Brane Tilings: NSVZ Beta Function

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NSVZ Beta Function

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$$\beta_{\frac{1}{g^2}} = \frac{1}{8\pi^2} \frac{3N - \sum_M \mu[\mathcal{R}_M](1 - \gamma_M(g))}{1 - g^2 N / 8\pi^2}$$

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$$D = 1 + \frac{\gamma}{2} = \frac{3}{2}r$$

Look at the numerator

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$$\mu(fund) = \frac{1}{2}$$

$$\mu(adj) = N$$

Matter in Adj & Bi-fund

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$$\beta_a = N_a - \sum_{A \in adj[a]} N_a(1 - r_{A,a}) - \frac{1}{2} \sum_{B \in bifund[a,b]} N_b(1 - r_{B,ab})$$

Vanishing Beta Function

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$$\sum_{B \in \text{bifund}[a,b]} N_b (1 - r_{B,ab}) = 2N_a$$

All ranks equal

All ranks equal

$$\sum_{B \in bifund} (1 - r_B) = 2$$

A superpotential term

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$$\sum_{B \in \text{monomial}} r_B = 2$$

Conditions for conformal invariance

$$\sum_{B \in bifund} (1 - r_B) = 2$$

$$\sum_{B \in monomial} r_B = 2$$

Look for a graphical representation

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Look for a graphical representation

- 3 objects in $N=1$ supersymmetry in $3+1$ d:
- gauge fields - vector multiplets
- matter fields - chiral multiplets
- interactions - superpotential

A Typical Quiver: Star of David

Quiver

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- no information on W

Improve by introducing Brane Tilings

Introduce arrows in alternating fashion

Brane Tilings Dictionary

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- Face (tile) - $U(N)$ Gauge group; $U(N)$ V-plet

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- Face (tile) - $U(N)$ Gauge group; $U(N)$ V-plet
- Edge - A bi-fundamental chiral multiplet
- Node - Interaction term in W
- $+(-)$ sign for a white (black) node
- $2+|d|$: Each Face - integer CS level

An Infinite class of SCFT's in $3+1d$ & $2+1d$

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Conditions for conformal invariance

$$\sum_{B \in \text{around face}} (1 - r_B)\pi = 2\pi$$

$$\sum_{B \in \text{monomial}} r_B = 2$$

Conditions for conformal invariance

$$\sum_{B \in \text{around face}} (1 - r_B)\pi = 2\pi$$

$$\sum_{B \in \text{around node}} r_B\pi = 2\pi$$

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- Locally flat tiles (NSVZ)
- Locally flat nodes (W has R charge 2)
- Periodic, bi-partite, 2d tilings

3 Hexagon tiling

$$CY_6 = \text{conifold}$$

brane tiling

quiver

$$W = X_{12}^{(1)} X_{21}^{(1)} X_{12}^{(2)} X_{21}^{(2)} - X_{12}^{(1)} X_{21}^{(2)} X_{12}^{(2)} X_{21}^{(1)}$$

plane tiling

Ex: Chessboard Tiling

Isoradial Embedding; dPl

Seiberg Duality

Brane Tilings

Moduli space of Vacua

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Moduli space of Vacua

- In 3+1d the moduli space is a non-compact singular toric CY3 cone
- In 2+1d with a choice of CS levels it is a non-compact singular toric CY4 cone

The 2+1d Lagrangian

$$\begin{aligned} & - \int d^4\theta \sum_{X_{ab}} X_{ab}^\dagger e^{-V_a} X_{ab} e^{V_b} \\ & + i \int d^4\theta \sum_{a=1}^G k_a \int_0^1 dt V_a \bar{D}^\alpha (e^{tV_a} D_\alpha e^{-tV_a}) \\ & + \int d^2\theta W(X_{ab}) + \text{c.c.} \end{aligned}$$

Vacuum Equations

$$\partial_{X_{ab}} W = 0$$

$$\mu_a(X) := \sum_{b=1}^G X_{ab} X_{ab}^\dagger - \sum_{c=1}^G X_{ca}^\dagger X_{ca} + [X_{aa}, X_{aa}^\dagger] = 4k_a \sigma_a$$

$$\sigma_a X_{ab} - X_{ab} \sigma_b = 0$$

Fibration of a CY3 over a complex line

Example: Chessboard Tiling; CS levels $(1, -1)$

Example: Chessboard Tiling CS levels (1,-1)

Example: Chessboard Tiling

CS levels (1,-1)

$$W = \text{Tr}(X_{12}^1 X_{21}^1 X_{12}^2 X_{21}^2 - X_{12}^1 X_{21}^2 X_{12}^2 X_{21}^1)$$

4 fields in the quiver

$$W_{(1)} = \text{Tr}(X_{12}^1 X_{21}^1 X_{12}^2 X_{21}^2 - X_{12}^1 X_{21}^2 X_{12}^2 X_{21}^1); \quad W_{(2)} = \text{Tr}(X_{12} [\phi_2^1, \phi_2^2] X_{21})$$

Figure 1: The quivers with 4 fields and 2 nodes. There are 2 solutions and the 2-term superpotentials are also given. The moduli space in both cases is just the trivial CY 4-fold $\mathbb{C}^4$.

1 hexagon; 1 double edge, $G=2$

Figure 3: (i) Quiver diagram for phase 2 of the $\mathbb{C}^4$ theory. (ii) Tiling for phase 2 of the $\mathbb{C}^4$ theory.

Toric Duality

2 hexagon tiling; (1-,1)

Conifold $(\mathcal{C}) \times \mathbb{C} \parallel$

$$W = \phi_1(X_{12}^1 X_{21}^2 - X_{12}^2 X_{21}^1) + \phi_2(X_{21}^1 X_{12}^2 - X_{21}^2 X_{12}^1)$$

Ex: 2 hexagon tiling

Conifold $\times \mathbb{C} \amalg$

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- $1/2$ for ϕ 's, $3/4$ for X 's
- Non-trivial SCFT in the IR

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- In 3+1d this is $C^2/Z_2 \times C$
- Master space - 2+1d mesonic moduli space
- Non trivial scaling dimensions
- 1/2 for ϕ 's, 3/4 for X 's
- Non-trivial SCFT in the IR
- a test of AdS/CFT

Toric Diagram $\mathcal{C} \times \mathbb{C}$

5 fields in the Quiver

Master space - $\mathbb{C}^5$

(1)

(3)

(4)

$$W_{(1)} = \text{Tr}(X_{21}X_{13}^1X_{31}X_{13}^2X_{32} - X_{21}X_{13}^2X_{31}X_{13}^1X_{32}) ;$$

$$W_{(3)} = \text{Tr}(X_{21}\phi_1X_{13}X_{31}X_{12} - X_{21}X_{13}X_{31}\phi_1X_{12}) ;$$

$$W_{(4)} = \text{Tr}(X_{21}[\phi_1^1, \phi_1^2]X_{13}X_{32}).$$

Chessboard tiling; I double edge; $(1, -1, 0)$

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- Combined mesonic baryonic space - $\mathbb{C}^5$
- Scaling dimensions $1/2$ for X_{12} , $3/8$ other

Conifold $\times$ $\mathbb{C}$

Phase III $(0,1,-1); (-2,1,1)$

Figure 13: [Phase III of $\mathcal{C} \times \mathbb{C}$] (i) Quiver diagram of the $\mathcal{D}_2\mathcal{H}_1$ model. (ii) Tiling of the $\mathcal{D}_2\mathcal{H}_1$ model.

Global symmetry conifold $\times \mathbb{C}$

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- $SU(2) \times SU(2) \times U(1)_q \times U(1)_R \times U(1)_B$

Conifold $\times$ $\mathbb{C}$

Table of charges

	$SU(2)_1$	$SU(2)_2$	$U(1)_q$	$U(1)_B$	$U(1)_R$	fugacity
p_1	1	0	1	1	$3/8$	$t^3 q b x_1$
p_2	-1	0	1	1	$3/8$	$t^3 q b / x_1$
p_3	0	1	1	-1	$3/8$	$t^3 q x_2 / b$
p_4	0	-1	1	-1	$3/8$	$t^3 q / (b x_2)$
p_5	0	0	-4	0	$1/2$	t^4 / q^4

Table 2: Charges under the global symmetry of the $\mathcal{C} \times \mathbb{C}$ theory. Here t is the fugacity associated with the $U(1)_R$ charges. The power of t counts R-charges in the unit of $1/8$, q is the fugacity associated with the $U(1)_q$ charges, and x_1, x_2 are respectively the $SU(2)_1, SU(2)_2$ weights.

Perfect matchings	Generator of Phase I	Generator of Phase II	Generator of Phase III
$p_1 p_3$	$X_{13} X_{32}^1$	X_{12}^1	$X_{21} X_{12}$
$p_2 p_3$	$X_{13} X_{32}^2$	X_{21}^1	$X_{21} X_{13}$
$p_1 p_4$	$X_{23} X_{32}^1$	X_{12}^2	$X_{31} X_{12}$
$p_2 p_4$	$X_{23} X_{32}^2$	X_{21}^2	$X_{21} X_{13}$
p_5	X_{21}	$\phi_1 = \phi_2$	ϕ_1

Toric Duality

conifold $\times \mathbb{C}$

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- Three phases

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- mesonic generators: linear | bi-linear | “

Hilbert Series

$\text{conifold} \times \mathbb{C}$

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conifold $\times \mathbb{C}$

1

$$(1 - t_1 x_1 b) \left(1 - \frac{t_1 x_2}{b}\right) \left(1 - \frac{t_1 b}{x_1}\right) \left(1 - \frac{t_1}{x_2 b}\right) (1 - t_2)$$

Hilbert Series conifold $\times \mathbb{C}$

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$$t_1 = t^3 q \quad t_2 = t^4 / q^4$$

Lattice of generators conifold $\times \mathbb{C}$

Figure 10: The lattice of generators of the $\mathcal{C} \times \mathbb{C}$ theory.

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- mesonic baryonic moduli space
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- Toric Duality

Happy Birthday Misha!