A MODEL OF RENTAL AND OWNER-OCCUPIED HOUSING

by

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This paper develops a partial equilibrium model of housing markets with special attention to the division of the housing stock between rental and owner-occupied housing. This work is motivated by my interest in developing a simple but complete model of the housing market that can simultaneously deal with what appear to be the stylized facts of the impact of inflationary expectations on housing markets, namely a substantial increase in the relative price of structures, a decline in the user cost of capital for owners, a decline in rents relative to prices in general, and an increase in the incidence of homeownership.

Much recent work has examined the impact of inflation on housing markets. For example see Hendershott, Hendershott and Hu, Villani, and Dougherty and Van Order. For the most part this work has concentrated on the implications of continuing inflation for the user cost of owner-occupied housing and has ignored links to rental housing. This paper, while drawing on that work, extends the analysis to include the endogenous determination of rents, the user cost of owner-occupied housing and housing tenure choice. The structure of the model is similar in spirit to recent work by Feldstein and Litztenberger and Sosin.

After first describing the model, the short and long run impacts of changes in exogenous variables, including the expected rate of inflation, are considered. The difference between the short and long run arises because of the time lag necessary to add to the stock of housing capital. Short run impacts of changes in exogenous variables are derived by holding the total stock of housing capital constant, but not the division between

rental and owner-occupied housing. Long run impacts are derived by holding construction costs constant and letting the quantity of housing adjust. Fixing construction costs is equivalent to an infinitely elastic long run supply curve of new housing units. In the short run, impacts of changes in exogenous variables are concentrated on prices and tenure choice. Changes in structure prices, when contrasted with construction costs, then give rise to net investment. Long run equilibrium is re-established when there is no longer any incentive to change the housing stock.

THE MODEL

The approach here is partial equilibrium in that, while demand and supply functions are specified for rental and owner-occupied housing, broader macroeconomic variables such as income, interest rates, the price of nonhousing goods, and expectations about the rate of inflation, are exogenous to the model. A number of important issues are not addressed in the formal model, including transactions costs and cash flow constraints from traditional forms of mortgage debt. It is also assumed that there is no physical depreciation of houses. The assumption is for analytic simplicity and has no effect on the qualitative results.

Before proceeding it will be helpful to define a number of terms. The term housing stock will refer to the physical stock of capital that provides housing services. The term unit of housing refers to the arbitrary amount of housing capital that is used to measure the housing stock. Thus one unit of housing is by definition equal to any other unit. Two

different houses on a block, often called housing units in everyday language, will differ here because they have a different number of units of housing capital.

The term housing will refer to the flow of shelter services from the housing stock. These services are provided, via a housing production function, from the stock of houses. It is assumed that one unit of housing capital, along with technologically determined operating costs, provides one unit of housing services.

The price of housing services depends upon the form of housing tenure. Renters pay an explicit market rent, denoted by R , for a unit of housing services for one period. For owner-occupants the price of a unit of housing services is an implicit, Jorgensonian user cost of capital.

Structure prices, P_H , refers to the current market price of a unit of the housing stock. All units of housing capital, regardless of their tenure classification, are assumed to sell for the same price. The price of individual houses or apartment buildings will differ as they contain different amounts of housing capital.

THE DEMAND FOR HOUSING

Utility maximizing behavior by individuals gives rise to a demand for housing that depends upon real income and the relative price of housing. It is assumed that individuals have no inherent preference for renting or owning. They are assumed to choose to own or rent on the basis of the market price of renting and the implicit after-tax cost of owning. Individuals choose the lowest cost form of housing. Differences in the quantity of housing services consumed by owners and renters reflect

differences in both incomes and after-tax prices.

The after-tax cost of homeownership is given by a risk adjusted Jorgensonian user cost of capital expression, equation (1).¹ Equation (1) incorporates the following assumptions: interest payments are tax deductible; income in kind from occupying one's home is not taxed; and capital gains on owner-occupied housing are tax free. Equation (1), and equation (6) below, assume that expectations of period by period capital gains are relevant to measures of user cost and rates of return. In actuality, the realization of capital gains is often postponed. Equations (1) and (6) implicitly assume that any such postponement does not affect the housing and investment decisions of owners and landlords.

As written, OCC may well be negative with appropriately large values for expected capital gains, g^e , and the marginal tax rate, t . In the absence of satiation, the demand for housing would then be infinite. One might either restrict the value of arguments in (1) or assume that the risk premium, ψ^o is positively related to π^e and g^e in a way that OCC is never negative.²

$$(1) \quad OCC(t) = [(1 - t)[i + \pi^e] + \psi^o - g^e]P_H + 0^o$$

where OCC - after-tax owner cost of capital;

t - marginal tax rate;

i - real interest rate;

¹For example, see Dougherty and Van Order who derive an expression analogous to equation (1) from both a consumption and investment viewpoint.

²See Feldstein 1979 and 1980 for discussion of a similar concern.

π^e - expected rate of inflation;

$i + \pi^e$ - nominal interest rate;

ψ^o - risk premium for owner-occupied housing;

g^e - expected rate of capital gains on structures;

P_H - market price of a unit of housing capital;

O^o - owner's operating costs.

Of the variables included in equation (1), OCC and P_H are endogenous to the model. All others are exogenous. OCC refers to the implicit price for owners of a single unit of housing services. A household's total housing costs will depend upon the number of units of housing services it chooses to consume. Interest, risk and capital gain elements are proportional to structure prices. Operating costs are assumed to be given by technical factors and need not be proportional to P_H . For example a change in P_H that reflected short run adjustment to a change in expected capital gains would not be expected to change operating costs.³

The cost of renting is simply the market rent, R , and is unaffected by an individual's marginal tax rate. Individuals decide to own or rent by comparing their own OCC and market rents.

Differentiating equation (1) shows that the cost of owning is lower for individuals with higher marginal tax rates. Assuming that there are both renters and owners, there will be a tax rate, and an associated owner cost of capital, at which individuals are indifferent between owning and

³Property taxes are not explicitly included in (1). Property taxes are tax deductible and are, except for lags in assessment, proportional to prices. Explicit inclusion of property taxes is a straightforward extension of equation (1).

renting. This tax rate is found by setting OCC from equation (1) equal to the market rent and solving for the marginal buy/rent marginal tax rate.

$$(2) \quad OCC(t_1) = R$$

where R - market rent for a unit of housing services;

t_1 - marginal buy/rent marginal tax rate.

All individuals with incomes such that their marginal tax rate is less than t_1 will be renters. All individuals with incomes such that their marginal tax rate is greater than t_1 will be owners. Changes in t_1 will signify a change in the incidence of homeownership as measured by the proportion of individuals who own rather than rent.

Once individuals have chosen to own or rent, their individual demand for housing will depend upon their income and the relevant price of housing. The total demand for owner and rental housing comes from aggregating across individuals above and below t_1 and is shown in equations (3) and (4).

$$(3) \quad H^O = \sum_{t > t_1} h[Y(t), OCC(t)]$$

$$(4) \quad H^R = \sum_{t < t_1} h[Y(t), R]$$

where $h(\cdot)$ - individual demand for housing, $h_1 > 0$, $h_2 < 0$;

H^O - aggregate demand for owner-occupied housing;

H^R - aggregate demand for rental housing;

$Y(t)$ - individual incomes.

Equations (3) and (4) represent aggregate demands for housing services.

Since a unit of housing services requires one unit of the housing stock,

equations (3) and (4) also represent aggregate demand for the housing stock, directly in the case of owner-occupants and indirectly in the case of renters.

THE SUPPLY OF HOUSING

The supply side of the model consists of equations that describe the behavior of landlord-investors, the suppliers of rental housing, equations (5), (6) and (7), and an equation that fixes the stock of houses, (8a), or the structure price of a unit of the housing stock, (8b).

$$(5) \quad ATR(t) = (1 - t)(R - O^R) + [tD(i + \pi^e) + (1 - \alpha t)g^e]P_H$$

where ATR - after-tax return from rental property;

O^R - operating expenses per unit of rental housing;

$D(\)$ - annuity equivalent after tax value of depreciation allowances, $D_1 < 0$;

α - ratio of marginal capital gains tax rate to marginal tax rate, $0 < \alpha < 1$.

$$(6) \quad ATR(t_2) = [(1 - t_2)(i + \pi^e) + \psi^R]P_H$$

where ψ^R - risk premium for investments in rental property;

t_2 - marginal landlord marginal tax rate.

$$(7) \quad H^R = S(t_2)$$

where S - supply function, $S_1 < 0$.

$$(8a) \quad H^0 + H^R = \bar{H}$$

$$(8b) \quad P_H = \bar{P}_H$$

Equation (5) defines a landlord-investor's expected after-tax nominal return from investing in rental housing, a return which is composed of rental income minus operating expenses plus the after-tax value of depreciation allowances and the expected after-tax value of capital gains.

Depreciation allowances are an important part of a landlord's return as they can be used to postpone taxes, increasing cash flow in the early years of an investment. The term $D(i + \pi)$ stands for the annuity equivalent of the present value of the tax postponement feature of a dollar's worth of depreciation allowances.⁴ This term is multiplied by P_H because depreciation allowances are proportional to the price of a depreciable asset. The after-tax value of depreciation allowances depends upon a landlord's marginal tax rate. By postponing taxes depreciation allowances work like an interest free loan, the present value of which depends upon nominal interest rates. The term $(1 - \alpha t)g^e P_H$ is the after-tax value of expected capital gains, taxed at the preferential tax rate αt .

It is assumed that individuals decide to become landlords after comparing the after-tax returns from investing in rental housing with the risk adjusted

⁴With accelerated depreciation, depreciation allowances vary from year to year. However, in equilibrium, units offering the same quantity of housing must have the same market rent, regardless of when the units were purchased. The impact of depreciation allowances on rents would be the present value annuity equivalent to the after tax impact of actual depreciation allowances. The $D(\cdot)$ term is this annuity equivalent.

after-tax returns from alternative investments, represented here by the after-tax return on a riskless bond, $(1 - t)(i + \pi^e)$.⁵ In the absence of financing limitations, a single landlord could supply all rental housing. With limitations on the amount of rental investments from particular individuals - either external, for example, downpayment constraints, or internal, for example, portfolio balance considerations - an expansion of the rental housing stock will require new landlords, typically with lower marginal tax rates, equation (7).

In equilibrium the risk-adjusted after-tax yield on rental housing will equal the after-tax yield on alternative investments for the marginal landlord, equation (6). An increase in t lowers the return on both bonds and rental houses. However the decline in the return to rental housing will be less than that on bonds, a result which establishes the superiority of rental housing as an investment option for high income individuals and a tax rate, t_2 , such that landlord-investors will all have marginal tax rates greater than or equal to t_2 . (See Appendix A for a further discussion of this issue.) Individuals with lower marginal tax rates will find that riskless bonds will offer them a more attractive return.

SHORT RUN

The short run version of the model consists of equations (1), (2), (3), (4), (5), (6), (7), and (8a). Endogenous variables are OCC , t_1 , H^O , H^R , R , ATR , t_2 , and P_H . While H^O and H^R , the stocks of

⁵In keeping with the partial equilibrium character of the analysis, the model could be easily extended to include a number of other investment alternatives, some or all of which may have favorable tax treatment. The result of such an extension would be to narrow the range of marginal tax rates for landlord-investors.

owner and rental housing, can change in the short run, the total housing stock, $H^O + H^R$, is fixed at $\bar{H}$. The solution to the model proceeds by considering subsectors of the model and solving summary representations of the subsectors simultaneously.⁷

The Aggregate Demand for Housing

Since the total stock of housing is fixed in the short run, equations (3) and (4) can be aggregated to yield a single relationship involving R , P_H , and t_1 . That is, in the short run, one or more of these three variables must adjust so that the aggregate demand for housing equals the aggregate supply. A small change in t_1 means that on the margin some individuals have switched between owning and renting. However, there will be no change in the aggregate demand for housing following this switch. At t_1 individuals find the cost of owning and renting equal and will demand the same quantity of housing regardless of tenure. Thus, a decision by these individuals to change their form of housing tenure will not change their demand for housing or the aggregate demand for housing. Since small changes in t_1 do not affect aggregate housing demand, R and P_H must adjust to see that aggregate demand equals aggregate supply.

In Figure 1 the DD curve shows combinations of R and P_H that leave the aggregate demand for housing unchanged. This relationship has a negative slope. Consider a lower R that increases the demand for rental housing and induces individuals previously on the margin to switch from owning to renting. The switch from owning to renting will not affect

⁷See Appendix B for a mathematical solution involving total differentiation.

the aggregate demand for housing, but the reduction in R will increase the aggregate demand for housing as it increases demand by intramarginal renting households. An increase in P_H is necessary to reduce the demand for owner-occupied housing and keep the aggregate demand for housing unchanged. The increase in P_H will increase the cost of owning. Again tenure switching at the margin will not affect aggregate demand, but the response by intramarginal owning households will.

The Rental Market

A second relationship between R and P_H comes from equilibrium in the rental market. Consider first the supply of rental housing. Substituting equation (5) into (6) results in a relationship showing how R , P_H and t_2 must adjust so that returns on investments in rental housing offer a competitive return to the marginal landlord. Equation (8) can now be used to eliminate t_2 . The result is the supply of rental housing. This supply curve shows combinations of rents, structure prices and the rental housing stock consistent with equilibrium landlord behavior. Holding structure prices constant, this supply curve slopes up, as seen in Figure 2a. A larger rental housing stock requires additional landlord investments which, according to equation (8), will come only from individuals with lower marginal tax rates. Individuals with lower marginal tax rates will, at constant structure prices, be induced to invest in rental housing only if rents rise. Any change in P_H will shift the supply curve. An increase in P_H shifts the supply curve up.

A second relationship between R , P_H and H^R can be derived from the behavior of renters. In equation (4) the demand for rental housing is seen to involve H^R , R and t_1 . As contrasted with the aggregate demand for housing, changes in t_1 are important for the demand for

rental housing as t_1 determines the number of renters. Equations (1) and (2) allow one to solve for t_1 as a function of R , P_H and exogenous variables. Substitution of this result into equation (4) yields the demand for rental housing as a function of R and P_H . This demand curve, which allows for changes in t_1 as R changes, is shown in Figure 2b for a given P_H . It has a negative slope as, at constant structure prices, any increase in R induces some individuals to switch to owning, and reduces demand by intramarginal renters. An increase in P_H will increase the demand for rental housing, shifting the demand curve to the right.

Figure 2c shows overall rental market equilibrium. It is clear from Figure 2c that overall rental market equilibrium involves a positive association between R and P_H . However, there is no presumption as to changes in the stock of rental housing at different rental market equilibrium combinations of R and P_H .

Overall Equilibrium

Overall equilibrium is found at the one combination of R and P_H that constrains the aggregate demand for housing services from Figure 1 and is consistent with rental market equilibrium in Figure 2c. This equilibrium can be seen graphically in Figure 3. The DD curve comes from Figure 1. The RR curve represents equilibrium in the rental market and comes from points such as E_1 and E_2 in Figure 2c.

Comparative Statics

To get a feeling for the short run implications of the model, consider a number of changes in exogenous variable. (Table 1 summarizes the fol-

lowing results.)

- A liberalization of depreciation allowances for rental property, that is an increase in D , shifts the RR curve to the right while leaving the DD curve unaffected. Rents fall while structure prices increase. Rents fall as more liberal depreciation allowances induce more individuals to become landlords. That is, in Figure 2c the RS curve shifts to the right. It is this shift in RS that shifts RR in Figure 3. Structure prices rise as landlord-investors compete to buy housing units to rent in order to take advantage of the change in depreciation allowances. The decline in R and increase in P_H imply an increase in t_1 , that is some owners switch to renting as it is now cheaper. More landlord-investors are needed, t_2 must fall, to meet the increased demand for rental housing by both new and intramarginal renters. Similar results would follow from a reduction in capital gains taxation on rental housing or a decrease in the operating cost of rental property.

A decrease in operating costs might come from a generalized subsidy to landlord-investors to, say, "help low-income renters".⁸ It may seem surprising that such a subsidy is not fully capitalized in structure prices, given that structures are in inelastic supply in the short run. The reason is that while all structures are in inelastic supply, the supply of structures to the rental sector of the market is not inelastic. Ceteris paribus, increases in structure prices will induce owner-occupants to give up some (intramarginal owners) or all (marginal owners) their housing capital to the rental sector. It is the supply of housing capital from intra-marginal owners that allows rents

⁸In keeping with the partial equilibrium character of the model, the discussion ignores possible impacts on income from financing such a subsidy.

Table I

Summary of Short-run and Long-run Impacts

Event		P_H	H	R	Variable t_1	t_2	H^0	H^R
Increase in D or α	Short-run	+	0	-	+	-	-	+
Decrease in O^R	Long-run	0	+	-	+	-	-	+
Increase in O^0	Short-run	-	0	-	?	- (?)	- (?)	+
	Long-run	0	-	+	+	-	-	+
Increase in i	Short-run	-	0	?	-	?	?	?
	Long-run	0	-	+	?	?	-	?
Increase in π^e and g^e	Short-run	+	0	+	- (?)	+	+	- (?)
	Long-run	0	+	-	-	?	+	?

to fall and prevents full capitalization of the subsidy.

- An increase in operating costs for owner-occupants, O^0 , shifts both the DD and RR curves to the left as illustrated in Figure 4. The shift in the DD curve comes from the direct impact of operating costs on the demand for owner-occupied housing, and, thus, the aggregate demand for housing. The increase in O^0 makes owning more expensive than before. While the switch to renting by individuals on the buy/rent margin will have no impact on the aggregate demand, the impact of the increase in O^0 on intra-marginal owners will reduce the aggregate demand for housing, shifting the DD curve to the left. The shift in the RR curve comes from the switch by individuals on the margin. In Figure 2c the RD curve shifts to the right implying a shift in the RR curve. The result is an unambiguous decline in P_H with what appear to be potentially ambiguous impacts on R , H^R , H^O , t_1 and t_2 .

Part of the ambiguity can be resolved by considering the horizontal shifts in both DD and RR following the increase in owner operating costs. Consider first the change in structure prices that would just offset the increase in operating costs for individuals who are on the margin of owning or renting, that is households with marginal tax rate t_1 . Call this change the reference change in structure prices. It is represented in Figure 4 as point A. It turns out that at the original level of rents, the horizontal shift in the DD curve is more than this reference change and the horizontal shift of the RR curve is less. As a result, R , as well as P_H , declines.

Consider first the shift in the DD curve. From inspection of equation (1) it is clear that the impact of a change in operating costs

on the cost of homeownership is independent of marginal tax rates while the impact of a change in structure prices is not. In particular the impact of a change in structure prices on the cost of owning declines with increases in the marginal tax rate. Thus, if following an increase in owner operating costs, structure prices decline just enough so that marginal households continue to own, then higher income households will find that, on net, their cost of owning has increased. For them, the decline in structure prices is not sufficient to offset the impact of higher operating costs. These intramarginal households will decrease their individual demand for housing, depressing the aggregate demand for housing. Therefore the decline in structure prices necessary to prevent any decrease in the aggregate demand for housing, point B in Figure 4, must be greater than the reference change in structure prices.⁹

Now consider the shift in the RR curve following the increase in 0° . If the decline in structure prices equals the reference change, there will, by definition, be no shift in the demand curve for rental housing. But the decline in structure prices will shift the supply curve for rental housing to the right. Thus the reference change in P_H is consistent with rental market equilibrium only if rents fall. That is the RR curve must pass through a point like C to the right of point A.

Following the shifts in DD and RR the new short run equilibrium shows declines in both R and P_H . The change in the incidence of homeownership, that is the change in t_1 , is still ambiguous. If the decline

⁹One can, at this point, ignore possible changes in rental demand because points A and B hold rents constant. As a result, there is no change in the demand for housing by intra-marginal renters. At point B, there will be an increase in the incidence of homeownership, but this switch will not affect the aggregate demand for housing.

in P_H is less than the reference change, t_1 increases, that is the incidence of homeownership declines. If the decline in P_H is sufficiently greater than the reference change, t_1 will decline.¹⁰ The marginal landlord-investor marginal tax rate, t_2 , must decrease as long as t_1 does not decrease since existing renters will increase their demand for rental housing following the reduction in rents.

It may appear surprising that rents decline following the increased demand for rental housing induced by the increase in owner operating costs. But the increased demand for rental housing comes with its own increase in supply as those making the switch to renting are simultaneously selling their homes. With the increase in operating costs affecting intramarginal owners as well, there is a net shift of housing capital out of the homeownership sector into the rental sector that exceeds the increase in the demand for rental housing.

- An increase in real interest rates, i , affects both the DD and RR curves. The increase in i increases the cost of homeownership, reducing the aggregate demand for housing because of its impact on intramarginal owners; the DD curve shifts to the left. The shift in the RR curve comes from shifts in both RD and RS. The shift from owning to renting, following the increase in the cost of homeownership, shifts the RD curve to the right. The increase in the return on alternative investments shifts the RS curve to the left. The result is that, at the original level of structure prices, equilibrium in the rental market re-

¹⁰ See the discussion of the $t_1 t_1$ curve in Figure 7 and footnote 13 for a consideration of factors that affect the incidence of homeownership and t_1 .

quires higher rents. As a result the RR curve shifts up. The result is an unambiguous short-run decline in P_H and some uncertainty about the impact on R and other variables.

As before, it is useful to compare the horizontal shifts in DD and RR to a more common reference. Again consider a reference change defined as that change in P_H that leaves the buy/rent decision, equation (2), unaffected following the increase in i . To consider the shift in the RR curve it is useful to first rewrite equations (2) and (6) in terms of rates of return.

$$(2') \quad (1 - t_1)(i + \pi^e) = \frac{R - 0^o}{P_H} + g^e - \psi^o$$

$$(6') \quad (1 - t_2)(i + \pi^e) = \frac{(1 - t_2)(R - 0^R)}{P_H} + t_2 D + (1 - \alpha t_2)g^e - \psi^R$$

From (2'), the reference change in P_H , following an increase in i , is

$$(9) \quad dP_H^* = \frac{-(1 - t_1)P_H^2}{R - 0^o} di.$$

This change in P_H would imply no shift in the RD curve. The net horizontal shift in the RR curve, relative to dP_H^* , then depends upon the impact of di and dP_H^* on the RS curve.

From (6') the shift in the RS curve, more or less than dP_H^* , depends upon whether

$$(10) \quad (1 - t_2)di + \frac{(1 - t_2)(R - 0^R)}{P_H^2} dP_H^*$$

is greater or less than zero. If expression (10) is greater (less) than

zero, then the increase in the return on riskless bonds from the increase in i exceeds (is less than) the increase in the return on rental housing from the reference change in P_H . Substituting from (9) for dP_H^* yields

$$(1 - t_2)di + (1 - t_2)\frac{(R - O^R)}{P_H^2} \cdot \left(\frac{-(1 - t_1)P_H^2}{R - O^0} \right) di =$$

$$(1 - t_2) \left[1 - (1 - t_1) \frac{R - O^R}{R - O^0} \right] di .$$

If rental operating costs are not less than owner operating costs then expression (10) is greater than zero and the RR curve shifts more than dP_H^* . That is, with no change in R , P_H must decline by more than the reference change in order that rental housing provide a competitive return.

As before, the shift in the DD curve depends upon the impact of di and dP_H^* on intramarginal owner-occupants. Some straightforward manipulations show that the impact of di and P_H^* on the cost of owner-occupied housing for intramarginal owners is

$$(11) \quad \frac{dOCC(t)}{di} = \frac{(t - t_1)(g^e - \psi^0)P_H}{(1 - t_1)(i + \pi^e) + \psi^0 - g^e} .$$

If, as is likely, $g^e > \psi^0$ then the cost of owning increases for intramarginal households and the horizontal shift in the DD curve is also greater than the reference change in P_H .¹¹

The net result of the likely shifts in DD and RR is an ambiguous

¹¹Villani has argued that the specification of equation (1) should be

(1') $OCC = [(1 - t)(i + \pi^e + \psi^0) - g^e]P_H + O^0$. Mechanically the ψ^0 term has been moved inside the parentheses with $i + \pi^e$. Villani argues that $i + \pi^e + \psi^0$ is the relevant nominal interest rate for risk averse consumers or investors. Note that with this specification the risk premium is tax deductible for borrowers and taxable to lenders. Using (i), the term $-\psi^0$ disappears from the numerator of equation (11) and the net impact of a change in real interest rates and the reference change in P_H is to unambiguously increase OCC for intra-marginal owners.

short-run impact on rents and the stocks of both rent and owner occupied housing. At the same time, considerations described in Figure 7 and footnote 13 suggest a short-run increase in the incidence of homeownership. The decline in P_H , greater than the reference change, tends to increase homeownership and a possible decline in rents will not be sufficient to offset the impact of the large decline in P_H .

- An increase in the rate of expected inflation is assumed to increase both π^e and g^e by the same amount. For the moment, hold ψ^o and ψ^R constant. The equal increases in π^e and g^e imply that, after any initial adjustment, all prices, including structure prices, are expected to increase at the same rate. Note that because of initial adjustments, induced by the change in expectations of inflation, some prices may appear to be increasing by more or less than the rate of inflation. For example see Figure 5 where it is assumed that at time $\hat{t}$ there is a change in actual inflation that is fully anticipated. In Figure 5 all prices, including structure prices, are assumed to have been rising at some common rate up to time $\hat{t}$. As shown below, the expectation of a higher rate of inflation will give rise to an instantaneous increase in structure prices, the movement from A to B in Figure 5. After the initial adjustment of structure prices, structure prices will continue to rise at the now higher, but common rate of inflation. Over any finite period of time, say $\hat{t}$ to t^* , it will appear that structure prices have been increasing at a rate faster than the rate of inflation. (In Figure 5 the slope of the dashed line exceeds the slope of the solid line to the right of point $\hat{t}$.)

The increase in the expected rate of inflation shifts both the DD

and RR curves to the right, unambiguously increasing P_H . When considering the DD curve, the incorporation of inflationary expectations into nominal interest rates increases the cost of owning but this increase is more than offset by the expectations of capital gains. Without considering impacts on P_H , the cost of owning declines, increasing the demand for owner-occupied housing by intramarginal owners. As a result structure prices or rents must rise in the short run. The DD curve shifts to the right.

When considering the rental market, the decline in the cost of owning associated with the expectation of more rapid inflation shifts the RD curve in Figure 2c to the left. At the same time the RS curve is likely to shift to the right. The net result is that at the initial level of P_H , rents must fall for rental market equilibrium; that is the RR curve shifts to the right.

Impacts on the supply of rental housing have been qualified by use of "likely" because the expectation of more rapid inflation has both negative and positive impacts on the returns to rental housing. The increase in nominal returns on other assets makes housing a less attractive investment. However, the preferential treatment of capital gains on rental housing more than offsets the increase in fully taxable nominal yields. But an increase in inflationary expectations also lowers the present value of future depreciation allowances making the final result theoretically ambiguous. Simulations reported by deLeeuw and Ozanne suggest that the net result will be to make rental housing a more attractive option, in which case the RS curve shifts to the right. Specifically deLeeuw and Ozanne find that, following an equal change in π^e and g^e , the net present value of a rental project increases somewhat (p. 58).

____ The final impact on rents of the change in inflationary expectations depends upon which curve, DD or RR, shifts further. Again it is useful to start with the reference change in P_H that holds R and t_1 constant.

but now following the increase in expected inflation. The change in π^e , g^e and the associated reference change in P_H imply that intramarginal owning households will experience a net reduction in their cost of homeownership. The net impact of increases in π^e and g^e reduces OCC by larger amounts at higher marginal tax rates, and, as noted above, any increase in P_H has a smaller impact at higher tax rates. Thus the horizontal shift in the DD curve at the original level of R must be more than the reference change in P_H in order to restrict the demand of intramarginal owners who would otherwise see a decline in their OCC and increase their demand for housing.

What about the shift in the RR curve? An analysis similar to that used above to analyze an increase in real interest rates suggests that the horizontal shift in the RR curve relative to the reference change in P_H will depend upon the sign of

$$\left\{ \left[t_1 (1 - t_2) \frac{(R - 0^R)}{(R - 0^0)} + (\alpha - D_1) t_2 \right] - t_2 \right\} d\pi^e.$$

The term in square brackets is positive, but it is theoretically unclear whether it is greater or less than t_2 . The deLeeuw-Ozanne simulations imply that the horizontal shift in the RR curve is less than the reference change in P_H , which in turn implies that the term in curly brackets is positive. The result is a short run rise in rents. Specifically, deLeeuw and Ozanne find that the percentage increase in net present value for their rental project with a marginal tax rate of .5 is less than the percentage increase in net present value for a homeowner with a marginal tax rate of .3 following a simultaneous change in π^e and g^e (p. 57-58).

The short run impact on the incidence of homeownership remains unclear. Rents are higher, implying an increase in homeownership, but one also needs to take account of the impact of the increase in structure

prices. One can determine that there will be a shift towards homeownership unless the change in structure prices is sufficiently in excess of the reference change that the impact of higher structure prices offsets the impact of higher rents. See Figure 7 and its extended comment.

Cash Flow and Risk Consideration

Equation (1) implicitly assumes that individuals are concerned only with real interest rates and are not affected by cash flow problems. An extensive literature on the form of the mortgage contract suggests that high nominal mortgage rates that come from the expectation of substantial inflation may impose significant cash flow problems for borrowers.¹² In this case, even though real interest rates may be unchanged, the immediate cash flow requirements of high nominal interest rates may depress the demand for owner-occupied housing. If this effect is significant, the result would be a smaller shift in the DD curve following an increase in π^e as cash flow concerns hold down demand by owner-occupants. If cash flow constraints are predominantly a problem for owner-occupants rather than landlord-investors, it may be that the shift in the DD curve is less than the shift in the RR curve. The net effect would then be an increase in P_H , a decline in R and an increase in t_1 .

Some have argued that increases in expected inflation may also be associated with increased risk. If one allows for a change in ψ^o along with the change in π^e and g^e , then the impact of increases in π^e and g^e will be moderated by the simultaneous increase in the required risk premium. In particular, the shift in the DD curve would be less than

¹²See for example, Kearl, Poole, Schwab, and Modigliani and Lessard.

without the inclusion of the risk term. The final impact on R and P_H would also depend upon the impact of risk consideration on the decisions of landlord-investors.

LONG RUN

In the long run version of the model equation (8b) replaces (8a). Structure prices are assumed to be held constant in the long run by exogenous construction costs. It is the comparison of changes in market prices, P_H , to construction costs that leads to adjustments in the housing stock. Long run equilibrium requires that the stock of houses adjust so P_H again equals construction costs.

The assumption of exogenous construction costs implies a horizontal long run supply curve for units of housing capital. While consistent with earlier research, this characterization of long run supply is inconsistent with more recent work, in particular work that focuses on the capitalization of tax effects in land prices. Nevertheless the assumption is used here partly for its analytical simplicity and partly as the logically polar alternative to the short run assumption of an unchanged stock.

The substitution of equation (8b) for (8a) has important implications for the structure of the model and the solution strategy. Now the whole model can be solved by concentrating on just the rental market. With structure prices held constant by exogenous construction costs, the quantity of owner occupied housing is recursive to the rest of the model. Formally one can solve the model by the use of Figure 2c which involves equations (5), (6) and (7), describing the supply of rental housing, and

equations (1), (2) and (4) describing the demand for rental housing. Equilibrium in the rental market implies an equilibrium value for t_1 . Once t_1 is determined one can use equation (3) to solve for the stock of owner-occupied housing. Because P_H is constant, the cost of owning for individuals is also constant, although it still varies inversely with marginal tax rates.

A liberalization in depreciation allowances, an increase in preferential capital gains taxation or a decrease in rental operating costs will shift the rental supply curve in Figure 2c to the right, lowering rents and increasing the stock of rental housing.¹³ The reduction in rents induces owners who were previously at the margin between renting and owning to shift to renting. The tenure shift by households at the margin will not affect the aggregate demand for housing, but the decrease in rents will affect demand by intramarginal renting households, increasing the total housing stock in the long run.

An increase in the costs of homeownership, O^o , shifts the demand for rental housing to the right. In the long run rents rise and the stock of rental housing increases. The increase in the rental housing

¹³In both the short and long run versions of the model, the shift in the RS curve in Figure 2c determines the shift in the RR curve in Figure 3. In the short run version of the model one must use Figure 3 as the change in P_H shifts both RS and RD in Figure 2c. In the long run version of the model there is, by assumption, no change in P_H and Figure 2c tells the whole story.

The transition to the long run can be seen in Figure 6. The short run increase in P_H , point Y, when contrasted to unchanged construction costs, will lead to an increase in the housing stock. The increase in the housing stock shifts the DD curve down along the RR curve in Figure 6. The DD curve continues to shift as long as there are incentives to add to the housing stock, that is until P_H returns to its long run equilibrium level, point Z, given by construction costs.

stock implies a reduction in t_2 . Even though rents rise the incidence of homeownership falls, that is t_1 increases.¹⁴ Finally there is an unambiguous decline in the total housing stock since everyone, owners and renters, faces higher housing prices in the new long run equilibrium.

This long run rise in rents is in contrast to the short run decline in rents. The increased demand for rental housing following the increase in 0° requires new landlord-investors with lower marginal tax rates. In the short run these new landlord-investors were willing to supply rental housing at lower rents because of the short-run decline in structure prices. In the long run, when structure prices return to their long run equilibrium level, these new landlords will supply rental housing only if rents rise.

An increase in real interest rates increases rents in the long run. The stock of owner-occupied housing declines in the long run as the increase in i coupled with the long run constancy of P_H increases the cost of housing for all owner-occupants. The long run change in the stock of rental housing and the incidence of homeownership are ambiguous as the increase in real interest rates shifts the demand curve for rental housing to the right and the supply curve to the left. However the total stock of housing clearly declines as all consumers, owners and renters, face higher housing costs.

¹⁴The increase in t_1 can be derived as follows: we know from Figure 2c that both rents and the quantity of rental housing increase. Since the increase in rents reduces the quantity demanded by intramarginal renters, it must be that the number of renters has increased.

¹⁵The short run decline in structure prices, below construction costs, induces a reduction in the stock of houses that restores structure prices to construction costs in the long run. In a model with physical depreciation the reduction could occur by natural attrition. In a model without physical depreciation the reduction requires conversion of units of the housing stock to nonresidential uses.

An increase in inflationary expectations lowers rents in the long run.¹⁶

With P_H constant there is an unambiguous long run decline in the cost of owning at all marginal tax rates. With declines in both R and $OCC(t)$ there is an unambiguous increase in the stock of houses. The long run change in the rental housing stock is unclear as the leftward shift in the RD curve and the rightward shift in the RS curve have conflicting impacts on the stock of rental housing. It is most likely that the incidence of homeownership increases in the long run. As explained in the description to Figure 7, the $t_1 t_1$ curve can be used to determine the change in the incidence of homeownership. If the new long run equilibrium, following an increase in inflationary expectations, is to the northwest of the relevant $t_1 t_1$ curve, then the incidence of homeownership will increase in the long run. It is likely that the $t_1 t_1$ curve is steeper than the RR curve.¹⁷ A steeper $t_1 t_1$ curve, together with a shift in the RR curve less than the reference change in P_H imply an increase in the incidence of homeownership in the long run.

¹⁶ A long run analysis of a change in π^e implicitly assumes that these expectations are in fact validated.

¹⁷ The slope of the $t_1 t_1$ curve can be compared to the slope of the RR curve by considering the impact of a change in P_H on rental market equilibrium. Call the change in R necessary to keep t_1 constant the reference change in R . This quantity can be found by differentiating equation (2) and together with the change in P_H determines the slope of $t_1 t_1$. (See equation B.6.) An increase in P_H shifts both the RD and RS curves up. The RD curve shifts up less than the reference change in R defined above due to the impact of higher rents on intramarginal owners. (See equation B.7.) As long as $O^R > 0^\circ$, the RS curve also shifts up by less than the reference change. (This result can be derived from equation B.10.) Thus, following a change in P_H , rental market equilibrium involves a smaller change in R than necessary to hold t_1 constant, or the slope of the RR curve is less than that of the $t_1 t_1$ curve.

Conclusion

The model of housing markets, described above, integrates demand and supply decisions about both rental and owner-occupied housing into a single model of housing markets. The short and long run impacts of changes in a number of exogenous variables are considered with particular attention given to a change in inflationary expectations. In the short run, a change in inflationary expectations will unambiguously increase structure prices. Following the initial adjustment of structure prices, the change in the cost of owner-occupied housing will depend upon an individual's position in the income distribution as measured by marginal tax rates. The very richest owners are quite likely to see a decline in their cost of housing while homeowners closer to the buy/rent margin may see an increase. In the short run there is pressure for rents to increase and it is likely that the incidence of homeownership will increase. In the long run, the cost of owner-occupied housing must decrease for all homeowners following an increase in inflationary expectations as structure prices return to their long run equilibrium level given by construction costs. Rents will decrease as capital gains form a larger part of a landlord's return. The incidence of homeownership will increase.

The model results are generally consistent with the stylized facts. At the same time, the model emphasizes the importance of marginal tax rates and distinguishing between the long and short run for a more complete understanding of the impact of inflation on owner-occupied and rental housing. In the model, the increase in the relative price of structures is a short run phenomenon. In the long run, the increase in structure prices induces an increase in the stock of houses that

Appendix A

Writing down equations (1) to (8) leaves unclear whether $0 < t_1 < 1$, $0 < t_2 < 1$, $t_2 > t_1$ and $\partial \text{ATR} / \partial t > -(i + \pi^e)P_H$. Because of the complexity of the model, it is difficult to derive conditions, solely in terms of exogenous variables and parameters, that guarantee these results. The following discussion is meant to be suggestive.

From equation (6)

$$(A.6) \quad t_2 = \frac{(i + \pi^e)P_H - [(R - O^R) + (g^e - \psi^R)P_H]}{[i + \pi^e]P_H - [(R - O^R) - (D - \alpha g^e)P_H]}$$

The following conditions are sufficient to establish that $0 < t_2 < 1$ and $\partial \text{ATR} / \partial t > -(i + \pi^e)P_H$

$$a) \quad 0 < (R - O^R) + (g^e - \psi^R)P_H < (i + \pi^e)P_H$$

$$b) \quad 0 < D + (1 - \alpha)g^e > \psi^R$$

Condition a) insures that the numerator of (A.6) is positive. Condition b) insures that the denominator of (A.6) is also positive and greater than the numerator. Together a) and b) imply that $t_2 < 1$. Condition b) implies that the "good" things about investing in rental housing, depreciation allowances and capital gains, outweigh the "bad", the capital gain tax liability and the riskiness. Conditions a) and b) together also imply that $R - O^R - (D - \alpha g^e)P_H < (i + \pi^e)P_H$ which is the condition that $\partial \text{ATR} / \partial t > -(i + \pi^e)P_H$.

From equation (2)

$$t_1 = \frac{(i + \pi^e)P_H - [(R - O^o) + (g^e - \psi^o)P_H]}{[i + \pi^e]P_H}$$

The following condition is sufficient to establish that $0 < t_1 < 1$.

$$c) \quad 0 < (R - O^o) + (g^e - \psi^o)P_H < (i + \pi^e)P_H.$$

Conditions a), b), and c) together with d) are sufficient to establish that $t_2 > t_1$.

$$d) \quad [(i + \pi^e)P_H][(O^o - O^R) + (\psi^o - \psi^R)P_H] < [(R - O^R) + (\alpha g^e - D)P_H] \times \\ [(i + \pi^e)P_H - [(R - O^o) + (g^e - \psi^o)P_H]]$$

As written d) appears too complex for meaningful interpretation. However note that the second large term on the right hand side of d) is the numerator of t_1 , which by c) is positive. If $(O^o - O^R) + (\psi^o - \psi^R)P_H \leq 0$, and if $[(R - O^R) + (\alpha g^e - D)P_H] > 0$ then $t_2 > t_1$. The first of these last two conditions says that operating costs plus risk premiums for owner occupied housing is not greater than for rental housing. The second condition is that average taxable income from a unit of rental housing is positive, where the averaging refers to the present value annuity impact of depreciation allowances.

Appendix B

I. The DD Curve

From equations (3) and (4)

$$(B.1) \quad dH^0 = \sum_{t > t_1} h_2 \frac{\partial OCC(t)}{\partial P_H} dP_H + \frac{\partial H^0}{\partial t_1} dt_1$$

$$(B.2) \quad dH^R = \sum_{t < t_1} h_2 dR + \frac{\partial H^R}{\partial t_1} dt_1$$

From equation (8a)

$$dH^0 + dH^R = 0.$$

Further

$$\frac{\partial H^0}{\partial t_1} = - \frac{\partial H^R}{\partial t_1}.$$

Thus

$$(B.3) \quad \sum_{t > t_1} h_2 \frac{\partial OCC(t)}{\partial P_H} dP_H + \sum_{t < t_1} h_2 dR = 0,$$

and

$$(B.4) \quad \frac{dR}{dP_H} = - \frac{\sum_{t > t_1} h_2 \frac{\partial OCC(t)}{\partial P_H}}{\sum_{t < t_1} h_2} < 0.$$

Equation B.4 is the slope of the DD curve.

II. The RD Curve

From equation (2)

$$(B.6) \quad \frac{\partial \text{OCC}(t_1)}{\partial t_1} dt_1 + \frac{\partial \text{OCC}(t_1)}{\partial P_H} dP_H = dR.$$

Solving (B.6) for dt_1 and substituting into (B.2) yields

$$(B.7) \quad dH^R = \left[\sum_{t < t_1} h_2 + \frac{\partial H^R / \partial t_1}{\partial \text{OCC}(t_1) / \partial t_1} \right] dR - \frac{\partial H^R}{\partial t_1} \left(\frac{\partial \text{OCC}(t_1) / \partial P_H}{\partial \text{OCC}(t_1) / \partial t_1} \right) dP_H$$

A consideration of the sign of partial derivatives establishes that the RD curve has a negative slope and shifts to the right following an increase in P_H .

III. The RS Curve

From equation (6)

$$(B.8) \quad \left[\frac{\partial \text{ATR}(t_2)}{\partial t_2} + (i + \pi^e) P_H \right] dt_2 + \frac{\partial \text{ATR}(t_2)}{\partial R} dR + \left[\frac{\partial \text{ATR}(t_2)}{\partial P_H} - (1 - t_2)(i + \pi^e) + \psi^R \right] dP_H = 0.$$

From equation (7)

$$(B.9) \quad dH^R = S_1 dt_2.$$

Substituting for dt_2 from (B.9) into (B.8) yields

$$(B.10) \quad \left[\frac{\partial \text{ATR}(t_2)}{\partial t_2} + (i + \pi^e) P_H \right] \frac{1}{S_1} dH^R + \frac{\partial \text{ATR}(t_2)}{\partial R} dR +$$

$$\left[\frac{\partial \text{ATR}(t_2)}{\partial P_H} - (1 - t_2)(i + \pi^e) + \psi^R \right] dP_H = 0.$$

The total coefficient on dH^R is negative. As discussed in Appendix A, the sign of the term in brackets is positive and S_1 is negative. The sign of the coefficient of dP_H is negative as long as $(R - O^R)$ is greater than zero. Then, since $\partial \text{ATR}(t_2)/\partial R > 0$, the RS curve has a positive slope and shifts to the left following an increase in P_H .

Figure 1

2.a

2.b

2.c

The Rental Market
 $(P_H(1) < P_H(2))$

Figure 2

Overall Equilibrium

Figure 3

An Increase in Owner Operating Costs

Figure 4

Figure 5

Transition to Long Run Equilibrium
Figure 6

Figure 7

The DD curve in Figure 7 is as before. The $t_1 t_1$ curve represents combinations of R and P_H that leave t_1 unchanged and hence leave the incidence of homeownership unchanged. The curve has a positive slope as higher rents would mean more homeownership and a higher P_H is necessary to leave t_1 unchanged. Points to the northwest of $t_1 t_1$ imply a decrease in t_1 while points to the southeast imply an increase. Following the increase in π^e and g^e , the $t_1 t_1$ curve shifts and passes through point A, the reference change in P_H . As discussed in the text the horizontal shift in the DD curve is greater than the shift from the original equilibrium to point A. The short run equilibrium will be somewhere along the dashed line, exactly where will be determined by the shift in the RR curve. The incidence of homeownership will rise, t_1 will fall, as long as final equilibrium is to the left of point Z. As seen in Figure 7 a short run increase in homeownership is correlated with a short run increase in rents. A decline in rents in the short run implies a short run decrease in homeownership.

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