Anomaly Detection of Time Series

A THESIS
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA
BY

Deepthi Cheboli

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
Master Of Science

May, 2010

Acknowledgements

First and foremost, I would like to thank my advisor, Prof.Vipin Kumar, for his valuable support and help throughout the course of my Masters. I would like to thank the Department of Computer Science and Engineering at University of Minnesota for providing me excellent facilities during my graduate studies. I would like to acknowledge Varun Chandola and Michael Steinbach for their valuable feedback for my work at all times. I would also like to thank all the members of our Data Mining Group for providing a challenging and friendly environment at work. I am indebted to my family members and friends who have constantly motivated me all through my life and made me what I am.

Abstract

This thesis deals with the problem of anomaly detection for time series data. Some of the important applications of time series anomaly detection are healthcare, eco-system disturbances, intrusion detection and aircraft system health management.

Although there has been extensive work on anomaly detection (1), most of the techniques look for individual objects that are different from normal objects but do not consider the sequence aspect of the data into consideration.

In this thesis, we analyze the state of the art of time series anomaly detection techniques and present a survey. We also propose novel anomaly detection techniques and transformation techniques for the time series data. Through extensive experimental evaluation of the proposed techniques on the data sets collected across diverse domains, we conclude that our techniques perform well across many datasets.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

Anomalies are patterns in data that do not conform to a well defined notion of normal behavior. The problem of finding these patterns is referred to as anomaly detection. The importance of anomaly detection is due to the fact that anomalies in data translate to significant and actionable information in a wide variety of application domains (1). For example, an anomalous traffic pattern in a computer network could mean that a hacked computer is sending out sensitive data to an unauthorized destination (2). An anomalous MRI image may indicate the presence of malignant tumors (3) or anomalies in credit card transaction data could indicate credit card or identity theft (4) . Detecting anomalies has been studied by several research communities to address issues in different application domains (1).

In many domains, such as flight safety, intrusion detection, fraud detection, health-care, etc., data is collected in the form of sequences or time-series. For example, in the domain of aviation or flight safety, the data collected from flights is in the form of sequences of observations from various aircraft sensors during the flight. A fault in the aircraft results in anomalous readings in sequences collected from one or more of the sensors. Similarly, in health-care domain, an abnormal medical condition in a patient's heart can be detected by identifying anomalies in the time-series corresponding to *Electrocardiogram* (ECG) recordings of the patient.

In this thesis, we identify different definitions of anomalies in time series. We focus on a specific problem formulation, semi-supervised anomaly detection (1). We study

the relationship between the anomaly detection techniques and the nature of time series. Depending on this understanding we propose a novel transformation technique for univariate time series which uses subspace based analysis.

1.1 Outline

This thesis is organized as follows.

- **Chapter 2** is a survey on anomaly detection techniques for time series data. It discusses the state of the art in this domain and categorizes the techniques depending on how they perform the anomaly detection and what transfomation techniques they use prior to anomaly detection.
- **Chapter 3** discusses our novel subspace based transformation for univariate time series data. It provides the motivation and methodology for this transformation.
- **Chapter 4** provides the extensive experimental framework we developed and the results obtained after transformation. We conclude with some observations and future directions of research.
- **Chapter 5** provides some discussions on various existing and proposed techniques and we conclude with future directions of research.

Chapter 2

Anomaly Detection for Time Series: A Survey

In this chapter we investigate the problem of anomaly detection for univariate time series. Although there has been extensive work on anomaly detection (1), most of the techniques look for individual objects that are different from normal objects but do not consider the sequence aspect of the data into consideration. Such anomalies are also referred to as point anomalies. Consider the example given in Figure 2.1 which corresponds to the ECG data of a patient, which is usually periodic. The highlighted region denotes an anomaly because the same low value exists for an abnormally long time (corresponding to an *Atrial Premature Contraction*). Note that the low value by itself is not an anomaly as it occurs at sevaral other places. Hence if the data is treated as a collection of amplitude values, while ignoring their temporal aspect, the anomaly cannot be detected.

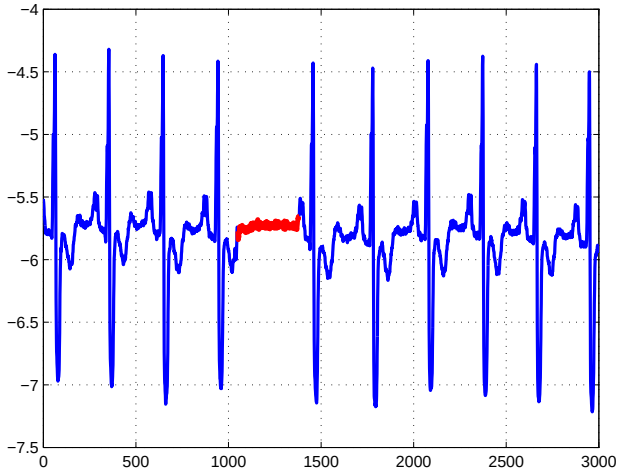


Figure 2.1 : Anomalous time series

The problem of anomaly detection for time series is not as well understood as the traditional anomaly detection problem. Multiple surveys: Chandola et al (1), Agyemang et al (5) and Hodge et al (6) discuss the problem of anomaly detection. For symbolic sequences, several anomaly detection techniques have been proposed. They are discussed in a survey by Chadola et al (7). While for the univariate and multivariate time series, limited number of techniques exist.

Existing research on anomaly detection for time series has been fragmented across different application domains, without a good understanding of how different techniques are related to each other and what their strengths and weaknesses are. The survey part of this thesis is an attempt to provide a comprehensive understanding and structured overview of the research on anomaly detection techniques for time series spanning multiple research areas and application domains. We try to understand how the performance of the techniques relates to the various aspects of the problem, such as nature of data, nature of anomalies, etc.

The organization of the chapter is as follows. In Section 2.1 we discuss some of the important applications of time series anomaly detection. Section 2.2 describes various formulations for the problem of anomaly detection of time series and Section 2.3 deals with the challenges involving this problem. We describe some of the different types of

time series data in section 2.4. In section 2.5 we give a brief overview of existing techniques and categorize them into two orthogonal dimensions (a) the process of finding anomalies (Procedural dimension, Section 2.7) and (b) the way the data is transformed prior to anomaly detection (Transformation dimension, Section 2.6). Out of the three problem formulations proposed in Section 2.2, we mainly deal with one of the formulation called semi-supervised setting, in this survey. Section 2.8 gives a brief overivew of another problem formulation called discord detection, which can be adapted to our problem formulation.

2.1 Applications

Some of the important applications of time series anomaly detection are :

1. Detecting anomalous heart beat pulses using ECG data (8; 9) : Usually ECG data can be seen as a periodic time series. An anomaly in this case would be the non-conforming pattern e.g., in terms of periodicity or amplitude, which could indicate a health problem.

2. Attack detection in recommender systems : Shilling attacks, in which the attackers introduce biased ratings in order to influence future recommendations (10).

3. Detection of anomalous flight sequences using sensor data from aircrafts: Typical system behavior of flights is characterized by the sensor data information of different parameters which change during the course of flight. Any deviation from the typical system behavior us anomalous (11).

4. Shape anomalies : Finding the shapes which interestingly differ from others, where each shape is converted to a time series (12; 13). In the field of medical data mining, given several shapes of a species, a shape that differs from others might indicate an anomaly caused by genetic mutation. In anthropological data mining, different shapes of interest can be pottery, bones, arrowheads etc (14; 13).

5. Outlier light curves in catalogs of periodic stars : Detection of outliers in periodic variable stars involves finding the statistical deviance from the rest. The outliers correspond to some interesting intrinsic physical differences, such as slowly

changing period or amplitude, which introduce noise in the light curve (15; 16).

6. Eco-system disturbances using earth science data such as vegetation or temperature (17).

2.2 Problem setting

The problem of anomaly detection for time series data can be viewed in different ways. Here we discuss three possible definitions/settings.

Problem setting 1 **: Detecting contextual anomalies in the time series.**

In this setting of anomaly detection in a time series, the anomalies are the individual instances of the time series which are anomalous in a specific context, but not otherwise. This is a widely researched problem in the statistics community (18; 19; 20). Figure 2.2 shows one such example for a temperature time series which shows the monthly temperature of an area over last few years. A temperature of 35F might be normal during the winter (at time t_1) at that place, but the same value during summer (at time t_2) would be an anomaly.

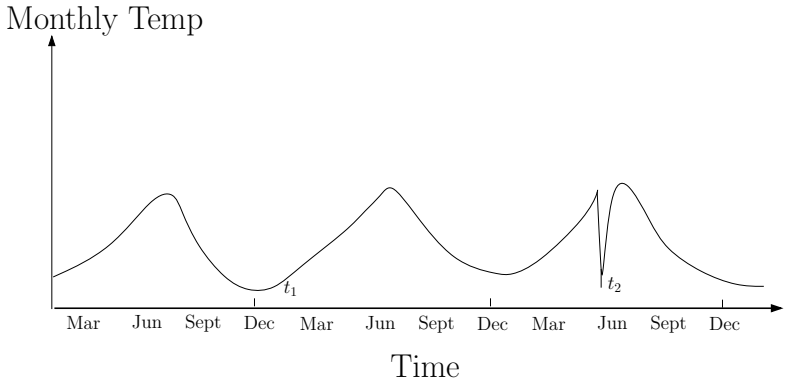


Figure 2.2 : Contextual Anomaly at t_2 in monthly temperature time series

***Problem setting 2* : Detecting anomalous subsequence within a given time series.**

A different setting of the anomaly detection problem tries to find an anomalous subsequence with respect to a given long sequence (time series). Figure 2.1 is an example of a time series containing an anomalous subsequence (the highlighted region), where in the same low value exists for an abnormally long time, though the low value by itself is not an anomaly as it occurs at several other places.

This problem corresponds to an unsupervised learning environment due to the lack of labeled data for training, but most part of the long sequence (time series) is assumed to be normal.

If the anomalous subsequence is of unit length, this problem is equivalent to finding contextual anomalies in the time series, which is the problem setting 1.

The anomalous subsequences are also called *discords*. The concept of discords was introduced by Keogh et al (21) in the context of finding anomalous subsequences within a large time series : "Discords are the subsequences of a longer time series that are maximally different from the rest of the sequence (22)."

***Problem setting 3* : Detecting anomalous time series w.r.t a time series data base.**

The third setting of anomaly detection problem tries to determine if a test time series is anomalous with respect to a database of training time series. This database can be of two types. One type consists of only normal time series, which is a semi-supervised setting (1). In the other variant it consists of unlabeled time series (unsupervised anomaly detection) of both normal and anomalous data, but it is assumed that the majority are normal.

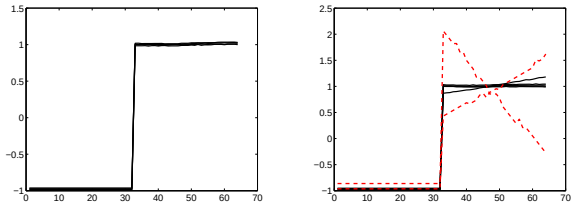


Figure 2.3 : Reference (left) and Test (right) time series for the NASA disk defect data (23)

In this thesis, we are primarily going to discuss the semi-supervised problem setting. Figure 2.3 shows one such example, where a set of reference time series (on the left) correspond to measurements from a healthy rotary engine disk (23) and a test set of time series (on the right) correspond to measurements from healthy (solid) and cracked (dashed) disks. Detecting when an engine disk develops cracks is crucial.

The normal time series in the reference database and the anomalous ones can vary among themselves due to one or more of following factors :

1. The normal time series are generated from a single generative process. The anomalous time series are generated by a completely different generative process. The generative process for normal time series might be periodic and the anomalous ones differ from the periodic ones in this aspect.

2. The normal time series are generated from a single generative process. In the anomalous time series, majority of the observations are generated by the same process, but a few observations are generated by a different process.

Many anomaly detection algorithms discussed in this survey are suitable for all the problem settings, but some are more specific to a particular setting.

2.3 Challenges for Time Series Anomaly Detection

Some major challenges associated with anomaly detection for time series are :

1. There are many ways in which an anomaly occurring in a time series may be defined. An event within a time series may be anomalous; a subsequence within a time series may be anomalous; or an entire time series may be anomalous with respect to a set of normal time series.

2. For detecting anomalous subsequences, the exact length of the subsequence is often unknown.

3. The training and test time series can be of different lengths.

4. Best similarity/distance measures which can be used for different types of time series is not easy to determine. Simple measures like Euclidean distance do not always perform well as they are highly sensitive to outliers and they also cannot be used when the time series are of different lengths.

5. Performances of many anomaly detection algorithms are highly susceptible to noise in the time series data, since differentiating anomalies from noise is a challenging task.

6. Time series in real applications are usually long and as the length increases the computational complexity also increases.

7. Many anomaly detection algorithms expect multiple time series to be at a comparable scale in magnitude while for most of the data it is not true.

2.4 Types of Time Series data

Most of the techniques in this survey use the training data to learn a model for normal behavior and assign an anomaly score to a test time series based on the model. Thus the performance of any technique depends on the nature of the normal time series as well as the anomalous time series. The differences between normal and anomalous time series are discussed in Section 2.2

We discuss two key characteristics of time series namely, periodicity and synchronous nature. The combination of these properties would give 4 different kinds of time series. We are given a dataset of n normal time series $T = \{t_1, t_2, \ldots, t_n\}$ which can be viewed as :

- Periodic and Synchronous : This is the simplest setting where every $t_i \, \epsilon \, T$ has a constant time period (p) and each of the time series are temporally aligned (start from the same time instance). The *power* data set (24) in Figure 2.4 corresponds to the weekly power usage by a research plant.

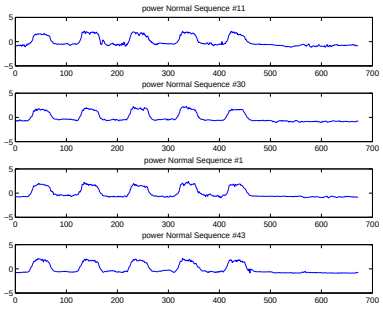


Figure 2.4 : Periodic - Synchronous time series

- Aperiodic and Synchronous : The time series do not have any periodicity, but they are temporally aligned. The *valve* data set (24) in Figure 2.5 corresponds to current measurements recorded on a valve on a space shuttle.

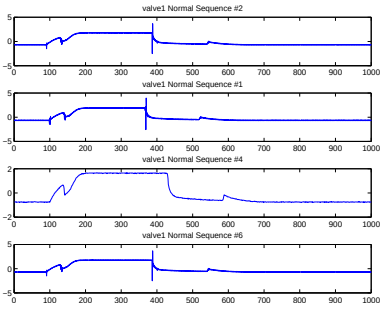


Figure 2.5 : Aperiodic - Synchronous time series

- Periodic and Asynchronous : Each time series has a specific time period, but they are not temporally aligned. The *motor* data set (24) in Figure 2.6 corresponds to functioning of an induction motor.

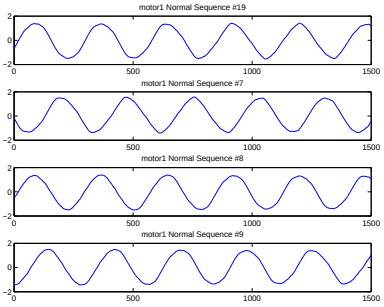


Figure 2.6 : Periodic - Asynchronous time series

- Aperiodic and Asynchronous : The time series neither have periodicity, nor are they temporally aligned. Figure 2.7 corresponds to physiological signals obtained from PhysioNet repository (25).

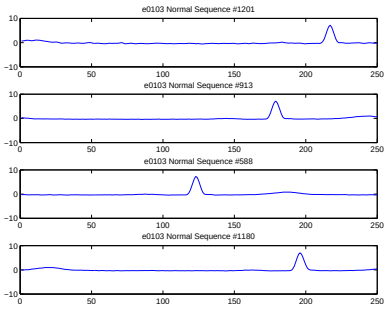


Figure 2.7 : Aperiodic - Asynchronous time series

2.5 Overview of Existing Techniques

Anomaly detection techniques for time series can be classified depending on the process of finding anomalies (Procedural dimension) or the way the data is transformed prior to anomaly detection (Transformation dimension). Both these dimensions are orthogonal; The techniques are listed in Table 2.1.

In procedural dimension, we discuss 5 different techniques that address the *Problem setting 3*. These techniques differ in many ways in their process of finding anomalies. Window based and similarity based methods build a lazy learning model which compares the test time series with the given training time series for assigning anomaly scores. HMM and Regression based methods build parametric models on the training data which probabilistically assign anomaly scores to a test time series. Segmentation based methods build a finite state automaton on the given training data and predict the state of the test time series. These techniques are discussed in detail in section 2.7.

To overcome some of the challenges described in section 2.3, transformation of the data is performed prior to applying anomaly detection techniques. Aggregation focusses on dimensionality reduction by aggregating consecutive values. Discretization and Signal processing based transformations reduce the dimensionality of the data in different

ways and transform the input data into a different domain which can be used to obtain computational efficiency. The transformation methods are discussed in detail in section 2.6.

As can be seen in Table 2.1, there are some domains which have been exploited by the researchers like Aggregation - Window based, Discretization - Window based etc., but there are some domains which are yet to be considered like Signal processing - Prediction based etc. This survey serves as a path for the upcoming research on the problem of anomaly detection for time series data.

Technique	Aggregation	Discretization	Signal Processing
Window Based	(26; 27)	(26)	
Similarity Based	(15; 16)		
Prediction Based	(19)	(28)	(29; 30)
HMM Based	(13)	(31; 32)	
Segmentation	(33; 34; 35)		

Table 2.1 : Techniques and Transformations

2.6 Transformation of Data

There exist many challenges associated with handling time series, such as high-dimensionality, noise, scaling etc. Transformation of data is essential in overcoming these challenges. Sometimes anomalies can be detected easily if the data is transformed and analyzed in a different space.

Another motivation for transformation of the data is to achieve computational efficiency. Some anomaly detection techniques use nearest neighbor approach to find the anomalous time series (or subsequences). These techniques transform the data into a different space to obtain a lower bound on the similarity measure in the original space. Finding the nearest neighbors using this lower bound on similarity would be computationally inexpensive as compared to working in the original space (21; 22; 36).

Many anomaly detection algorithms expect multiple time series to be at a comparable scale. Thus one needs to normalize the data so that each attribute contributes uniformly for the similarity. This is a pre-processing step before any kind of further transformation on the data.

In this section we discuss three different transformations that are commonly used on continuous data to make them suitable for anomaly detection algorithms.

2.6.1 Aggregation

Aggregation methods compress a time series by replacing a set of consecutive values by a representative value of them (usually their average). Aggregation can provide a number of benefits. It reduces dimensionality of the data. In addition the resulting time series is smoother and thus masks noise and missing values. However aggregation can also mask some critical features of the data that may make it harder to detect anomalies. This transformation deals with the time domain of the time series.

Lin et al (37) presented one of the simplest forms of aggregation called Piecewise Aggregate Approximation. To reduce the time series from n dimensions to w dimensions, the data is divided into w equal sized frames. The mean value of the data falling within a frame is calculated and a vector of these values produces Piecewise Aggregate Approximation (PAA) representation.

More formally, given a time series $C = \{c_1, \ldots c_n\}$ of length n, it is transformed to a w-dimensional space vector $\bar{C} = \bar{c}_1, \ldots \bar{c}_w$. The i^{th} element of $\bar{C}$ is calculated by the following equation :

$$\bar{c}_i = \frac{w}{n} \sum_{j=\frac{n}{w}\times(i-1)+1}^{\frac{n}{w}\times i} c_j$$

If w is large and relatively close to n, then the aggregated time series would be almost similar to the original one. If w is too small then there can be huge loss of information.

A variation of the PAA representation is Adaptive piecewise constant approximation (APCA) proposed by Keogh et al (38) which produces variable length frames. The motivation behind this approach is that there can be a single segment in the area of low variance and multiple segments in the areas of high variance instead of uniform segments for all the areas(PAA).

Given a time series $C = \{c_1, \ldots c_n\}$, an APCA representation for this would be $C = \{< cv_1, cr_1 >, \ldots, < cv_m, cr_m >\}$, where cv_i is the mean value of datapoints in the i^{th} frame and cr_i is the right endpoint of the i^{th} frame. The algorithm works

by first converting the problem into a wavelet compression problem, for which there are well known optimal solutions, and then converting the solution back to the APCA representation.

The drawback of this approach is that the obtained time series is no longer in the standard format. A value in the output time series can correspond to varying number of events in the original time series or just one instance value. Thus this transformation is not yet used in any of the traditional anomaly detection algorithms, since they cannot deal with the non-uniformity of the time series.

2.6.2 Discretization

The primary goal of discretization is to convert the given time series into a discrete sequence of finite alphabets. This transformation deals with the amplitude domain of the time series. Primary motivation behind discretization is to utilize the existing symbolic sequence anomaly detection algorithms (39; 37). Another motivation is to improve the computational efficiency (21; 22; 36). However discretization can also cause loss of information.

Discretization involves the following steps : (i) Divide the amplitude range (max-min) of the time series into different bins (ii) Assign a symbol to each of the bin which can be an alphabet or an integer etc. (iii) Transform the time series by replacing every data point with the symbol that is associated with the bin in which it lies. Figure 6 below shows an example of a simple discretization, where the amplitude range is divided into equal sized bins.

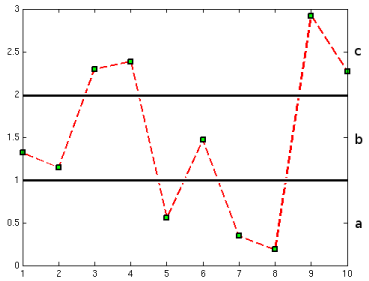


Figure 2.8 : The time series amplitude (0-3) is divided into 3 equal sized bins and assigned a, b, c. The symbolic representation of this time series would be *bbccabaacc*

The discretization techniques vary according to how the bins and symbols are chosen.

Let us consider the variants in terms of symbols. Choices for the symbols can either be integers or alphabets. Although the use of unordered alphabets is quite common (37; 22), the use of integers retains more information. For example if integers 1, 2, 3 are used in the place of a, b, c in the example of Figure 2.8, one could easily determine that the data points in bin c (integer 3) are closer to bin b (integer 2) than to bin a (integer 1), which is not possible if unordered alphabets are used.

Forrest and DasGupta (40) discuss a discretization scheme that makes use of integers. They number the bins using integers and then encode them into a binary form. Each bin is thus associated with a binary number. Now each data point is given a binary symbol depending on the bin where it falls. The 2 bins with all 0's and all 1's is not considered during training so that the observations of the test time series which fall outside the range of training time series are mapped to them depending on which side of the range they cross. Thus there are $2^m - 2$ bins for m bits.

There are number of ways in which the intervals can be chosen. In the following, we discuss three ways in which this can be done :

1. Equal bin size :

The amplitude range can be divided into n equal bins and each bin is assigned a unique symbol. Figure 2.8 provides an illustration.

2. Equal frequency :

The amplitude range is divided into bins such that each bin has equal number of data points (37).

3. Clustering :

The amplitude range is divided into different bins such that each bin has closely related values and thus a unique discrete symbol can be assigned to each bin (41).

One of the most widely used discretization technique called SAX (Symbolic Aggregate approXimation) (37) uses the Piecewise Aggregate Approximation discussed in

2.6.1 followed by equal frequency binning. The advantage of this method over many discretization techniques is that it is the only one that allows both dimensionality reduction and lower bounding of L_p norms (distance measures) (21; 22; 36).

If the data distribution is known, equal frequency binning can be seen as the division of this distribution into equal sized areas. Since the time series are normalized before PAA, they tend to have a highly Gaussian distribution. In SAX transformation, a notion called *breakpoints* is introduced which divide the gaussian into equal-sized areas. *Breakpoints* are a sorted list of numbers $B = \beta_1, \ldots \beta_{\alpha-1}$ such that the area under a $N(0,1)$ Gaussian curve from β_i to $\beta_{i+1} = 1/\alpha$.

βi \ a	3	4	5
β_1	-0.43	-0.67	-0.84
β_2	0.43	0	-0.25
β_3		0.67	0.25
β_4			0.84

Figure 2.9 : A lookup table that contains the breakpoints that divides a Gaussian distribution in an arbitrary number (from 3 to 5) of equiprobable regions (21)

Once the breakpoints are obtained, the PAA representation of the time series is discretized considering symbols as alphabets. All PAA coefficients that are below the smallest breakpoint are mapped to the symbol "a", all coefficients greater than or equal to the smallest breakpoint and less than the second smallest breakpoint are mapped to the symbol "b", etc.

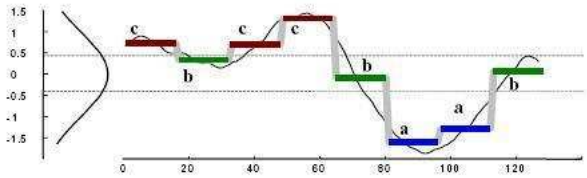


Figure 2.10 : A time series (thin black line) is discretized by first obtaining a PAA approximation (heavy gray line) and then using predetermined breakpoints to map the PAA coefficients into symbols (bold letters). In the example above, with $n = 128$, $w = 8$ and $\alpha = 3$, the time series is mapped to the word *cbccbaab*. (21)

Discussion:

Though transformation of the continuous data to discrete/segmented form helps there exist a lot of discretization techniques which have some basic drawbacks with them. Firstly, the dimensionality of the symbolic representation is the same as the original data after some transformations, which is a challenge for the anomaly detection algorithms. Secondly, although some distance measures can be defined on the symbolic sequences, these measures do not correlate well with the distance measures of the original time series. Finally, many approaches need access to the entire data for the creating the representation.

2.6.3 Signal Processing Based

Sometimes anomalies can be detected more easily if the data is used in a different space. Signal processing techniques like Fourier transforms, wavelet transforms help to obtain this entirely different space of coefficients where the data can be analyzed (42; 29). These techniques are also used to get a lower dimensional representation of time series. Many of these transformations are useful in achieving lower bounding of norms, in order to make the algorithm computationally efficient (43; 44).

One such signal processing technique that has been used in the context of anomaly detection for time series data is Haar Transform. A 1-dimensional vector (x_0, x_1) is transformed as (s_0, s_1) by Haar transform, using

$$(s_0\, s_1)^T = D(x_0\, x_1)^T, \text{ where } D = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

It can be seen that s_0, s_1 are the sum and difference of x_0, x_1 and scaled by $\frac{1}{\sqrt{2}}$ to preserve energy. In general Haar transform can be seen as a sequence of averaging and differencing operations on the consecutive values of a discrete time function. This transformation preserves the Euclidean distance between two time series and is therefore a useful technique for compression. If a prefix of the transformed time series is considered instead of the entire sequence, the Euclidean distance between these prefixes will be the lower bounding estimate of the actual Euclidean distance. Since the distance is preserved even after the transformation, these prefixes which have lower dimension than

the original time series, can be considered for the anomaly detection problem and hence get a better computational time. These lower bounding estimates are clearly discussed by Chan et al (45).

Shahabi et al. (46; 47) proposed a wavelet-based data structure called TSA-Tree to efficiently retrieve trends and surprises in spatio-temporal data. The wavelet based approaches tend to outperform the other methods : DFT and SVD, since these methods only consider the frequency components of the time series, where as wavelets process the data at different resolutions.

Ma et al (27) use one class SVMs for prediction which need a set of vectors as input instead of a time series. Thus they convert the time series into a phase-space using time-delay embedding process, i.e., create overlapping subsequences from a given long sequence. These vectors are projected into an orthogonal subspace which acts as a high pass filter used to filter out the low frequency components and allow only high frequency ones (anomalies).

2.7 Detection Techniques

The discussion in the previous sections provided understanding on how the time series look and how they can be transformed for the application of anomaly detection techniques. This section discusses the various anomaly detection techniques, some of which have been proposed and evaluated in the literature, many others are new or slight variations of the existing techniques, to suit the *Problem setting 3* mentioned in section 2.2. In general, the process of anomaly detection consists of the following steps :

1. Compute the anomaly scores of individual observations or subsequences of a given test time series using a detection technique.

2. Aggregate these anomaly scores to calculate the anomaly score of the given test time series. This aggregation can be done in different ways, for example : (1) mean of all the anomaly scores, (2) mean of top k anomaly scores (3) mean of log of anomaly scores (4) number of times the running average of the anomaly scores exceeds a threshold etc.

A test time series with anomaly score greater than a threshold is labeled anomalous.

2.7.1 Window Based

The techniques in this category divide the given time series into fixed size windows (subsequences) to localize the cause of anomaly within one or more windows. The motivation behind this technique is that an anomaly in a time series can be caused due to the presence of one or more anomalous subsequences.

Window based techniques (26) extract fixed length (= m) windows (subsequences) from training and test time series by moving one or more symbols (hop) at a time. The anomaly score of a test time series is calculated by aggregating the anomaly scores of its windows. A formal description of a generic window based scheme is as follows :

1. Given a training database, $S_{training} = \{S_1, S_2 \ldots, S_n\}$, extract p windows of each time series S_i , s_{i_1}, $s_{i_2}, \ldots s_{i_p}$, where p can be calculated as $|S_i| + m - 1$ when sliding a window of size m, one step at a time. Similarly for the test database $S_{test} = \{T_1, T_2, \ldots, T_n\}$, divide each test time series T_i into $|T_i| + m - 1$ windows , t_{i_1}, $t_{i_2}, \ldots t_{i_{p'}}$.

2. The anomaly score for each test window ($A(t_{i_j})$) is calculated using its similarity to the training windows. This similarity function can be distance measures like Euclidean, Manhattan or Correlation values, etc.

If the subsequences are obtained by sliding one step at a time, considering all possible overlaps, it gets computationally inefficient since the number of windows is nearly equal to the length of the time series. Hence one alternative to obtain the subsequences is to slide a window of fixed length (m) across the time series and move it with a certain hop (h) each time, that is skip h observations from the initial position of the current window and start the next window. A special case would be $hgeqm$, where there is no overlap between the windows. The disadvantage of the hop being too large is that there can be some loss of information. For an appropriate value of window size m, if $h = 1$ then the probability that the anomaly is captured by atleast one window is 1, but when $h > 1$ this probability might reduce. Consider the training database containing n similar time series *abcabcabc*, where each symbol is a real value. If the window size is 3, the training windows for any value of h would include only the following subsequences : *abc*, *bca*, *cab*. Let a test time series be *abccabcabc*, where the occurrence of *c* after the

first *abc* is anomalous. Table 2.2 shows the test windows generated with different values of hop.

hop (h)	Windows
1	*abc*, *bcc*, *cca*, *cab*, *abc*, *bca*, *cab*, *abc*
2	*abc*, *cca*, *abc*, *cab*
3	*abc*, *cab*, *cab*
4	*abc*, *abc*

Table 2.2 : Windows of size 3 for different hop values

Both $h = 1, 2$ capture the anomalous occurrence of c by the windows *bcc* and *cca* as there will not be any neighbors of these among the training windows. Hops of 3, 4 fail to capture anomaly as they miss the c during the window shift. Thus the value of h has to be carefully chosen.

The window based techniques can differ in their score assignment to a test window. For example, the anomaly score of a test window can be the distance to its k^{th} nearest neighbor among the training windows (26). Ma et al (27) use the training windows to build one class SVMs for classification. The anomaly score of a test window is 0 or 1 depending on whether it is classified as normal or anomalous using the trained SVM.

Once the windows are extracted from the training and test time series, one can apply any traditional anomaly detection technique for multivariate data to assign an anomaly score to each window of the test time series (1).

Strengths and Weaknesses : The drawback of window based techniques is that the window size has to be chosen carefully so that it can explicitly capture the anomaly. The optimal size of the window depends on the length of the anomalous region in the anomalous time series. For example, in the *power* data set in Figure 2.11, the anomalous region has the same length as the periodicity of the time series. Thus if m is chosen to be smaller than the cycle length, the performance will be poor, while the performance will improve if m is larger than the cycle length. Another drawback of window based techniques is that they are computationally expensive. Since every pair of test and training windows are compared, the complexity is $O((nl)^2)$, where l is the average length of the time series, n is the number of test and training time series in

the database. Most of the window based techniques are proposed for problem setting 2, the discord detection problem. These drawbacks of window size and computational complexity are addressed by some of the discord detection techniques and are discussed in section 2.8.

Window based techniques can capture all different kinds of anomalies mentioned in the challenges (Section 2.3) : an anomalous observation in a time series, an anomalous subsequence in a time series, an anomalous time series as a whole. Since the entire time series is broken into smaller subsequences, we can easily identify if an observation or the subsequence is anomalous. If the entire time series is anomalous then all the subsequences are also anomalous, hence window based techniques would capture it well.

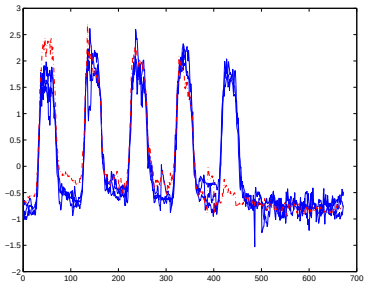


Figure 2.11 : An anomalous time series (red), a normal time series (blue) of the *power* data set. The anomalous time series has the last cycle missing (anomalous region).

2.7.2 Proximity Based

These techniques make use of the pairwise proximity between the test and training time series using an appropriate distance or similarity kernel to compute the anomaly score of the test time series. The assumption behind these techniques is that the anomalous time series are "different" from the normal ones and this difference can be captured using a proximity measure.

The anomaly score of a test time series with respect to the training database is calculated using the following methodologies:

1. k-NN : The distance of the test time series to its k^{th} nearest neighbor in the training data set is its anomaly score.

2. Clustering : The training time series are clustered into a specified number of clusters and the cluster centroids are computed. The distance of the test time series to its closest cluster centroid is its anomaly score.

The similiarity based techniques primarily differ in the choice of similarity measures for the calculation of anomaly score. Different similarity/distance measures can be adopted such as correlation, Euclidean, Cosine, DTW etc.

Simple measures like Euclidean, though are well understood and are faster to compute, they cannot be used when the time series are of different lengths. Also, consider the physiological signals in Figure 2.7 which are all normal, but they have the spikes at different time stamps. These time series are non-linearly aligned which is one of the challenges in handling time series data. Euclidean distance measure would give high anomaly score to these time series in case of similarity based techniques.

Dynamic time warping (DTW) (48) is a distance measure which is more suitable for comparing time series which are non-linearly aligned or of different lengths. DTW aligns the time series which are similar but with small variations in the time axis in such a way that the distance between them is minimal. This minimal distance can be used to characterize the similarity between the time series. The disadvantage of DTW is it performs excessive matchings which can make the algorithm computationally expensive and can also lead to distortion of the actual distance between time series.

Another challenge with time series data is that different time series can be generated at different conditions and times and hence they may not be synchronous. Hence, measures like Euclidean and DTW may consider two phase misaligned time series to be dissimilar though they are almost similar. For example, consider the motor data set in Figure 2.6, where all the time series are normal but they are not in phase (phase-misaligned). While DTW addresses the issue of non-linear alignment of the time series, Cross Correlation, a similarity measure, addresses the issue of phase mis-alignments (15). Given two time series, different phase-shifts of the pair are considered and the correlation between them for each alignment is computed. The maximum of these correlations is used as the cross correlation measure for these two time series. Thus, two time series with phase misalignments will have high correlation in one of their phase-shifted alignments, which shows that they have high similarity. Revisiting the motor data set in Figure 2.6, the first two time series will have maximum correlation

when one of them is considered with a $\frac{\pi}{2}$ phase shift.

Anomaly detection techniques in this category include a k-nearest neighbor based anomaly detection technique, which has been proposed by Propotopapas et al (15) to compare catalogs of periodic light curves to obtain the anomalies. They handle the issues of phase-misaligned data using cross correlation measure which is computed efficiently in Fourier space using convolution theorem.

For the same problem of comparing periodic light-curves, Rebba Pragada et al (16) use PCAD (Periodic Curve Anomaly Detection) clustering algorithm which is a variant of k-means. The similarity of the test time series is a function of maximum cross correlation measure between the time series and the cluster centroids obtained from PCAD.

Strengths and Weaknesses : The drawbacks of similarity based techniques are that, they can determine if the entire time series is anomalous or not, but cannot exactly locate the anomalous subsequence. To localize the exact region(s) within the time series which causes the anomaly, one needs to do post processing of the time series. Also the phase misalignments and the non-linear alignments of different time series, which are some common challenges for the time series data, restrict the usage of different proximity measures for these class of techniques. Thus the performance of these techniques is highly dependent on the proximity measure, which is often not easy to choose. In detecting different types of anomalies, similarity based techniques might fail when a single observation is anomalous in the time series as its effect might not be prominent when the entire time series is considered at once.

These techniques would detect anomalies such as anomalous subsequences in a time series or an anomalous time series as a whole.

2.7.3 Prediction Based

Anomaly detection using predictive models has been mostly investigated in the statistics community (49; 50; 18), most of which look for single observations as outliers. The motivation behind these techniques is that the normal time series are generated from a statistical process and the anomalous time series do not fit this process. So the key step is to learn the parameters of this process from the training database of normal time

series and then estimate the probability that a test time series is generated from the learnt process.

A basic predictive model based anomaly detection technique consists of the following steps (26):

1. Learn a predictive model on the given training time series, which uses m observations (history) to predict the $(m+1)^{th}$ observation following them.

2. For a test time series, using the predictive model built in step 1, forecast the observation at each time step using the observations seen so far (previous m observations). The prediction error corresponding to an observation is a function of the difference between the forecasted value and the actual observation and certain model parameters such as the variance of the model.

The techniques differ in the prediction models used and can be broadly categorized as follows.

1. Time series based models such as Moving Average (MA) (51), AutoRegression (AR) (52; 51), ARMA (53), ARIMA (54), Kalman filters (55) etc. The input to these models is the entire time series and the length of the history (m), also denoted as the order of the model. These models differ in the kind of filters they use to generate the output.

   (a) Moving Average (MA) : The MA-models represent time series that are generated by passing the input through a linear filter which produces the output $y(t)$ at any time t using only the input values $x(t-\tau)$, $0 \leq \tau \leq m$, also called a non-recursive filter.

   $$y(t) = \sum_{i=0}^{m} b(i)x(t-i) + \epsilon(t)$$

   where, $b(1)$, $b(2)$, $\ldots b(m)$ are the coefficients of the non-recursive filter, $\epsilon(t)$ is the noise at time t. If every instance t of a time series has a value equal to the mean of its previous m values then it can be represented by a moving average model, in which case $b(i) = \frac{1}{m}$ and $\epsilon(t) = 0$.

   (b) Autoregression (AR) : The AR-models represent time series that are generated by passing the input through a linear filter which produces the output

$y(t)$ at any time t using the previous output values $y(t-\tau)$, $1 \leq \tau \leq m$, also called a recursive filter.

$$y(t) = \sum_{i=1}^{m} a(i)y(t-i) + \epsilon(t)$$

where, $a(1)$, $a(2)$, $\ldots a(m)$ are the autoregressive coefficients of the recursive filter (autoregressive coefficients), $\epsilon(t)$ is the noise at time t.

(c) Autoregressive Moving Average (ARMA) : The ARMA models represent time series that are generated by passing the input through a recursive and through a nonrecursive linear filter , consecutively . In other words, the ARMA model is a combination of an autoregressive (AR) model and a moving average (MA) model . The orders of AR part of the model and MA part of the model can differ.

$$y(t) = \sum_{i=0}^{m} a(i)y(t-i) + \sum_{i=0}^{m} b(i)x(t-i) + \epsilon(t)$$

where, $a(1)$, $a(2)$, $\ldots a(m)$ are the autoregressive coefficients of the recursive filter (autoregressive coefficients), $b(1)$, $b(2)$, $\ldots b(m)$ are the coefficients of the non-recursive filter, $\epsilon(t)$ is the noise at time t.

(d) Autoregressive Integrated Moving Average (ARIMA) : The ARIMA models which extend ARMA models, apply the ARMA model not immediately to the given time series, but after its preliminary differencing, which is the time series obtained by computing the differences between consecutive values of the original time series.

(e) Kalman Filters : The basic idea of Kalman filter is described as follows - Consider, a time series with Markov property, described by the following equation:

$$x(t+1) = Ax(t) + \epsilon(t)$$

where $x(t)$ represents a hidden state of the system and A is a matrix describing the causal link between current state ($x(t)$) and next state ($x(t+1)$). The Kalman filter for time series $X = (x(1)\, x(2)\, \ldots x(n))$ is described as follows :

$$y(t+1) = Ax(t) + K(x(t) - y(t))$$

where K is called the Kalman gain.

2. General Regression (non-time series based) : Linear regression (56), Gaussian process regression (57), Support vector regression (19), etc. The subsequences of length m (history length), extracted from the original time series are the input to these models. The training set would a set of subsequences given by, $T = \{(X(t), y(t)),\ t = m, \ldots n-1\}$, where $X(t) = [x(t-m+1) \ldots x(t)]$ and $y(t) = x(t+1)$. A linear regression function is constructed using a weight vector W and a mapping function $\Phi(X(t))$, $y = W^T\Phi(X(t)) + b$. Different regression models differ in how they fit the function.

   (a) Linear regression : For simple linear regression the above equation is solved by minimizing the sum of squared error of the residue $(y(i) - x(i+1))$. The mapping function here is identity,$\Phi(X(t)) = X(t)$.

   (b) Support vector regression : These models use the ϵ-insensitive loss function proposed by Vapnik (58). They solve the above equation by solving the following objective function.

$$\text{minimize } P = \tfrac{1}{2}||W||^2 + C\Sigma_{i=1}^{l}(\zeta_i + \zeta_i^*)$$

$$\text{such that, } y_i - (W^T\Phi(X(t)) + b) \leq \epsilon + \zeta_i$$

$$-y_i + (W^T\Phi(X(t)) + b) \leq \epsilon + \zeta_i^*$$

$$\zeta_i,\ \zeta_i^* \geq 0$$

   This optimization criterion penalizes data points whose y values differ from $f(x)$ by more than ϵ. The slack variables ζ, ζ^* represent the amount of excess deviation. Different kernel functions (59) such as polynomial, RBF and sigmoid can be used.

Ma et al (19) use m-length training subsequences and use support vector regression on them for building an online novelty detection model which also uses statistical tests to determine the anomalies with some fixed confidence.

Chandola et al (26) use AR model on the original time series while Lotze et al (30) use wavelet transformations of the training time series to produce multiple-resolution data and then use AR model, Neural Networks on them for prediction.

Zhang et al (29) propose a simple prediction model built on the lowest wavelet resolution (called trend) of the subsequences. Let the original subsequence be $X_i = x(1)\,x(2)\,\ldots x(i-1)$, and its trend be $Y_i = y(1)\,y(2)\,\ldots y(i-1)$. The distribution of the difference of X_i and Y_i, called residual, is constructed. If the difference of the $x(i)$ and the trend $y(i-1)$ is statistically insignificant as per the residual distribution, then the observation $x(i)$ is termed anomalous.

Strengths and Weaknesses : The issues with prediction based techniques are as follows. Similar to the window based techniques, the length of the history chosen here is critical to exactly locate the anomaly. Referring to Figure 2.11, the anomalous region has the same length as the periodicity of the time series. Thus if history length m is chosen to be smaller than the cycle length, the performance will be poor, while the performance will improve if m is larger than the cycle length. But if the value of m is too large then we have a very high dimensional data, which might increase the computational complexity. Also, because of the sparse nature of high dimensional data, the *curse of dimensionality* comes into effect, i.e, the observations which are closer in the smaller dimensional spaces will seem very far in high dimensional space because of its sparsity.

Since these techniques assume that the data is generated from a statistical process, if this assumption is true (Ex : *motor* (Figure 2.6), *power* (Figure 2.4)) these techniques perform well, though the challenge is to find the right process and estimate its parameters, else if the data is not generated by a process (Ex : physiological signals (Figure 7)) they might fail to capture the anomalies.

All the prediction based techniques use fixed length histories. For the same time series, sometimes a smaller history is sufficient for prediction but other times we might need a longer history. Thus, one can use dynamic length history where in, if an observation cannot be predicted with high confidence given an m length history then increase or decrease the history length to predict the observation with higher confidence.

Prediction based techniques calculate anomaly score for each of the observations

in the time series. Hence they can capture all kinds of anomalies : an anomalous observation in a time series, an anomalous subsequence in a time series, an anomalous time series as a whole.

2.7.4 Hidden Markov Models Based

Hidden Markov models (HMM) (60) are powerful finite state machines that characterize a system using its observable parameters. HMMs are widely used for sequence modeling (60) and sequence anomaly detection(61; 62). They have also been applied to anomaly detection in time series (63; 13).

The assumption behind HMM based techniques is that a given time series $O = O_1 \dots O_n$ is the indirect observation of an underlying (hidden) time series $Q = Q_1 \dots Q_n$, where the process creating the hidden time series is Markovian, even though the observed process creating the original time series may not be so. Thus the normal time series can be modeled using a HMM, while the anomalous time series cannot be.

Given a training series, we can build a single HMM (λ) model, which consists of parameters that describe the normal data, such as the initial state distribution, state transition probabilities etc., (64). Every training time series can have a specific model or there can be a unified model for all the training time series. A basic HMM based anomaly detection technique operates as follows :

1. Given a training time series, $O_{train} = O_1 \dots O_n$, it is considered as a sequence of indirect observations of the HMM model. There exists a well-established procedure which is able to determine the HMM parameters by maximizing the probability $P(O_{train}|\lambda)$ using a technique called the Baum-Welch (65) re-estimation procedure.

2. In the testing stage, given an unknown time series $O_{test} = O_1^{'} \dots O_{n'}^{'}$, the probability $P(O_{test}|\lambda)$ is computed using the trained model. Among the test time series, we can say that the anomalous ones are those which have the minimum value of $P(O_{test}|\lambda)$.

The existing techniques differ slightly in the HMM parameters used. He et al (63) use the standard parameters like initial state distribution, state transition probabilities

etc. Liu et al (13) use segmental HMM where the additional variable is the probability distribution over the duration of each hidden state.

Strengths and Weaknesses : The issue of HMM based techniques is their assumption that there exists a hidden process which is markovian and generates the normal time series. In the absence of an underlying markovian process, these techniques might fail to capture the anomalies.

HMM based techniques build a markov model for the time series. Hence they estimate the anomalous behavior for each of the observations in the time series which helps in capturing all kinds of anomalies (as long as the assumption holds) : an anomalous observation in a time series, an anomalous subsequence in a time series, an anomalous time series as a whole.

2.7.5 Segmentation Based

The basic approach of segmentation based anomaly detection techniques consists of partitioning the training time series into a series of segments and learning an FSA to model the transition between these segments. The assumption behind these techniques is that the series of segments obtained from an anomalous time series will not "fit" the FSA, while a normal time series will. Given a test time series, its segments are identified and the probabilites of transitions between these segments (computed using the FSA) are used to predict its anomalous nature. The standard framework of segmentation based techniques is as follows :

1. Training phase : Given one or more training time series, construct a linear FSA in which each successive state represents a homogeneous segment of the given time series (one or more).

2. Testing phase : Given a test time series $X = \{x_1, x_2, \ldots x_n\}$, FSA is used to predict whether it is anomalous or not as follows :

   (a) For x_1, the current state is set to be the first state.

   (b) For $i = 2 : n$

(i) If x_i (the current input) *matches* the characteristics of the current state, then remain in the current state.

(ii) Else if x_i *matches* the characteristics of the next state, then transition to next state.

(iii) Else compute the anomaly score of x_i and remain in the current state.

Three prominent techniques in this category are proposed by Chan et al (33; 34; 35). Below is the discussion of two different segmentation based anomaly detection techniques proposed by Mahoney et al (34) .

1. Path Modelling : This approach generates a d-dimensional space from the training time series using feature extraction and uses bottom up segmentation where in, a given d-dimensional time series X is approximated with $k-1$ line segments defined by k vertices using a greedy bottom-up approach. The other $n-k$ vertices are removed by vertex removal algorithm and a path is fitted. Figure 2.12 shows how the path is fitted after removal of the vertex B. The error because of removing B and fitting the path is approximately $|A'C'||3BB'/4|^2$. The vertex to be removed should have the minimum error.

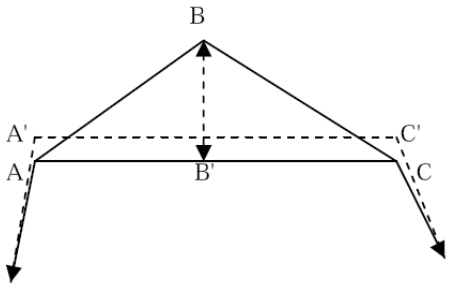


Figure 2.12 : Path Fitting - After removing vertex B, A and C are shifted $1/4^{rth}$the distance from B' to B. (34)

The states are defined by these $k-1$ segments. Hence when a test sequence is presented, the input x_i is tested whether it is closest to the current or the next segment (state) and its anomaly score is the closest distance to either of these segments.

2. Box modelling : This approach also generates a d-dimensional space from the training time series using feature extraction and uses bottom up segmentation which is an extension of the MBR (minimum bounding rectangle) approach to form boxes which minimize the volume expansion. A sequence of n points in a feature space is first approximated by a sequence of $n-1$ boxes, each enclosing a pair of adjacent points. Then pairs of adjacent boxes are merged by greedily selecting the pair that minimizes the increase in volume after merging. This greedy split attempts to find a sequence of k boxes with the least total volume. The boxes have the d-dimensional co-ordinates which characterize the states. Hence the standard FSA is built on the states. To determine if a test point x_i belongs to a state, one has to check if it falls into the box corresponding to the state. If the input does not fall into the current or next box then the anomaly score of it is defined as the minimum distance to either of those boxes. The box modelling approach has been extensively discussed in (33) by Chan et al. They use a 3D feature space obtained from the data.

From the above illustrations, one could see that the variations in this category come from (i) the methods used for constructing the FSA or the segmentation of time series, (ii) the *matching* functions used to determine if an observation belongs to a certain state and (iii) the anomaly score computation for each observation.

To construct an FSA, one or more training time series are partitioned into a series of homogeneous segments, where each segment is termed as a state and the homegeneity of a segment is characterized using a criterion function. There are different approaches for segmenting a time series to obtain discrete states (66). Some traditional approaches are : (i) Sliding window : Grow the segments from the time series until the error is above a specified threshold, then start a new segment. (ii) Top-down : The entire time series is recursively split until the desired number of segments is reached, or an error threshold is reached (35). (iii) Bottom-up : The approaches by Mahoney et al (33; 34) use this segmentation where they start with $\frac{n}{2}$ (or $n-1$) segments, the 2 most similar adjacent segments are repeatedly joined until a stopping criterion is reached .

Salvador et al (35) propose a top-down approach using Gecko clustering to form the initial groups. The rules which characterize these clusters are generated using RIPPER (67). All the above techniques generate one FSA per time series. Box Modelling (33) approach was extended to multiple time series by initially finding the boxes for one time

series and expanding the boxes to fit in the rest of the training time series.

The other variation in terms of *matching* function for a time series and anomaly score assignment relate closely to how a state is characterized. The approaches described above (33; 35; 34) use the following scheme. (1) If the state is enclosed, then check if x_i lies in that state, in which case its anomaly score is 0; If it does not lie in that state or the next state, then it is either termed anomalous (35) or its anomaly score is computed as the shortest distance to either of the states (current or next) (33). (2) If the state is not enclosed, then the anomaly score of x_i is its shortest distance to the current or state.

The testing phase of above FSA is not robust to slight variations in the data which are effected by the rigid boundaries of a state. Assume $x_l, \ldots, x_m$ and $x_{m+2}, \ldots, x_t$ match a state s_p , while x_{m+1} which is slightly outside s_p and matches the next state s_{p+1}. According to the above FSA, once x_{m+1} falls into s_{p+1} the current state is modified to $s_{p+1}$and all the following observations, $x_{m+2}, \ldots, x_t$ are considered anomalous as they do not match either the current state, s_{p+1} or the next state, s_{p+2}, though they are not anomalous. This issue is handled by Chan et al (33) by modifying step (b) of the FSA, such that a transition to the next state (s_{next}) occurs only if a specified number of consecutive observations *match* the next state. Thus in our example, though x_{m+1} matches s_{p+1}, $x_{m+1}, \ldots x_t$ do not match s_{p+1}, hence x_{m+1} is considered anomalous and the current state would still be s_p.

Strengths and Weaknesses : The assumption of segmentation based techniques is that the all the training time series can be partitioned into a group of homogeneous segments. In cases where this cannot be done, segmentation based approaches might fail. Also these techniques are highly inefficient as both the training phase of segmentation and testing phase of checking each observation of each test time series with the FSA are computationally intensive.

Segmentation based techniques build an FSA for the time series and compute anomaly score for each of the observations in the time series. This helps in capturing all kinds of anomalies : an anomalous observation in a time series, an anomalous subsequence in a time series, an anomalous time series as a whole.

2.8 Discord Detection

The other important problem setting in anomaly detection as discussed in Section 2.2 is **discord detection**. The general approach for finding the discord in a given test time series S_q is as follows.

1. Divide S_q into t windows(subsequences), w_1, $w_2, \ldots w_t$, where $t = |S_q| + m - 1$ by sliding a window of size m, one step at a time.

2. Calculate the anomaly score for each window ($A(w_i)$) by calculating its distance from the other windows. Different distance/similarity measures like Euclidean, Manhattan or Correlation can be chosen.

3. The window with maximum anomaly score is declared as the discord for the given time-series.

As discussed in section 2.7.1, the windows can be obtained by sliding one or more steps at a time, also called hop (h). If $h < m$, then the windows are overlapping. An important issue to be considered in discord detection is : while comparing subsequences with overlap, there is a chance that significant anomalies might be missed. This can be seen in the following example. Consider the time series *abcabcDDDabcababc* , where each symbol represents a real value and a sliding window of length 3. The significant anomaly as can be seen is DDD. If the discord is defined as a subsequence farthest from its nearest neighbour, then *bab* becomes our first discord as DDD has a close match DDa within one step, which is also called a partial self-match, whereas *bab* does not have a closer nearest neighbor. One way to avoid this problem, proposed by Keogh et al (22), is to only consider non-self matches, i.e, a window is compared only with the windows which do not overlap with it. Hence in the above example, DDD will not be compared with cDD or DDa and will be considered as a discord.

A non-self match is formally defined as : "Given a time series T, containing a subsequence C of length m beginning at position p and a matching subsequence M beginning at q, we say that M is a non-self match to C at distance of Dist(M, C) if $|p - q| > m$." (22)

The issues with the window based approaches in discord detection are described below. The techniques in this category mainly differ from each other in terms of how

they handle these issues.

1. Window size (m) : As discussed in Section 2.7.1, choosing the window size is critical as it should be able to explicitly capture the anomaly. Most of the techniques have a user defined window size parameter. Some of the techniques, as described in (68; 69; 70) use the window size equal to the periodicity of the data which is computed using the correlogram of the time series. A correlogram is known to be a graphical representation of the auto correlation functions of the time series plotted with different lags. The assumption is that if the time series is periodic then the correlation of the original time series with a phased lagged version of it would give a maximum value, if the phase lag is equal to its periodicity . Also the correlogram would have the same periodicity as the process generating the original time series (69). Following the same idea, Chuah and Fu (8) analyze ECG data using the window size as one heartbeat's length.

2. Anomaly score ($A(s_q)$) : Calculation of anomaly score depends on which similarity/distance measure is chosen and how the discord is defined. Palshikar et al (71) define discord as the window which has maximum number of nearest neighbours whose distance is greater than a user defined threshold. Their anomaly score is the distance of a window to its nearest match including partial self-matches. For many other techniques, discord is defined as the window which has the largest distance to its nearest non-self match (72; 22; 39; 14). The anomaly score in this case is the distance of a window to its nearest non-self match. Most of the discord detection techniques consider the Euclidean distance measure. Das Gupta et al(40) discretize the time series to obtain a binary representation and use simple matching of the windows, that is two windows are identical if and only if r continuous bits match between the windows.

Cheng et al (17) proposed a graph based technique with the vertices being windows and edge weights being the distances between the windows. The anomaly score, $A(s_q)$, of a vertex is calculated as the connectivity of that vertex measured by a random walk on the graph. An anomalous vertex would be highly different from others, so its similarity (edge weight) with the other vertices in the graph would be small. Hence the probability of visiting this vertex when performing a random walk is small. The authors generalized

this work to multivariate time series (17), where the same graph based priniciples are applied in finding anomalous subsequences.

3. Efficiency : The general approach for window based anomaly detection problem is $O(n^2)$, where n is the length of the time-series. This is because, each possible subsequence is considered (outer loop) and its distance to all the other subsequences (inner loop) is calculated to find the nearest among them (71).

Most of the window based techniques for discord detection (72; 22; 39; 14) use an approximation to the following heuristic ordering of the subsequences in order to make the algorithm linearly scalable. For the outer loop : the subsequences are sorted by descending order of the non-self distance to its nearest neighbor so that the true discord is the first subsequence in the loop. For the inner loop: the subsequences are sorted in ascending order of their distance to the current candidate. For such a heuristic, since the first subsequence of the outer loop is the true discord, the largest nearest neighbor distance (l_{max}) is found in the first pass of the inner loop. The subsequent invocations of the inner loop will terminate in $O(1)$ time as the distance of the outer loop subsequence to its first inner loop subsequence is less than the existing threshold of the maximum nearest neighbor distance (l_{max}) thus the outer loop subsequence cannot be a discord, thus giving an $O(n)$ algorithm. The approximation to this heuristic ordering was obtained by specific data structures built on the SAX representation (22), wavelet representation (44) of time series and these representations are also useful to improve the efficiency further.

Instead of finding the most unusual discord, few approaches generalize it to finding top-k discords (44). Also an important application of window based approaches is the detection of shape anomalies (14). 2D shapes are converted to time series which are shape invariant and the anomalies are detected using the same approaches mentioned above.

2.9 Conclusions

This chapter summarizes the state of the art of anomaly detection techniques for univariate time series. In Chapter 4, we investigate the behavior of these techniques on

publicly available data sets, each with different properties as discussed in Section 2.4.

Chapter 3

Subspace based transformation for univariate timeseries

In the previous chapter, we discussed the problem of anomaly detection for time series data and categorized the existing techniques based on their methodology and the way they transform the data prior to anomaly detection. In this chapter we discuss a novel transformation technique for univariate time series which uses concepts from the window based scheme discussed in section 2.7.1. We present the motivation behind this transformation and the methodology in the following sections.

3.1 Motivation

In the discussion of window based techniques (section 2.7.1), we suggested that, once the univariate test and training time series are divided into windows, any traditional anomaly detection technique for multivariate data can be applied to these sets of windows. The reason for this application is that if a normal univariate time series is generated by a process, the windows of the normal time series also follow a generative process in the multivariate space. Hence the set of fixed-length windows obtained from a univariate time series can be considered as a multivariate time series. Univariate anomalous time series follow a different process or do not follow any process; hence the windows obtained from them need not follow the same process as the windows obtained from normal time series.

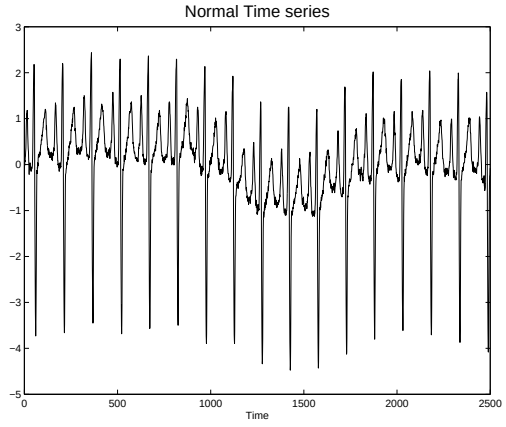
We explain this with an example. Consider the normal time series in Figure 3.1(a). The first few windows of it are shown in Figure 3.1(c) (obtained with parameters: window length $n = 250$ and number of steps (hop) $h = 10$). We consider each of the time instance of a window as a variable, i.e., if the window length is n then the multivariate time series has n variables. In Figure 3.2 we show the time series corresponding to a sample of these variables. Figure 3.2(a) shows the time series corresponding to time instances (variables) 50 , 150 and 200 of the windows obtained from normal time series in Figure 3.1(a). We will refer to these variables as normal variables. Similarly Figure 3.2(b) shows the time series corresponding to same time instances of the windows obtained from the anomalous time series in Figure 3.1(b). As can be seen, the time series of normal variables follow a periodic pattern, while the time series of anomalous variables does not follow the periodic pattern. Hence these windows from univariate time series can be considered as multivariate time series as they follow a specific process in the multivariate space.

With this motivation we proceed to present a brief overview of multivariate time series and anomaly detection techniques in this domain in section 3.2.

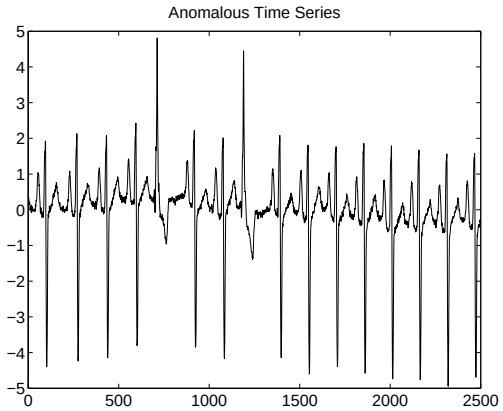
3.2 Detecting Anomalies in Multivariate Time Series

Data collected in many application domains consists of multivariate time series. For example, a single aircraft generates different univariate time series, which are considered as a single multivariate time series. Each of these univariate time series correspond to data coming from a single sensor or a switch on the aircraft (73). Similarly the daily network log stored for network intrusion detection depicts a multivariate time series, where each variable measures certain aspects of the time series (74).

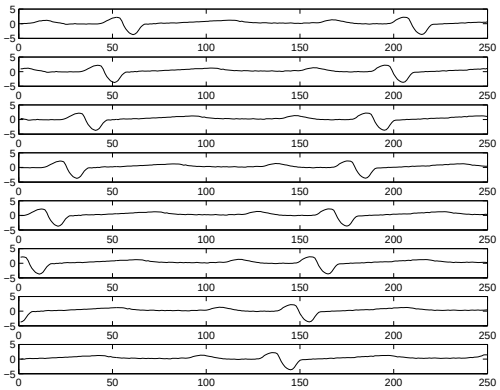
Anomaly detection for multivariate time series data is distinct from traditional anomaly detection for multivariate data as well as anomaly detection for univariate time series data. This is because the anomaly detection techniques for multivariate data analyse only the multivariate aspect of the data, while the techniques for univariate time series data analyse the sequence aspect of each variable independently. Often, the anomaly in a multivariate time series can be detected only by analyzing the sequence of all (or a subset of) variables.



(a) Univariate Normal Time Series

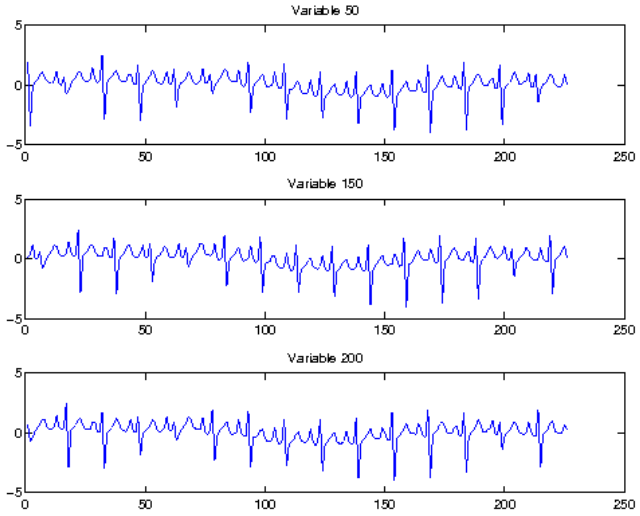


(b) Univariate Anomalous Time Series

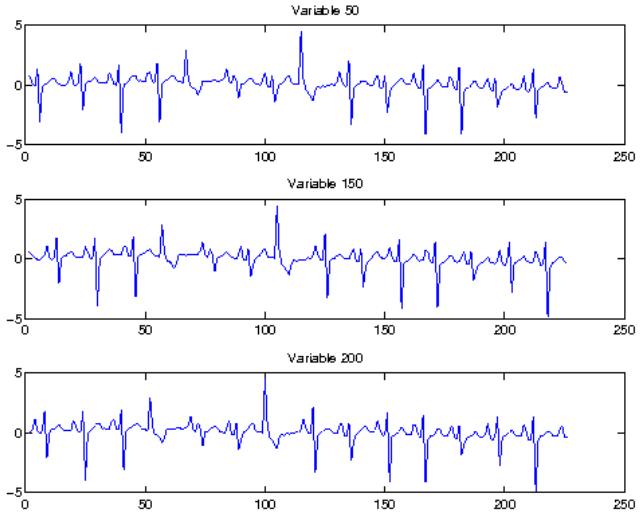


(c) First Windows

Figure 3.1: Windows of Univariate Time series



(a) Variables of Normal Time Series



(b) Variables of Anomalous Time Series

Figure 3.2: Time series corresponding to normal and anomalous variables

A real life instance of anomalies in multivariate time series data can be found in aviation safety domain (73). During the course of the flight of an aircraft, multiple sensors measure different aspects of the flight, thereby generating a multivariate time series. An fault or accident during the flight of the aircraft can be detected by finding anomalies in the multivariate time series data. To detect such anomalies in multivariate sequences, multivariate aspect as well as the sequence aspect needs to be simultaneously modeled.

A multivariate anomaly detection technique proposed by Chandola (75), is a window based technique which accounts for both the multivariate and sequence aspects of the data while detecting anomalies. The key underlying idea is to reduce a multivariate time series into a univariate time series by exploring the change in the correlation structure of the time series, using subspace monitoring. We discuss subspace monitoring in section 3.2.1 and the anomaly detection technique for multivariate time series using subspace monitoring, WIN_{SS} in section 3.2.2.

3.2.1 Subspace Monitoring for Multivariate Time Series

Subspace monitoring for damage detection and other related areas falls under the broad purview of *statistical process control* (76; 77). Jordan et al (78) proposed the original concept of comparing two subspaces using the angles between their principal components, which was later explained as *canonical correlation* by Hotelling (79). Chandola et al (75) provide a detailed discussion of the angle between subspaces using principal angles.

Comparison of multivariate time series often use subspace based analysis, so as to compare their generative models such as vector AR models (80), ARMA models (81), and linear dynamical systems (76). It has been shown that the modes and modal shapes of vibrating structures (76) coincide with their system eigenstructure. Any change in this structure can be computed using covariance driven identification methods which use the change in subspaces.

The anomaly detection technique proposed by Chandola et al (75) measures the change between two subspaces, S_A and S_B, which is defined as the following (77): *Change in subspace when S_A changes to subspace S_B is equal to the maximum distance between an arbitrary unit vector $\hat{x}$ in S_B and the subspace S_A.* This change δ_{AB} is given

as:

$$\delta_{AB} = \sqrt{1 - \lambda_{min}} \tag{3.1}$$

where λ_{min} is the minimum eigen value of $B^T(AA^T)B$. A and B are the m-dimensional basis vectors of subspaces A and B, respectively. For proof see (77).

3.2.2 Converting a Multivariate Time Series to Univariate Time Series

Chandola et al (75) propose a novel anomaly detection technique, WIN_{SS} which consists of two steps. Each training and test multivariate time series is converted into a univariate time series in the first step and the second step involves using an existing univariate anomaly detection technique on the transformed univariate time series to detect anomalies.

Let $S \in \Re^{|S| \times m}$ be a multivariate time series of length $|T|$ and consisting of m variables.

A w length window of S starting at time t is denoted as $W_t = S[t]S[t+1] \ldots S[t+w-1]$. Thus $T - w + 1$ such windows are extracted from S. Consider two successive windows $W_t, W_{t+1} \in \Re^{w \times m}$. V_t denotes the subspace spanned by the top few[1] principal components of W_t. Similarly V_{t+1} denotes the subspace spanned by the top few principal components of W_{t+1}. Note that W_t and W_{t+1} can have a different number of basis vectors. The change between the two successive subspaces, $\delta_{t,t+1}$ can be defined using 3.1 as:

$$\delta_{t,t+1} = \sqrt{1 - \lambda_{min}} \tag{3.2}$$

where λ_{min} is the minimum eigenvalue of the matrix $V_t^T V_{t-1} V_{t-1}^T V_t$. Thus the multivariate time series S can be transformed into a univariate time series $\delta_{1,2}\delta_{2,3} \ldots$.

The transformed univariate time series captures the dynamics of the subspace structure of a multivariate time series, such that the normal time series are expected to follow similar dynamics, while anomalous time series are different. Any traditional univariate anomaly detection technique can be applied to this transformed time series. Chandola et al (75) use a window based anomaly detection technique in the second step.

[1] Capturing $\alpha\%$ of the total variance.

3.3 Transformation

Section 3.2.2 describes how the anomaly in the multivariate space can be captured using the subspace based analysis by exploring the correlation structure of the multivariate time series. With this motivation, we proceed further to apply the subspace analysis to the windows obtained from univariate time series which can be considered as multivariate time series. The process of this subspace based transformation is : converting a univariate time series to a multivariate time series and back to another univariate time series.

3.3.1 Methodology

For a given univariate time series T of length N, $T = t_1 t_2 \dots t_N$, overlapping windows of length n are created. Let w_i be the i^{th} window which can be represented as $t_i t_{i+1} \dots t_{i+n-1}$. These windows are stacked and are considered as multivariate time series, W, represented as :

$$\begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_{N-n+1} \end{pmatrix}$$

$=$

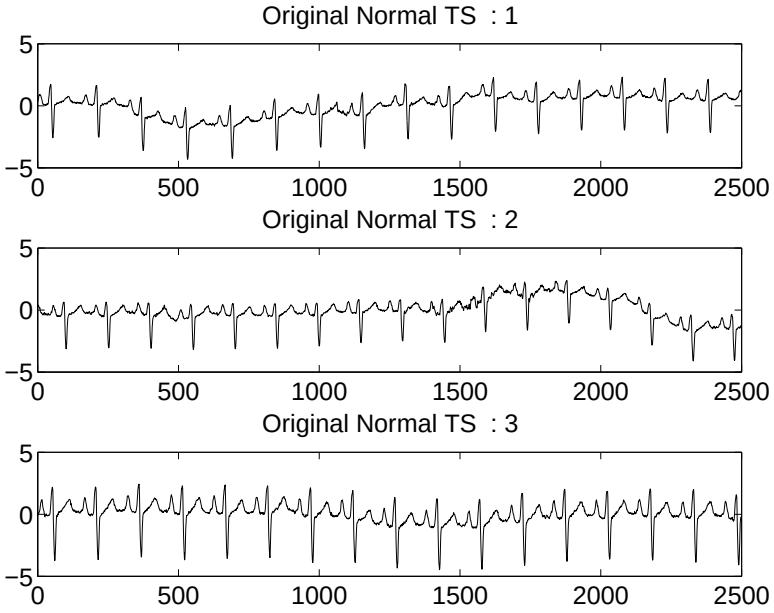
$$\begin{pmatrix} t_1 & t_2 & \dots & t_n \\ t_2 & t_3 & \dots & t_{n+1} \\ \vdots & \vdots & \vdots & \vdots \\ t_{N-n+1} & t_{N-n+2} & \dots & t_N \end{pmatrix}$$

Each time instance of a window is considered as a variable, so the columns correspond to time series of different variables. The time series corresponding to k^{th} variable is $t_k t_{k+1} t_{k+2} \dots$ and the length of the multivariate time series is the number of windows extracted. As discussed in section 2.7.1, the windows from a univariate time series can be obtained by sliding one or more steps at a time, also called hop (h). In the above case we considered $h = 1$. If $h < n$, the windows are overlapping. For any general hop h, the time series corresponding to k^{th} variable is $t_k t_{k+h} t_{k+2h} \dots$.

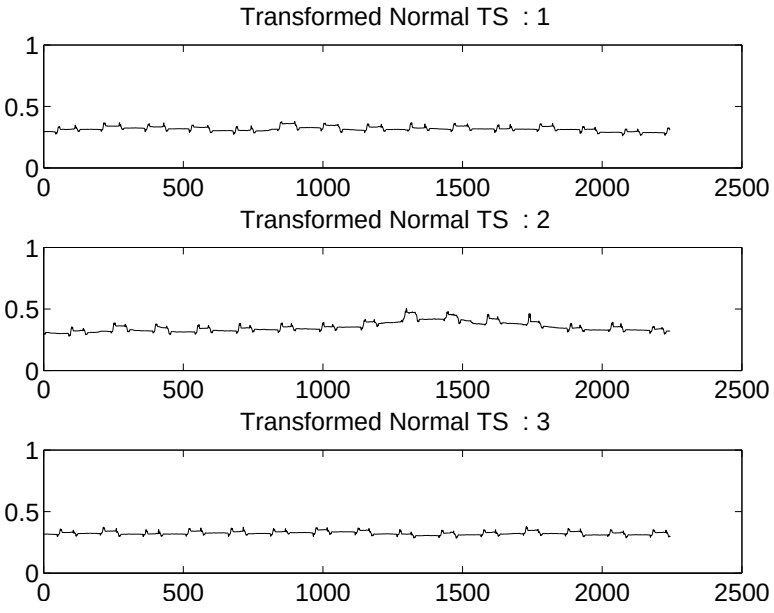
Each training and test time series in the database is converted to a multivariate time series which is further transformed to another univariate time series by using the subspace based transformation described in 3.2.2. This approach explores the change in correlation structure of the variables and the sequence aspect of a multivariate time series. Overlapping windows of a multivariate time series are created and the change in subspaces of consecutive windows is computed to obtain the univariate time series. Once the transformation is applied on all the time series, the existing univariate anomaly detection techniques can be applied.

Figure 3.3 shows a set of univariate normal time series and their corresponding transformations. As can be seen the transformed time series capture the periodicity of the original time series well. That is the angle between consecutive subspaces keeps repeating with similar periodicity.

Figure 3.4 shows a set of univariate anomalous time series and their corresponding transformations. The transformed anomalous time series do not have similar characteristics as transformed normal time series. The change in subspaces corresponding to two normal windows is different from the change in subspaces corresponding to two anomalous windows, which highlights the existence of anomaly. The properties of the transformation are clearly discussed in the Chapter 4.

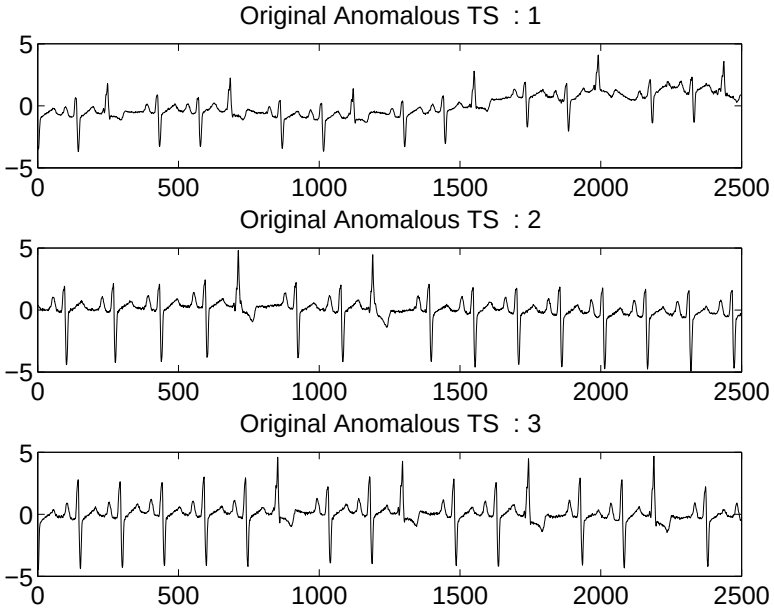


(a) Univariate Normal Time Series

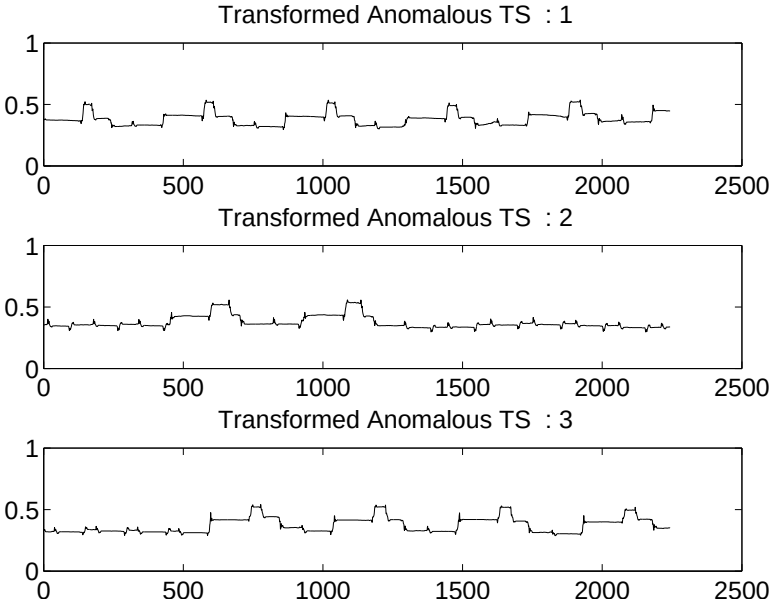


(b) Transformed Univariate Normal Time Series

Figure 3.3: Original and Transformed normal time series



(a) Univariate Anomalous Time Series



(b) Transformed Univariate Anomalous Time Series

Figure 3.4: Original and Transformed Anomalous time series

Chapter 4

Experiments and Discussion

In this chapter we investigate the behavior of the anomaly detection techniques discussed in Chapter 2 and also the transformation technique discussed in Chapter 3. The experimental evaluation of the existing anomaly detection techniques on various publicly available datasets has been thoroughly discussed by Chandola et al (26).

This chapter provides results and discussions from two different experiments. One is a brief discussion on the strengths and weaknesses of the existing anomaly detection techniques on the original time series and their discretized versions. The other one, which is the focus of the thesis, is on the comparison of existing anomaly detection techniques (see chapter 2) on a variety of publicly available datasets, with the same techniques after applying the subspace based transformation (see chapter 3) on these datasets.

We provide a thorough experimental framework for understanding these techniques on the transformed time series obtained from the subspace based transformation. The transformation essentially converts a given univariate time series to a multivariate time series and back to another univariate time series. We provide the experimental evaluation using data sets with different properties to understand the types of time series on which the transformation works best.

The rest of the chapter is organized as follows. Section 4.1 briefly describes the different anomaly detection techniques for time series data. Section 4.2 describes the different kinds of data sets used for this experimental evaluation. Section 4.3 summarizes our experimental results of using different anomaly detection techniques on the publicly

available data sets. Section 4.4 shows the results of using different anomaly detection techniques on the transformed time series.

4.1 Anomaly Detection Techniques

The following techniques were investigated by Chandola et al (26).

- **Proximity Based Techniques** These techniques make use of a proximity measure defined for every pair of time series. We evaluate a nearest neighbor based technique, called $KNNC$ (k-nearest neighbor for continuous time series) (82), that utilizes this proximity measure. $KNNC$ assigns an anomaly score to a test time series, as equal to its distance to its k^{th} nearest neighbor in the training database, $\mathbf{X}$.

  A discrete version of $KNNC$ called $KNND$ is also evaluated (where 'D' stands for Discrete), where a continuous time series is initially discretized into a sequence of symbols using symbolic approximation (SAX) technique (22).

  For $KNNC$, we evaluate three distance measures, viz., *Euclidean Distance*, *Dynamic Time Warp* (DTW) (83), and *Cross Correlation* (84).

- **Window Based Techniques** These techniques extract fixed length (w) windows from a test time series, and assign an anomaly score to each window. The per-window scores are then, aggregated to obtain the anomaly score for the test time series. The score assignment to a window and the score aggregation can be done in different ways.

  We denote the window based techniques for continuous time series as $WINC$. The experimented version of $WINC$ assigns an anomaly score to each window of a test time series equal to the *Euclidean* distance between the window and its k^{th} nearest neighbor in the set of windows extracted from the training time series. The anomaly score for the test time series is equal to the average score of all its windows. $WINC$ estimates the density of w-dimensional windows using the windows extracted from the training time series and then determine if the windows extracted from a test time series lie in the dense regions or not.

The discrete version of this called $WIND$ is also evaluated. We evaluated two variants of $WIND$: $WIND$ and $tSTIDE$. $WIND$ is similar to $WINC$ with the only difference being that the score for each window from a test sequence is a likelihood score (inverse of anomaly score), and is equal to the similarity between the window and its k^{th} nearest neighbor in the set of windows extracted from the training sequences. For $tSTIDE$ (proposed by Forrest et al (85) for symbolic sequences), the likelihood score for each window is equal to, the number of times the window occurs in the set of training windows (windows extracted from the training sequences), divided by the total number of training windows.

- **Prediction Based Techniques** These techniques learn a predictive model from the training time series. Testing involves forecasting the next observation in a test time series, using the predictive model and the test time series observed so far, and comparing the forecasted observation with the actual observation to determine if an anomaly has occurred.

  We evaluate three predictive techniques: AR (using Auto Regressive models (52)), SVR (using Support Vector Regression (86)), and $FSAz$ (using Finite State Automata based technique for symbolic sequences (87)). For AR and SVR, the anomaly score for each observation in a test time series is equal to the difference between the forecasted value and the actual observation. The anomaly score of the test time series is equal to the average anomaly scores for all of its observations. For $FSAz$, a likelihood score is obtained for each symbol in the discretized test sequence which is equal to the conditional probability of observing the symbol in the training sequences, given the previous few symbols. The anomaly score for the discretized test sequence is equal to the inverse of the average likelihood scores for all of its symbols.

4.2 Data Sets Used

To evaluate the performance of each algorithm, 18 publicly available data sets are used. Table 4.1 summarizes the different data sets for the cross-domain experimental evaluation. These datasets are grouped into different categories, each with distinct characteristics as discussed in Section 2.4. For the periodic time series, the last column denotes

			$\lvert\mathbf{Y}\rvert$						
Name (#)	L	$\lvert\mathbf{X}\rvert$	$\lvert\mathbf{Y_N}\rvert$	$\lvert\mathbf{Y_A}\rvert$	λ	A	P	S	l
disk (2)	64	10	500	50	0.09	s	$\times$	$\surd$	–
motor (4)	1500	10	10	10	0.50	p	$\surd$	$\times$	250
power (1)	672	11	33	8	0.19	s	$\surd$	$\surd$	96
valve (1)	1000	4	4	8	0.67	s	$\times$	$\surd$	300
shape1 (1)	1614	10	10	10	0.50	p	$\times$	$\times$	–
shape2 (1)	1614	30	30	10	0.25	p	$\times$	$\times$	–
l-ecg (4)	2500	250	250	25	0.09	s	$\surd$	$\times$	250
s-ecg (4)	360	500	500	50	0.09	p	$\times$	$\times$	50
evi (2)	220	500	500	50	0.09	p	$\surd$	$\surd$	12

Table 4.1: Details of different data sets used in the experiments. # - number of data sets in each group, L - length of sequences, $\mathbf{X}$ - Training database, $\mathbf{Y}$ - Test database, $\mathbf{Y_N}$ - Normal test time series, $\mathbf{Y_A}$ - Anomalous test time series, λ - Baseline Accuracy, A - Anomaly Type (Process - p, Subsequence - s), P - Periodic (Yes - $\surd$, No - $\times$), S - Synchronized (Yes - $\surd$, No - $\times$), l - Cycle/Characteristic Pattern Length.

the cycle length and for the non-periodic time series, the last column denotes the length of a definitive pattern in the time series. For example, for the valve data set, the bump between time instances 100 and 400 can be considered as a pattern. For some non-periodic data sets (edb1-2, shape12), it was not possible to define a characteristic pattern. For some data sets, we adapted the actual data sets to create suitable evaluation time series. The general methodology to create the data sets was the following: For each data collection, a normal database, **N**, and an anomalous database, **A**, of time series is created. A training (reference) database, **X**, is created by randomly sampling a fixed number of time series from **N**. A test database, **Y**, is created by randomly sampling m normal time series from **N-X** and n anomalous time series from **A**. All time series were normalized to have a zero mean and unit standard deviation.

The ratio $\frac{n}{m+n}$ determines the "baseline level" of accuracy for the given test database, e.g., if baseline accuracy is 0.50, a "dumb" technique that declares all time-series to be anomalous will perform correctly 50% of the time. Thus a real technique should perform significantly better than the baseline to be considered effective. The different data sets are:

- Motor Current Data Set (motor) (24). The normal time series correspond to the current signals from the normal operation of a induction motor. The anomalous time series correspond to signals obtained from a faulty motor. Different data sets

(motor1 motor4) correspond to different kinds of faults in the motor.

- Power Usage Data (power) (24). The normal time series correspond to weekly time series of power consumption at a research facility in 1997 for weeks with no holidays during the weekdays. The anomalous time series correspond to power consumption during weeks with a holiday during the week.

- NASA Valve Data (valve) (24). The normal time series consists of TEK solenoid current measurements recorded during the normal operation of a Marrotta series valves located on a space shuttle. The anomalous time series correspond to the faulty operation of the valve.

- Shape Data (shape1 and shape2) (24). The normal time series correspond to one or more shapes, while the anomalous time series correspond to other shapes. For the shape1 database, the normal time series correspond to a pencil shape, while the anomalous time series correspond to other similar shapes (e.g., fork). For the shape2 database, the normal time series correspond to shapes of cups, glasses, and crosses, while the anomalous time series correspond to distinctly dissimilar shapes.

- Electrocardiogram Data (l-ecg and s-ecg) (25). Each data set corresponds to an ECG recording for one subject suffering with a particular heart condition. The ECG recording is segmented into short time series of equal lengths. Each short time series is added to the normal database if it does not contain any annotations of a heart condition2 , and is added to the anomalous database if it contains one or more annotations indicating a heart condition. The l-ecg databases contain 10 second long time series. Four such databases (l-ecg1l-ecg4) were constructed from BIDMC Congestive Heart Failure Database and Long-Term ST Database. The s-ecg databases contain 1 second long time series. Four such databases (s-ecg1s-ecg4) were constructed from European ST-T Database and MIT-BIH Arrhythmia Database.

- EVI Data (evi1 and evi2) (88). Enhanced Vegetation Index essentially serves as a measure of the amount and "greenness" of vegetation at a particular location. Each land location has a time series of length T which corresponds to T monthly

observations at that location. A normal time series corresponds to periodic cycles of vegetation with high values in summer and low values in winter. An anomalous time series corresponds to places where there was land cover change because of a natural calamity or forest fires. This dataset is used only for the experiments on the transformation technique.

- **Disk Defect Data Set (disk)** (89). The normal time series corresponds to blade tip clearance measurements obtained from a simulated aircraft engine disk and the anomalous time series correspond to the measurements obtained from a disk with a crack. Different data sets (disk1 – disk3) correspond to different speeds at which the disk is rotating. This dataset has not been used for transformation evaluation but used for the generic comparison of anomaly detection techniques.

4.3 Comparison of Anomaly Detection Techniques

In this section we first provide the results for the experimental evaluation of anomaly detection techniques discussed by Chandola et al (26). For each technique, we use the parameters that result in the best average performance over all datasets. The accuracy results (or precision on normal class) of the techniques using these best parameter settings are reported in Table 4.3. The results reveal several interesting insights into the performance of the different techniques. At a high level, our results indicate that none of the techniques are superior to others across all data sets, but have certain characteristics that make them effective for certain types of data sets, and ineffective for certain others. Here we summarize some high level conclusions:

- Techniques that operate on the continuous time series are generally superior to techniques that operate on discrete sequences. Moreover, techniques for continuous time series are more robust to parameter choices such as distance measure (KNNC vs. KNND) or window length ($WINC$ vs. $WIND$).

- Overall, kernel and window based techniques, which are model independent, tend to outperform predictive and segmentation based techniques, that try to build a model for the time series data. This seems to indicate that building a predictive model for time series data is challenging.

- For kernel based techniques, the choice of distance or similarity measure is critical, and for window based techniques, the choice of window length is critical. Kernel based techniques are faster than window based techniques, though indexing techniques, that were originally proposed to improve the time complexity of discord detection techniques (21; 22; 43; 44), can be employed for $WINC$ and $WIND$ to make them faster. If online anomaly detection is desired, techniques such as KNNC and KNND, which require the knowledge of entire test time series are not suitable, while window based and predictive techniques can be adapted to operate in an online setting.

- For periodic time series data, window based techniques are superior to other techniques. The reason is that if the training time series are periodic, they can be represented using a small set of windows which form dense regions in the w dimensional space. Thus nearest neighbor based techniques can learn a tight boundary around the dense regions, and can differentiate well between the windows from a normal test time series and those from an anomalous time series. On the other hand, for non-periodic time series data, a larger set of windows is required to represent the training time series, and hence the windows form sparse regions in the w dimensional space. This results in relatively poor performance for $WINC$ compared to other techniques on non-periodic time series data, e.g., shape2.

- If the time series data contains process anomalies, kernel based techniques give the best performance, e.g., KNND for shape2, while the performance is poor if the time series data contains subsequence anomalies, e.g., KNNC for l-ecg. The reason is that the kernel based techniques assume that the anomalous test time series are significantly different from the normal time series, which is true for data with process anomalies, but not for data with subsequence anomalies. For the latter type of data, window based and predictive techniques are better suited since they analyze windows within the time series and are more likely to detect the anomalous subsequences.

- We learnt several relationships between the nature of the normal and anomalous data and the performance of different techniques. For example, KNNC with the DTW measure is suited for non-periodic time series while $WINC$ is more suited for periodic time series. $WINC$ performs poorly for data sets in which the normal time series belong to multiple modes (e.g., shape2), while KNNC and KNND are better suited to

handle such data sets.

Data	KNNC	KNND	WINC	WIND	tSTIDE	SVR	AR	FSAz	BOX
disk1	0.88	0.26	0.98	0.09	0.32	0.09	0.74	0.08	0.94
disk2	0.96	1.00	0.96	0.09	1.00	1.00	0.40	1.00	0.92
disk3	0.96	0.96	1.00	0.52	0.96	0.98	0.48	0.96	0.94
motor1	0.60	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.80
motor2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.80
motor3	0.90	1.00	1.00	1.00	1.00	1.00	1.00	0.90	0.90
motor4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.90
power	0.88	0.88	0.88	0.62	0.75	0.62	0.50	0.62	0.50
valve1	0.75	1.00	0.75	0.75	0.75	0.75	0.75	1.00	0.62
shape1	1.00	1.00	1.00	0.50	0.80	1.00	1.00	0.80	0.80
shape2	0.80	0.90	0.50	0.25	0.20	0.30	0.00	0.40	0.70
chfdb1	0.20	0.36	0.40	0.72	0.24	0.48	0.20	0.52	0.72
chfdb2	0.40	0.12	0.32	0.24	0.04	0.04	0.00	0.16	0.04
ltstdb1	0.52	0.12	0.56	0.40	0.16	0.04	0.00	0.16	0.60
ltstdb2	0.44	0.28	0.28	0.20	0.32	0.04	0.20	0.24	0.08
edb1	0.74	0.76	0.78	0.74	0.56	0.74	0.02	0.74	0.66
edb2	0.30	0.30	0.16	0.14	0.12	0.36	0.00	0.22	0.10
mitdb1	0.78	0.70	0.90	0.66	0.32	0.70	0.00	0.38	0.18
mitdb2	0.94	0.86	0.94	0.84	0.56	0.90	0.02	0.62	0.18
Avg	0.74	0.71	0.76	0.57	0.58	0.63	0.44	0.62	0.60
time (s)	8	291	73	609	2	7	1	18	857

Table 4.2: Comparing results across all techniques.

4.4 Comparison of Original and Transformed Time Series

In this section we provide the experimental results of some univariate anomaly detection techniques, applied on the original time series and on their subspace based transformations. Another set of experiments show the effect of the transformation on noise induced data. Uniform gaussian noise has been added to all the data sets in Table 4.1 and the performance of techniques on the noisy data and the transformed noisy data have been observed. As per the results in Table 4.3, we can see that $WINC$ and $KNNC$ performed best for the continuous time series. Therefore we also wanted to investigate the behavior of these techniques on transformed data also.

We denote the techniques applied on original time series as $WINC$, $KNNC$ with Euclidean measure ($KNNC_{EUC}$) and $KNNC$ with Cross Correlation measure ($KNNC_{CORR}$). The techniques applied on transformed time series are represented as T_{WINC}, $T_{KNNC_{EUC}}$ and $T_{KNNC_{CORR}}$ respectively. All the 6 techniques are compared on the public data sets using the *Accuracy* evaluation metric. The results of Accuracy (or precision on normal class) on original and transformed time series are shown in Table 4.3 and the results on

their noisy counterparts are shown in 4.4. These tables show the best accuracies of the techniques on all data sets over different parameter settings.
Figure 4.1 shows the average accuracies of the techniques over the periodic and non-periodic datasets separately and Figure 4.2 the same for the noise induced datasets. These tables show the best accuracies of the techniques on all data sets over different parameter settings.

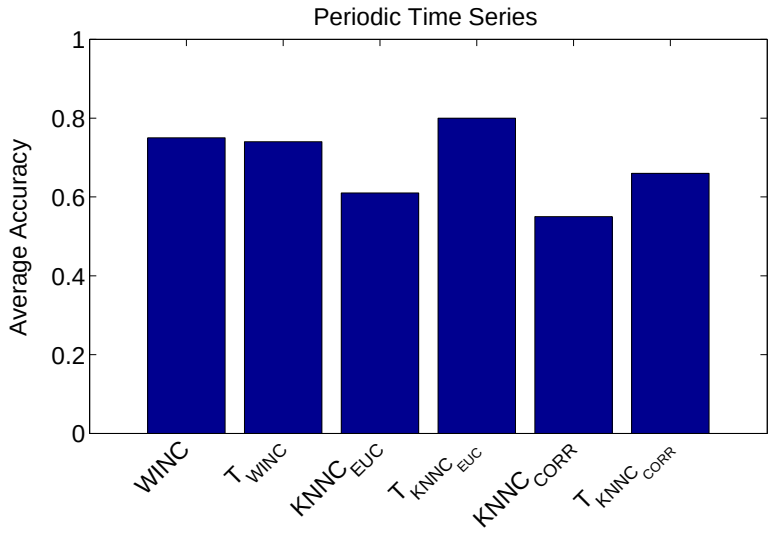
For all the techniques, we experimented with different parameter settings listed below :

- Window size (w_1) to extract windows to form the multivariate time series : $(\frac{l}{2}, l, 2l)$, where l is the Cycle/Characteristic Pattern Length in the time series.
- Window size (w_2) to extract windows from the multivariate time series for the transformation : $(4, 6, 8, 10)$
- The percentage of total variance captured by the subspace, α : $(0.75 - 0.95)$

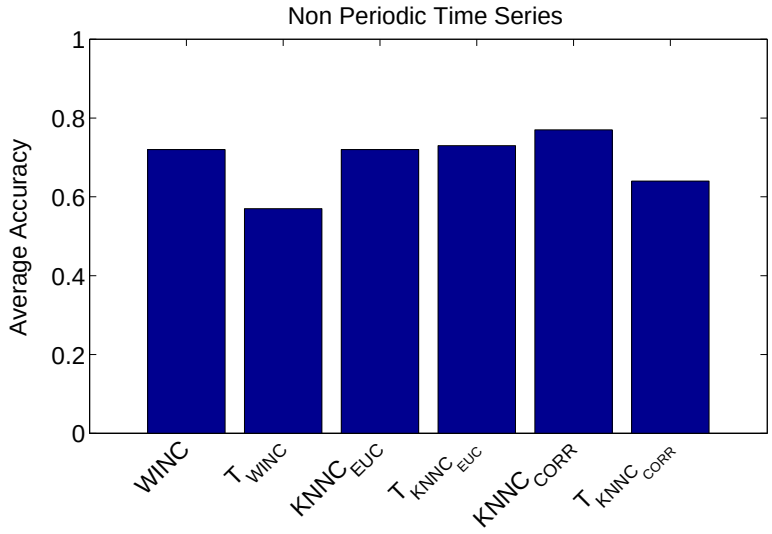
The effect of these parameters settings and the characteristics of the data on the transformation are discussed in the section 4.5.

	DataSet	$WINC$	T_{WINC}	$KNNC_{EUC}$	$T_{KNNC_{EUC}}$	$KNNC_{CORR}$	$T_{KNNC_{CORR}}$
1.	motor1	1.00	1.00	1.00	1.00	1.00	1.00
2.	motor2	1.00	1.00	1.00	1.00	1.00	1.00
3.	motor3	1.00	1.00	1.00	1.00	0.90	1.00
4.	motor4	1.00	1.00	1.00	1.00	1.00	1.00
5.	power	0.88	0.75	0.88	0.88	0.88	0.88
6.	chfdb1	0.72	0.64	0.12	0.76	0.20	0.68
7.	chfdb2	0.32	0.44	0.16	0.52	0.40	0.40
8.	ltstdb1	0.80	0.80	0.12	0.80	0.52	0.76
9.	ltstdb2	0.28	0.32	0.24	0.44	0.44	0.32
10.	evi1	0.98	0.75	1.00	0.98	0.30	0.36
11.	evi2	0.88	0.83	1.00	0.96	0.10	0.39
12.	valve1	0.75	0.75	0.75	1.00	0.75	0.75
13.	edb1	0.78	0.50	0.68	0.52	0.74	0.40
14.	edb2	0.16	0.22	0.22	0.30	0.38	0.28
15.	mitdb1	0.84	0.64	0.70	0.60	0.80	0.50
16.	mitdb2	0.94	0.70	0.90	0.70	0.94	0.60
17.	shapes1	1.00	1.00	0.90	1.00	1.00	1.00
18.	shapes2	0.50	0.20	0.90	1.00	0.80	0.80
	Avg	0.74	0.64	0.65	0.78	0.63	0.65
	Time(s)	74	80	6	15	9	17

Table 4.3: Accuracy Results

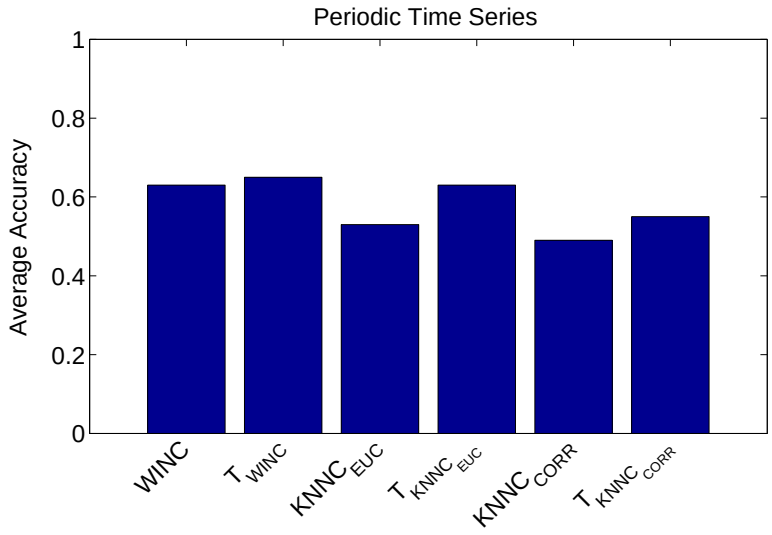


(a) Average accuracies of Periodic datasets

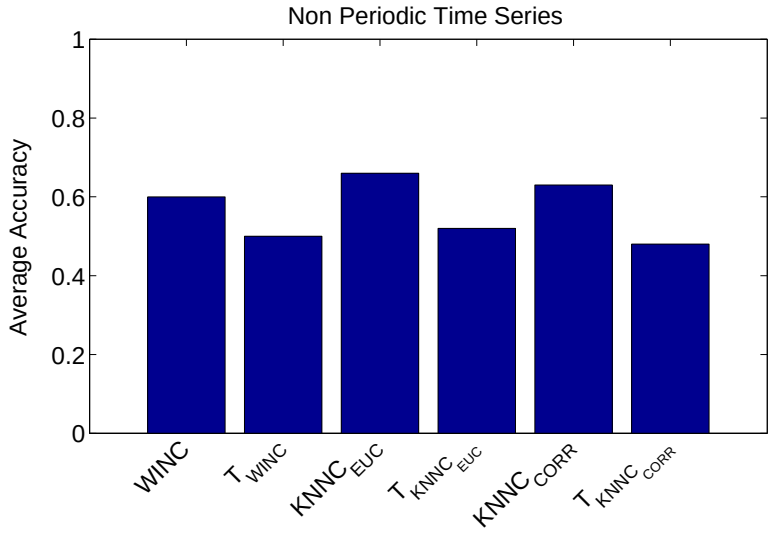


(b) Average accuracies of Non-Periodic datasets

Figure 4.1: Average accuracies of original time series



(a) Average accuracies of Periodic datasets



(b) Average accuracies of Non-Periodic datasets

Figure 4.2: Average accuracies of noise induced time series

	DataSet	$WINC$	T_{WINC}	$KNNC_{EUC}$	$T_{KNNC_{EUC}}$	$KNNC_{CORR}$	$T_{KNNC_{CORR}}$
1.	motor1	0.70	0.90	0.70	0.90	0.70	0.90
2.	motor2	0.90	0.90	0.70	0.90	0.90	0.90
3.	motor3	0.80	0.90	0.50	0.90	0.70	0.90
4.	motor4	0.90	0.90	0.90	0.80	0.70	0.80
5.	power	0.63	0.75	0.88	0.88	0.88	0.88
6.	chfdb1	0.36	0.28	0.12	0.32	0.24	0.24
7.	chfdb2	0.36	0.36	0.24	0.30	0.32	0.32
8.	ltstdb1	0.76	0.70	0.16	0.68	0.48	0.60
9.	ltstdb2	0.20	0.40	0.08	0.36	0.44	0.40
10.	evi1	0.92	0.70	1.00	0.98	0.14	0.38
11.	evi2	0.90	0.72	1.00	0.96	0.12	0.32
12.	valve1	0.63	0.88	0.75	1.00	0.75	0.88
13	edb1	0.68	0.30	0.58	0.34	0.74	0.32
14.	edb2	0.16	0.18	0.20	0.20	0.20	0.20
15.	mitdb1	0.64	0.36	0.68	0.34	0.70	0.34
16.	mitdb2	0.92	0.40	0.82	0.36	0.80	0.34
17.	shapes1	0.80	0.80	0.90	0.80	0.60	0.70
18.	shapes2	0.40	0.60	0.70	0.60	0.75	0.60
	Avg	0.57	0.56	0.52	0.58	0.53	0.51
	Time(s)	73	80	6	16	11	18

Table 4.4: Accuracy Results for noisy data

4.5 Observations

The results show significant promise for application of the transformation on univariate time series to improve anomaly detection. As per the properties of the transformation, it behaves differently on different types of time series. The wide range of public data sets considered in our analysis enable us to study the performance of this transformation under different circumstances. We discuss the performance of different algorithms on original and transformed data below.

The transformation behaves differently on periodic and non-periodic data sets and hence they are studied separately. The first 11 datasets in Table 4.3 are periodic and the others are non-periodic.

For the data without noise, we make the following high level conclusions.

- For periodic time series :
  - $T_{KNNC_{EUC}}$ performs significantly better than $KNNC_{EUC}$. $T_{KNNC_{CORR}}$ performs comparably to $KNNC_{CORR}$ and similar relation holds with T_{WINC} and $WINC$. $T_{KNNC_{EUC}}$ performs consistently better than $WINC$ which performs the best on the original data. These observations are justified below.

- For window based technique, $WINC$, the optimal size of the window depends on the length of anomalous region in the anomalous time series. Anomalies in periodic time series can vary such as irregular periods, anomalous values etc. Hence if the parameters such as window length, hop and nearest neighbors are set appropriately, $WINC$ performs well.
- The phase-misalignments and non-linear alignments, which are some common challenges of periodic time series data, restrict the usage of different proximity measures for the proximity based techniques. Thus $KNNC_{EUC}$ performs well only when the time series are synchronous, while $KNNC_{CORR}$ works moderately well given that it tries to align phases and compute similarity.
- After the transformation, the normal periodic time series become periodic or constant valued time series. Since the normal periodic time series would not have varying periods or anomalous values, the change in angle between consecutive subspaces of windows nearly remains constant or varies with some periodicity. The transformed anomalous time series highlight the existence of anomaly. Since the change in angle between consecutive subspaces is governed by the maximum distance between an arbitrary unit vector in one subspace and the other suspace, even a small change in one of the subspaces will change the angle significantly. This property of the transformation enables it to highlight the existence of the anomaly in the time series. Thus the transformed anomalous time series is signigicantly different from the normal time series. In figure 3.4, the spikes in the transformed anomalous time series correspond to the entry and exit of the anomalous portion of the original time series in the windows of the multivariate time series.
- The above aspects of the transformed time series make them more suited for the proximity based scheme with Euclidean distance measure ($T_{KNNC_{EUC}}$) as it performs consistently the best when (i) the normal time series are synchronous (the constant valued time series) while the anomalous portions are highlighted in the anomalous time series, or (ii) when the normal time series are periodic and the portions of anomalous time series are highly different from the normal ones.

  - T_{WINC} is not always better than $WINC$ because the performance of a window based scheme is highly dependent on the appropriate parameter settings.
  - Performance of $T_{KNNC_{CORR}}$ is typically next to $T_{KNNC_{EUC}}$ in most cases. This is because for the anomalous time series, the correlation measure finds the best "phase-shifted" version of the normal time series or vice-versa. Even if the anomalous portion is highlighed in the transformed anomalous time series, it might happen that its "phase-shifted" version is similar to a normal time series and hence the anomalous time series appears to be normal. This property of correlation measure makes it unsuitable for *evi* data as they originally have noise and their anomalous time series might be phase-aligned with some noisy normal time series and hence appear normal.

- For non-periodic datasets :
  - $WINC$ performs consistently better than $KNNC_{EUC}$ or $KNNC_{CORR}$. This is because $KNNC_{EUC}$ is incapable of capturing the anomaly especially when the time series are not synchronous and $KNNC_{CORR}$ might not perfectly capture the anomaly given the above reasons of phase-alignments of anomalous and normal time series, while for $WINC$ if the parameters are set appropriately, the anomalies can be captured better.
  - The transformation of a non-periodic time series does not result in either constant valued time series or a periodic time series. Also the transformed anomalous time series need not be significantly different from normal ones. Hence the T_{WINC}, $T_{KNNC_{EUC}}$ or $T_{KNNC_{CORR}}$ need not necessarily be better than their original counterparts.

For noise induced data, we make the following high level conclusions.

- The performances of all the algorithms are affected by noise. $WINC$ performs consistently best among all the algorithms including the transformed ones.
- The reason for best performance of $WINC$ even though there is noise in the time series is because the time series is broken down into smaller windows and the

normal windows would find some nearest neighbors while the anomalous windows might not find any.

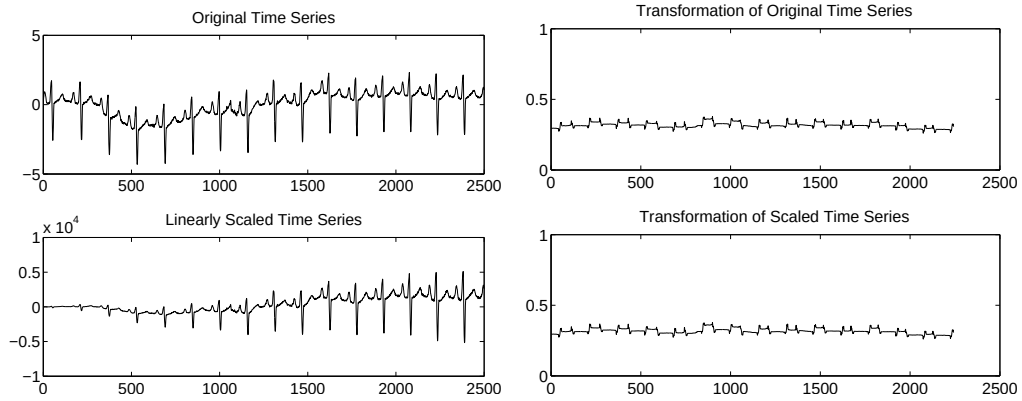
- $KNNC_{EUC}$ and $KNNC_{CORR}$ are affected by noise because they consider the entire time series at once and suffer with the $curse of dimensionality$.

- For periodic time series, $T_{KNNC_{EUC}}$ still performs better than $KNNC_{EUC}$ because the euclidean distance measure fails on the original periodic data, while the transformed time series still maintain the properties of nearly constant valued time series or periodic time series with some noise.

Another significant property of this transformation is that it is resistant to scaling of time series. Consider the Figure 4.3(a) which shows a normal time series and its scaled version obtained after multiplying with a linearly increasing factor. Though there is significant difference between these two time series, the transformation results in same time series. Similarly 4.3(c) shows a normal time series and its scaled version obtained after adding a linearly increasing factor but the transformation, as shown in Figure 4.3(d), results in the same time series. This is because the transformation calculates the change in subsequent subspaces, which remains the same even after scaling. This property could also be a drawback of the transformation as the linear scale difference could anomalous.

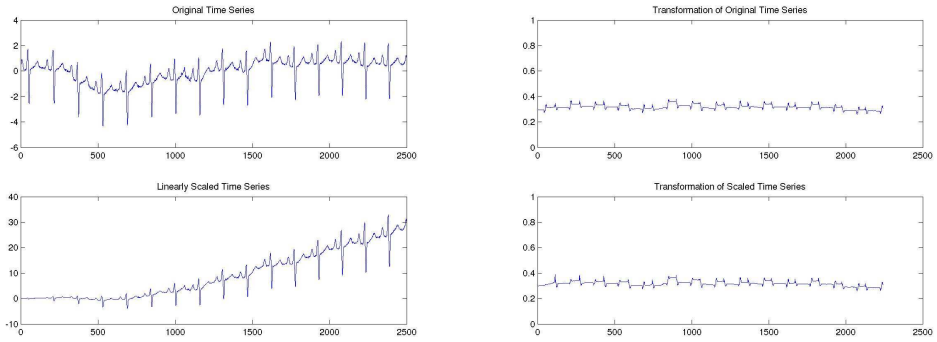
4.6 Conclusions and Future Work

We proposed a novel transformation for univariate time series using change in subspace structures. The properties of this transformation result in significant improvements of performance for anomaly detection techniques on periodic time series. Though window based schemes perform well on the original periodic time series, the parameter settings might be tedious. On the other hand experimental results indicate that all the techniques perform well on the transformed time series and the parameter settings for the transformation are consistent across many datasets. The transformation performs best when the window size $w_1 = l$, where l is the Cycle/Characteristic Pattern Length in the time series, $w_2 \approx 8$. The performance is not sensitive to the choice of α for $0.85 \leq \alpha \leq 0.95$. For $\alpha > 0.95$, the successive subspaces for normal windows become



(a) Original and Scaled Time Series obtained after multiplying with a linearly increasing factor

(b) Transfomation of Original and Scaled Time Series



(c) Original and Scaled Time Series obtained after adding a linearly increasing factor

(d) Transfomation of Original and Scaled Time Series

Figure 4.3: Effect of Transformation on Scaled Time Series

unstable and hence result in a high false alarm rate. For $\alpha < 0.85$, the successive subspaces for anomalous windows do not differ significantly and hence result in a low detection rate.

A critical aspect of the proposed technique is the computation of eigen vectors for the moving window which could become a computational bottleneck, especially when the dimensionality and length of the time series becomes large. A potential solution is to update the eigen vectors for sucessive windows incrementally using techniques such as *Matrix Perturbation* (90) to avoid computation of the eigen vectors for every window, and is suggested as a future direction of research.

Chapter 5

Conclusions and Future Work

The existing research on anomaly detection for time series is limited to developing techniques that are suitable for specific application domains. In this thesis we provided a comprehensive understanding and structured overview of the research on anomaly detection techniques for time series spanning multiple research areas and application domains. Each application domain possesses unique characteristics in terms of the data on which the anomaly detection techniques operate. Our study reveals that the performance of the anomaly detection techniques is closely tied to the nature of the underlying data, and hence the techniques exhibit varying performance across application domains. The results of our thorough experimental studies for time series data can help a domain scientist find the technique which is best suited for an application domain.

We also proposed a novel transformation for univariate time series using change in subspace structures. The properties of this transformation result in significant improvements of performance for anomaly detection techniques on periodic time series. Though window based schemes perform well on the original periodic time series, the parameter settings might be tedious. On the other hand, the transformation removes the periodicity of the time series and highlights the anomalies, hence is a better scheme. Since the transformation is window based, there is the aspect of redundancy, since successive windows, are often highly similar to each other and do not provide any additional information. Removing this redundancy can speed up the performance of the technique. This aspect of redundancy needs to be understood and is suggested as a future direction of research. Also as mentioned in chapter 4, computation of eigen vectors for successive

windows can be made efficient using *Matrix Perturbation* (90).

Since we are dealing with periodic time series, a direction of future work could be using signal processing techniques to handle the periodicity of the time series. One can remove the periodicity from the time series, i.e., transform them using signal processing techniques, such that the normal and anomalous time series look differently.

The subspace based transformation proposed, though motivated from the work for multivariate time series anomaly detection, is essentially a transformation from a unvariate time series to another univariate time series. Thus the process of the transformation could be seen in this perspective and better understood.

References

[1] Arindam Banerjee Varun Chandola and Vipin Kumar. Anomaly detection : A survey. *To Appear in ACM Computing Surveys, 2009*.

[2] Eilertson E. Lazarevic A. Tan P.N. Kumar V. Srivastava J. Ertoz, L. and P. Dokas. Minds - minnesota intrusion detection system. *In Data Mining - Next Generation Challenges and Future Directions. MIT Press.*, 2004.

[3] Parra L. Spence, C. and P. Sajda. Detection, synthesis and compression in mammographic image analysis with a hierarchical image probability model. In *In Proceedings of the IEEE Workshop on Mathematical Methods in Biomedical Image Analysis. IEEE Computer Society, Washington, DC, USA, 3*, 2001.

[4] Freisleben B. Aleskerov, E. and Rao. Cardwatch: A neural network based database mining system for credit card fraud detection. In *In Proceedings of IEEE Computational Intelligence for Financial Engineering. 220-226*, 1997.

[5] Malik Agyemang, Ken Barker, and Rada Alhajj. A comprehensive survey of numeric and symbolic outlier mining techniques. *Intell. Data Anal.*, 10(6):521–538, 2006.

[6] Victoria Hodge and Jim Austin. A survey of outlier detection methodologies. *Artif. Intell. Rev.*, 22(2):85–126, 2004.

[7] Arindam Banerjee Varun Chandola and Vipin Kumar. Anomaly detection for discrete sequences : A survey. *Unpublished Work*.

[8] F. Chuah, M. C. Fu. Ecg anomaly detection via time series analysis. *LECTURE NOTES IN COMPUTER SCIENCE*, pages 123–135.

[9] S. Akhavan and G. Calva. Automatic anomaly detection in ecg signal by fuzzy decision making. In *In Proceedings of 6th International Conference on Fuzzy Theory and Technology: Association for Intelligent Machinery, 23-28*, 1998.

[10] Sheng Zhang, Amit Chakrabarti, James Ford, and Fillia Makedon. Attack detection in time series for recommender systems. In *KDD '06: Proceedings of the 12th ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 809–814, New York, NY, USA, 2006. ACM.

[11] David L. Iverson. Inductive system health monitoring. *International Conference on Artificial Intelligence*, 2004.

[12] Manuele Bicego and Vittorio Murino. Investigating hidden markov models' capabilities in 2d shape classification. *IEEE Trans. Pattern Anal. Mach. Intell.*, 26(2):281–286, 2004.

[13] Zheng Liu, J.X. Yu, Lei Chen, and Di Wu. Detection of shape anomalies: A probabilistic approach using hidden markov models. *IEEE 24th International Conference on Data Engineering*, pages 1325–1327, April 2008.

[14] Li Wei, E. Keogh, and Xiaopeng Xi. Saxually explicit images: Finding unusual shapes. *Data Mining, 2006. ICDM '06. Sixth International Conference on*, pages 711–720, Dec. 2006.

[15] P. Protopapas, J. M. Giammarco, L. Faccioli, M. F. Struble, R. Dave, and C. Alcock. Finding outlier light-curves in catalogs of periodic variable stars. *MON.NOT.ROY.ASTRON.SOC.*, 369:677, 2006.

[16] U. Rebbapragada, P. Protopapas, C. E. Brodley, and C. Alcock. Finding anomalous periodic time series. Submitted to the Machine Learning Journal, 2008.

[17] Haibin Cheng, Pang-Ning Tan, Christopher Potter, and Steven Klooster. Detection and characterization of anomalies in multivariate time series. In *Proceedings of the ninth SIAM International Conference on Data Mining*, 2009.

[18] A. J. Fox. Outliers in time series. *Journal of the Royal Statistical Society. Series B(Methodological)*, 34(3):350–363, 1972.

[19] Junshui Ma and Simon Perkins. Online novelty detection on temporal sequences. In *KDD '03: Proceedings of the ninth ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 613–618, New York, NY, USA, 2003. ACM.

[20] Qingtao Wu and Zhiqing Shao. Network anomaly detection using time series analysis. In *Proceedings of the Joint International Conference on Autonomic and Autonomous Systems and International Conference on Networking and Services*, page 42, Washington, DC, USA, 2005. IEEE Computer Society.

[21] Eamonn Keogh, Jessica Lin, Sang-Hee Lee, and Helga Van Herle. Finding the most unusual time series subsequence: algorithms and applications. *Knowledge and Information Systems*, 11(1):1–27, 2006.

[22] Eamonn Keogh, Jessica Lin, and Ada Fu. Hot sax: Efficiently finding the most unusual time series subsequence. In *Proceedings of the Fifth IEEE International Conference on Data Mining*, pages 226–233, Washington, DC, USA, 2005. IEEE Computer Society.

[23] NASA DashLink. In *https://dashlink.arc.nasa.gov.*

[24] E. Keogh and T. Folias. In *The ucr time series data mining archive.*

[25] A. L. Goldberger et al. Physiobank physiotoolkit and physionet: Components of a new research resource for complex physiologic signals. 2000.

[26] Deepthi Cheboli Varun Chandola and Vipin Kumar. Detecting anomalies in a time series database. *Technical Report, 09-004.*

[27] J. Ma and S. Perkins. Time-series novelty detection using one-class support vector machines. volume 3, pages 1741–1745 vol.3, 2003.

[28] C. C. Michael and A. Ghosh. Two state-based approaches to program-based anomaly detection. In *ACSAC '00: Proceedings of the 16th Annual Computer Security Applications Conference*, page 21, Washington, DC, USA, 2000. IEEE Computer Society.

[29] Zhang J, Tsui FC, Wagner MM, and Hogan WR. Detection of outbreaks from time series data using wavelet transform. *AMIA Annu Symp Proc*, 2003.

[30] Thomas Lotze, Galit Shmueli, Sean Murphy, and Howard Burkom. A wavelet-based anomaly detector for early detection of disease outbreaks. 2008.

[31] Y. Qiao, X. W. Xin, Y. Bin, and S. Ge. Anomaly intrusion detection method based on hmm. *Electronics Letters*, 38(13):663–664, 2002.

[32] Xiaoqiang Zhang, Pingzhi Fan, and Zhongliang Zhu. A new anomaly detection method based on hierarchical hmm. In *Proceedings of the 4th International Conference on Parallel and Distributed Computing, Applications and Technologies*, pages 249–252, 2003.

[33] Philip K. Chan and Matthew V. Mahoney. Modeling multiple time series for anomaly detection. In *Proceedings of the Fifth IEEE International Conference on Data Mining*, pages 90–97, Washington, DC, USA, 2005. IEEE Computer Society.

[34] Matthew V. Mahoney and Philip K. Chan. Trajectory boundary modeling of time series for anomaly detection. In *Proceedings of the KDD Workshop on Data Mining Methods for Anomaly Detection*, 2005.

[35] Stan Salvador and Philip Chan. Learning states and rules for detecting anomalies in time series. *Applied Intelligence*, 23(3):241–255, 2005.

[36] Li Wei, Nitin Kumar, Venkata Lolla, Eamonn J. Keogh, Stefano Lonardi, and Chotirat Ratanamahatana. Assumption-free anomaly detection in time series. In *Proceedings of the 17th international conference on Scientific and statistical database management*, pages 237–240, Berkeley, CA, US, 2005. Lawrence Berkeley Laboratory.

[37] Jessica Lin, Eamonn Keogh, Stefano Lonardi, and Bill Chiu. A symbolic representation of time series, with implications for streaming algorithms. In *DMKD '03: Proceedings of the 8th ACM SIGMOD workshop on Research issues in data mining and knowledge discovery*, pages 2–11, New York, NY, USA, 2003. ACM.

[38] Eamonn Keogh, Kaushik Chakrabarti, Sharad Mehrotra, and Michael Pazzani. Locally adaptive dimensionality reduction for indexing large time series databases. In *In proceedings of ACM SIGMOD Conference on Management of Data*, pages 151–162, 2001.

[39] Eamonn Keogh, Stefano Lonardi, and Bill 'Yuan chi' Chiu. Finding surprising patterns in a time series database in linear time and space. In *Proceedings of the eighth ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 550–556, New York, NY, USA, 2002. ACM Press.

[40] Stephanie Forrest and Dipankar Dasgupta. Novelty detection in time series data using ideas from immunology. In *Proceedings of the 5th International Conference on Intelligence Systems*, Reno, Nevada, USA, 1996. IEEE Computer Society.

[41] Vipin Kumar Pang-Ning Tan, Michael Steinbach. Introduction to data mining.

[42] Xiao yun Chen and Yan yan Zhana. Multi-scale anomaly detection algorithm based on infrequent pattern of time series. *Journal of Computational and Applied Mathematics*, 214(1):227–237, April 2008.

[43] Ada Wai-Chee Fu, Oscar Tat-Wing Leung, Eamonn J. Keogh, and Jessica Lin. Finding time series discords based on haar transform. In *Proceeding of the 2nd International Conference on Advanced Data Mining and Applications*, pages 31–41. Springer Verlag, 2006.

[44] Y. Bu, T-W Leung, A. Fu, E. Keogh, J. Pei, and S. Meshkin. Wat: Finding top-k discords in time series database. In *Proceedings of 7th SIAM International Conference on Data Mining*, 2007.

[45] Kin-Pong Chan and Ada Wai-Chee Fu. Efficient time series matching by wavelets. *Data Engineering, 1999. Proceedings., 15th International Conference on*, pages 126–133, Mar 1999.

[46] Cyrus Shahabi, Xiaoming Tian, and Wugang Zhao. Tsa-tree: A wavelet-based approach to improve the efficiency of multi-level surprise and trend queries on time series data. In *Proceedings of the 12th International Conference on Scientific and*

Statistical Database Management, page 55, Washington, DC, USA, 2000. IEEE Computer Society.

[47] Cyrus Shahabi, Seokkyung Chung, Maytham Safar, and George Hajj. 2d tsa-tree: A wavelet-based approach to improve the efficiency of multi-level spatial data mining. In *Proceedings of the 13th International Conference on Scientific and Statistical Database Management*, page 59, Washington, DC, USA, 2001. IEEE Computer Society.

[48] Michail Vlachos, Marios Hadjieleftheriou, Dimitrios Gunopulos, and Eamonn Keogh. Indexing multi-dimensional time-series with support for multiple distance measures. In *KDD '03: Proceedings of the ninth ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 216–225, New York, NY, USA, 2003. ACM.

[49] B. Abraham and A. Chuang. Outlier detection and time series modeling. *Technometrics*, 31(2):241–248, 1989.

[50] B. Abraham and G. E. P. Box. Bayesian analysis of some outlier problems in time series. *Biometrika*, 66(2):229–236, 1979.

[51] Chris Chatfield. *The analysis of time series : an introduction*. Chapman and Hall CRC, 6th edition, 2004.

[52] Ryohei Fujimaki, Takehisa Yairi, and Kazuo Machida. An anomaly detection method for spacecraft using relevance vector learning. *Advances in Knowledge Discovery and Data Mining*, 3518:785–790, 2005.

[53] B. Pincombe. Anomaly detection in time series of graphs using arma processes. *ASOR BULLETIN*, 24(4):2–10, 2005.

[54] H. Zare Moayedi and M.A. Masnadi-Shirazi. Arima model for network traffic prediction and anomaly detection. *International Symposium on Information Technology*, 4:1–6, Aug. 2008.

[55] F. Knorn and D.J. Leith. Adaptive kalman filtering for anomaly detection in

software appliances. *IEEE Conference on Computer Communications Workshops*, pages 1–6, April 2008.

[56] David Moore and George McCabe. *Introduction to the Practice of Statistics*. 5th edition, 2004.

[57] Carl Edward Rasmussen. Gaussian processes in machine learning. *Machine Learning Summer School*, 2003.

[58] Vladimir N. Vapnik. *The nature of statistical learning theory*. Springer-Verlag New York, Inc., New York, NY, USA, 1995.

[59] Christopher Bishop. *Pattern Recognition and Machine Learning*. Springer Science Business Media LLC, 2006.

[60] L. Rabiner and B. Juang. An introduction to hidden markov models. *ASSP Magazine, IEEE*, 3(1):4–16, Jan 1986.

[61] Veselina Jecheva. About some applications of hidden markov model in intrusion detection systems. In *International Conference on Computer Systems and Technologies - CompSysTech*, 2006.

[62] Shrijit S. Joshi and Vir V. Phoha. Investigating hidden markov models capabilities in anomaly detection. In *ACM-SE 43: Proceedings of the 43rd annual Southeast regional conference*, pages 98–103, New York, NY, USA, 2005. ACM.

[63] Hai-Tao He and Xiao-Nan Luo. A novel hmm-based approach to anomaly detection. 2004.

[64] Ninad Thakoor and Jean Gao. Hidden markov model based 2d shape classification, 2005.

[65] Leonard E. Baum, Ted Petrie, George Soules, and Norman Weiss. A maximization technique occurring in the statistical analysis of probabilistic functions of markov chains. *The Annals of Mathematical Statistics*, 41(1):164–171, 1970.

[66] Eamonn J. Keogh, Selina Chu, David Hart, and Michael J. Pazzani. An online algorithm for segmenting time series. In *ICDM '01: Proceedings of the 2001 IEEE*

International Conference on Data Mining, pages 289–296, Washington, DC, USA, 2001. IEEE Computer Society.

[67] William W. Cohen. Fast effective rule induction. In *In Proceedings of the Twelfth International Conference on Machine Learning*, pages 115–123. Morgan Kaufmann, 1995.

[68] Jamal Ameen and Rawshan Basha. Mining time series for identifying unusual subsequences with applications. In *ICICIC '06: Proceedings of the First International Conference on Innovative Computing, Information and Control*, pages 574–577, Washington, DC, USA, 2006. IEEE Computer Society.

[69] Basha R and Ameen JRM. Unusual sub-sequence identifications in time series with periodicity. In *IJICIC '07: International Journal on Innovative Computing, Information and Control*, volume 3, 2007.

[70] Jamal Ameen and Rawshan Basha. Hierarchical data mining for unusual subsequence identifications in time series processes. In *Proceedings of the Second International Conference on Innovative Computing, Information and Control*, page 177, Washington, DC, USA, 2007. IEEE Computer Society.

[71] G. Keshav Palshikar. Distance-based outliers in sequences. 3816/2005.

[72] E. Keogh, J. Lin, A.W. Fu, and H. VanHerle. Finding unusual medical time-series subsequences: Algorithms and applications. *Information Technology in Biomedicine, IEEE Transactions on*, 10(3):429–439, July 2006.

[73] Ashok N. Srivastava. Discovering system health anomalies using data mining techniques. In *Proceedings of 2005 Joint Army Navy NASA Airforce Conference on Propulsion*, 2005.

[74] Ling Huang, Xuanlong Nguyen, Minos Garofalakis, Michael Jordan, Anthony Joseph, and Nina Taft. In-network pca and anomaly detection. Technical support, U.C. Berkeley, Berkeley, CA, 01/2007 2007.

[75] Varun Chandola. Anomaly detection for symbolic sequences and time series data : Ph.d dissertation.

[76] M. Basseville, M. Abdelghani, and A. Benveniste. Subspace-based fault detection algorithms for vibration monitoring. *Automatica*, 36:101–109, 2000.

[77] Manabu Kano, Shinji Hasebea, Iori Hashimotoa, and Hiromu Ohno. A new multivariate statistical process monitoring method using principal component analysis. *Computers & Chemical Engineering*, 25(7–8):1103–1113, 2001.

[78] C. Jordan. Essai sur la géométrie à n dimensions. *Buletin de la Societe Mathematique de France*, 3:103–174, 1875.

[79] H. Hotelling. Relation between two sets of variates. *Biometrika*, 28:322–377, 1936.

[80] K. De Cock and B. De Moor. Subspace angles between arma models. *Systems and Controls Letters*, 46(4):265–270, July 2002.

[81] Jeroen Boets, K. De Cock, M. Espinoza, and B. De Moor. Clustering time series, subspace identification and cepstral distances. *Communications in Information Systems*, 5(1):69–96, 2005.

[82] Dragomir Yankov, Eamonn J. Keogh, and Umaa Rebbapragada. Disk aware discord discovery: Finding unusual time series in terabyte sized datasets. In *Proceedings of International Conference on Data Mining*, pages 381–390, 2007.

[83] D. Berndt and J. Clifford. Using dynamic time warping to find patterns in time series. In *AAAI Workshop on Knowledge Discovery in Databases*, pages 229–248, 1994.

[84] P. Protopapas, J. M. Giammarco, L. Faccioli, M. F. Struble, R. Dave, and C. Alcock. Finding outlier light curves in catalogues of periodic variable stars. *Monthly Notices of the Royal Astronomical Society*, 369(2):677–696, 2006.

[85] Stephanie Forrest, Christina Warrender, and Barak Pearlmutter. Detecting intrusions using system calls: Alternate data models. In *Proceedings of the 1999 IEEE ISRSP*, pages 133–145, Washington, DC, USA, 1999. IEEE Computer Society.

[86] Klaus-Robert Müller, Alex J. Smola, Gunnar Rätsch, Bernhard Schölkopf, Jens Kohlmorgen, and Vladimir Vapnik. Predicting time series with support vector

machines. In *Proceedings of the 7th International Conference on Artificial Neural Networks*, pages 999–1004, London, UK, 1997. Springer-Verlag.

[87] V. Chandola, V. Mithal, and V. Kumar. A comparative evaluation of anomaly detection techniques for sequence data. In *Proceedings of International Conference on Data Mining*, 2008.

[88] Shyam Boriah. Time series change detection: Algorithms for land cover change : Ph.d dissertation.

[89] Ashok Srivastava. Nasa dashlink. https://dashlink.arc.nasa.gov, 2008.

[90] G. W. Steward and Ji guang Sun. *Matrix Perturbation Theory*. Academic Press, 1990.