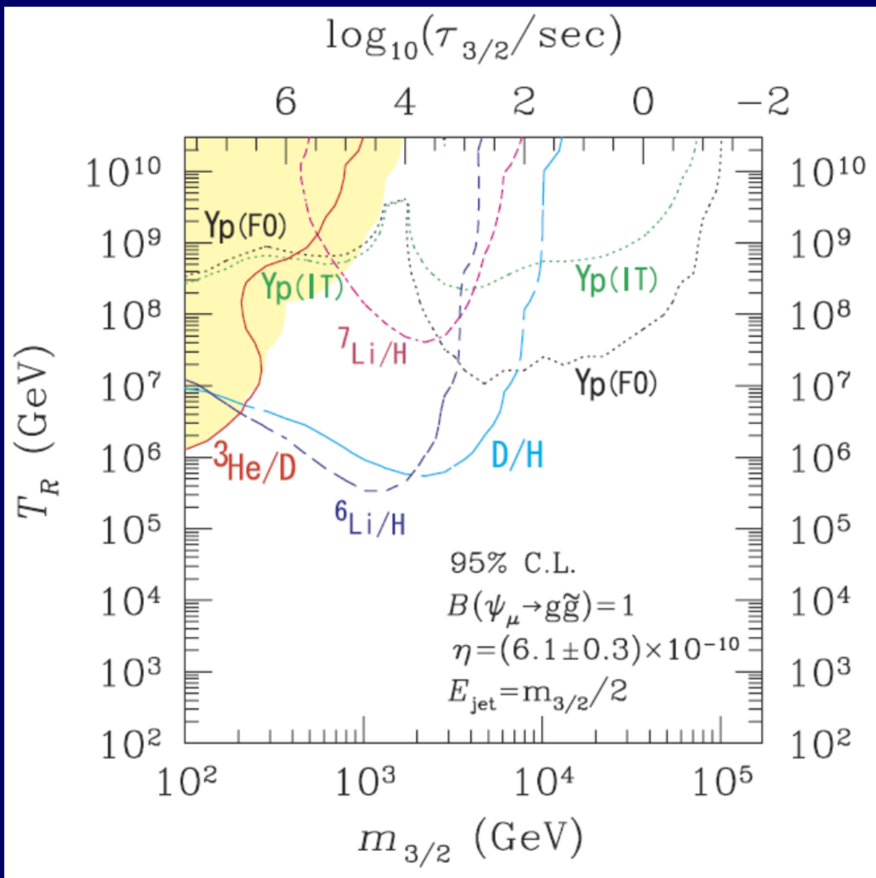
- This range of the temperature interestingly coincides with the constraints on the reheat temperature imposed by the gravitino problem.

  The abundance of gravitinos is determined by the reheat temperature modulo the dilution factor due to subsequent entropy production.

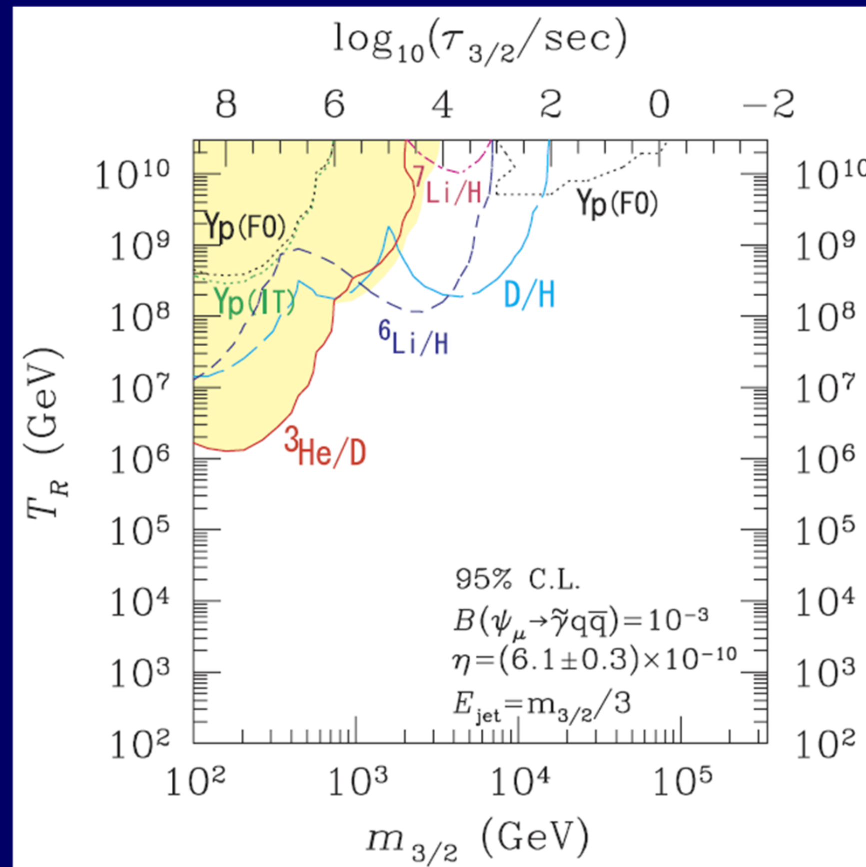
  Unstable gravitinos dissociate light elements.
  Stable gravitinos could overdominate the Universe.

Constraints on the reheat temperature from unstable gravitinos

$$T_R \leq 10^{6-8}\,\mathrm{GeV}$$



$B_h = 1$ Large hadronic branching ratio (gluino mode open)



$B_h \sim 10^{-3}$ Small hadronic branching ratio (gluino mode closed)

(Kawasaki, Kohri, & Moroi 05)

If, on the contrary, the gravitino is light and stable (LSP), then its abundance should be equal to or smaller than the dark matter.

$$T_R < 7 \times 10^6 \,\mathrm{GeV} \left(\frac{m_{\tilde{g}}}{1\ \mathrm{TeV}} \right)^{-2} \left(\frac{m_{3/2}}{1\ \mathrm{GeV}} \right)$$

for $m_{3/2} \sim 10^{-4} - 10\mathrm{GeV}$

(Moroi, Murayama & Yamaguchi 93)

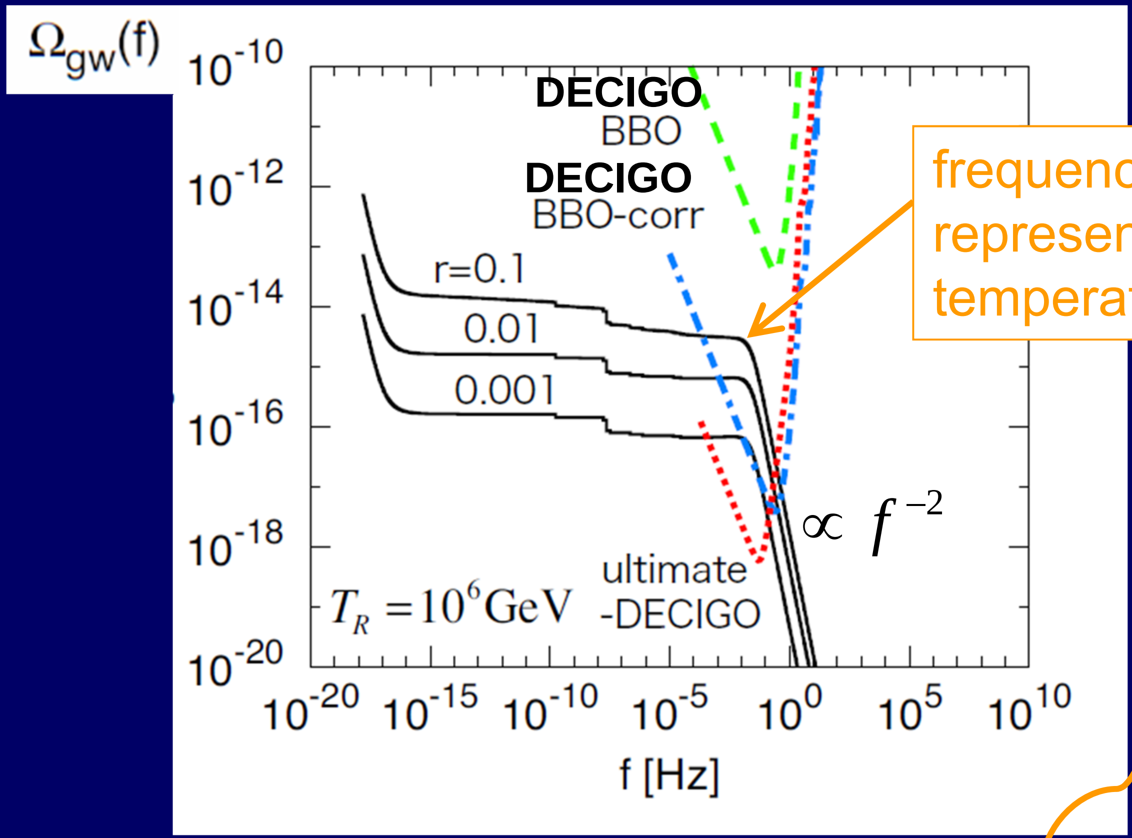
includes DECIGO band

Non-thermal production of gravitinos gives a lower-bound on the reheat temperature.

Model dependent

(Kawasaki, Takahashi & Yanagida 06
Endo, Takahashi & Yanagida 06)

Then we should find a trace of reheating epoch in GW spectrum.

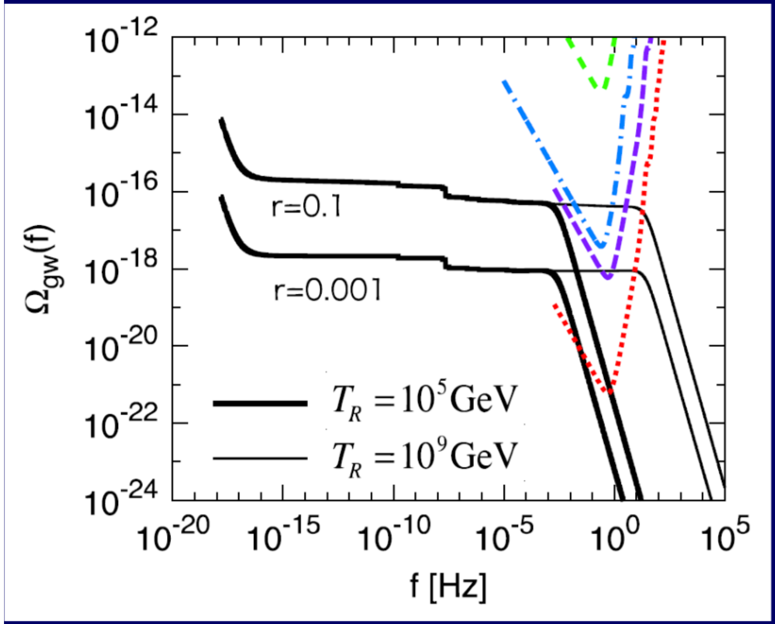


frequency of the bend represents the reheat temperature.

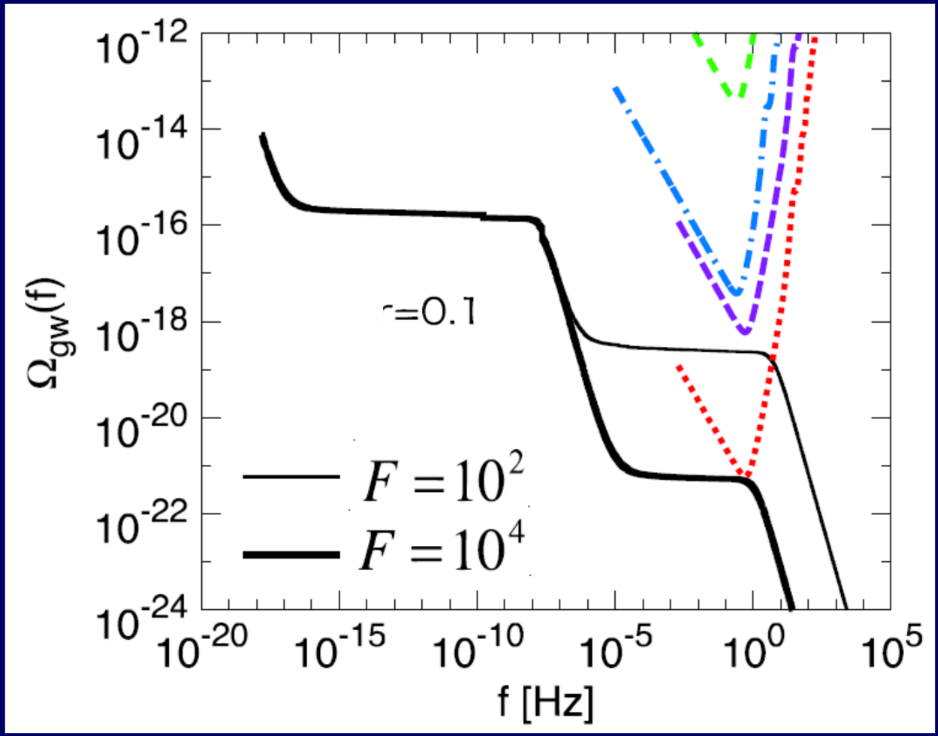
$$\Omega_{GW}(f) \propto \Delta_h^2(f)\left[1 - 0.32x_R + 0.99x_R^2\right]^{-1}$$

$$x_R \equiv \frac{f}{f_R} \qquad f_R = 0.1\mathrm{Hz}\left(\frac{T_R}{3.8\times10^6\,\mathrm{GeV}}\right)\left(\frac{g_*(T_R)}{106.75}\right)^{\frac{1}{6}}$$

If late-time entropy production occurs after reheating, the spectrum is further modified to exhibit multistep structures.



No entropy production after reheating



Entropy production w/ dilution factor

$$F \equiv \frac{s(T_d)a^3(T_d)}{s(T_R)a^3(T_R)}$$

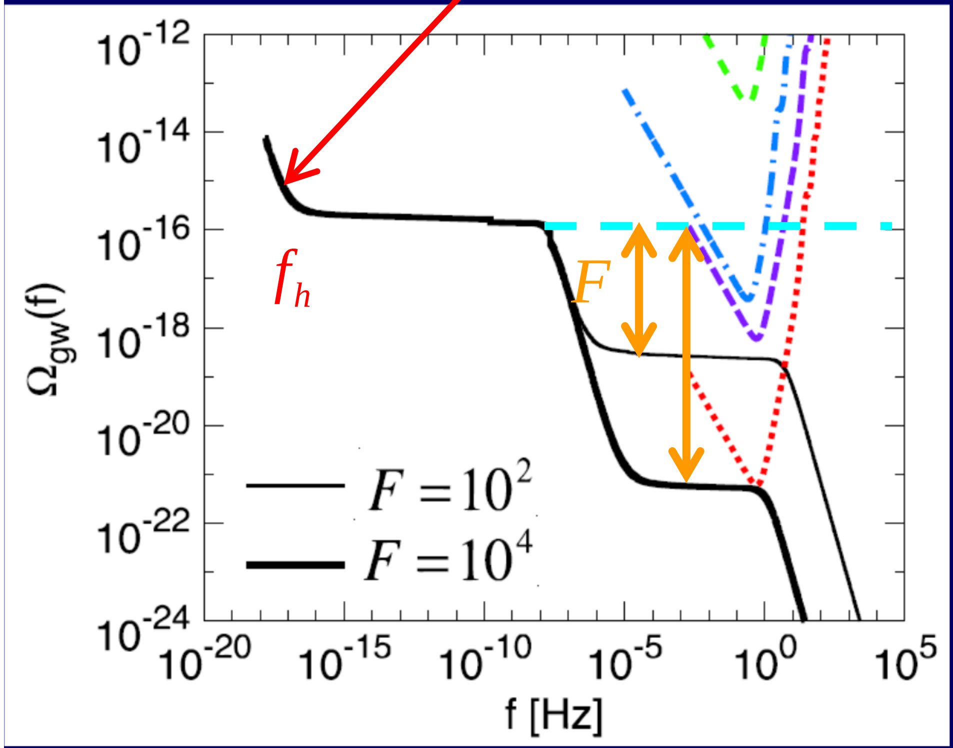
T_d: temperature after entropy production

High frequency part is modified as

$$\Omega_{GW}(f,t) \to F^{-4/3}\Omega_{GW}(f,t) \quad \text{and} \quad f_R \to F^{-1/3} f_R$$

If we can measure the amplitude at two distinct frequency bands, we can in principle determine both the dilution factor and reheat temperature simultaneously.

Low frequency component can be measured by B-mode polarization of CMB



We can extrapolate the B-mode observation result to higher frequencies using the slow-roll parameters of the standard inflation model which can be determined by observation of CMB temperature anisotropy.

$$\varepsilon = \frac{M_G^2}{2}\left(\frac{V'[\phi]}{V[\phi]}\right)^2 \qquad \eta = M_G^2 \frac{V''[\phi]}{V[\phi]}$$

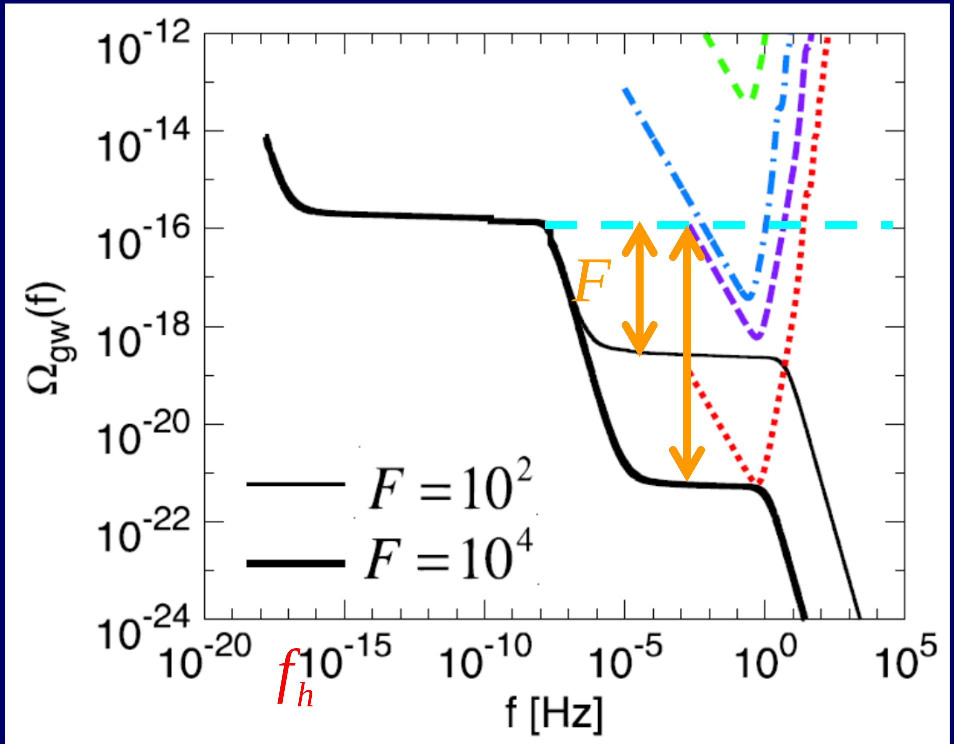
$$\xi = M_G^4 \frac{V'[\phi]V'''[\phi]}{V^2[\phi]}$$

Observables: $r = \frac{\Delta_h^2}{\Delta_R^2} = 16\varepsilon \qquad n_s - 1 = -6\varepsilon + 2\eta, \qquad \frac{dn_s}{d\ln k} = 16\varepsilon\eta - 24\varepsilon^2 - 2\xi$

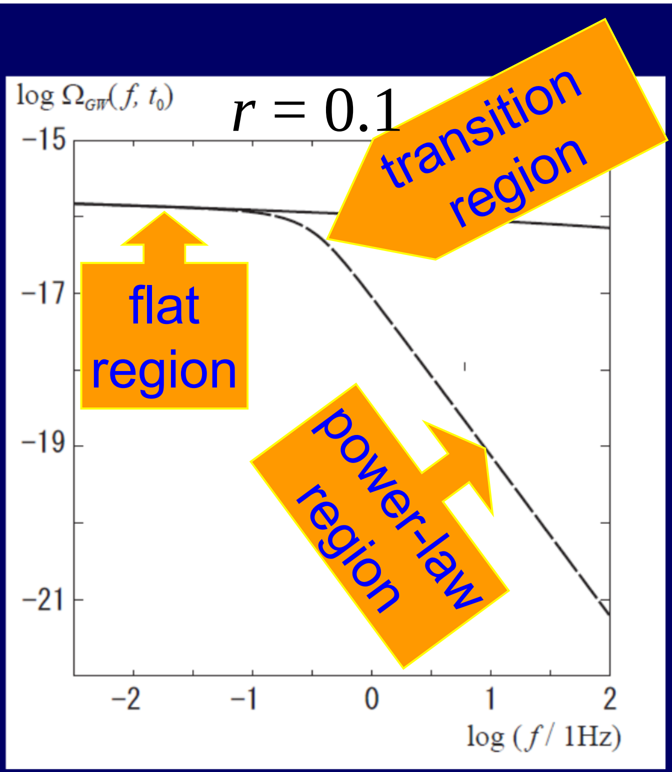
★ We can extrapolate $\Delta_h^2(f_h)$ to $\Delta_h^2(f)$ using slow-roll parameters at $k = 0.002\mathrm{Mpc}^{-1}$.

$$\Delta_h^2(f) = 8\left(\frac{H(\phi(f))}{2\pi M_G}\right)^2 = \Delta_h^2(f_h)\left[1 - 2\varepsilon \ln\frac{f}{f_h} + 2\varepsilon(\eta - \varepsilon)\left(\ln\frac{f}{f_h}\right)^2 - \frac{1}{3}\varepsilon(12\varepsilon^2 - 16\varepsilon\eta + 4\eta^2 + 2\xi)\left(\ln\frac{f}{f_h}\right)^3\right]$$

★ This gives the initial amplitude of gravitational radiation when each frequency mode reentered the Hubble radius at $t \equiv t_{in}(f)$.



By comparing it with the actual observation, we can determine the dilution factor F.



Depending on which region is observed, we obtain different information due to the possible degeneracy between T_R and F .

1) Flat region only

F is determined by the height of $\Omega_{GW}(f)$.

$T_R > 2.4\times10^9 F^{1/3}$ GeV lower bound

Gravitino mass should satisfy

$$m_{\frac{3}{2}} > 0.4\left(\frac{F}{3\times10^4}\right)^{-2/3}\left(\frac{m_{\tilde{g}}}{1\ \mathrm{TeV}}\right)^2 \mathrm{GeV} \quad \text{(stable)}$$

$$m_{\frac{3}{2}} > 10\ \mathrm{TeV} \quad \text{(unstable)}$$

2) Transition region

$$7.1\times10^5 F^{1/3}\ \mathrm{GeV} < T_R < 2.4\times10^9 F^{1/3}\ \mathrm{GeV}$$

F is determined from the amplitude and T_R from the spectral shape.

Thermal history is essentially fixed!

3) Power-law region $T_R < 7.1\times10^5 F^{1/3}$ GeV

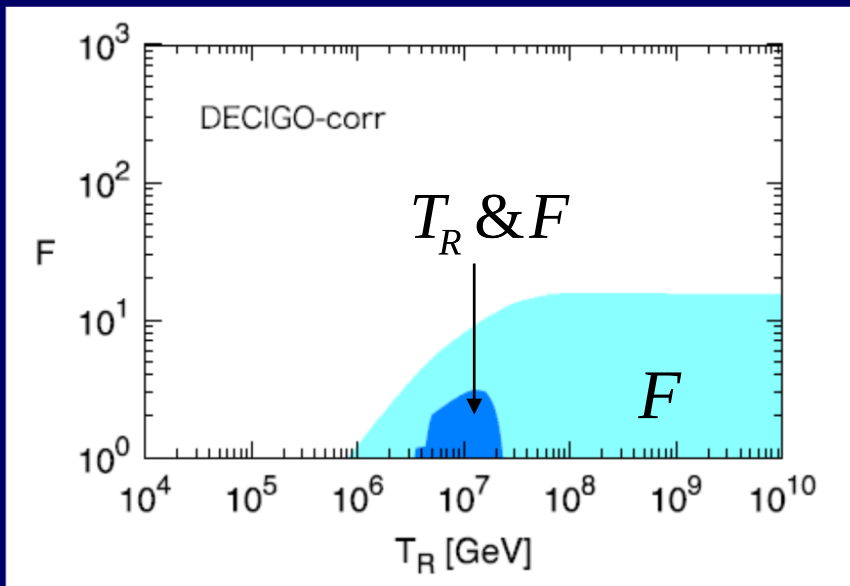
We can determine only the ratio T_R/F but it fixes $n_{\frac{3}{2}}/s$ uniquely.

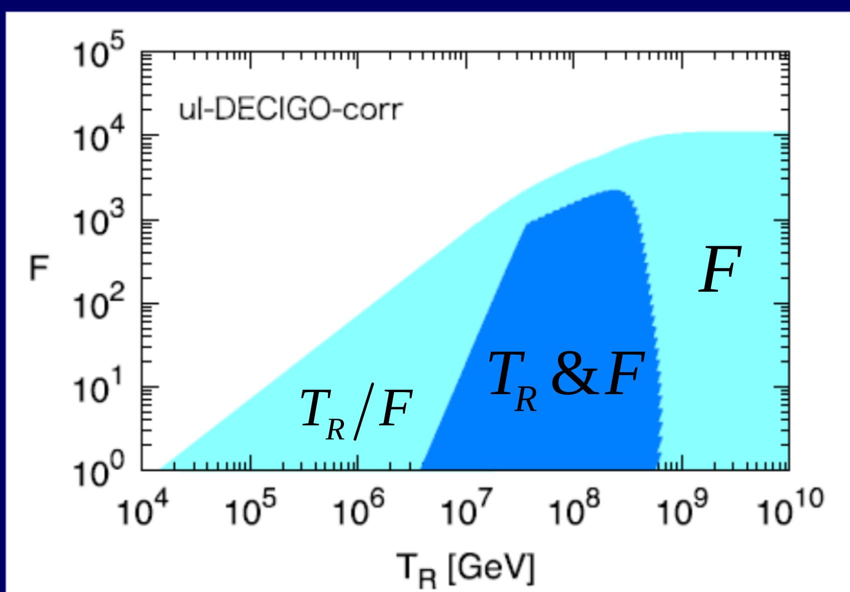
Stable gravitino should satisfy

$$m_{\frac{3}{2}} > 0.01\left(\frac{T_R/F}{7\times10^5\mathrm{GeV}}\right)\left(\frac{m_{\tilde{g}}}{1\ \mathrm{TeV}}\right)^{-1} \mathrm{GeV}$$

No constraint on unstable gravitino mass.

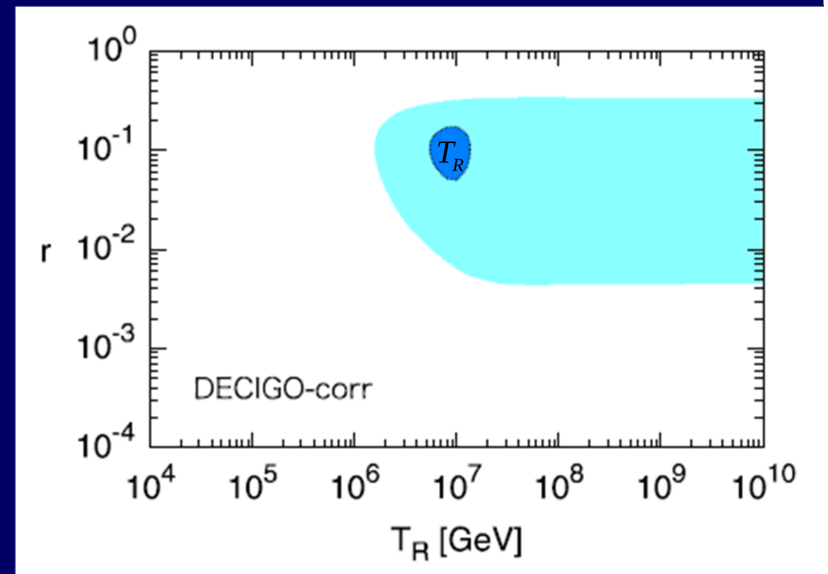
$r = 0.1$

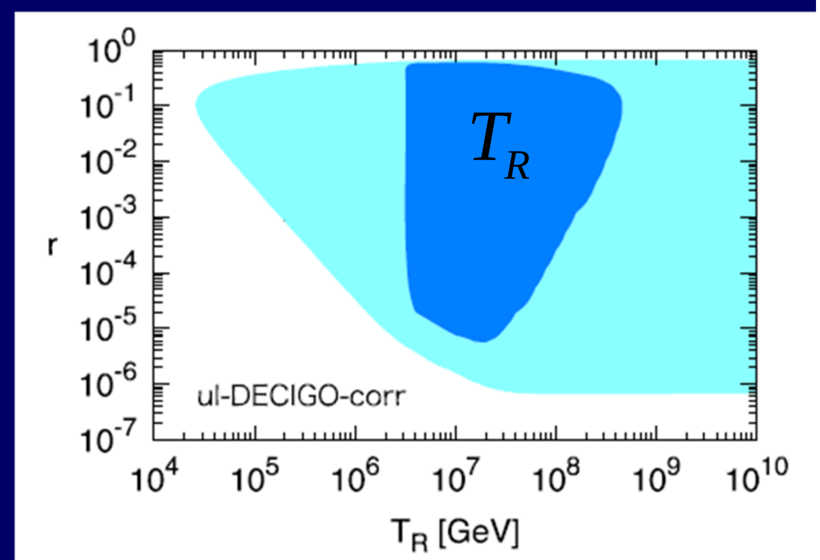

DECIGO-corr
T_R & F
F
F
T_R [GeV]


ul-DECIGO-corr
F
T_R & F
T_R / F
F
T_R [GeV]

S/N=5

$F = 1$


T_R
DECIGO-corr
r
T_R [GeV]


T_R
ul-DECIGO-corr
r
T_R [GeV]

So far:

- ⋆ Standard nearly scale-invariant spectrum of density/curvature fluctuations from the inflaton
- ⋆ with a non-standard Thermal History in the Early Universe

From now on:

- ⋆ The case density/curvature fluctuations are entirely generated by a curvaton.
- ⋆ By assumption, density/curvature fluctuations from the inflaton is negligibly small.

The Curvaton σ

(Mollerach 90, Linde & Mukhanov 97, Lyth & Wands 02 Moroi & Takahashi 01, Enqvist & Sloth 02)

- ★ A light scalar field during inflation which acquires quantum fluctuation $\delta\sigma = \frac{H_{\inf}}{2\pi}$ each Hubble time.
- ★ Let us assume its potential is just quadratic $V[\sigma] = \frac{m_\sigma^2}{2}\sigma^2$ with its initial amplitude of the homogeneous part σ_i.
- ★ It starts coherent field oscillation at $H \approx m_\sigma$, and its energy density gradually increases relative to radiation.
- ★ It contributes to the comoving curvature perturbation according to the delta-*N* formula up to second order,

$$\mathcal{R}_c = N_\sigma \delta\sigma + \frac{1}{2} N_{\sigma\sigma} (\delta\sigma)^2$$

- ★ $\delta\sigma$ is random Gaussian.

★ More explicitly, it is given by the relative energy density of the curvaton at its decay as

$$\mathcal{R}_c = \frac{2R}{3}\left(\frac{\delta\sigma}{\sigma_i}\right) + \left(\frac{R}{3} - \frac{4R^2}{9} - \frac{2R^3}{9}\right)\left(\frac{\delta\sigma}{\sigma_i}\right)^2$$

$$R = \left.\frac{3\rho_\sigma}{4\rho_r + 3\rho_\sigma}\right|_{\sigma\ \mathrm{decay}}$$

If *R* =1 at decay, the curvaton increases the entropy density.

★ COBE／WMAP normalization

$$\sqrt{\Delta^2_{\mathcal{R}_c}(k_*)} = \frac{R}{3}\left(\frac{H_{\rm inf}}{\pi\sigma_i}\right) = 5 \times 10^{-5}$$

★ Tensor-scalar ratio in this scenario

$$r \equiv \frac{\Delta^2_h(k_*)}{\Delta^2_{\mathcal{R}_c}(k_*)} = \frac{18}{R^2}\left(\frac{\sigma_i}{M_{\rm Pl}}\right)^2 = 0.14\left(\frac{H_{\rm inf}}{10^{14}{\rm GeV}}\right)^2$$

★ Spectral indices in the curvaton scenario

Tensor

$n_t = -2\varepsilon$

Scalar

$n_s = 1 - 2\varepsilon + \frac{2m_\sigma^2}{3H_{\inf}^2}$

$$m_\sigma^2 = \frac{3}{2}(n_s - n_t - 1)H_{\rm inf}^2 = 11(n_s - n_t - 1)r \times (10^{14}{\rm GeV})^2$$

Usually the curvaton mass is so small that we would find $n_s - n_t \simeq 1$.

Smoking gun for the curvaton scenario ?

★ Non-Gaussianity: nonlinearity parameter: f_{NL}

$$\mathcal{R}_c = \frac{2R}{3}\left(\frac{\delta\sigma}{\sigma_i}\right) + \left(\frac{R}{3} - \frac{4R^2}{9} - \frac{2R^3}{9}\right)\left(\frac{\delta\sigma}{\sigma_i}\right)^2$$

VS

$$\mathcal{R}_c = \mathcal{R}_c{}^{(\rm g)} + \frac{3}{5} f_{\rm NL} \mathcal{R}_c{}^{(\rm g)2}$$

$$f_{\rm NL} = \frac{5}{4R}\left(1 - \frac{4}{3}R - \frac{2}{3}R^2\right)$$

currently constrained as $-10 < f_{NL} < 74$ by WMAP7.

★ Large non-Gaussianity for small *R* $\quad f_{NL} \cong \frac{5}{4R}$

If non-Gaussianity is large, say, $f_{NL} > 10$ it could be measured by Planck and the curvaton parameters could be fixed.

★ Large entropy production if *R* is close to 1

Dilution factor $F = \frac{\sigma_i^2}{6M_G^2}\frac{T_R}{T_\sigma} \square\ 1$ $\quad T_\sigma$: radiation temperature just after the curvaton decay

If we could measure the dilution factor by DECIGO combined with B-mode data, we could determine the curvaton parameters as well.

These two cases are complementary to each other.

σ_i : initial amplitude of the curvaton

T_σ : related with the decay rate

2 important parameters

★ The case with large non-Gaussianity

$$r \equiv \frac{\Delta_h^2(k_*)}{\Delta_{\mathcal{R}_c}^2(k_*)} = \frac{18}{R^2}\left(\frac{\sigma_i}{M_{\rm Pl}}\right)^2$$ and $$R \cong 5/(4f_{NL})$$ yield

$$\boxed{\frac{\sigma_i}{M_{\rm Pl}} = \frac{5}{12 f_{NL}}\left(\frac{r}{2}\right)^{1/2}}$$

Since no significant entropy production occurs in this case,

$$\left.\frac{\rho_\sigma}{\rho_r}\right|_{\sigma\ \rm decay} = \frac{1}{6}\left(\frac{\sigma_i}{M_{\rm Pl}}\right)^2 \frac{a(T_\sigma)}{a(T_{\rm R})} = \frac{1}{6}\left(\frac{\sigma_i}{M_{\rm Pl}}\right)^2 \frac{T_{\rm R}}{T_\sigma} \cong \frac{4}{3}R$$

The ratio at the onset of σ oscillation

$$\boxed{\frac{T_\sigma}{T_{\rm R}} = \frac{5r}{576 f_{NL}}}$$

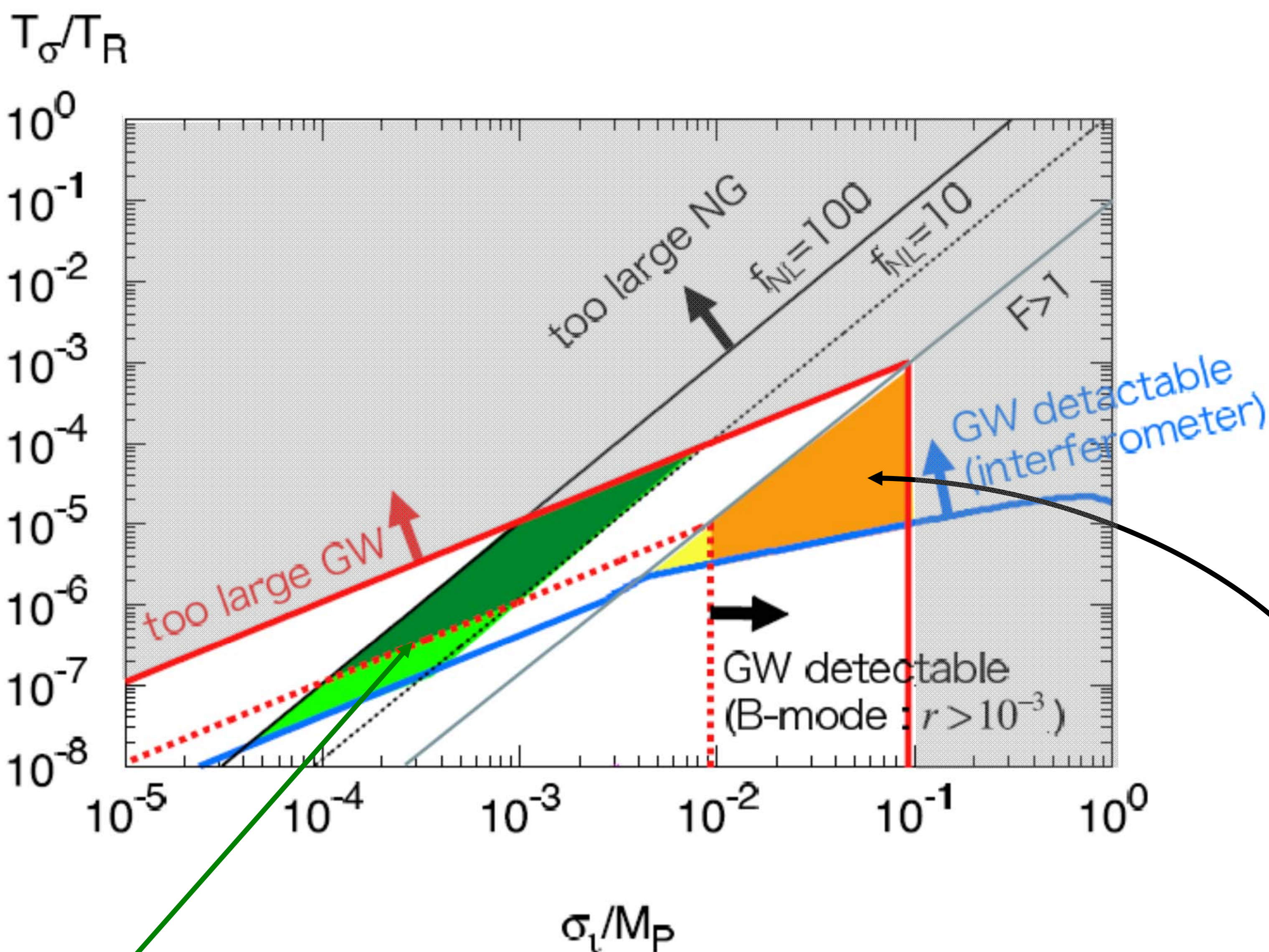
Both parameters can be determined by the observables.

★ The case curvaton is dominant at decay: $R = 1$

$$r \equiv \frac{\Delta_h^2(k_*)}{\Delta_{\mathcal{R}_c}^2(k_*)} = \frac{18}{R^2}\left(\frac{\sigma_i}{M_{\rm Pl}}\right)^2$$ yields $$\boxed{\frac{\sigma_i}{M_{\rm Pl}} = \left(\frac{r}{18}\right)^{1/2}}$$

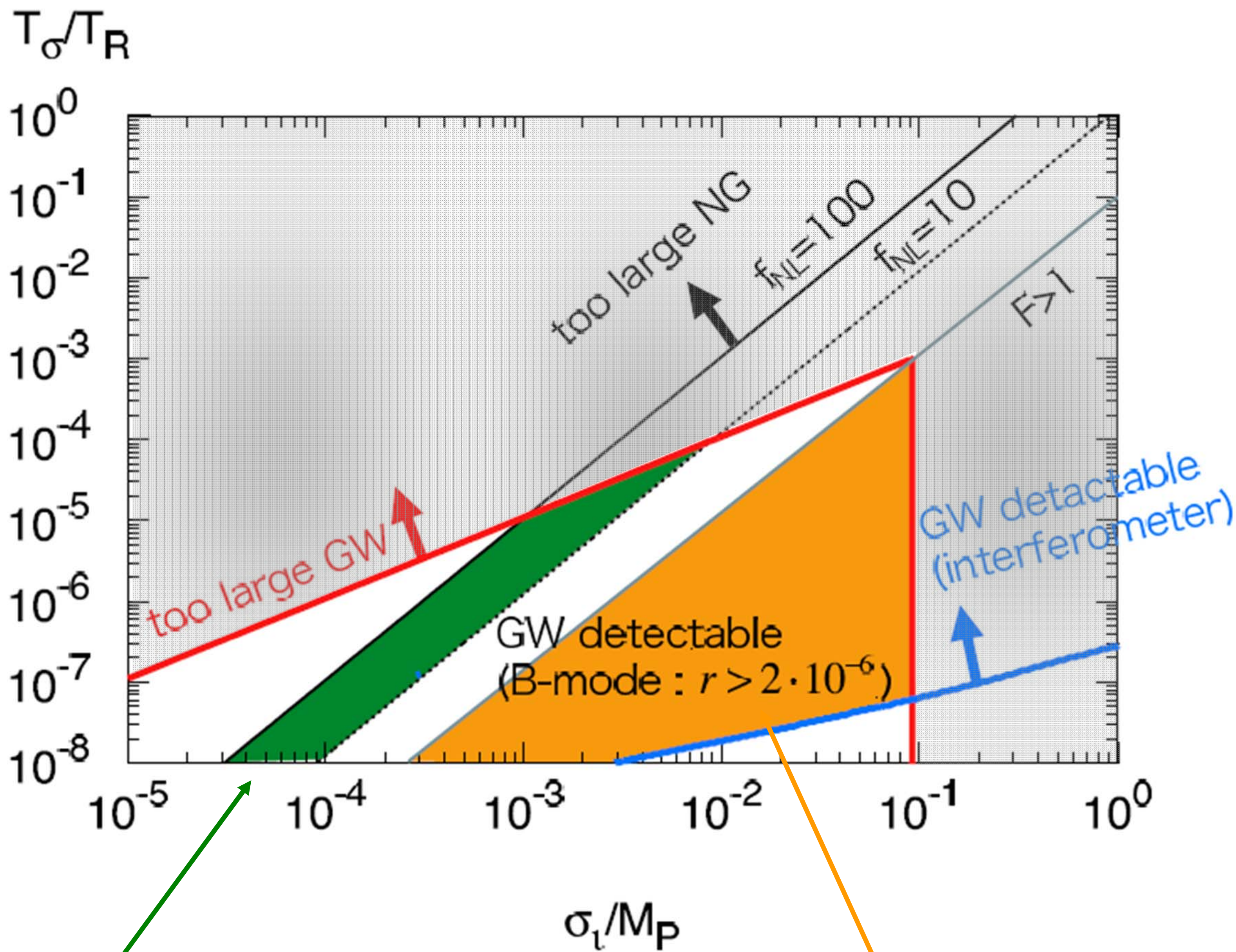
$$F = \frac{\sigma_i^2}{6M_G^2}\frac{T_R}{T_\sigma}$$ leads to $$\boxed{\frac{T_\sigma}{T_{\rm R}} = \frac{r}{108F}}$$

Both parameters can be determined by the observables.



Model parameters can be determined using non-Gaussianity

Model parameters can be determined using dilution factor

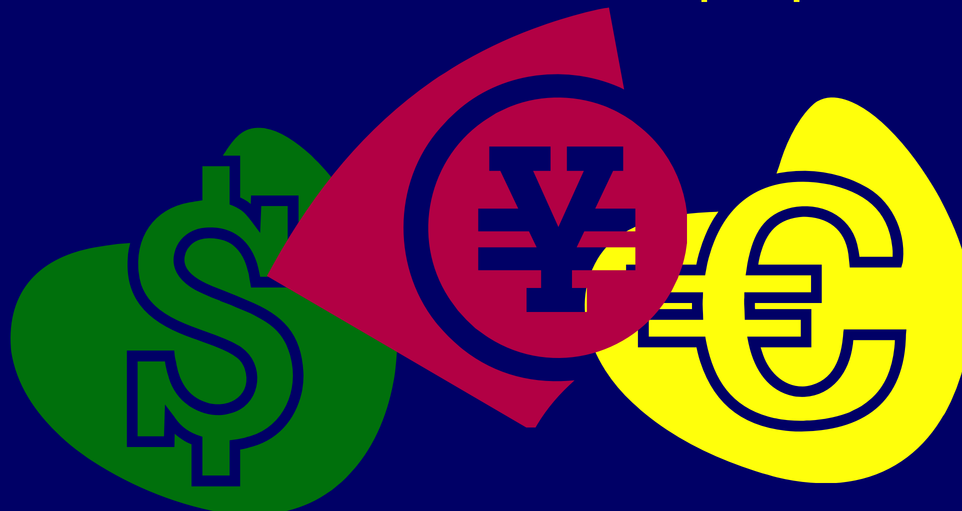


Model parameters can be determined using non-Gaussianity

Model parameters can be determined using dilution factor

Conclusion

Observation of cosmological gravitational wave background can bring about useful information on the thermal history of the early Universe, the nature of dark matter, and properties of the curvaton.



Since GWs associated with PBH-DM production is so large that the first scientific achievement of DECIGO/BBO/LISA would be to rule out (or confirm?) PBH-Dark Matter.

TOBA may also do it even earlier.