INVENTORY DYNAMICS IN A SIMPLE DISEQUILIBRIUM

MACROECONOMIC MODEL †

by

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Abstract

The paper formulates and analyzes a simple piecewise linear macroeconomic model with buffer stock inventories. Buffer stocks partly eliminate the rationing of the demand for goods, since rationing appears only when a stock-out is possible. This means that the region of Classical Unemployment necessarily shrinks, and it even disappears when buffer stocks absorb all the randomness of aggregate demand for goods. The inventory dynamics with constant wages and prices have steady states either in the region of Keynesian Unemployment or Repressed Inflation. In the former case the motion is oscillatory and either converges or ends in a limit cycle, while in the latter it is monotonic. Some of the complications caused by flexible wages and prices are sketched but the results are heavily dependent on the nature of wage and price adjustments.

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I Introduction

In the 1970's there has developed a literature, see especially Barro and Grossman (1976) and Malinvaud (1977), which has reconsidered some of the foundations of the theory of unemployment. This literature has become known as disequilibrium theory, because it is postulated that prices do not instantaneously adjust to clear the markets so that imbalances between supply and demand can persist. These imbalances should obviously show themselves in the behavior of inventories, but unfortunately most of the literature has assumed that goods are perishable so that the role of inventories has been neglected. (The exceptions are discussed below.)

Since inventories raise several distinct issues, it is necessary to review the three roles of inventories, discussed e.g. in Arrow (1958). The transaction motive to hold inventories stems from the differences in the timing of production and sales. If there are set-up or adjustment costs in production while sales fluctuate, then inventories are used to smooth out the production schedule. The transaction motive of inventories arises even if the demand for the commodity obeys deterministic or perfectly foreseen fluctuations. However, no transaction motive exists when there are no adjustment costs in the production level. Secondly, the speculative motive for holding inventories emerges (i) if the producers expect that the value of the product is rising and (ii) if there are decreasing returns to scale or increases in input prices are expected. Under this motive inventories are held as wealth to achieve capital gains. Thirdly, when the demand for the product is uncertain there is a precautionary motive to hold inventories. The optimal level of inventories

is determined by the trade-off between the inventory cost and the penalty or opportunity cost of stock-outs. Inventories are therefore held as a buffer stock.

In this paper our aim is to analyze the static and dynamic features of a disequilibrium macroeconomic model with inventories, emphasizing their role as buffer stocks. First, the introduction of inventories affects the classification of the short-run states of the economy into different regimes. Malinvaud (1977, appendix) and Muellbauer and Portes (1978) argue that the introduction of inventories to a disequilibrium macro model makes it possible to have the regime of Underconsumption, in addition to the well-known three regimes of Keynesian Unemployment, Classical Unemployment and Repressed Inflation, discussed by Barro and Grossman (1976) and Malinvaud (1977). However, they view inventories as wealth and overlook the role of buffer stocks. We shall argue that inventories as buffer stocks can alter spillover effects by eliminating some of the rationing of the demand for goods. Consequently, the classification criteria and the regimes are liable to change. It turns out that the region of Classical Unemployment will shrink and even disappear in certain cases. Secondly, we shall analyze the dynamic behavior of inventories and employment beyond the short run. In simple aggregate models e.g. Metzler (1941) and Lovell (1962) stability of inventory fluctuations necessitates that the adjustment speed of inventories be relatively small. These classic contributions do not take into account the problem of switching regimes and the non-negativities of employment and inventories. Incorporating them results in significant modifications. Assuming fixed wages and prices, steady states are in general either in the region of Keynesian Unemployment or Repressed Inflation and both possibilities are stable for many parameter configurations even if inventories adjust to their optimal level within the

period (excluding the complications caused by rationing in hiring labor). Moreover, the non-negativity of employment and the regime of Underconsumption exert stabilizing influences so that otherwise unstable oscillations result in limit cycles. Finally, we sketch some modifications to these conclusions, when wages and prices are permitted to vary.

The paper is organized as follows. Section II describes the basic framework. The types of short-run equilibria are analyzed in section III. Section IV is devoted to the analysis of the dynamics of inventories at constant prices, while complications caused by flexible prices are discussed in section V. Concluding remarks are offered in section VI.

II The Basic Model

We shall consider a two-market macroeconomic model with inventories as a buffer stock. The markets are assumed to operate sequentially so that labor transactions are first carried out, and after them production and trade of the goods take place. For the sake of brevity, in describing the model we shall not formulate the maximization problems of the consumer and the firm but simply postulate piecewise linear behavioral functions. They should be seen as plausible outcomes of expected profit maximization by the firm and of expected utility maximization by the consumer.¹⁾ The level of employment is determined as the minimum of the supply of and the demand for labor. The firms produce the consumption good by using labor as the only input. The initial inventories and current production make up the current supply of goods.²⁾ The trading of goods is in turn determined as the minimum of supply and demand, while their difference determines the initial inventory of the following period. The latter will be the main source of dynamics in the model.

The consumption good Y is produced out of employed labor L with constant returns to scale $Y = \delta L$. Naturally, it is assumed that the price of the good P is always kept high enough relative to the wage W in order to induce a positive level of production. Throughout this and the next two sections wages and prices are held constant and they are therefore suppressed into the parameters of the structural equations.

As the labor market operates first the notional aggregate demand is a linear function of employment

$$(2.1) \quad Y_t^d = a + bL_t + \varepsilon_t,$$

where the disturbance terms ε_t are independent, identically distributed random variables with the properties, $E\varepsilon_t = 0$, $-\alpha \leq \varepsilon_t \leq \alpha$. The latter states that they are bounded.

The optimal level of inventory is postulated, following eg. Lovell (1962), to be a linear function of expected sales Y_t^e

$$I_t^* = \alpha + \beta Y_t^e,$$

where $Y_t^e = EY_t^d \equiv \bar{Y}_t^d$ ie. the firms has rational expectations of sales. It is to be noted that the constant term of I_t^* is equal to the maximum disturbance in sales. This can be justified by rational expectations and high stock-out costs, relative to inventory holding costs, so that the precautionary demand for inventories can meet any random fluctuations. Finally, unlike Lovell (1962), we assume that the period is long enough so that planned inventories are always equal to the optimal level.³⁾

These considerations imply that the notional demand for labor is given by

$$(2.2) \quad L_t^d = \max \left\{ 0, \frac{1}{\delta} (I_t^* + \bar{Y}_t^d - I_{t-1}) \right\} \\ = \max \left\{ 0, \frac{1}{\delta} (\alpha + (\beta+1)\bar{Y}_t^d - I_{t-1}) \right\}$$

since the firms aim at having the quantity $I_t^* + \bar{Y}_t^d$ to meet actual sales. Again, since labor transactions take place first the effective supply of goods is given by

$$(2.3) \quad Y_t^S = I_{t-1} + \delta L_t,$$

where L_t is the actual employment of labor.

Regarding the supply of labor we shall, for the sake of simplicity, assume that it is fixed so that

$$(2.4) \quad L^S = d > 0.$$

The labor market transactions are determined by the usual minimum rule so that employment is given by ⁴⁾

$$(2.5) \quad L_t = \min (L_t^S, L_t^d).$$

If either the demand for or the supply of labor is expected to be rationed, the constrained side will change its behavior and either exaggerate the rationing or be discouraged by it. This is justified by the theory of decision-making under stochastic rationing, which is elaborated in Honkapohja and Ito (1979 a, b).

The constrained or effective demands and the states of the markets are as follows.

If $L_t^s > L_t^d$, then unemployment prevails and

$$(2.6) \quad L_t^s = d + e (L_t^s - L_t^d), \quad e < 1$$

$$L_t^d = (2.2) = L_t.$$

Here $e < 0$ means that labor supply is discouraged while $1 > e > 0$ implies that there exists an over-bidding effect. Similarly, if $L_t^s < L_t^d$, excess demand for labor prevails and

$$(2.7) \quad L_t^s = d = L_t.$$

$$L_t^d = (2.2) + \gamma (L_t^d - L_t^s), \quad \gamma > 1,$$

where the interpretation of γ is analogous to that of e .

Considering the product market, it is evident that the demand side is always satisfied as long as firms can fulfill their notional demand for labor. On the other hand, rationing of labor demand creates the possibility of a stock-out which in turn has repercussions on aggregate demand. Thus the relevant criterion for classifying the states of the product market is the possibility or impossibility of a stock-out. Therefore, we have the following two cases:

- 1) If $Y_t^s > \bar{Y}_t^d + \alpha$ a stock-out is impossible and the effective demand and supply are given by (2.1) and (2.3) respectively.
- 2) If $Y_t^s < \bar{Y}_t^d + \alpha$ stock-out is possible, and aggregate supply is given by (2.3), but aggregate demand is modified to

$$(2.8) \quad Y_t^d = (2.1) + c (Y_t^s - \bar{Y}_t^d - \alpha), \quad c > -1.$$

Here $c < 0$ means that consumers increase their demand because of the possibility of a stock-out, while $c > 0$ implies that they are discouraged.

III The Types of Short-Run Equilibria

The description of the model above revealed two special features which have to be taken into account when the classification of the short-run outcomes is attempted.⁵⁾ First, with buffer-stock inventories the relevant criterion for the goods market is the possibility or impossibility of a stock-out. Secondly, if firms are not rationed in their demand for labor, a stock-out is impossible. This means that the region of Classical Unemployment disappears. This follows from the two hypotheses that the optimal inventory is sufficient to cover the random variation in aggregate demand and that firms adjust to the optimum within the period.⁶⁾ Thus, apart from the boundaries (which are largely ignored below) the short-run equilibrium falls into one of the three remaining possibilities, Keynesian Unemployment, Underconsumption and Repressed Inflation, and they are discussed separately below.⁷⁾

A. Keynesian Unemployment (K) prevails if $L^d < L^s$ and the stock-out in the goods market is impossible. The relevant system of equations is given by (2.1), (2.3) and (2.6), from which the level of employment is determined as

$$(3.1) \quad L_t = \begin{cases} \frac{\alpha + (\beta+1) a - I_{t-1}}{k - b(\beta-1)} & , \text{ if } \alpha + (\beta+1) a > I_{t-1} \\ 0 & , \text{ otherwise.} \end{cases}$$

It is necessary to assume that $k > b(1+\beta)$ i.e. the productivity of labor is

larger than the sum of the marginal propensities to consume and hold inventories. The condition $L_t^d < L_t^s$ gives us the restriction

$$(3.2) \quad \bar{I} \equiv \alpha + (\beta+1) a - d \{k-b (\beta+1)\} < I_{t-1}$$

which defines the boundary of the Keynesian region in terms of initial inventories, since at $I_{t-1} = \bar{I}$ the labor market would exactly clear. From (3.1) it is evident that employment in the region of Keynesian Unemployment is negatively related to the level of initial inventories, unless it is zero. For later reference denote by $\hat{I} = \alpha + (\beta+1) a$ this critical level of initial inventories.

B. Underconsumption (U) is defined by the conditions that a stock-out is impossible and that $L_t^d > L_t^s$. In this case the system consists of (2.1), (2.3) and (2.7), from which the level of employment is

$$(3.3) \quad L_t = d.$$

The boundaries of the region are given by the requirements

$$\bar{I} > I_{t-1},$$

where $\bar{I}$ is defined in (3.2), and

$$(3.4) \quad \bar{\bar{I}} \equiv a + \alpha + (b-k)d < I_{t-1},$$

the latter being obtained from the requirement $\bar{Y}_t^d + \alpha < Y_t^s$, with $L_t = d$.

Indeed, $\bar{\bar{I}}$ is the boundary between Underconsumption and Repressed Inflation.

Note that $\hat{I} > \bar{I} > \bar{\bar{I}}$. It may also be observed that in the region U the level of employment is independent of the level of initial inventories.

C. Repressed Inflation (R) prevails if, first, $L_t^S < L_t^d$ and, second, stock-outs are possible. The relevant model now consists of (2.3), (2.7) and (2.8). Note that in principle labor supply might be altered by the probability of a stock-out, but we ignore this complication. It would be straight-forward to include the general case, but the conclusions for the dynamics become less clear. By solving the system we obtain

$$(3.5) \quad \bar{Y}_t^d = \{a + bd + ckd - c\alpha + c I_{t-1}\} / (1+c)$$

with the domain of validity $0 \leq I_{t-1} < \bar{\bar{I}}$, where $\bar{\bar{I}}$ is defined in (3.4) above. The level of employment is determined by supply

$$(3.6) \quad L_t = d,$$

which is again independent of the level of initial inventories.

IV Inventory Dynamics with Constant Prices

Let us now start to consider how the short-run equilibria shift over time. Even if wages, prices and wealth are held constant this movement will occur, because the level of inventories is changing from period to period. Indeed, we have the general equation

$$(4.1) \quad I_t = Y_t^S - Y_t^d = I_{t-1} + \delta L_t - Y_t^d,$$

from which we can obtain the difference equation for inventories in the different regions. They take the following forms.

First, in the Keynesian region $I_{t-1} > \bar{I}$ we get

$$(4.2) \quad I_t = \begin{cases} \frac{-b\beta}{k-b(\beta+1)} I_{t-1} + \frac{\alpha(k-b) + k a \beta}{k-b(\beta+1)} - \epsilon_t, & I_{t-1} < \hat{I} \\ I_{t-1} - a - \epsilon_t, & I_{t-1} > \hat{I} \end{cases}$$

This is a piecewise linear stochastic difference equation of first order.

The mean path of I_t is obtained from the same equation with the error term ϵ_t deleted. The system is stable, when $b(1+2\beta) < k$, since by assumption $k - b(\beta+1) > 0$. The mean stationary state in this region is given by

$$(4.3) \quad I^k = \alpha + \frac{k a \beta}{k - b}.$$

The condition for its existence will be given below.

In the region of Underconsumption, defined by the inequalities $\bar{I} < I_{t-1} < \bar{I}$, the equation (4.1) takes the form

$$(4.4) \quad I_t = I_{t-1} + d(k-b) - a - \epsilon_t$$

from which it is evident that no steady states exist in the regime of Underconsumption. Depending on the sign of $d(k-b) - a$ the mean level of inventories is either rising or falling.

Finally, in the region of Repressed Inflation, $I_{t-1} < \bar{I}$ the difference equation (4.1) has the form

$$(4.5) \quad I_t = \max \{0, (\frac{1}{1+c}) I_{t-1} + (\frac{1}{1+c}) [\bar{c}\alpha + d(k-b) - a] - \varepsilon_t\},$$

which is stable for a positive steady state if $c > 0$ ie. consumer demand for goods is discouraged by the possibility of stock-out. The mean steady state can be calculated to be

$$(4.6) \quad I^R = \alpha + \frac{1}{c} \{d(k-b) - a\}.$$

The analysis of the piecewise linear equation defined by (4.2), (4.4) and (4.6) can be conveniently summarized by constructing the phase diagram for the resulting mean path. Recalling that $\bar{\bar{I}} < \bar{I} < \hat{I}$ we first locate these points in the (I_{t-1}, I_t) - space and then draw the graphs of the equations appropriately. This leads us to distinguish between two possibilities.

A. $d(k-b) > a$:

In this case the graph of the equation in the U region, i.e. $\bar{\bar{I}} < I_{t-1} < \bar{I}$ is above the 45° line and parallel to it. On the other hand, for $I_{t-1} > \hat{I}$ the graph is again parallel to the 45° line but definitely below it. Since the graph can easily be seen to be continuous, there exists a Keynesian steady state, as is evident from the diagram.

Diagram 1: A Keynesian stationary state.

Moreover, if in the R region $c > 0$ it is clear that no other steady states exist. In any case steady states in the R region will be unstable. The movement of I_t is monotonically increasing in the regions R and U and decreasing in the subset of K where no production takes place. On the other hand, in the remaining part of K the movement is necessarily oscillatory. These findings give the following theorem.

Theorem 1. If $d(k-b) > a$, there exists a Keynesian steady state. The motion around it is always oscillatory, and the stability condition is $b(1+2\beta) < k$. If the oscillations are unstable, the adjacent regions ($\bar{I} < I_{t-1} < \bar{I}$, $I_{t-1} > \bar{I}$) create a limit cycle. Any steady state in region R is definitely unstable.

B. $d(k-b) < a$:

In this event the graph of the difference equation in U lies below the 45° degree line and is parallel to it. Therefore, the R region will contain all steady states. In them the level of inventories is either positive or zero. The former type will be stable and entail monotonic convergence if $c > 0$, while the latter are trivially stable. These results are summarized in the following theorem and diagram.

Diagram 2: A stationary state in the region of Repressed Inflation

Theorem 2. If $d(k-b) < a$, no steady states exist in the Keynesian region. A steady state in the region of Repressed Inflation may have a positive or zero mean level of inventories. A necessary, but not sufficient, condition for the existence of the former type is $c > 0$, and then it will be stable with monotonic convergence. The latter type is trivially stable.

These two theorems point out the importance of the sign of $d(k-b)-a$. It is in general uncertain and evidently depends on the level of prices, wages and wealth which have been kept constant in the analysis. In the next section we shall analyze some of the consequences of variable wages and prices.

We may also note that the evolution of employment over time can be derived from the behavior of inventories. The evolution varies a great deal from regime to regime and in our simple model employment is constant in the regions of R and U but is liable to fluctuate, when a Keynesian steady state exists.

V The Complications from Price Flexibility⁸⁾

Let us now briefly discuss some of the modifications to the previous analysis which result from wage and price flexibility. They seem to raise two new issues. First, as was already noted earlier, the parameters in the behavioral functions

are dependent on the level of wages, prices and wealth. Secondly, the behavior of optimal inventories may undergo a major change, since in inflationary and deflationary circumstances the speculative motive for holding inventories appears.⁹⁾ The former issue will be dealt with below, but the latter would seem to require a more drastic change in the modelling of optimal inventories, and limitations of space do not permit an analysis of speculative inventory behavior.

Looking back to the model we make the assumptions $a'(w) > 0$, $b'(w) > 0$, $b'(w) \leq 0$, where $w = W/P$, the real wage. All of these are quite natural: the exogenous and employment-dependent components of aggregate demand are increasing with the real wage, while transaction related part of inventories is non-increasing in w , because the holding costs increase with w . Given these assumptions it is easy to plot in the (w, I) - space the classification of the regimes and the locus $\Delta I = 0$. It is straightforward to calculate the derivatives

$$(4.1) \quad d\bar{I}/dw = (1+\beta)(a'+db')+(a+bd)\beta' \gtrless 0$$

$$(4.2) \quad d\bar{\bar{I}}/dw = a' + db' > 0$$

$$(4.3) \quad dI^K/dw = \frac{1}{(k-b)^2} \{ (k-b)(ak\beta' + \beta ka') + ak\beta b' \} \gtrless 0$$

$$(4.4) \quad dI^R/dw = \frac{d}{c} (-b' -a') < 0.$$

Let us also assume that $\exists \hat{w}$ such that $\beta(\hat{w}) = 0$. (We also have $\beta'(\hat{w}) = 0$, since $\beta \geq 0$.) From this we see that the slope of the locus $\bar{I}$ is in general uncertain but it becomes positive as $w \rightarrow \hat{w}$. The same is true of the locus I^K .

Notice also that for $w > \hat{w}$ there is no possibility of the U region i.e. inventories are purely buffer stocks, and the curves $\bar{I}$ and $\bar{\bar{I}}$ coincide.

These considerations result in diagram 3.

Diagram 3. The regimes and steady states at different level of the real wage.

On this diagram we can also draw the locus of the steady states $\Delta I = 0$, of which I^K and I^R are parts. Its location is generally unknown, as it also depends on wealth. In order to describe the different possibilities define $\tilde{w}$ by the equation $d\{k-b(\tilde{w})\} - a(\tilde{w}) = 0$. Note that the left-hand side was critical for the existence of the different types of steady states. After this we draw the locus $\Delta I = 0$ to the diagram assuming $\tilde{w} < \hat{w}$, $\tilde{w}_2 = \hat{w}$, $\tilde{w}_3 > \hat{w}$. From diagrams 1 and 2 we know that above the locus $\Delta I = 0$ inventories are decreasing and below it increasing.¹⁰⁾

To complete the dynamics one needs to describe the evolution of real wages over time. In most of the earlier work on disequilibrium dynamics the behavior of real wages is either specified separately for each region in a fairly ad hoc way, or it has been derived from the law of supply and demand type equations for the nominal magnitudes.¹¹⁾ Acknowledging the need for better stories about the behavior of wages and prices in disequilibrium, let us sketch the consequences of the above-mentioned alternatives.

A conceivable specification, analogous to the ones used by Ito (1978, 1979) and Böhm (1978), is

$$(4.5) \quad \begin{aligned} \Delta w &< 0 \text{ in } K \\ \Delta w &> 0 \text{ in } U \text{ and } R. \end{aligned}$$

In this case the curve $\bar{I}$ becomes the locus of stationary real wages, and we have diagram 4 in which suggests behavior of the saddle-point type.

Diagram 4

Another specification, similar to one of those suggested by Böhm (1978), is

$$(4.6) \quad \begin{aligned} \Delta w &> 0 \text{ in } K \text{ and } U \\ \Delta w &< 0 \text{ in } R. \end{aligned}$$

This implies that the curve $\bar{I}$ becomes the stationary locus for the real wage and we have diagram 5.

Diagram 5

With this specification the system tends to exhibit oscillatory behavior, but stability would require a further analysis which we omit here.

The third alternative, based on a law of supply and demand type of specification for nominal wages and prices, leads to a locus $\Delta w = 0$ which passes through the point A of diagram 3 and goes through regions R and K, since nominal wages and prices can be expected to rise in R and fall in K.

Unfortunately, it is impossible to determine in general the slope of this locus as conflicting terms appear in the calculations. Consequently, the evolution may have any features. However, it may be noted that different types of steady states can occur, depending on the location of the locus $\Delta I = 0$. Using the earlier classification in terms of $\tilde{w}$ we have that the

steady state is (a) in region R if $\tilde{w} < \hat{w}$, (b) in region K if $\tilde{w} > \hat{w}$ and (c) at the point A of diagram 3, if $\tilde{w} = \hat{w}$. The last case is illustrated in diagram 6 under the assumption that the curve $\Delta w = 0$ is upward-sloping. The system tends to oscillate, while for stability a further analysis is needed.

Diagram 6

VI Concluding Remarks

The paper has focussed on the role of buffer stocks in models of market disequilibria. In such models some of the rationing of the demand for goods vanishes and this reduces the likelihood of Classical Unemployment.

The dynamics of inventories tends to have oscillatory behavior around a steady state in the Keynesian region even when the mechanic higher-order lags of classic models of the inventory cycle are not present. Moreover, the switching-regimes phenomenon of disequilibrium models will dampen explosive oscillations to limit cycles.

The correlation between inventories and employment varies from one regime to another, since employment is either supply- or demand-determined. In the former case the correlation is very indirect, but in the latter a direct negative correlation between employment and beginning-of-period inventories should appear.

The applicability of these results is limited by the assumption of constant wages and prices. The modifications caused by wage and price flexibilities accord well with intuitions if one follows the earlier literature

on disequilibrium dynamics and if persistent inflation or deflation is not present so that speculative inventory holding plays little role. Unfortunately, the postulated wage and price adjustment rules do not have strong justifications, so that the results must be considered very tentative. Better theories of price and wage setting with inventories are acutely needed.

Footnotes

1) This expected utility and profit maximization has been spelled out in recent studies utilizing the idea of trading uncertainty or stochastic rationing to represent the situation confronting individual agents in market disequilibria. See Svensson (1976), Gale (1979), Green (1979), Honkapohja and Ito (1979 a,b) for these models. In the linearized version the differences to the usual approach of Barro and Grossman (1976), Benassy (1976) and Malinvaud (1977) are not very big. Some of them will be noted below.

2) We are therefore largely abstracting for speculative holdings of inventories. This assumption is clearly appropriate as long as wages and prices are expected to remain unchanged.

3) Allowing for a stock adjustment process of the type $I_t^p = q \{I_t^* - I_{t-1}\} + I_{t-1}$, $0 < q < 1$, that Lovell (1962) does, would result in higher-order lags and create considerable complications, since the difference equations will be only piecewise linear. See Feldstein and Auerbach (1976) for empirical evidence of $q = 1$.

4) We do not make notational distinctions between notional and effective demands. It is to be remembered that unemployment and excess demand for labor are defined in terms of effective demands and supplies. Similarly in the classification criteria for the goods market effective demand and supply are the relevant ones.

5) See Barro and Grossman (1976), Malinvaud (1977) and especially Muellbauer and Portes (1978) for this classification in the standard model, where the possibilities are Keynesian Unemployment ($L^d < L^s$, $Y^d < Y^s$) Repressed Inflation ($L^d > L^s$, $Y^d > Y^s$) Classical Unemployment ($Y^d > Y^s$, $L^d < L^s$) and Underconsumption ($Y^d < Y^s$, $L^d > L^s$).

6) If these assumptions are violated stock-outs can occur even when firms are not rationed in the labor market, but in any case it is clear that buffer stocks of goods reduce the importance of Classical Unemployment.

7) Notice that if $\beta = 0$, then the U region would disappear, and impossibility of stock-outs would be in one-to-one correspondence with no rationing of labor demand. However, in this case inventory dynamics become a pure random process.

8) Quite recently, Green and Laffont (1979) have started to analyze disequilibrium dynamics and inventories by making the interesting though special assumption that wages and prices are set to their expected market-clearing levels before any random shocks are known. However, their model is a linearized version of the Muellbauer and Portes (1978) framework, and inventories are not treated as buffer stocks.

9) Speculative inventories are likely to be a function of the rates of change of wages and prices, not of their level. On the other hand, transaction inventories should be a function of the levels of wages and prices, since holding costs depend on them.

10) To be precise mathematically, this statement is not quite true, as we are dealing with difference equations and I can jump across the line $\Delta I = 0$. Indeed, the phase diagram technique used in this section is incomplete and cannot characterize all aspects of the dynamics, but since there is no fully satisfactory phase diagram technique for a plane system of difference equations we use the diagrams to illustrate some general features.

11) See Böhm (1978), Honkapohja (1979, a,b), Ito (1978, 1979), for examples of these. A different line of attack is followed in the above-mentioned work by Green and Laffont (1979).

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Diagram 1: A Keynesian stationary state

Diagram 2: A stationary state in the region of Repressed Inflation

Diagram 3: The regimes and the steady states at different levels of the real wage and wealth.

Diagram 4

Diagram 5

Diagram 6