LABOR CONTRACTS WITH VOLUNTARY QUITs

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ABSTRACT

This paper considers characteristics of labor contracts between the risk-neutral firm and risk-averse workers who have heterogeneous outside opportunities (alternative wages). The alternative wage, not verifiable by the firm, becomes known to the worker after costly on-the-job search. The worker voluntarily quits if the outside wage is higher than the contract wage in the firm.

If the alternative wage is deterministic, then self-selection among workers over a menu of contract wages would achieve the first-best with efficient allocation of workers in different firms (industries), i.e., productive efficiency, and perfect risk-sharing.

If the alternative wages are stochastic, the second-best contract would emerge on a tradeoff between productive efficiency and risk-sharing. Workers who are induced to search in the second-best contracts are fewer than in the first-best; and workers given search are less likely to quit than the first-best. The severance payment for a voluntary leaver serves only incomplete insurance because the exact outcome of search is not known to the firm; and unsuccessful search which force workers to stay is only partially compensated. If search effort is not monitored, even fewer workers conduct search.

In sum, workers' stochastic alternative wages, which is a private information, yield a contract which induces workers to conduct less search and quits less often than the first-best.

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1. Introduction

Inefficiency in the labor market can take various forms. Among others, involuntary unemployment has been a popular topic of investigations. A form of inefficiency, which is as important as involuntary unemployment but often neglected in the literature, arises when workers are employed in the wrong firm (or industry). A worker should be working in a firm where his productivity, after search and moving costs, is highest. Hence, for example when the productivity of one firm declines, it is productively efficient that those workers with better opportunities outside the firm should quit the firm. If the risk-neutral firm and risk-averse workers have agreed on the Azariadis-type fixed-wage labor contract, which guarantees a constant wage regardless of the productivity of the firm, there is no incentive for a worker to move to other firms. A perfect insurance is possible only if a labor contract is modified so that severance payment is contingent on the productivity of the firm and alternative opportunities of the worker. However, it would be difficult to implement such contingent severance payments since (i) an alternative opportunity for a worker is typically not verifiable by the firm; and (ii) an exact payoff of an alternative opportunity is usually found only after costly search by a worker, where search activity is not monitored by the firm. This observation shows that it is not straightforward to achieve both productive efficiency and perfect risk-sharing between the firm and workers, when worker's alternative opportunity is private information.

The purpose of this paper is to construct a simple contract model with on-the-job search, and characterize the (second-) best contract given the informational constraint. In particular, we are

interested in finding out how productive efficiency and risk-sharing are distorted in such a contract. It will be shown that too few search and too few severance are a main consequence of informational asymmetry between the firm and workers.¹

Implicit contract models, initiated by Azariadis (1975), Baily (1974), and Gordon (1974), have shown that wages are equalized across the good and bad times as a result of risk-sharing between the risk-neutral firm and risk-averse workers.² Workers in the low-productivity firm are enjoying the wage higher than marginal products, so that they do not have incentives to move in the spot market to the firm with a higher marginal product, which nonetheless is short of the contract wage level. However, a mere existence of such contracts does not imply a failure of productive efficiency. It will be shown in Section 3 that if alternative opportunities are deterministic and known to workers and if the right amount of contingent severance payments are taken into consideration at the time of contracting, workers voluntarily move in and out of the firm achieving productive efficiency (equal marginal products), while incomes (wages plus severance payments) are equalized across different states.

Sections 4 and 5 are a core of this paper. In both sections, alternative wages are stochastic and the exact result of search is known only to the worker after costly search. It will be shown that the informational asymmetry will cause "too few severance" from the firm in the bad times.³ There are two factors, both contributing to distortion in the sense of too few severance of workers from the low productive firms. First, the severance payment is only an incomplete insurance, because an outcome of search is not verifiable by the firm. Second, unsuccessful search, though costly, cannot be

compensated efficiently because search intensity (all or nothing in this paper) is not monitored. In order to achieve (productively) efficient separation, search with positive (net) expected value must be undertaken. In order to induce a risk-averse worker to conduct the search risk of unsuccessful (low alternative wages) search must be covered by the firm. However, there is a moral hazard problem in this search activity. If costs of unsuccessful search were fully compensated, workers would lie about the search intensity. That is, workers would claim unsuccessful search and collect compensation, without conducting costly search. Thus, unsuccessful search cannot be fully covered. The firm can only insure those who return to the firm after finding out that their alternative wages are relatively low. Since insurance for search is not perfect, some workers who would barely prefer to search in the first best would not be searching in the second best. It will also be shown that the second-best contract compensates those who return more than those who quit after the search. This implies that severance is not encouraged, given search is done. As a result, both induced search and severance from the low productivity firm is too few, compared to the first best.

Throughout this paper, a correspondence between the contract solution and an Arrow-Debreu competitive market solution is emphasized and illustrated. The contract wage is interpreted as the combination of spot market wage and insurance premium or coverage. Any inefficiency in an allocation of workers in optimal contracts will be understood to be caused by imperfection of markets.

Since there have been many implicit contract models with asymmetric information, some remarks on the novelty of this paper in

relative to others are in order. Many implicit contract models consider asymmetric information in the sense that workers cannot observe the productivity of the firm in which they are employed. (See, for example, Azariadis (1983), Green and Kahn (1983), Grossman and Hart (1983), Grossman, Hart, and Maskin (1983), and Hall and Lilien (1979).) A viable contract in that framework has to be incentive compatible, in that the contract, as a combination of the level of employment and the wage, induces the firm to reveal the true state of nature despite its superior information. It is well-known that in these models one may obtain "over-employment" or "under-employment" depending on firm's risk-aversion and the inferior/normal good property of leisure. (See, for a summary of these combination, Cooper (1983).) Most existing implicit contract models consider why the firm may lay off workers in the low productivity state, but fail to explain why workers prefer to stay laid off instead of taking up another job. Most contracts must require an assumption that workers are completely immobile after contracts are written, since worker's decision problem with respect to finding out or moving to outside opportunities are not modelled at all.

The present paper proposes to consider a case in which workers, not the firm, have superior information. Asymmetric information in this paper means that an alternative opportunity for a worker outside the contracted firm is known only to that worker. Since workers have better information, an efficient contract will require workers to take actions (quit or stay) to reveal his information as much as possible. This is the reason why I focus voluntary quits instead of layoffs in this paper. Recently, several attempts have been made to construct a model with informational asymmetry with respect to

worker's alternative opportunity. Arnott, Hosios and Stiglitz (1984), Geanakoplos and Ito (1982), Kahn (1985) and Nalebuff and Zeckhauser (1984) belong to this category. However, this paper is unique in this category of the literature, in that alternative opportunities are known to workers only after costly search and that both search and quit decisions are endogenously derived in the model. For example, Kahn (1985) also considers stochastic alternative (outside) opportunities observed only by workers. However, outside opportunities became known to the worker without search costs. Since search behavior is an important ingredient, this paper can be viewed as a step toward synthesizing contract theory and search theory, following a seminal work by Burdett and Mortensen (1980).

A few remarks about involuntary unemployment and voluntary quits are in order. First, note that changing jobs does not mean that workers have to experience unemployment. In fact, among those who change jobs, a majority of them do so without experiencing unemployment. Mattila (1974) and Clark and Summers (1979) estimate that about 50 to 60 percent of job changers line up their new jobs before quitting and leaving their old jobs. This suggests that a model with voluntary severance and on-the-job search, as opposed to a model of involuntary unemployment, is a meaningful framework. Second, as Stiglitz (1984) points out, models with asymmetric information with respect to worker's alternative opportunity, such as Geanakoplos and Ito (1982) and Kahn (1985), often produce a result of involuntary employment rather than involuntary unemployment, i.e., the laid-off workers are better off than the retained workers. This paper, unlike others, does not produce an unreasonable contract which requires workers to stay in the firm when they prefer to separate.

2. The Model

Suppose a risk-neutral firm with a stochastic linear production function and many risk-averse workers whose marginal productivities are equal in this firm for a given state of nature. The firm's production function is represented simply by a linear production function with stochastic coefficients: $m(s)L$, where s denotes the state of nature, $s = g$ (for good), b (for bad), i.e., $m(g) > m(b)$. The probability of the good state of nature, $\text{prob}(s=g)$ is denoted by p . Thus, $\text{Prob}(s=b) = 1-p$. The firm is willing to hire an infinitely large number of workers if the spot market wage is less than $m(s)$, and to shut down if the spot market wage is higher than $m(s)$.⁴

Each worker inelastically supplies one unit of labor. Workers are homogeneous within the firm, but heterogeneous with respect to their expected alternative opportunities (wages) available outside this firm. Some have general (compatible) skills which can be employed or easily retained in other firms, while some have very firm-specific skills. There are finitely many types of workers with respect to expected alternative wages. There are ℓ_j workers for the j -th type, $j = 1, \dots, n$. The j -th type worker has an alternative wage opportunity $a_j + \epsilon_j$, where a_j is deterministic independent from s and known ex ante to the worker and ϵ_j , a random variable with zero mean, is known only if search occurs. Neither a_j nor ϵ_j is observed by the firm. Whether a worker stays, $q=0$, or quits, $q=1$, is publicly observed. Workers have identical monotone increasing, strictly concave, differentiable utility functions $u(y)$, where y is net income at the end of the period: $u' > 0$, $u'' < 0$. Arrange workers in ascending order with respect to a_j :

$$a_1 < \dots < a_j < a_k < \dots < a_n.$$

The present model is a one-period static model with the following evolution of information revelation and agents' decisions. A contract, consisting of the state-contingent wage, which is paid to a worker who stays, and the state-contingent severance payment, which is paid to a worker who leaves, is made before the state of nature for this firm is revealed. The state revelation is followed by search decisions of workers. If the worker declines to search and stays, then the wage, $w_j(s)$, is paid. If the worker decides to search for an alternative job, the cost $z \geq 0$, which is independent of s and j , is incurred. The worker then finds out the realization of ϵ_j and becomes eligible for $a_j + \epsilon_j$ outside the present firm. The worker still has the option of going back to the original job if the ϵ_j is relatively low. The alternative wage, $a_j + \epsilon_j$ can be thought of as an offer as a result of an on-the-job search, which does not decrease their productivity in the contracted firm. This assumption can be justified by the fact that most job changes are believed to take place without experiencing unemployment. The contract is written before s and ϵ_j are revealed, and is strictly enforced.⁵

I also assume that the firm is under competition in the same industry so that the expected profit is always driven to zero. Thus, a competitive equilibrium in the insurance market, or the ex ante contract market, is defined by one which maximize worker's expected utility subject to the zero expected profit. This corresponds to the usual analysis where the competitive insurance firm offers an actuarially-fair (hereafter, "fair" for brevity) insurance policy.

3. Self Selection: Case of Deterministic Alternative Wages

In this section, I assume interior solutions for the deterministic case ($\epsilon=0$), in that some workers stay in this firm: $m(b) \geq a_1 - z$;

and some leave, $m(g) < a_n - z$. It is assumed that even in the case of $\epsilon_j = 0$, the worker still has to pay z , the cost of obtaining the letter of confirming the offer a_j .

Although results obtained in this section are not main conclusions of this paper, a detailed discussion is provided because it demonstrates two important facts: (i) that contracts mimic the Arrow-Debreu solution; and (ii) that a mere existence of asymmetric information, in that the firm does not observe worker's alternative wage, does not cause inefficiency. Self selection of workers over the menu of combinations of contingent wages/severance payments would achieve the first best, i.e., productive efficiency and perfect risk sharing.

If the worker searches and leaves to another firm, it costs the worker z . Net income fluctuates in the absence of contracts for those workers whose $a_j - z$ is lower than $m(g)$ but higher than $m(b)$. Contracts cannot be contingent on a_j , which is unobservable by the firm.

First, the spot market solution without contracts is considered. Fair insurance policies contingent on observable events are offered and workers can choose the amount to purchase. Combining the spot market solution and fair insurance, we claim that the Arrow-Debreu allocation is Pareto efficient since the contingent claims span the entire contingent commodity space. Second, the implicit contract solution is shown to mimic the first best allocation.⁶

Suppose the j -th type worker is currently hired at the firm, with an option to move to another. The worker chooses the higher of the wage at this firm and the alternative wage net of the search costs, once the state of nature become known. The wage at the firm is equal to the marginal productivity by the assumption of zero profit. The net income of this worker is:

$$y_j(s) = \max [m(s), a_j - z].$$

The firm, being risk-neutral, is willing to offer fair insurance before the state of nature becomes known. Namely, the firm is ready to promise a payment c_j when $s = b$, in return to the worker's payment of a "premium" $(1-p)/p)c_j$ when $s = g$. This insurance for the state of nature is not contingent on whether or not the worker stays at the firm. Since the expected value of this insurance is zero, the firm is indifferent as to how many units of c_j are sold. The number of units of c_j purchased is determined by the j -th worker's expected utility maximization. Note that the worker's decision to stay or to quit is solely dependent on whether $a_j - z$ is larger than $m(s)$. The stay-quit decision is not affected by how many units of this insurance are purchased, because the insurance policy only stabilizes the incomes of the worker in different states of nature without playing any roles in achieving the productive efficiency.

Proposition 3.1

An equilibrium with insurance (state-contingent payments) described above achieves the first best, i.e., productive efficiency and perfect risk sharing, despite the (deterministic) alternative wages are private information of workers. The units of fair insurance chosen by the worker reveals the private information.

Proof of Proposition 3.1:

Since workers with $(a_j - z)$ higher than $m(g)$ would not be hired by this firm in either state of nature, we concentrate on workers whose $(a_j - z)$ is lower than $m(g)$. The worker's optimization problem is,

$$\text{Max}_{c_j} \quad pu[m(g) - ((1-p)/p)c_j] + (1-p)u[\max[m(b), a_j - z] + c_j].$$

the first order condition is the equality of the marginal utilities:

$$u'(m(g) - ((1-p)/p)c_j) = u'(\max[m(b), a_j - z] + c_j).$$

Therefore, the net income in both states of nature is equalized by the optimal decision and denoted by $\hat{y}_j$, where

$$\hat{y}_j \equiv m(g) - ((1-p)/p)c_j = \max[m(b), a_j - z] + c_j.$$

Thus, the optimal insurance coverage and net income for the j -th type worker, respectively, are

$$\begin{aligned}\hat{c}_j &= pm(g) - p\max[m(b), a_j - z], \\ \hat{y}_j &= pm(g) + (1-p)\max[m(b), a_j - z].\end{aligned}$$

The worker purchases the insurance policy to the extent that the net income is equalized in the two states of nature. Observe that the optimal amount of insurance reveals the information of the worker's alternative wage, because $\hat{c}_j$ is a non-decreasing function of a_j and strictly increasing when $a_j - z > m(b)$, i.e., the worker is quitting in the bad state of nature. It is easy to confirm that the worker does not have any incentive to misrepresent his alternative opportunities, because doing so implies a mean preserving spread of his net income across the state of nature, which is utility decreasing for the risk-averse worker. Q.E.D.

What is shown above is that worker's self selection results in the resource (labor) allocations and the income distribution in the Arrow-Debreu complete markets economy. In other words, so long as the alternative wage is deterministic, asymmetric information, that the firm cannot verify the alternative wage, is overcome by self selection of workers.

Note that there is no "cross-subsidization" between worker

types, so that the firm is break-even for each of a_j types. At the same time, the firm would not be able to offer any contracts which yield a positive profit, because another firm could offer a (break-even) contract more attractive to workers.

Next let us investigate whether the implicit contract can mimic the efficient allocation. The standard procedure is followed to calculate the optimal contract. The expected profit is maximized subject to the condition that the j -th worker's expected utility is at least as high as some guaranteed level. The firm offers the contingent wage, $w_j(s)$, and the severance pay $c_j(s)$ which is paid only when the worker quits the firm.

The firm maximizes the following Lagrangean with respect to $\lambda_j(s)$, $w_j(s)$, $s=g,b$ and $c_j(s, q=1)$, knowing that those who are offered $a_j > w_j(s) - c_j(s)$ will leave.

$$\begin{aligned}
 H = & pm(g)L_G + (1-p)m(b)L_B - p \sum_{J_G} w_j(g)\lambda_j(g) - (1-p) \left[\sum_{J_B} w_j(b)\lambda_j(b) - \sum_{J_G - J_B} c_j(b)\lambda_j(b) \right] \\
 & + \sum_{J_B} \lambda_j [pu(w_j(g)) + (1-p)u(w_j(b)) - \bar{U}_j] \\
 & + \sum_{J_B - J_B} \lambda_j [pu(w_j(g)) + (1-p)u(c_j(g) + a_j - z) - \bar{U}_j]
 \end{aligned}$$

where $L_s = \sum_{J_s} \lambda_j(s)$, where $J_s = \{j \mid a_j - z \leq w_j(s)\}$,

$$J_G - J_B = \{j \mid (j \in J_G) \text{ but not } (j \in J_B)\},$$

and $\bar{U}_j$ is the ex ante guaranteed level of expected utility available from another firm. The determination of $\hat{U}_j$ is regarded as the determination of the division of gains from risk sharing. An assumption that ex ante competition among firms forces the expected profit to be zero makes it possible to determine uniquely the value of $\hat{U}_j$.⁸

From the first order conditions, we can derive the optimal solutions satisfying the following properties.

$$\begin{aligned} w_j(g) &= w_j(b), & \text{for } j \in J_G. \\ w_j(g) &= c_j + a_j - z, & \text{for } j \in J_G - J_B. \end{aligned}$$

The optimal contract is found to equalize net incomes across the states of nature, the familiar result. Using an assumption that the expected profit is equal to zero, the level of the equalized income is shown to equal $\hat{y}_j$, calculated in the Arrow-Debreu economy. Thus, the implicit contract will obtain the first-best allocation. Although different contracts have to be tailored for workers with different a_j 's, the firm does not have to know the value of a_j for a particular worker. The firm should offer a menu of contracts with respect to different a_j 's, and let the workers pick a particular contract. The contract wage, which is equal to the spot wage plus (minus, resp.) insurance coverage (premium, resp.), is shown in Figure 1, where a_j , which is in fact a step function of L , is approximated by a continuous function in L .

 Insert Figure 1 about here

In Figure 1, those workers between 0 and L_1 would not search in both states of natures. (Recall that workers are counted in an ascending order of their a_j .) There are payed at the stabilized wage which is equal to the mean value of marginal products in two states of nature. Workers with a_j 's that place them right of L_2 will not have any associations with this firm.

In the rest of this paper, we will concentrate on workers who are between L_1 and L_2 , i.e., searching when $s=B$, but not searching

when $s=G$. When workers quit, they collect severance payments, c_j , which make up the difference between the equalized wage level, y_j , and the alternative wage after search cost, a_j-z . Note that the slope of y is flatter than the slope of a_j-z , which implies that a worker with a higher a_j purchases lesser units of "insurance" for the bad state of nature. In other words, both the "premium" (the difference between $m(g)$ and y) and the "coverage" (the difference between y and a_j-z) are less for a worker with a higher a_j (one toward L_2).

Three remarks are in order. First, note that an introduction of severance payments are enough to achieve a complete-market Arrow-Debreu optimum in the present environment. This feature is not surprising. In a different environment investigated in the seminal work by Azariadis (1975), the complete stabilization of income across the states of nature, thus no involuntary unemployment, would be obtained if severance payments were introduced in that model.⁹

Second, observe that a uniform size of severance payments would not achieve efficiency of worker reallocation. However, the optimal size of the severance payment for a particular worker will be revealed by workers' decision. Thus, it is important that the size of severance payments should be determined through workers' decisions.

Third, in the good state of nature, two workers with the same productivity within the firm are paid differently: A worker who commands a higher alternative wage is paid more when $s=g$, even if his alternative wage when $s=g$ is less than the marginal productivity of this firm.¹⁰ The reason is that one with a higher alternative wage will voluntarily quit in the bad state of nature with smaller severance payments, and his contract wage reflect the lesser premium in the good state of nature.

4. Stochastic Alternative Wages and Costly Search

In the preceding section, it was shown that self selection among workers over the menu of packages of the contingent wages, and the severance payment achieves the efficient allocation of workers over different firms. Consider now a more realistic case in that alternative wages are stochastic, i.e. ϵ_j is non-degenerate. A worker has to engage in costly search in order to find out the best alternative wages available to him. In this environment, it will be shown that whether or not the firm knows the exact alternative wage would matter. Let us assume, for the sake of simplicity, a uniform distribution of ϵ_j with range of $[-v, v]$, for all j .

Assume that the exact alternative wage for a worker is not known to the worker without a search activity which costs the monetary value z . Workers, however, know the true probability distribution of alternative wages before the search.

4.1 Symmetric Information: First Best

In this subsection, the outcome of search $a_j + \epsilon_j$ is assumed to be observable by the firm as well as the worker. The (first-best complete information) optimal contracts are calculated just like in the deterministic case with a twist in the worker's quit-stay decision. First the insurance problem is solved. Insurance policies on search and on the state of nature. Then the corresponding contract which supports the allocation will be described. The logic is the same as the one in the preceding section.

Let us recall the evolution of timing. Search decisions have to be made using information about the realization of s , the value of a_j and the distribution of ϵ_j only. Once the search is done, the cost z is sunk and the realization of ϵ_j becomes known. Then the worker decides

whether to move to the alternative job, or to stay at the contracted firm. We solve the problem backwards: first, solve the decision problem concerning quitting or staying, given the search; second, knowing expected gains from search, consider the problem of whether or not to search. Note that two kinds of insurance policies are needed in this economy to span the contingent commodity space: Search activities as well as the state of nature have to be insured.

A worker whose ϵ_j is found to be larger than $m(s) - a_j$ should go to the alternative firm. However, the gain from the search $a_j + \epsilon_j - m(s)$ may turn out to be less than the search cost, z . But, the search cost is already "sunk" when a decision about whether to stay at the firm, $m(s) \geq a_j + \epsilon_j$, or to go to the alternative job is made. Having an option of staying the original firm, the gross gain from the search is defined as,

$$\begin{aligned} g_j &= \max[a_j + \epsilon_j, m(s)] - m(s). \\ &= \max[a_j + \epsilon_j - m(s), 0]. \end{aligned}$$

The distribution of g_j is a distribution of $a_j + \epsilon_j - m(s)$ truncated at zero, with a probability mass at zero. Formally, with the cost of z , the search will yield the expected gain

$$Eg_j = \int_{-v}^v (1/2v) \max[a_j + \epsilon_j - m(s), 0] d\epsilon_j. \quad (4.1)$$

In the following, I consider only the case $s=B$ and $v > z$. Then Eg_j is illustrated in Figure 2 as the difference between the thick line and $m(b)$. (If $v < z$ was the case, then all searching workers would quit, because the search costs were higher than the worst deviation of alternative wages from the expected value. The search decision becomes redundant.)

 Insert Figure 2 about here

The fair insurance policy which guarantees the expected gain for the j -th worker can be devised: the insurance premium/coverage is equal to the deviation of gain from search from its expected value, $[Eg_j - g_j]$. Insurance can equalize the benefit from search across the outcomes of ϵ_j , with zero expected profit to the insuring firm. It is important to recognize that this insurance needs information on ϵ_j .

Having this search insurance, workers will search when $Eg_j > z$ and receive the deterministic income, at the state of nature s ,

$$y_j(s) = m(s) + \max[0, Eg_j - z]. \quad (4.2)$$

There exists such $a_j^*(s)$, that if $a_j \leq a_j^*(s)$ then search is not conducted, and if $a_j > a_j^*(s)$ then search is conducted. As shown in Figure 2, $a_j^*(s)$, is found where $Eg = z$. Workers between 0 and L^* will not search in the bad state of nature. Define the threshold of search, $a_j^*(s)$, as a solution to the following equation:

$$Eg_j = z. \quad (4.3)$$

Since Eg_j is an increasing function of a_j , as evident from (4.1), and independent of z , it is easy to verify that threshold of search a_j^* is an increasing function of the search cost, z .

Having carried out search does not necessarily mean that the worker leaves the firm. The threshold $a_j(s)$ in this case is less than the threshold in the deterministic case $\epsilon = 0$ since a low realization of ϵ is not relevant to the resource allocation because of the "stay" option.¹¹ That is, marginal workers who would not search in the deterministic world (i.e., $\epsilon=0$, as in Section 3) are now induced to search. In reference to Figure 1, threshold moves to left: $L^* < L_1$.

Now consider insurance regarding the state of nature as in Sec-

tion 3: c_j is paid to the worker if $s=B$, and $c_j(1-p)/p$ is paid to the firm if $s=G$, where the size of c_j is chosen by the worker's ex ante expected utility maximization. Having both the state-of-nature and the search insurance policies, the worker is guaranteed

$$y_j^* = py_j(g) + (1-p)y_j(b), \quad (4.4)$$

where $y_j(s)$ is defined in (4.2). This is the level of income achieved for any realizations of s and any ϵ_j , by optimal contingent contracts with a full set of insurance markets. In sum, the worker's income, consisting of spot wages and the insurance premium/coverage with respect to both states of nature and search outcomes, can optimally be fixed at y_j level for the j -th worker.

Let us discuss how to support the Arrow-Debreu insurance solution by labor contracts. The contract specifies y_j calculated in (4.4) as the wage for the good state of nature. In the bad state of nature, the wage contingent upon a_j , s , and ϵ_j can be written so that the Arrow-Debreu allocations can be mimicked: those workers with $a_j > a_j^*$ are required to conduct search, if $a_j + \epsilon$ exceeds $m(b)$, then workers are released with severance pay $c(q) = Eq_j - \epsilon_j$, otherwise asked to stay with the wage y^* . Search is fully insured in the sense that payments to those who search and quit receive the fixed net income, that is the severance payment is perfectly negatively correlated with realization of ϵ . The threshold of ϵ which separates quitting from staying is dependent on the level of insurance, which is in turn dependent on a_j . Those who should be searching can actually be induced to search since wages without search is set very low. The solution in this subsection will be referred to as "first-best". In the rest of the paper, deviations from the first-best solutions caused by informational asymmetry will be investigated.

4.2 Asymmetric Information: Second Best

It is not realistic that the realization of the actual alternative wage offer is observed or verifiable by the firm, so that search outcomes are insurable.¹²

Suppose now that the firm knows the distribution of alternative wage for each worker but cannot verify the exact outcome of search, ϵ_j . Then there would be an obvious problem to implement the first-best solution: workers would lie about the outside offer, should the severance payment depend on what workers announced to the firm about their alternative wages. Note a contrast between Section 3, where workers would report truthfully the expected alternative wage ex ante for insurance of the states of nature and this section, where workers would not report the realized actual alternative wage when insurance for the search outcome is designed.

Given the structure of asymmetric information, severance payments are contingent only on whether workers quit or stay and on the state of nature. If ϵ_j is found to be relatively low, then the worker will stay in the firm, and the firm can infer from worker's behavior that ϵ_j is relatively low. However, if a worker leaves the firm, there is no way of finding out the exact realization of ϵ_j . Therefore, search insurance, contingent on quitting or staying, will be only partial in that only the lower end of the ϵ_j probability distribution is insurable.

The decision and event tree for insurance and contract solutions are illustrated in Figure 3.

Insert Figure 3 about here

We solve this problem first by taking the spot market allocation

with fair insurance and then showing how to support the allocation by contracts. The solution of this problem is only second best, in that it is a constrained optimum and worse in expected utility of workers than the first-best solution shown in the preceding subsection.

Suppose that as the spot market solution, a worker staying in the firm is paid $m(s)$, and if the worker leaves the firm then net income in the spot market is $a_j + \epsilon_j - z$. Now consider the following fair insurance policies with respect to states of nature and search results. First, if the worker does not search in either state of nature s , then c is paid to the worker if $s=b$, and $c(1-p)/p$ is collected from the worker if $s=g$. Second, for workers who do not search when $s=G$ but do search when $s=B$, the following policy is offered. The worker pays to the firm $c_j(g)$ when $s=g$; the firm pays to the worker $c_j(R)$ when $s=g$ and the worker searches and quits; and the firm pays to the worker $c_j(R)$ when $s=b$ and the worker searches but stays at the firm. The former is most naturally regarded as severance payments, while the latter may be interpreted as an incentive pay to stay after the on-the-job search, both being posted before worker's search.¹³

After the worker carries out a search for alternative wages, the worker faces a decision whether to quit the firm and take $c(q)$ or to stay at the firm and take $c(R)$. Given that search is done (z being sunk) and given that the insurance policy ($c_j(q)$ and $c_j(R)$) is provided for the search, workers behavior with respect to the quit-stay decision is summarized as follows:

the j -th worker $\left\{ \begin{array}{l} \text{stays at} \\ \text{quits} \end{array} \right\}$ the firm, if $a_j + \epsilon_j + c_j(q) \left\{ \begin{array}{l} \leq \\ > \end{array} \right\} m(b) + c_j(R)$.

Since this decision depends on the realization of ϵ_j , we can calculate the probability of staying. Recall that ϵ_j is distributed uniformly between v and $-v$. Consider a non-trivial case, in that the worker who for some ϵ_j quits and for some ϵ stays, i.e.,

$$-v \leq m(s) + c_j(R) - c_j(q) - a_j \leq v,$$

Denoting $v^*(s) \equiv m(s) + c_j(R) - c_j(q) - a_j$, the probability of staying is defined by

$$r \equiv \int_{-v}^{v^*(s)} (1/2v) d\epsilon_j = [v + m(s) + c_j(R) - a_j - c_j(q)]/2v. \quad (4.5)$$

Confirm that the probability to quit is

$$1-r = \int_{v^*(s)}^v (1/2v) d\epsilon_j = [v + a_j + c_j(q) - m(s) - c_j(r)]/2v.$$

Consider a type of workers who will stay without search in the good state of nature, $m(g) - c_j(g) > a_j + v$, but will search in the bad state of nature. The assumption of zero expected profit is

$$c_j(g) = r(1-p)/pc_j(R) + (1-r)(1-p)/pc_j(q). \quad (4.6)$$

The expected utility from search is now defined by

$$Eu(.| z > 0) = ru(m(b) + c_j(R) - z) + \int_{v^*(b)}^v (1/2v) u(a_j + c_j(q) + \epsilon_j - z) d\epsilon_j. \quad (4.7)$$

Those who search thus command the ex ante expected utility $pu(m(g) - c_j(g)) + (1-p)Eu(.| z > 0)$. Since $Eu(.| z > 0)$ is an increasing function of a_j , we in general have a threshold in a_j , that is, if a_j is such that $pu(m(g) - c_j(g)) + (1-p)Eu(.| z > 0) > u(pm(g) + (1-p)m(b))$, then the worker searches for alternative opportunities. Now denote the threshold by $\bar{a}_j$, defined by

$$u(pm(g) + (1-p)m(b)) = pu(m(b) - c_j(g)) + (1-p)Eu(.| z > 0). \quad (4.8)$$

It is easy to verify that the threshold $\bar{a}_j$ is increasing with respect to the search cost z . For the worker with $a_j > \bar{a}_j$, the ex ante (before s is known) expected utility maximization, given the optimal search and quit-stay decisions, can be written as follows:

$$\text{Max}_{(c_j(g), c_j(R), c_j(q))} H^I = pu(m(g)-c_j(g)) + (1-p)Eu(.| z>0). \quad (4.9)$$

subject to (4.6), the zero expected profit condition,

where $Eu(.| z>0)$ is defined in (4.7). In order to find an optimal fair insurance policy, $(c_j(g), c_j(q), c_j(R))$, take the first order conditions of (4.9) with respect to $c_j(q)$ and $c_j(R)$, with $Eu(.| z>0)$ and $c_j(g)$ being substituted from (4.6) and with r being substituted from (4.5). Note that $\partial r / \partial c(q) = -1/2v$, and $\partial r / \partial c(R) = 1/2v$.

Since the best insurance is devised for each type of a_j , we may omit subscript j without fear of confusion. Denoting $y(g) = m(g)-c(g)$, $y(q) = a + \epsilon + c(q) - z$, and $y(R) = m(b) + c(R) - z$, the first order conditions are

$$0 = \frac{\partial H^I}{\partial c(R)} = (1-p)[-ru'(y(g))-c(R)u'(y(g))/2v+c(q)u'(y(g))/2v-u(y(R))/2v+u(y(R))/2v+ru'(y(R))], \quad (4.10)$$

$$0 = \frac{\partial H^I}{\partial c(q)} = (1-p)[-(1-r)u'(y(g))+c(R)u'(y(g))/2v-c(q)u'(y(g))/2v+u(y(R))/2v+\int_{v^*(b)}^v (1/2v)u'(y(R))d\epsilon -u(y(R))/2v]. \quad (4.11)$$

Cancelling out and rearranging terms, (4.9) and (4.10) become,

$$u'(\bar{y}(R)) - u'(\bar{y}(g)) + u'(\bar{y}(g))(\bar{c}(q)-\bar{c}(R))/2rv = 0 \quad (4.12)$$

$$u'(\bar{y}(g)) = ru'(\bar{y}(R)) + \int_{v^*(b)}^v (1/2v)u'(\bar{y}(q)) d\epsilon. \quad (4.13)$$

The above first order conditions, combined with (4.6) and definitions of $y(g)$, $y(q)$, $y(R)$, determine the second-best insurance solution, $(\bar{c}(g), \bar{c}(q), \bar{c}(R))$.

Before implications of the second-best optimal solution are investigated, let us confirm that the contract solution would produce the same allocation with the insurance solution, in a manner parallel to one in Section 3. Consider the profit maximization problem subject to the expected utility constraint for the type j worker whose a_j is high enough to prompt search in the bad state of nature. The contract consists of $\delta \equiv \{w(g), w(b), c(Q)\}$, the wage in the good state of nature, the wage in the bad state of nature provided that the workers search and return, and the severance payment provided that the workers search and leave.

$$\begin{aligned} \text{Max}_{\delta} \quad Eu &= pm(g) - w(g) + (1-p)r(m(b) - w(b)) + (1-p) \int_{w(b)-a_j-c(Q)}^v (-c(Q)/2v) d\epsilon_j \\ &+ \lambda (pu(w(g)) + (1-p)ru(w(b) - z) + (1-p) \int_{w(b)-a_j-c(Q)}^v [u(a+\epsilon+c(Q)-z)/2v] d\epsilon - \bar{U}). \end{aligned} \quad (4.14)$$

It is easy to verify that solutions to the optimal contract problem, (4.14) is equivalent to solutions to the optimal (partial) insurance problem. First order conditions to the contract problem yield the first order conditions which are summarized by (4.12) and (4.13) with $w(g)$, $w(b)$, and $c(Q)$ substituting $y(g)$, $y(R)+z$ and $c(q)$, respectively. Now in the following, we use the second-best contracts interchangeably with the second-best insurance solution.

Equation (4.12) shows that unless $c(q)=c(R)$ the net income for the good state of nature is not equal to the net income for the returning worker after an unsuccessful search in the bad state of nature. Equation (4.13) shows that the marginal utility in the good state of nature equals the expected marginal utility in the bad state of nature. This is a typical condition for incomplete insurance. From these equations, the following theorems are derived.

Theorem 4.1 [Too Few Severance]

The second-best solution produces less severance than the first-best solution: (i) In the second-best solution, fewer workers are engaged in search than in the first-best solution; (ii) For a worker who searches, they are more probable to stay in the second-best solution than in the first-best solution.

Proof of the theorem is provided in Appendix. The first part of the theorem implies that $a_j^* < \bar{a}_j$. In Figure 2, it implies $L^* < \bar{L}$, where $\bar{L}$ is the threshold in the second best. Of course, different threshold levels correspond to different levels of search cost. However, given the search cost, fewer types of workers are searching in the second-best solution than the first-best solution. The relationship between the thresholds of the second-best and first-best contracts is illustrated in Figure 4.

 Insert Figure 4 about here

The second part of the theorem considers the particular type of workers who search even in the second best. The theorem states that the probability of staying given search is higher in the second-best solution than in the first-best solution. In Figure 2, realizations

of ϵ in interval ϵ_1 would induce workers to stay in the first-best contracts, while ϵ in interval ϵ_2 would do that in the second-best contract, with the difference between ϵ_1 and ϵ_2 being $c(q)-c(R)$.

In the contract version, the theorem implies $c(Q) < w(R)-m(b)$, i.e., the severance payment is less than the recall subsidization (wage payment in excess of their marginal products) to the staying workers. This is intuitively straightforward, since workers who are leaving after search are on average better off than workers who stay. Since ϵ_j is not observable, it is a matter of course that incomes for those who quit the firm are not equalized. It may be of some interest to compare ex post utility levels with different states of nature for the worker who stays at the firm.

Theorem 4.2

Net income for the returned worker in the bad state of nature is less than net income for the good state of nature.

Proof: In the proof of theorem 4.1, we have $u'(y(G)) < u'(y(R))$. Thus by the concavity of utility functions, $u(y(G)) > u(y(R))$. Q.E.D.

Therefore, net income is not equalized across the states of nature even if the worker stays at the same firm. This differential treatment is a result from a spillover of the nonobservability of ϵ_j . Returning after the search plays the role of an incomplete signal about the realization of ϵ . In order to stabilize incomes partially (i.e., equalized marginal utilities in (4.13)), payments are distorted from the complete stabilization.

Theorems 4.1 and 4.2 characterize incomplete insurance (contract with asymmetric information) in that upper-end realizations of ϵ are not insured. The search is encouraged by insurance coverage in the

bad state of nature: if the worker quits the firm after the search, then $c(Q)$ is paid; and if the worker returns after the search, then $w(R)=c(R)+m(B)$ is paid. Quitting shows that ϵ is relatively good; and staying after search reveals that ϵ is relatively bad. The optimal amount of $c(R)$ is shown to be larger than $c(Q)$, because this is a partial compensation for the unsuccessful (low ϵ) search. The optimal amount of $c(R)$ is not quite as high as the returned worker hopes for in the sense that net income for the returned is lower than net income in the case of the good state of nature.

4.3 Voluntary Quits, Involuntary Unemployment, and Paradox

This paper is unique in the contract literature which emphasizes the notion of involuntary unemployment. There are four reasons why voluntary quits are an important phenomenon to be analyzed. First, as emphasized in Introduction, statistics show that job changes without unemployment are more common than job changes with an unemployment spell. Second, workers in the "wrong" firm (industry) are as inefficient as involuntarily unemployed workers. Third, this model is designed to emphasize an asymmetric information structure where workers have private information. In this case, it is optimum that workers make decisions of separation and self-select contracts. The firm's layoff decision becomes necessary when the firm has a private information, as in the Hart and Grossman (1983). If we were to consider both layoffs and quits, we would need a model with bilateral informational asymmetry, as in Hall and Lazear (1984). Fourth, when stochastic alternative wages are introduced in the usual framework of implicit contracts with layoff decisions by the firm, it creates a "paradoxical" situation that workers prefer to be laid off. This feature is a basis of Stiglitz (1984; pp. 39-41) renouncing implicit

contract theory.

Suppose an implicit contract model where the firm has a right to nominate those to be laid off with a uniform severance payments, which is incomplete insurance for uncertain alternative wages. As established in Imai, Geanakoplos and Ito (1981), Ito and Machina (1983), and Stiglitz (1984: appendix) a laid-off worker has a higher expected utility than a retained worker, under plausible assumptions on risk aversion. If workers prefer being laid off, workers would try to be dismissed. The only way to keep workers honestly working in the firm is to differentiate the severance payment of disciplinary dismissal, the payment for voluntary job leavers, and the payment by lay-off. However, the distinction between job losers (involuntary separation) and job leavers (voluntary separation) is fuzzy. If both those who voluntarily leave the firm and those who are dismissed for disciplinary reasons are denied the severance payments to which job losers are entitled, then firms would try forcing workers out by making working conditions uncomfortable or by finding out small mistakes for disciplinary actions. Expecting such behavior, workers would not sign contracts which differentiate between job leavers and job losers. Therefore, Stiglitz was right to be concerned about this paradoxical feature of the implicit contract model with worker's asymmetric information. However, this does not mean that all of implicit contract models fail to be a sound framework of analyzing the labor market. In models where workers have a superior information set, workers, not the firm, should make a separation decision, such as demonstrated and commented on in Section 3. From results in preceding results we know that Stiglitz' criticism is not applicable to the present model where the worker makes a decision of separation.

5. Moral Hazard in Search: Third-Best

In subsection 4.2, the (second-best) contract was investigated in an environment where outcomes of search $(a_j + \epsilon_j)$ are not observable by the firm. Since staying in the firm after search reveals that ϵ_j was relatively low, unsuccessful search is partially insured in the second-best contract. It was implicitly assumed that the firm could monitor workers' search effort (all or nothing).

Suppose now that the firm cannot verify whether the worker actually searches. Would the worker still carry out costly search which is only partially insured? The answer is negative if the worker finds staying in the firm without search (but claiming an unsuccessful search) yielding a higher utility than taking a chance in costly search activity. The second-best solution with $\bar{w}(b)$ and $\bar{c}(Q)$ is not implementable because of moral hazard in workers' search activity, if the following inequality holds for the second-best:

$$u(\bar{w}(b)) > ru(\bar{w}(b) - z) + \int_{v^*(b)}^v (1/2v) u(a + \epsilon + \bar{c}(q) - z) d\epsilon, \quad (5.1)$$

If the firm asked the workers whether they search or not, workers would lie and say that they had searched but found low realizations of ϵ , i.e., a case of "phantom search." If the firm does not have an ability to monitor worker's search activity, the firm would not ask the question, but modify the contract to prevent the possible loss from insurance. In other words, "moral hazard" in search activities has to be prevented.¹⁴

Let us define a "third-best" contract by one which is written taking into account firm's inability of knowing search intensity (all or nothing) as well as realizations of outcomes of search (ϵ) . In

order to solve for the third best contract, the expected utility (4.8) is maximized with the zero expected profit condition and the "incentive compatible" search constraint:

$$u(m(b)+c(R)) \leq ru(m(b)+c(R)-z) + \int_{v^*(b)}^v (1/2v) u(a+\epsilon+c(q)-z) d\epsilon \quad (5.2)$$

Denoting the solution to the third best contract by $(\bar{c}(G), \bar{c}(R), \bar{c}(q))$, or in the contract notation, $(w(G), w(B), c(q))$, where $w(G)=m(B)-c(G)$, $w(B)=m(B)+c(G)$, $c(Q)=c(q)$, the first-order condition corresponding to (4.12) of the second-best contract is, for $\mu \geq 0$,

$$\begin{aligned} u'(y(g)) = & ru'(m(b)+c(R)-z) + \int_{v^*(b)}^v (1/2v) u'(a+\epsilon+c(q)-z) d\epsilon \\ & + \mu [-u'(m(b)+c(R)) + ru'(m(b)+c(R)-z) + \int_{v^*(b)}^v (1/2v) u'(a+\epsilon+c(q)-z) d\epsilon], \end{aligned} \quad (5.3)$$

where μ is the (Kuhn-Tucker) Lagrangean multiplier for the constraint (5.2). First, observe that condition (5.1) is not redundant, by showing that it is binding when a_j is barely high enough to justify search in the the second-best contract, i.e., the threshold of search, BB in Figure 4. Second, the property of the third-best contract in relation to the second-best contract will be discussed.

Theorem 5.1

Suppose that the utility function has the property of decreasing absolute risk aversion. For the worker with $a_j = \bar{a}_j$, the second best optimum has the property of (5.1), i.e., violates the incentive compatibility of search.

Proof of Theorem 5.1 is given in Appendix. By the continuity

of the utility function, workers with a_j close enough to $\bar{a}$, given $z > 0$, would find the incentive compatibility constraint to be binding. However, if search is costless ($z=0$), then no one loses from searching and (5.2) is automatically satisfied, i.e., those workers with a_j high enough to make them search in the second-best contract should also search in the third-best contract. Combining this observation with Theorem 5.1, we have the threshold of search with the third-best contract BC as drawn in Figure 4.

Now, the third-best contract can be characterized. Since staying in the firm should be discouraged, the difference between insurance coverages of staying $c(R)$ and quitting $c(q)$ should be narrower in the third-best than in the second-best contract (noting that $c(R) > c(q)$ in the second-best contract).

A question is how $c(q)$ or $c(G)$, or both increases as the incentive compatibility constraint becomes binding. In particular, we are interested in knowing and whether workers are more likely to quit in the third-best contract.

Theorem 5.2 [Too Few Search]

Suppose that the utility function has the property of decreasing absolute risk aversion. The third-best (search incentive compatible) contracts produces less search than the second-best solution, that is, fewer workers are engaged in search in the third-best contract than in the second-best contract.

Proof of Theorem 5.2

By Theorem 5.1, for worker with $\bar{a}_j$, incentive compatibility constraint is binding. Therefore, with the third-best insurance, search would not be better off than the no search option. The logic

here parallels Theorem 4.1.(i), and it is obvious from Figure 4.

Q.E.D.

The above theorem parallels the first part of Theorem 4.1. However, the second part of Theorem 4.1 does not carry over to the third-best contract. For a worker who searches in the third-best solution, it is ambiguous whether they are more probable to stay in the third best solution than in the second best solution.

6. Concluding Remarks

We have shown optimal implicit contracts in an environment where search intensity and outcomes are not verifiable by the firm. When the severance payment cannot be contingent on the outcomes of the search for alternative wages, perfect risk-sharing is not possible. Under the second-best insurance, fewer workers are shown to search and quit. When the firm cannot verify whether a worker carries out the search activity in addition to search outcome, even the second-best contracts are not viable, if the utility is of decreasing absolute risk aversion. The third best contract is calculated with the incentive compatibility constraint preventing a "phantom search."

Key results are summarized as follows: informational constraints would distort workers allocation toward too few severance. Workers are induced to stay in the low-productivity firm: fewer (types of) workers are engaged in search with the second-best contracts than the first-best contracts; and given searching, workers with the second-best contracts are more likely to choose to stay than those with the first-best contracts. Similarly under the third-best contracts fewer workers are engaged in search than in the second-best contracts. However, it is ambiguous whether the probability of staying at the

firm, given the search is made, is greater in the third-best contract than the second-best contract.

Although our main purpose is to derive theoretical predictions of informational asymmetry, there are obvious policy implications of results obtained in this paper. If search activities and outcomes of workers are better monitored by an outside agency (the government or a consortium of firms) rather than the individual firm, then unemployment compensations should be managed by the outside agency instead of private companies. The government center for matching workers for another jobs, on the one hand monitoring search activity and on the other hand subsidizing to pay for z , might improve allocations.

One might want to stretch implications of this model a little further. The model implies that if several firms which are subject to different technology or demand shocks, i.e., in different industries, have an incentive to form a "group" in which workers are transferred from one firm to another depending on which firm is in the good times. Since the firms, within a group, can monitor the worker's alternative wage and search/moving costs, they can offer a better contract to workers than otherwise, given that workers agree to a job assignment in a different firm. There are several groups of big firms in Japan, such as Mitsubishi, Mitsui and Sumitomo, which are known to transfer workers within each group. The so-called lifetime employment in Japan is only possible with an arrangement that workers may be transferred to other firms within a group when the originally contracted firm is under the bad times.

One may wonder how simplifying assumptions in this paper could be generalized: a linear production function, a trivial search

technology, and a univariate utility function. A number of extensions toward more general cases obviously are possible. First, components of the utility function can include leisure during layoff. Cooper (1983) and Kahn (1985) show, in different context, that some results are sensitive to the specification of the utility function. Second, search technology in this paper has been treated as being either "on" or "off." More realistic the modelling of search activity would call for a continuous choice variable for search intensity. With a continuously variable search intensity, the moral hazard problem, which appears as the search incentive constraint in this paper, will take a more realistic form. These considerations are left for future investigations.

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Footnotes

1. Of course, there are various reasons, other than considered here, why productive inefficiency in the form of too few severance might arise. First, different kinds of human capital (skills) may be required in different industries, and it requires time to retrain. However, the difference in human capitals are embedded in the value of (net) alternative opportunity for a worker in the present model. Second, if the firm and worker cannot determine whether a productivity shock is temporary or permanent, it is difficult to characterize an optimal separation of workers.

2. Surveys of the literature include Azariadis and Stiglitz (1983), Hart (1983), and Ito (1982).

3. An emphasis on "too few severance" here is at odds with a notion of "excess sensitivity of layoffs and quits" to demand by Hall and Lazear (1984). Note that in the framework of this paper, if the alternative wages are deterministic, which Hall and Lazear maintain, the efficient separation is a result (section 3 of this paper). On the other hand, in Hall and Lazear's framework, unilateral asymmetric information, which this paper assumes, will yield efficient solution (Hall and Lazear (1984; pp. 253-254). Thus, each of the two models are internally consistent, while difference results reflect what authors think important in the labor market.

4. A more realistic concave production function is considered in Geanakoplos and Ito (1982) which emphasizes involuntary unemployment among homogeneous workers unlike this paper. Since workers' decision

problem is the focus of this paper, the technology of production and firm's behavior is simplified as much as possible. There are, however, marked differences between Geanakoplos and Ito (1982) and this paper. The former paper needs a layoff mechanism as opposed to a voluntary quit mechanism, because of the marginal productivity and homogeneous (even with respect to a_j) workers. Therefore, the former paper is subject to a paradox described in Section 4.3 in this paper, questions asked in Section 3 of this paper are not present. On the other hand, the former paper constructed a general equilibrium framework, where the difference in ϵ_j is generated by compatibility of skills between a worker and (ex ante) unknown booming industry.

5. The problem of enforcement is an interesting and important question in implicit contract theory. If the firm is bound to honor the contract, but not workers, then a two-period model a la Holmstrom (1983), and a multi-period model a la Harris and Holmstrom (1982), can be applied to implement an enforcement mechanism with characteristics of this paper. See also Ito (1984) for results in the case of declining degree of worker's mobility.

6. In fact, interpreting implicit contracts as the sum of the spot market solution is not new. See Akerlof and Miyazaki (1980) and Harris and Holmstrom (1981) for earlier remarks on this point. In the latter paper, the utility is maximized subject to the constant expected profit. This is equivalent to assuming that spot market wages and fair insurance policies are available to consumers.

7. This result also can be viewed as an application of the so-called revelation principle in a direct mechanism in the allocation of incomes between the firm (principal) and workers (agents) in the

sense of Myerson (1979), or of the efficient mechanism in the sense of Holmstrom and Myerson (1983).

8. How U_j is determined in the implicit contract model of Azariadis (1975) is illustrated in Ito (1982).

9. See Akerlof and Miyazaki (1979), Sargent (1979; chapter 8), Geanakoplos and Ito (1982), and Ito (1982) for pointing out and demonstrating a features of contract with severance payments in the Azariadis-type model.

10. The difference in the wage among homogeneous workers within the firm in the good state of nature can be understood as a reflection of the differences in the insurance premium. We observe that professors in different fields command different salaries. Those whose skills are easily adopted outside a university environment, e.g., those in finance and medicine, tend to command higher salaries than others. One of the reasons for this differential treatment is that those with "practical" skills would quit a university with a small severance payment at the time of university budget crunch.

Some people may still wonder whether we actually observe the theoretical prediction of differential payments reflecting the different size of premia. It could be the case that each firm employs only one type (a_j) of workers. Then those firms with less fringe benefits (pension and severance pay plan) pay higher wages than those with higher fringe benefits.

11. Note that if $v < z$, then the threshold of search of this (complete information, stochastic wage) case is the same with the one for the deterministic case. The reason is that if the expected gain

exceeds the search cost, then $a_j - v$ would be already above $m(B)$, that is everybody quits once search is conducted even for the worst realization. It is not possible to insure the lower end of the ϵ , because everybody quits.

12. One might think that the firm could know the outcome of search by requiring workers to submit letters of job offers after the search. However, a worker would have an incentive to arrange a (fraudulent) letter stating an alternative wage which is "lower" than the truth, but high enough to make the worker quit. The worker could thus collect higher severance payments than the optimum level. It would be prohibitively costly to verify whether letters are authentic or fraudulent. In this point, I tend to agree with Hall and Lazear (1984; pp. 247-248).

13. One might question the existence of corresponding institution of the incentive pay to stay, especially as coexisting with the severance payments. First, this may not have to be explicit in the labor contract. As will be shown shortly, the contract wage is a sum of the spot wage, $m(s)$, and the subsidy, $c(R)$. Thus the incentive pay to stay is the portion of the contract wage in excess of the marginal products. Second, we can implement the following sequential mechanism which serves the purpose. If the state of nature is bad, then $\min[c(r), c(q)]$ is paid as an insurance policy with respect to the state of nature. Then an additional subsidy, $|c(R) - c(q)|$, is paid to a returning (quitting, resp.) worker, if $c(R)$ is larger (smaller, resp.) than $c(q)$. It will be shown that $c(q) < c(R)$, so that the coverage for those who stay, $c(R)$, may seem to be decomposed, according to the two-step implementation, into an insurance coverage for

the state of nature, $s=B$, $c(q)$ and an incentive pay to stay, $c(R)-c(q)$. However, the point here is that in this environment, we cannot separate insurance for the state of nature from insurance for search.

14. Although the moral hazard problem associated with search behavior due to severance payments has been recognized in the literature, such as Baily (1980) and Stiglitz (1984), no implicit contract model, to my best knowledge, has been developed to investigate the moral hazard problem associated with search for alternative opportunities. A correct way to study such a model is to devise the incentive-compatible severance payments and to let workers choose (voluntarily) to quit as will be demonstrated in the next section.

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APPENDIX

Proof of Theorem 4.1:

(i) At the threshold in the perfect information case, a_j^* , complete insurance with respect to search as well as states of nature is available. The second-best insurance policy does not provide complete coverage against search outcomes. An increase in variance of incomes across the state of nature has to be compensated by increase in the mean. However, the constraint of fair insurance makes it impossible to do so without decreasing the income in the good state of nature. Thus, for workers at the threshold of search of the first-best contract, search would not pay off with the second-best contract.

(ii) Recall that worker's decision between returning to the original firm and quitting:

(first best) Quit if and only if $m(b) < a_j + \epsilon_j$.

(second best) Quit if and only if $m(b) + \bar{c}_j(R) < a_j + \epsilon_j + \bar{c}_j(q)$.

Thus, it suffices to show that the insurance coverage for the re-turning worker, $c(R)$, is larger than the insurance coverage (severance payments) of quitting workers, $c(q)$. That is, the second best solution distorts the incentives expressed in a combination of $c(R)$ and $c(q)$ toward favoring staying at the firm, rather than to let a worker quit, because the quitting workers are better off than re-turning workers.

Note that the optimal stay-quit decision after ϵ is known implies that the utility of returning is less than the utility of quitting for the realization of ϵ_j . It implies $u(\bar{y}(R)) < u(\bar{y}(q))$

for any $\epsilon > m(b) + \bar{c}(R) - \bar{c}(q) - a$. Therefore, taking the marginal utility yields $u'(y(R)) > u'(y(q))$ for any $\epsilon > m(b) + \bar{c}(R) - \bar{c}(q) - a$. From (4.12), this implies that the value of the left-hand side of (4.12) should be less than the upper bound of u' in the right-hand side, i.e., $u'(\bar{y}(R))$. Thus, $u'(\bar{y}(g)) < u'(\bar{y}(R))$. This inequality implies, in (4.11), $c(R) > c(q)$. Q.E.D.

Proof of Theorem 5.1:

By the definition of the threshold level of alternative wage, a worker with $a_j = \bar{a}_j$ is indifferent between a search-contract and a non-search contract:

$$u(m(b) + \bar{c}) = pu(m(g) - c(g)) + (1-p)[ru(m(b) + c(R) - z) + (1-r) \int (1/2v)u(.)d\epsilon] \quad (5.4)$$

$$\text{where } m(b) + \bar{c} = m(g) - ((1-p)/p)c. \quad (5.5)$$

By Theorem 4.3, we know that the right-hand side of (5.2) is larger than the utility in the good state of nature which does not involve search:

$$u(m(b) + c) > u(m(g) - c_j(g)) \quad (5.6)$$

where, by (4.5),

$$\bar{c}_j(g) = ((1-p)/p)(r\bar{c}_j(R) + (1-r)\bar{c}_j(q)). \quad (5.7)$$

Therefore, substituting (5.5) and (5.7) into (5.6), we find

$$m(g) - ((1-p)/p)\bar{c} > m(g) - ((1-p)/p)(r\bar{c}_j(R) + (1-r)\bar{c}_j(q)).$$

By subtracting $m(g)$ from both sides and multiplying (-1) on both sides,

$$\bar{c} < r\bar{c}_j(R) + (1-r)\bar{c}_j(q), \quad 0 < r < 1.$$

Recall the result of Theorem 4.1, i.e., $\bar{c}_j(R) > \bar{c}_j(q)$. Thus,

$$\bar{c}_j(R) > \bar{c} > \bar{c}_j(q). \quad (5.8)$$

Taking the first inequality of (5.8), we find

$$u(m(b) + \bar{c}_j(R)) > u(m(B) + \bar{c}). \quad (5.9)$$

Combining (5.4) and (5.9),

$$u(m(b) + \bar{c}_j(R)) > (1-p)[ru(m(b) + c(R) - z) + (1-r) \int (1/2v)u(.) d\epsilon.$$

violating the incentive compatibility of search.

Q.E.D.

Figure 1 [Deterministic Case]

Figure 2
[stochastic case]

FIGURE 3: EVENT AND DECISION TREE

Workers with high a_j

Workers with low a_j

Figure 4

a^* , defined in equation (4.2), is illustrated by AA

$\bar{a}$, defined in equation (4.7), is illustrated by BB

I: Search if and only if the first-best contract is available

I_A: Search and sometimes quit (for high ϵ)

I_B: Search and always quit

II: Search if the second/first-best contract is available, but no-search if only the third-best contract is available.

III: Search if the third/second/first-best contract is available.